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Office-seeking politicians, interest groups and split contributions in a campaign finance model

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The present paper investigates an extended version of Prat's campaign finance models. In this model, interest groups make contributions to politicians to influence policy decisions. Voters are assumed to judge candidates on two aspects: policy promises and nonpolicy personal qualities referred to as valence. There are two types of voters. Among these, uninformed voters only observe campaign contributions that take the role of a signaling medium. We solve the equilibrium of the game between politicians and interest groups. We then specify conditions under which a separating equilibrium exists and study the effect of split contributions on the welfare of the median voter.

Key words campaign finance, common agency, median voter, welfare

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1 Introduction

The main aims of the present paper are to analyze an extended model of campaign finance in which multiple interest groups, politicians and voters rationally interact through voting and campaign finance, and to study the welfare effects of the well-known phenomenon of "split contributions". Split contributions occurs where a donor gives money to more than one candidate in a single race. The present paper tackles this technically challenging problem (see Prat 2002a; Morton and Myerson 2003) and takes a further step towards a microfounded model of political advertising.

In two influential papers, Prat (2002a,b) constructs a campaign finance model, where campaign money has value as a medium of signaling. It is difficult to construct an intuitively reasonable and logically complete model of elections that integrates two competitions: one for fund-raising and the other for votes. Prat's work develops a model of voting with campaign advertising (see Prat 2002a,b) by incorporating the idea of advertising proposed by Milgrom and Roberts (1986).

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However, these models depend on several simplifying assumptions: in Prat (2002b), only an incumbent can obtain money and in Prat (2002a) there is only one interest group. The present paper extends the models in Prat (2002a,b) by allowing multiple interest groups and split contributions. The paper then considers under what conditions split contributions occur and how this affects voters' welfare. The paper also considers the relationship between split contributions and political parties' fund-raising abilities.

The model in the present paper is a retrospective voting model. Voters judge candidates on two aspects: policy promises and non-policy personal qualities referred to as "valence".[1] Informed voters observe the incumbent's valence, the campaign contributions and the policy promises. Uninformed voters only observe the campaign contributions, and then form a belief about the type of the incumbent and the policy promises. There are a large number of organized interest groups. Policy is multidimensional, but each interest group is only interested in a small subset of policy dimensions. Given the large number of interest groups, each interest group is small enough that the influence of its contributions on electoral outcomes is negligible. The interest groups are assumed to be characterized by two features: a bliss point and fund-raising ability. The bliss points of interest groups and the median voter are assumed to differ. They offer money to politicians in exchange for favors that take the form of beneficial policy promises. In addition, how much an interest group cares about policy in monetary terms is modeled by an exogenous parameter denoted as fund-raising ability.

The analysis first solves the equilibrium in a game between politicians and interest groups. A separating equilibrium is then considered where a good-type incumbent receives contributions and a bad-type incumbent does not. This provides conditions under which the separating equilibrium exists. The present paper also provides conditions under which split contributions occur in the separating equilibrium. The paper lastly provides a welfare analysis of the effect of split contributions on the median voter. The main results of the analysis are as follows. First, sufficient conditions are provided for the separating equilibrium and for split contributions to arise in equilibrium. Second, how small each interest group's fund-raising ability should be in order for split contributions to occur is discussed. Finally, it is shown that split contributions can be good for the median voter if the valence of a good-type incumbent is greater than the expected valence of a challenger.

The structure of the paper is as follows. Section 2 presents the model. Section 3 considers an equilibrium in a game between interest groups and politicians. In Section 4 a separating equilibrium in a voting stage in which a bad-type incumbent does not receive money is constructed. Sufficient conditions under which split contributions arise in a separating equilibrium are then established and a welfare analysis of the median voter is provided.

2 The model

The description of the model is mainly based on Prat (2002a). In this model, each candidate is characterized by two aspects: policy stance and valence. A valence variable benefits all

[1] Examples of valence include integrity, educational background and a "clean" past.

voters independently of their political opinions. The expected value of the challenger's valence variable is perfectly known to everybody, but the model includes asymmetric information about the incumbent's valence. The incumbent's valence can take the form of two variables: good or bad. There are two types of voters: informed and uninformed. Informed voters, the challenger, and an interest group can see if an incumbent is of a good type, whereas uninformed voters cannot observe this feature.

We assume that there are n policy issues and that the policy space is $[-1, 1]^n$. We assume that all players are risk-neutral. Moreover, we assume that interest groups' bliss points are common knowledge to all. We assume that politicians favor an interest group only in exchange for campaign contributions and spend contributions only on campaign advertising.

2.1 Voters

There is a continuum of voters uniformly distributed on the policy space with mass 1. Voter i is described by his or her preferred policy $p_i \in [-1, 1]^n$. Let $e \in \{I, C\}$ denote the candidate who wins the election, and whose policy promise is $p_e \in [-1, 1]^n$. Let $v_e \in \mathbb{R}$ be a valence of a politician e and $||\cdot||$ denote the Euclidean distance. The utility of voter i is:

$$u_i(v_e, p_e) = v_e - ||p_e - p_i||. \tag{1}$$

As stated in Prat (2002a), due to Davis *et al.* (1972), the median voter theorem holds in this multidimensional policy space. All voters equally benefit from valence but have different preferences regarding policy. We assume that each voter votes for a politician to maximize his or her expected utility and that all voters must vote for one of the two politicians.

Informed voters have full information regarding campaign contributions, policy promises and the incumbent's valence. Uninformed voters do not have direct information; that is, they only observe campaign contributions. We assume that voters become informed with a probability of ρ.

2.2 Interest groups

There are $2n$ interest groups. Typically, each interest group cares about a limited set of issues. Therefore, we assume that each interest group cares about one policy dimension and for each dimension exactly two interest groups are active. We assume that they have opposing interests. The right interest group for each policy dimension is assumed to have a favorable policy, 1, and the left interest group for each policy dimension is assumed to have a favorable policy, -1.

Let π denote the probability of I winning. This is determined in equilibrium. We assume that there are a large number of interest groups, each of which is small enough to take this probability as given. We denote the p_I's jth element by p_I^j and the p_C's jth element by p_C^j. Let a function $f_j^l : [-1, 1] \to \mathbb{R}$ and $h_j^l : [-1, 1] \to \mathbb{R}$ for $l \in \{L, R\}$ denote a contribution schedule for I and C, respectively. A contribution schedule from an interest group to a politician is a function of a policy promise that each politician will implement if elected. We assume that for each $i \in \{I, C\}$ and $l \in \{L, R\}$, these contribution functions are continuously differentiable. Let $Q_l^j(p_I^j, p_C^j) = f_l^j(p_I^j) + h_l^j(p_C^j)$ for $l \in \{L, R\}$. We assume that each interest group has ω as its initial endowment. The total contribution from each interest group cannot exceed this amount. We suppose that each politician's policy promise is honored when he or she is elected.

2.3 Incumbent

In the first stage, a politician is exogenously placed in power. This politician is called the "incumbent" and denoted by I. The politician's valence is denoted by $v_I = g + \theta$. A part of the valence, g, is observed by all players. $\theta \in \{B, G\}$ is perfectly observed by informed voters, interest groups and politicians, but imperfectly observed by uninformed voters. We assume that $B < G$. Because the median voter theorem holds, each politician aims to position his or her policy stance as close as possible to the median voter's favorable point zero. Let a_I denote the campaign funds that the incumbent obtains.

2.4 Challenger

At the end of the first stage, another politician appears with a probability of 1. This politician is called the "challenger" and denoted by C. The expected value of the challenger's valence is denoted by x; that is, $\mathrm{E}[v_C] = x$. Let F denote the cumulative distribution function of x. We assume that F is continuously differentiable and has full support on $(-\infty, \infty)$. The expected valence of the challenger is common knowledge among all players. Let a_C denote the campaign funds that the challenger obtains.

2.5 Information structure

We assume that F is common knowledge to everyone, including the challenger and voters. Informed voters observe (p_I, p_C), (a_I, a_C) and θ. Uninformed voters only observe (a_I, a_C) and form beliefs regarding (p_I, p_C) and θ. We assume that uninformed voters form identical beliefs regarding each politician's policy promise and the politician's valence after the campaign contributions are observed.

2.6 Timing of the game

The timing of the race is summarized as follows.

Stage 1 Nature chooses I's valence. An incumbent I is in office. A challenger C appears. Everybody observes g. The challenger observes θ.

Stage 2 A large number of interest groups observe θ. Each interest group chooses the contribution schedules. The incumbent I sets p_I^* and the challenger C sets

p_C^*. The incumbent receives $a_I^* = \sum_{l\in\{L,R\}} \sum_{j=1}^n f_l^{j*}(p_I^{j*})$ and the challenger receives $a_C^* = \sum_{l\in\{L,R\}} \sum_{j=1}^n h_l^{j*}(p_C^{j*})$. With a probability ρ, voters become informed and they observe (p_I^*, p_C^*), (a_I^*, a_C^*) and θ. With a probability of $1-\rho$, voters become uninformed and they only observe (a_I^*, a_C^*) and then form a belief regarding (p_I^*, p_C^*) and θ.

Stage 3 Voters cast their ballot. If I is elected, p_I^* is implemented. If C is elected, p_C^* is implemented. The payoffs are then made to voters.

For any given π, each interest group determines its contribution schedule to each politician. We suppose that both politicians spend all contributions on their campaigns. Each politician decides on their policy promises and campaign spending by maximizing the probability of being reelected. Formally, a voting equilibrium in the third stage is described by the following.

Definition 1 *A voting equilibrium consists of each voter's choice of a candidate such that each voter i votes for a candidate e if and only if* $Eu_i(\theta_e, p_e) \geq Eu_i(\theta_{e'}, p_{e'})$ *for* $e, e' \in \{C, I\}$.

3 Truthful political equilibrium

In this section, we consider the equilibrium of the game between interest groups and politicians in the second stage. The politician's objective is to get elected and so minimize the distance between their policy promises and the median voter's bliss point. With respect to each contribution schedule, the incumbent's problem is described by:

$$(IP) \qquad \min_{p_I} \quad ||p_I|| \qquad \text{sub. to} \quad \sum_{l\in\{L,R\}} \sum_{j=1}^n f_l^{j*}(p_I^j) \geq a_I.$$

With respect to each contribution schedule, the challenger's problem is described by:

$$(CP) \qquad \min_{p_C} \quad ||p_C|| \qquad \text{sub. to} \quad \sum_{l\in\{L,R\}} \sum_{j=1}^n h_l^{j*}(p_C^j) \geq a_C.$$

Let (IP^*) and (CP^*) denote a set of solutions for the incumbent problem (IP) and the challenger's problem (CP). Now we define a political equilibrium in the game between politicians and interest groups. Because each politician's concern is to maximize the probability that the median voter votes for them and they obtain the campaign funds, there is no incentive for a politician to choose a position outside a range between the right interest group's policy stance and the left interest group's policy stance.

Let $\alpha_l^j \in \mathbb{R}_+$ denote the relative value of policy and valence for each interest group l^j in monetary terms. If α_l^j is larger, the policy promise and valence is more valuable for interest group l^j and vice versa. In Prat (2002a), α^i is interpreted as the fund-raising ability of interest group *i*. An interest group's maximization problem can then be described by the following:

$$
\begin{aligned}
\left(GP_L^j\right) \quad & \max \Pi_L^j\left(p_I^{j*}, p_C^{j*}, Q_L^j\right) \\
& = -\alpha_L^j \cdot \left[\pi \cdot \left(p_I^{j*} - (g+\theta)\right) + (1-\pi)\cdot\left(p_C^{j*} - x\right)\right] - Q_L^j\left(p_I^{j*}, p_C^{j*}\right), \\
\text{subject to} \quad & Q_L^j\left(p_I^{j*}, p_C^{j*}\right) \le \omega \\
& p_I^{j*} \in (IP^*), \quad p_C^{j*} \in (CP^*)
\end{aligned}
$$

and

$$
\begin{aligned}
\left(GP_R^j\right) \quad & \max \ \Pi_R^j\left(p_I^{j*}, p_C^{j*}, Q_R^j\right) \\
& = \alpha_R^j \cdot \left[\pi \cdot \left(p_I^{j*} + g + \theta\right) + (1-\pi)\cdot\left(p_C^{j*} + x\right)\right] - Q_R^j\left(p_I^{j*}, p_C^{j*}\right) \\
\text{subject to} \quad & Q_R^j\left(p_I^{j*}, p_C^{j*}\right) \le \omega \\
& p_I^{j*} \in (IP^*), \quad p_C^{j*} \in (CP^*).
\end{aligned}
$$

Here, interest groups maximize their utility in the second stage. The incumbent should implement a policy in the first stage. Although we do not include this policy in our problem, unlike Prat (2002a), it does not affect an important part of the analysis and should just involve rescaling of the equilibrium variables. We can now formally define a political equilibrium in the second stage. By using the above problems, a political equilibrium is defined as follows.

Definition 2 *A political equilibrium consists of a pair of policy promises* (p_I^*, p_C^*) *and a bundle of contribution schedules* $f_l^{j^*}$ *and* $h_l^{j^*}$ *for each* $j \in \{1, \ldots, n\}$ *and* $l \in \{L, R\}$ *such that:*

(i) with respect to $f_l^{j^*}$ *for each* $j \in \{1, \ldots, n\}$ *and* $l \in \{L, R\}$, $p_I^* \in (IP^*)$;
(ii) with respect to $h_l^{j^*}$ *for each* $j \in \{1, \ldots, n\}$ *and* $l \in \{L, R\}$, $p_C^* \in (CP^*)$;
(iii) each $f_l^{j^*}(\cdot)$ *and* $h_l^{j^*}(\cdot)$ *solves* $(GP_l^{j^*})$.

In a political equilibrium, each interest group decides their contribution schedule contingent on each policy promise so as to let politicians implement their policy closer to their favorable policy. Given this contribution schedule, each politician chooses a policy promise to minimize the distance between their policy promise and the median voter's favorable point 0. A common agency framework such as this is known to have multiple equilibria. However, it is known that one class of equilibria, called truthful equilibria, exists uniquely and maximizes the sum of all agents' payoffs. In the present paper, we use this equilibrium concept. We define a truthful equilibrium in the following way. In the definition, by abuse of notation, we denote the payment schedule that gives the utility $K_l^j = K_l^j(I) + K_l^j(C)$ to an interest group l^j by $Q_l^j(p_I^j, p_C^j; K_l^j)$. Also, we denote each payment schedule for the incumbent or the challenger when an interest group l^j obtains the utility $K_l^j = K_l^j(I) + K_l^j(C)$ by $f_l^j(p_I^j; K_l^j(I))$ and $h_l^j(p_C^j; K_l^j(C))$, respectively.

Definition 3 *A truthful contribution schedule is defined to be a pair of contribution-payment functions* $(f_l^{j^*}, h_l^{j^*})$ *for each* $l \in \{L, R\}$ *and* $j \in \{1, \ldots, n\}$ *relative to the constant* K_l^j *if:*

$$f_l^{j*}\big(p_I^j; K_l^j(I)\big) = \max\big\{0, \psi_l^j\big(p_I^j, p_C^j; K_l^j(I)\big)\big\}, \tag{2}$$

where $\psi_l^j(p_I^j, p_C^j; K_l^j(I))$ *is implicitly defined by* $\Pi_l^j(p_I^j, p_C^j, \psi_l^j(p_I^j, p_C^j; K_l^j(I)) = K_l^j(I)$, *and:*

$$h_l^{j*}\big(p_C^j; K_l^j(C)\big) = \max\big\{0, \psi_l^j\big(p_I^j, p_C^j; K_l^j(C)\big)\big\}, \tag{3}$$

where $\psi_l^j(p_I^j, p_C^j; K_l^j(C))$ *is implicitly defined by* $\Pi_l^j(p_I^j, p_C^j, \psi_l^j(p_I^j, p_C^j; K_l^j(C)) = K_l^j(C)$.

As argued in Bernheim and Whinston (1986), a truthful contribution schedule for an interest group rewards the politicians for every change in the action by exactly the amount of change in the interest group's welfare. From Bernheim and Whinston, it is proved that a truthful equilibrium is equivalent to the set of coalition-proof equilibrium with respect to its payoff. This is an equilibrium in which the contribution schedule of each interest group follows the shape of the payoff function of that interest group, less a constant and save for a nonnegativity constraint. The constant K_l^j is then the equilibrium welfare level of each interest group.

Definition 4 *A truthful political equilibrium is defined to be a political equilibrium in which all contribution schedules are truthful, relative to the equilibrium welfare levels* K_l^j *for all j and l.*

Now we assume that ω is large so that we can ignore the budget constraint of each interest group. We then solve for the truthful political equilibrium. To solve the equilibrium, we first consider the politicians' problem. Now, we assume that there are some dimensions on which $\alpha_L^j \neq \alpha_R^j$. This assumption is necessary to avoid the extreme case of λ_I or λ_C going to 0 where the following analysis would not be well-defined. We form the Lagrangian for (IP):

$$L_I(p_I, \lambda_I) = -\sum_{j=1}^{n}\big(p_I^j\big)^2 + \lambda_I\left(\sum_{j=1}^{n}\big[f_L^{j*}(p_I^j) + f_R^{j*}(p_I^j)\big] - a_I\right). \tag{4}$$

Moreover, we form the Lagrangian for (CP):

$$L_C(p_C, \lambda_C) = -\sum_{j=1}^{n}\big(p_C^j\big)^2 + \lambda_C\left(\sum_{j=1}^{n}\big[h_L^{j*}(p_C^j) + h_R^{j*}(p_C^j)\big] - a_C\right). \tag{5}$$

Each single interest group takes the contribution schedules of other interest groups, and the Lagrange multipliers, λ_I and λ_C, as given. From the point of view of interest group

l on dimension j, I's problem becomes:

$$L_I^j(p_I^j, \lambda_I) = -(p_I^j)^2 + \lambda_I(f_L^{j*}(p_I^j) + f_R^{j*}(p_I^j)). \tag{6}$$

Also, C's problem becomes:

$$L_C^j(p_C^j, \lambda_C) = -(p_C^j)^2 + \lambda_C(h_L^{j*}(p_C^j) + h_R^{j*}(p_C^j)). \tag{7}$$

Now, we solve the equilibrium by using the defined Lagrangian for each politician. The following two lemmas describe the equilibrium policy promise and contribution schedules. The proofs of the lemmas are provided in the appendix.

Lemma 1 *Each interest group's campaign fund in the equilibrium is as follows:*

$$a_I^* = \frac{\lambda_I \cdot \pi^2}{4} \sum_{j=1}^{n} \left[(\alpha_L^j)^2 + (\alpha_R^j)^2\right], \tag{8}$$

$$a_C^* = \frac{\lambda_C \cdot (1-\pi)^2}{4} \sum_{j=1}^{n} \left[(\alpha_L^j)^2 + (\alpha_R^j)^2\right]. \tag{9}$$

Lagrangian multipliers, λ_I and λ_C are determined by (8) and (9), respectively.

Lemma 2 *The equilibrium policy stance in each dimension j is as follows:*

$$p_I^{j*} = \frac{2a_I^*(-\alpha_L^j + \alpha_R^j)}{\pi \cdot \sum_{j=1}^{n}\left[(\alpha_L^j)^2 + (\alpha_R^j)^2\right]}, \tag{10}$$

$$p_C^{j*} = \frac{2a_C^*(-\alpha_L^j + \alpha_R^j)}{(1-\pi) \cdot \sum_{j=1}^{n}\left[(\alpha_L^j)^2 + (\alpha_R^j)^2\right]}. \tag{11}$$

It may well be counterintuitive that politicians' valences do not affect equilibrium policy stances or contribution, although interest groups care about politician's valences in maximization problems. This is because of the equilibrium concept of truthful equilibrium. As described, in a truthful equilibrium each interest group pays each politician for additional favors in policy promise by exactly the amount of change in the interest group's welfare. Therefore, only the additional change in policy stance to the interest group's welfare is counted in a truthful contribution schedule. As a result, valences that are constantly counted in the interest group's welfare are not included in the truthful contribution schedule.

4 A voting equilibrium and welfare analysis

Here we consider a game between the politicians and determine each politician's winning probability by analyzing an expected outcome in the third stage. In the second stage, uninformed voters observe (a_I, a_C) and form a belief. Because the median voter theorem holds, I will be elected if and only if the median voter votes for him or her. For simplicity of notation, we first define δ by:

$$\delta = \frac{2\sqrt{\sum_{j=1}^{n}\left(-\alpha_L^j + \alpha_R^j\right)^2}}{\sum_{j=1}^{n}\left[\left(\alpha_L^j\right)^2 + \left(\alpha_R^j\right)^2\right]}. \tag{12}$$

Remember that α^i is interpreted as the fund-raising ability of interest group i. Therefore, Prat (2002a) views δ as the concentration index of fund-raising abilities. To guarantee the existence of equilibrium, we suppose that δ is not exactly zero. However, when the number of interest groups is sufficiently large, it is very likely that δ is very small. Therefore, we assume the following.

Assumption 1 *Suppose that δ is nonzero and sufficiently small.*

Assumption 1 can also mean that in each dimension each interest group's fund-raising ability is similar to the others, but not exactly the same. By using δ, from (10) and (11), we can rewrite each politician's policy stance as follows. Let $a_\theta \in IR_+$ denote campaign contributions that type $\theta \in \{B, G\}$ incumbent receives. For $\theta \in \{B, G\}$ and π_θ:

$$P_\theta(a_\theta; \pi_\theta) = \frac{a_\theta \delta}{\pi_\theta}, \tag{13}$$

$$P_C(a_C(\theta); \pi_\theta) = \frac{a_C(\theta)\delta}{1-\pi_\theta}. \tag{14}$$

By adopting the idea in Milgrom and Roberts (1982) and Prat (2002a), we assume the following.

Assumption 2 *Suppose that uninformed voters have beliefs such that:*

$$\beta(a_I, a_C) = \begin{cases} B - P_B(a_I; \pi_B) + P_C(a_C; \pi_B) & : \text{ if } a_I < t^* + a_C \\ G - P_G(a_I; \pi_G) + P_C(a_C; \pi_G) & : \text{ if } a_I \geq t^* + a_C \end{cases} \tag{15}$$

for some t^*.

Assumption 2 implies that there is a critical level above which uninformed voters believe that the incumbent is of a good type if he she spends more when compared to the challenger's spending. When $a_C = 0$, the form of the belief becomes exactly the same as that in Prat (2002a). This is because in that model, by assumption, the challenger does not obtain any money. As we describe in what follows, the incumbent's winning probability π_θ will be determined by $a_C(\theta)$ and a_θ in equilibrium. Then, the probability that I is elected can be written as:

$$\begin{aligned} e_\theta(a_\theta, a_C(\theta)) &= \rho\cdot\Pr(g + \theta - P_\theta(a_\theta;\pi_\theta) + P_C(a_C(\theta);\pi_\theta) \geq x) + (1-\rho)\cdot\Pr(g + \beta(a_\theta, a_C(\theta)) \geq x) \\ &= \rho\cdot F(g + \theta - P_\theta(a_\theta;\pi_\theta) + P_C(a_C(\theta);\pi_\theta)) + (1-\rho)F(g + \beta(a_\theta, a_C(\theta))). \end{aligned} \tag{16}$$

Notice that when a_θ increases, $P_\theta(a_\theta; \pi_\theta)$ increases provided that the challenger chooses $a_C^*(\theta)$. Under Assumption 2, any $a_\theta \in (0, t^* + a_C^*(\theta))$ is dominated by $a_\theta = 0$ and any $a_\theta \in (t^* + a_C^*(\theta), \infty)$ is dominated by $a_\theta = t^* + a_C^*(\theta)$. Therefore, we can focus on $a_\theta \in \{0, t^* + a_C^*(\theta)\}$.

Definition 5 *An equilibrium in a voting stage consists of* $(a_\theta^*, a_C^*(\theta))$ *for* $\theta = \{B, G\}$ *such that:*

(I) *with respect to* $a_C^*(\theta)$, *each type of incumbent chooses* $a_\theta^* \in \arg\max_{a_\theta} e_\theta(a_\theta, a_C^*(\theta))$;
(C) *with respect to* a_θ^*, *a challenger chooses* $a_C^*(\theta) \in \arg\min_{a_C(\theta)} e_\theta(a_\theta^*, a_C(\theta))$.

In the second stage, interest groups calculate their campaign contributions to each politician, given the winning probability of each politician. Each politician then considers their winning probability upon accepting or not accepting the campaign contribution. The definition clarifies the consistency of a winning probability between the second and third stages.

Definition 6 *An equilibrium in a voting stage is consistent if* $\pi_\theta^* = e_\theta(a_\theta^*, a_C^*(\theta))$.

In a separating equilibrium where the good-type politician spends some strictly positive amount $a_G^* = t^* + a_C^*(G)$ and the bad-type politician spends 0, the following conditions must hold: $e_G(t^* + a_C^*(G), a_C^*(G)) \geq e_G(0, a_C^*(G))$ and $e_B(t^* + a_C^*(B), a_C^*(B)) \leq e_B(0, a_C^*(B))$. These are incentive compatibility constraints. In order to proceed, following Prat (2002a) we impose the following assumption.

Assumption 3 *Suppose that:* $e_B(t^* + a_C^*(B), a_C^*(B)) = e_B(0, a_C^*(B))$.

Assumption 3 means that for the bad-type politician, the winning probability is the same whether or not he or she spends the money. Next, we turn our attention to the good type's incentive compatibility constraint.

Lemma 3 *Suppose that* $(1 - \rho) \cdot (G - B)$ *is sufficiently large. Then,* $e_G(t^* + a_C^*(G), a_C^*(G)) > e_G(0, a_C^*(G))$.

The good type's incentive compatibility constraint holds if the valence difference of the two types is sufficiently large, and the probability that voters become informed is low. Intuitively, it has to be beneficial for the good-type politician to spend money. Therefore, if the good type spends money, by being recognized as the good type the good type should be better off in terms of valence and also in terms of signals. The condition that $(1 - \rho) \cdot (G - B)$ is sufficiently large combines these two concepts. Before we proceed to the equilibrium analysis, we provide a lemma that is necessary for the subsequent analysis.

Lemma 4 *In a consistent equilibrium, if* $\pi_G^* > \frac{1}{2}$, *then* $\frac{d\pi_G^*}{da_C^*(G)} > 0$ *and otherwise,* $\frac{d\pi_G^*}{da_C^*(G)} \leq 0$.

The following lemma specifies conditions that $(a_\theta^*, a_C^*(\theta))$ must satisfy in a separating equilibrium.

Lemma 5 *Suppose that* $(1-\rho)\cdot(G-B)$ *is sufficiently large and Assumptions 1 to 3 hold. Then, a consistent separating equilibrium solves the following:*

(Good-type Case) *with respect to* $a_C^*(G)$ *satisfying:*

$$a_C^*(G) \in \arg\min F\left(g+G-\frac{\big(t^*+a_C^*(G)\big)\delta}{\pi_G^*}+\frac{a_C^*(G)\delta}{1-\pi_G^*}\right), \tag{17}$$

the good type chooses $a_G^* = t^* + a_C^*(G)$, *which satisfies:*

$$F(g+B)=\rho F\left(g+B-\frac{t^*\delta}{\pi_B^*}\right)+(1-\rho)F\left(g+G-\frac{t^*\delta}{\pi_G^*}\right). \tag{18}$$

(Bad-type Case) *with respect to* $a_B^*=0$, *the challenger chooses* $a_C^*(B)=0$.

(Consistency)

$$\pi_G^* = F\left(g+G-\frac{(t^*+a_C^*(G))\delta}{\pi_G^*}+\frac{a_C^*(G)\delta}{1-\pi_G^*}\right) \tag{19}$$

and

$$\pi_B^* = F(g+B). \tag{20}$$

The separating equilibrium described in Lemma 5 generates a fully revealing equilibrium in which the bad-type incumbent receives no money and so both politicians choose the median voter's ideal policy, while the good type receives enough money to credibly signal that he she is not of the bad type. From Lemma 4 and (17) we obtain the following three cases: if $\pi_G^* > \frac{1}{2}$, then $a_C^*(G)=0$ (split contributions do not occur); if $\pi_G^* = \frac{1}{2}$, then $a_C^*(G)$ is arbitrary; and if $\pi_G^* < \frac{1}{2}$, then $a_C^*(G) = \omega n - \frac{t^*}{2}$ (split contributions occur). If $\pi_G^* = \frac{1}{2}$, then (18) would be:

$$F(g+B)=\rho F\left(g+B-\frac{t^*\delta}{F(g+B)}\right)+\frac{1-\rho}{2}; \tag{21}$$

and (19) would be:

$$\frac{1}{2} = F(g+G-2t^*\delta). \tag{22}$$

The next lemma describes the equilibrium in a special case of $\pi_G^* = \frac{1}{2}$.

Lemma 6 *Suppose that* $(1-\rho)\cdot(G-B)$ *is sufficiently large and Assumptions 1 to 3 hold. Then, if there exists* t^* *that satisfies* (21) *and* (22) *simultaneously, then there exists a consistent separating equilibrium* $(a_G^*, a_C^*(G)), (a_B^*, a_C^*(B))$ *such that:*

(Good-type Case) $a_G^* = t^* + a_C^*(G)$ *and arbitrary* $a_C^*(G)$, *in which* t^* *solves (21) and (22);*

(Bad-type Case) $a_B^*=0$, *and* $a_C^*(B)=0$.

Proposition 1 *Suppose that* $(1-\rho)\cdot(G-B)$ *is sufficiently large and Assumptions 1 to 3 hold. Suppose that* $F(g+B) > \frac{1+\rho}{2}$. *Then there exists a consistent separating equilibrium* $(t^* + a^*_C(G), a^*_C(G)), (a^*_B, a^*_C(B))$ *such that:*

(Good-type Case)

$$a^*_C(G) = 0, \tag{23}$$

and t^* *satisfies:*

$$F(g+B) = \rho F\left(g + B - \frac{t^*\delta}{F(g+B)}\right) + (1-\rho)F\left(g + G - \frac{t^*\delta}{\pi^*_G}\right); \tag{24}$$

(Bad-type Case) $a^*_B = 0$, *and* $a^*_C(B) = 0$;

(Consistency) $\pi^*_G > \frac{1}{2}$ *and* $\pi^*_B = F(g+B)$.

Proof of Proposition 1: By Lemma 3, $a^*_G = t^* + a^*_C(G)$ in a consistent separating equilibrium. Because the maximum amount that each interest group can contribute is ω and there are $2n$ interest groups, $a^*_C(G) \in [0, \omega n - \frac{t^*}{2}]$. The logic here is as follows. First, we show that when $\pi^*_G > \frac{1}{2}$, $a^*_C(G) = 0$ and in equilibrium the probability that the incumbent gets reelected is actually greater than $\frac{1}{2}$. Then, we show that there exists $\pi^*_G \in (\frac{1}{2}, 1]$ and t^* that satisfies the equilibrium condition described in Lemma 5.

Suppose that $F(g+B) > \frac{1+\rho}{2}$. By (18):

$$\begin{aligned} F\left(g + G - \frac{t^*\delta}{\pi^*_G}\right) &= \frac{F(g+B)}{1-\rho} - \frac{\rho}{1-\rho} F\left(g + B - \frac{t^*\delta}{F(g+B)}\right) \\ &> \frac{1+\rho}{2(1-\rho)} - \frac{\rho}{1-\rho} F\left(g + B - \frac{t^*\delta}{F(g+B)}\right) > \frac{1}{2}. \end{aligned} \tag{25}$$

Then, notice that for $\pi^*_G \in (1/2, 1]$:

$$F\left(g + G - \frac{\left(t^* + a^*_C(G)\right)\delta}{\pi^*_G} + \frac{a^*_C(G)\delta}{1-\pi^*_G}\right) \geq F\left(g + G - \frac{t^*\delta}{\pi^*_G}\right). \tag{26}$$

Therefore, in equilibrium, by (19), for $\pi^*_G \in (1/2, 1]$, $a^*_C(G) = 0$ and $\pi^*_G = F(g + G - \frac{t^*\delta}{\pi^*_G}) > \frac{1}{2}$. From Lemma 3, any $a_C(G)$ increases π^*_G. Then:

$$t^*\delta = \pi^*_G\left(g + G - F^{-1}(\pi^*_G)\right). \tag{27}$$

Now, (18) can be rewritten as:

$$F(g+B) = \rho F\left(g + B - \frac{\pi^*_G\left(g + G - F^{-1}\left(\pi^*_G\right)\right)}{F(g+B)}\right) + (1-\rho)\pi^*_G. \tag{28}$$

When π^*_G goes to 1, then the right-hand side of (28) goes to $\rho F(g + B - \frac{g+G-F^{-1}(1)}{F(g+B)}) + (1-\rho)$. When π^*_G goes to 1/2, then the right-hand side of (28) goes to $\rho F(g + B - \frac{g+G-F^{-1}(1/2)}{F(g+B)}) + \frac{(1-\rho)}{2}$.

Because $F(g+G) < 1$, we have:

$$\rho F\left(g + B - \frac{g+G-F^{-1}(1)}{F(g+B)}\right) + (1-\rho) > F(g+B). \tag{29}$$

Because $F(g+B) > \frac{1+\rho}{2}$:

$$\rho F\left(g + B - \frac{g+G-F^{-1}(1/2)}{F(g+B)}\right) + \frac{(1-\rho)}{2} < \rho + \frac{1-\rho}{2} < F(g+B). \tag{30}$$

Because the right-hand side of (28) is continuous for $\pi_G^* \in (1/2, 1]$, by the intermediate value theorem, there exists a $\pi_G^* \in (1/2, 1]$, which satisfies (28). This completes our proof. □

In the previous proposition, we considered the sufficient conditions under which split contributions do not occur in the separating equilibrium. The next Proposition describes the sufficient conditions under which split contributions occur in equilibrium.

Proposition 2 *Suppose that* $(1-\rho)\cdot(G-B)$ *is sufficiently large and Assumptions 1–3 hold. Suppose that* $F(g+B) \leq \frac{1-\rho}{2}$. *Then there exists a consistent separating equilibrium* $(t^* + a_C^*(G), a_C^*(G)), (a_B^*, a_C^*(B))$ *such that:*

(Good-type Case)

$$a_C^*(G) = \omega n - \frac{t^*}{2}, \tag{31}$$

and t^* *satisfies:*

$$F(g+B) = \rho F\left(g + B - \frac{t^*\delta}{F(g+B)}\right) + (1-\rho) F\left(g + G - \frac{t^*\delta}{\pi_G^*}\right); \tag{32}$$

(Bad-type Case) $a_B^* = 0$, *and* $a_C^*(B) = 0$;

(Consistency) $\pi_G^* < \frac{1}{2}$ *and* $\pi_B^* = F(g+B)$.

Proof of Proposition 2: Suppose that $F(g+B) \leq \frac{1-\rho}{2}$. Then, first we show that $\pi_G^* < \frac{1}{2}$. Now, by (18):

$$F\left(g + G - \frac{t^*\delta}{\pi_G^*}\right) = \frac{F(g+B)}{1-\rho} - \frac{\rho}{1-\rho} F\left(g + B - \frac{t^*\delta}{F(g+B)}\right) < \frac{1}{2}. \tag{33}$$

Then, because $a_C^*(G)$ minimizes $F(g + G - \frac{(t^* + a_C^*(G))\delta}{\pi_G^*} + \frac{a_C^*(G)\delta}{1-\pi_G^*})$, we have:

$$F\left(g + G - \frac{\big(t^* + a_C^*(G)\big)\delta}{\pi_G^*} + \frac{a_C^*(G)\delta}{1-\pi_G^*}\right) \leq F\left(g + G - \frac{t^*\delta}{\pi_G^*}\right). \tag{34}$$

Therefore, in equilibrium, from (19):

$$\pi_G^* = F\left(g + G - \frac{\big(t^* + a_C^*(G)\big)\delta}{\pi_G^*} + \frac{a_C^*(G)\delta}{1-\pi_G^*}\right) < \frac{1}{2}. \tag{35}$$

From Lemma 3 we conclude $a_C^*(G) = \omega n - \frac{t^*}{2}$. Then, given (19) we have:

$$\pi_G^* = F\left(g + G - \frac{(\omega n + t^*/2)\delta}{\pi_G^*} + \frac{(\omega n - t^*/2)\delta}{\left(1 - \pi_G^*\right)}\right). \tag{36}$$

Therefore:

$$t^*\delta = \frac{2\pi_G^*\left(1 - \pi_G^*\right)}{1 - 2\pi_G^*}\left[g + G - F^{-1}\left(\pi_G^*\right)\right] - \omega n\delta \equiv V\left(\pi_G^*\right). \tag{37}$$

Now, (18) can be rewritten as:

$$F(g + B) = \rho F\left(g + B - \frac{V\left(\pi_G^*\right)}{F(g + B)}\right) + (1 - \rho)F\left(g + G - \frac{V\left(\pi_G^*\right)}{\pi_G^*}\right). \tag{38}$$

When π_G^* goes to 0, then the right-hand side of (18) goes to $\rho F(g + B + \frac{\omega n\delta}{F(g+B)}) + (1 - \rho)$. When π_G^* goes to 1/2, then the right-hand side of (18) goes to 0. Note that:

$$0 < F(g + B) < \rho F\left(g + B + \frac{\omega n\delta}{F(g + B)}\right) + (1 - \rho). \tag{39}$$

Notice that $V(\pi_G^*)$ is continuous for $\pi_G^* \in [0, 1/2)$ and, therefore, the right-hand side of (38) is also continuous for $\pi_G^* \in [0, 1/2)$. Given the intermediate value theorem, there exists a $\pi_G^* \in [0, 1/2)$, which satisfies (38). This completes our proof. □

It is worth mentioning that in Prat (2002a), split contributions may arise because if the interest groups have no insider information about the candidates, they offer money to both candidates and both candidates accept. Propositions 1 and 2 provide a completely different implication for split contributions. If the good type is good enough when compared to the bad type, the good type needs to obtain money for advertising. Otherwise, the candidate would be considered as the bad type. However, this means that the good-type politician's policy promise deviates from the median voter's bliss point.

From Lemma 4 and (17) we know that: if $\pi_G^* > \frac{1}{2}$, then $a_C^*(G) = 0$ and if $\pi_G^* < \frac{1}{2}$, then $a_C^*(G) > 0$. Thus, if $\pi_G^* > \frac{1}{2}$, then split contributions do not occur and if $\pi_G^* < \frac{1}{2}$, then split contributions occur. The condition $F(g + B) > \frac{1+\rho}{2}$ is sufficient for $\pi_G^* > \frac{1}{2}$ and the condition $F(g + B) \leq \frac{1-\rho}{2}$ is sufficient for $\pi_G^* < \frac{1}{2}$.

Next, we consider the relationship between δ and split contributions. This is an interesting problem to consider because δ is the concentration index of fund-raising abilities, α^i, and it will answer the question of how small each interest group's influence should be for split contributions to occur. The following theorem provides conditions on the relationship of the exogenous parameters that are sufficient for split contributions to occur in equilibrium. This result yields the whole picture of the model and the split contributions.

Theorem 1 *Suppose that* $\frac{1-\rho}{4\omega n}(G - B) \geq \delta > 0$ *and Assumptions 2 and 3 hold. Suppose that* $F(g + B) \leq \frac{1-\rho}{2}$. *Then, in a separating equilibrium, split contributions occur.*

Proof of Theorem 1: From the proof of Lemma 3, the good type's incentive compatibility holds if:

$$(1-\rho)(G-B) \geq \left[(1-\rho)\left(\frac{a_C^*(G)}{1-\pi_B} - \frac{a_C^*(G)}{1-\pi_G}\right) + \frac{t^* + a_C^*(G)}{\pi_G}\right]\delta. \tag{40}$$

From the proof of Proposition 2, we know that if $F(g+B) < \frac{1-\rho}{2}$, then $\pi_G^* < \frac{1}{2}$. Also, in a consistent equilibrium, $\pi_B^* = F(g+B)$. Therefore, we have:

$$(1-\rho)\left(\frac{a_C^*(G)}{1-\pi_B} - \frac{a_C^*(G)}{1-\pi_G}\right) + \frac{t^* + a_C^*(G)}{\pi_G} \leq (1-\rho)\frac{a_C^*(G)}{1/2} + \frac{t^* + a_C^*(G)}{1/2}. \tag{41}$$

Because $a_C^*(G) + t^* + a_C^*(G) \leq 2\omega n$ given the feasibility constraint imposed on the whole economy, we have:

$$(1-\rho)\frac{a_C^*(G)}{1/2} + \frac{t^* + a_C^*(G)}{1/2} < \frac{2\omega n\delta}{1/2} = 4\omega n. \tag{42}$$

Therefore, we can conclude that the good type's incentive compatibility holds if:$(1-\rho)(G-B) \geq 4\omega n\delta$. This completes our proof. □

The condition $\frac{1-\rho}{4\omega n}(G-B) \geq \delta > 0$ is necessary for the good type's incentive compatibility to hold and the condition $F(g+B) \leq \frac{1-\rho}{2}$ is necessary for $\pi_G^* < 1/2$ and for the challenger to receive money. When both conditions hold, then split contributions occur in equilibrium. The intuition behind the theorem is as follows. When $\frac{1-\rho}{4\omega n}(G-B) \geq \delta > 0$ holds, then the cost of receiving money is higher than the benefit of receiving money for the good-type incumbent and, therefore, the incumbent receives money.

We now turn our attention to the robustness of the separating equilibrium. Similar arguments in Prat (2002a) can also be applied here. We consider the following situation where for uninformed voters campaign spending does not matter and, therefore, politicians have no reason to undertake any spending. This pooling equilibrium is supported by voters' belief $\beta(a_I, a_C) = qG$ for any (a_I, a_C), where q denotes the probability that the good-type incumbent appears in the game. Following the proof in Prat (2002a), we can also prove that the pooling equilibrium does not survive the intuitive criterion in our model.

Remark 1 *The pooling equilibrium stated above is not robust.*

Proof of Remark 1: See Prat (2002a). □

Finally, we study how campaign spending affects voters' welfare in the separating equilibrium described in Proposition 2. More specifically, we consider the median voter's welfare. Because in this model voters are symmetrically distributed over the policy space, the expected utility of the median voter is a measure of utilitarian social welfare.

Let W denote the median voter's utility function in each state and EU denote the median voter's ex ante utility function. Then, the median voter's ex ante welfare in the separating equilibrium described in Proposition 2 is:

$$\mathrm{E}U\big((a_B^*, a_C^*(B)), \big(a_G^*, a_C^*(G)\big)\big) = (1-q)\cdot W\big(a_B^*, a_C^*(B)\big) + q\cdot W\big(a_G^*, a_C^*(G)\big), \tag{43}$$

where:

$$W\big(a_B^*, a_C^*(B)\big) = e_B\big(a_B^*, a_C^*(B)\big)(g + B) + \big(1 - e_B\big(a_B^*, a_C^*(B)\big)\big)x; \tag{44}$$

$$\begin{aligned} W\big(a_G^*, a_C^*(G)\big) \;=\; & e_G\big(a_G^*, a_C^*(G)\big)\left[(g + G) - \frac{a_G^* \cdot \delta}{e_G\big(a_G^*, a_C^*(G)\big)}\right] \\ & + \big(1 - e_G\big(a_G^*, a_C^*(G)\big)\big)\left[x - \frac{a_C^*(G) \cdot \delta}{1 - e_G\big(a_G^*, a_C^*(G)\big)}\right]. \end{aligned} \tag{45}$$

The ultimate goal is to consider if split contributions could increase the median voter's welfare. To do so, we change the relative campaign spending of each politician by making total spending constant. In other words, we consider what would happen to the median voter's welfare when the incumbent's spending increases by a and the challenger's spending decreases by a. Because of the following:

$$\begin{aligned} W\big(a_G^* + a, a_C^*(G) - a\big) - W\big(a_G^*, a_C^*(G)\big) \;=\; & \big[e_G\big(a_G^* + a, a_C^*(G) - a\big) \\ & - e_G\big(a_G^*, a_C^*(G)\big)\big](g + G - x), \end{aligned} \tag{46}$$

the effect of the change in spending appears on the change of the winning probability. Then, we can obtain the following proposition.

Proposition 3 *If $g + G$ is larger than x, split contributions increase the median voter's welfare. If $g + G$ is smaller than x, split contributions decrease the median voter's welfare.*

Proof of Proposition 3: Because in the case where an incumbent is of a bad type, no politician receives money whether or not split contributions are allowed, we focus on the case where an incumbent is of a good type. We focus on the equilibrium described in Proposition 2. When $a_C^*(G)$ increases by a and a_G^* decreases by a, D_G decreases. Therefore, the good-type incumbent's winning probability increases. We denote this change by $\Delta\pi_G^*$. Then:

$$\Delta W_G := W\big(a_G^*, a_C^*(G)\big) - W\big(a_G^* + a, a_C^*(G) - a\big) = \Delta\pi_G^* \times (g + G - x). \tag{47}$$

Therefore, because $\Delta\pi_G^* > 0$, if $g + G - x < 0$, then $\Delta W_G < 0$. Otherwise, $\Delta W_G \geq 0$. By making $a = a_C^*(G)$ we can conclude that if $g + G$ is larger than x, split contributions can increase the median voter's welfare. □

When the challenger receives less money, the challenger's policy stance then deviates less from the median voter's bliss point. This means that the incumbent's winning probability decreases. Therefore, if the incumbent's valence is higher when compared to the challenger's valence, then by decreasing the good incumbent's winning probability the median voter can become worse off and vice versa. The ultimate case of this argument is that split contributions are prohibited and the challenger does not receive any money. If the incumbent is bad when compared to the challenger in terms of valence, then the median voter becomes better off due to the prohibition of split contributions.

Appendix

PROOF OF LEMMA 1: To avoid lengthy calculations, we explain the steps for calculating the equilibrium. We consider each politician's maximization problem. By taking the first derivative of the maximization problem with respect to p_I^j or p_C^j, we obtain: $\frac{\partial f_L^j(p_I^j)}{\partial p_I^j} = -\pi\alpha_L^j$, $\frac{\partial h_L^j(p_C^j)}{\partial p_C^j} = -(1-\pi)\alpha_L^j$, $\frac{\partial f_R^j(p_I^j)}{\partial p_I^j} = \pi\alpha_R^j$, $\frac{\partial h_R^j(p_C^j)}{\partial p_C^j} = (1-\pi)\alpha_R^j$. From the politicians' problems, we obtain the following first-order conditions:

$$\lambda_I \pi\left(-\alpha_L^j + \alpha_R^j\right) = 2p_I^j, \text{ and } \lambda_C(1-\pi)\left(-\alpha_L^j + \alpha_R^j\right) = 2p_C^j.$$

Define $\bar{p}_l^j$ for $l \in \{L, R\}$ by $\bar{p}_l^j = \arg\max_{p_I^j} \lambda_I \cdot f_l^{j*}(p_I^j) - (p_I^j)^2$. We assume that $f_L^{j*}(p_I^{j*})$, $f_R^{j*}(p_I^{j*})$, $f_L^{j*}(\bar{p}_L^j)$ and $f_R^{j*}(\bar{p}_R^j)$ are all nonnegative and that ω is sufficiently large. We can then disregard the nonnegativity and budget constraints. We will check the nonnegativity condition later. From the properties of the truthful equilibrium, the following must hold: $\lambda_I[f_L^{j*}(p_I^{j*}) + f_R^{j*}(p_I^{j*})] - (p_I^{j*})^2 = \lambda_I f_L^{j*}(\bar{p}_L^j) - (\bar{p}_L^j)^2 = \lambda_I f_R^{j*}(\bar{p}_R^j) - (\bar{p}_R^j)^2$. By solving the above, we can find $K_L^j(I)$ and $K_R^j(I)$.

Next, we consider C's problem. Similar to I's problem, define $\hat{p}_l^j$ for $l \in \{L, R\}$ by $\hat{p}_l^j = \arg\max_{p_C^j} \lambda_C \cdot h_l^{j*}(p_C^j) - (p_C^j)^2$. Similar to I's problem, given the property of the truthful equilibrium for C's problem, we obtain: $\lambda_C[h_L^{j*}(p_C^{j*}) + h_R^{j*}(p_C^{j*})] - (p_C^{j*})^2 = \lambda_C h_L^{j*}(\hat{p}_L^j) - (\hat{p}_L^j)^2 = \lambda_C h_R^{j*}(\hat{p}_R^j) - (\hat{p}_R^j)^2$. Then, we can find $K_L^j(C)$ and $K_R^j(C)$ and, accordingly, $f_L^{j*}(p_I^{j*})$, $f_R^{j*}(p_I^{j*})$, $f_L^{j*}(\bar{p}_L^j)$, $f_R^{j*}(\bar{p}_R^j)$, $f_L^{j*}(\hat{p}_L^j)$ and $f_R^{j*}(\hat{p}_R^j)$. After we check nonnegativity of these variables in equilibrium, we conclude that for each given λ_I, λ_C and π, there exists a unique truthful political equilibrium. □

PROOF OF LEMMA 2: Since $a_I^* = \sum_{j=1}^n (f_L^{j*}(p_I^{j*}) + f_L^{j*}(p_I^{j*}))$, from Proposition 1, we calculate λ_C and λ_I. Then, substituting λ_C and λ_I into the pair of the policy promises that each politician would choose in equilibrium, (p_I^{j*}, p_C^{j*}), we obtain the desired result. □

PROOF OF LEMMA 3: The incentive compatibility holds if:

$$\begin{aligned} &e_G\left(t^* + a_C^*(G), a_C^*(G)\right) = F\left(g + G - P_G\left(t^* + a_C^*(G); \pi_G\right) + P_C\left(a_C^*(G); \pi_G\right)\right), \\ &\quad > e_G\left(0, a_C^*(G)\right) = \rho F\left(g + G + P_C\left(a_C^*(G); \pi_G\right)\right) + (1-\rho) F\left(g + B + P_C\left(a_C^*(G); \pi_B\right)\right). \end{aligned} \tag{48}$$

Because F is strictly concave, the above holds if:

$$\begin{aligned} &g + G - P_G\left(t^* + a_C^*(G); \pi_G\right) + P_C\left(a_C^*(G); \pi_G\right) \\ &\quad \geq \rho\left(g + G + P_C\left(a_C^*(G); \pi_G\right)\right) + (1-\rho)\left(g + B + P_C\left(a_C^*(G); \pi_B\right)\right). \end{aligned} \tag{49}$$

This gives us the desired result. □

PROOF OF LEMMA 4: For simplicity of notation, we write $a_C^*(G) = a_C^*$ in this proof. Let $y_G = \frac{d\pi_G^*}{da_C^*}$. Notice that in a consistent equilibrium, π_G^* is equal to $e_G(t^* + a_C^*(G), a_C^*(G))$ by Lemma 3. So, we focus on the case where $\pi_G^* = e_G(t^* + a_C^*(G), a_C^*(G))$. Let $D_G(a_G, a_C(G)) = P_G(a_G; \pi_G) - P_C(a_C(G); \pi_G)$. Let $f(x) = \frac{dF(x)}{dx}$. Then, observe that:

$$y_G = \frac{f\left(g + G - D_G\left(t^* + a_C^*, a_C^*\right)\right) \cdot \left(\frac{1}{1-\pi_G^*} - \frac{1}{\pi_G^*}\right) \cdot \delta}{1 - f\left(g + G - D_G\left(t^* + a_C^*, a_C^*\right)\right) \cdot \left(\frac{t^* + a_C^*}{\left(\pi_G^*\right)^2} + \frac{a_C^*}{\left(1-\pi_G^*\right)^2}\right) \cdot \delta}. \tag{50}$$

Therefore, if δ is sufficiently small, we obtain the desired result. □

PROOF OF LEMMA 5: Because for $a_C^*(B) \geq 0$ we have:

$$e_B\left(0, a_C^*(B)\right) = F\left(g + B + P_C\left(a_C^*(B); \pi_B^*\right)\right) \geq F(g + B) = e_B(0, 0), \tag{51}$$

we can conclude that if $a_B^* = 0$, the challenger chooses $a_C^*(B) = 0$, because the objective of the challenger is to minimize e_B. Then, by Assumption 3 and (13), we obtain condition (18). Because the challenger chooses $a_C^*(G)$ to minimize π_G^*:

$$a_C^*(G) \in \arg\min F\left(g + G - \frac{\left(t^* + a_C^*(G)\right)\delta}{\pi_G^*} + \frac{a_C^*(G)\delta}{1 - \pi_G^*}\right).$$

This completes our proof. □

PROOF OF LEMMA 6: The proof is done by Lemma 4 and Lemma 5. □

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