Sectoral Risk Weights and Macroprudential Policy

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Abstract

This paper analyses bank capital requirements in a general equilibrium model by evaluating the implications of different designs of such requirements in terms of their impact on the tendency of banks to amplify the business cycle. Three capital frameworks are tested: a risk sensitive Internal Ratings-Based (IRB) approach, a risk insensitive leverage ratio approach, and an alternative macroprudential approach which sets risk-weights in response to sectoral measures of leverage. The different methods are compared in a crisis scenario, where the crisis originates in the housing sector, moves on to the banking sector and then infects the wider real economy. We investigate both boom and bust phases of the crisis by simulating an unrealized news shock that leads to a gradual build up and rapid crash in the economy. Our results suggest that the IRB approach creates procyclicality in regulatory capital requirements and thereby works to amplify both boom and bust phases of the financial cycle. On the other hand, our proposed macroprudential approach to setting risk-weights leads to counter-cyclicality in regulatory capital requirements and thereby attenuates the financial cycle.

Keywords: macroprudential policy, risk weighted assets, sectoral and countercyclical capital requirements, basel-established internal ratings-based (IRB) approach, DSGE models.

JEL Classifications: C68, E44, E58, E61

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1 Non Technical Summary

Banks have a tendency to amplify relatively small shocks into major macroeconomic fluctuations. In particular, negative asset price shocks in an individual sector can generate a broad based credit crunch across the economy. This procyclical tendency has recently become more widely understood by policy-makers and bank supervisors especially following the global financial crisis, which to a large extent began in the housing market and then spread across the economy. As a result, some steps have been taken in order to attenuate the procyclical behaviour of banks for example via the addition of macroprudential tools to the regulatory framework, such as the counter-cyclical capital buffer.

Our paper seeks to add to the debate on how best to reduce the procyclicality that arises from the banking sector, specifically by exploring the potential merit of integrating a macroprudential perspective directly into microprudential rules. We assess different potential designs for calculating bank capital requirements in order to understand how they impact on the banking systems tendency to amplify the business cycle. Within this, we propose a macroprudential approach to set risk-based capital requirements which results in capital requirements that vary inversely to the progression of the credit cycle for each asset class. Risk based capital requirements for a certain asset class increase as credit volumes into that asset class increase, and decrease as credit volumes recede from that asset class.

The proposed macroprudential approach to setting capital requirements has the effect of integrating the macroprudential and microprudential elements of banks capital requirements into a single system. It thereby hard-wires a counter-cyclical element within the rules, reflecting the idea that all financial regulation should seek to stabilize credit conditions and reduce the volatility of credit flows. Such an approach might offer particular advantages in an environment where one has doubts that discretionary macroprudential tools will be used sufficiently intensively due to potential inaction biases arising from the surrounding political economy. It would also offer potential benefits over the existing countercyclical capital buffer system, including via more precise targeting of capital requirements on the most macroprudentially risky sectors.

In the analysis, the macroprudential approach is tested against two alternatives the IRB approach, and the leverage ratio approach. The IRB approach is based upon banks own estimates of the probability of default (PD) and loss given default (LGD) of their assets. The estimates of PDs and LGDs are based on historical observations of real data either from the banks own portfolio, or potentially drawing from representative external data if justified. The leverage ratio approach is not risk-based and sets capital requirements as a simple ratio of capital / assets.

Our results show that in both boom and bust phases of the crisis the IRB approach leads to procyclical capital requirements. In the boom phase, capital requirements become looser thereby reinforcing the boom by making loans cheaper and more available. In recessions, higher PDs lead to higher risk-weights and tighter capital regulation thereby depressing bank lending and economic activity. The IRB approach thereby exacerbates a recession by making loans more expensive just at the point where firms need to raise finance. On the other hand, we find that our macroprudential approach to risk-weighting leads to countercyclicality of capital regulation in both boom and bust phases of the crisis - thereby attenuating the business cycle. The negative impact of a bad financial shock on the real economy turns out to be smaller and the eventual recovery happens faster in comparison to an IRB scenario.
These findings tend to support the view that there is room for improvement in the current design of risk-based capital requirements, in particular regarding the IRB approach. As the design of the financial framework evolves, an emphasis on less procyclical mechanisms would be potentially beneficial for the sake of mitigating the banking sectors tendency to exacerbate the real economy impact of financial shocks. The non-risk-based leverage ratio setup could be an improvement in this regard. A macroprudential approach, which encourages banks to continue lending in a recession and discourages banks from lending excessively in the boom phase may be better still.

2 Introduction

Bernanke et al. (1999) established the financial accelerator framework that theoretically characterizes how the financial sector can amplify relatively small shocks into major macroeconomic fluctuations. In particular, negative asset price shocks can generate substantially tighter credit conditions through the impact these shocks have in reducing collateral availability, thereby reducing borrowing and further depressing asset prices. Banking regulation needs to be designed with this problem in mind, and should where possible work to mitigate the inherently procyclical lending behaviour of banks.

This paper assesses different potential designs for calculating bank capital requirements in order to understand how they impact on the banking system’s tendency to amplify the business cycle. We test the IRB approach, the leverage ratio approach, and a hypothetical macroprudential approach. The macroprudential application of sectoral capital requirements is based on a Taylor-type rule, which varies capital requirements with the credit cycle in each asset class. Capital requirements for a certain asset class increase as credit volumes into that asset class increase, and decrease as credit volumes recede from that asset class. The analysis takes place in a DSGE model with a banking sector, financial frictions and risky loans.

We start from the hypothesis that the ideal financial regulation should seek to stabilize credit conditions and reduce the volatility of credit flows.\(^1\) Stable credit conditions help to avoid inefficient sectoral asset price booms and busts, and generally reduce the role of the financial system in amplifying the business cycle.

The IRB approach is not, at first glance, well designed for the purpose of stabilizing credit flows. It is based upon banks’ own estimates of the probability of default (PD) and loss given default (LGD) of their assets. The estimates of PDs and LGDs are based on historical observations of real data either from the banks’ own portfolio, or potentially drawing from representative external data if justified. At the outset of a recession, it can be expected that the preceding period’s PD and LGD observations will be increasing. This backward-looking approach to risk-estimation can therefore result in capital requirements increasing just as a recession is starting.\(^2\) Such an effect is not optimal from the perspective of stabilizing capital flows as it could lead a bank towards further deleveraging at the start of a recession. Likewise, on the other side of the business cycle there may be a tendency for capital requirements to decrease just as a boom phase begins. PDs and LGDs are expected to fall as the economy improves, and this will filter through into reduced capital requirements for banks. Concerns about the procyclicality of the IRB

\(^1\)This hypothesis is supported, for example, by Brunnermeier et al. (2009) - where Chapter 4 describes how financial regulation should pursue counter-cyclical approaches.

\(^2\)On average short horizons, i.e. the past five years, are considered to compute the PD and LGD.
framework are well established, and have been well explored in the academic literature, e.g. within Gordy and Howells (2006), Repullo and Suarez (2013), and Kashyap and Stein (2004).

In the following analysis we deploy a New-Keynesian general equilibrium model with financial frictions of the type Bernanke et al. (1999), hereafter referred to as BGG. The main departure from the BGG set-up is that interest rates are predetermined as in Quint and Rabanal (2014). However, unlike Quint and Rabanal (2014), our analysis models both the banking sector and bank capital dynamics. Concerning the basic structure of the banking sector, our paper is closely related to Gerali et al. (2010) and Angelini et al. (2014). However, we extend the banking sector by adding defaulting loans and asset risk-weights in the bank capital requirements.

All aforementioned papers analyse the merits of macroprudential policy. However, our paper is novel as it introduces bank asset risk-weights in capital requirements in order to analyse their impact on the business cycle. Gerali et al. (2010) introduce capital requirements but regard the assets as equally weighted with a weight of one: thereby corresponding to a leverage ratio. Angelini et al. (2014) study the interaction between capital requirements and monetary policy. In contrast to our set-up, where risk weights on assets are determined endogenously, Angelini et al. (2014) introduce asset risk-weights for the capital requirements according to an ad-hoc rule which relates risk-weights to current and past output procyclically.

Concerning the research question, our paper is closest to the empirical papers Behn et al. (2014) and Goodhart et al. (2004). Behn et al. (2014) analyses the impact of model-based capital regulation on the credit risk of financial institutions. Goodhart et al. (2004) studies among other things the impact of the Basel II reform on the procyclicality of regulatory system. Both find empirical evidence for procyclical capital charges resulting from model-based (IRB) regulation. Similarly, Borio et al. (2001) find that bank provisions and capital ratios contribute to financial system procyclicality by increasing the cyclicality of bank profitability.

Our results show that in both boom and bust phases of the crisis the IRB approach leads to procyclical capital requirements. In the boom phase, capital requirements become looser thereby reinforcing the boom by making loans cheaper and more available. In recessions, higher PDs lead to higher risk-weights and tighter capital regulation thereby depressing bank lending and economic activity. The IRB approach thereby exacerbates a recession by making loans more expensive just at the point where firms need to raise finance. On the other hand, we find that our macroprudential approach to risk-weighting leads to countercyclicality of capital regulation in both boom and bust phases of the crisis - thereby attenuating the business cycle. The negative impact of a bad financial shock on the real economy turns out to be smaller and the eventual recovery happens faster in comparison to an IRB scenario.

These findings tend to support the view that there is room for improvement in the current design of risk-based capital requirements, in particular regarding the IRB approach. As the design of the financial framework evolves, an emphasis on less procyclical mechanisms would be potentially beneficial for the sake of mitigating the banking sectors tendency to exacerbate the real economy impact of financial shocks. The non-risk-based leverage ratio setup could be an improvement in this regard. A macroprudential approach, which encourages banks to continue lending in a recession and discourages banks from lending excessively in the boom phase may be better still.

Our macroprudential approach to setting risk weights is an alternative means of ad-
justing capital requirements for the state of the financial cycle in comparison to the Countercyclical Capital Buffer (CCB) approach that is set out within Basel III. The macroprudential risk weights we apply could offer two sets of benefits in comparison to the CBB approach. First, our Taylor type rule eliminates any discretion from the policymaker within the setting of macroprudential capital requirements. This automaticity ensures a counter-cyclical approach is pursued. By contrast, the maintenance of discretion in the application of the CCB risks that distortions arising from the surrounding political economy lead to inaction bias in the use of the CCB, and thereby to the maintenance of procyclical requirements.

Second, the macroprudential risk weights we apply adjust to the sectoral financial cycle thereby providing a more precise tool for reacting to sector-specific bubbles and credit booms. Historically, financial crises tend to arise as a result of credit booms and bubbles that are focused on particular sectors. To name just a few examples the recent Great Recession originated in the housing sector, around the turn of the millennium there was the Dotcom bubble in the technology sector, and in the 1840s there was a spectacular boom in the US railroad sector. The sectoral risk weights that we put forward in this analysis could helpfully focus macroprudential controls over capital requirements on those sectors that are experiencing booms, whilst avoiding unwarranted impacts on other non-bubble sectors. This would potentially concentrate the impact of the tool where it needs to be to stabilise capital flows, and reduce unintended spillover costs to other non-bubbly sectors.

This paper is structured as follows. Section 2 outlines the model design, section 3 explains our approach to calibrating or estimating the model parameters, section 4 sets out our results and provides accompanying policy analysis, and the final section concludes.

3 Model

This paper provides a dynamic stochastic general equilibrium model with a banking sector, closely related to Gerali et al. (2010) and Angelini et al. (2014). The model is used as a laboratory for the comparison of the IRB approach versus a macroprudential asset-risk weight setting rule in stabilizing the economy during recessions. The model is populated by entrepreneurs, heterogeneous households, and monopolistically competitive banks and firms. This section describes the agents in the model as well as the direct impact of the macroprudential policy rule.

3.1 Banks

The banking sector consists of a wholesale branch and two retail branches. The wholesale branch manages the capital-asset position of the bank as it accumulates bank capital out of retained earnings and pays a quadratic cost whenever it deviates from a risk weighted capital-asset requirement. As bank capital can only be accumulated through retained earnings, the supply of credit is constrained as imposed by the Basel regulation. The two retail branches obtain funds from the wholesale branch and lend them to households and firms respectively. The two types of loans are non-recourse with pre-determined interest rates - this allows for unexpected changes in the default rates or collateral prices. These unexpected changes lead to profits/losses that affect the capital-asset position of the banking sector.
3.1.1 Wholesale branch

The wholesale branch collects deposits $D$ at the gross policy rate $R$ which together with the accumulated bank capital $K^b$ is used to fund its loans $B$, leading to a balance sheet identity

$$B_t = D_t + K^b_t$$

(3.1)

where the two sources of funding, $K^b$ and $B$, are perfect substitutes. Bank capital is accumulated through retained earnings

$$K^b_t = (1 - \delta^b)K^b_{t-1} + \Pi_t$$

where $\delta^b$ is the depreciation rate of bank capital, and should be interpreted as the costs of managing bank capital. $\Pi_t$ denotes the realized overall profits of all bank branches, including the profits of the wholesale $\Pi^{ws}_t$ and the two retail branches profits $\Pi^I_t$ and $\Pi^F_t$

$$\Pi_t = \Pi^{ws}_t + \Pi^I_t + \Pi^F_t$$

The overall loans $B_t$ in the economy consist of the loans $B^I_t$ and $B^E_t$ that the two retail branches lend to households and firms, respectively. The retail branches obtain the funds to lend from the wholesale branch at the gross interest rates $R^{b,I}_t$ and $R^{b,E}_t$ respectively. The wholesale branch maximizes profits taking into account a quadratic cost $QC_t$ whenever the risk-weighted capital-asset ratio $K^b_t/RWA_t$ deviates from an exogenous level $\nu^b_t$ which represents the regulatory capital requirement.

$$QC_t = \frac{\kappa^b}{2} \left( \frac{K^b_t}{RWA_t} - \nu^b_t \right)^2 K^b_t$$

where $RWA_t$ denotes the risk-weighted assets and is given by the weighted sum of each asset type. The asset specific weights $w^I_t$ and $w^E_t$ represent a regulatory instrument that allows for adjusting the risk-weight of a specific asset class.

$$RWA_t = w^I_t B^I_t + w^E_t B^E_t$$

(3.2)

Thus the wholesale branch maximization problem is given by

$$\max_{\{D_t, B^I_t, B^E_t\}} \sum_{i=0}^{\infty} \Lambda_0, t \left[ \left( R^{b,I}_t - 1 \right) B^I_t + \left( R^{b,E}_t - 1 \right) B^E_t - (R_t - 1) D_t - QC_t \right]$$

s.t. $B_t = D_t + K^b_t$

The wholesale branch maximizes its profits subject to the balance sheet identity (3.1) by taking $R^{b,I}_t$, $R^{b,E}_t$ and $R_t$ as given. Using the FOCs, we can write

$$R^{b,j}_t - R_t = \kappa^b \left( \nu^b_t - \frac{K^b_t}{RWA_t} \right) \left( \frac{K^b_t}{RWA_t} \right)^2 w^j_t \quad \text{for } j \in \{I, E\}$$

(3.3)

Equation (3.3) links the interest rate spread $R^{b,j}_t - R_t$ for each loan type $j \in \{I, E\}$ to the degree of deviation of the capital-asset ratio from its requirement $\nu^b_t$, as well as to the loan specific risk weight $w^j_t$. The LHS of equation (3.3) represents the marginal benefit from increasing lending of type $j$ (an increase in profits equal to the interest rate spread), while the RHS represents the marginal cost from doing so (an increase in the costs for deviating from $\nu^b_t$). Therefore, the wholesale branch chooses a level of each type of lending $j$ which, at the margin, equalizes costs and benefits of changing the capital risk-weighted asset ratio.\footnote{The optimal leverage ratio in this context can be thought of as capturing the trade-offs that would arise in the decision of how much own resources to hold, or alternatively as a simple shortcut for studying the implications and costs of regulatory capital requirements.” (Gerali et al., 2010)}
3.1.2 The retail branches

The retail branches face endogenous loan defaults due to an idiosyncratic shock to the collateral value of borrowers and the non-recourse contract with predetermined interest rates. Unlike the wholesale branch, each retail branch has the necessary and specialized expertise for it’s type of lending - to evaluate expected collateral prices and default rates. One retail branch is specialized in mortgage loans - the branch denoted by $j = I$ provides loans to households against housing collateral. The second retail branch $j = E$ lends to firms (entrepreneurs) against capital collateral. Apart from their different specialization both retail branches are identical. For future reference, ex-ante expected and ex-post realized variables are denoted with $a$ and $p$ superscripts respectively. There are neither agency problems nor asymmetric information, hence bank retail branches are perfectly competitive and make zero profits in expected terms. The retail branch $j$ obtains funds from the wholesale bank branch and takes the interest rate $R_{b,j}$ that the wholesale branch charges as given. According to the probability of default the retail branch sets the interest rate of it’s loans $r^j$.

The default condition for borrower $j$ is given by

$$\frac{r^j_{t-1}B^j_{t-1}}{\pi_t} \leq \omega^j_{t-1}q^j_{t-1}h^j_{t-1} \quad (3.4)$$

If the inequality (3.4) is satisfied the borrower does not default, i.e. when the amount to repay is smaller than the value of the collateral after the realization of the idiosyncratic shock $\omega_t$. The idiosyncratic shock is log-normally distributed with CDF $F(\omega)$, PDF $f(\omega)$ and mean $E(\omega_t) = 1$. At period $t$, high enough realizations of $\omega_{t-1}$ will induce the borrower to repay his loan in full: $r^j_{t-1}B^j_{t-1}/\pi_t$, where $r^j$ is the gross borrowing rate and $B^j$ the quantity borrowed from retail branch $j$. Low enough realizations will cause the borrower to default and give up his collateral after the realization of the shock: $\omega^j_{t-1}q^j_{t-1}h^j_{t-1}$, where $q^j$ denotes the collateral price and $h^j$ denotes the amount of collateral.

In period $t$, the cut-off value of $\overline{\omega}^j_{t-1}$, i.e. the ex-post realized threshold value $\overline{\omega}^j_t$, that separates borrowers that default and those that do not can be expressed as

$$\overline{\omega}^j_{t-1} \equiv \overline{\omega}^j_{t,a} = \frac{r^j_{t-1}B^j_{t-1}}{q^j_t h^j_{t-1}} \pi_t$$

At period $t$, the retail branch extends loans at a rate $r^j_t$ without knowing the exact value of the default threshold, since it will also depend on the period $t + 1$ collateral price $q^j_{t+1}$ and next period inflation. The retail branch forms ex-ante expectations on $\overline{\omega}^j_{t,a}$

$$\overline{\omega}^j_{t,a} = \frac{r^j_t B^j_t}{E(\pi_{t+1}q^j_{t+1})h^j_t} \quad (3.5)$$

Note that $\overline{\omega}^j_{t,a}$ equals the retail branch’s expected LTV ratio of loan type $j$.

Unlike the wholesale branch, retail branches do not maximize profits but simply

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4This implies that the log of $\omega$ is normally distributed: $\log(\omega_t) \sim N(-\sigma^2_\omega, \sigma^2_\omega)$. There is idiosyncratic but no aggregate risk.

5Although the retail branches do not maximize profits, since we consider each bank as composed of one wholesale and two retail branches we can say that each bank operates under monopolistic competition with profit maximization occurring at the wholesale level.
require that the expected return from one unit of a loan equals the cost of funds (the rate at which the funds are obtained from the wholesale branch rate $R_{b,j}^t$). This leads to the following participation constraint

$$R_{b,j}^t = (1 - \mu)G_{j,a}^t \frac{E_t(\pi_{t+1}^{j_a})h_t^j}{B_t^j} + (1 - F_{j,a}^t)r_{j}^t$$

(3.6)

where the RHS of (3.6) consists of the expected return in the case of default (i.e. the repossessed collateral) and the expected return in the case of non-default (i.e. the repayment of the loan). $G_{j,a}^t \equiv G(\bar{\omega}_{j,a}^t, \sigma_{\omega,t}^j) = \int_{0}^{\bar{\omega}_{j,a}^t} \omega dF(\omega, \sigma_{\omega}^j)$ denotes the expected value of the idiosyncratic shock, conditional on the shock being less than $\bar{\omega}_{j,a}^t$; and $1 - F_{j,a}^t \equiv 1 - F(\bar{\omega}_{j,a}^t, \sigma_{\omega,t}^j) = \int_{\bar{\omega}_{j,a}^t}^{\infty} f(\omega, \sigma_{\omega}^j)d\omega$ being the probability that the shock exceeds the ex-ante threshold $\bar{\omega}_{j,a}^t$, i.e. the probability of non-default. Banks can repossess only a fraction $1 - \mu$ of the collateral as the remainder is assumed to be lost as a cost of default.

Rearranging the participation constraint (3.6) yields

$$\frac{r_{j}^t}{R_{b,j}^t} = \frac{1}{(1 - \mu)G_{j,a}^t \frac{\bar{\omega}_{j,a}^t}{\omega} + (1 - F_{j,a}^t)}$$

(3.7)

where the retail spread of each type of loan $j \in \{I, E\}$ is expressed as a function of the expected default threshold $\bar{\omega}_{j,a}^t$. Due to the properties of the log-normal distribution with $E_t(\omega) = 1$, it can be shown that the denominator of the RHS of (3.7) is a decreasing function in ex-ante threshold $\bar{\omega}_{j,a}^t$, and hence, the interest rate spread becomes an increasing function of $\bar{\omega}_{j,a}^t$.

$$\frac{r_{j}^t}{R_{b,j}^t} = f(\bar{\omega}_{j,a}^t); \quad f'(\bar{\omega}_{j,a}^t) > 0$$

The intuition behind this relationship is the following: For a larger expected LTV ratio (RHS of equation (3.5)), a larger proportion of loans is expected to default, and hence the ex-ante threshold $\bar{\omega}_{j,a}^t$ increases. Since the threshold separates the defaulting from non-defaulting loans, the bank would expect a larger default area and a smaller non-default area given by $(1 - F_{j,a}^t)$. In order to compensate for the larger expected defaults, the retail branches increase the loan rate $r_{j}^t$.

### 3.1.3 Bank profits

The participation constraint (3.6) ensures that retail branches make zero profits in expected terms. However, due to the predetermined interest rate and as a consequence of shocks, the participation constraint does not hold ex-post. This can occur due to the aggregate risk that cannot be insured by the retail branches. An example: An unexpected increase of the collateral price would lead to lower ex-post threshold than the one expected last period when the loan was issued: $\bar{\omega}_{j,a}^{t-1} < \bar{\omega}_{j,a}^t$. Hence, a smaller fraction of borrowers would be below the expected threshold and default. The decrease in the default rate and the price increase of the repossessed collateral would lead to positive profits for the respective retail branch, these profits would be accumulated as bank capital.

Thus ex-post profits of loan type $j$ is given by

$$\Pi_{j}^t = (1 - \mu)G_{j,a}^{tp} q_{t-1}^{j} \pi_{t-1} + (1 - F_{j,a}^{tp})r_{j}^{t-1} B_{t-1}^{j} - R_{j,b}^{t-1} B_{t-1}^{j}$$

$^6$See appendix 7.2
the sum of the average repossession value of collateral for the defaulted loans and the loan repayment of non-defaulted loans, minus the cost of funds for the bank.

It can be shown that the profits of each branch are a function of the difference between last period’s ex-ante expected and current period’s ex-post realized thresholds.\(^7\) Whenever the two thresholds are equal, profits are zero. When the ex-post threshold is smaller than expected (i.e. a smaller proportion of loans default than expected) profits are positive

\[
\Pi^j_t = f(\bar{\omega}^j_{t-1} - \bar{\omega}^j_t), \quad f'(\cdot) > 0
\]

Figure (1) gives an overview of the different interest rate spreads in the economy and the factors that affect them. The asset specific interest rate spreads determine bank profits and hence quantities of specific loan types in the economy. The wholesale spreads \((R_b^b - R, R_b^b - R_b^b, R_b^b - R_b^b)\) are affected by the capital asset position of the banking sector and the composition of the loan portfolio. The two retail spreads \((r^I - R_b^b, r^E - R_b^b)\) are affected by the expected collateral values and expected default threshold of each type of loan.

![Figure 1: Interest rate spreads structure](image)

The above two-level representation of spreads can also be interpreted from the perspective of the Basel capital regulation. While the retail level spread arises due to provisioning of expected losses by retail branches, the wholesale level spread arises due to capital regulation which aims to address the possibility of unexpected losses which are covered by bank capital.

### 3.2 Heterogeneous Households

**Savers**

Each saver (or patient household) \(i\) maximizes expected lifetime utility subject to the budget constraint

\[
\begin{align*}
\max E_0 \sum_{t=0}^{\infty} \beta^t & \left[ (1 - \alpha^p) \varepsilon^c_t \log(C^P_t(i)) - \alpha^c C^P_{t-1}(i) + \varepsilon^h_t \log(H^P_t(i)) - \frac{(L^P_t(i))^{1+\phi}}{1+\phi} \right] \\
\text{s.t. } & C^P_t(i) + q^h_t \Delta H^P_t(i) + D_t(i) = W^P_t L^P_t(i) + \frac{R_{t-1} D_{t-1}(i)}{\pi} + T_t(i)
\end{align*}
\]

\(^7\)See appendix 7.3
Expected lifetime utility depends on current individual (and lagged aggregate) consumption $C^P_t$, housing $H^P_t$ and hours worked.$^8$ $L^P_t$ denotes labour disutility where $\phi$ denotes the inverse elasticity of labour supply. There are two preference shocks present, $\varepsilon^c_t$ affects the marginal utility of consumption, and $\varepsilon^h_t$ the marginal utility of housing.

The patient household spends his income on current consumption, accumulation of housing (with $q^h_t$ denoting real house prices), and on saving via real deposits $D_t$. The income side consists of wage earnings $W_t L^P_t$ (where $W_t$ is the real wage), and gross interest income from last period deposits $R_{t-1}D_{t-1}/\pi_t$, where $\pi_t = P_t/P_{t-1}$ is gross inflation and $R_{t-1}$ denotes the gross interest rate on deposits. $T_t$ includes profits from intermediate goods producers and from debt repossesson agencies.

**Borrowers**

Borrowers (or impatient households) differ from savers in several aspects. First, their discount factor is smaller than the one of savers ($\beta^I < \beta^P$) which means that they are more impatient to consume. Due to their impatience, in equilibrium, savers are willing to accumulate assets as deposits, and borrowers are willing to offer their housing wealth as collateral to obtain loans. Second, borrowers don’t earn profits from goods producers. And third, borrowers are subject to a quality $\omega^j$ shock to the value of their housing stock which can lead to loan defaults.

Analogously to savers, each borrower $i$, maximizes expected lifetime utility subject to the budget and collateral constraint$^9$

$$\max E_0 \sum_{t=0}^{\infty} (\beta^I)^t \left[ (1 - \alpha^I)\varepsilon^c_t \log(C^I_t(i)) - \alpha^I C^I_{t-1}) + \varepsilon^h_t \log(H^I_t(i)) - \frac{(L^I_t(i))^{1+\phi}}{1 + \phi} \right]$$

s.t. $C^I_t + q^h_t \Delta H^I_t + q^h_t H^I_{t-1} G^I_p + \frac{(1 - F^I_p)r^I_{t-1}B^I_{t-1}}{\pi_t} = B^I_t + W^I_t L^I_t$ \hspace{1cm} (3.8)

The budget constraint for borrowers differs among those who default and those who repay their loans. Aggregating borrowers’ budget constraints$^{10}$ and dropping the $i$ superscripts, yields (3.8).$^{11}$

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$^8$Pre-multiplying by the habit coefficient $\alpha^P$ offsets the impact of external habits on the steady-state marginal utility of consumption.

$^9$All variables and parameters with the superscript $I$ indicate that they are specific to borrowers.

$^{10}$We assume that the households are members of a dynasty and insure themselves after the realization of the shock, thus becoming ex-post identical ensuring the representative agent solution.

$^{11}$The last two terms on the LHS of (3.8) denote the average repossessed value of collateral of those who default, and repayment of credit by those who don’t default. Since those terms arise from the aggregated budget constraint and not from the individual one, we assume that the individual agent does not take into account the probability of not repaying the loan tomorrow when borrowing today. Similarly we assume that the agents do not consider the probability to defaulting tomorrow when choosing collateral stock today. A similar assumption is made for entrepreneurs. See appendix 7.4. The latter terms are calculated using the ex-post realized threshold separating defaulting from non-defaulting households $\bar{\omega}^p_{p,t}$.  

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3.3 Firm sector

Entrepreneurs

Entrepreneurs maximize the sum of expected lifetime utility subject to the budget constraint, production function and the collateral constraint

\[
\max_{\{c_t^E,K_t^E,B_t^E,L_t^P,L_t^I\}} \sum_{t=0}^{\infty} (\beta^t)\left[(1 - \alpha^E)\log(C_t^E(i) - \alpha^E C_{t-1}^E)\right]
\]

s.t. \[ C_t^E + W_t^P L_t^P + W_t^I L_t^I + \frac{(1 - F_t^E) r_t^E B_{t-1}^E}{\pi_t} + q_t^k [K_t^E - (1 - \delta)K_{t-1}^E] \]
\[ + q_t^k K_{t-1}^E G_t^{E_p} = \frac{Y_t^E}{X_t} + B_t^E \] \hspace{1cm} (3.9)
\[ Y_t^E = A_t^E K_{t-1}^E \alpha L_t^{1-\alpha} \] \hspace{1cm} (3.10)

The entrepreneur \( i \)'s utility depends on the deviations of his consumption \( C_t^E(i) \) from the aggregated lagged level. The entrepreneur chooses consumption \( C_t^E \), physical capital \( K_t^E \), loans from banks \( B_t^E \), and labour \( \{L_t^P, L_t^I\} \). Entrepreneurs have the same discount factor as borrower households, such that entrepreneurs become net borrowers in equilibrium, willing to pledge capital used for production as a collateral.

The depreciation rate of capital is denoted by \( \delta \), \( q_t^k \) denotes the price of capital and \( P_t^W / P_t = 1 / X_t \) is the relative competitive price of the wholesale good \( Y_t^E \) that is produced according to the Cobb-Douglas production technology (3.10), where \( A_t^E \) denotes a stochastic productivity shock. Aggregate labour, denoted by \( L_t^E \), is given by \( L_t^E = (L_t^P)^\nu (L_t^I)^{1-\nu} \), where \( \nu \) measures the labour income share of patient households.

Capital Producers

Capital producers are a modeling device to derive the price of capital. Capital producers are perfectly competitive. To produce capital, capital producers buy two inputs. First, last-period undepreciated capital \( (1 - \delta)K_{t-1} \) at price \( Q_t^k \) (the nominal price of capital) from entrepreneurs. Second, \( I_t \) units of the final consumption good from retailers at price \( P_t \). The accumulation of capital is given by \( \Delta \bar{x}_t = K_t - (1 - \delta)K_{t-1} \). The new stock of effective capital \( \bar{x}_t \) is sold back to entrepreneurs at price \( Q_t^k \). In addition, the transformation of the final good into new capital is subject to adjustment costs \( \kappa_t \). Capital producers maximization problem is given by

\[
\max_{\{\bar{x}_t,I_t\}} \sum_{t=0}^{\infty} \lambda_t^{E}(q_t^k \Delta \bar{x}_t - I_t)
\]

s.t. \[ \bar{x}_t = \bar{x}_{t-1} + \left[1 - \frac{\kappa_t}{2}\left(I_t \bar{\varepsilon}_t^{qk} - 1\right)\right]I_t \] \hspace{1cm} (3.11)

where \( \bar{\varepsilon}_t^{qk} \) denotes a shock to investment efficiency, and \( q_t^k \equiv \frac{Q_t^k}{\bar{x}_t} \) the real price of capital.

Retailers

We follow Bernanke et al. (1999) regarding the structure of the retail good market. We assume monopolistic competitive. Retail prices are sticky and are indexed to a combination of past and steady-state inflation, with relative weights parameterized by \( \gamma_p \).

\[ \gamma_p \] Group habits are parameterized by \( \alpha^E \).
Whenever retailers want to change prices beyond this indexation allowance, they face a quadratic adjustment cost parameterized by $\kappa_p$. Retailer $i$ chooses $P_t(i)$ to maximize subject to the consumers demand function (3.12)

$$\max_{P_t(i)} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P \left[ P_t(i) Y_t(i) - P_t^W Y_t(i) - \frac{\kappa_p}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \pi_{t-1} \pi_{t-1}^{1-i_p} \right) P_t Y_t \right]$$

s.t. $Y_t(i) = \frac{P_t(i)}{P_t} - \varepsilon_t Y_t$ (3.12)

where $\pi$ denotes steady state inflation, and $\varepsilon_t$ the stochastic demand price elasticity.

3.4 Policy

3.4.1 Monetary Policy

The central bank sets the deposit interest rate according to the following Taylor rule

$$R_t = (R)(1-\phi_n)(R_{t-1})^{\phi_R} \left( \frac{\pi_t}{\pi} \right)^{\phi\pi(1-\phi_n)} \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi\pi(1-\phi_n)} \varepsilon_t$$

where $\phi_\pi$ and $\phi_\pi$ denote the weights of inflation and output, $R$ the steady state policy rate and $\varepsilon_t$ the monetary policy shock. Changes in policy rate $R_t$ will affect all interest rates equally, without affecting any of the interest rate spreads shown in Figure (1).

3.4.2 Macroprudential policy

The macroprudential instruments we analyse include the capital-asset requirement $\nu_b^j$, and the asset-specific risk weights $w_j^j$. Equation (3.3) allows the analysis of how different macroprudential instruments impact the asset specific interest rate spreads. In turn, these interest rate spreads determine bank profits and hence the volumes of loans to different sectors of the economy. For convenience, equation (3.3) is repeated here

$$R_{t}^{b,j} - R_t = \kappa_b \left( \nu_b^j - \frac{K_t^b}{RWA_t} \right) \left( \frac{K_t^b}{RWA_t} \right)^2 w_j^j$$

for $j \in \{I,E\}$

Keeping everything else constant, an increase of the capital-asset requirement $\nu_b^j$ increases the interest rate spread $R_{t}^{b,j} - R$ for all loan types $j$. The impact of this instrument is not asset type specific, it affects the provision of both loan types alike.

In contrast, an increase of the risk weight $w_j^j$ of a specific loan type $j \in \{I,E\}$ will have a stronger impact on interest rate spread $(R_{t}^{b,j} - R_t)$ of the loan type $j$ relative to $j' \neq j$. However, the interest rate spread of loan type $j'$ will also be affected through an increase in the risk weighted assets ($RWA$) defined by (3.2). This creates the possibility for macroprudential policy to conduct tailored interventions in order to influence bank lending behaviour. For example, by increasing the risk-weight on mortgages and maintaining or decreasing the risk weight for corporate loans, the macroprudential regulator can alter the relative cost of the two types of lending.

Under our macroprudential approach, the policy maker sets risk weight $w_j^j$ for asset $j$ according to a Taylor-type rule that responds to credit-to-GDP measures. According to (ESRB, 2014), the credit-to-GDP ratio is an empirically sound basis for designing
macroprudential interventions.\textsuperscript{13} Fluctuations in this ratio are historically associated with episodes of financial instability whereby the banking sector can destabilise the real economy. In our setup, the macro-prudential Taylor-type rule takes the form

\[ w_t = (\bar{w}_t)^{(1-\rho_w)}(w_{t-1})^{\rho_w} \left( \frac{B_t}{Y_t} \right)^{\chi_w(1-\rho_w)} \]

The risk weight of loans to households and firms are set according to the deviation of the loan-specific measure of leverage ($\frac{B_t}{Y_t}$) from its steady state, where the parameters $\chi_w$ and $\rho_w$ represent the responsiveness of the instrument to the sectoral leverage measure and its autoregressive properties. In Section 4, we discuss the macro-prudential setting of risk weights in further detail and compare its results to the leverage ratio capital requirements and the IRB approach.

3.5 Market clearing and shock processes

The equilibrium in the good market can be expressed by the resource constraint, i.e. the aggregated budget constraint of the entrepreneurs, equation (3.9), where $C_t$ denotes aggregate consumption and is given by $C_t = C^E_t + C^I_t + C^P_t$, while $Y^E = A^E_t K^E_t L^E_t$.

The assumption that the housing stock exists in fixed supply, $\bar{H}$, leads to the house market clearing condition

\[ \bar{H} = H^P_t + H^I_t \]  

(3.14)

Shock processes

The shock processes we employ, are specified in Table 3, and are of an AR(1) form. The scenario with news shocks is simulated by a negative shock to the expected exogenous term four periods in the future. Then at period 4 a positive shock is simulated and the two impulse responses are added. This cancels the shock itself and the resulting responses of the variables are entirely due to changes in expectations. In particular, the shock to idiosyncratic risk to mortgages takes the form $\sigma_t^i = \bar{\sigma}^i + \rho^\sigma (\sigma_{t-1}^i - \bar{\sigma}^i) + \epsilon^\sigma_{t-4}$

and $\sigma_{t+4}^i = \bar{\sigma}^i + \rho^\sigma (\sigma_{t+3}^i - \bar{\sigma}^i) + \epsilon^\sigma_{t+4}$.

4 Calibration and Estimation

4.1 Calibration

Table (1) summarizes the calibration of the model parameters. Some model parameters are calibrated to match data or have been taken directly from the literature. The model is calibrated such that each period represents a quarter.

The discount factor of patient households is set to 0.9939 which pins down a quarterly steady state policy (deposit) interest rate of 0.60 percent (2.5 percent annualized), which is consistent with the policy rate average of our data sample. Discount factors for impatient

\textsuperscript{13}Schularick and Taylor (2012) find that nowadays, credit aggregates play an important role for the macroeconomy, as they contain valuable information about the likelihood of future financial crisis. Lagged credit growth turns out to be a highly significant predictor of financial crises. Adding other variables adds very little explanatory power. Borio and Drehmann (2009) show that the credit-to-GDP gap did perform very well in predicting the recent housing bubble in real time. The credit gap indicates a potential build-up of vulnerabilities at least as early as 2001. Borio and Drehmann (2002) find that more than 80% of all financial crises can be predicted on the basis of a credit boom at one year horizon.
households and entrepreneurs are calibrated such that we match steady state quarterly borrowing rates of 0.98 and 1.1 percent (4 and 4.5 percent annualized), respectively. These borrowing rates are consistent with the average borrowing rates for mortgages and corporate loans in our data sample.

For the calibration of the LTV steady-state ratios we follow Gerali et al. (2010). We set the LTV of households loans (i.e. mortgages) $\bar{\omega}^I$ to 0.7 and for entrepreneurs $\bar{\omega}^E$ to 0.35. In the steady-state, the two LTVs together with the standard deviations of the idiosyncratic shock $\tilde{\sigma}_j^\omega$ pin down the default rates of loan type $j$. Hence, similarly to Quint and Rabanal (2014) we set the standard deviation of households’ idiosyncratic shock $\tilde{\sigma}_I^\omega$ such that we match the average default rate of mortgages for the Euro area of 2.5 percent. For firms we calibrate the standard deviation of entrepreneurs’ idiosyncratic shock $\tilde{\sigma}_E^\omega$ to 0.47 to match a default rate 2.5 percent.\footnote{Due to data availability, we cannot differentiate between default rates of mortgages and corporate loans in the data. The average default rate of all types of loans is 2.5 percent for the Euro area.}

The collateral repossession cost parameters of households and firms of $(\mu^I, \mu^E)$ are implied by the interest rates, LTV ratios and standard deviations of idiosyncratic shocks.

The calibration values for the capital share, frisch elasticity, depreciation rates, and mark-ups are taken from the literature. We follow Gerali et al. (2010) and set the capital share to 0.25 and the depreciation rate to 0.025. As common in the literature, we assume a mark-up of 20\% in the good market, and hence set $\epsilon_Y$ to 6. For the calibration of the markup in the labor market we follow Gerali et al. (2010) and set $\epsilon_Y$ to 5, implying a mark-up of 15\%.

The steady state capital-asset requirement $\bar{\nu}^b$ is set to 0.08, consistent with the the Basel II regulation. The parameter $\delta^b$, the bank capital depreciation rate, is set to 0.0061.\footnote{In our model, banks make profits in the steady state and the depreciation rate $\delta^b$ is set such that it consumes the steady state profits so that bank capital stays constant at the steady state.}

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^P$</td>
<td>Patient households discount factor</td>
<td>0.9939</td>
</tr>
<tr>
<td>$\beta^I$</td>
<td>Impatient households discount factor</td>
<td>0.9902</td>
</tr>
<tr>
<td>$\beta^E$</td>
<td>Entrepreneurs discount factor</td>
<td>0.9890</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Inverse Frisch elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in the production function</td>
<td>0.25</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\epsilon_Y$</td>
<td>$\frac{\epsilon_Y}{\epsilon_{Y}}$ markup in the goods market</td>
<td>6</td>
</tr>
<tr>
<td>$\epsilon_f$</td>
<td>$\frac{\epsilon_f}{\epsilon_{f}}$ markup in the labour market</td>
<td>5</td>
</tr>
<tr>
<td>$\bar{\omega}^I$</td>
<td>Households LTV ratio</td>
<td>0.7</td>
</tr>
<tr>
<td>$\bar{\omega}^E$</td>
<td>Firms LTV ratio</td>
<td>0.35</td>
</tr>
<tr>
<td>$\tilde{\sigma}_I^\omega$</td>
<td>Stddev of households’ idiosyncratic shock</td>
<td>0.17</td>
</tr>
<tr>
<td>$\tilde{\sigma}_E^\omega$</td>
<td>Stddev of households’ idiosyncratic shock</td>
<td>0.47</td>
</tr>
<tr>
<td>$\delta^b$</td>
<td>Bank capital depreciation rate</td>
<td>0.0061</td>
</tr>
<tr>
<td>$\mu^I$</td>
<td>Collateral repossession cost, households</td>
<td>0.093</td>
</tr>
<tr>
<td>$\mu^E$</td>
<td>Collateral repossession cost, firms</td>
<td>0.049</td>
</tr>
<tr>
<td>$\bar{\nu}^b$</td>
<td>SS capital-asset requirement</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 1: Calibration of model parameters
4.2 Data

The dataset includes 12 variables for the Euro Area with quarterly frequency covering the time period 2000:1 to 2014:4. Data is collected on real consumption, real investment, real house prices, real loans to households and firms, real deposits, real wages, inflation, interest rates to households and firms and the policy (deposit) rate. Variables involving a trend component (i.e. consumption, investment, house prices, wages, borrowing of households and firms, and deposits) are made stationary using the HP filter (smoothing parameter set to 1600) and are transformed to log deviations from their HP-filtered trend. Interest rates and the inflation rate are de-meaned. The time-series of the variables is shown in Figure (2).

4.3 Estimation (Metropolis-Hastings algorithm)

Model parameters that cannot be calibrated and are therefore estimated using Bayesian methods are listed in Tables (2) and (3). We are using a Monte-Carlo based optimization technique for computing the mode with 10 parallel chains for the Metropolis-Hastings algorithm with 20000 replications each. The scale parameter of the jumping distribution’s covariance matrix is set to 0.4 which leads to an average acceptance ratio of 33%.

---

16A full description of the data is provided in Appendix 7.1.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distribution</td>
<td>Mean</td>
</tr>
<tr>
<td>$\kappa_b$ Bank capital adj. cost</td>
<td>Gamma</td>
<td>10</td>
</tr>
<tr>
<td>$\kappa_i$ Capital adj. cost</td>
<td>Gamma</td>
<td>2.5</td>
</tr>
<tr>
<td>$\kappa_p$ Retailers’ price adj. cost</td>
<td>Gamma</td>
<td>50</td>
</tr>
<tr>
<td>$\iota_p$ Retailers’ price index</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha_h$ Habit coefficient</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi^R$ TR AR coeff.</td>
<td>Beta</td>
<td>0.75</td>
</tr>
<tr>
<td>$\phi^\pi$ TR inflation coeff.</td>
<td>Gamma</td>
<td>2</td>
</tr>
<tr>
<td>$\phi^Y$ TR output coeff.</td>
<td>Normal</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2: Estimated structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distribution</td>
<td>Mean</td>
</tr>
<tr>
<td>$\rho_c$ Cons. pref.</td>
<td>Beta</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_h$ Housing. pref.</td>
<td>Beta</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_k$ Capital adj. cost</td>
<td>Beta</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_A$ Technology</td>
<td>Beta</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_{\sigma_i}$ HHs idiosync.</td>
<td>Beta</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_{\sigma_e}$ Es idiosync.</td>
<td>Beta</td>
<td>0.8</td>
</tr>
<tr>
<td>$\sigma_c$ Cons. pref.</td>
<td>Inv. gamma</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_h$ Housing. pref.</td>
<td>Inv. gamma</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_k$ Capital adj. cost</td>
<td>Inv. gamma</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_A$ Technology</td>
<td>Inv. gamma</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_{\tau}$ Monetary Policy</td>
<td>Inv. gamma</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_{\sigma_i}$ HHs idiosync.</td>
<td>Inv. gamma</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_{\sigma_e}$ Es idiosync.</td>
<td>Inv. gamma</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 3: Estimated exogenous processes’ parameters
4.4 Historical Variance Decomposition

Estimating the model with real data allows to conduct a historical variance decomposition. The variance decomposition assesses the importance of different shocks by determining the relative share of variance that each structural shock contributes to the total variance of each variable. Figures (3)-(7) visualize the variance decomposition for the following variables: real consumption, interest rates charged on mortgages, real house prices, real investment, and interest rates charged for corporate loans.

In Figure (3) the variance decomposition of consumption shows that the model identifies the productivity shock and the shock to idiosyncratic risk in mortgage lending to be the main drivers of the build-up and fall in real consumption during the Great Recession. The main channel through which the shock to idiosyncratic risk of mortgages can have a procyclical effect on consumption is through lending and house prices. Figure (4) demonstrates that this idiosyncratic risk shock contributed negatively to mortgages interest rates in the build up phase, and positively in the crash period. Figure (5) shows that the same shock contributed positively to house prices in the build up phase, and negatively in the crash period.

The variance decomposition of investment indicates that the dynamics of real investment can be well explained by shocks to idiosyncratic firm default risk, refer to Figure (6). The channel works as follows: A lower firm idiosyncratic default risk shock leads to lower expected default rates of firm loans, and hence lower interest rates as shown in Figure (7). Lower interest rates lead to higher investment.

In summary, the model is able to identify both the build up-phase and the crash of the recent crisis as originating from mortgage and firm lending. In the build up phase, lower mortgage risk leads to lower mortgage interest rates, higher house prices and higher consumption. At the same time a lower firm lending risk leads to lower firm interest rates and higher investment. The 2008 crash is explained as a rapid increase in the risk of both types of lending (mortgages and firm loans) leading to higher interest rate spreads, and a decline in both types of borrowing and house prices. As a result consumption, investment and output all decline.

The results of the variance decomposition motivate the comparison of different policies in a crisis scenario. In particular we simulate the crisis as originating from a shock to the idiosyncratic risk in the mortgage market.
Figure 5: Variance decomposition - Real house prices

Figure 6: Variance decomposition - Real investment
5 Policy analysis

In this section we analyze three alternative asset-risk weight setting policies: the leverage ratio, the internal ratings-based models (IRB) approach which was introduced by Basel II, and a hypothetical new macroprudential risk weight setting rule. The impacts of the policy instruments and their effectiveness in stabilizing the economy are compared under two different scenarios. The first scenario represents the crash phase of the crisis, and the second consists of a simulated boom and bust cycle.

5.1 Internal ratings-based models (IRB) risk weighting

The related literature that seeks to analyse the effects of macroprudential capital requirements in general equilibrium models generally regard the regulatory capital requirement as a leverage ratio, thereby abstracting from any risk-weighting of assets.\textsuperscript{17} Such a setup is equivalent to our benchmark case in which the risk weights are constant and equal to one.

Large banks (> €100bn in assets) generally calculate their risk weighted assets following the Internal ratings-based (IRB) approach.\textsuperscript{18} In contrast to e.g. Angelini et al. (2014) and Gerali et al. (2010), our model allows for risky defaulting loans and hence can be used as a means to study the impacts of the IRB approach on the real economy.

The purpose of the IRB framework is to guarantee financial stability by imposing a bank capital requirement that is sufficient to absorb any unexpected losses arising from a bank’s assets. The capital charge that the bank has to hold for each loan type is

\textsuperscript{17}E.g., see Angelini et al. (2014) and Gerali et al. (2010).

\textsuperscript{18}The current Basel reform introduces both, a leverage ratio and asset risk weight-based constraints on bank capital.
proportional to the loan’s probability of incurring unexpected losses.

Below, we discuss the IRB approach in detail and then apply it to our theoretical model presented in section 3.

According to the IRB approach, expected losses (EL) should be covered by bank provisions and are entered on the banks balance sheet directly as a cost associated with its lending. In the model, bank provisioning is represented by the retail level of the banking system. Retail bankers set the interest rate spread by taking into account the probability of default. Unexpected losses (UL) arise in exceptional circumstances, and hence are not taken into account by bank provisioning at the retail bank level. In the model, the unexpected losses are taken into account on the wholesale level of the banking system. The wholesale bank makes sure that the capital-asset requirement is met, i.e. assets that are more volatile / more prone to generating unexpected losses require the bank to hold more capital to absorb those unexpected losses.

The IRB framework allows banks to calculate the risk weight of a specific loan type in order to ensure it has enough capital to cover the unexpected loss region shown in Figure (8). The expected loss (EL) per unit of a loan is defined as the expected annual probability of default (PD) times the loss-given-default (LGD), \( EL = PD \cdot LGD \). The expected total losses \( (TL = EL + UL) \) are rather higher than the pure EL, as some unexpected losses (UL) are also likely to occur in some scenarios where systematic factors (e.g. large economy-wide recessions) make the realised annual default rate higher than the expected PD. To model the UL, and thereby derive capital requirements, one must therefore condition the PD and LGD to increase them beyond their simple historical average levels. In the IRB approach, the conditioning of the PD is designed to increase the unconditioned PD to the point where the bank is able to absorb the unexpected losses on its assets in all but the absolute most severe (top 0.1%) negative scenarios that may occur in the following year.

\[ UL = TL - EL = LGD^c \cdot PD^c - PD \cdot LGD \]  

(5.1)

where \( PD^c \) denotes the conditional probability of default and \( LGD^c \) the conditional loss-given-default. Hence, the risk-weight that would ensure enough capital to cover the unexpected losses of loan type \( j \) can be calculated as

\[ w^j_t = \frac{1}{\nu^b} UL^j_t \]  

(5.2)

Figure 8: Loss distribution of a loan portfolio

Unexpected losses can be expressed as

Unemployment losses can be expressed as

\( UL = TL - EL = LGD^c \cdot PD^c - PD \cdot LGD \)  

(5.1)
where $\nu^b$ is the regulatory risk weighted capital-asset ratio requirement. As a result the risk-weight of a particular loan type becomes a function of the respective default probability PD and loss-given-default LGD.

In our theoretical model, we are able to use the true model values for the PD and LGD, thus eliminating any measurement errors. In terms of our notation, the PD is simply $F_{j,a}^t$ and the expected loss in the event of default of loan type $j$ is given by

$$EL_j^t = \frac{r_j^t F(\bar{w}_j^{a,a}, \sigma^j_\omega)}{\pi_{t+1}} - \frac{(1 - \mu_j^t)G(\bar{w}_j^{a,a}, \sigma^j_\omega)q_{t+1}H_j^t}{B_j^t}$$

(5.3)

The expected losses in (5.3) are expressed as the value of foregone interest minus the value of repossessed collateral. We calculate the loss-given-default as $LGD = \frac{EL}{PD}$, and the conditional $PD$ and $LGD$ values according to the Basel methodology.19 Using the latter we calculate the total losses as $TL = LGD \cdot PD$. Using equation (5.1), we compute the loan specific, time varying risk weight according to equation (5.2).

5.2 Macroprudential risk weighting

The IRB risk weight setting approach presented in the previous section, creates a positive relationship between the risk weight of a particular type of loan and its probability of default which makes risk weights procyclical. For example, in the boom phase of the economy, asset prices are high and lending conditions are lax, hence the default probability of loans decreases, leading to lower risk-weights. Similarly, in the downturn, asset prices are low and lending conditions tighten, the default rate of loans increases leading to higher risk-weights. In both phases of the credit cycle the IRB approach may lead to risk weights that reinforce economic fluctuations thereby increasing financial fragility. This procyclicality of capital requirements is consistent with the empirical evidence found by Behn et al. (2014) and Goodhart et al. (2004).

As an alternative policy setting we introduce macroprudential interventions that aim to attenuate the business cycle and minimize its vulnerability to financial distress.20 For this purpose we employ a Taylor-type rule that set the risk weight of a loan type responding to an indicator. We have chosen the indicators following the regulatory guidelines and set our instruments to respond to credit-to-GDP measures (ESRB, 2014). Therefore, in our macroprudential setting we substitute the risk weights of equation (5.2) with the following Taylor-type rule

$$w_t^j = (\bar{w}_j)^{(1-\rho_w)}(w_{t-1}^{j})^{\rho_w} \left(\frac{B_j^t}{Y_t}\right)^{\chi_w(1-\rho_w)}$$

(5.4)

The risk weight of loans to households and firms are set according to the deviation of the loan-specific measure of leverage ($B_j^t/Y_t$) from its steady state, where the parameters $\chi_w$ and $\rho_w$ represent the responsiveness of the instrument to the leverage measure and its autoregressive properties.

19See BCBS (2005), for the $LGD^c$ we use the unconditional LDG increased by 10% as a downturn estimate.

20The Capital requirements regulation - CRR IV allows for regulatory setting of higher risk-weights due to "financial stability considerations", see Article 124(4)(b).
5.3 Analysis in the crisis scenario **Bust Phase**

This scenario allows us to represent the crash phase of the crisis, and is therefore suitable to assess different policies in terms of their effect in the aftermath of a crisis. We study the impulse responses to an unexpected increase in the standard deviation of the idiosyncratic shock to mortgages. In Section 3.4., the variance decomposition identified this shock to be a driving factor at the peak of the crisis. The direct impact of this shock consists of increasing the proportion of loans below the ex-ante default threshold. This leads to a larger default rate for mortgages than was expected by the banks when the loans were issued. This in turn leads to losses to banks and the destruction of bank capital resulting in the capital-asset ratio falling below the regulatory requirement.

Figures (9) and (10) highlight the differences in the impulse responses to the shock due to the different policy settings of capital requirements. In the Benchmark case (static and equal risk weights, i.e. the leverage ratio), the destruction of bank capital reduces the capital-asset ratio and the capital-to-risk weighted-asset ratio below the regulatory requirements. In order to adjust their balance sheet to ensure the regulatory requirement is met, banks increase the wholesale interest rate spread, thereby leading to higher interest rates on loans. The higher interest rates depress economic activity and lead to a long recession.

In contrast to the benchmark case, the IRB approach increases the risk weights of mortgages as the estimate of default probability increases. The risk weights decrease following the process of household deleveraging (which results in the default probability falling). The higher risk weight on mortgages leads to a higher value for the risk weighted assets (RWAs) on the mortgage book in comparison to the other policies. This in turn leads to a larger decline in the Capital/RWA measure and hence to a higher increase in spreads and interest rates. Ultimately, this results in a larger decline in investment and output following the shock, and a slower recovery.

Finally, the macroprudential approach to setting risk-weights has a countercyclical effect as it decreases the risk weights on both types of lending as a result of the de-risking effect of the lower sectoral leverage levels in the bust phase of the crisis. This leads to lower risk weighted assets (RWA) and a higher Capital/RWA ratio, and thereby to a relatively lower increase in spreads and interest rates on bank lending. Ultimately, this results in the stimulation of investment and thereby to a relatively fast recovery.

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21 Figures (9) and (10) shows the responses of the variables in percentage deviation from steady-state values except for the responses of variables denoted with *. These variables are plotted as absolute responses due to different steady states or variables already being in percentage form.
Figure 9: IRF - unexpected shock to $\sigma^i$
5.4 Analysis in the crisis scenario *Boom and Bust*

In this crisis scenario we aim to represent both the build-up and crash phases of the crisis and thus to examine how the different policy approaches perform in terms of their amplifying or attenuating effects on the boom phase of the cycle. The scenario is simulated as a positive news shock in the initial period whereby the agents in the economy expect the default rate of mortgages 4 periods in the future to be lower than before the shock. This thereby leads to optimism and buoyancy in both lending and asset markets. However, when period 4 arrives, the shock does not occur and agents expectations of lower default rates do not materialize. As a result, the default rate of mortgages is higher than expected and banks realize losses thereby leading to a destruction of bank capital.

Figure (11) and (12) show the various impacts on agents behaviour associated with the positive news shock. In the Benchmark case (static and equal risk weights, i.e. the leverage ratio), optimism leads to higher borrowing and decreases in the Capital-Assets and Capital-Risk-weighted-Assets ratios. Banks respond to these decreased regulatory capital ratios by increasing the wholesale spread in order to stay in line with the regulatory requirement. However, the higher wholesale spread to mortgages is not enough to offset the lower retail spread which is driven by the lower default probability in the boom phase. As a result, mortgages face lower interest rates and sectoral leverage is increased further.

Unlike the results of the leverage ratio approach, the IRB approach results in decreases to the risk weights on loans due to lower PD estimates in the optimistic phase. As a result, risk weighted assets (RWA) decline and the Capital/RWA measure increases leaving the impression that banks are better capitalised when, in reality, the pure Capital/Asset measure has decreased. During this phase, IRB banks decrease their wholesale spreads and further reinforce lower interest rates and higher sectoral leverage.

As in the previous scenario, the macroprudential approach to setting risk-weights has a countercyclical effect during the boom phase of the crisis as it increases risk weights on both types of lending in response to the increases in leverage in both sectors. This leads to higher risk weighted assets (RWA) and a lower Capital/RWA ratio - and hence to an increase in wholesale spreads, leading to higher interest rates and lower borrowing than is observed under the other capital measurement approaches.

At period 4 the positive shock does not materialize, and the economy faces less favourable financial conditions than expected. From that point forward, the crisis proceeds in a similar way to the bust phase in section 5.3. The difference between the scenarios is that the negative shock here is driven by unmaterialized expectations rather than actual changes in financial outcomes.\(^{22}\)

In the benchmark (leverage ratio) setting, the destruction of bank capital reduces the Capital/Assets and Capital/RWA ratios below the regulatory requirement. In order to meet their regulatory requirement, banks increase wholesale spreads resulting in higher interest rates to loans. The higher rates depress economic activity and lead to a relatively long recession.

Unlike the leverage ratio case, the IRB approach increases the risk weight on mortgages at the point where the negative shock arises due to higher resulting estimates of PDs. Subsequently, risk weights then fall as households deleverage, and PDs decline. The higher the risk weight to mortgages leads to a larger measure of risk weighted assets on

\(^{22}\)Note that in the unrealized news shock (boom and bust) scenario the dynamics are entirely driven by expectations while the impulse response of the shock remains flat.
banks mortgage books in comparison to the other policies which in turn leads to larger decline of the Capital/RWA ratio and hence to a greater increase in spreads and interest rates on lending.

In the case of our macroprudential approach to setting risk-weights, lending conditions are tight before the shock due to the stricter capital requirements that result from high risk weights during the phase where sectoral leverage is increasing. The destruction in bank capital is therefore lower when the shock hits, and therefore the negative impacts of the shock are also lower. After the shock, the economy faces relatively favourable credit conditions in comparison to the IRB and leverage ratio regulatory cases - and the economic recovery is therefore faster, as investment can be sustained through the cycle.

Figure 11: IRF - unrealized news shock to $\sigma^i$ at period 4
6 Conclusion

Bank capital regulation has evolved through time to incorporate risk-sensitivity, i.e. the idea that different asset classes contribute to credit risk to a different degree and hence capital charges should be proportionate to the riskiness of each asset class. This framework is incorporated by the introduction of risk-weights in capital-asset requirements. However, there has been an ongoing debate on the correct approach to measure the riskiness of assets that would allow for effective risk-weighting.

While most studies on this topic rely on empirical evidence and econometric models, we incorporate different methods to asset risk-weighting in a general equilibrium macro model. This approach allows us to investigate the effect of the different methods for setting risk-weights on the financial cycle, the macroeconomy, and on the resilience of the financial system in a crisis. We compare three approaches: the IRB approach, the leverage ratio approach, and a hypothetical macroprudential approach which sets risk-weights for each asset type inversely to the development of aggregate sectoral leverage. We compare each method for setting capital requirements in terms of their response to a crisis originating from mortgage lending.

Our results show that in both boom and bust phases of the crisis the IRB approach leads to procyclical capital requirements. In the boom phase, the IRB approach leads to looser capital requirements and thereby to lending conditions that reinforce market exuberance. In the bust phase, higher PD estimates lead to higher risk-weights and tighter capital requirements that depress bank lending and push down on economic activity. The IRB approach therefore reinforces the financial cycle in the event of a crisis.

By contrast, our macroprudential approach to setting risk weights leads to counter-
cyclicality in capital requirements in both the boom and bust phases of the crisis thereby serving to attenuate the financial cycle. The negative impact of the financial crash to the real economy is smaller and the recovery happens faster.

Our macroprudential approach to setting risk weights is an alternative means of adjusting capital requirements for the state of the financial cycle in comparison to the Counter-cyclical Capital Buffer (CCB) approach that is set out within Basel III. The macroprudential risk weights we apply could offer two sets of benefits in comparison to the CBB approach. First, our Taylor type rule eliminates any discretion from the policymaker within the setting of macroprudential capital requirements. This automaticity ensures a counter-cyclical approach is pursued. By contrast, the maintenance of discretion in the application of the CCB risks that distortions arising from the surrounding political economy lead to inaction bias in the use of the CCB, and thereby to the maintenance of procyclical requirements.

Second, the macroprudential risk weights we apply adjust to the sectoral financial cycle thereby providing a more precise tool for reacting to sector-specific bubbles and credit booms. Historically, financial crises tend to arise as a result of credit booms and bubbles that are focused on particular sectors. To name just a few examples the recent Great Recession originated in the housing sector, around the turn of the millennium there was the Dotcom bubble in the technology sector, and in the 1840s there was a spectacular boom in the US railroad sector. The sectoral risk weights that we put forward in this analysis could helpfully focus macroprudential controls over capital requirements on those sectors that are experiencing booms, whilst avoiding unwarranted impacts on other non-bubble sectors. This would potentially concentrate the impact of the tool where it needs to be to stabilise capital flows, and reduce unintended spillover costs to other non-bubbly sectors.

Overall, the findings in this paper tend to support the view that there is room for improvement in the current design of bank capital regulation. As the design of the financial framework evolves, an emphasis on less procyclical mechanisms would be potentially beneficial for the sake of mitigating the banking sectors tendency to exacerbate the real economy impact of financial shocks. The non-risk-based leverage ratio setup could be an improvement over the IRB approach in this regard. A move to a hard-wired sector-specific macroprudential approach may be better still.
References


International Monetary Fund, "Lessons from Asset Price Fluctuations for Monetary Policy", Chapter 3 of World Economic Outlook, April (2009).


7 Appendix

7.1 Data description

- **Consumption**: Household and NPISH final consumption expenditure, chain linked volumes (2010), seasonally adjusted and adjusted data by working days. Transformation: log deviation from HP-filtered mean. Source: Eurostat.

- **Investment**: Gross fixed capital formation, chain linked volumes (2010), seasonally adjusted and adjusted data by working days. Transformation: log deviation from HP-filtered mean. Source: Eurostat.

- **House prices**: Residential Property Valuation, new and existing dwellings, neither seasonally nor working day adjusted. Transformation: deflated by HICP inflation, log deviation from HP-filtered mean. Source: ECB.

- **Wages**: Labour cost index, whole economy excluding agriculture, fishing and government sectors, working day and seasonally adjusted. Transformation: deflated by HICP inflation, log deviation from HP-filtered mean. Source: Eurostat.

- **Inflation**: Harmonised Index of Consumer Prices (HICP), seasonally adjusted, not working day adjusted. Transformation: deviation from mean. Source: ECB.

- **Policy Rate**: Euribor 3-month - historical close, average of observations through period. Transformation: in gross quarterly form, deviation from mean. Source: ECB.

- **Borrowing rate - households**: Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks), Lending for house purchase excluding revolving loans and overdrafts, convenience and extended credit card debt, Up to 1 year initial rate fixation, New business coverage, Households and NPISH. Transformation: in gross quarterly form, deviation from mean. Source: ECB.
- **Borrowing rate - firms**: Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks) reporting sector - Loans other than revolving loans and overdrafts, convenience and extended credit card debt, Up to 1 year initial rate fixation, Up to and including EUR 1 million amount, New business coverage, Non-Financial corporations. Transformation: in gross quarterly form, deviation from mean. Source: ECB.

- **Borrowing volume - households**: Lending for house purchase, households and NPISH, outstanding amounts at the end of the period (stocks), neither seasonally nor working day adjusted. Transformation: deflated by HICP inflation, log deviation from HP-filtered mean. Source: ECB.

- **Borrowing volume - firms**: Loans to non-financial corporations, outstanding amounts at the end of the period (stocks), neither seasonally nor working day adjusted. Transformation: deflated by HICP inflation, log deviation from HP-filtered mean. Source: ECB.

- **Deposits**: Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Deposits with agreed maturity, Over 1 and up to 2 years maturity, All currencies combined - Euro area (changing composition) counterpart, Households and NPISH, denominated in Euro, neither seasonally nor working day adjusted. Transformation: deflated by HICP inflation, log deviation from HP-filtered mean. Source: ECB.

### 7.2 Spread expression

Given the spread equation (3.9) we have that the denominator is the following function of the ex-ante threshold \( \tilde{\omega}^{j,a} \):

\[
X(\tilde{\omega}^{j,a}) = \frac{(1-\mu)G(\tilde{\omega}^{j,a}, \sigma^j)}{\tilde{\omega}^{j,a}} + (1 - F(\tilde{\omega}^{j,a}, \sigma^j)) \tag{7.1}
\]

or expressed with integrals:

\[
X(\tilde{\omega}^{j,a}) = \frac{(1-\mu)\int_0^{\tilde{\omega}^{j,a}} \omega f(\omega)d\omega}{\tilde{\omega}^{j,a}} + 1 - F(\tilde{\omega}^{j,a}) \tag{7.2}
\]

where \( f(\omega) \) is the PDF and \( F(\omega) \) is the CDF of the log-normal distribution. In fact, the second term in the RHS which is the probability of non-default, expressed as 1 - the probability of default, where the latter is just the CDF evaluated at \( \tilde{\omega}^{j,a} \). Then it is straightforward to see that as the CDF is increasing function in \( \tilde{\omega}^{j,a} \) then:

\[
\frac{d(1 - F(\tilde{\omega}^{j,a}))}{d\tilde{\omega}^{j,a}} < 0 \tag{7.3}
\]

is a decreasing function in \( \tilde{\omega}^{j,a} \). Then calculating the derivative of the of \( X(\tilde{\omega}^{j,a}) \) wrt \( \tilde{\omega}^{j,a} \) we obtain:

\[
\frac{dX(\tilde{\omega}^{j,a})}{d\tilde{\omega}^{j,a}} = \frac{(1-\mu)\tilde{\omega}^{j,a}f(\tilde{\omega}^{j,a}) - f(\tilde{\omega}^{j,a})}{(\tilde{\omega}^{j,a})^2} - \frac{(1-\mu)\int_0^{\tilde{\omega}^{j,a}} \omega f(\omega)d\omega}{(\tilde{\omega}^{j,a})^2} + \frac{d(1 - F(\tilde{\omega}^{j,a}))}{d\tilde{\omega}^{j,a}} \tag{7.4}
\]
which simplifies to:
\[
dX(\bar{\omega}^{j,a}) = -\mu f(\bar{\omega}^{j,a}) - \left(1 - \mu\right)\int_{0}^{\bar{\omega}^{j,a}} \omega f(\omega) d\omega + \frac{d(1 - F(\bar{\omega}^{j,a}))}{d\bar{\omega}^{j,a}}
\] (7.5)
which is negative, meaning that \(X(\bar{\omega}^{j,a})\) is decreasing function of the ex-ante threshold \(\bar{\omega}^{j,a}\). Then as we have from equation (10) the spread is:
\[
\frac{r^{j}_{t}}{R^{b}_{t}} = \frac{1}{(1 - \mu)G(\bar{\omega}^{j,a}, \sigma^{j}_{\omega}) + (1 - F(\bar{\omega}^{j,a}, \sigma^{j}_{\omega}))} = \frac{1}{X(\bar{\omega}^{j,a})}
\] (7.6)
meaning that the spread is an increasing function of the ex-ante threshold such that:
\[
\frac{r^{j}_{t}}{R^{b}_{t}} = f(\bar{\omega}^{j,a}), f'(\cdot) > 0
\] (7.7)
which is equation (11).

7.3 Profits expression

Starting from the equation (15) of profits, dividing by the borrowing quantity \(b_{t-1}^{j} - 1\) and substituting the ex-post threshold (14), we obtain profits per unit of loans as:
\[
\Pi^{j}_{t} = \left(1 - \mu\right)G(\bar{\omega}^{j,p}, \sigma^{j}_{\omega}) \frac{r^{j}_{t-1}}{\bar{\omega}^{j,p}} + (1 - F(\bar{\omega}^{j,p}, \sigma^{j}_{\omega}))r^{j}_{t-1} - R^{b}_{t-1}
\] (7.8)
then from evaluating the participation constraint (7) in period \(t - 1\) and substituting the ex-ante threshold (6) in period \(t - 1\), \(\bar{\omega}^{j,a}_{t-1}\) in it we have that:
\[
R^{b}_{t-1} = \left(1 - \mu\right)G(\bar{\omega}^{j,a}_{t-1}, \sigma^{j}_{\omega}) \frac{r^{j}_{t-1}}{\bar{\omega}^{j,a}_{t-1}} + (1 - F(\bar{\omega}^{j,a}_{t-1}, \sigma^{j}_{\omega}))r^{j}_{t-1}
\] (7.9)
which can be substituted in (24) leading to:
\[
\Pi^{j}_{t} = r^{j}_{t-1} \left[\left(1 - \mu\right)G(\bar{\omega}^{j,p}, \sigma^{j}_{\omega}) \omega^{j,p} + (1 - F(\bar{\omega}^{j,p}, \sigma^{j}_{\omega})) - \left(1 - \mu\right)G(\bar{\omega}^{j,a}_{t-1}, \sigma^{j}_{\omega}) \omega^{j,a} + (1 - F(\bar{\omega}^{j,a}_{t-1}, \sigma^{j}_{\omega}))\right]
\]
Then using the formulation of \(X(\bar{\omega}^{j,a})\) in (17), the last equation becomes:
\[
\Pi^{j}_{t} = b_{t-1}^{j}r^{j}_{t-1} \left[-X(\bar{\omega}^{j,a}_{t-1}) - X(\bar{\omega}^{j,p}_{t-1})\right]
\] (7.10)
And since we have showed in 5.1 that \(X(\bar{\omega}^{j,a})\) is a decreasing function in \(\bar{\omega}^{j,a}\), then for any \(\bar{\omega}^{j,a}_{t-1} = \bar{\omega}^{j,p}_{t-1}\) the above expression would be zero, and for any \(\bar{\omega}^{j,a}_{t-1} > \bar{\omega}^{j,p}_{t-1}\) we would have that \(X(\bar{\omega}^{j,p}_{t-1}) > X(\bar{\omega}^{j,a}_{t-1})\) and that \(\Pi^{j}_{t} > 0\) leading to:
\[
\Pi^{j}_{t} = f(\omega^{j,a}_{t-1} - \omega^{j,p}_{t-1}), f'(\cdot) > 0
\] (7.11)
which is equation (16).
### 7.4 Model first order conditions

**Patient households (Savers)**

PHHs choose: $C_t^P$, $H_t^P$, and $L_t^P$ to maximize:

$$E_0 \sum_{t=0}^{\infty} (\beta^P)^t U_t = E_0 \sum_{t=0}^{\infty} (\beta^P)^t \left[ (1-\alpha^P)\varepsilon_t^c \log(C_t^P(j)-\alpha^P C_{t-1}^P) + \varepsilon_t^h \log(H_t^P(j)) - \frac{(L_t^P(j))^{1+\phi}}{1+\phi} \right]$$

subject to:

$$C_t^P(j) + q_t^h \Delta H_t^P(j) + D_t(j) = W_t L_t^P(j) + \frac{R_{t-1} D_{t-1}(j)}{\pi_t} + T_t(j) \quad (7.12)$$

If we denote marginal utility of consumption with:

$$U_{C_t^P} = \Lambda_t^P = \frac{(1-\alpha^P)\varepsilon_t^c}{C_t^P - \alpha^P C_{t-1}^P} \quad (7.13)$$

then substituting eq(31) for $C_t$ and $C_{t+1}$ into eq(30) and differentiating wrt. $D_t$ we obtain the following Euler equation:

$$\Lambda_t^P = \beta^P \Lambda_{t+1}^P \frac{R_t}{\pi_{t+1}} \quad (7.14)$$

Then differentiating the infinite sum of discounted utility wrt. $H_t^P$ gives the demand for housing:

$$\Lambda_t^P q_t^h = \frac{\varepsilon_t^h}{H_t^P} + \beta^P \Lambda_{t+1}^P q_{t+1}^h \quad (7.15)$$

Finally differentiating wrt. leisure $L_t^P$, we obtain the labour supply:

$$\Lambda_t^P = \frac{(L_t^P)^\phi}{W_t^P} \quad (7.16)$$

**Impatient households (Borrowers)**

IHHs choose: $C_t^I$, $H_t^I$, and $L_t^I$ to maximize:

$$E_0 \sum_{t=0}^{\infty} (\beta^I)^t U_t = E_0 \sum_{t=0}^{\infty} (\beta^I)^t \left[ (1-\alpha^I)\varepsilon_t^c \log(C_t^I(i)-\alpha^I C_{t-1}^I) + \varepsilon_t^h \log(H_t^I(i)) - \frac{(L_t^I(i))^{1+\phi}}{1+\phi} \right]$$

subject to the budget constraint:

$$C_t^I + q_t^h \Delta H_t^I + \frac{(1-F_t^P)r_{t-1} B_{t-1}^I}{\pi_t} + q_t^h H_{t-1}^I C_t^P = B_t^I + W_t L_t^I \quad (7.17)$$

and collateral constraint:

$$r_t^I B_t^I / \pi_{t+1} = m_t^I q_{t+1}^h H_t^I \quad (7.18)$$
If we denote marginal utility of consumption with:

\[ U_{C_t} = \Lambda_t = \frac{(1 - \alpha^t)\varepsilon_t^c}{C_t - \alpha^t C_{t-1}} \]  

(7.19)

By constructing a Lagrangian with the collateral constraint and \( S_t^I \) being its shadow value we obtain:

\[ L^I = E_0 \sum_{t=0}^{\infty} (\beta^t)^t [U_t + S_t^I (m_t^h q_{t+1}^h H_t^I - \frac{r_t^B_t}{\pi_{t+1}})] \]  

(7.20)

substituting the budget constraint for \( C_t^I \) and \( C_{t+1}^I \) and differentiating wrt. \( B_t^I \) we obtain the following Euler equation:

\[ \Lambda_t = \frac{\beta^t \Lambda_{t+1} r_t^I}{\pi_{t+1}} + \frac{S_t^I r_t^I}{\pi_{t+1}} \]  

(7.21)

Differentiating wrt \( H_t \) gives the following housing demand:

\[ \Lambda_t^I q_t^h = \beta^t \Lambda_{t+1}^I q_{t+1}^h + \frac{\varepsilon_h}{H_t^I} + S_t^I m_t^h q_t^h \]  

(7.22)

Lastly, labour supply:

\[ \Lambda_t = \frac{(L_t^I)^{\phi}}{W_t^I} \]  

(7.23)

**Entrepreneurs**

Choose consumption \( C_t^E \), physical capital \( K_t^E \), loans from banks \( B_t^E \), degree of capital utilization, and labour inputs from patient and impatient households \( L_t^P, L_t^I \) to maximize:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[(1 - \alpha^E)\log(C_t^E(i) - \alpha^E C_{t-1}^E)\right] \]  

(7.24)

subject to:

\[ C_t^E + W_t^P L_t^P + W_t^I L_t^I + \frac{(1 - F_t^{p,E}) r_{t-1}^E B_{t-1}^E(i)}{\pi_t} + q_t^k K_{t-1}^E - (1 - \delta) K_{t-1}^E + \]  

\[ q_t^k K_{t-1}^E G_{t-1}^{p,E} = \frac{Y_t^E}{X_t} + B_t^E \]  

(7.25)

with production function:

\[ Y_t^E(i) = A_t^E K_{t-1}^E(i) \alpha L_t^E(i)^{1-\alpha} \]  

(7.26)

where: \( L_t^E = (L_t^P)^{\nu}(L_t^I)^{1-\nu} \)

subject to a budget constraint:

\[ r_t^E B_t^E / \pi_{t+1} \leq m_t^k q_{t+1} K_t^E \]  

(7.27)
Denoting marginal utility of consumption as:

\[ \Lambda_t^E = \frac{(1 - \alpha^e)}{C_t^E - \alpha^e C_{t-1}^E} \]  

(7.28)

Constructing Lagrangian with \( S^E \) being the shadow value of the collateral constraint, then differentiating wrt. \( K_t^E \) leads to:

\[ \Lambda_t^E q^k_t = \Lambda_{t+1}^E \beta^E (q^k_{t+1} (1 - \delta) + r^k_{t+1}) + S^E m^E_t q^k_{t+1} \]  

(7.29)

where \( r^k_t \) is the rental rate of capital: \( r^k_t = \frac{\alpha Y^E_t}{K_{t-1}^E X_t} \)

For labour demand we have MP of each labour type equal to its MC:

\[ W_t^P = \frac{\nu (1 - \alpha) Y^E_t}{L^P_t X_t} \quad W_t^I = \frac{(1 - \nu)(1 - \alpha) Y^E_t}{L^I_t X_t} \]  

(7.30)

Finally the Euler equation is:

\[ \Lambda_t^E = \frac{\Lambda_{t+1}^E \beta^E r^E_t}{\pi_{t+1}} + \frac{S^E r^E_t}{\pi_{t+1}} \]  

(7.31)

**Capital Producers**

Using the discount factor of entrepreneurs (as being owned by them), capital producers maximize:

\[ E_0 \sum_{t=0}^{\infty} \Lambda_t^E (\beta^E)^t \left[ q^k_t \Delta x_t - I_t \right] \]  

(7.32)

by choosing \( \Delta x_t \) and \( I_t \) subject to the following constraint:

\[ \Delta x_t = \left[ 1 - \frac{\kappa_i}{2} \left( \frac{I_{t+1} \varepsilon^k_t}{I_{t-1} - 1} \right)^2 \right] I_t \]  

(7.33)

Where, \( \Delta x_t = K_t - (1 - \delta)K_{t-1} \). Differentiating wrt. \( I_t \) we obtain:

\[ \Lambda_t^E \left[ q^k_t \frac{\partial \Delta x_t}{\partial I_t} - 1 \right] + \Lambda_{t+1}^E \beta^E \left[ q^k_{t+1} \frac{\partial \Delta x_{t+1}}{\partial I_t} \right] = 0 \]  

(7.34)

for the partial derivatives we obtain:

\[ \frac{\partial \Delta x_t}{\partial I_t} = 1 - \frac{\kappa_i}{2} \left( \frac{I_{t+1} \varepsilon^k_t}{I_{t-1} - 1} \right)^2 - \kappa_i \left( \frac{I_{t+1} \varepsilon^k_t}{I_{t-1} - 1} \right) \frac{I_{t+1} \varepsilon^k_t}{I_{t-1}} \]  

(7.35)

\[ \frac{\partial \Delta x_{t+1}}{\partial I_t} = \kappa_i \left( \frac{I_{t+1} \varepsilon^k_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1} \varepsilon^k_{t+1}}{I_{t+1}} \right)^2 \]  

(7.36)

substituting the last two into 52 we obtain the optimality condition:

\[ 1 = q^k_t \left[ 1 - \frac{\kappa_i}{2} \left( \frac{I_{t+1} \varepsilon^k_t}{I_{t-1} - 1} \right)^2 - \kappa_i \left( \frac{I_{t+1} \varepsilon^k_t}{I_{t-1} - 1} \right) \frac{I_{t+1} \varepsilon^k_t}{I_{t-1}} \right] + \beta^E E_t \left[ \frac{\Lambda_{t+1}^E q^k_{t+1} \varepsilon^k_{t+1}}{\Lambda_t^E} \kappa_i \left( \frac{I_{t+1} \varepsilon^k_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \]

**Retailers**

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Thus retailers choose $P_t(j)$ to maximize:

$$E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[ P_t(j)Y_t(j) - P_tW_tY_t(j) - \frac{\kappa_p}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{i_p} \pi_{t-1}^{-i_p} \right)^2 P_tY_t \right]$$  \hspace{1cm} (7.37)

subject to: $Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^\epsilon Y_t$.

Thus the part of the infinite sum that includes $P_t(j)$ is:

$$\sum_{t=1}^{R} = \Lambda_{t} \left[ Y_t(j)(P_t(j) - P_tW_t) - \frac{\kappa_p}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{i_p} \pi_{t-1}^{-i_p} \right)^2 P_tY_t \right] + \Lambda_{t+1} \beta \left[ Y_{t+1}(j)(P_{t+1}(j) - P_{t+1}W_{t+1}) - \frac{\kappa_p}{2} \left( \frac{P_{t+1}(j)}{P_t(j)} - \pi_{t}^{i_p} \pi_{t-1}^{-i_p} \right)^2 P_{t+1}Y_{t+1} \right]$$  \hspace{1cm} (7.38)

Differentiating wrt. $P_t(j)$ and imposing $P_t(j) = P_t$ leads to:

$$\Lambda_{t} \left[ -\epsilon Y_t + \frac{\epsilon Y_t}{X_t} + Y_t - \kappa_p(\pi_t - \pi_{t-1}^{i_p} \pi_{t-1}^{-i_p})P_tY_t \frac{1}{P_{t-1}(j)} \right] + \Lambda_{t+1} \beta \left[ \frac{\kappa_p(\pi_{t+1} - \pi_{t}^{i_p} \pi_1^{-i_p})P_{t+1}Y_{t+1}}{Y_t} \frac{P_{t+1}(j)}{P_t(j)} \right] = 0$$  \hspace{1cm} (7.39)

which after dividing by $Y_t$ and $\Lambda_{t}$ simplifies to:

$$1 - \epsilon Y_t + \frac{\epsilon Y_t}{X_t} - \kappa_p(\pi_t - \pi_{t-1}^{i_p} \pi_{t-1}^{-i_p})\pi_t + \frac{\Lambda_{t+1} \beta}{\Lambda_t} \kappa_p(\pi_{t+1} - \pi_{t}^{i_p} \pi_1^{-i_p}) \frac{Y_{t+1}}{Y_t} \frac{P_{t+1}(j)}{P_t(j)} = 0$$  \hspace{1cm} (7.40)

where we use that $1/X = P_t^W/P_t$ and $\pi_t = P_t/P_{t-1}$

The profits of retailers that are transferred back to savers are:

$$J_t^R = Y_t \left( 1 - \frac{1}{X_t} \right) - \kappa_p(\pi_t - \pi_{t-1}^{i_p} \pi_{t-1}^{-i_p})^2$$  \hspace{1cm} (7.41)