# The Dynamic Strategy of an Informed Trader with Market Manipulation

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**Abstract.** This paper considers stock price manipulation by a dynamic informed trader. We provide a simple proof of the existence of manipulation in a market in a standard sequential trade model. We also give the lower bound of the number of trading periods for the existence of manipulation in equilibrium and show that if the number of trading periods is larger than that lower bound, every equilibrium involves stock price manipulation. Irrespective of the prior of the market maker, if the informed trading probability is high enough, every equilibrium involves manipulation.

**Key Words:** Information asymmetry
**JEL Classification Numbers:** D82, G12.

# 1 Introduction

In asymmetric information models of financial markets, trading behavior imperfectly reveals the private information held by traders. Dynamic informed traders thus have an incentive not only to trade less aggressively but also to manipulate the market by trading in the wrong direction, undertaking short-term losses to confuse the market and then recouping the losses in the future. Chakraborty and Yilmaz (2004) adopt the framework presented in Glosten and Milgrom (1985) and show that when the market faces uncertainty about the existence of informed traders in the market and there are a large number of trading periods, long-lived informed traders will manipulate in every equilibrium.

This paper gives the lower bound of trading periods, above which, if trades continue, any equilibrium involves stock price manipulation using the model presented in Chakraborty and Yilmaz (2004). The contribution of the paper is to describe this lower bound in terms of exogenous parameters associated with the information structure of the model. At the same time, the paper provides a very simple proof of the existence of manipulation and gives an interesting intuition about stock price manipulation.

The paper adopts a sequential trade framework that considers markets where a risky asset is traded between a market maker, strategic traders and liquidity traders. At the beginning of the trading process, either a strategic trader or a liquidity trader arrives at the market in a random manner. There are two types of strategic traders. The first, a high type, knows that the value of the asset is high. The other, called a low-type, knows the value of the asset is low. One of these three types of traders will trade with the market maker, who is not informed of the risky asset payoff, and quotes the bid and ask price according to orders received.

Within this framework, Chakraborty and Yilmaz (2004) proved the existence of manipulation by obtaining a bound on the number of periods of trading above which every equilibrium must be manipulative. Their bound depends on the prior probabilities of noise trading, informed trading in the market, and the probabilities that the risky asset is high or low. This paper provides the lower bound that only depends on the probability of informed trading. By doing this, the paper directly shows the relationship between the possible event of manipulation and informed trading in the market. Interestingly, the lower bound that we provide shows that, irrespective of the prior of the market maker, if the informed trading probability is high enough, every equilibrium involves manipulation.

The paper is organized as follows. The second section presents the model in Chakraborty and Yilmaz (2004). The third section analyzes an equilibrium strategy of the informed trader and provides the lower bound on the number of trading periods above which every equilibrium must be manipulative.

## 2 The Model of the Dynamic Informed Trader

In this section, we will describe the model presented in Chakraborty and Yilmaz (2004). In this model, there is a risky stock and a numeraire in terms of which the stock price is quoted. The terminal value of the risky stock denoted by $\tilde{v}$ is a random variable, which can take the value 0 or 1. To abstract from issues of time preference, the risk-free interest rate is assumed to be zero. The expected value of the asset is denoted by $\lambda$.

There are three classes of risk-neutral market participants: a competitive market maker, an informed trader and a liquidity trader. Knowledge of the game structure and of the parameters of the joint distribution of the investor state variables is common to all market participants. In each period, the market maker posts bid and ask prices equal to the expected value of the asset, conditional on the observed history of trades in equilibrium. The trader trades at those prices or possibly chooses not to trade. Trading happens for $T$ successive periods, after which all private information is revealed.

The private information, or type, of the trader is denoted by $\theta \in \Theta = \{0, 1, N\}$. When $\theta = 0$, the trader is informed and knows that the value of the asset is low, $v = 0$. When $\theta = 1$, the trader is informed and knows that the value of the asset is high, $v = 1$. When $\theta = N$, the trader is a liquidity trader and his trading is driven by exogenous liquidity needs. Only one type of trader is actually chosen by nature to trade in any given play of the game.

We consider the following game: with probability $\mu$, the informed trader will be chosen and with probability $1 - \mu$, the liquidity trader will be chosen. Moreover, we suppose that the prior distribution of $\theta$ is specified by:

$$\Pr(\theta = 0 | v = 0) = \Pr(\theta = 1 | v = 1) \equiv \mu, \tag{1}$$

and

$$\Pr(\theta = N|v = 0) = \Pr(\theta = N|v = 1) \equiv 1 - \mu, \tag{2}$$

where $\mu \in (0, 1)$.

The timing structure of the $T$-period trading game is as follows:

1. In period 0, nature chooses the realization $v \in \{0, 1\}$ of the risky asset payoff $\tilde{v}$ and the type of trader $\theta$. The informed trader observes $\theta$.

2. In successive periods, indexed by $t = 1, ..., T$, having observed the realized trades in periods $1, ..., t-1$, the competitive market maker posts bid and ask prices and the trader chooses his trade.

3. In period $T + 1$, the realization $\tilde{v}$ is publicly disclosed, and consumption takes place.

Let $E = \{-1, 0, 1\}$ denote the set of possible trades available to the trader in each period, with $e$ its generic element. That is, $e = 1$ denotes a buy order, $e = -1$ a sell order and $e = 0$ no trade. Let $E^t$ denote the $t$-fold Cartesian product of $E$, which is the set of possible $t$-period histories of trades. Let $e^t$ denote the generic element $\{e_1, \cdots, e_t\}$ of $E^t$. Let $\Delta(E)$ denote the set of probability distributions on $E$.

In every period $t$, the market maker posts an ask price and a bid price. The trader can choose to buy the asset at the ask price or sell the asset at the bid price or choose not to trade. Let $p_{1,t}$ be the ask price posted in period $t$ and $p_{-1,t}$ the bid price. Let $p_t \equiv (p_{1,t}, p_{-1,t}) \in P \equiv [0, 1]^2$. Denote by $P^t$ the set of possible $t$-period histories of bid and ask prices with $p^t$ denoting its generic element. Denote by $P^0$ the null history of prices with $p^0$ its unique element.

Let $H^t = (P \times E)^t$ denote the set of $t$-period histories of prices and trades with $h^t$ its generic element. Let $H^0$ be the null history set with $h^0$ its unique element. Given a history $h^t$, the (t+1)-period history generated by prices $p_{t+1}$ and a trade $e_{t+1}$ will be denoted as $\{h^t, p_{t+1}, e_{t+1}\}$. Let $\mathbf{e}(h^t)$ be the history of trades associated with the $t$-period history $h^t \in H^t$ of trades and prices, $t = 0, \cdots, T$. Finally, let $\overline{H} = \cup_{t=0}^{T-1} H^t$.

For each type of trader, a trading strategy specifies a probability distribution over trades in period $t+1$ with respect to the ask and bid prices $p_{t+1}$ posted in period $t+1$ conditional on an observed history of past prices and trades $h^t$. A strategy for the trader is defined as a function $\sigma : \overline{H} \times P \to \Delta(E)$. Let $\sigma(e|h^t, p_{t+1})$ be the probability that $\sigma$ assigns to action $e$ conditional on $p_{t+1}$ after history $h^t$.

A price rule is defined as a function $\mathbf{p} : \overline{H} \to P$ specifying a bid and an ask price that will be posted by the market maker after every history. Denote by $\mathbf{p}_1(h^t)$ the ask price and $\mathbf{p}_{-1}(h^t)$ the bid price that the market maker posts in period $t+1$ after history $h^t \in H^t$.

Now, we consider the market maker's belief. Let $\beta_t \in \Delta(\Theta)$ denote the market maker's prior belief at the beginning of period $t$; that is, $\beta_t(\theta = 0|h_{t-1})$ denotes the market maker's belief that the trader is the low type after observing history $h_{t-1}$. Then, the (Bayesian) market maker's belief is updated through Bayes' rule; that is, for all $e \in E$,

$$\begin{aligned}\beta_{t+1}(\theta = 1|h_t) &:= \Pr(\tilde{v} = 1|h_{t-1}, e_t = e) \\ &= \frac{\beta_t(\theta = 1|h_{t-1})\sigma_1^*(e|h_{t-1}, p_t)}{\sum_{\theta \in \Theta} \beta_t(\theta|h_{t-1})\sigma_\theta^*(e|h_{t-1}, p_t)}. \end{aligned} \tag{3}$$

**Definition 1** *An informed trader's strategy $\sigma$ is optimal for price rule $\mathbf{P}$ if it prescribes a probability distribution $\sigma_\theta^*(h_{t-1}, p_t) \in \Delta(E)$ over $E$ for each $\theta \in \{0, 1\}$ and history $h_{t-1} \in H^{t-1}$ such that*

$$\sigma_\theta^*(h_{t-1}, p_t) \in \arg \max_{\sigma_t \in (\Delta(E))^t} \sum_{t'=t}^{T} \left[ \sum_{e \in E} \sigma_t(e|h^{t'-1}, p_{t'})(\theta - p_{e,t'})e \right].$$

Next, we define an equilibrium for our economy:

**Definition 2** *An equilibrium consists of the market maker's price rule $\{\mathbf{p}^*\}$, informed trader's trading strategies $\{\sigma_\theta^* : \theta = 0, 1\}$ such that for all $t \in \{1, \cdots, T\}$ and for all $h_{t-1} \in \cup_{t'=1}^{t-1} H^{t'}$*

**(P1)** *the price rule $\mathbf{p}_e^*$ satisfies the zero-profit condition with respect to the posterior belief:*

$$\mathbf{p}_e^*(h_{t-1}) = \mathrm{E}[v|h^{t-1}, e_t = e],$$

*for all $e \in \{-1, 1\}$;*

**(P2)** *$\sigma_\theta^*(h^{t-1}, p_t^*)$ is the informed trader's optimal trading strategy for price rule $\mathbf{p}^*$ for all $\theta \in \{0, 1\}$;*

**(B)** *for all $e \in E$, $p_{e,t}^*$ satisfies Bayes' rule (3).*

We define a manipulative strategy. In principle, there should be two kinds of manipulation. One is that an informed trader abstains from trading. The other is that an informed trader trades against his information. In the first kind, the informed trader would not incur loss but just postpone his trade. In this sense, we can say that this kind of manipulation is "weak" kind of manipulation. Although it will be interesting to consider this kind of manipulation, i.e., how abstaining trades affects the market, we will only consider the second kind of manipulation.

**Definition 3** *Given a price rule $\mathbf{p}$; a strategy $\sigma$ is called manipulative for type $\theta \in \{0, 1\}$; if, for all $t \in \{0, \cdots, T\}$, $h_{t-1} \in H^{t-1}(\mathbf{p})$, $e \in \{-1, 1\}$,*

$$\sigma_\theta(e|h_{t-1}, \mathbf{p}(h_{t-1})) > 0 \;\; \textit{and} \;\; (\theta - \mathbf{p}_e(h_{t-1}))e < 0.$$

As mentioned above, we say that a strategy is manipulative if it involves the informed trader undertaking a trade in any period that yields a strictly negative short-term profit. If this occurs in equilibrium, the it means that manipulation enables the informed trader to recoup the short-term losses.

In the model, $\sigma_N$ is exogenous, and we impose the following condition on it.

**Assumption 1** *We assume the following about the distribution of liquidity trades* [1]*: For all* $t \in \{0, \cdots, T\}$*,* $h_t \in H^{t-1}(\mathbf{p})$*,* $p_t \in \mathbf{p}$*, we assume that* $\sigma_N(e|h^{t-1}, p_t) > 0$ *for* $e \in \{-1, 0, 1\}$*.*

The condition states that if the liquidity trader arrives in the market, then his selling, buying or not trading has strictly positive probability. In other words, all the possible trades order arise from the liquidity trader as well as the informed traders.

# 3 Equilibrium Analysis

In this section, we will provide our results on the number of trading periods for manipulative strategies in equilibrium. Before stating our results, we would like to talk about more details of the results in Chakraborty and Yilmaz (2004). Within the framework presented in the last section, first they provided a bound on the expected payoff for each type of the informed trader in any candidate equilibrium involving nonmanipulative strategies. In order to find the bound, they defined the notion of "price-independent strategy," which only depends on a history of orders that the market maker received up to the present period. Then, they proved that if the informed trader, say the high type sells in the first period instead of buying in equilibrium, then this deviation from the candidate equilibrium involving nonmanipulative strategies will give him a higher payoff than the bound. By using this logic, they concluded that if the number of trading periods is sufficiently large, then every equilibrium has manipulative strategies.

In the following analysis, our logic is similar to theirs except here we do not distinguish price-independent strategies from price-dependent strategies. Since the history in the model includes the history of past prices, Chakraborty and Yilmaz (2004) had to consider all the possible paths after all possible bid and ask prices in a history. In our proof here, we do not need to do so, because we first suppose that in equilibrium, neither type of informed traders adopts a manipulative strategy, irrespective of the prices. Second, we calculate the payoff from this non-manipulative strategies,

[1] In order to prove existence of equilibrium, Chakraborty and Yilmaz (2004) also imposed the condition that $\sigma_N$ is independent of current and past prices. More formally, they assumed that for all $t \in \{0, \cdots, T\}$, $h'_t, h_t \in H^{t-1}(\mathbf{p})$, $\sigma_N(h^t, p_{t+1}) = \sigma_N(h'^t, p_{t+1})$ for all $p_t \in \mathbf{p}$. Since we do not use this condition in this paper, we will not impose this.

if they constitute equilibrium and then show that one of the informed traders becomes better off by deviating from this equilibrium. In this way, we directly find a lower bound on the number of trading periods above which, if trades continue, any equilibrium involves manipulation. If both types are always honest and do not adopt manipulative strategies, then two completely opposite trades in a history lead the market maker's beliefs to assign probability one to the noise trader. So, the high type can make profits of $1-\lambda$ and the low type can make profits of $\lambda$ for the consecutive periods. When the number of consecutive periods is large enough to compensate for the loss at the beginning, then manipulation should be the optimal choice for both informed traders. In the end, we will find the lower bound for the number of trading periods by using the conditions for either type to deviate from the equilibrium.

**Proposition 1** *Suppose that at period $\bar{t}$, the market maker receives his first purchase or sell order. Then, if $T - \bar{t} > L(\mu, \lambda) \equiv \frac{2(1-\mu)}{\mu \cdot \max\{\lambda, 1-\lambda\}} + 1$, every equilibrium involves a manipulative strategy.*

**Proof:** Suppose that in equilibrium the informed trader always trades on his information; that is, the high type always buys and the low type always sells. First, we consider the decision problem for the high type. In order for the high type to make profits, he has to start to trade. In other words, he cannot always postpone his trade by choosing not to trade. Thus, we assume that in period $\bar{t}$, he starts to buy. Then, if the high type always buys in consecutive periods, the equilibrium price after period $\bar{t}$ is: for $t > \bar{t}$,

$$p_{1,t}^* = \frac{\mu\lambda + (1-\mu)\prod_{t'=1}^{t} \sigma_N(1|h^{t'-1}, p_{t'}) \cdot \lambda}{(1-\mu)\prod_{t'=1}^{t} \sigma_N(1|h^{t'-1}, p_{t'}) + \mu\lambda}, \tag{4}$$

and

$$p_{-1,t}^* = \lambda. \tag{5}$$

Consider the following deviation from the equilibrium. Suppose that the high type sells once in period $\bar{t}$. Then, for $t > \bar{t}$, $p'_{-1,t} = \lambda$; and $p'_{1,t} = \lambda$. Then, the deviation is more profitable if the following holds:

$$\sum_{t'=\bar{t}+1}^{T} (1 - p'_{1,t'}) + (1 - p^*_{-1,\bar{t}}) \cdot (-1) > \sum_{t'=\bar{t}}^{T} (1 - p^*_{1,t'}). \tag{6}$$

Notice:

$$\frac{(1-\lambda)(1-\mu)\prod_{t'=1}^{t} \sigma_N(1|h^{t'-1}, p_{t'})}{(1-\mu)\prod_{t'=1}^{t} \sigma_N(1|h^{t'-1}, p_{t'}) + \mu\lambda} \leq \frac{(1-\lambda)(1-\mu)}{(1-\mu)+\mu\lambda}. \tag{7}$$

Therefore, the deviation is more profitable if the following holds:

$$(1-\lambda)(T-\bar{t}) > \frac{(1-\lambda)(1-\mu)}{(1-\mu)+\mu\lambda}(T - \bar{t} + 1) + 1 - \lambda. \tag{8}$$

Thus, the deviation is more profitable if the following holds:

$$T - \bar{t} > \frac{2(1-\mu)}{\mu\lambda} + 1. \tag{9}$$

Second, similarly with the high type's case, we consider the low type's decision problem. Suppose that the low type starts to sell at period $\bar{t}$. Then, if he always sells in consecutive periods, the equilibrium price after period $\bar{t}$ is: for $t > \bar{t}$,

$$p^*_{1,t} = \lambda, \tag{10}$$

and

$$p^*_{-1,t} = \frac{(1-\mu)\prod_{t'=1}^{t} \sigma_N(-1|h^{t'-1}, p_{t'}) \cdot \lambda}{(1-\mu)\prod_{t'=1}^{t} \sigma_N(-1|h^{t'-1}, p_{t'}) + \mu(1-\lambda)}. \tag{11}$$

Now, we consider the deviation for the low type. Suppose that the low type buys in period $\bar{t}$. Then, for $t > \bar{t}$, $p'_{1,t} = \lambda$; and $p'_{-1,t} = \lambda$. If the deviation is profitable, the following must be true:

$$\sum_{t'=\bar{t}+1}^{T} p'_{-1,t'} - p^*_{1,\bar{t}} > \sum_{t'=\bar{t}}^{T} p^*_{-1,t'}. \tag{12}$$

Therefore, the following must be true:

$$\lambda(T-\bar{t}) - \lambda > \sum_{t'=\bar{t}}^{T} \frac{(1-\mu)\prod_{t'=1}^{t} \sigma_N(-1|h^{t'-1}, p_{t'}) \cdot \lambda}{(1-\mu)\prod_{t'=1}^{t} \sigma_N(-1|h^{t'-1}, p_{t'}) + \mu(1-\lambda)}.$$

Observe that:

$$\frac{(1-\mu)\prod_{t'=1}^{t} \sigma_N(-1|h^{t'-1}, p_{t'}) \cdot \lambda}{(1-\mu)\prod_{t'=1}^{t} \sigma_N(-1|h^{t'-1}, p_{t'}) + \mu(1-\lambda)} \leq \frac{(1-\mu)\cdot\lambda}{(1-\mu) + \mu(1-\lambda)}.$$

Thus, the deviation must be profitable if the following holds:

$$\lambda(T-\bar{t}) > \frac{(1-\mu)\cdot\lambda}{(1-\mu)+\mu(1-\lambda)}(T-\bar{t}+1) + \lambda. \tag{13}$$

The inequality (13) holds if and only if the following holds:

$$T - \bar{t} > \frac{2(1-\mu)}{\mu(1-\lambda)} + 1. \tag{14}$$

By using the bounds defined in (9) and (14), we conclude that if $T - \bar{t} > L(\mu, \lambda) \equiv \frac{2(1-\mu)}{\mu \cdot \max\{\lambda, 1-\lambda\}} + 1$, then either type should choose a manipulative strategy. ■

Notice that $\max\{\lambda, 1-\lambda\} \geq \frac{1}{2}$ always holds. Thus, $L(\mu, \lambda) < \frac{4(1-\mu)}{\mu} + 1$ also always holds for all $\lambda \in [0, 1]$. Therefore, we can obtain the following result.

**Corollary 1** *If* $T - \bar{t} > L'(\mu) \equiv \frac{4(1-\mu)}{\mu} + 1$*, then every equilibrium involves manipulative strategy.*

Chakraborty and Yilmaz (2004) show an example where the high type chooses a manipulative strategy and is better off compared with the nonmanipulative case. In their case, $\mu = 0.9$, $\lambda = 0.25$, $T = 3$ and $\bar{t} = 1$. By a simple calculation, we can show that this example is consistent with Proposition 1 and also with Corollary 1.

The key assumption for the proof is that in the beginning of the whole game, nature chooses one of the three trees. The first tree is the high type, the second the low type, and the third the liquidity type. If no informed trader chooses a manipulative strategy, then a history including sell and buy orders makes the market maker believe that the tree must be the liquidity trader's. This, then, gives the space for the informed traders to make profits. Therefore, when the number of trading periods becomes large enough to compensate for the loss at the beginning, one of the informed traders should choose a manipulative strategy.

Notice that if $\mu$ increases, $L$ will go down. This means that if the probability of informed trades increases, the number of trading periods needed for manipulation will decrease. An intuition is as follows. When $\mu$ is higher and the informed traders trade on their information, the market maker's quote will be closer to the realized value of the asset. Therefore, profits that the informed traders can make in one trade will be smaller. So, by taking manipulative strategies, the informed traders can more easily exploit their profits compared with the profits that they can receive by trading on their information.

Interestingly, the lower bound in Corollary 1 shows that regardless of the prior belief of the market maker, if the informed trading probability is high enough, every equilibrium involves manipulation. It is interesting to see that if the condition in Corollary 1 is satisfied, for any prior belief of the market maker, one or both of the informed traders chooses a manipulative strategy. In other words, no trader chooses a non-manipulative strategy. Whether or not informed traders manipulate solely depends on the number of trading periods $T - \bar{t}$ and the informed trading probability $\mu$.

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