Abstract

There is a strong negative cross-country correlation between the share of consumption that households spend on housing services and house price bubbles. Countries that spend less on housing services as a share of total consumption, experienced significantly more house price booms and busts during the period 1970 - 2014, and the associated housing boom-bust cycles were larger and more volatile. This paper proposes an overlapping generation (OLG) model that replicates this fact by separating the consumption and investment side of a real estate asset. When agents have weaker preferences for housing services, this model economy will be characterized by lower consumption shares for housing services. In that case the economy is more prone to experience house price bubbles and faces larger house price bubbles. The model offers novel policy implications. While help-to-buy schemes make the economy more bubble-prone, rental subsidies are an effective tool to reduce the prevalence of house price bubbles.

Keywords: house price bubbles, housing booms and busts, preference for housing services, expenditure shares for housing services, rental and real estate purchase subsidies, OLG models

JEL Classifications: E32, E44, E52, E58
1 Introduction

Large house price bubbles can be devastating for the real economy.\textsuperscript{1} The bursting of housing bubbles played an important role in generating the financial crisis that led to the Great Recession of the 21\textsuperscript{st} century. This episode has raised interest among policy makers and researchers to understand which economic environments are more prone to produce such house price bubbles.\textsuperscript{2}

This paper explores a novel channel: the demand for housing consumption. In particular, I test the hypothesis that housing consumption drives economies’ vulnerability to house price bubbles. The hypothesis is tested from three angles: a theoretical overlapping generations model that disentangles the consumption and investment demand for housing, empirical data analysis, and with a laboratory experiment.

Housing is very different to other assets given its duality - the consumption and investment demand for housing. Empirical studies have shown that times of intensive housing investment are often associated with bubbly episodes.\textsuperscript{3} The existing literature explores channels that work through the investment demand for housing.\textsuperscript{4} The role housing consumption plays in generating housing bubbles, however, remains underexplored.

This paper aims to fill that gap. Housing consumption constitutes a large fraction of total consumption and is measured by the consumption of housing services. A household receives housing services from living in a house, independent of whether the dwelling is owned or rented. The demand for housing consumption determines the relative price of housing services and hence drives the fundamental value of real estate in the economy. This has important implications for the bubble size and for economies’ vulnerability to house price bubbles.

This paper highlights two main results. First, if the demand for housing consumption is low, countries are more prone to experience a housing bubble. Second, these countries where housing consumption is low face larger and more volatile housing bubbles.

\textsuperscript{1}Claessens \textit{et al.} (2012), Claessens, Kose, and Terrones (2009) and IMF (2003) show that recessions associated with house price busts are more than twice as long and twice as deep compared to normal recessions or recessions associated with equity busts.


\textsuperscript{3}Housing investment e.g. measured by turnover rates. The strong relationship between turnover and prices was first illustrated in Stein (1995). Subsequently, papers by Leung (2004), Andrew and Meen (2003), Hort (2000), and Berkovec and Goodman (1996) have confirmed the results.

\textsuperscript{4}The credit channel is widely accepted to play an important role for bubble formation, e.g. Drudi \textit{et al.} (2009), Agnello and Schuknecht (2011), Schularick and Taylor (2012), Igan and Loungani (2012), Agnello and Schuknecht (2011), Claessens \textit{et al.} (2009), Borio and Lowe (2002). Transaction costs are found to matter for bubble formation.
The first section of this paper explores the implications of housing consumption on house price bubbles through the lens of an overlapping generations model. Crucially, this paper takes a two-dimensional approach to model housing demand - considering the consumption and investment demand for housing separately. This recognition of the duality of housing distinguishes my model from existing papers of housing demand and price formation. It therefore allows the specific analysis of the impact of the preference for housing services on house price bubble occurrence.

In the model I assume cross-country differences in the preference for housing services relative to all other consumption goods. This preference parameter determines the share of consumption spent on housing services as an equilibrium outcome. Assuming cross-country differences in the preference for housing is soundly justified by empirical evidence, as provided in the companion paper Huber and Schmidt (2016). In that paper, we show that large cross-country differences in housing preferences exist and that these cross-country differences are persistent over time.

Two main results emerge from the analysis of the model. First, economies characterized by high housing consumption, are those economies that face smaller housing bubbles. The mechanism behind this result is intuitive: strong preferences for housing services (relative to other consumption goods) imply a large demand for housing services, and this drives high relative prices of those housing services. This implies that a large share of the consumption expenditure is spent on housing services. The fundamental value of real estate is given by expected discounted stream of the price for housing services. Therefore, stronger preferences for housing services imply a larger fundamental value of real estate, all else equal. It follows from the economy’s resource constraint that the maximum bubble size is smaller in such an environment, as there is less room left for a housing bubble.

5The log-specification over composite consumption and housing services is supported by e.g. Davis and Ortalo-Magne (2011), who find that the expenditure share on housing services is constant over time and across cities in the United States. Further, I find that cross-country differences in the expenditure share on housing services are constant over time (for a sample of 18 OECD countries).

6Huber and Schmidt (2016) study the impact of culture on living arrangements, using data on the tenure choice decision of second generation immigrants in the United States - holding constant institutional factors. We find that cultural housing preferences transmitted by parents play an important role for the housing tenure choice of these second generation immigrants. For concreteness, we show that differences in preferences for homeownership exist across countries. In my model, I implement our empirical finding by assuming cross-country differences in preferences for housing services. This is in line with other theoretical papers, e.g. Kaplan et al. (2016) that calibrates the preference for housing services such that it matches homeownership rates in the data.

7Empirically, I measure the expenditure share of housing consumption using actual and imputed rents. What are imputed rents? Homeowners do not pay for the consumption of housing services the owned dwelling provides. The imputed rents of these housing services are valued at the estimated rent that a tenant pays for a dwelling of the same size and quality in a comparable location.
The second theoretical result shows that economies with strong preferences for housing services are less vulnerable to housing bubbles in the first place. I show that the existence condition for housing bubbles becomes tighter, the larger the demand for housing services. This means that the set of possibilities for bubble occurrence is reduced.

The model is also used to study the impact of two prominent alternative policies aimed at fostering the affordability of housing - rental subsidies and help-to-buy schemes. I focus on analysing the extent to which each policy impacts economies’ vulnerability to housing bubbles. I find that a proportional rental subsidy is an effective tool to control housing bubbles, while the help-to-buy scheme makes the economy more bubble prone.

The second section of this paper evaluates the extent to which the model’s predictions can be reconciled with empirical evidence. To do so, I first provide an empirical characterization of housing cycles, bubbles and housing consumption using a large database covering 18 OECD countries during 1970-2014. Interestingly, I find large cross-country differences in both housing consumption (i.e. housing services), and house price bubbles (number, amplitude, volatility) that have occurred during 1970-2014. The number of house price bubbles is interpreted as a measure of the vulnerability of an economy to housing bubbles. Second, the interaction between housing consumption and house price fluctuations is investigated. In line with the model’s predictions, two novel empirical regularities are identified across countries: housing consumption is highly negatively correlated with (1) the frequency and (2) the intensity and amplitude of house price bubbles. Thus, countries with a lower share of consumption spent on housing services experienced not only more, but also larger and more volatile house price bubbles during 1970-2014.

The companion paper Huber et al. (2016) complements the empirical analysis with the evaluation of the theoretical model predictions using a laboratory macro experiment. In contrast to the empirical analysis, this technique allows to isolate and test the causal effect of the preference for housing services on house price bubbles. The empirical work proxies these preferences by the expenditure share of housing consumption - an equilibrium outcome of the OLG model. Further, the experimental setup allows to quantify housing bubbles without measurement error. The results of the macro experiment provide strong support for the model’s predictions. In the treatment where we induce low preferences for housing services we consistently observe significantly larger house price bubbles.

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8Rental subsidies are a common policy to promote the affordability of housing in many countries. In e.g. France, the proportion of assisted households is large. According to Laferriere and Blanc (2004), 51.6% of the private sector tenants received a subsidy in 1996. The help-to-buy scheme is most common in English speaking countries, like the United States and the United Kingdom.
Related Literature - Theory: This paper is related to a growing literature on rational bubbles. The model is based on an overlapping generations (OLG) structure, drawing upon seminal work on bubbles by Samuelson (1958) and Tirole (1985). Most rational bubble models adopt an OLG structure. However, there is a small, but growing literature on rational bubbles using infinite-horizon models. My model is related to Galí (2014) in terms of how I introduce the bubbly asset in the model economy. This paper also relates to the theoretical literature on rational house price bubbles. The closest papers are Arce and López-Salido (2011), Basco (2014), and Basco (2016), who investigate housing bubbles in overlapping generation models. These papers study the interaction between financial market imperfections and rational housing bubbles.

Apart from the different research question, one main difference between existing work and my own lies in how the housing bubble is modeled. In the related literature, the housing bubble is modeled as a shortage of assets in the economy. In my model the housing bubble is a part of the housing price. A second crucial difference between the existing literature on rational house price bubbles and my own, concerns the duality of housing. This paper takes a two-dimensional approach to model housing demand - considering both the demand for consumption and the demand for investment. This recognition of housing duality distinguishes my model from the existing rational housing bubble literature. Likewise, most studies of housing prices in standard macroeconomic models without bubbles do not consider simultaneously the two-dimensional aspect of housing.

Given that housing services constitute a large part of total consumption, and its potential to drive bubbles, my model is a useful extension to the literature. My model makes

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9It is nontrivial to introduce rational bubbles into an infinite-horizon model, due to the transversality conditions (Santos and Woodford (1997)). As Kocherlakota (1992) points out, infinite-horizon models with trading frictions or borrowing constraints can generate bubbles. Kocherlakota (2008) and Hellwig and Lorenzoni (2009) provide infinite-horizon endowment economies with such features.

10Brunnermeier and Julliard (2008) and Burnside et al. (2016) present models of housing bubbles based on heterogeneous beliefs or irrational behaviors. There exist many studies of housing prices in the standard macroeconomic models without bubbles, e.g. Iacoviello (2005), Kiyotaki et al. (2011), Liu et al. (2013), among many others.

11Iacoviello (2005)’s seminal paper develops an infinite horizon monetary model and introduces a financial accelerator that works through the housing sector. Housing enters the utility function and the budget constraint. In contrast to my paper, there is no clear distinction between the consumption and investment aspect of a house. Further, housing bubbles are ruled out by a transversality condition. Many papers studying house prices and or policy interventions, use or extend Iacoviello (2005)’s framework without the distinction between investment and consumption demands for housing. Iacoviello and Neri (2010) investigate the nature of the shocks that hit the housing market and access the magnitude of the spillovers resulting from the housing market to the wider economy. Iacoviello and Pavan (2013) study housing and mortgage debt over the cycle. Rubio and Carrasco-Gallego (2014) study how the interaction of macro prudential and monetary policies affect the economy.
it possible to study housing policy interventions that target either the consumption or investment demand of housing and allows the assessment of their impact on bubble formation.

Related Literature - Empirics: This paper contributes to the empirical literature investigating why some countries experience a larger number (and more extreme) house price bubbles than others. While this paper highlights a new channel: housing consumption, the existing literature explores channels that work through the investment demand for housing, such as the credit supply, transaction costs or property taxes. Many studies highlight the credit channel. This channel is widely accepted to play an important role for bubble formation. Drudi et al. (2009) analyze the main developments in housing finance in the euro area over the last decade and evaluate cross-country differences in mortgage markets. This includes relative differences in variable versus fixed rate mortgages, bankruptcy laws, tax regimes etc. Agnello and Schuknecht (2011) study the credit related determinants of house price booms and busts for a sample of 18 countries over the period 1980-2007. The driving factors this study considers and finds to be important are all credit related: the level of short term interest rates, credit growth to the private sector, global liquidity growth, and a mortgage market regulation dummy. Housing policies that increase transaction costs are sometimes claimed to reduce speculative behaviour in the housing market. However, Hau (2001) suggests that transaction costs have only a minor impact in preventing asset price bubbles. On the other hand, Andrews et al. (2011) and Catte et al. (2004) find that house price volatility is smaller in countries with greater transaction costs in property markets. Similarly, Ikromov and Abdullah (2012) find that transaction costs reduce the magnitude of experimental asset price bubbles and push prices closer to fundamentals. In the empirical part of this paper, I control for the above described channels that work through the investment demand for housing. The novel channel of housing consumption remains highly significant and seems to play a large role in driving economies’ vulnerability to house price bubbles.

The remainder of this paper is organized as follows: Section 2 describes the overlapping generation model, emphasizing the existence condition for house price bubble occurrence. Section 3 provides comparative statics and shows the impact of the preference for housing

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12 Following the empirical literature, credit related variables perform well in predicting crises in real time, e.g. the private sector credit to GDP ratio or credit growth as measures for the leverage of an economy (Schularick and Taylor (2012), Igan and Loungani (2012), Agnello and Schuknecht (2011), Claessens et al. (2009), Borio and Lowe (2002)).

13 In contrast to stock prices, Hau (2006) studies the French stock market and finds that higher transaction costs should be considered as volatility increasing.
services on choice variables, prices and housing bubbles. Section 4 outlines the policy analysis. Section 5 describes the methodology used to identify and measure house price bubbles empirically. This section also provides comprehensive descriptive statistics of all housing cycles and housing booms and busts that have occurred during 1970-2014 in 18 OECD countries. Further, this section presents the novel empirical cross-country regularities. Finally, section 6 concludes. Appendix A provides a detailed derivation of the model equations. A description of the data used for the empirical analysis can be found in Appendix B. For more details on the empirical justification for the model assumption that housing preferences differ across countries, I refer to the first companion paper *Cross-Country Differences in Homeownership Rates: A Cultural Phenomenon?* For the macro-experiment, I refer to the second companion paper *Preference for Housing Services and House Price Bubble Occurrence: Evidence from a Macro-Experiment.*

2 Model

This paper provides a highly stylized overlapping generations model with housing, without capital and where labor is supplied inelastically. In equilibrium, aggregate employment and output are constant. However, this framework allows to study why countries with a weaker preference for housing services experienced significantly larger and a larger number of house price bubbles over the time period 1970-2014. The model is used as a laboratory for the qualitative analysis of the impact of the preference of housing services on (1) house price bubble occurrence, and (2) the amplitude of those bubbles. Further, I study the impact of two prominent, but very different policies aiming to foster the affordability of housing - rental subsidies and help-to-buy schemes. Thereby I investigate the potential for these policies to be used for mitigating house price bubbles occurrence.

2.1 Households

I assume an overlapping generations structure where a continuum of households lives for two periods. The size of each generation (young and old) is normalized to unity. After dying, the old generation is replaced by a new, young one. Hence total population remains constant. Households born at time $t$ maximize the expected lifetime utility

$$u(C_{1,t}) + \xi^k v(S_t) + \gamma E_t\{u(C_{2,t+1})\}$$

(2.1)
where $C_t$ denotes the non-durable composite consumption good.  

Consuming housing stock of size $S_t$ yields housing service utility $v(S_t)$. $\xi^k$ denotes the aggregate preference for housing service of country $k$ relative to all other consumption $C_{1,t}$ when young. I will use log utility as the functional form for what will follow, i.e. $u(\cdot) = v(\cdot) = \log(\cdot)$.  

Young households supply their labor service inelastically for a real wage $W_t$, and allocate their net wealth between consuming the bundle $C_{1,t}$, housing services of size $S_t$, save/invest in an one period riskless bond of value $Z_t$ and purchasing housing stock of size $H_t$. The return to saving $Z_t$ is given by the nominal interest rate $(1 + i_t)$. For future reference, I define the real interest rate as

$$R_t \equiv (1 + i_t)E_t \left\{ \frac{P_t}{P_{t+1}} \right\}. \quad (2.2)$$

In this paper, I consider the two-dimensional aspect of housing, the demand for consumption and the investment demand. I disentangle the dual motives of housing behavior by modeling the consumption aspect (consuming housing services) and investment aspect (investing in housing) separately. This assumption distinguishes this model from existing models of housing demand e.g. Iacoviello (2005), and allows the separate analysis of the impact of housing services on house price bubble occurrence.

For concreteness, when young households buy housing services $S_t$, they do so by renting housing stock $S_t$ from the old generation. Young households that invest into housing, buy housing stock $H_t$ when young from the old generation. The housing asset yields a dividend payment next period - a rental income when old. Before the old household dies, he sells the remaining housing stock to the new young generation.  

14 $C_{1,t} \equiv \left( \int_0^1 C_{1,t}^{1-i}(i)di \right)^{\frac{1}{1-i}}$ and $C_{2,t+1} \equiv \left( \int_0^1 C_{2,t+1}^{1-i}(i)di \right)^{\frac{1}{1-i}}$ are the bundles consumed when young and old, respectively. In each period, there exists a continuum of differentiated goods, each produced by a different firm, and with a constant elasticity of substitution denoted by $\varepsilon$. Henceforth I assume $\varepsilon > 1$. Differentiated consumption goods (and the firms producing them) are indexed by $i \in [0, 1]$.

15 As in Iacoviello (2005), I assume that housing service and all other composite consumption are separable. The decision of choosing a log specification over housing service and composite consumption is based e.g. on Davis and Ortalo-Magne (2011), who find that the expenditure share on housing is constant (over time and across U.S. cities). Further, I find that cross-country differences in the expenditure share on housing services are constant over time (for a sample of 18 OECD countries). Bernanke (1984) studies the joint behavior of the consumption of durable and non-durable goods, and finds that a separable log specification is a good approximation. Note the functions $u(\cdot)$, $v(\cdot)$ are continuous and twice differentiable, with $\lim_{C^i \to 0^+} u(C^i) = -\infty$ and $\lim_{C^i \to 0^+} v(C^i) = \infty$, $\lim_{S \to 0} v(S) = -\infty$ and $\lim_{S \to 0} v'(S) = \infty$.

16 As Henderson and Ioannides (1983) argued, "...before the introduction of institutional considerations there is no reason for people to actually owner-occupy their consumption-investment demands, as opposed to being landlords of their asset holdings and renting their consumption demand from some other landlords".
When born, households are endowed with \( \delta \in [0, 1) \) units of housing stock whose price is \( Q_{t|t} > 0 \). Households can buy and trade houses.\(^{17}\) Each period, the housing stock depreciates by the fraction \( \delta \); it follows that the total housing stock in the economy remains constant.

Young households are endowed with know-how to set up a new firm producing a differentiated consumption good. That firm only becomes productive after one and for one period only (i.e. when the founder is old), generating profits, \( D_t \), for the owner when old.

Accordingly, the budget constraint of the young household at time \( t \) is given by

\[
C_{1,t} + \frac{Z_t}{P_t} + \sum_{k=0}^{\infty} q_{t|t-k} H_{t|t-k} + p_t r_t S_t \leq W_t + \delta q_{t|t},
\]

where \( P_t \) is the price of the composite consumption good in period \( t \). The rental and purchasing price of one unit of housing stock is denoted by \( P_t^r \) and \( Q_t \), respectively. With prices written in lowercase letters, I define prices relative to the consumption bundle, so \( q_t = \frac{Q_t}{P_t} \) and \( p_t = \frac{P_t^r}{P_t} \). Further, \( H_{t|t-k} \) denotes the quantity of the housing stock purchased in \( t \), introduced by the cohort born in period \( t-k \), and whose relative current price is \( q_{t|t-k} \) for \( k = 0, 1, 2, \ldots \) The budget constraint when old is given by equation (2.4). By purchasing the consumption bundle \( C_{2,t+1} \), the household consumes all his wealth. The household’s wealth consists of (1) the rental income from renting his housing stock to the young generation, which is given by \( \sum_{k=0}^{\infty} p_{t+1} r_{t+1} H_{t|t-k} \), (2) the re-selling value of his housing stock, (3) the payoff from his maturing bond holding and (4) real profits generated by his intermediate firm, \( D_{t+1} \). Formally, for each old household we have

\[
C_{2,t+1} \leq \left( \frac{1 + i_t}{P_{t+1}} \right) Z_t + \sum_{k=0}^{\infty} p_{t+1} r_{t+1} H_{t|t-k} + (1 - \delta) \sum_{k=0}^{\infty} q_{t+1|t-k} H_{t|t-k} + D_{t+1},
\]

where \( H_t = \sum_{k=0}^{\infty} H_{t|t-k} \).

\(^{17}\)Assuming that housing is a partially bubbly asset, it follows that households are endowed with a partially bubbly asset as in Galí (2014). With the difference that in Galí (2014) households are endowed with a pure bubbly asset, that is intrinsically useless.

\(^{18}\)At the end of the period the old household sells his remaining housing stock, i.e. \( (1 - \delta) \sum_{k=0}^{\infty} q_{t+1|t-k} H_{t|t-k} \), to the young generation.
2.1.1 Household Optimality Conditions

The Euler Equation is derived using FOCs (A1), (A2) and (A5)

$$1 = \gamma (1 + i_t) E_t \left\{ \left( \frac{C_{1,t}}{C_{2,t+1}} \right) \left( \frac{P_t}{P_{t+1}} \right) \right\}$$  \hspace{1cm}  (2.5)

The intra-temporal Optimality Condition is derived using FOCs (A1) and (A2)

$$\frac{\xi k C_{1,t}}{S_t} = p_t^r$$  \hspace{1cm}  (2.6)

The optimal saving/investment decision is derived using FOCs (A3) and (A5)

$$q_t|_{t-k} = E_t \left\{ \frac{P_{t+1}}{(1 + i_t)P_t} \left( p_{t+1}^r + (1 - \delta)q_{t+1|t-k} \right) \right\}$$  \hspace{1cm}  (2.7)

Using the Euler Equation, the previous equation can be rewritten as

$$q_t|_{t-k} = \gamma E_t \left\{ \left( \frac{C_{1,t}}{C_{2,t+1}} \right) \left( p_{t+1}^r + (1 - \delta)q_{t+1|t-k} \right) \right\}$$  \hspace{1cm}  (2.8)

2.2 The Price of Housing: Definitions and Assumptions

I define the House price as

$$q_t \equiv q_t^F + q_t^B,$$  \hspace{1cm}  (2.9)

where the fundamental price component is defined as the present discounted value of expected rental income the house generates, and hence is given by

$$q_t^F \equiv E_t \left\{ \sum_{k=1}^{\infty} \prod_{j=0}^{k-1} \frac{1}{R_{t+j}} (1 - \delta)^{k-1} p_{r,t+k} \right\}.$$  \hspace{1cm}  (2.10)

The bubbly price component is defined as

$$q_t^B \equiv B_t + U_t^b,$$  \hspace{1cm}  (2.11)

with $B_t \equiv \delta \sum_{k=1}^{\infty} (1 - \delta)^k q_{t|t-k}^B$ and $U_t^b \equiv \delta q_{t|t}^B$, where $B_t$ denotes the value of pre-existing bubbles in the economy and $U_t^b$ the value of the newly introduced bubbles in $t$. I assume that $U_t^b$ will follow an exogenous i.i.d. process with mean $U^b$. 

9
It can be shown that (2.10) satisfies

\[ q_{t|-k}^F = E_t \left\{ \frac{1}{R_t} (p_{r,t+1} + (1 - \delta)q_{t+1|-k}^F) \right\}. \]  \hspace{1cm} (2.12)

Using (2.9), (2.12) and (2.8), it follows that the bubble component must satisfy

\[ q_{t}^B = B_t + U_t^b = E_t \left\{ \frac{1}{R_t} B_{t+1} \right\}. \]  \hspace{1cm} (2.13)

Hence, an increase in the interest rate will raise the expected growth of the bubble (as long as \( U_t^b > 0 \)), while the fundamental component of the housing price will be affected negatively by a rise in the interest rate, refer to equation (2.10).

2.3 Firms

2.3.1 Final Production Sector

The final consumption good production is perfectly competitive, hence final consumption good producers earn zero profits. Each final consumption good producer has the following production function

\[ y_t \equiv \left( \int_0^1 y_t(i)^{(\frac{\varepsilon - 1}{\varepsilon})} \, di \right)^{(\frac{\varepsilon}{\varepsilon - 1})} \text{ with } \varepsilon > 1, \]  \hspace{1cm} (2.14)

where \( y_t(i) \) is the quantity of the intermediate good \( i \) with the demand function

\[ y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} y_t \forall i \in [0, 1]. \]  \hspace{1cm} (2.15)

It follows that the price of the final consumption good is given by

\[ P_t \equiv \left( \int_0^1 P_t(i)^{(1-\varepsilon)} \, di \right)^{(\frac{1}{1-\varepsilon})}. \]  \hspace{1cm} (2.16)

The optimization problem of the representative final producer is therefore

\[
\begin{align*}
\text{max} & \quad P_t y_t - \left( \int_0^1 P_t(i)y_t(i) \, di \right) \\
\text{s.t.} & \quad y_t = \left( \int_0^1 y_t(i)^{(\frac{\varepsilon - 1}{\varepsilon})} \, di \right)^{(\frac{\varepsilon}{\varepsilon - 1})}
\end{align*}
\]
2.3.2 Intermediate Production Sector

The production function uses labor as the only input and is given by

\[ y_t(i) \equiv L_t(i) \quad \forall i \in [0, 1] \]  

(2.17)

Every firm has monopolistic power in the production of his own variety. The monopolist sets his price \( P_t(i) \) in order to maximize his profits subject to the demand constraint (2.15). The optimization problem of the monopolistic firm is given by

\[
\max_{P_t} \quad E_{t-1} \left\{ \Lambda_{t-1,t} \left( P^*_t y_t(i) - \Psi_t(y_t(i)) \right) \right\} \\
\text{s.t.} \quad y_t(i) = \left( \frac{P^*_t}{P_t} \right)^{-\varepsilon} y_{e,t} \tag{2.19}
\]

where the price \( P^*_t \) is set at the end of \( t-1 \) (prices set in advance), which introduces nominal rigidities in the model. \( \Psi_t(y_t(i)) \) denotes the nominal cost function of firm \( i \). \( \Lambda_{t-1,t} \) denotes the discount factor. As households own the intermediate production firms, they will get the profits as a lump-sum payment when old.\(^{19}\)

The first order condition (FOC) is given by:

\[
E_{t-1} \left\{ \Lambda_{t-1,t} \left( y_t(i) + P^*_t (-\varepsilon) \left( \frac{P^*_t}{P_t} \right)^{-\varepsilon-1} \frac{y_t}{P_t} - \Psi'_t(y_t(i)) (-\varepsilon) \left( \frac{P^*_t}{P_t} \right)^{-\varepsilon-1} \frac{y_t}{P_t} \right) \right\} = 0
\]

After some manipulations, we get the optimal pricing condition:

\[
E_{t-1} \left\{ \Lambda_{t-1,t} y_t(i) \left( P^*_t - \frac{\varepsilon}{\varepsilon - 1} \Psi'_t(y_t(i)) \right) \right\} = 0 \tag{2.20}
\]

Each firm chooses its new price equal to a fixed markup over its current nominal marginal cost, i.e. \( M = \frac{\varepsilon}{\varepsilon - 1} \).

\(^{19}\)Note that households take prices as given, therefore the discount factor used in the firm maximization problem must be slightly different than the one of the household. But as the difference just has to be infinitesimally small, the discount factor can be approximated by the discount factor of the household. So, the relevant discount factor will be derived from the Euler Equation and will be given by \( \Lambda_{t-1,t} \approx \frac{E_t\{w(C_{2,t})\}}{\gamma w(C_{1,t-1})} \).
In the case of flexible prices and no uncertainty, the FOC (2.20) is satisfied with

\[ P^*_t = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \Psi'_t(y_t(i)) \quad \iff \quad P^*_t = MW_tP_t \] (2.21)

Hence, the real wage is given by \( W = \frac{1}{N} \) and aggregate (real) profits by \( D = (1 - \frac{1}{M}) = (1 - W) \) \( \forall t \).

2.4 Equilibrium

In this section, I describe the equilibrium of the economy.

Aggregate consumption good market clearing requires\(^{21}\)

\[ Y_t = (C_{1,t} + C_{2,t}). \] (2.22)

From the income side, I can write

\[ Y_t = D_t + W_t. \] (2.23)

Labor Market Clearing

Given the assumption that only young households supply inelastically one unit of labor it follows that total labor employed is given by\(^{22}\)

\[ L_t = \int_0^1 L_t(i) \, di = 1. \] (2.24)

Labor market clearing implies

\[ L_t = \int_0^1 Y_t(i) \, di = Y_t = 1. \] (2.25)

The second equality follows because I assume that in a symmetric equilibrium all firms set the same price and produce the same amount.

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\(^{20}D = \frac{1}{M} \left( P^*_t \int_0^1 y_t(i) \, di - \int_0^1 \Psi_t(y_t(i)) \, di \right) = \frac{1}{M} (MW_tP_t - W_tP_t) = (1 - \frac{1}{M}) \quad \forall t.

\(^{21}\)The market clearing for each consumption good \( i \) requires that \( Y_t(i) = C_{1,t}(i) + C_{2,t}(i) \) for all \( t \) and \( i \in [0, 1] \). Using the aggregate output \( Y_t = \int_0^1 y_t(i) \left( \frac{t}{t_i} \right) \, di \) and the demand functions for each consumption good \( i \), i.e. \( C_{1,t}(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_{1,t} \) and \( C_{2,t+1}(i) = \left( \frac{P_{t+1}(i)}{P_{t+1}} \right)^{-\varepsilon} C_{2,t+1} \), we can derive the aggregate good market clearing condition (2.22).

\(^{22}\)Given that \( Y_t(i) = L_t(i) \), it follows that \( \int_0^1 Y_t(i) \, di = \int_0^1 L_t(i) \, di \), hence \( Y_t = L_t = 1 \).
**Housing Market Clearing**

Houses exist in fixed supply. The aggregate supply of the housing stock is given by

\[
\bar{H}_t = \delta + \delta(1 - \delta) + \delta(1 - \delta)^2 + ... = \delta \sum_{k=0}^{\infty} (1 - \delta)^k = 1 \quad \forall t. \tag{2.26}
\]

with \( \bar{H}_{t|t-k} = \delta(1 - \delta)^k \). The total supply of houses as to equal demand each period. Hence,

\[
H_t = 1 \quad \text{and} \quad H_{t|t-k} = \delta(1 - \delta)^k \quad \forall t. \tag{2.27}
\]

**Rental Market Clearing**

The supply of houses \( \bar{H}_t \) is constant and normalized to one. The aggregate supply of the housing stock that is available for rent is given by the aggregate housing stock itself and is denoted by \( \bar{S}_t \). Formally,

\[
\bar{S}_t = \delta + \delta(1 - \delta) + \delta(1 - \delta)^2 + ... = \delta \sum_{k=0}^{\infty} (1 - \delta)^k = 1 \quad \forall t. \tag{2.28}
\]

The supply of rental-homes has to equal the demand each period. It follows that

\[
S_t = 1 \quad \forall t. \tag{2.29}
\]

**Bond market Clearing**

Market clearing implies that the aggregate value of the bond market must equal zero,

\[
Z_t = 0 \quad \forall t. \tag{2.30}
\]

Market clearing conditions (2.22), (2.25), (2.27), (2.29), (2.30) and the optimal price setting equation (2.20) together with the optimality conditions of the household (2.5)-(2.8) and the definition of the housing price (2.9) with (2.12) describe the equilibrium of the economy.
Equilibrium Equations\textsuperscript{23}

\begin{align*}
L_t &= Y_t = 1 \\
Y_t &= (C_{1,t} + C_{2,t}) \\
Y_t &= D_t + W_t = 1 \\
S_t &= H_t = 1 \\
C_{1,t} &= \frac{1}{1+\xi} (W_t - F_t - B_t) \\
p_r^t &= \frac{\xi}{1+\xi} (W_t - F_t - B_t) \\
C_{2,t+1} &= E_t \left\{ D_{t+1} + \frac{\xi}{(1+\xi)} W_{t+1} + \frac{1}{(1+\xi)} (F_{t+1} + B_{t+1}) \right\} \\
q_t &= F_t + B_t + U_t \\
q_t^B &= B_t + U_t^B = \gamma E_t \left\{ \left( \frac{C_{1,t}}{C_{2,t+1}} \right) B_{t+1} \right\} \\
q_t^F &= F_t + U_t^F = \gamma E_t \left\{ \left( \frac{C_{1,t}}{C_{2,t+1}} \right) \left( p_{t+1}^r + (1 - \delta) q_{t+1}^F \right) \right\} \\
1 &= \gamma (1 + i_t) E_t \left\{ \left( \frac{C_{1,t}}{C_{2,t+1}} \right) \left( \frac{p_t}{p_{t+1}} \right) \right\}
\end{align*}

Next, I will characterize the deterministic equilibrium for which an exact analytical solution exists. Emphasis is placed on the existence conditions for a bubbly equilibrium with positive fundamental. Subsequently, I will discuss comparative statics.

\textsuperscript{23}The derivation can be found in the Appendix.
2.5 Equilibrium Dynamics

In the deterministic case, where \( U_t = U > 0 \), and \( B_t - E_{t-1}\{B_t\} = 0 \), and \( F_t - E_{t-1}\{F_t\} = 0 \) for all \( t \). The optimal price setting equation implies that \( W_t = W = (1/\mathcal{M}) \), and it follows from market clearing condition that \( D_t = 1 - W \) for all \( t \). Recall that:

\[
C_{1,t} = \frac{1}{1 + \xi} (W_t - F_t - B_t) \tag{2.31}
\]

\[
p_t^\varepsilon = \frac{\xi}{1 + \xi} (W_t - F_t - B_t) \tag{2.32}
\]

\[
C_{2,t} = D_t + \frac{\xi}{(1 + \xi)} W_t + \frac{1}{(1 + \xi)} (F_t + B_t) \tag{2.33}
\]

\[
= 1 - \frac{1}{(1 + \xi)} (W_t - F_t - B_t)
\]

Using the Euler Equation and the consumption levels (2.31) and (2.33), the real interest rate can be expressed as

\[
R_t = \frac{(1 - W) + \xi + F_{t+1} + B_{t+1}^u}{\gamma (W - F_t - B_t)} \equiv R(B_t, B_{t+1}, F_t, F_{t+1}) \tag{2.34}
\]

The previous conditions determine the deterministic equilibrium allocations given the equilibrium path for the fundamental and the bubble \( \{B_t, F_t\} \). The latter two must satisfy the following difference equations:

\[
B_{t+1} = \frac{(1 - W)(1 + \xi)(B_t + U^B)}{\gamma W - (1 + \xi + \gamma)(B_t + F_t) - (1 + \xi)U}
\equiv H(B_t, F_t, U) \tag{2.35}
\]

\[
F_{t+1} = \frac{(1 - W)(F_t + U^F) + \xi^2(B_t + F_t + U)}{\gamma W - (1 + \xi + \gamma)(B_t + F_t) - (1 + \xi)U}
+ \frac{\xi}{\gamma W - (1 + \xi + \gamma)(B_t + F_t) - (1 + \xi)U}
\left[ \gamma W(B_t + F_t - W) + (B_t + U^B) + (2 - W)(F + U^F) \right]
\equiv G(B_t, F_t, U) \tag{2.36}
\]

A deterministic bubbly equilibrium with positive fundamental value is defined by a sequence \( \{B_t, F_t\} \) satisfying the two difference equations (2.35) and (2.36), where \( B_t \in \left( W - F_t - \frac{(1 + \xi)}{1 + (1 - \delta)\gamma}, W - F_t - \frac{(1 + \xi)}{1 + \gamma} \right) \) for all \( t \) and a range of \( U \in [u_{R_t}, \tilde{u}_1] \). The aggregate bubble is then given by \( Q_t^B = B_t + U^B \). Given the \( \{B_t, F_t\} \), we can determine the equilibrium values for all variables. The derivation of all equation in is section is provided in appendix. In the next section, I derive the range of \( U \) consistent with equilibrium.
Equilibrium Dynamics of Bubbly Steady State with positive Fundamental

Figure (1) plots the transition dynamics of \( \{B_t, F_t\} \), with \( U \in \{\tilde{u}_1, u_1\} \). There exist two sets of bubbly steady states with positive fundamental, one set of stable and one set of unstable steady states.

I define a steady state by the triple \((B,F,U)\) such that \( B = H(B,U) \) and \( F = G(F,U) \) with \( B \in \left(W - F - \frac{(1+\xi)}{1+(1+\xi)^y}, W - F - \frac{(1+\xi)}{1+y}\right) \) and \( U \in \{\tilde{u}_1, u_1\} \). The steady state \((B^*, F^*, U)\) depicted in figure (1) is locally stable. It can be shown numerically that \( \partial[H(B,F,U) - B] / \partial B < 0 \) for \( B > B^* \) and \( \partial[G(B,F,U) - F] / \partial F < 0 \) for \( F > F^* \), while \( \partial[H(B,F,U) - B] / \partial B > 0 \) for \( 0 < B < B^* \) and \( \partial[G(B,F,U) - F] / \partial F > 0 \) for \( 0 < F < F^* \). The steady state \((B^u, F^u, U)\) depicted in figure (1) is globally unstable. It can be shown numerically that \( \partial[H(B,F,U) - B] / \partial B > 0 \) for \( B > B^u \) and \( \partial[G(B,F,U) - F] / \partial F > 0 \) for \( F > F^u \), while \( \partial[H(B,F,U) - B] / \partial B < 0 \) for \( 0 < B < B^u \) and \( \partial[G(B,F,U) - F] / \partial F < 0 \) for \( 0 < F < F^u \).

For each \( U \in \{\tilde{u}_1, u_1\} \), the mappings \( B_{t+1} = H(B_t, F_t, U) \) and \( F_{t+1} = G(B_t, F_t, U) \) have two fixed points, given by \((B^*, F^*, U)\) and \((B^u, F^u, U)\). Given an initial condition \( B_0 \in ]0, B^*[ \) and \( F_0 \in ]0, F^*[ \), the solutions to \( B_{t+1} = H(B_t, F_t, U) \) and \( F_{t+1} = G(B_t, F_t, U) \) correspond to a bubbly equilibrium path that converges to \((B^*, F^*, U)\). For any initial condition \( \{B_0 > B^u, F_0 > F^u\} \) the constraint (2.38), \( B_t < W - F_t \), would be violated in finite time, hence not consistent with equilibrium. Hence for initial condition \( B_0 \in ]0, B^*[ \) and \( F_0 \in ]0, F^*[ \), the system of two difference equations has a globally stable steady state given by \((B^*, F^*, U)\).

![Figure 1: Two sets of Bubbly Steady States with positive Fundamental](image-url)
Equilibrium Dynamics of Bubbless Steady State

A deterministic bubbleless equilibrium ($B_t = 0$) with a positive and real fundamental value is defined by a sequence $\{F_t\}$ satisfying the difference equation (2.37), where $F_t \in \left(W - \frac{(1+\xi)}{1+(1-\delta)}, W\right)$ for all $t$ and a range of $U^F \in [u_F, \bar{u}_F]$.

The aggregate fundamental is then given by $Q_t^F = F_t + U^F$. Given $\{F_t\}$, we can determine the equilibrium values for all variables.

$$F_{t+1} = \frac{(1-W)(F + U^F) + \xi^2(F_t + U^F) + \xi \left[\gamma W(F_t - W) + (2 - W)(F + U^F)\right]}{\gamma W - (1 + \xi + \gamma)F_t - (1 + \xi)U^F} = G'(F_t, U^F)$$

(2.37)

Figure (2) plots the transition dynamics of $F_t$, with $U^F \in [u_F, \bar{u}_F]$. There exist two sets of bubbleless steady states, one set of stable $F^* = G'(F^*, U^F)$ and one set of unstable steady states $F^u = G'(F^u, U^F)$. The steady state is stable (unstable) if $\partial G'(F, U^F)/\partial F < 1 (> 1)$.

In the bubbleless steady state the real interest rate $R(F, B = 0) < 1$. This implies that $u'(C_1) < \gamma u'(C_2)$, meaning that all households would be better off by transferring resources from the young to the old.

**Proof: see Appendix.**
2.6 Conditions for the Existence of Bubbles

In this section I discuss the conditions for the existence of such bubbly equilibria and steady states with positive fundamental value \((F, B > 0)\) in detail. Second, I will show the restrictions on the real interest rate and the resulting constraints on the bubble size and the size of the fundamental. In the last part of this section, I derive the range of \(U\) consistent with such an equilibrium.

2.6.1 Affordability Constraint

The investment in housing has to be affordable. Given that the young households are the only agents that buy houses, the affordability constraint is derived from the budget constraint of the young. In a bubbly equilibrium it must hold that

\[
B_t \in \left[0; W-q^f_t\right] \text{ for all } t
\]

Lemma 2.1. The larger the fundamental value of real estate today, the smaller the maximum pre-existing aggregate bubble value today. Proof: See Appendix.

2.6.2 Bubbly Equilibrium: Existence Condition

Proposition 2.2. A necessary condition for the existence of a deterministic bubbly steady state with a positive fundamental and bubble value is given by

\[
W > F(\xi^k, \gamma, \delta) + \left(\frac{1 + \xi^k}{1 + (1 - \delta)\gamma}\right)
\]

where \(W = \frac{1}{M}\) is pinned down by the exogeneous parameter \(M\). Proof: See Appendix

Corollary 2.3. The higher \(\xi^k\), i.e. the larger the population’s preference for housing services \(\xi\) relative to other consumption in country \(k\), the tighter the inequality - hence, the smaller the set of possibilities that a positive bubble exists (that Proposition 2.2 holds).

A higher \(\xi^k\) implies that a larger share of consumption expenditure is spent on housing services as an equilibrium outcome in country \(k\). Hence according to Proposition 2.2, countries with a larger share of consumption expenditure spent on housing services, are less prone to experience housing bubbles.26

26In section 5, I provide empirical cross-country evidence for both model implications. First, I show that countries that have a larger fundamental value of housing, experienced smaller housing bubbles. Second, I show that countries that spend a larger share of their consumption expenditure on housing services experienced significantly less housing bubbles over 1970-2014.
Corollary 2.4. The higher the fundamental component of the housing price, the tighter the inequality - hence, the smaller the set of possibilities that a positive bubble exists (that Proposition 2.2 holds).

Proposition 2.5. A necessary and sufficient condition for the existence of a deterministic pure bubbly steady state without fundamental value is given by $W > \left( \frac{1}{1+\gamma} \right)$.
Proof: See Appendix.

Proposition 2.6. A necessary condition for the existence of a deterministic bubbleless steady state with a positive fundamental value is given by $W > \frac{1+e^k}{1+(1-\delta)\gamma}$.
Proof: See Appendix.

2.6.3 Interest Rates and Bubble Sizes

Deterministic Steady State Interest Rate
Case 1: Bubble World with positive Fundamental ($F(u^f) > 0, B(u^b) > 0$)
With $u^b > 0$, it follows from (2.13) that in a bubbly steady state, the interest rate has to be $R(B(u^b), F(u^f)) < 1$. It follows from (2.10) that $R(F(u^f), B(u^b)) > (1 - \delta)$ must hold in any deterministic steady state. Consequently, in a bubbly deterministic steady state with a positive fundamental value, the real interest rate lies between $(1 - \delta) < R(F(u^f), B(u^b)) < 1$.

Case 2: Bubbleless World ($F(u^f) > 0, B(u^b) = 0$)
It follows from (2.10) that $R(F(u^f), B(u^b)) > (1 - \delta)$ must hold in any deterministic steady state Consequently, in a bubbleless deterministic steady state with a positive fundamental value, the real interest rate has to be larger than $(1 - \delta)$, i.e. $R(F(u^f)) > (1 - \delta)$.

Case 3: Pure Bubble World ($\xi = F = 0, B(u^b) > 0$)
In a pure bubble world, the model collapses to the economy in Galí (2014). If $u^b = 0$ and $\xi = f(0) = 0$, the deterministic steady state interest rate is given by $R(0, B(0)) = 1$, the interest rate corresponding to the upper bound of the unstable steady state bubble size.
Note if $u^b > 0$ and $\xi = F(u^f) = 0$, it follows from (2.13) that $R(B(u^b)) < 1$.

\footnote{In the deterministic steady state, the definition of the fundamental price (2.10) becomes $q_t^F \equiv F + u^f = \frac{\xi(W - F - B)}{(1+\gamma)|R-(1-\delta)|}$. Hence, for the price of the fundamental component to be positive, it must hold $R(F(u^f), B(u^b)) > (1 - \delta)$.}
The Bubble size and the size of the Fundamental

Using the *deterministic version* of the Euler equation (2.5) and the definition of the real interest rate (2.2), I can write:

\[
R_t = \frac{(1 - W) + \xi^k + F_{t+1} + B_{t+1}}{\gamma(W - F_t - B_t)}
\] (2.39)

Using (2.39), the affordability constraint (2.38), and the conditions on the real interest rate derived above (cases 1-3), it can be shown that the bubble size and the size of the fundamental are given by

**Case 1**: Bubble World with positive Fundamental \((F(u^f) > 0, B(u^b) > 0)\)

\[
B \in \left( W - F - \frac{(1 + \xi)}{1 + (1 - \delta)\gamma}, W - F - \frac{(1 + \xi)}{1 + \gamma} \right) \quad \text{where} \quad (1 - \delta) < R < 1
\] (2.40)

**Case 2**: Bubbleless World \((F(u^f) > 0, B(u^b) = 0)\)

\[
F \in \left( W - \frac{1 + \xi}{1 + (1 - \delta)\gamma}, W - \frac{(1 + \xi)}{1 + \gamma} \right) \quad \text{where} \quad (1 - \delta) < R < 1
\] (2.41)

**Case 2**: Pure Bubble World \((\xi = F = 0, B(u^b) > 0)\)

\[
B \in \left( 0, W - \frac{1}{1 + \gamma} \right) \quad \text{where} \quad R \leq 1
\] (2.42)

### 2.6.4 Conditions on the U-Range for Steady States

We determine the region of compatible \(U\) via the steady state expression of the real interest rate. The corresponding derivations can be found in the Appendix.

The continuum of bubbly deterministic steady state with a positive fundamental value \((B, F)\) are describe by:

\[
\begin{cases}
\exists \text{ two sets of steady states with } R_1(U) \neq R_2(U) & \text{for } U \in [u_{R_1}, \tilde{u}_1]. \\
\exists \text{ one set of steady states with } R_2(U) & \text{for } U \in (u_{R_2}, \tilde{u}_1).
\end{cases}
\]

where

\[
u_{R_1} = \left( \frac{\xi^k + \delta [W(1 + \gamma) - (1 + \xi^k)] - W\gamma\delta^2}{[1 + \gamma(1 - \delta)][1 - \delta]} \right) \quad \nu_{R_2} = \left( \frac{\xi^k}{1 + \gamma} \right)
\]
and
\[ \tilde{u}_1 \equiv (\gamma + \xi) + (1 + \gamma)(1 - W) - 2\sqrt{\gamma(1 - W)(1 + \gamma + \xi)} \]

**Proof:** see appendix.

Figure (3) plots the two sets of steady states with \( R_1(U) \neq R_2(U) \) for \( U \in [\tilde{u}_R, \tilde{u}_1] \). Figure (4) plots one set of steady states with \( R_2(U) \) for \( U \in (\tilde{u}_R, \tilde{u}_1) \).

**Figure 3:** Set of Steady State interest rates \( R_1(U), R_2(U) \) over full solution space

**Figure 4:** Set of Steady State interest rates \( R_2(U) \) over full solution space
3 Comparative Statics

3.1 Impact of the Preference of Housing Services on Steady State Allocations

Figures (5)-(7) depict the impact of \( \xi^k \), the aggregate preference for housing services of country \( k \), on the set of stable steady state allocations for a given \( U \in [\bar{U}, \bar{\bar{U}}) \).

A rise in \( \xi \) captures an increase in the preference for housing services relative to \textit{all other} consumption goods when young, hence the relative price for housing services (the rental price) increases. This leads to an increase in the fundamental value of the house, \( Q^F \) increases. The fundamental price \( Q^F \), is defined as the discounted stream of rental prices. Figure (5) shows that the price of houses \( Q \) increases, while its bubble component \( Q^B \) decreases with the preference of housing services.

Figure (6) shows that the price-rent-ratio (PRR) decreases with the preference for housing services \( \xi \). In Figure (5) we have seen that the bubble component of the house price \( Q^B \) decreases with \( \xi \). In policy debates the PRR is often referred to as a good indicator for the detection of housing bubbles. Likewise in the model economy, the a larger PRR indicates larger bubbles.

An increase in \( \xi \) captures an increase in the preference for housing services relative to \textit{all other} consumption goods when young. Figure (7) shows that an increase in \( \xi \) induces a decrease in all other consumption when young, \( C_1 \) decreases. Consumption when old \( C_2 \) increases, the larger the preference for housing services \( \xi \).

This analysis of comparative statics implies that countries characterized with a lower aggregate preference for housing services (and hence a lower share of consumption spent on housing services) will experience larger housing bubbles (if any), all else equal. This model implication is investigated empirically in section 5.

In addition to the empirical evidence, I evaluate the theoretical model prediction by the means of a laboratory macro-experiment, where we can isolate and directly test the causal effect of the preference of housing services on the size of house price bubbles. We find strong support for the model’s prediction. In the treatment where we induce a low preference for housing services, we consistently observe significantly larger house price bubbles. These results are robust to a wide range of robustness checks. We rule out alternative explanations. The corresponding results can be found in the companion paper Huber et al. (2016).
Figure 5: Price Components - Stable Steady State: Effect of increase in $\xi$

Figure 6: Price Rent Ratio - Stable Steady State: Effect of increase in $\xi$
Policy Analysis

Tools aiming to reduce and contain systemic risks are known as macro prudential policies. Many macroprudential tools focus on housing markets. The borrower based regulatory instruments that are most commonly discussed include loan-to-value, debt-service-to-income and loan-debt-to-income ratios. This paper instead explores tax policies and analyzes their impact on economies’ vulnerability to house price bubbles.\(^{28}\) Many governments are concerned about the affordability of housing. This section discusses two different types of tax policy that both aim to foster the affordability of housing. I study the consequences of these different tax policies on housing bubble occurrence.

First, I study a subsidy fostering the demand for housing consumption (i.e. housing services). Second, I consider the implications of a subsidy to promote the demand for housing investment.

### 4.1 Rental Subsidies

A rental subsidy could be paid proportionally to the rental price or as a flat payment that is independent of the rental price.\(^{29}\) The impact on bubble occurrence is investigated for a proportional subsidy that is financed by a lump-sum payment of the young.

---

\(^{28}\) According to Hartmann (2015) tax policies fall into the category of macro prudential policies as well.

\(^{29}\) In the case of a flat payment that is independent of the rental price and financed by a lump-sum payment by the young, the existence condition for bubble occurrence is not affected.
In France, there is a housing subsidy in place that is proportional to the rental price.\textsuperscript{30} The proportion of assisted households in France is large.\textsuperscript{31} The subsidy incentivizes households to consume more housing services relative to other consumption goods. In France, the proportional subsidy depends on households’ characteristics and is given to the household by a transfer.\textsuperscript{32}

Abstracting from the details of the French case, implementing a proportional rental subsidy leads to the following changes in the budget constraint when young:

\[ C_{1,t} + \frac{Z_t}{P_t} + \sum_{k=0}^{\infty} q_{t|-k} H_{t|-k} + (1 - \tau_s) p_t^s S_t \leq W_t + u_t - T_t \]

The government finances the rental subsidy by income taxation (lump-sum of the young), hence \( T_t = \tau_w W_t \), leading to the following budget constraint of the government

\[ \tau_s p_t^s S_t = \tau_w W_t \]

Does the proportional rental subsidy make the economy more or less prone to housing bubbles? Deriving the corresponding existence condition for housing bubbles yields:

\[ W > q^f (\tau_s) + \left( \frac{1 - \tau_s + \xi}{(1 - \tau_s)(1 + (1 - \delta)\gamma)} \right) \]

where \( \frac{\partial q^f}{\partial \tau_s} > 0 \) and \( \frac{\partial q^f}{\partial \tau_s} > 0 \). The larger \( \tau_s \), the tighter the existence condition. Hence, the smaller the set of possibilities that a positive bubble exists (that the existence condition is satisfied).

Implementing a proportional rental subsidy makes the economy less prone to housing bubbles. The intuition behind this result is as follows. The subsidy on housing consumption will induce a substitution away from other consumption goods towards housing services. This leads to a higher relative price for housing services and therefore to a higher fundamental value of the housing asset in the economy. This has implications for the existence of bubbles and the bubble size. The larger the fundamental value, the less likely

\textsuperscript{30}In contrast to the United States, in France the rental subsidy is granted independent of the rent or type of house. In the United States, rental subsidy beneficiaries are not eligible to the subsidy if they pay a rent that is above a so-called fair market rent (this market rent can be very low).

\textsuperscript{31}51.6\% of the private sector tenants received a subsidy in 1996 (LaFerrere and Blanc (2004)).

\textsuperscript{32}A few notes: First, there exists a ceiling for the rent above which the subsidy does not vary, we ignore the ceiling in this theoretical study. Second, I abstract from the fact that the subsidy depends on the geographical region (France is divided into four regions). For details on the computation of individual proportional rental subsidies in France refer to LaFerrere and Blanc (2004).
that a house price bubble arrives, and the smaller the maximum bubble size.

4.2 Help-to-Buy Scheme

The previous section has shown that a subsidy fostering the demand for housing consumption (i.e. housing services) is an effective tool to reduce the prevalence of housing bubbles. Next, I consider the implications of a subsidy that promotes the demand for housing investment. Such investment subsidies are found predominantly in English speaking countries. The UK Government introduced a Help-to-Buy scheme. Help to Buy takes two forms: one part offers buyers the opportunity to take an interest-free loan from the government; the other sees the government acting as guarantor for some of a borrower’s debt. Alongside Help to Buy there is also the newly launched Help to Buy ISA. The ISA is only available to first-time buyers, who will receive a tax-free bonus from the government to help such first time buyers in buying a home. It is equivalent to a 25 per cent subsidy for first-time buyers on the savings to pay for the deposit. In the United states, there are a number of Government schemes to increase the affordability of house purchases for first time buyerst. In addition, favorable tax treatments are available for homeowners. To name a few, mortgage interest and property tax deductions.

Abstracting from the details in the real world, implementing a proportional housing investment subsidy (financed by lump-sum of the young) leads to the following changes in the budget constraint when young:

\[ C_{1,t} + \frac{Z_t}{P_t} + (1 - \tau_h) \sum_{k=0}^{\infty} q_{t|-k} H_{t|-k} + p_t^* S_t \leq (1 - \tau_w) W_t + u_t, \]

and the budget constraint of the government becomes 

\[ \tau_h \sum_{k=0}^{\infty} q_{t|-k} H_{t|-k} = \tau_w W_t. \]

Does the proportional help-to-buy subsidy make the economy more or less prone to housing bubbles? Deriving the corresponding existence condition for housing bubbles yields:

\[ W > q^f(\tau_h) + \left( \frac{1 + \xi}{1 + (1 - \delta) \gamma} \right) \]

(4.2)

where \( \frac{\partial q^f}{\partial \tau_h} < 0 \). Hence, the larger \( \tau_h \), the looser the existence condition. Hence, the larger the set of possibilities that a positive bubble exists (that the existence condition is

\[33\text{Refer to https://www.gov.uk/affordable-home-ownership-schemes/help-to-buy-isa} \]

\[34\text{Schwartz (2014) provides a detailed overview on the housing policy in the United States.} \]
satisfied). Implementing a help-to-buy subsidy makes the economy more prone to housing bubbles.

The mechanism behind the help-to-buy subsidy can be illustrated as follows. The subsidy for housing investment induces an inter-temporal substitution, away from consuming today towards investing for tomorrow. This leads to a higher real interest rate. The real interest rate has a direct impact on the housing price. The housing price consists of its fundamental component and a bubble component. The fundamental component decreases with the real interest rate, given that the fundamental value is defined by the net present value of the price for housing services. The bubble component of the housing price is growing with the interest rate. This is a common feature of OLG generation models with rational bubbles. Hence, implementing a help to buy scheme decreases the fundamental value of real estate in an economy and therefore creates more room for larger bubbles. Given the existence condition for bubbles, the economy also becomes more likely to experience a house price bubble in the first place.
5 Empirical Findings

This section is devoted to test the model’s main predictions empirically. Two main results have emerged from the analysis of the model. First, economies characterized by a larger share of consumption spent for housing services, are those economies that allow for smaller housing bubbles. Second, these economies are less vulnerable to housing bubbles in the first place.

This section provides an empirical characterization of housing cycles, bubbles and housing consumption using a large database covering 18 OECD countries during 1970-2014. Interestingly, I find large cross-country differences in both, housing consumption (i.e. housing services), and in the number of house price bubbles that have occurred during 1970-2014. The number of house price bubbles is interpreted as a measure for the vulnerability of an economy to housing bubbles.

Second, the interaction between housing consumption and house price bubbles is investigated. In line with the model’s predictions, two novel empirical regularities are identified across countries: housing consumption is highly negatively correlated with (1) the frequency and (2) the intensity and amplitude of house price bubbles. Thus, countries with a lower share of consumption spent on housing services experienced not only more, but also larger and more volatile house price bubbles during 1970-2014.

The remainder of this section is organised as follows: In section 5.1., I provide a very detailed explanation on how I measured housing cycles and bubbles for 18 OECD countries during 1970 to 2014. Housing bubbles are measured empirically with two indicators, independent housing booms, and boom-bust cycles. Section 5.2. provides the corresponding descriptive statistics. And section 5.3. presents the empirical cross-country regularities.

Empirically, I proxy the consumption demand for housing by two indicators. First by national CPI weights on housing services, this is a good measure for the relative importance of housing services in the total consumption basket. And second by household spending on housing services (% of disposable income). In my sample of 18 OECD countries, the national CPI weights on housing services varies from around 10.3% in Portugal to around 28% in Denmark. Both indicators include actual and imputed rents. Household spending on housing (% of disposable income) varies from 14% in Portugal to 30% in Denmark. Housing bubbles I measure empirically by independent house price booms as well as boom-bust cycles.

A description of the data used for this part of the analysis can be found in Appendix B.
5.1 Methodology

This section provides a detailed explanation of how housing cycles, house price booms and busts are identified in this paper. Defining and thus measuring a housing bubble proves to be more challenging. Section 5.1.2 discusses my choice of independent housing booms and boom-bust cycles as a potential indicator for housing bubbles.

5.1.1 Methodology: Identifying House Price Cycles

Housing cycles are identified with a method that falls into the category of classical approaches. Cycles are identified in the level of the reference variable. An alternative concept of measuring housing cycles is that of growth cycles, fluctuations in economic activity around a long-run trend. For this study, the classical approach is more suitable in order to achieve the desired objectives - as it offers the following advantages: (1) turning points are robust to the inclusion of newly available data, in contrast to methods that require detrending (where the inclusion of new data can affect the estimated trend and hence the identification of a cycle); and most importantly (2) detrending involves an arbitrary distinction between trend and cycle, where there is no clear consensus on the best method for this distinction, and (3) turning point analysis does not require a pre-specified frequency range at which the house price cycle is assumed to operate. Since this paper redominately aims to uncover new empirical regularities between house price fluctuations and housing services, I want to avoid restrictive parametric assumptions and chose to look at cycles in the level of real house prices.

Harding and Pagan (2002)'s BBQ algorithm is used to detect turning points in quarterly house price data. This algorithm belongs to the strand of pattern-recognition methods pioneered by Burns and Mitchell (1946) in their work on business cycles for the National Bureau of Economic Research (NBER), and later formalized by Bry and Boschan (1971). The dating procedure consists in finding a series of local maxima and minima that allow a segmentation of the series into expansions and contractions. In order

---

37Cycles are identified by changes in the level of economic activity and hence describe absolute increases and declines.

38See King et al. (1991) among others. The identification of cycles does clearly depend on the detrending method (parametric assumptions) chosen. As a result, key growth cycle characteristics depend on the detrending method employed, see Canova (1998).

39Growth cycles require this pre-specified frequency range and are therefore not suited for the analysis and comparison of empirical regularities across countries, as research has shown that characteristics of financial cycles (e.g. duration) are indeed very different across countries, see Hiebert et al. (2015).

40Following Bracke (2013) “The algorithm is denominated BBQ because it is a quarterly (Q) application of the Bry and Boschan (1971) algorithm designed to find business cycles in monthly data.”

As well illustrated in Bracke (2013), the algorithm requires the implementation of the following three steps on a quarterly series:

1. Identification rule. Identification of points which are higher or lower than a window of surrounding observations. Using a window of \( j \) quarters on each side, a local maximum \( q_{t}^{\text{max}} \) is defined as an observation of the house price series such that \((q_{t-j},...,q_{t-1}) < q_{t}^{\text{max}} > (q_{t+1},...,q_{t+j})\). Symmetrically, a local minimum \( q_{t}^{\text{min}} \) satisfies \((q_{t-j},...,q_{t-1}) > q_{t}^{\text{min}} < (q_{t+1},...,q_{t+j})\).

2. Alternation rule. A local maximum must be followed by a local minimum, and vice versa. In the case of two consecutive maxima (minima), the highest (lowest) \( q_{t} \) is chosen.

3. Censoring rule. The distance between two turning points has to be at least \( x \) quarters, where \( x \) is chosen by the analyst in order to retrieve only the significant series movements and avoid some of the series noise.\(^{41}\)

As housing cycles are longer than GDP cycles, the threshold parameter for the identification and censoring rule should be set higher to avoid spurious cycles. For housing cycles, Borio and McGuire (2004) suggest a rolling window of 13 quarters to be appropriate, which implies \( j = 6 \). For the censoring rule I follow Giroud et al. (2006). The distance between two turning points has to be at least 6 quarters, i.e. \( x = 6 \).\(^{42}\) The decisions over the length of the rolling window \( (j) \) and the minimum phase duration \( (q) \) correspond to the choices made by Bracke (2013).

5.1.2 Methodology: Identifying Housing Bubbles

There is no clean or generally accepted definition for the term asset price bubble in the literature. Researchers often focus on a single specific aspect of a generally vague concept: rapid and large price increases\(^{43}\), unrealistic expectations of future price increases\(^{44}\), the

\(^{41}\) Harding and Pagan (2002) choose \( x = 2 \) for U.S. GDP.
\(^{42}\) It follows that a housing cycle has a minimum duration of 3 years.
\(^{43}\) Baker (2002).
\(^{44}\) Case and Shiller (2003).
departure of prices from fundamentals\textsuperscript{45}, or large drops in prices after the bubble pops\textsuperscript{46}.

The empirical literature measuring housing bubbles can be decomposed into two main strands. Firstly, the \textit{fundamental analysis}, tries to explicitly measure the departure of the housing price from fundamental values that are inferred from the residual of an error-correction framework with real house prices regressed on fundamental variables.\textsuperscript{47} The selection of variables that are seen as fundamental to housing prices is subjective and varies significantly across studies.\textsuperscript{48} The selection of fundamental variables is crucial when measuring housing bubbles with this approach. This is very problematic and I will therefore not follow this route.\textsuperscript{49}

The second strand of literature that identifies housing bubbles empirically uses the \textit{technical analysis}, and this has a strong descriptive character. This method is intuitive and has the big advantage that fundamental factors do not need to be chosen. Researchers simply need to have data on the evolution of housing prices to identify housing bubbles. According to this method, a necessary feature of a housing bubble is a "dramatic price increase", the literature calls this phenomenon an asset price boom.\textsuperscript{50} An obvious criticism follows from the fact that a rapid price increase could also result from a pure change in fundamentals.\textsuperscript{51} Given this criticism, researchers extended the concept to boom-bust cycles, i.e. a rapid price increase has to be directly followed by a dramatic bust.\textsuperscript{52} However, for the identification of a housing bubble, many researchers do not require booms to be followed by busts. Allowing booms to be disconnected from busts is appropriate from a theoretical perspective as well, as bubbles do not need to burst. Despite the debates concerning the measurement of housing bubbles, there is a widespread consensus

\textsuperscript{45}Garber (2000) and Lansing (2006).
\textsuperscript{46}Siegel (2003), p.3.
\textsuperscript{47}Theoretically, researchers would need to quantify the unobserved expected future values of fundamentals on which the fundamental asset price depends.
\textsuperscript{48}Examples for fundamental variables included in empirical studies are (1) short run factors like current real GDP per capita, construction costs, the real interest rate, investment demand, (2) long run factors like population and economic growth and (3) institutional factors as supply of land, taxes, financial deregulation (...).
\textsuperscript{49}The selection of fundamental factors will determine the unexplained residual of the regression and hence the bubble size.
\textsuperscript{50}Detken and Smets (2004), pp.9. However, it should be noted that from a theoretical perspective, bubbles do not have to involve past rise in prices.
\textsuperscript{51}Case and Shiller (2003): "The mere fact of rapid price increases is not itself conclusive evidence of a bubble." Helbling (2005): "However, large price increases - which will be referred to as booms - are only a sufficient but not a necessary condition for bubbles."
\textsuperscript{52}Following Garber (2000) the general criticism also applies to the boom-bust cycle, it is still just "an empirical statement about the pattern of prices." This aspect is also highlighted by Haines and Rosen (2007): "Thus, what appears to be a bubble in some markets might just be a reflection of normally high volatility in those markets".
that many boom-bust cycles in housing prices were accompanied by financial instabilities and recessions. Moreover it is widely accepted that recessions associated with house price busts are not only longer but at least twice as deep as normal recessions or recessions that are associated with other types of asset price busts.\textsuperscript{53} This study will use both concepts to identify housing bubbles; independent booms, and boom-bust cycles.

In summary, the technical analysis can only provide indications for housing bubbles. Nevertheless, advantages of this method include that (1) it is clearly defined and economically intuitive concept, (2) it has a low requirement for information, and (3) it allows exact dating of housing bubbles. I conclude that its advantages outweigh its disadvantages - and I therefore choose to proceed using this method.

**House Price Booms and Busts: An Indicator for Housing Bubbles**

The identification of housing booms and busts requires two steps. The first step, the determination of housing price cycles, was described in detail in section 5.1.\textsuperscript{54} The second step, the identification of housing price booms and busts, involves the choice of a cut-off value for a house price increase (decrease) to be considered as large enough to denote a boom (bust). Such a threshold for the identification of booms and busts is clearly rather arbitrary and varies across studies.\textsuperscript{55} This analysis will therefore consider four different cut-off values, leading to four different bubble identification methods. A housing price boom (bust) is defined as an upturn (downturn) that is accompanied by at least a 10\%, 15\%, 20\% or 80\% price increase (decrease). The stylized facts presented in this study remain robust across these threshold options.\textsuperscript{56}

Recall that independent booms are the first measure for housing bubbles. The second approach (boom-bust cycles) considers only those booms that are followed by busts. The empirical regularities are robust to both types of housing bubble measurements.

\textsuperscript{53}Refer to e.g. Claessens et al. (2009), and Claessens et al. (2011), IMF (2003).

\textsuperscript{54}The described dating procedure was employed among others by Bordo and Landon-Lane (2014), Bracke (2013), Igan and Loungani (2012), Claessens et al. (2011), Andre (2010), Girouard et al. (2006) and Borio and McGuire (2004).

\textsuperscript{55}E.g. Girouard et al. (2006) identifies booms and busts episodes when a real price change exceeds 15\%. Claessens et al. (2011), Helbling (2005), IMF (2003) chose the quartile as cutoff value. Bordo and Landon-Lane (2014) define an upturn as a boom if the price increase is at least 10\% within 2 years. IMF (2009) chooses a methodology similar to Bordo and Jeanne (2002) where turning points are not determined. Busts (booms) are defined as periods when the four-quarter trailing moving average of the annual growth rate of the housing price, in real terms, falls below (above) 5\%, equivalent to an accumulated (decrease) increase of 20\%.

\textsuperscript{56}This two-step procedure does not require booms to be followed by busts, as these two events are determined independently. Bordo and Jeanne (2002) and Helbling (2005) among others also use a procedure whereby booms and busts are determined independently.
5.2 Descriptive Statistics: Housing Cycles, Booms and Busts, and Housing Services

This section provides descriptive statistics of the cross-country differences in the consumption of housing service. Second, this section provides a descriptive analysis of housing cycles, booms and busts, that have occurred between 1970:1-2013:4 for 18 OECD countries in the sample. For the descriptive part of the housing cycles and bubble indicators, I focus on four main characteristics: (1) the frequency, (2) the amplitude, (3) the duration, and (4) the intensity. However, in the analysis that will follow, special emphasis is placed on the frequency and the amplitude. Frequency is measured by the number of completed up- and downturns (booms, busts) in the sample. Amplitude is measured by the change in real house prices from peak (trough) to trough (peak), expressed in %. Duration is measured in quarters. Intensity is a good proxy for the violence of an episode and is given by the amplitude divided by duration.

A comprehensive summary of all real house price peaks and troughs for all OECD countries is given in Table (B3) in the appendix. For each housing up- and downturn in the sample, the four characteristics (frequency, amplitude, duration, and intensity) are listed separately. Figure (B2) plots the house price indices and shows the peaks and troughs for all twenty countries. The grey shaded areas show downturns (peak to trough) and the white areas symbolize upturns (trough to peak).

The structure of the dataset is such that it has an on-going upturn or downturn at the time of the last observation (2013q4). I will compute the statistics and analysis without those right-censored phases. Left-censored phases are excluded as well.57

House Price Cycles:

Table (1) gives an overview of the frequency, duration, amplitude and intensity of all up- and downturns in the sample. The dataset contains 55 completed downturns and 50 completed upturns.58 I find that housing cycles are on average 11.7 years long with notable dispersions across countries.59 Table (1) shows that on average, upturns last longer and display more duration variability (measured by the standard deviation) than downturns.

---

57 Phases for which the starting date precedes 1970q1 and is unknown.
58 The dataset contains additionally 7 right-censored downturns and 10 right-censored upturns. Including these, does not alter the statistics much. The average upturns are slightly shorter and larger. While downturns become slightly shorter and are of smaller magnitude (on average).
59 The average housing cycle length is in line with e.g. Schueler et al. (2015), Bracke (2013) and Drehmann et al. (2012). Bracke (2013) finds that the average house price cycle lasts 10.6 years, while Drehmann et al. (2012) finds an average duration of 10.5 years. Schueler et al. (2015) find an average financial cycle length of 12 years, using 13 European countries, a time period spanning over 1970-2013, and a novel spectral approach to identify financial cycles.
The amplitude of upturns is larger on average and displays a much larger variability than the amplitude of downturns. These findings are in line with e.g. Claessens et al. (2011), Drehmann et al. (2012), Igan and Loungani (2012) and Bracke (2013). This related literature does not consider the intensity measure. The intensity and its variability (measured by the standard deviation) of housing cycles is much larger for upturns than for downturns. In summary and on average, upturns are larger, longer, more violent and more volatile than downturns.

**House Price Booms and Busts**

Table (2) provides descriptive statistics for independent house price booms and boom-bust cycles. House price booms that are followed by a bust are on average shorter than upturns, while independent house price booms tend to be longer than upturns. Since independent house price booms last longer than booms that are followed by busts, it is not surprising that the amplitude of independent booms is larger than the amplitude of booms that are followed by busts.

Interestingly, the intensity of booms that are followed by busts (measured by amplitude divided by duration) is larger than the intensity of independent booms. This reinforces the point, that the intensity of house price increases might inherit more valuable information on future house price busts compared to the amplitude of house price increases. This intensity measure of house price booms is not yet widely used in the literature. However, policy makers might want to keep track of the intensity measure as a warning signal.

Table (3) gives an detailed overview on how many completed independent booms and boom-bust cycles each country experienced during 1970:1 to 2013:4. While table (B2) in the appendix provides the average amplitude of all individual house price booms and boom-bust cycles for each country in the sample.

The cross-country variation in the number of house price booms (as well as the cross-country variation of the average amplitude) is substantial.

---

60 How many booms end in a bust? I find that 80.9% (74.4%, 69.2%, 25%) of all booms that involve at least a 10% (15%, 20%, 80%) price increase are followed by a bust.

61 The corresponding table listing for each country the number of completed and ongoing housing booms and boom-bust cycles is shown in the Appendix, table (B1).
<table>
<thead>
<tr>
<th>Completed upturns</th>
<th>Frequency</th>
<th>Duration</th>
<th>Amplitude</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>Mean</td>
<td>StDev</td>
<td>Mean</td>
</tr>
<tr>
<td>Completed downturns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>27.89</td>
<td>20.14</td>
<td>72.54</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>19.08</td>
<td>6.62</td>
<td>-22.96</td>
</tr>
</tbody>
</table>

Frequency is measured by the number of quarters from peak (trough) to trough (peak). Amplitude measured by real house prices % change from peak (trough) to trough (peak). Duration is measured in quarters from peak (trough) to trough (peak). Intensity is obtained by $I_i = \frac{\text{Amplitude}}{\text{Duration}}$.

Table 1: Descriptive Statistics of Housing Cycles for 18 OECD Countries

| Upturns | Completed upturns
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>50</td>
</tr>
<tr>
<td>A</td>
<td>72.5</td>
</tr>
<tr>
<td>D</td>
<td>27.9</td>
</tr>
<tr>
<td>I</td>
<td>2.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Booms</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 10%</td>
</tr>
<tr>
<td>&gt; 15%</td>
</tr>
<tr>
<td>&gt; 20%</td>
</tr>
<tr>
<td>&gt; 80%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Boom-Buster</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 10%</td>
</tr>
<tr>
<td>&gt; 15%</td>
</tr>
<tr>
<td>&gt; 20%</td>
</tr>
</tbody>
</table>

F stands for the number of upturns, independent booms and boom-busters. A for the amplitude, D for duration and I for intensity. Frequency is measured by the number of quarters from peak (trough) to trough (peak). Amplitude measured by real house prices % change from peak (trough) to trough (peak). Duration is measured in quarters from peak (trough) to trough (peak). Intensity is obtained by $I_i = \frac{\text{Amplitude}}{\text{Duration}}$.

Table 2: Descriptive Statistics of Housing Booms for 18 OECD Countries

<table>
<thead>
<tr>
<th>Independent Booms</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 10%</td>
</tr>
<tr>
<td>&gt; 15%</td>
</tr>
<tr>
<td>&gt; 20%</td>
</tr>
<tr>
<td>&gt; 80%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Boom-Bust Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 10%</td>
</tr>
<tr>
<td>&gt; 15%</td>
</tr>
<tr>
<td>&gt; 20%</td>
</tr>
<tr>
<td>&gt; 80%</td>
</tr>
</tbody>
</table>

Table 3: Number of completed Housing Booms and Busts for each OECD Country
The Preference for Housing Services

In the model, the preference for housing services implies the share of consumption on housing services as an equilibrium outcome. Empirically, housing services constitute a large fraction of total consumption. I use two indicators to measure housing services empirically. The first indicator is the national consumer price index (CPI) weight on housing services. This indicator is a good measure for the relative importance of housing services in the total consumption basket. The CPI weight on housing services varies from 11% in Spain to 29% in Denmark. As a second indicator I use spending on housing services as % of disposable income. This measure varies from 15% in Spain to 30% in Denmark. The variation of both indicators across countries is shown in Figure (8). The cross-country differences in expenditure shares for housing services are large, and persistent over time.

Importantly, both indicators include imputed rents. What are imputed rents? Homeowners do not pay for the consumption of housing services the owned dwelling provides. The imputed rents of these housing services are valued at the estimated rent that a tenant pays for a dwelling of the same size and quality in a comparable location with similar neighborhood amenities. In the model we do not distinguish between homeowners and renters, therefore the expenditure on housing services includes imputed and actual rents. Hence, it is important that the empirical indicators for housing services include imputed and actual rents.

(a) CPI weight on housing services
(b) Fraction of disposable income spent on HS

Figure 8: Indicators for the Preference for Housing Services (HS) across countries

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62 The indicator includes (Housing, Electricity, Gas and other Fuels). Annual frequency over time period 1992 to 2013 for 17 countries. Measured as per thousand of National CPI total. Source: OECD.stat.
63 Point estimates for the years 1995 and 2005 for 18 OECD countries available. Source OECD Outlook No. 86 and OECD National Accounts.
64 Refer to appendix B, figure (B1). Appendix B also provides detailed information on the data sources.
65 When markets for rented accommodation are virtually non-existent or unrepresentative, the value of imputed rents is derived by some other objective procedure such as the user-cost method. Refer to the OECD glossary of statistical terms, imputed rents.
5.3 Empirical Cross-Country Regularities

This paper highlights two novel empirical regularities identified across countries:

First, the consumption of housing services is highly and negatively correlated with the number of independent house price booms and the number of boom-bust cycles. For instance, the number of completed independent house price booms (boom-bust cycles) that are associated with at least a 80% price increase, displays a cross-country correlation with the share of consumption on housing services of -0.72 (-0.30).66

Second, I find that consumption of housing services is highly and negatively correlated with the amplitude and especially with the intensity of independent house price booms and boom-bust cycles across countries.67

Thus, countries with a lower share of consumption spent on housing services experienced not only more frequent, but also larger and more violent independent house price booms as well as boom-bust cycles. And therefore potentially more frequent, larger and more violent house price bubbles during 1970 to 2014.

5.3.1 The Frequency of Housing Bubbles

In this section, I analyze the cross-country relationship between housing services and the frequency of house price bubble occurrence during 1970-2014. I find that the share of consumption spent on housing services is highly and negatively correlated with the number of independent house price booms and the number of boom-bust cycles. Table (4) presents these cross-country correlations.

OLS regressions show that housing services inherit a high explanatory power for the frequency of independent house price booms, see regression table (5). How important is the impact of housing services quantitatively?

An increase in the level of the CPI weight on housing services by one standard deviation (across countries) is associated with a decrease in the average number of independent booms (associated with a price increase of at least 20%) by 0.66 which accounts for 52% of the variation of the number of such independent house price booms across countries.68

This is remarkable.

66 Please refer to table (4) in section 5.3.1.
67 Please refer to table (7) in section 5.3.2.
68 An increase in the level of the CPI weight on housing services by one standard deviation (across countries) is associated with a decrease in the average number of booms (associated with a price increase of at least 80%) by 0.50 which accounts for 72% of the variation of the number of booms per country. The regression results for independent housing booms defined by a different threshold are very similar. The results are henceforth robust to various housing bubble identification rules.
Adding more control variables to the regression does not alter the results - countries with a lower share of consumption spent on housing services experienced more frequent independent house price booms and boom-bust cycles. Table (6) illustrates the regression results including measures for cross-country differences in the taxation of properties, transaction costs, percentage of the population living in urban areas and the typical loan-to-value (LTV) ratio. For very large independent house price booms (associated with a price increase of at least 80%), the controls transaction costs, percentage of the population living in urban areas and the LTV ratio are statistically significant. As expected, countries with lower transaction costs, higher urban density, and easier access to credit (higher LTV ratios), experienced more frequent independent house price booms during 1970-2014. However and in comparison, the explanatory variable housing services is not only the most significant regressor, the quantitative impact is also largest.\(^{69}\)

In summary and in line with the theoretical model’s prediction, cross-country differences in housing services pick up a large part of the cross-country variation of house price bubble occurrence over the time period 1970-2014.

<table>
<thead>
<tr>
<th>Number of Booms</th>
<th>CPI weight on housing services larger than</th>
<th>Number of Boom-Busters</th>
<th>CPI weight on housing services larger than</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt; 80%</td>
<td>&gt; 20%</td>
<td>&gt; 15%</td>
</tr>
<tr>
<td>CPI weight</td>
<td>-0.72</td>
<td>-0.52</td>
<td>-0.52</td>
</tr>
<tr>
<td>Constant</td>
<td>2.111****</td>
<td>4.006****</td>
<td>4.006****</td>
</tr>
</tbody>
</table>

Table 4: Correlation of housing service CPI weight with number of Booms (Boom-Busters)

<table>
<thead>
<tr>
<th>Dependent variable: Number of independent Booms</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 80%</td>
</tr>
<tr>
<td>CPI weight (on housing services)</td>
</tr>
<tr>
<td>(-4.30)</td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td>(5.98)</td>
</tr>
</tbody>
</table>

| N     | 17    | 17    | 17    | 17    |
| R\(^2\) | 0.515 | 0.272 | 0.272 | 0.070 |
| adj. R\(^2\) | 0.483 | 0.223 | 0.223 | 0.008 |

\(t\) statistics in parentheses. * \(p < 0.1\), ** \(p < 0.05\), *** \(p < 0.01\), **** \(p < 0.001\). Booms denoted by \(> x\%\) involve real house price changes (trough to peak) larger than \(x\%.\)

Table 5: OLS: Number of independent Housing Booms

\(^{69}\) An alternative set of control variables is used to account for cross-country differences in mortgage markets and institutions, unemployment and income. Table (7) provides the regression results.
<table>
<thead>
<tr>
<th>Booms</th>
<th>Boom-Busts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt; 80%</td>
</tr>
<tr>
<td>CPI weight</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>(-0.0898****)</td>
<td>(-0.131***)</td>
</tr>
<tr>
<td>(-6.50)</td>
<td>(-3.59)</td>
</tr>
<tr>
<td>typical LTV</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.97)</td>
</tr>
<tr>
<td>0.0183*</td>
<td>0.0383</td>
</tr>
<tr>
<td>(2.12)</td>
<td>(-1.33)</td>
</tr>
<tr>
<td>Urban population</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.12)</td>
</tr>
<tr>
<td>Transaction cost</td>
<td>-0.0747**</td>
</tr>
<tr>
<td></td>
<td>(-2.21)</td>
</tr>
<tr>
<td>Property tax</td>
<td>0.0331</td>
</tr>
<tr>
<td></td>
<td>(1.16)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.286*</td>
</tr>
<tr>
<td></td>
<td>(1.98)</td>
</tr>
</tbody>
</table>

| N   | 17  | 17  | 17  | 17  | 17  | 17  |
| R²  | 0.779 | 0.432 | 0.432 | 0.259 | 0.368 | 0.310 |
| adj. R² | 0.678 | 0.174 | 0.174 | -0.077 | 0.080 | -0.004 |

* t statistics in parentheses. ** p < 0.1, *** p < 0.05, **** p < 0.01, ***** p < 0.001. Booms denoted by > x% are those booms that involve real house price changes (trough to peak) larger than x%. The variable of interest is the national consumer price index (CPI) weight on housing services. This indicator is a good measure for the relative importance of housing services in the total consumption basket. Urban Pop: % of national population living in urban regions, 2012. Typical LTV for 1992 and 2002, taken from Calza et al. (2013), Catte et al. (2004). Transaction costs measured as a percentage of property value, 2009. It includes notary fees, typical real estate agent fees, legal fees, registration fees, and transfer taxes. Property tax, (%) of GDP. Defined as recurrent and non-recurrent taxes on the use, ownership or transfer of property. These include taxes on immovable property or net wealth, taxes on the change of ownership of property through inheritance or gift and taxes on financial and capital transactions. This indicator relates to government as a whole (all government levels) and is measured in percentage of total taxation. Macro variables: averages 1970-2013, if not noted otherwise.

Table 6: OLS: Frequency of independent Booms and Boom-Bust Cycles
5.3.2 The Amplitude, Intensity and Duration of House Price Bubbles

In this section, I analyze the relationship between housing services and house price bubble characteristics, such as the amplitude, intensity and duration of house price booms and busts. Table (7) illustrates the cross-country correlations between these bubble characteristics and housing services (measured by the CPI weight on housing services).

I find that the cross-country correlations are negative for the amplitude and especially large (and negative) for the intensity of house price bubbles. Countries with a lower CPI share on housing services experienced larger independent housing booms as well as boom-bust cycles. Further, these independent booms and boom-bust cycles have been shorter, the amplitude of the price increase (decrease) has been reached faster in these countries. It follows that these countries lived larger and more violent independent house price booms and boom-bust cycles.\footnote{For policy makers, the intensity (violence) of housing bubbles might be more important than the amplitude, as the impact of a housing boom (bust) on the macroeconomy will depend on the speed of the price increase (decrease). A smooth build up (drop) in house prices can be managed by macro-prudential policy makers - as this will provide time to put policies in place to mitigate risks to the real economy. More violent booms (busts) are more dangerous - as policymakers have less time to effectively mitigate risks, and it is therefore more likely that risks spillover from the housing market into the financial sector and on into the rest of the economy.}

For the regression analysis, I include measures for cross-country differences in the taxation of properties, transaction costs, percentage of the population living in urban areas and the typical loan-to-value (LTV) ratio. To account for cross-country differences in income, I include GDP per capita. The fraction of the population in the working age is also included as an additional measure of housing demand. The OLS regression analysis shows that our proxy for the preference for housing services (CPI weight on housing services) has a large and significant explanatory power for both the intensity and amplitude of moderate independent housing booms and boom-bust cycles that involve a price increase of at least 10%. In line with the empirical literature, the larger the LTV ratio, the larger the independent booms. Interestingly, the LTV ratio has no explanatory power for the intensity of booms nor for boom-bust cycles. Table (8) shows the regression output.

How large is the impact for the amplitude? An increase in the CPI by one standard deviation (across countries) is associated with a decrease in the amplitude of independent booms (> 10%) by 27.98, which accounts for 58 percent of the cross-country variation in the amplitude of such booms. This impact is remarkable.

How large is the impact for the intensity? An increase in the CPI by one standard deviation (across countries) is associated with a decrease in the intensity of independent
booms (> 10%) by 0.94, which accounts for 99 percent of the cross-country variation in the intensity of such booms.\footnote{An increase in the CPI by one standard deviation (across countries) is associated with a decrease in the intensity of boom-bust cycles (> 10%) by 1.08, which accounts for 95 percent of the cross-country variation in the intensity of such boom-bust cycles.} The impact is very large.

In summary, cross-country differences in housing services pick up a large part of the cross-country variation of the amplitude and intensity of house price bubbles that have occurred during the time period 1970-2013. This is in line with the theoretical model’s prediction.

<table>
<thead>
<tr>
<th>Booms</th>
<th>Amplitude</th>
<th>Intensity</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom (&gt; 10%)</td>
<td>-0.34</td>
<td>-0.51</td>
<td>0.02</td>
</tr>
<tr>
<td>Boom (&gt; 15%)</td>
<td>-0.20</td>
<td>-0.28</td>
<td>0.10</td>
</tr>
<tr>
<td>Boom (&gt; 20%)</td>
<td>-0.20</td>
<td>-0.28</td>
<td>0.10</td>
</tr>
<tr>
<td>Boom (&gt; 80%)</td>
<td>0.06</td>
<td>-0.53</td>
<td>0.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Boom Phase of Boom-Buster</th>
<th>Amplitude</th>
<th>Intensity</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom-Bust (&gt; 10%)</td>
<td>-0.44</td>
<td>-0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>Boom-Bust (&gt; 15%)</td>
<td>-0.11</td>
<td>-0.28</td>
<td>0.10</td>
</tr>
<tr>
<td>Boom-Bust (&gt; 20%)</td>
<td>-0.11</td>
<td>-0.25</td>
<td>0.16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bust Phase of Boom-Buster</th>
<th>Amplitude</th>
<th>Intensity</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom-Bust (&gt; 10%)</td>
<td>0.03</td>
<td>-0.43</td>
<td>0.28</td>
</tr>
<tr>
<td>Boom-Bust (&gt; 15%)</td>
<td>-0.14</td>
<td>-0.38</td>
<td>0.30</td>
</tr>
<tr>
<td>Boom-Bust (&gt; 20%)</td>
<td>0.03</td>
<td>-0.41</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Booms denoted by > x% are those booms that involve real house price changes (trough to peak) larger than x%. To qualify for a boom-buster of > x%, the amplitude of the boom (trough to peak) has to be larger than x%, the threshold for qualifying as a subsequent bust is chosen such that the bust falls into the same percentile than the booms that have larger price increases than > x%. Amplitude is measured by the change in real house prices from peak (trough) to trough (peak), expressed in %. Duration is measured in quarters. Intensity is is a good proxy for the violence of an episode and is given by the amplitude divided by duration. Intensity is measured from trough (peak) to peak trough (through) by $I_t = \frac{\text{amplitude}}{\text{duration}}$. 

Table 7: Correlations: Amplitude, Intensity, Duration of Bubbles with CPI weight
<table>
<thead>
<tr>
<th></th>
<th>Amplitude</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>indep. Boom</td>
<td>Boom-Buster</td>
</tr>
<tr>
<td>CPI weight (of housing services)</td>
<td>-3.613**</td>
<td>-0.941</td>
</tr>
<tr>
<td></td>
<td>(-2.47)</td>
<td>(-0.78)</td>
</tr>
<tr>
<td>Urban population</td>
<td>1.088**</td>
<td>-0.381</td>
</tr>
<tr>
<td></td>
<td>(2.31)</td>
<td>(-0.88)</td>
</tr>
<tr>
<td>Working population</td>
<td>13.73</td>
<td>1.081</td>
</tr>
<tr>
<td></td>
<td>(0.68)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>GDP (head, PPP)</td>
<td>-0.00364</td>
<td>0.000862</td>
</tr>
<tr>
<td></td>
<td>(-1.66)</td>
<td>(0.43)</td>
</tr>
<tr>
<td>Property tax</td>
<td>1.498</td>
<td>1.206</td>
</tr>
<tr>
<td></td>
<td>(0.46)</td>
<td>(0.34)</td>
</tr>
<tr>
<td>Transaction cost</td>
<td>-1.015</td>
<td>2.065</td>
</tr>
<tr>
<td></td>
<td>(-0.31)</td>
<td>(0.71)</td>
</tr>
<tr>
<td>Typical LTV</td>
<td>2.183**</td>
<td>-0.119</td>
</tr>
<tr>
<td></td>
<td>(2.46)</td>
<td>(-0.11)</td>
</tr>
<tr>
<td>Constant</td>
<td>-22.00</td>
<td>49.66</td>
</tr>
<tr>
<td></td>
<td>(-0.24)</td>
<td>(0.59)</td>
</tr>
<tr>
<td>N</td>
<td>17</td>
<td>15</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.733</td>
<td>0.309</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.526</td>
<td>-0.383</td>
</tr>
</tbody>
</table>

$t$ statistics in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. GDP measure per head, constant prices, constant PPPs, OECD base year. Urban Pop: % of national population living in urban regions, 2012. Working population measured by the ratio working age (20-64) per pension age (+65). Typical LTV for 1992 and 2002, taken from Calza et al. (2013), Catte et al. (2004). Transaction costs measured as a percentage of property value, 2009. It includes notary fees, typical real estate agent fees, legal fees, registration fees, and transfer taxes. Property tax, (%) of GDP. Defined as recurrent and non-recurrent taxes on the use, ownership or transfer of property. These include taxes on immovable property or net wealth, taxes on the change of ownership of property through inheritance or gift and taxes on financial and capital transactions. This indicator relates to government as a whole (all government levels) and is measured in percentage both of GDP and of total taxation. Macro variables: averages 1970-2013, if not noted otherwise.

Table 8: OLS: Amplitude and Intensity of Housing Booms including macro variables.
Housing is very different to other assets given its duality - the consumption and investment demand. Housing consumption constitutes a large fraction of total consumption and is measured by the consumption of housing services. However, the role housing consumption plays in generating housing bubbles was until now underexplored in the literature.

The present paper should be viewed as part of an effort to enhance our understanding of the relation between housing consumption and housing bubbles. In particular, I tested the hypothesis that housing consumption drives economies' vulnerability to housing bubbles from three angles: a theoretical overlapping generation model, empirical data analysis, and a laboratory experiment.

The first part of this paper explored the implications of housing consumption on house price bubbles through the lens of an OLG model. I disentangle the dual motives of housing demand by modeling the consumption aspect (consuming housing services) and investment aspect (investing in housing) independently. In the model I assume cross-country differences in the preference for housing services relative to all other consumption goods. Assuming cross-country differences in the preference for housing is soundly justified by empirical evidence, as provided in the companion paper Huber and Schmidt (2016).

Two main results emerged from the analysis of the model. First, if the demand for housing consumption is low, countries are more prone to housing bubbles. Second, these countries where housing consumption is low face larger and more volatile housing bubbles.

The second part of this paper evaluated the extent to which the model's predictions can be reconciled with empirical evidence. In line with the model's predictions, novel empirical regularities are identified across countries: countries with a lower share of consumption spent on housing services experienced more frequent housing bubbles during 1970-2014. These bubbles have been larger and more volatile.

The companion paper Huber et al. (2016) complements the empirical analysis with the evaluation of the OLG model's predictions using a laboratory macro experiment. In contrast to the empirical analysis, this technique allows to isolate and directly test the causal effect of the preference for housing services on house price bubbles. The empirical work proxies these preferences by the expenditure share of housing consumption - an equilibrium outcome of the OLG model. Further, the experimental setup allows to quantify housing

\footnote{Huber and Schmidt (2016) study the impact of culture on living arrangements, using data on the tenure choice decision of second generation immigrants in the United States - holding constant institutional factors. We find that cultural housing preferences transmitted by parents play an important role for the housing tenure choice of these second generation immigrants.}
bubbles without measurement error. The results of the macro experiment provide strong support for the model’s predictions. In the treatment where we induce a low preference for housing services, we consistently observe significantly larger house price bubbles.

Finally, the model was used to study the impact of two prominent alternative policies aimed at fostering the affordability of housing - rental subsidies and help-to-buy schemes. I found that a proportional rental subsidy is an effective tool to control housing bubbles, while the help-to-buy scheme makes the economy more prone to housing bubbles. The results arising from the model should not be used unadulterated as the basis for designing housing policy in the real world, as certain elements of the policy making balancing act are not directly considered in the analysis. In particular, the model above abstracts from distributional consequences, hence inequality concerns. Nonetheless, the undertaken policy analysis may provide some relevant insight on the relative merits of alternative policies for enhancing house- affordability from the perspective of their different consequences for the vulnerability of the economy to housing bubbles.

More generally, the paper should be viewed as an effort to enhance our understanding of the relationship between housing consumption and rational house price bubbles. This topic has been underexplored to this point. For policy makers, this paper provides a new perspective on how to reduce the economy’s vulnerability to house price bubbles. Vulnerability to house price bubbles can potentially be reduced by incentivising households to increase their housing consumption (housing services) relative to other consumption goods. Such an increase in housing consumption could be achieved in various ways. For example, the government could subsidize the renovation of owner-occupied dwellings or provide incentives to landlords to improve the quality of their properties.\textsuperscript{73} These interventions would increase the amount of housing services provided by one unit of housing stock.\textsuperscript{74} Alternatively, governments could seek to impose reforms to improve the efficiency of housing service production - thereby resulting in more housing services being provided to the economy per unit of housing stock.\textsuperscript{75}

\textsuperscript{73}Germany provides examples of such subsidies. Subsidies are given to homeowners that improve the quality of their dwelling, e.g. improvements of insolation. Renovation of dwellings that have been originally build prior to the Second World War are also subsidized. Landlords that improve the quality of their real estate can subtract all costs from their income subject to taxation.

\textsuperscript{74}In the model this could be captured by an increase in the parameter $\xi$.

\textsuperscript{75}In the model, a unit of housing stock $S$ provides housing services $\zeta u(S)$. The relative preference parameter $\xi$ can also be interpreted as an efficiency parameter in transforming housing stock into housing services. A proxy for many things, e.g. the ability of the legal framework to enforce and hence promote rental contracts.
References


Definition of Variables

\( Y_t \) \hspace{2cm} Total Output of Final Consumption Good in period \( t \)
\( H_t \) \hspace{2cm} Total Housing Stock in period \( t \)
\( H_{t|t} \) \hspace{2cm} Total Housing Stock in period \( t \) that was introduced into the economy in \( t \)
\( S_t \) \hspace{2cm} Total Rental Housing Stock in period \( t \)
\( C_t \) \hspace{2cm} Composite consumption good in period \( t \)
\( Z_t \) \hspace{2cm} Value of a one period riskless Bond in period \( t \)
\( R_t \) \hspace{2cm} Real Interest Rate in period \( t \)
\( (1 + i_t) \) \hspace{2cm} Nominal Interest Rate in period \( t \)
\( Q_{t|t} \) \hspace{2cm} Nominal Price of a unit of Housing Stock in \( t \), introduced into the economy in \( t \)
\( P_{c,t} \) \hspace{2cm} Nominal Price of the final Consumption good
\( P_{r,t} \) \hspace{2cm} Nominal Rental Price
\( q_t \) \hspace{2cm} Real Price of Housing Stock in terms of final Consumption good
\( p_{r,t} \) \hspace{2cm} Real Rental Price of Housing Stock in terms of final Consumption good
\( B_t \) \hspace{2cm} Value of pre-existing bubbles in the economy in period \( t \)
\( F_t \) \hspace{2cm} Value of pre-existing fundamentals in the economy in period \( t \)
\( u_{t}^{b} \) \hspace{2cm} Value of newly introduced bubbles in the economy in period \( t \)
\( u_{t}^{f} \) \hspace{2cm} Value of newly introduced fundamentals in the economy in period \( t \)
\( W_t \) \hspace{2cm} Real Wage
\( D_t \) \hspace{2cm} Firm’s profit in period \( t \), received by Household as a lump-sum payment
\( \xi^k \) \hspace{2cm} Aggregate Preference for Housing Service relative to Consumption in country \( k \)
\( \gamma \) \hspace{2cm} Discount rate of Households
\( \delta \) \hspace{2cm} Depreciation rate of Housing
\( \Psi_t \) \hspace{2cm} Nominal Cost Function of Monopolistic Firm in period \( t \)
\( \varepsilon \) \hspace{2cm} Elasticity of Substitution between differentiated intermediate inputs
Appendix A: Model Derivations

First Order Conditions

Households - First Order Conditions: Lagrangian and First Order Conditions (FOCs)

\[
\max_{C_{1,t}, C_{2,t+1}, H_t, S_t} \begin{cases}
    u(C_{1,t}) + \xi v(S_t) + \gamma E_t \{ \log(C_{2,t+1}) \} \\
    -\lambda_t \left( C_{1,t} + \frac{Z_t}{P_t} + \sum_{k=0}^\infty q_{t|t-k} H_t|t-k + p_t' S_t - W_t - \delta q_t|t \right) \\
    -\phi_t \left( E_t \{ C_{2,t+1} \} - \frac{(1+i_t)Z_t}{E_t(P_{t+1})} - \sum_{k=0}^\infty E_t \{ (p_t' + (1-\delta)q_{t+1|t-k}) H_t|t-k - D_{t+1} \} \right) \\
    +\mu_t C_{1,t} \\
    +\gamma_t C_{2,t+1} \\
    +\kappa_t H_t \\
    +\varphi_t S_t \\
    +\psi_t Z_t
\end{cases}
\]

The households first order conditions (FOCs) and complementary slackness conditions (CSCs) are given by

\[
C_{1,t}: \quad u'(C_{1,t}) - \lambda_t + \mu_t = 0 \quad \text{(A1)}
\]

with \( \mu_t, C_{1,t} \geq 0 \) and \( \mu_t C_{1,t} = 0 \)

\[
C_{2,t+1}: \quad \gamma E_t \{ u'(C_{2,t+1}) \} - \phi_t + \gamma_t = 0 \quad \text{(A2)}
\]

with \( \gamma_t, C_{2,t+1} \geq 0 \) and \( \gamma_t C_{2,t+1} = 0 \)

\[
H_t|t-k: \quad -\lambda_t q_{t|t-k} + \phi_t (1-\delta) E_t \{ q_{t+1|t-k} \} + \phi_t E_t \{ p_{t+1}' \} + \kappa_t = 0 \quad \text{(A3)}
\]

with \( \kappa_t, H_t|t-k \geq 0 \) and \( \kappa_t H_t|t-k = 0 \)

\[
S_t: \quad \xi v'(S_t) - \lambda_t p_t' + \varphi_t = 0 \quad \text{(A4)}
\]

with \( \varphi_t, S_t \geq 0 \) and \( \varphi_t S_t = 0 \)

\[
Z_t: \quad -\frac{\lambda_t}{P_t} + \frac{\phi_t (1+i_t)}{E_t(P_{t+1})} + \psi_t = 0 \quad \Rightarrow \lambda_t = \phi_t (1+i_t) \frac{P_t}{E_t(P_{t+1})} + \psi_t \quad \text{(A5)}
\]

with \( \psi_t, Z_t \geq 0 \) and \( \psi_t Z_t = 0 \)

Note: I focus on the case where consumption is positive in both periods of life, i.e. \( C_{1,t}, C_{2,t+1} > 0 \), this is a realistic assumption as it is empirically motivated. One time period corresponds to around 35 years. Hence, \( \mu_t = \gamma_t = 0 \). Further I assume that the constraints on \( H_t, S_t, Z_t \) are not binding, i.e. \( \kappa = \varphi_t = \psi_t = 0 \).
6.1 Equilibrium Dynamics

In the deterministic case, where \( U_t = U > 0 \), and \( B_t - E_{t-1}\{B_t\} = 0 \), and \( F_t - E_{t-1}\{F_t\} = 0 \) for all \( t \). The optimal price setting equation implies that \( W_t = W = (1/M) \), and it follows from market clearing condition that \( D_t = 1 - W \) for all \( t \). Recall that:

\[
C_{1,t} = \frac{1}{1+\xi} (W_t - F_t - B_t) \quad \text{(A6)}
\]

\[
p_t^* = \frac{\xi}{1+\xi} (W_t - F_t - B_t) \quad \text{(A7)}
\]

\[
C_{2,t} = D_t + \frac{\xi}{(1+\xi)} W_t + \frac{1}{(1+\xi)}(F_t + B_t) \quad \text{(A8)}
\]

\[
= 1 - \frac{1}{(1+\xi)} (W_t - F_t - B_t)
\]

Using the Euler Equation and (A6) and (A8), we get

\[
R_t = \frac{(1 - W_{t+1}) + \xi + F_{t+1} + B_{t+1}}{\gamma(W_t - F_t - B_t)} \equiv R(B_t, B_{t+1}, F_t, F_{t+1}) \quad \text{(A9)}
\]

The latter must satisfy the deterministic version of the intertemporal optimality condition, which was given by:\footnote{The deterministic version of the intertemporal optimality condition is given by \( q_t \equiv F_t + B_t + U_t = \frac{p_{r,t+1} + (1-\delta)q_{t+1,t-k}}{R_t} \). Multiplying this equation by \( \delta \sum_{k=0}^{\infty}(1-\delta)^k \) and recalling that \( q_t = \delta \sum_{k=0}^{\infty}(1-\delta)^k q_{t|t-k} \) yields (A10)}

\[
q_t = F_t + B_t + U_t = \frac{p_{r,t+1} + B_{t+1} + F_{t+1}}{R_t}
\]

(A10)

Plugging (A7) in the latter equation and solving for \( R_t \) gives

\[
R_t = \frac{\xi W_{t+1} + F_{t+1} + B_{t+1}}{(1+\xi)(F_t + B_t + U_t)} \equiv R(B_t, B_{t+1}, F_t, F_{t+1}) \quad \text{(A11)}
\]

Setting (A9) and (A11) equal and solving for \( F_{t+1} \) gives

\[
F_{t+1}(B_t, B_{t+1}, F_t) = -\frac{1}{\gamma(B_t + F_t - W)} + \frac{1}{\gamma(B_t + F_t - W)}
\]

(A12)

Recall that the intertemporal optimality condition was given by:

\[
q_{t|t-k} = \frac{p_{r,t+1}}{R_t} + \frac{(1-\delta)}{R_t} q_{t+1|t-k}
\]
Solving forward gives

\[ q_{t|t-k} = \frac{p_{r,t+1}}{R_t} + \frac{(1 - \delta)p_{r,t+2}}{R_t R_{t+1}} + \frac{(1 - \delta)^2 p_{r,t+3}}{R_t R_{t+1} R_{t+2}} + (\ldots) \]

\[
= \sum_{k=1}^{\infty} \prod_{j=0}^{k-1} \frac{1}{R_{t+j}} (1 - \delta)^{k-1} p_{r,t+k} + \lim_{m \to \infty} \left( \prod_{j=0}^{m-1} \frac{1}{R_{t+j}} (1 - \delta)^{m} q_{t+m|t-k} \right)
\]

Rewriting \( q_{t|t-k}^F \) gives

\[
q_{t|t-k}^F = \frac{p_{r,t+1}}{R_t} + \sum_{k=2}^{\infty} \prod_{j=0}^{k-1} \frac{1}{R_{t+j}} (1 - \delta)^{k-1} p_{r,t+k}
\]

\[
= \frac{p_{r,t+1}}{R_t} + \sum_{k=1}^{\infty} \prod_{j=0}^{k} \frac{1}{R_{t+j}} (1 - \delta)^{k} p_{r,t+1+k}
\]

\[
= \frac{p_{r,t+1}}{R_t} + (1 - \delta) \sum_{k=1}^{\infty} \prod_{j=0}^{k} \frac{1}{R_{t+j}} (1 - \delta)^{k-1} p_{r,t+1+k}
\]

\[
= \frac{p_{r,t+1}}{R_t} + (1 - \delta) \sum_{k=1}^{\infty} \prod_{j=0}^{k-1} \frac{1}{R_{t+1+j}} (1 - \delta)^{k-1} p_{r,t+1+k} \tag{A13}
\]

Recall that \( q_t = \delta \sum_{k=0}^{\infty} (1 - \delta)^k q_{t|t-k} \). Multiplying (A13) with \( \delta \sum_{k=0}^{\infty} (1 - \delta)^k \) gives

\[
q_t^F = \frac{p_{r,t+1}}{R_t} + \frac{1}{R_t} \delta \sum_{k=0}^{\infty} (1 - \delta)^{k+1} q_{t+1|t-k}^F
\]

\[
= \frac{p_{r,t+1}}{R_t} + \frac{1}{R_t} \delta \sum_{k=1}^{\infty} (1 - \delta)^{k} q_{t+1|t-1-k}^F \tag{A14}
\]

Recall the definition \( q_t^F = F_t + u_t^F \). Plug (A7) in (A14) and solve for \( R_t \):

\[
R_t = \frac{\xi}{(1+\xi)} \left( W - F_{t+1} - B_{t+1} \right) + \frac{F_{t+1}}{F_t + u^F} \tag{A15}
\]
Substracting from (A10), (A14) gives
\[ q_t - q_t^F = \frac{(p_{r,t+1} + F_{t+1} + B_{t+1}) - (p_{r,t+1} + F_{t+1})}{R_t} \]
\[ \equiv q_t^B \equiv B_t + U^B = \frac{B_{t+1}}{R_t} \]
\[ \equiv R_t = \frac{B_{t+1}}{B_t + U^B} \tag{A16} \]

Setting (A16) equal to (A15) and solving for \( B_{t+1} \) gives
\[ B_{t+1} = \frac{(1 - W)(1 + \xi)(B_t + U^B)}{\gamma W - (1 + \xi + \gamma)(B_t + F_t) - (1 + \xi)U} \equiv H(B_t, F_t, U) \tag{A17} \]

Now plugging (A17) into (A12) gives
\[ F_{t+1} = \frac{(1 - W)(F_t + U^F) + \xi^2(B_t + F_t + U) + \xi \left[ \gamma W(B_t + F_t - W) + (B_t + U^B) + (2 - W)(F_t + U^F) \right]}{\gamma W - (1 + \xi + \gamma)(B_t + F_t) - (1 + \xi)U} \equiv G(B_t, F_t, U) \tag{A18} \]

A deterministic bubbly equilibrium with positive fundamental value is defined by a sequence \( \{B_t, F_t\} \) satisfying the two difference equations (A17) and (A18), where \( B_t \in \left( W - F_t - \frac{(1 + \xi)}{1 + \eta \delta}, W - F_t - \frac{(1 + \xi)}{1 + \gamma} \right) \) for all \( t \) and a range of \( U \in [\underline{u} R_t, \bar{u}_t] \). The aggregate bubble is then given by \( Q_t^B = B_t + U^B \). Given the \( \{B_t, F_t\} \), we can determine the equilibrium values for all variables.

### 6.2 Existence Conditions

Derivation of Proposition 2.2:

Using the deterministic versions of the Euler equation (2.5), the definition of the real interest rate (2.2), the equilibrium equations for the consumption levels, \( C_{1,t} = \frac{1}{1+\xi} (W_t - F_t - B_t) \) and \( C_{2,t+1} = D_{t+1} + \frac{\xi}{(1+\xi)} W_{t+1} + \frac{1}{(1+\xi)} (F_{t+1} + B_{t+1}) \), the fact that in the flexible price equilibrium we have \( D_t = 1 - W_t \), as well as the necessary condition that the real interest rate has to be larger than \((1 - \delta)\), we can show that for the existence of a deterministic bubbly steady state with a positive fundamental and bubble the following inequality has to hold:
\[ W > F(\xi^k, \gamma, \delta) + \left( \frac{1 + \xi^k}{1 + (1 - \delta) \gamma} \right) \]

Given the necessity that the real interest rate has to be smaller than one, we derive the restriction \( W > F(\xi^k, \gamma, \delta) + \left( \frac{1 + \xi^k}{1 + \gamma} \right) \). Hence, the existence condition for a deterministic bubbly steady state with a positive fundamental and bubble is given by
\[ W > F(\xi^k, \gamma, \delta) + \left( \frac{1 + \xi^k}{1 + (1 - \delta) \gamma} \right) \]
Derivation of Proposition 2.5:
The derivation is very similar Proposition 2.2, with the difference that $\xi = 0$ and hence no housing services $S$ enter the utility function. It follows that the fundamental value $q^F(u^F = 0) = 0$. In this case, the necessary condition on the real interest rate demands the interest rate to be smaller than one. Using this condition, we can derive the necessary and sufficient condition for the existence of a deterministic pure bubbly steady state without fundamental value, which is given by

$$W > \left( \frac{1}{1+\gamma} \right)$$

Derivation of Proposition 2.6:
The derivation is very similar Proposition 2.2, with the difference that the bubble component $q^B(u^b = 0) = 0$. Then the necessary condition for the existence of a deterministic bubbleless steady state with a positive fundamental value is given by

$$W > \left( \frac{1+\xi k}{1+(1-\delta)\gamma} \right).$$

6.3 Conditions on the U-Range for Steady States

The steady state interest rate solves the equation

$$R^2 + \frac{(1 - \gamma)W + U - (1 + \xi)}{\gamma(W + U)} R + \frac{(1 - W)}{\gamma(W + U)} = 0 \quad \text{for } U > \tilde{u}_1.$$  \hspace{1cm} (A19)

where $W$ is a constant. Solving for $R$ gives

$$R_{1,2}(U) = \frac{(1 + \xi) - U - W(1 - \gamma) \pm \sqrt{4\gamma(1 - W)(U + W) + [U + (1 - \gamma)W - (1 + \xi)]^2}}{2\gamma(U + W)}$$

Solving if $Z^2 - 4X = 0$ for $u$ gives

$$\tilde{u}_{1,2} \equiv (\gamma + \xi) + (1 + \gamma)(1 - W) \pm 2\sqrt{\gamma(1-W)(1+\gamma+\xi)}$$  \hspace{1cm} (A20)

Resulting in two real solutions $R_1(\tilde{u}_1) = R_2(\tilde{u}_1)$ and $R_1(\tilde{u}_2) = R_2(\tilde{u}_2)$.\footnote{Solving the version of the Euler Equation (2.5) for $R$ yields this quadratic equation.} In the following we focus on the range $U \in (0, \tilde{u}_1)$.

Bubbly deterministic steady state with a positive fundamental value:

\footnote{For $U > \tilde{u}_2$, two real, non-positive solutions $R_1(U) \neq R_2(U)$. For $\tilde{u}_1 < U < \tilde{u}_2$, two complex solutions $R_1(U) \neq R_2(U)$. For $U < \tilde{u}_1$, two real, positive solutions $R_1(U) \neq R_2(U)$.}
\[
\begin{align*}
\exists \text{ two sets of steady states with } & \quad R_1(U) \neq R_2(U) \quad \text{for } U \in [u_{R_1}, \bar{u}_1]. \\
\exists \text{ one set of steady states with } & \quad R_2(U) \quad \text{for } U \in (u_{R_2}, \bar{u}_1).
\end{align*}
\]

where

\[
\begin{align*}
u_{R_1} &= \left(\frac{\xi^k + \delta [W(1 + \gamma) - (1 + \xi^k)] - W\gamma\delta^2}{1 + \gamma(1 - \delta)}\right) \\
u_{R_2} &= \left(\frac{\xi^k}{1 + \gamma}\right)
\end{align*}
\]

Proof 1: \exists two sets of steady states with \( R_1(U) \neq R_2(U) \) for \( U \in [u_{R_1}, \bar{u}_1] \)

\( R_2(U) > R_1(U) \) and \( \frac{\partial R_1}{\partial U} > 0 \) and \( \frac{\partial R_2}{\partial U} < 0 \) for \( U < \bar{u}_1 \). Given the restriction that \( (1 - \delta) < R(U) \leq 1 \), recall (2.40), the lower bound on \( U \) for both real interest rates can be derived and is given by \( R_1(u_{R_1}) = (1 - \delta) \) and \( R_2(u_{R_2}) = 1 \). Hence, \( u = \max\{u_{R_1}, u_{R_2}, 0\} \) where \( u_{R_1} = \left(\frac{\xi^k + \delta [W(1 + \gamma) - (1 + \xi^k)] - W\gamma\delta^2}{1 + \gamma(1 - \delta)}\right) \), \( u_{R_2} = \left(\frac{\xi^k}{1 + \gamma}\right) \).

Using the necessary condition (Proposition 1.2.) for the existence of a deterministic bubbly steady state with positive fundamental, it can be shown that \( u_{R_1} > u_{R_2} \) and hence \( u = u_{R_1} \).\(^79\)

Proof 2: \exists one set of steady states with \( R_2(U) \) for \( U \in (u_{R_2}, \bar{u}_1) \)

\( R_2(U) \) is decreasing in \( U \), hence a sufficient condition is \( R_2(\bar{u}_1) - (1 - \delta) > 0 \). The solution set, where all parameter restrictions and the existence condition (Proposition 2.1) hold is given by the explicit representation of the following region: \( 0 < \delta < 1 \quad \land \quad 0 < \gamma < 1 \quad \land \quad 0 < \xi < -\frac{\gamma(\delta - 1)\delta}{\gamma(\delta^2 - 3\delta + 2) + 1} \quad \land \quad \frac{\xi + 1}{\gamma - \delta + \gamma + 1} < W < \frac{1 - \gamma(\delta - 1)(\delta + \xi - 1)}{(\gamma(\delta - 1) - 1)^2} \).

### 6.4 Proportional Rental Subsidy

Budget constraint when young:

\[
C_{1,t} + \frac{Z_t}{P_t} + \sum_{k=0}^{\infty} q_{t-k}H_{t-k} + (1 - \tau_s)p_tS_t \leq (1 - \tau_u)W_t + u_t
\]

(A21)

Budget constraint when old:

\[
C_{2,t+1} \leq \frac{(1 + i_t)Z_t}{P_{t+1}} + \sum_{k=0}^{\infty} (p_{t+1} + (1 - \delta)q_{t+1|t-k})H_{t+1-k} + D_{t+1}
\]

(A22)

\(^{79}\)This follows from Proposition 1.2. and the fact that \( \frac{\partial u_t}{\partial U} = \frac{\delta}{1 - \delta} > 0 \).
where $H_t = \sum_{k=0}^{\infty} H_{t|t-k}$. The inter-temporal budget constraint (IBC) is thus given by

$$C_{1,t} + \sum_{k=0}^{\infty} q_{t|t-k} H_{t|t-k} + (1 - \tau_s) p_{t}^{S} S_t + \frac{1}{R_t} \left( C_{2,t} - \sum_{k=0}^{\infty} (p_{t+1}^{F} + (1 - \delta) q_{t+1|t-k}) H_{t|t-k} \right) \leq (1 - \tau_w) W_t + \delta q_{t|t} + \frac{D_{t+1}}{R_t} \tag{A23}$$

**Financed by income taxation (lump-sum of the young)**

Budget constraint of the government

$$\tau_s p_t^S S_t = \tau_w W_t \tag{A24}$$

The steady state interest rate solves the following quadratic equation

$$R^2 + \frac{(1 + \tau_u \xi - \gamma (1 - \tau_w)(1 - \tau_s) - \tau_s (1 + \xi)) W + (1 - \tau_s (1 + \xi)) U - (1 + \xi) (1 - \tau_s) \gamma (1 - \tau_w) W + U}{\gamma (1 - \tau_s) (1 - \tau_w) W + U} R$$

$$+ \frac{(1 - \tau_s (1 + \xi))(1 - W)}{\gamma (1 - \tau_s) (1 - \tau_w) W + U} = 0 \equiv \bar{z}_1$$

$$\equiv \bar{f}_1$$

where $W$, $\xi$, $\gamma$, $\tau_u$, and $\tau_s$ are constants. Solving for $R$ gives

$$R_{1,2}(U) = \frac{(1 + \xi) - U - W (1 - \gamma) + \tau_s (1 + \xi) (U + W) - W (\tau_w \gamma (1 - \tau_s) + \gamma \tau_s + \tau_w \xi)}{2 \gamma ((1 - \tau_s) U + (1 - \tau_s) - (1 - \tau_s) \tau_w W)}$$

$$\equiv \sqrt{-4 \gamma (1 - W - \tau_s (1 + \xi) + \tau_s (1 + \xi) W) [(U + W) - \tau_s (W + U) - \tau_w (1 - \tau_s) W]^{2}}$$

$$2 \gamma ((1 - \tau_s) U + (1 - \tau_s) - (1 - \tau_s) \tau_w W)$$

Solving for $Z_1^2 - 4F_1 = 0$ for $u$ gives

$$\bar{u}_{1,2} \equiv \frac{(1 + \xi) + \tau_s^2 (\xi^2 + 2 \xi + 1) - \tau_s (\xi^2 + 3 \xi + 2) - W (1 - \tau_s (1 + \xi)) (1 - \gamma (1 + \tau_w) + \tau_w \xi - \tau_s (1 + \xi))}{((1 + \xi) \tau_s - 1)^2}$$

$$\equiv 2 \sqrt{\gamma (\tau_s - 1)^2 (1 - W + \xi + 1) (\tau_s (\xi + 1) - 1)^2 (1 - \tau_s W)}$$

$$((1 + \xi) \tau_s - 1)^2$$

Resulting in two real solutions $R_1(\bar{u}_1) = R_2(\bar{u}_1)$ and $R_1(\bar{u}_2) = R_2(\bar{u}_2)$.

For $U > \bar{u}_2$, two real, non-positive solutions $R_1(U) \neq R_2(U)$. For $\bar{u}_1 < U < \bar{u}_2$, two complex solutions $R_1(U) \neq R_2(U)$. For $U < \bar{u}_1$, two real, positive solutions $R_1(U) \neq R_2(U)$. 

---

80For $U > \bar{u}_2$, two real, non-positive solutions $R_1(U) \neq R_2(U)$. For $\bar{u}_1 < U < \bar{u}_2$, two complex solutions $R_1(U) \neq R_2(U)$. For $U < \bar{u}_1$, two real, positive solutions $R_1(U) \neq R_2(U)$. 

57
we focus on the range $U \in (0, \tilde{u}_1)$.

Bubbly deterministic steady state with a positive fundamental value:

$$
\begin{aligned}
\exists \text{ two sets of steady states with } & R_1(U) \neq R_2(U) \quad \text{for } U \in [u_{R_1}, \tilde{u}_1), \\
\exists \text{ one set of steady states with } & R_2(U) \quad \text{for } U \in (u_{R_2}, \tilde{u}_1).
\end{aligned}
$$

where

$$
u_{R_1} = \left( (1 - \tau_w W) \xi + \delta [W(1 + \tau_w \xi + \gamma(1 - \tau_w)(1 - \tau_s) - (1 + \xi)] - W \gamma \delta^2 (1 - \tau_w)(1 - \tau_s) \right)
\left[ 1 + \gamma (1 - \delta)(1 - \tau_s) + (1 + \xi) \tau_s \right] (1 - \delta)
$$

$$
u_{R_2} = \frac{\xi(1 - \tau_w W)}{1 + \gamma (1 - \tau_s) - \tau_s (1 + \xi)}
$$

Note that these expressions boil down to those in the baseline scenario on page 19, when setting $\tau_s = \tau_w = 0$.

Derivation of the Bubble Existence Condition (4.1):

Using the deterministic versions of the Euler equation (2.5), the definition of the real interest rate (2.2), the adjusted equilibrium equations for the consumption levels,

$$
C_{1,t} = \left( 1 - \tau_h \xi \right) W_t + \delta [W(1 + \tau_h \xi + \gamma (1 - \tau_h)(1 - \tau_s) - (1 + \xi)] - W \gamma \delta^2 (1 - \tau_w)(1 - \tau_s) \right)
\left[ 1 + \gamma (1 - \delta)(1 - \tau_s) + (1 + \xi) \tau_s \right] (1 - \delta)
$$

Derivation of the Bubble Existence Condition (4.1):

Using the deterministic versions of the Euler equation (2.5), the definition of the real interest rate (2.2), the adjusted equilibrium equations for the consumption levels, $C_{1,t} = \left( 1 - \tau_h \xi \right) W_t - F_t - B_t$ and $C_{2,t+1} = D_{t+1} + \frac{\xi}{(1 - \tau_s + \xi)} W_{t+1} + \frac{1 - \tau_s}{(1 - \tau_s + \xi)} (F_{t+1} + B_{t+1})$, the fact that in the flexible price equilibrium we have $D_t = 1 - W_t$, as well as the necessary condition that the real interest rate has to be larger than $(1 - \delta)$, we can show that the existence of a deterministic bubbly steady state with a positive fundamental and bubble value is given by (4.1) in the text.

6.5 Help-to-Buy Scheme - Proportional Buying Subsidy

Budget constraint when young:

$$
C_{1,t} + \frac{Z_t}{P_t} + (1 - \tau_h) \sum_{k=0}^{\infty} q_{t|t-k} H_{t|t-k} + p^t S_t \leq (1 - \tau_w) W_t + u_t \quad (A25)
$$

Budget constraint when old:

$$
C_{2,t+1} \leq \frac{(1 + i_t) Z_t}{P_{t+1}} + \sum_{k=0}^{\infty} \left( p^t_{t+1} + (1 - \delta) q_{t+1|t-k} H_{t|t-k} + D_{t+1} \right) \quad (A26)
$$

Budget constraint of the government

$$
\tau_h \sum_{k=0}^{\infty} q_{t|t-k} H_{t|t-k} = \tau_w W_t \quad (A27)
$$
The steady state interest rate solves the following quadratic equation

\[
R^2 + \frac{[(1 + \tau_w \xi)(1 - \tau) - \gamma(1 - \tau_w)]W + [1 - \tau(1 + \gamma)]U - (1 + \xi)(1 - \tau)\gamma(1 - \tau)(1 - \tau_w)W + U}{R} = 0
\]

where \( W, \xi, \gamma, \tau_w \) and \( \tau \) are constants. Solving for \( R \) gives

\[
R_{1,2}(U) = \frac{(U + W)[(1 - \tau)(1 + \xi) - U(1 - \tau(1 + \gamma)) - W[(1 - \tau_w \xi)(1 - \tau) - \gamma(1 + \tau)]]}{2\gamma(1 - \tau)(U + W)(U + (1 - \tau_w)W)}
\]

\[
\mp \sqrt{\frac{(U + W)[-(1 - (1 + \gamma)\tau)U - (1 - \gamma(1 - \tau) + \tau_w \xi(1 - \tau) + \gamma)W + (1 + \xi)(1 - \tau)]^2}{2\gamma(1 - \tau)(U + W)(U + (1 - \tau_w)W)}}
\]

\[
\mp 2\sqrt{\frac{\gamma(1 - W)(\tau - 1)^2(1 - \xi \tau)(X)}{(1 + \xi)(\tau - 1)^2}}
\]

where

\[
X = (1 - \tau)(1 - \tau_w(1 - \xi)W + \xi) - \tau(1 - \tau_w)\gamma^2W + \gamma(1 - \tau(1 + 2\xi) - W(\tau_w(1 - 2\tau) + (1 + \xi(\tau_w - 1))\tau_w).
\]

Resulting in two real solutions \( R_1(\tilde{u}_1) = R_2(\tilde{u}_1) \) and \( R_1(\tilde{u}_2) = R_2(\tilde{u}_2) \).\(^{81}\) In the following we focus on the range \( U \in (0, \tilde{u}_1) \).

Bubbly deterministic steady state with a positive fundamental value:

\[
\begin{cases}
\exists \text{ two sets of steady states with } R_1(U) \neq R_2(U) \quad \text{for } U \in [\bar{u}, \tilde{u}_1], \\
\exists \text{ one set of steady states with } R_2(U) \quad \text{for } U \in (\bar{u}, \tilde{u}_1).
\end{cases}
\]

\(^{81}\)For \( U > \tilde{u}_2 \), two real, non-positive solutions \( R_1(U) \neq R_2(U) \). For \( \tilde{u}_1 < U < \tilde{u}_2 \), two complex solutions \( R_1(U) \neq R_2(U) \). For \( U < \tilde{u}_1 \), two real, positive solutions \( R_1(U) \neq R_2(U) \).
where

\[
\begin{align*}
  u_{R1} &= \left( 1 - \tau_w W \right) \xi + \delta \left[ W (1 + \tau_w \xi + \gamma (1 - \tau_w)(1 - \tau_s) - (1 + \xi)) - W \gamma \delta^2 (1 - \tau_w)(1 - \tau_s) \right] \\
  &\quad \div \left[ 1 + \gamma (1 - \delta)(1 - \tau_s) + (1 + \xi) \tau_s \right] (1 - \delta) \\
  u_{R2} &= \left( \frac{\xi (1 - \tau_w W)}{1 + \gamma (1 - \tau_s) - \tau_s (1 + \xi)} \right)
\end{align*}
\]

Note that these expressions boil down to those in the baseline scenario on page 19, when setting \( \tau_s = \tau_w = 0 \).
7 Appendix B: Empirical Work

7.1 Data Sources and Descriptive Statistics

This section outlines the data sources and provides a short descriptive statistics of the data used in the forthcoming analysis.

House Price Data
The dataset consists of 22 OECD countries and contains real and nominal prices for housing markets and are reported from national statistical sources. It includes: Australia, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, the United Kingdom, Greece, Ireland, Israel, Italy, Japan, Korea, Portugal, the Netherlands, Norway, New Zealand, Sweden, and the United States. The series are provided on a quarterly basis, are seasonally adjusted, and the average of the observations in 2010 is indexed to 100. Most of the series contain observations from 1970Q1 to 2013Q4 except for 5 countries that have later starting points.\textsuperscript{82}

Due to the much shorter sample sizes I discard Greece, Israel, Korea and Portugal from the analysis. Spain is included, thereby leaving a total of 18 OECD countries.

Preference for Housing Services measured with two proxies:

- National CPI weights (Housing, Water, Electricity, Gas and other Fuels)
  Per thousand of the National CPI Total. Annual frequency over the time period 1992 to 2013 (if available) for 17 countries, data for Australia is missing. Source: OECD.stat

- Household spending on housing (% of disposable income)
  Point estimate for the years 1995 and 2005 for 18 OECD countries. Source: OECD Outlook No 86 and OECD National Accounts.
  (varies from 14% in Portugal to 30% in Denmark)

Preference of housing services differs significantly across OECD countries.

Figure (7a) plots for a sample of 17 OECD countries the initial observation of CPI weight for housing (year 2001) against the last observation of homeownership available (year 2013). The fitted line is close and parallel to the 45 degree line. Hence, CPI weights remained constant in these OECD countries.

Figure (7b) plots for 18 OECD countries the initial observation of fraction of income spent on housing services (year 1995) against the last observation of fraction of income spent on housing services available (year 2005). Cross-country differences in the preference of housing services are very persistent over time.

\textsuperscript{82}These are Spain (1971Q1), Greece (1997Q1), Israel (1994Q1), Portugal (1988Q1) and Korea (1986Q1).
I thank Natalie Girouard (OECD) for providing me with the house price data.
### 7.1.1 Descriptive Statistics of Housing Cycles

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Table B2: Descriptive Statistics of Amplitude of Cycles for 20 OECD countries
Note: Prices in 2010 are normalized to 100.

Figure B2: House Price Indices and Turning Points

Figure B2: House Price Indices and Turning Points (cont.)
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Notes: Turning points of real housing price cycles are based on the cycle-dating procedure of Harding and Pagan (2002) with the minimum duration between two turning points being set to six quarters. Duration is the number of quarters from peak (trough) to trough (peak). Amplitude is the change in real house prices from peak (trough) to trough (peak) and expressed in %.

*: On-going upturn (downturn), no peak (trough) identified, amplitude calculated from last trough (peak) to 2013q4.

Table B3: House Price Cycles by Country - Turning Points with BBQ Procedure
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Notes: Turning points of real housing price cycles are based on the cycle-dating procedure of Harding and Pagan (2002) with the minimum duration between two turning points being set to six quarters. Duration is the number of quarters from peak (trough) to trough (peak). Amplitude is the change in real house prices from peak (trough) to trough (peak) and expressed in %.

*: On-going upturn (downturn), no peak (trough) identified, amplitude calculated from last trough (peak) to 2013q4.

Table B3: House Price Cycles by Country - Turning Points with BBQ Procedure - continued
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Notes: Turning points of real housing price cycles are based on the cycle-dating procedure of Harding and Pagan (2002) with the minimum duration between two turning points being set to six quarters. Duration is the number of quarters from peak (trough) to trough (peak). Amplitude is the change in real house prices from peak (trough) to trough (peak) and expressed in %. *: On-going upturn (downturn), no peak (trough) identified, amplitude calculated from last trough (peak) to 2013q4.

Table B3: House Price Cycles by Country - Turning Points with BBQ Procedure - continued
<table>
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<tr>
<th>Country</th>
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<th>Amplitude</th>
<th>Number</th>
<th>Intensity</th>
<th>Duration</th>
<th>Amplitude</th>
<th>Number</th>
<th>Intensity</th>
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Notes: Turning points of real housing price cycles are based on the cycle-dating procedure of Harding and Pagan (2002) with the minimum duration between two turning points being set to six quarters. Duration is the number of quarters from peak (trough) to trough (peak). Amplitude is the change in real house prices from peak (trough) to trough (peak) and expressed in %. *: On-going upturn (downturn), no peak (trough) identified, amplitude calculated from last trough (peak) to 2013q4.

Table B3: House Price Cycles by Country - Turning Points with BBQ Procedure - concluded
7.2 Additional empirical results

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<th>Boom-Busts</th>
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<td>&gt; 20%</td>
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<td>(2.86)</td>
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<td>adj. $R^2$</td>
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$t$ statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Booms denoted by > x% are those booms that involve real house price changes (trough to peak) larger than x%. The variable of interest is the national consumer price index (CPI) weight on housing services. This indicator is a good measure for the relative importance of housing services in the total consumption basket. Typical LTV for 1992 and 2002, taken from Calza et al. (2013), Catte et al. (2004). Maximum LTV taken from Heitor et al. (2006). GDP measure per head, constant prices, constant PPPs, OECD base year. IMF Mortgage Index is taken from World Economic Outlook, April 2008: Housing and the Business Cycle. The index includes: typical refinancing option, mortgage equity withdrawal option (yes, no), typical LTV, covered bond issues (% of residential loans outstanding), mortgage backed security issues (% of residential loans outstanding). Index not available for New Zealand and Switzerland. Macro variables: averages 1970-2013, if not noted otherwise.

Table B4: OLS: Frequency of independent Booms and Boom-Bust Cycles (2)