

4 F-Tests

Reading:

Kennedy (1998) “A Guide to Econometrics”, Chapters 4 and 6

Maddala, G.S. (1992) “Introduction to Econometrics” p. 170-177

Moore and McCabe, chapter 12 (a good explanation of the more general use of F-tests)

Field 2001 section 4.1.3

Field 2005 (2nd ed.) chapter 8 for background reading on ANOVA

Field 2009 (3rd ed.) chapter 10 for background reading on ANOVA

Aim:

The aim of this section is to consider the use of F-tests in regression analysis.

Objectives:

By the end of this chapter, students should be aware of the basic formula for F-tests run in the context of regression analysis; be able to test for the existence of a relationship; test for structural breaks and test for a set of linear restrictions.

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4.1 Introduction

It just so happens that the ratio of two residual sums of squares has an F distribution. A number of tests have been developed that make use of this principle. Since they all are based on the same basic formula (one that compares an “*unrestricted model*” with a “*restricted model*”), it makes sense to gather the most common F-tests into one place. So the purpose of this chapter is to introduce F-tests and their application to testing for the existence of a relationship, the occurrence of structural breaks, and the effect of linear restrictions. It is particularly recommended that students familiarise themselves with the procedures for testing for structural breaks since these are most useful as a diagnostic tool.

4.2 Testing a set of linear Restrictions - The General Procedure

Suppose we want to test whether there are any country specific effects in the relationship between inflation and the money supply. We can do this by testing the following null hypothesis:

$$H_0: g_1 = g_2 = g_3 = \dots = g_{10} = 0$$

where g_1, g_2, \dots, g_{10} are the slope coefficients on country dummies in the following regression:

$$INFL = a + b MS + g_1 COUNTRY1 + \dots + g_{10} COUNTRY10 \quad [1]$$

We can conceive of this as being equivalent to comparing two regressions, one *Restricted* and one *Unrestricted*. The *Unrestricted* regression is equation [1] above with the country dummies included. The *Restricted* regression, on the other hand, is:

$$INFL = a + b MS \quad [2]$$

To compare these two regressions (equation [1] and equation [2]) we use an F test based on comparing the residual sum of squares from each regression. The **general formula** for the F test on regressions is given by:

$$F_{\frac{df_{\text{numerator}}}{df_{\text{denominator}}}} = F_{df_U}^r = \frac{(RSS_R - RSS_U) / r}{RSS_U / df_U}$$

Where:

$$\begin{aligned} RSS_U &= \text{unrestricted residual sum of squares} \\ &= \text{RSS under } H_1 \end{aligned}$$

$$\begin{aligned} RSS_R &= \text{restricted residual sum of squares} \\ &= \text{RSS under } H_0 \end{aligned}$$

$$\begin{aligned} r &= \text{number of restrictions} \\ &= \text{difference in the number of parameters between restricted and unrestricted equations} \end{aligned}$$

$$\begin{aligned} df_u &= df \text{ from unrestricted regression} = n - k \text{ where } k \text{ is all coefficients} \\ &\text{including the intercept.} \end{aligned}$$

If the null hypothesis is true (i.e. restrictions are satisfied) then we would expect the restricted and unrestricted regressions to give similar results. In other words, RSS_R and RSS_U will be similar, so we accept H_0 when the test statistic gives a small value for F . In contrast, if restrictions are not valid, then the restricted regression will be misspecified. This results in more residual variation, and a higher restricted residual sum of squares (**NB** RSS_R will **always** be greater than RSS_U since imposing a restriction on an equation can never reduce the residual sum of squares). Consequently, we reject H_0 when the test statistic gives a large value.

4.2.1 Exercise:

1. Suppose the results from the unrestricted regression described above (of inflation on money supply and ten country dummies) are as follows: $RSS_U = 8000$, $n = 212$, k (the number of coefficients including the intercept) = 12. And suppose the results

from the restricted regression (inflation regressed on money supply only) are: $RSS_R = 10,000$, $n = 212$, $k = 2$. State the null and alternative hypotheses and calculate the value of F . Use the F -table provided in the **Ftests.xls** template (open the spreadsheet and click on the “ F -table extract” worksheet) to complete the hypothesis test at the 5% significance level.

2. Suppose you repeat the above exercise using a different dataset and estimate $RSS_U = 2,000$ and $RSS_R = 2,100$ (sample sizes and explanatory variables are the same as before). Conduct a hypothesis test of no country effects at the 1% significance level.
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4.3 Test Procedure:

There are many different hypothesis tests that can be conducted as an F -test. In the example above, we looked at one of the simplest forms – when we test the hypothesis that a group of coefficients equal zero. But we can also run more complicated tests such as,

$$H_0: b_1 + b_2 = 4. \quad (\text{i.e. the sum of coefficients } b_1 \text{ and } b_2 \text{ equals } 4).$$

or that the coefficients from regressions run separately on two subsamples of our data (e.g. small countries vs large countries) will be the same as coefficients from a single regression run on the entire sample (this is called a “structural break test” – see below).

Given the variety of possible restrictions we can test, it can be confusing how to set up the F -test in particular situations. So, it might help if you follow the following recipe for running F -tests.

There are four basic steps to executing an F -test:

(i) Compute RSS_U

Run the unrestricted form of the regression in SPSS and take a note of the residual sum of squares = RSS_U

(ii) Compute RSS_R

Run the restricted form of the regression in SPSS and take a note of the residual sum of squares = RSS_R . If the restrictions are not homogenous (i.e. if they imply that the restricted and unrestricted regressions have ***different dependent variables***) then you need to:

- ❑ substitute the restrictions into the equation;
- ❑ rearrange the equation so that each parameter appears only once;
- ❑ create new variables where necessary and estimate by OLS;
- ❑ take a note of the residual sum of squares from this regression = RSS_R ;

(iii) Calculate r and df_U

r is the number of restrictions and is equal to the difference in the number of parameters between the restricted regression and the unrestricted regression. df_u is the degrees of freedom from unrestricted regression = $n - k$ where k is all coefficients including the intercept. The df_u is listed in the ANOVA table (for the unrestricted regression) as the degrees of freedom of the residual. For example, in the following ANOVA table, the degrees of freedom is 511. Note also that $n = 516 = \text{degrees of freedom of the Total Sum of Squares} + 1$ and that k (as we have defined it, i.e. the total number of coefficients including the intercept) = $5 = \text{degrees of freedom of the Regression Sum of Squares} + 1$.

ANOVA^b

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	296.772	4	74.193	20.652	.000 ^a
	Residual	1835.811	511	3.593		
	Total	2132.583	515			

a. Predictors: (Constant), CNTRY_3, CNTRY_2, CNTRY_1, money supply

b. Dependent Variable: inflation

(iv) Substitute RSS_U , RSS_R , r and df_U

Once you have computed RSS_U , RSS_R , r and df_U , enter them in the equation for F and find the associated significance level.

NB you can find the sig value of any F value that you have calculated by using the following syntax in Excel:

=FDIST(F value that you've calculated, $df_{\text{numerator}}$, $df_{\text{denominator}}$)

Alternatively, you can use the Excel Template which will calculate the test F value and sig level for you from SPSS ANOVA tables (see below).

4.4 Linear restrictions where algebraic manipulation is involved (advanced)

F-tests can not only be used to test whether a group of variables in the model are jointly significant. They can also be used to test more sophisticated restrictions on the values of particular coefficients. You might, for example, wish to test whether two coefficients are equal, or whether a number of coefficients sum to one, or whether two different regressions have similar coefficients. To do these tests you need to do some algebraic manipulation of your original regression equations, as the following examples demonstrate:

Example 1

In a regression of x_2 and x_3 on y , test the hypothesis that the coefficients sum to one.

The *Unrestricted* regression is given by:

$$y = b_1 + b_2x_2 + b_3x_3 + u$$

The null hypothesis is $H_0: b_2 + b_3 = 1$;

Now we need to substitute the null hypothesis into the equation to derive the restricted regression. If H_0 is true, then: $b_3 = 1 - b_2$ and:

$$y = b_1 + b_2x_2 + (1-b_2)x_3 + u$$

$$\begin{aligned}
 &= b_1 + b_2x_2 + x_3 - b_2x_3 + u \\
 &= b_1 + b_2(x_2 - x_3) + x_3 + u \\
 y - x_3 &= b_1 + b_2(x_2 - x_3) + u
 \end{aligned}$$

Restricted regression can now be written as:

$$z = b_1 + b_2(v) + u \quad \text{where } z = y - x_3; \text{ and } v = x_2 - x_3$$

Example 2

For the same regression as the one considered in Example 1, test the hypothesis that $b_2 = b_3$

First note that this is a homogenous restriction because, as we shall see in a moment, the restricted and unrestricted regressions have the same dependent variable.

Unrestricted regression:

$$y = b_1 + b_2x_2 + b_3x_3 + u$$

$H_0: b_2 = b_3;$

If H_0 is true, then:

$$\begin{aligned}
 y &= b_1 + b_2x_2 + b_2x_3 + u \\
 &= b_1 + b_2(x_2 + x_3) + u
 \end{aligned}$$

Restricted regression is simply:

$$y = b_1 + b_2(w) + u \quad \text{where } w = x_2 + x_3;$$

Example 3

In a regression of : MS_GDP and MP_GDP on Inflation, test the hypothesis that the coefficients on MS_GDP equals that of MP_GDP plus one.

In this case, we will find that Restricted and Unrestricted regressions have the different dependent variables and so the restrictions are not homogenous:

Unrestricted regression:

$$Infl = b_1 + b_2MS_GDP + b_3MP_GDP + u$$

$H_0: b_3 = b_2 + 1$

If H_0 is true, then:

$$\begin{aligned}
 Infl &= b_1 + b_2MS_GDP + (b_2+1)MP_GDP + u \\
 &= b_1 + b_2MS_GDP + b_2MP_GDP + 1 \times MP_GDP + u \\
 &= b_1 + b_2(MS_GDP + MP_GDP) + MP_GDP + u
 \end{aligned}$$

$$Infl - MP_GDP = b_1 + b_2(MS_GDP + MP_GDP) + u$$

Restricted regression:

$$z = b_1 + b_2(v) + u$$

where,

$$\begin{aligned}
 z &= Infl - MP_GDP ; \\
 v &= MS_GDP + MP_GDP
 \end{aligned}$$

4.5 Using the Excel Template

You should familiarize yourself with calculating F -tests manually, but it is worth checking your results using the **Ftests.xls** template. Open up the **Ftests.xls** spreadsheet, and go to the first worksheet (tab label: **General Test for Linear Restrcs**). The general F -test procedure using this spreadsheet is as follows:

- (i) Run the *Unrestricted* regression and copy the ANOVA table from this regression. Place the cursor in cell E17 of the spreadsheet (the blue shaded cell) and paste as text (in Excel, select Edit, Paste Special, Text). This will displace any existing entry with your new ANOVA *Unrestricted* regression results.
- (ii) Now run the *Restricted* regression and copy the ANOVA table from this regression. Place the cursor in cell E6 of the spreadsheet (the green shaded cell) and paste as text (in Excel, select Edit, Paste Special, Text). This will again displace any existing entry with your new ANOVA *Restricted* regression results.
- (iii) Excel should automatically calculate the r , df_U , F and significance level (see cells N25 to N35) but you should check the answer manually the first few times to ensure that you are using the spreadsheet correctly and that you understand the procedure. Remember that the null hypothesis is that there is no difference between the restricted and unrestricted regressions. Therefore, if F is large and the significance level small, you can reject the null with only a small probability of a Type I Error (i.e. that you have rejected H_0 incorrectly).

4.5.1 Exercise:

1. Open the **sovdebt.sav** data. Create dummies for the first **ten** countries.
2. Test whether there are country effects for Argentina, Bolivia and Brazil in a regression of money supply on inflation:

$$INFL = a + b MS + g_1 COUNTRY1 + g_2 COUNTRY2 + g_3 COUNTRY3$$

3. (Optional) In a regression explaining imports per capita (mp_pc), test the hypothesis that the coefficients on GDP per capita (gdp_pc) and exports per capita (xp_pc) sum to one. That is, for:

$$mp_pc = b_1 + b_2 gdp_pc + b_3 xp_pc$$

test the null hypothesis that $b_2 + b_3 = 1$

Hint: first you need to create the new variables:

```
compute xp_pc = xp/pop.
VARIABLE LABELS xp_pc 'exports per capita'.
execute.

compute gdp_pc = gdp/pop.
VARIABLE LABELS gdp_pc 'GDP per capita'.
execute.

compute mp_pc = mp/pop.
VARIABLE LABELS mp_pc 'Imports per capita'.
execute.
```

Then run the unrestricted regression ($mp_pc = b_1 + b_2 \text{gdp_pc} + b_3 \text{xp_pc}$).

Then you need to derive the restricted regression. Before you know what regression to run, you need to work through the algebraic implications of the null hypothesis:

- substitute the restrictions into the equation (i.e. substitute $b_2 + b_3 = 1$ into the original unrestricted equation: $mp_pc = b_1 + b_2 \text{gdp_pc} + b_3 \text{xp_pc}$).
- rearrange the equation so that each parameter appears only once
- create new variables where necessary and estimate by OLS

Once you've done this, and you know what the restricted regression looks like, run the restricted regression.

Then paste the ANOVA tables from the restricted and unrestricted regressions into the Excel template. If the significance level is small, you can reject the null that $b_2 + b_3 = 1$. You should check your results by calculating the F -ratio manually.

4. (Optional) Test whether, in the previous question, the two slope coefficients are equal.

4.6 Testing a set of linear Restrictions when the Restrictions are Homogenous

When the linear restrictions we want to test are homogenous:

e.g. $H_0: b_2 = b_3 = 0$

e.g. $H_0: b_2 = b_3$

we will find that we will not need to transform the dependent variable of the restricted equation in order to specify the restricted equation.

For restrictions of this type, where the dependent variable is the same in the restricted and unrestricted regressions, we can re-write our F -ratio test statistic in terms of R^2 s:

$$F_{df_{denominator}}^{df_{numerator}} = F_{df_U}^r = \frac{(R_U^2 - R_R^2) / r}{(1 - R_U^2) / df_U}$$

where:

$R_U^2 = R^2$ from the unrestricted regression,

$R_R^2 = R^2$ from the restricted regression

r = number of restrictions = difference in the number of parameters between restricted and unrestricted equations.

df_u = degrees of freedom from unrestricted regression = $n - k$ where k is the total number of coefficients in the unrestricted regression including the intercept.

5. (Optional) Look again at the restrictions suggested in the previous questions and consider whether they are homogenous or not. If they are homogenous, try running the test again, but this time use the simplified F -ratio test statistic.

4.7 Testing for the existence of a relationship

If we are testing a set of linear restrictions where the restrictions are that all slope coefficients are zero ($b_j = 0$ for all j), in other words we are testing for the existence of a relationship, then we can simplify the F -ratio test statistic further. Suppose the *Unrestricted regression* is as follows:

$$y = b_1 + b_2x_2 + b_3x_3 + u$$

$$H_0: b_2 = b_3 = 0;$$

If H_0 is true, then $y = b_1$. In this case, the *Restricted regression* does no explaining at all and so $R_R^2 = 0$. As a result, the homogenous restriction F -ratio test statistic reduces to:

$$F_{\frac{df_{\text{numerator}}}{df_{\text{denominator}}}} = F_{df_U}^r = \frac{(R_U^2 - 0) / r}{(1 - R_U^2) / df_U}$$

$$= \frac{(R_U^2) / r}{(1 - R_U^2) / df_U}$$

where,

df_u = degrees of freedom from unrestricted regression = $n - k$ where k is the total number of coefficients in the unrestricted regression including the intercept.

R_U^2 = R^2 from the unrestricted regression,

r = number of restrictions = $k - 1$.

6. This is the F-test we came across in Session 2, and is the one automatically calculated in the SPSS ANOVA table. Try calculating the test manually and compare it with the ANOVA table output.

4.8 Testing for Structural Breaks

The F -test also comes into play when we want to test whether the estimated coefficients change significantly if we split the sample in two at a given point. These tests are sometimes called “Chow Tests” after one of its early proponents. There are actually two versions of the test: *Chow’s First Test* and *Chow’s Second Test*.

Chow’s First Test

Use where $n_2 > k$

- (1) Run the regression on the first set of data ($i = 1, 2, 3, \dots, n_1$) and let its RSS be labeled RSS_{n_1}
- (2) Run the regression on the second set of data ($i = n_{1+1}, n_{1+2}, \dots, \text{end of data}$) and let its RSS be labeled RSS_{n_2}
- (3) Run the regression on the two sets of data combined ($i = 1, \dots, \text{end of data}$) and let its RSS be $RSS_{n_1 + n_2}$
- (4) Compute RSS_U, RSS_R, r and df_U where:

$$RSS_U = RSS_{n1} + RSS_{n2}$$

$$RSS_R = RSS_{n1 + n2}$$

$$r = k = \text{total number of coefficients including the constant}$$

$$df_U = n_1 + n_2 - 2k$$

- (5) Use RSS_U , RSS_R , r and df_U to calculate F using the general formula for F and find the sig. Level:

Chow's Second Test

Use where $n_2 < k$ (I.e. when you have insufficient observations on the second sub-sample to run Chow's 1st test)

- (1) Run the regression on the first set of data ($i = 1, 2, 3, \dots, n_1$) & let its RSS be RSS_{n1}

- (2) Run the regression on the two sets of data combined ($i = 1, \dots, \text{end of data}$) & let its RSS be $RSS_{n1 + n2}$

- (3) Compute RSS_U , RSS_R , r and df_U :

$$RSS_U = RSS_{n1}$$

$$RSS_R = RSS_{n1 + n2}$$

$$r = n_2$$

$$df_U = n_1 - k$$

- (4) Use RSS_U , RSS_R , r and df_U to calculate F using the general formula for F and find the significance.

7. In the imports per capita regression you estimated above, use Chow's First Test to test the hypothesis that slope coefficients changed (i.e. there was a structural break) before and after 1986.

8. Test the same hypothesis using Chow's Second Test. Which test is more reliable do you think in this instance?

9. In the same regression, test the hypothesis that the slope coefficients are different for smaller countries ($\text{pop} < 15$).