A Hierarchical Agency Model of Deposit Insurance

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Abstract. This paper develops a hierarchical agency model of deposit insurance. The main purpose of the analysis is to propose a micro-founded model of deposit insurance schemes and study their effects on the behavior of depositors and the monitoring problem for a bank. This paper also characterizes a risk-based premium in equilibrium, and conducts a comparative statics analysis of depositors' optimal actions. The results supply the basic theoretical foundation for designing deposit insurance schemes. Our findings are consistent with the empirical research on depositor behavior.

Key Words: Deposit Insurance; Banks; Regulation; Hierarchical Agency Model.

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1 Introduction

In the paper, we develop a framework for studying deposit insurance schemes and depositors' behavior. The key feature underlying our model that differentiates it from the literature is that we model the relationships between banks, depositors, and firms within the framework of a hierarchical agency model. The paper studies the effect of deposit insurance on the problem of monitoring and depositors' behavior and provides theoretical results about depositor behavior that are consistent with the extant empirical research. Our motivation is to provide a simplest possible model to study the relationship between risk covered by deposit insurance and depositors' behavior.

We show that as the amount of insured deposit increases, the risk dependent premium increases and also that the risk dependent premium is monotone in the risk. Moreover, by assuming a specific form of depositors' utility function, we prove that the incentive for firms to monitor also increases. Finally, we show how aggregate risk affects depositors' behavior. In our model, it is assumed that depositors pay risk dependent premiums for insurance and thus market risk affects depositors decision through two channels; current payment for premiums and future payment upon banks failure. In other words, as the uncertainty they face is smaller, the premium could be higher while expected payment upon bank failure is larger.

Our model works in the following manner. In our economy, there are four classes of agents: firms, banks, depositors, and the government. We assume that a bank invests money in firms and then decides whether to monitor firms. The identical depositors are risk averse and wish to maximize their utility as a function of their consumption. The government balances its budget in the case of bank failure. The model formulates these agents as follows: firms as an agent; banks as a supervisor; the depositors as principals; and the government as a balanced budget in the case of bank failure.

Boyd et al. (2002) and Boyd et al. (2004) have studied the incentives of deposit insurance. Boyd et al. (2002) focus specifically on how banks deal with moral hazard in the presence of deposit insurance. In a partial equilibrium context, they show that the pricing of deposit insurance affects a bank's decision to address moral hazard. However, when allowing for a general equilibrium framework in which interest rates are endogenously determined, it is shown that neither the deposit insurance premium nor the degree of governmental losses are of any importance to any agent in equilibrium. These studies assume that there is no aggregate uncertainty and so it is feasible for the insurer to fully insure depositors.

In this paper, aggregate uncertainty is captured by the amount of insured deposit and as a part of the insurance plan, the amount insured is given. In this sense, our model does not assume that deposits are always fully insured. This allows us to study the relationship between the amount insured, aggregate risk and the premium in equilibrium. Another difference of our model with Boyd et al. (2002) is that we allow depositors to not deposit with a bank. Boyd et al. (2002) and Boyd et al. (2004) assume that depositors care only about second period of consumption and, thus, supply their funds to the banks inelastically. Our analysis abstracts from this by explicitly modeling the depositors decision to save. Indeed, the paper establishes that the assumption that depositors necessarily supply funds may not be innocuous. We show that depositors may not deposit money unless the interest rate on deposits is sufficiently high or the risk of bank failure is sufficiently low. This result is consistent with the empirical research on depositor behavior (see Park (1995) or Park and Peristiani (1998)).

Much of the banking literature emphasizes the various types of moral hazard problems confronted by banks. Indeed, theories of bank capital increasingly focus on how agency and moral hazard problems influence bank capital management and more broadly the bank's entire balance sheet[1]. Berger et al. (1995) also provide an extensive discussion of various rationales for regulator-determined versus market capital ratios in light of the Modigliani–Miller theorem. Following earlier contributions by Diamond and Dybvig (1983) and Casamatta and Haritchabalet (1991), Diamond and Rajan (2001) develop a theory of banking in which fragility to bank runs commits banks to the creation of liquidity. Alternatively, Diamond and Rajan (2000) develop a theory of bank capital and trace decision trees for entrepreneurs (borrowers), banks (lenders), capital (debt holders), and depositors. Akerlof and Romer (1993) argue that deposit insurance can also create an incentive for managers to bankrupt the bank intentionally.

However, despite this large body of work, the literature produces mixed predictions on the effects of deposit insurance and capital regulation on asset risk and the safety and soundness of the banking system as a whole. Overall, the theoretical literature suggests that deposit insurance may have either a positive or a negative effect on financial stability. These broadly ambiguous results suggest that understanding the effects of deposit insurance in diverse banking systems is an important agenda for theoretical research in banking.

In contrast, a sizeable empirical literature exists on moral hazard and deposit insurance (see Hooks and Robinson (2002) or Grossman (1992)). An increasing number of studies have considered moral hazard and deposit insurance using more recent or international data sets. Brewer and Mondschean (1994) use 1980s data on US savings and loans associations, arguing that the increase in investment in junk bonds is directly related to the moral hazard caused by deposit insurance, though Karels and McClatchey (1999) find no evidence of a decline in asset quality or liquidity using data on US credit unions in 1970. Nevertheless, much of the literature supports the link between deposit insurance and moral hazard. For instance, Yilmaz and Muslumov (2008) find evidence of moral hazard in the Turkish banking system, especially among locally owned banks. Likewise, Gonzales (1992) employs global data to compare bank balance sheets and finds that deposit insurance directly increases bank incentives for risk taking. However, the findings also suggest that a decrease in risk-taking incentives from the increased charter value resulting from deposit insurance at least partly offsets this detrimental effect. Lastly, based on data from 61 countries between 1980 and 1997, Demirguc-Kunt and Detragiache (2002) argue that explicit deposit insurance tends to increase the likelihood of banking crises and is more generally more likely when bank interest rates are deregulated.

There is also some empirical evidence to suggest that deposit insurance changes the behavior of depositors as well as banks. Importantly, where deposits are not insured, depositors may discipline banks for taking on extra risk by either withdrawing their deposits or requiring that the banks pay a higher rate of interest on their deposits. Demirguc-Kunt and Huizinga (2004) find that explicit deposit insurance reduces the deposit interest rates required. Similarly, Park (1995) employs data on US banks between 1985 and 1992 and finds that riskier banks had a slower rate of growth in uninsured deposits when compared with banks that were less likely to fail but still required to pay large depositors a higher

[1] See Van Hoose (2007) for an extensive survey.

interest rate. In the same vein, and using data from US thrifts after 1987, Park and Peristiani (1998) find that even though riskier thrifts paid higher interest rates on uninsured deposits, they attracted a smaller quantity of deposits.

This paper aims to provide a basic framework to study the direct and indirect effects of deposit insurance on depositors' optimal actions and its relationship with the problem of moral hazard. The outline of the remainder of the paper is as follows. The second section describes the environment and the model of deposit insurance. The third section undertakes the equilibrium analysis of the decisions by depositors, a firm and a bank, and conducts comparative statics. The final section concludes.

2 The Model

In our economy, there are four classes of agents: firms, banks, depositors, and the government. We assume that there is a unit mass of identical firms and we focus on a representative firm. Moreover, we assume that there is a unit mass of identical banks and we study a decision by a representative bank.[2] We assume the economy lasts for two periods: day 0 and 1. The firm borrows money from the bank and undertake a project with risky outcomes in the next period. The project requires a certain amount of funds, say L. The bank invests L into the firm and then decides whether to monitor the firm. The firm could make a high or low level of effort, such that we denote the level of effort $e > 0$ for $e \in \{e_l, e_h\}$ with $e_l << e_h$.

There are n identical depositors and we assume that n is sufficiently large so that each depositor is a price taker. We also assume depositors are risk averse and wish to maximize their utility as a function of the amount of consumption. In the model, we formulate the agents as follows: the firm as an agent; the bank as a supervisor; the depositors as principals; and the government as a balanced budget in the case of bank failure.

There are three exogenous interest rates. Let r_L be the interest rate on loans, and r_D the rate on deposits. We assume that r_L is determined through negotiation between the firm and the bank, while r_D is determined by negotiation between depositors and the bank. We do not explicitly model these negotiations. In addition there is a general interest rate where the bank can invest money in securities other than the firm, which we call a "market interest rate." Let $r = (r^h, r^l)$ where each r^j denote a market interest rate when $e = e_j$. We assume that all interest rates are strictly positive.

Depositors

Depositors receive initial wealth ω on day 0 but nothing on day 1. We denote depositors' action by $a \in \{No, De\}$, which No stands for "no deposit" and De stands for "deposit," respectively. Let $c^n = (c_0^n, c_1^n)$ and c_t^n denotes the consumption plan on day $t \in \{0, 1\}$ when $a = No$. Let $c_j = (c_{0,j}, \{c_{1,j}^s\}_s)$ and for each $j \in \{h, l\}$, $c_{0,j}$ denotes the consumption plan on day 0 and $c_{1,j}^s$ denotes the plan on day 1 for state s when $a = De$. Let k indicate the portion of deposits guaranteed by deposit insurance when

[2] As described later, depositors are identical and a price taker. In this way, we focus on the relationship between representative agents. We can consider this situation as the case where each depositor saves money for each bank with the same probability or each bank invests money in each firm with the same probability. Introducing the heterogeneity in each class of agents is an interesting extension and is discussed in the conclusion.

the bank fails and $P(\phi^j)$ be the premium for the insurance, which is paid by depositors. Let β denote a time discount rate for depositors. The depositors then choose the deposit and consumption plan $(a, \mathbf{c}, \{D^j\})$ with $\mathbf{c} = (c^n, \{c_j\}_j)$ to solve the following maximization problem. For each $j \in \{h, l\}$:

$$\max \mathrm{E}U(\mathbf{c}|a, j) = u(c_{0,j}) + \beta \sum_s \Pr(s|j) \cdot u(c_{1,j}^s), \tag{1}$$

in which $u(c)$ is strictly increasing and strictly concave, subject to:

$$\begin{array}{rcll} c_{0,j} + D^j + P(\phi^j) & \leq & \omega & \text{for } a = De; \\ c_0^n & \leq & \omega & \text{for } a = No; \\ c_1^n \leq \omega - c_0; \quad c_{1,j}^G & \leq & (1 + r_D) \cdot D^j; & c_{1,j}^B \leq k \cdot D^j, \end{array} \tag{2}$$

and all variables are strictly positive. To avoid a complicated mixed strategy analysis, we assume that, if depositors are indifferent between depositing and not depositing, they choose to deposit money.

Firms

Firms are the productive unit. The profit created by the firm's production in each state depends on the effort e. There is a single project that should be undertaken with the necessary funds of L. The profit of the project is denoted by π and the state of the world, which relates to the efforts made by the firm in a probability that will be defined later. If the firm exerts a high level of effort (e_h), the probability of the project's success is ϕ^h. If the firm exerts a low level of effort (e_l), the probability of the project's success is ϕ^l. More specifically, for each $j \in \{h, l\}$ when $e = e_j$, the repayment from the firm to the bank, $f(j)$, is assumed to be:

$$f(j) = \begin{cases} (1 + r_L)L & : \text{ with probability } \phi^j \\ 0 & : \text{ with probability } 1 - \phi^j. \end{cases} \tag{3}$$

Naturally, we assume $\phi^h >> \phi^l$. Suppose that in the good state the firm's output is y and in the bad state the firm's output is 0. Define $\pi^G = y - (1 + r_L)L$, which is the profit in the good state and note that the profit in the bad state is 0. Suppose that the firm's disutility of high effort is equal, in monetary terms, to $g_h = g$ and the disutility of low effort is $g_l = 0$.

Now we consider firms maximization problem. We assume that if depositors do not deposit in the first period, firms do not earn any profit and thus their payoff in the second period is 0. Firms only obtain payoff from the successful project and so they do not earn anything in the first period. Let U_F denote the payoff for the firm in the second period. If depositors do not deposit, firms obtain $zero$ in the second period and U_F is equal to the expected payoff when depositors deposit. Let $\Pr(a)$ denote the probability that depositors choose action $a \in \{De, No\}$ and δ denote their discount factor. Then the expected payoff in the present value is given by $\delta \Pr(De) U_F$. Under the assumption that there are identical many agents, each firm does not have control over $\Pr(De)$. Therefore, the key issue for their maximization problem is to maximize the profit from the project, U_F. Let W_0 be the firm's reservation utility to participate in the debt contract. To keep our analysis simple, let us assume that W_0 is sufficiently low and for any firms discount factor δ firms prefer to implement the project.

The firm's problem is to choose an effort level e_j for $j \in \{h, l\}$ to maximize:

$$\mathrm{E}W(e_j) := \phi^j \cdot [y - f - g_j] - (1 - \phi^j) \cdot g_j, \tag{4}$$

subject to $\delta \mathrm{E}W(e_j) \geq W_0$.

Banks

Banks are risk neutral and have access to deposits at a market interest rate. If depositors do not deposit, banks payoff in the second period is assumed to be 0. Otherwise, in the second period, there are two states of the world: either the project is successful or the project is not successful. Depending on the project outcome, we refer to $s = G$ as the good state and $s = B$ as the bad state. Suppose that if the bank pays M then the bank can monitor the effort employed by the firm to which it lends and enforce a contract requiring from the firm that the effort level is high. If depositors do not deposit in the first period, the bank is assumed to obtain $zero$. Each bank does not have control over depositors' decision on saving. Then the bank's optimal decision[3] chooses $m \in \{0, M\}$ to maximize their expected payoff $\mathrm{E}\Pi(D^j, m)$ where for each $j \in \{h, l\}$:

$$\mathrm{E}\Pi(D^j, m) := \frac{\phi^j (1 + r_L) L}{1 + r^j} - L + \frac{r^j - r_D}{1 + r^j} \cdot nD^j - m. \tag{5}$$

The first term $\frac{\phi^j \cdot (1+r_L)L}{1+r^j} - L$ represents the present value of profit made from making loans to the firm and the second term $\frac{r^j - r_D}{1+r^j} nD$ represents the implicit profit made from attaining a deposit and thus allowing equity to be used to buy market securities. We say that the bank fails when the payoff it receives falls lower than Π_0. Moreover, assume that for each $j \in \{h, l\}$:

$$-L + \frac{r^j - r_D}{1 + r^j} \cdot nD^j < \Pi_0. \tag{6}$$

This means the bank will fail in the event that the firm is unable to return the loan. We assume that if the bank is indifferent between monitoring and not monitoring, the bank chooses to monitor the effort level of the firm.

Government

The government collects the premium $P(\phi^j)$ as a lump sum[4]. The amount the government then collects as a premium is $(1 + r^j)P(\phi^j)$ in the next period. This forms the maximum amount the government can pay when the bank fails, as the government cannot insure a greater amount. Alternatively, the government is unable to retain any positive amount when the bank fails. In other words, if the government's objective were to maximize depositors' welfare, then it would be natural to assume that the government

[3] Freixas and Rochet (2008) define the expected gain for the bank's shareholders by looking at the bank's balance sheet.

[4] It is true that insurers are only able to use proxies, such as CAMEL ratings in the US or information given by rating agencies in Australia. While judging the best proxy for risk is outside the scope of this paper, valuing deposit insurance as a put option and then using market data to price this put has been discussed by Pennacchi (2006), Marcus and Shaked (1984), and Ronn and Verma (1986).

insures as much as it can given the premium. Accordingly, we suppose that given r^j and $k \leq 1$, $P(\phi^j)$ for each j satisfies:

$$(1 + r^j) \cdot P(\phi^j) = k \cdot D^j. \tag{7}$$

Note that $k < 1$ captures aggregate uncertainty in this model.

Timing

More formally, the timing is as follows.

Day 0 Given r, the government announces k. Accordingly, depositors, the firm and the bank move as follows.

- The firm decides the effort level.
- The bank decides whether to monitor the firm's decision upon lending.
- Depositors receive the initial endowment ω, decide their consumption plan c and whether to deposit money, and make a consumption for day 0.

If depositors do not deposit, the game ends here and no profit is made for banks and firms in the next period, Day 1.

Day 1 One of the two states is realized and the profit is determined. The firm returns the loan if possible. Otherwise, the bank fails and the insurance is paid to depositors.

Now we define the equilibrium in this economy as follows.

Definition 1. The equilibrium in this economy *consists of: the firm's effort* e^**, the bank's decision on monitoring* $m^* \in \{0, M\}$*, the premium* $\{P^*(\phi^j)\}_j$*, and the depositors' plan* $(a^*, c^*, \{D^{j*}\}_j)$*, of which each variable solves each maximization problem as stated above.*

To close the description of our model, we should note two things. We assume that the amount of insured deposit k and the loan size L are assumed to be exogenous. In other words, we do not explicitly model the optimal decision for k by the government and also firms and banks do not negotiate for L. Making these choices endogenous is interesting as it involves extension of this model to a general equilibrium framework. However, our focus in this paper is to provide a simplest possible model and to study how changing those variables affects depositors' behavior.

We intend to study the relation between the depositors' welfare and the monitoring problem along with deposit insurance. The key players in this model are the bank, the firm, and depositors. Therefore, we start with depositors' optimal decision and characterize the equilibrium premium.

3 Equilibrium Analysis

3.1 Optimal decision by depositors

In this subsection, we consider the optimal action and consumption plan for depositors. Depositors decide whether they deposit and choose the optimal consumption plan if they deposit. We consider the expected utility for the depositors following their decision a in order to study their optimal action. First, if they do not deposit, the bank cannot lend money to the firm and the firm cannot produce anything. A depositor's utility is then:

$$\mathrm{E}U(c^{n*}|No) = u(c^{n*}) + \beta u(\omega - c^{n*}), \tag{8}$$

where c^{n*} satisfies $u'(c^{n*}) = \beta u'(\omega - c^{n*})$.

Second, we consider the case of deposit. For each effort level $j \in \{h, l\}$, with probability ϕ^j the good state occurs and the bad state occurs with probability $1 - \phi^j$. If a depositor deposits, the expected utility function becomes:

$$\mathrm{E}U(c_j|De, j) = u(c_{0,j}) + \beta\phi^j \cdot u(c^G_{1,j}) + \beta(1 - \phi^j) \cdot u(c^B_{1,j}). \tag{9}$$

Further, given the utility function is increasing in consumption, at the optimal level depositors use all available resources and so the budget constraint (2) implies that in equilibrium $c_{0,j} = \omega - D^j - P(\phi^j)$, $c^G_{1,j} = (1 + r_D) \cdot D^j$ and $c^B_{1,j} = k \cdot D^j$ for each $j \in \{h, l\}$. Substituting these into (9), taking the first derivative of the expected utility, and setting it to zero, then for each $j \in \{h, l\}$, the expected utility in equilibrium is:

$$\mathrm{E}U(c^*_j|De, j) = u(\omega - D^{j*} - P^*(\phi^j)) + \beta\phi^j u((1 + r_D)D^{j*}) + \beta(1 - \phi^j)u(kD^{j*}), \tag{10}$$

where $c^*_{0,j} = \omega - D^{j*}$ and D^{j*} satisfies the first order condition:

$$-u'(\omega - D^{j*} - P^*(\phi^j)) + \beta\phi^j(1 + r_D)u'((1 + r_D)D^{j*}) + \beta(1 - \phi^j)ku'(kD^{j*}) = 0. \tag{11}$$

Comparing (8) with (10), we can conclude that for each $j \in \{h, l\}$ depositors deposit money if:

$$\mathrm{E}U(c^*_j|De, j) \geq \mathrm{E}U(c^{n*}|No), \tag{12}$$

where D^{j*} satisfies (11). Thus, we obtain the following.

Lemma 1. *In equilibrium, depositors deposit if (12) holds when the firm chooses $e^* = e^j$.*

Before proceeding with the monitoring problem, let us characterize the equilibrium premium and the relationship between the premium and the amount insured. As a shorthand, we write $(1+r^j)P^*(\phi^j) = X(\phi^j)$ and $\frac{(1+r^j+k)P^*(\phi^j)}{k} = Y(\phi^j)$. Then, by substituting $D^{j*} = \frac{(1+r^j)P^*(\phi^j)}{k}$ from (7) into (11):

$$\beta\left[\phi^j(1 + r_D)u'(\frac{(1 + r_D)X(\phi^j)}{k}) + (1 - \phi^j)ku'(X(\phi^j))\right] = u'(\omega - Y(\phi^j)). \tag{13}$$

Next, we consider how the equilibrium premium changes as the risk ϕ^j changes. To further investigate this, we use a measure of risk-aversion. Let $A(c)$ denote an absolute Arrow–Pratt risk-aversion measure $-\frac{u''(c)}{u'(c)}$ and $R(c) := cA(c)$ denote a relative risk-aversion measure.

Proposition 1. *If* $R(X(\phi^j)) \geq 1$*, then* $\frac{\mathrm{d}P^*(\phi^j)}{\mathrm{d}\phi^j} \leq 0$ *with equality for a logarithmic utility function. If* $R(X(\phi^j)) < 1$*, then* $\frac{\mathrm{d}P^*(\phi^j)}{\mathrm{d}\phi^j} > 0$.

Proof of Proposition 1. Found in Appendix.

By applying monotonicity of the equilibrium premium shown in Proposition 1, we obtain the following corollary.

Corollary 1. *If* $R(X(\phi^j)) \geq 1$ *for each* $j \in \{l, h\}$, $P^*(\phi^h) \leq P^*(\phi^l)$ *with equality for a logarithmic utility function. If* $R(X(\phi^j)) < 1$ *for each* $j \in \{l, h\}$, $P^*(\phi^h) > P^*(\phi^l)$.

Intuitively, when a consumer is risk averse enough so that a deposit insurance is important for a consumer, the equilibrium premium increases with the risk involved. Otherwise, depositors pay more on insurance along with their expected income. So, the premium is monotonically increasing in this situation. Proposition 1 states that a relative risk aversion measure equal to one is the boundary for the two consequences.

Here, we further study a particular class of utility functions for which Proposition 1 and Corollary 1 hold so that we can obtain clear intuitions on how these results work. As discussed previously, the results hold for any strict concave utility functions. To make an example, we consider a hyperbolic utility function (HARA), which is a general class of utility functions that are used in practice. For a hyperbolic utility functions, $A(c) = \frac{1}{ac+b}$ holds and a constant absolute risk averse utility function (CARA) or a constant relative risk averse utility function (CRRA) belongs to this class.[5] When a is zero, then u is a CARA utility function and $A(x) = \frac{1}{b}$ holds. When b is zero, then u is a CRRA utility function and $R(x) = \frac{1}{a}$. So as an example of Proposition 1, we summarize this observation as follows:

Example 1. *The equilibrium premium is decreasing in* ϕ^j *if a utility function* u *satisfies:*

- $a = 0$ *and* b *is small enough (a CARA utility function with relatively large risk aversion); or*
- $b = 0$ *and* $a \leq 1$ *(a CRRA utility function with relatively large risk aversion).*

By using ideas in Example 1, we obtain three specific utility functions to show the implications in Proposition 1. Each example describes each case of Proposition 1 where the equilibrium premium is constant, decreasing and increasing in ϕ^j.

1. Let $u(c) = \log c$. This is the boundary case. Note that in this case, u is a CRRA utility function as $R(c) = -c\frac{u''(c)}{u'(c)} = 1$ is constant. Then, by substituting $u'(c) = \frac{1}{c}$ into (13), we obtain $P^*(\phi^j) = \frac{k\omega\beta}{(1+\beta)(1+r^j)+k\beta}$. As Proposition 1 indicates, we can see that the equilibrium premiums is constant, when a utility function u is $\log c$, which is a CRRA utility function with $R(c) = 1$.

[5] When $a > 0$, this class of utility functions is called a DARA utility function and Friend and Blume (1975) states that experimental and empirical evidence is mostly consistent with this class of utility functions. For example, Goodhard et. al. (2005) use a log function to model household's loan demand and proposes a model to assess risk for banks.

2. Let $u(c) = -\frac{1}{c}$. This is a CRRA utility function with $R(c) = 2$. By (13), we obtain:

$$\beta\left[\frac{\phi^j k^2}{(1+r_D)(1+r^j)^2} + \frac{(1-\phi^j)k}{(1+r^j)^2}\right] = \left(\frac{P^*(\phi^j)}{\omega - \frac{(1+r^j+k)}{k}P^*(\phi^j)}\right)^2. \quad (14)$$

As the coefficient of ϕ^j is $\left(\frac{k}{(1+r_D)} - 1\right)\frac{k}{(1+r^j)^2} < 0$, the LHS of (14) decreases as ϕ^j increases. So, as Proposition 1 indicates, $P^*(\phi^j)$ decreases by (14).

3. Let $u(c) = 2\sqrt{c}$. This is a CRRA utility function with $R(c) = \frac{1}{2}$. By (13), we obtain:

$$\beta\left[\phi^j\sqrt{\frac{k(1+r_D)}{(1+r^j)}} + (1-\phi^j)k\sqrt{\frac{1}{(1+r^j)}}\right] = \sqrt{\frac{P^*(\phi^j)}{\omega - \frac{(1+r^j+k)}{k}P^*(\phi^j)}}. \quad (15)$$

As the coefficient of ϕ^j is $\sqrt{\frac{k}{(1+r^j)}}\left(\sqrt{(1+r_D)} - \sqrt{k}\right) > 0$, we can see that $P^*(\phi^j)$ is increasing in ϕ^j by (15).

In many areas of economics, the Arrow–Pratt absolute and relative risk aversion measures play a significant role for analyzing choice problems under uncertainty. The literature has found that the comparative static results in many expected utility models crucially depend on the value of relative risk aversion (see Choi and Menezes (1992) for a further discussion). In this model, as Proposition 1 indicates, relative risk-aversion is a key factor to determine the relationship between the equilibrium premium and the risk. Intuitively, it is understandable that when depositors are more risk-averse, they value insurance more and the equilibrium premium becomes higher in a riskier situation. On the other hand, if the probability of success ϕ^j is higher, then the expected income for depositors is higher. So, they can pay more on insurance. Proposition 1 states that a relative risk-aversion measure draws the line between these two forces and as depositors become more risk-averse, then they would hedge the risk by spending more on insurance. Proposition 1 conveys this intuition.

The next result shows that as the amount insured increases, the equilibrium premium also increases. It then compares the equilibrium associated with k_1 and one associated with k_2 with $k_1 > k_2$ and proves that the premium for k_1 is higher than one for k_2.

Proposition 2. *As k increases, $P^*(\phi^j)$ also increases.*

Proof of Proposition 2. Found in Appendix.

Note that Proposition 2 holds for any risk-averse utility function. In (13), the LHS is a change of day-1 utility and the RHS is a change of day-0 utility. The first order condition (13) dictates that in equilibrium, change in utility from a change of deposit must be equal to change in utility associated with deposit. To balance these two changes in (13), strict concavity of a utility function implies that the equilibrium premium is higher while expected payment in the next period is larger.

Proposition 2 or Proposition 1 cannot say whether or not D^{*j} increases as k or ϕ^j increases because it depends on how much $P^*(\phi^j)$ changes. Moreover, one may ask if D^{h*} is greater than D^{l*}. Even in the environment where Proposition 1 holds, we cannot draw a general conclusion about this relationship because it also depends on the premiums and the ratio $\frac{1+r^h}{1+r^l}$ by the government budget (7). At the end of the next subsection, we study this in more detail by using a specific utility function and discussing its connection with the monitoring problem.

3.2 Monitoring problem

In this subsection, we consider the optimal action by the firm and then the bank. Now consider the firm's problem and the firm has two alternatives, namely, either a low or a high effort level. Now we consider the incentive-compatible contract that makes the firm exert high effort. The condition states that:

$$\phi^h \cdot [y - (1 + r_L)L - g] - (1 - \phi^h) \cdot g \geq \phi^l \cdot [y - (1 + r_L)L], \tag{16}$$

and the individual rationality condition states that:

$$\phi^h \cdot [y - (1 + r_L)L - g] - (1 - \phi^h) \cdot g \geq W_0. \tag{17}$$

By assumption, W_0 is low enough and so (17) holds. This will allow us to focus on the incentive compatibility condition (16). Now suppose that L takes value in the parameter space such that (16) does not hold.[6] We consider the bank's monitoring problem. If the bank does or does not monitor, the expected payoff for the bank in each case is:

$$\begin{aligned} \mathrm{E}\Pi(D^h, M) &= \frac{\phi^h(1 + r_L)L}{1 + r^h} - L + \frac{r^h - r_D}{1 + r^h} \cdot nD^h - M; && (18) \\ \mathrm{E}\Pi(D^l, 0) &= \frac{\phi^l(1 + r_L)L}{1 + r^l} - L + \frac{r^l - r_D}{1 + r^l} \cdot nD^l. && (19) \end{aligned}$$

Thus, we can state the result as follows.

Lemma 2. *The bank monitors if* (18) $\geq$ (19).

Now we study how the amount insured affects the bank's balance sheet and then the monitoring problem. The government's budget (7) connects them in our model. Here we restrict our attention to one class of equilibria by imposing one condition. To define this condition, consider the relationship between the market interest rate (r^j) and loan risk that is proxied by ϕ^j by looking at the expected profits from the loan (5). Equation (5) could be restated as:

$$E\Pi(D^j, M) = \frac{\phi^j(1 + r_L) - (1 + r^j)}{1 + r^j} L + \frac{r^j - r_D}{1 + r^j} nD^j - M. \tag{20}$$

Equation (20) makes it obvious that $(1 + r^j)$ is the opportunity cost for every dollar loaned to the firm, with $\phi^j(1 + r_L)$ being the expected return on the dollar loaned. Therefore, $\frac{\phi^j(1+r_L)-(1+r^j)}{1+r^j}$

[6]We can make y, or the probabilities ϕ as a function of L and under some technical assumptions, we can prove that there exists such an interval. However, in this model, the choice of L is not endogenous and to keep our analysis as simple as possible, we assume that for L the incentive compatibility does not hold. Moreover, if (16) holds, then monitoring would not really be a problem.

is the marginal increase in the bank's expected present valued payoffs for every dollar loaned. If $\phi^j(1+r_L)-(1+r^j)<0$, then the bank is strictly better off not lending any money to the firm, and instead, investing all available funds (nD^j) into market securities paying r^j. For the bank to lend L to the firm, for each j, it must be that $\phi^j(1+r_L)\geq(1+r^j)$. On the other hand, if $(1+r^j)<\phi^j(1+r_L)$, there is an ex ante arbitrage opportunity for other banks. For example, we can think of Bertrand-type competition in r_L between banks. Thus, we assume that perfect competition between many banks brings the market interest rate r^j and the risk proxy ϕ^j into the following condition. For all L and $j\in\{h,l\}$:

$$(1+r^j)L=\phi^j(1+r_L)L. \tag{21}$$

Definition 2. *We say that there is* no arbitrage opportunity for investors *when (21) holds.*

We would like to know when k changes, how D^{j*} changes for each j and if possible, what is the difference between D^{h*} and D^{l*}. Note that by (7) and (21):

$$\frac{\mathrm{E}\Pi(D^h,M)-\mathrm{E}\Pi(D^l,0)}{n}=\frac{r^h-r_D}{1+r^h}\cdot D^h-\frac{r^l-r_D}{1+r^l}\cdot D^l-\frac{M}{n}. \tag{22}$$

Looking at the first-order condition (11), it would be very complicated to compute a general result on how D^{j*} moves in equilibrium as k changes. To obtain a simple intuition, again let $u(c)=\log(c)$. The log-utility function allows us to compute the equilibrium deposit and consumption in day 0 as a fraction of $\omega-P^*(\phi^j)$ and simplifies the calculation. The following proposition shows that when k increases, D^{j*} decreases when $u(c)=\log(c)$ and it decreases more for $j=l$ than $j=h$. In other words, the effect on deposit by changing k is larger when $j=l$.

Proposition 3. *Suppose $u(c)=\log(c)$ and there is no arbitrage opportunity for investors. As k increases, $\mathrm{E}\Pi(D^{h*},M)$ and $\mathrm{E}\Pi(D^{l*},0)$ decrease, and also $0>\frac{\mathrm{d}D^{h*}}{\mathrm{d}k}>\frac{\mathrm{d}D^{l*}}{\mathrm{d}k}$.*

Proof of Proposition 3. Found in Appendix.

Note that by (21) we obtain $r^h>r^l$ when there is no arbitrage opportunity for investors. As explained in the previous section, r can be thought of as a market interest rate for other securities available to the bank. It is intuitive that when the good state occurs with a higher probability, the interest rate for the market, r^h, is higher. One interesting case is where $r^h>r^D>r^l$. This means that if the firm exerts a high effort, then the bank could gain more from the markets, but if the firm exerts a low effort, then the bank could make a loss from their investment in other securities because the difference in the interest rates on deposits and other securities is less beneficial to the bank. To see this more clearly, note that (5) could be restated as:

$$E\Pi(D^j,M)=\left(\frac{\phi^j(1+r_L)-(1+r_D)}{1+r^j}\right)L+\left(\frac{r^j-r_D}{1+r^j}\right)(nD^j-L)-M, \tag{23}$$

where $\left(\frac{r^j-r_D}{1+r^j}\right)(nD^j-L)$ represents the net interest earnings on market securities. For a sufficiently large n, $L<\min_{j\in\{h,l\}}(nD^{j*})$ holds and thus for $j=h$, this interest earnings is strictly positive while for $j = l$, this term is negative. The following proposition considers this case and shows that when the firm makes high efforts, the bank makes profits in investing in other securities, and so the bank's payoff decreases because of the reduction in deposits after the change in k. Conversely, when the firm makes low efforts, the bank incurs a loss from investing in other securities, and so the bank gains from the reduction in deposits after the change in k.

Proposition 4. *Suppose $u(c) = \log(c)$ and there is no arbitrage opportunity for investors. As k increases, $\mathrm{E}\Pi(D^{h*}, M) - \mathrm{E}\Pi(D^{l*}, 0)$ decreases if $r^h > r_D > r^l$.*

Proof of Proposition 4. Found in Appendix.

Note that the same result in Proposition 4 holds when $r^l < r^h \leq r_D$. Given $r^l < r^h \leq r_D$, the net interest spread $(r^j - r_D)$ on market securities is negative for $j \in \{h, l\}$, in which case the rate of net interest expense in current valued terms is higher for $j = l$ than $j = h$, that is $\frac{r_D - r^l}{1+r^l} > \frac{r_D - r^h}{1+r^h} > 0$. Thus, changing k has a larger effect when the firm makes low effort ($j = l$) because the reduction in net interest expense on residual deposits not committed to the loan is greater when $j = l$. At the same time, as k increases, the difference between (18) and (19) decreases and hence Lemma 2 is less likely to hold. In other words, the bank would be less likely to monitor as k increases.

3.3 Equilibrium and comparative statics

So far, we have considered optimal decisions by the bank, the firm, and depositors. In this subsection, we first consider the equilibrium as a whole. By Lemma 2, we know that when $(18) \geq (19)$ the bank monitors and otherwise it does not. Given Lemma 1 specifies the condition under which depositors deposit, there will be four cases depending on whether or not each of Lemma 1 and Lemma 2 holds. First, we analyze the equilibrium under the following two cases **Case 1.** $(18) \geq (19)$ and **Case 2.** $(18) < (19)$ because this condition specifies if the firm makes low or high effort, which specifies the level of effort (12) that should hold for depositors to deposit. The next proposition is a simple consequence of Lemma 1 and Lemma 2.

Proposition 5. *In equilibrium, there are two possible cases:*

Case 1. *Depositors deposit, the firm exerts high effort and the bank monitors, if (12) holds for $j = h$;*

Case 2. *Depositors deposit, the firm exerts low effort and the bank does not monitor, if (12) holds for $j = l$.*

We can study the implications of Proposition 5 on the depositors' behavior further. First, if ϕ^j is higher, then it is more likely for (12) to hold for each $j \in \{l, h\}$. Second, if the bank does not monitor and the firm only exerts a low level of effort, then the depositors deposit at most D^{l*}or not at all. This depends on whether (12) holds for $j = l$. Compared with the case of high firm effort, the expected utility of depositing is lower in the case of low effort (i.e., $EU(c^{l*}|De, l) < EU(c^{h*}|De, h)$). Therefore, we can say that (12) for $j = l$ is less likely to hold than for $j = h$, and hence depositors are more likely to deposit in the high effort level case. We more formally state this argument about market risk and the depositors' decision in the following result.

Proposition 6. *For an n that is sufficiently large, the following holds.*

Case 1. *For a sufficiently high ϕ^h and a sufficiently small k, depositors deposit money;*

Case 2. *For a sufficiently low ϕ^l and $k < 1$, depositors do not deposit money.*

Proof of Proposition 6. Found in Appendix.

Proposition 6 provides the implication about depositors' behavior and the market risk ϕ^j for each j. The result for Case 1 might be counter-intuitive. This result holds as premium is related to k. If k is higher, premium is more costly. Thus even if ϕ^h is high, depositors do not deposit if k is high as there is still a risk to lose deposits and in addition they have to pay more costly premiums. Therefore in this situation, depositors do not deposit.

Empirically, and as mentioned in the introduction, we observe that depositors may require a higher rate of interest on their deposit, given the risk taken on by the bank in committing to the loan. If r_D is sufficiently close to 0, then $u(\omega - D^* - P^*) + \beta \cdot u((1+r_D)\cdot D^*)$ is close to $u(\omega - D^* - P^*) + \beta \cdot u(D^*)$, while $u(c^{n*}) + \beta \cdot u(\omega - c^{n*})$ does not change as it is independent of r_D. One may wonder how the range of ϕ^h that makes it possible for (41) to hold would change as r_D increases. The answer also depends on the equilibrium premium as depositors are price takers. Now we consider this question by conducting a comparative statics analysis. To do so, we define the maximized expected utility for depositing by $\mathscr{X}$ as the function of k, r_D and ϕ^j:

$$\mathrm{E}U(c_j^*|De, j) := \mathscr{X}(r_D, k, \phi^j). \tag{24}$$

We define the certainty equivalent $I(D, r_D, k, \phi^j)$[7], which will be useful in the following analysis, as:

$$u(I(D, r_D, k, \phi^j)) = \phi^j u((1 + r_D)D) + (1 - \phi^j)u(kD). \tag{25}$$

By using the certainty equivalent, in equilibrium the maximization problem chooses $D^{*j} = \mathscr{D}^j(r_D, k, \phi^j)$, which satisfies for given P^*, we have:

$$u'(\omega - D^{*j} - P^*) = \beta u'(I(D^{*j}, r_D, k, \phi^j)) \frac{\mathrm{d}I(D^{*j}, r_D, k, \phi^j)}{\mathrm{d}D}, \tag{26}$$

and $P^* = \mathscr{P}(r_D, k, \phi^j)$ satisfies (7).

Calligraphic letters $\mathscr{X}$, $\mathscr{D}$ and $\mathscr{P}$ are functions of exogenous variables (r_D, k, ϕ^j) and represent the associated equilibrium variables if depositors deposit in equilibrium. As r_D increases, $\mathscr{X}(r_D, k, \phi^j)$ may increase and the upper bound of ϕ^l, which makes it possible for any D to satisfy (43) could decrease, although it also depends on how P^* changes. As discussed in Proposition 1, we cannot say how the equilibrium premium changes in general as ϕ^j changes. To study this relationship in more detail, we take the derivative of $\mathscr{X}$ with respect to r_D and ϕ^j. To discuss the relationship between the interest rate on deposit, risk, and depositors' behavior, we define the following two sets:

$$\Phi^h(r_D) = \{\phi^h : \exists D^{h*} \text{ satisfying (41) for a certain } r_D \text{ and the equilibrium } P^*\};$$
$$\Phi^l(r_D) = \{\phi^l : \forall D^{l*}, \text{ (43) holds for a certain } r_D \text{ and the equilibrium } P^*\}.$$

[7] We thank Terence Yeo for pointing out that certainty equivalence could be used for a simpler proof.

Then we obtain the following result.

Proposition 7. *As r_D increases, Φ^h expands and Φ^l shrinks, if and only if* $\frac{\mathrm{d}\mathscr{P}}{\mathrm{d}r_D} < 0$ *and* $\frac{\mathrm{d}\mathscr{P}}{\mathrm{d}\phi} < 0$.

Proof of Proposition 7. Found in Appendix.

The intuition of Proposition 7 is straightforward. If the equilibrium premium is lowered and the interest rate on deposits is increased, then the range of risk in which consumers would deposit in equilibrium expands, and the range of risk for which consumers would not deposit contracts. We could also interpret Proposition 7 as saying that if for a certain level of risk and interest rate on deposits, consumers prefer not depositing to depositing, then to induce consumers to deposit, the interest rate on deposits needs to be higher.

Now note that Proposition 7 holds for any $k < 1$. In an extreme case of $k = 0$ where deposits are uninsured, we must set $P = 0$. Then a similar result to Proposition 7 still holds. The following result states this. Take two distinct interest rates on deposit r_1 and r_2. Denote by D^{1j} and D^{2j} the equilibrium deposits for each j associated with r_1 and r_2, respectively.

Proposition 8. *Suppose that* $k = 0$ *and* $P = 0$. *If* $\mathscr{X}(r_1, 0, \phi) = \bar{u}$ *and* $\phi \in \Phi^h(r_2)$, *then* $r_1 \leq r_2$. *If* $\mathscr{X}(r_1, 0, \phi) = \bar{u}$ *and* $\phi \in \Phi^l(r_2)$, *then* $r_1 > r_2$.

Proof of Proposition 8. Found in Appendix.

It is worth mentioning that the theoretical observations in Propositions 7 and 8 are consistent with the empirical research on depositor behavior (see Park (1995) or Park and Peristiani (1998)). Market discipline should result in higher interest rates and slower growth of uninsured deposits at risky banks. When the bank's situation is riskier, the interest rate on deposits needs to be higher to make depositors willing to deposit. As r_D is exogenously given, it could be misleading to use the word, "depositors' discipline." However, by using comparative statics, we can say that holding everything else constant, if the bank is more likely to fail, the interest rate needs to be higher to attract deposits. Propositions 7 and 8 give a theoretical explanation for this, and at the same time provide a testable hypothesis on depositors' behavior and partially insured deposits.

4 Conclusion

Banking operations may be varied and complex. According to Freixas and Rochet (2008), a simple operational definition of a bank that regulators use is: *A bank is an institution whose current operations comprise granting loans and receiving deposits from the public.* Along with this definition, this paper has developed the simplest possible model to discuss the effects of deposit insurance and the moral hazard problem. This paper has also characterized a risk-based premium in equilibrium and conducted comparative statics analysis of depositors' optimal actions. Moreover, the analysis showed that depositors might not deposit money unless the interest rate on deposits is sufficiently high or the risk of bank failure is sufficiently low. This result is consistent with the extant empirical work on depositor behavior about riskiness and the interest rate.

Various extensions are possible from our model. First, one may introduce heterogeneous banks or firms. Then, in equilibrium depositors could choose which bank they deposit money with. Another possibility is to make the choice of the insured amount or interest rates for deposits endogenous. Those research would require us to extend our framework to a general equilibrium framework. Our framework is sufficiently simple and well behaved and provides a path to the future research, such as the design of an optimal deposit insurance.

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Appendix: Proofs

Lemma 3. *The equilibrium exists uniquely.*

Proof of Lemma 3. First fix $P(\phi^j)$. Given u is strictly increasing and concave and given $P(\phi^j)$, there exists a unique consumption plan $c = c(P(\phi^j))$, which maximizes (1) subject to the set of the budget constraints (2). As u is increasing, (2) holds with equalities in equilibrium and then we obtain a unique deposit $D^j = D^j(P(\phi^j))$. As $P(\phi^j)$ is fixed arbitrarily, we obtain the optimal deposit plans as a function of $P(\phi^j)$, which we denote by $D^{j*}(P(\phi^j))$. Substituting $D^j = D^{j*}(P(\phi^j))$ into (7) we obtain a unique $P^*(\phi^j)$ by solving $P^*(\phi^j) = \frac{kD^{j*}(P(\phi^j))}{(1+r^j)}$. Substituting $P^*(\phi^j)$ into $c^* = c(P^*(\phi^j))$ and $D^{j*} = D^{j*}(P^*(\phi^j))$, we obtain a unique c^* and D^{j*} for each j. In this way, we obtain the following result. □

Lemma 4. *If* $R(x) \geq 1$, $\frac{\mathrm{d}xu'(x)}{\mathrm{d}x} \leq 0$ *holds. If* $R(x) < 1$, $\frac{\mathrm{d}xu'(x)}{\mathrm{d}x} > 0$ *holds.*

Proof of Lemma 4. The result is direct by noting that $\frac{\mathrm{d}xu'(x)}{\mathrm{d}x} = u'(x) + xu''(x)$. □

Proof of Proposition 1. By Lemma 3, the equilibrium exists uniquely for each ϕ^j. Therefore there must exist $X(\phi^j)$ and $Y(\phi^j)$ for each ϕ^j that satisfy (13). Suppose that:

$$(1 + r_D)u'(\frac{(1 + r_D)X(\phi^j)}{k}) - ku'(X(\phi^j)) \neq 0, \tag{27}$$

so that the LHS of (13) becomes a function of ϕ^j. Taking the derivative of the LHS of (13) w.r.t. ϕ^j[8] and dividing it by β, we obtain:

$$\begin{aligned}(1+r_D)\left[u'(\tfrac{(1+r_D)X(\phi^j)}{k}) + \tfrac{(1+r_D)X'(\phi^j)\phi^j}{k}u''(\tfrac{(1+r_D)X(\phi^j)}{k})\right] \\ -k\left[u'(X(\phi^j)) - (1-\phi^j)X'(\phi^j)u''(X(\phi^j))\right].\end{aligned} \tag{28}$$

As $A(\frac{(1+r_D)X(\phi^j)}{k}) = -\frac{u''(\frac{(1+r_D)X(\phi^j)}{k})}{u'(\frac{(1+r_D)X(\phi^j)}{k})}$, (28) becomes

$$\begin{aligned}(1+r_D)\left[1 - \tfrac{(1+r_D)X'(\phi^j)\phi^j}{k}A(\tfrac{(1+r_D)X(\phi^j)}{k})\right]u'(\tfrac{(1+r_D)X(\phi^j)}{k}) \\ -k\left[1+(1-\phi^j)X'(\phi^j)A(X(\phi^j))\right]u'(X(\phi^j)).\end{aligned} \tag{29}$$

Now on the contrary, suppose that $R(X(\phi^j)) \geq 1$ and P^* is increasing so that $X'(\phi^j)$ is positive. Then, because

$$1 - \frac{(1+r_D)X'(\phi^j)\phi^j}{k}A(\frac{(1+r_D)X(\phi^j)}{k}) < 1 + (1-\phi^j)X'(\phi^j)A(X(\phi^j)),$$

$$(29) < \left[1+(1-\phi^j)X'(\phi^j)A(X(\phi^j))\right]\cdot\left[(1+r_D)u'(\frac{(1+r_D)X(\phi^j)}{k}) - ku'(X(\phi^j))\right]. \tag{30}$$

By Lemma 4, we obtain:

$$(1+r_D)u'(\frac{(1+r_D)X(\phi^j)}{k}) - ku'(X(\phi^j)) \leq 0. \tag{31}$$

Therefore (28) is negative and so the LHS of (13) decreases when ϕ^j increases. On the other hand, we have:

$$\begin{aligned}\frac{\mathrm{d}u'(\omega - Y(\phi^j))}{\mathrm{d}\phi^j} &= -u''(\omega - Y(\phi^j))\cdot Y'(\phi^j) \\ &= A(\omega - Y(\phi^j))u'(\omega - Y(\phi^j))\cdot Y'(\phi^j).\end{aligned} \tag{32}$$

Given the LHS of (13) decreases when ϕ^j increases, which indicates the RHS of (32) is strictly negative, it must be the case that $Y'(\phi^j) = \frac{(1+r^j+k)}{k}\cdot\frac{\mathrm{d}P^*(\phi^j)}{\mathrm{d}\phi^j} < 0$. Thus, we conclude $\frac{\mathrm{d}P^*(\phi^j)}{\mathrm{d}\phi^j} < 0$. This is a contradiction to our assumption that P^* is increasing. Thus, we conclude that P^* is decreasing in ϕ^j.

Second on the contrary, suppose that $R(X(\phi^j)) < 1$ and P^* is decreasing so that $X'(\phi^j)$ is negative. Then, similarly with the above, because

$$1 - \frac{(1+r_D)X'(\phi^j)\phi^j}{k}A(\frac{(1+r_D)X(\phi^j)}{k}) > 1 + (1-\phi^j)X'(\phi^j)A(X(\phi^j)),$$

we obtain

$$\begin{aligned}(29) &> \left[1+(1-\phi^j)X'(\phi^j)A(X(\phi^j))\right]\cdot\left[(1+r_D)u'(\tfrac{(1+r_D)X(\phi^j)}{k}) - ku'(X(\phi^j))\right] \\ &> 0,\end{aligned} \tag{33}$$

[8]The derivative is well defined because both sides of (13) are continuously differentiable.

where the last inequality holds by Lemma 4. Similarly to the first case, we can obtain a contradiction as (33) implies that the LHS of (13) increases in ϕ^j, which indicates the RHS of (32) is strictly positive. This implies that $Y'(\phi^j) = \frac{(1+r^j+k)}{k} \cdot \frac{\mathrm{d}P^*(\phi^j)}{\mathrm{d}\phi^j} > 0$, which is a contradiction to our assumption that P^* is decreasing. Thus, we conclude that P^* is increasing in ϕ^j.

Finally, we consider the case where (27) holds. We can see that in this case, $u'(x)$ is linear with respect to $\frac{1}{x}$. The only function that satisfies this is logarithmic functions. In this case, the equilibrium premium is constant as the LHS becomes constant. This completes the proof. □

Proof of Corollary 1. Notice that $\frac{1-\phi^l}{\phi^l} > \frac{1-\phi^h}{\phi^h}$. Thus, if $\frac{(1+r_D)}{k} < \frac{1-\phi^h}{\phi^h}$, then the same condition holds for ϕ^l. By applying Lemma 1, we obtain the desired result. □

Proof of Proposition 2. Suppose that $k_1 > k_2$ and on the contrary suppose that the equilibrium premiums associated with k_1 and k_2, $P^1(\phi^j)$ and $P^2(\phi^j)$, satisfy $P^1(\phi^j) \leq P^2(\phi^j)$. Thus, we obtain $\frac{P^1(\phi^j)}{k_1} < \frac{P^2(\phi^j)}{k_2}$. Then, by strict concavity of u:

$$\begin{aligned}&\phi^j(1+r_D)u'(\tfrac{(1+r_D)(1+r^j)P^1(\phi^j)}{k_1}) + (1-\phi^j)k_1u'((1+r^j)P^1(\phi^j)) \\ &\quad > \phi^j(1+r_D)u'(\tfrac{(1+r_D)(1+r^j)P^2(\phi^j)}{k_2}) + (1-\phi^j)k_2u'((1+r^j)P^2(\phi^j)).\end{aligned} \tag{34}$$

On the other hand, $\frac{(1+r^j)P^1(\phi^j)}{k_1} + P^1(\phi^j) < \frac{(1+r^j)P^2(\phi^j)}{k_2} + P^2(\phi^j)$, which by strict concavity of u implies:

$$u'(\omega - \frac{(1+r^j+k_1)P^1(\phi^j)}{k_1}) < u'(\omega - \frac{(1+r^j+k_2)P^2(\phi^j)}{k_2}). \tag{35}$$

Notice that (34) contradicts (35) because $(k_1, P^1(\phi^j))$ and $(k_2, P^2(\phi^j))$ must satisfy (13). □

Proof of Proposition 3. By (7) we obtain $D^{j*} = \frac{(\omega - P^*(\phi^j))\beta}{1+\beta} = \frac{(1+r^j)P^*(\phi^j)}{k}$. Thus, we obtain $P^*(\phi^j) = \frac{k\omega\beta}{\beta k + (1+r^j)(1+\beta)}$ and $D^{j*} = \frac{\omega\beta(1+r^j)}{\beta k + (1+r^j)(1+\beta)}$; therefore:

$$\frac{\mathrm{d}D^{j*}}{\mathrm{d}k} = -\frac{\omega\beta^2(1+r^j)}{[\beta k + (1+r^j)(1+\beta)]^2} = -\frac{\omega(1+r^j)}{[k + (1+r^j)(1+\beta)/\beta]^2}. \tag{36}$$

Then:

$$-\frac{\mathrm{d}\frac{\omega(1+r)}{[k+(1+r)(1+\beta)/\beta]^2}}{\mathrm{d}r} = -\frac{\omega}{[k+(1+r)(1+\beta)/\beta]^2} + 2\frac{\omega(1+r)(1+\beta)/\beta}{[k+(1+r)(1+\beta)/\beta]^3} > 0, \tag{37}$$

because $k \leq 1 < (1+r)(1+\beta)/\beta$ implies $k + (1+r)(1+\beta)/\beta < 2(1+r)(1+\beta)/\beta$.

Thus, $0 > \frac{\mathrm{d}D^{h*}}{\mathrm{d}k} > \frac{\mathrm{d}D^{l*}}{\mathrm{d}k}$, which completes the proof.

□

Proof of Proposition 4. As $0 > \frac{\mathrm{d}D^{h*}}{\mathrm{d}k} > \frac{\mathrm{d}D^{l*}}{\mathrm{d}k}$ as in Proposition 3, and $\frac{r^h - r_D}{1+r^h} > 0 > \frac{r^l - r_D}{1+r^l}$, by Proposition 3 we obtain:

$$\frac{\mathrm{d}D^{h*}}{\mathrm{d}k} \cdot \frac{r^h - r_D}{1+r^h} < 0 < \frac{\mathrm{d}D^{l*}}{\mathrm{d}k} \cdot \frac{r^l - r_D}{1+r^l}. \tag{38}$$

Therefore, by (22) we conclude:

$$\frac{\mathrm{d}\,\frac{\mathrm{E}\Pi(D^h,M)-\mathrm{E}\Pi(D^l,0)}{n}}{\mathrm{d}k} = \frac{\mathrm{d}D^{h*}}{\mathrm{d}k}\cdot\frac{r^h - r_D}{1+r^h} - \frac{\mathrm{d}D^{l*}}{\mathrm{d}k}\cdot\frac{r^l - r_D}{1+r^l} < 0. \tag{39}$$

□

Proof of Proposition 6. First, we consider Case 1. Take a sequence of $\{\phi^h_m\}_m$ for each j and denote the equilibrium deposits and premium by (D^{*j}_m, P^{j*}_m) associated with each ϕ^j_m for $j \in \{h, l\}$. Suppose that ϕ^h_m converges to 1 and that given r_D and P^*, $x = D^*$ maximizes $u(\omega - x - P^*) + \beta \cdot u((1+r_D)\cdot x)$ where P^* satisfies $P^* = \frac{kD^*}{(1+r^h)}$. By Lemma 3, the equilibrium exists uniquely and by the same argument with Lemma 3 (D^*, P^*) exists uniquely, because $x \leq \omega$ by feasibility and u is strictly concave, there is a unique x^* that satisfies $u'(\omega - x - P) = \beta(1+r_D)\cdot u'((1+r_D)\cdot x)$ for given P. Notice that (D^{h*}_m, P^{h*}_m) converges to (D^*, P^*) as $m \to \infty$. Otherwise, there would be a different $(\hat{D}, \hat{P})$ that (D^{h*}_m, P^{h*}_m) converges to and this violates the unique existence of (D^*, P^*). Then, as $m \to \infty$, the LHS of (12) for $j = h$ converges to $u(\omega - D^* - P^*) + \beta \cdot u((1+r_D)\cdot D^*)$. When k is sufficiently small, by the government budget (7), P^* is sufficiently small. Then we must have:

$$u(\omega - D^* - P^*) + \beta \cdot u((1+r_D)\cdot D^*) \geq u(c^{n*}) + \beta \cdot u((1+r_D)\cdot(\omega - c^{n*})),$$

and because u is strictly increasing and $r_D > 0$:

$$> u(c^{n*}) + \beta \cdot u(\omega - c^{n*}) := \bar{u}. \tag{40}$$

Given the LHS of (12) is continuous in both D and ϕ^h, there exists an $\bar{m}$ such that for all $m \geq \bar{m}$, (D^{h*}_m, P^{h*}_m) satisfies:

$$u(\omega - D^{h*}_m - P^{h*}_m) + \beta\phi^h_m u((1+r_D)D^{h*}_m) + \beta(1-\phi^h_m)u(kD^{h*}_m) > \bar{u}, \tag{41}$$

because the LHS of (12) for $j = h$ converges to $u(\omega - D^* - P^*) + \beta \cdot u((1+r_D)\cdot D^*)$ and by (40). Therefore, we can say for a sufficiently high ϕ^h and a sufficiently small k, the condition (12) for $j = h$ holds, which proves the statement for Case 1.

We next prove the statement for Case 2. Similarly with the previous case, as $\phi^l \to 0$, the LHS of (12) for $j = l$ converges to $u(\omega - D^{**} - P^{**}) + \beta \cdot u(k \cdot D^{**})$ where $D = D^{**}$ maximizes $u(\omega - D - P^{**}) + \beta \cdot u(k \cdot D)$ for given P^{**} and P^{**} satisfies $P^{**} = \frac{kD^{**}}{(1+r^l)}$. On the other hand, as $x \leq \omega$ by feasibility and u is strictly concave, there is a unique x^* that maximizes $u(x) + \beta u(\omega - x)$ by satisfying $u'(x) = \beta \cdot u'(\omega - x)$. Given $x = c^{n*}$:

$$\begin{aligned} u(c^{n*}) + \beta u(\omega - c^{n*}) &\geq u(\omega - D^{**}) + \beta \cdot u(D^{**}) && (x = c^{n*} \text{ is the maximizer}) \\ &> u(\omega - D^{**} - P^{**}) + \beta \cdot u(D^{**}) && (P^{**} > 0) \\ &> u(\omega - D^{**} - P^{**}) + \beta \cdot u(kD^{**}). && (k < 1) \end{aligned} \tag{42}$$

As the LHS of (12) for $j = l$ converges to $u(\omega - D^{**} - P^{**}) + \beta \cdot u(kD^{**})$ when $m \to \infty$, by (42) there exists a $\bar{m}$ such that for all $m \geq \bar{m}$, (D^{l*}_m, P^{l*}_m) satisfies:

$$\bar{u} > u(\omega - D^{l*}_m - P^{l*}_m) + \beta\phi^l_m u((1+r_D)D^{l*}_m) + \beta(1-\phi^l_m)u(kD^{l*}_m). \tag{43}$$

This completes our proof. □

Proof of Proposition 7. As with the argument in the proof of Proposition 1, for any r_D, (13) has to be satisfied in equilibrium. The derivatives are well defined as u and both sides of (13) are continuously differentiable.[9] Then, we obtain:

$$\begin{aligned}
\frac{\mathrm{d}\mathscr{X}}{\mathrm{d}r_D} &= -u'(\omega-\mathscr{D}-\mathscr{P})\frac{\mathrm{d}(\mathscr{D}+\mathscr{P})}{\mathrm{d}r_D}+\beta u'(I(D,r_D,k,\phi^j))\frac{\mathrm{d}I(D,r_D,k,\phi^j)}{\mathrm{d}r_D}\\
&= \beta u'(I(D,r_D,k,\phi^j))\left(\frac{\mathrm{d}I(D,r_D,k,\phi^j)}{\mathrm{d}r_D}-\frac{\mathrm{d}I(D,r_D,k,\phi^j)}{\mathrm{d}D}\frac{\mathrm{d}(\mathscr{D}+\mathscr{P})}{\mathrm{d}r_D}\right)\\
&= \beta u'(I(D,r_D,k,\phi^j))\frac{\mathrm{d}I(D,r_D,k,\phi^j)}{\mathrm{d}D}\left(\frac{\mathrm{d}\mathscr{D}}{\mathrm{d}r_D}-\frac{\mathrm{d}(\mathscr{D}+\mathscr{P})}{\mathrm{d}r_D}\right)\\
&= -\beta u'(I(D,r_D,k,\phi^j))\frac{\mathrm{d}I(D,r_D,k,\phi^j)}{\mathrm{d}D}\frac{\mathrm{d}\mathscr{P}}{\mathrm{d}r_D}.
\end{aligned} \tag{44}$$

Similarly, we obtain:

$$\begin{aligned}
\frac{\mathrm{d}\mathscr{X}}{\mathrm{d}\phi} &= -u'(\omega-\mathscr{D}-\mathscr{P})\frac{\mathrm{d}(\mathscr{D}+\mathscr{P})}{\mathrm{d}\phi}+\beta u'(I(D,r_D,k,\phi^j))\frac{\mathrm{d}I(D,r_D,k,\phi^j)}{\mathrm{d}\phi}\\
&= -\beta u'(I(D,r_D,k,\phi^j))\frac{\mathrm{d}I(D,r_D,k,\phi^j)}{\mathrm{d}\phi}\frac{\mathrm{d}\mathscr{P}}{\mathrm{d}\phi}.
\end{aligned} \tag{45}$$

Note that $\frac{\mathrm{d}I(D,r_D,k,\phi^j)}{\mathrm{d}D}>0$ because if D increases, then the RHS increases as u is increasing and thus the LHS has to increase to satisfy (25) and similarly $\frac{\mathrm{d}I(D,r_D,k,\phi^j)}{\mathrm{d}\phi^j}>0$. Thus, if $\mathscr{X}(\bar{r},k,\bar{\phi})=\bar{u}$, $\mathscr{X}(r_D,k,\bar{\phi})>\bar{u}$ for $r_D>\bar{r}$ if and only if $\frac{\mathrm{d}\mathscr{P}}{\mathrm{d}r_D}<0$ and $\mathscr{X}(\bar{r},k,\phi)>\bar{u}$ for $\phi>\bar{\phi}$ if $\frac{\mathrm{d}\mathscr{P}}{\mathrm{d}\phi}<0$. □

Proof of Proposition 8. On the contrary, supposing that $r_1>r_2$ while $\phi\in\Phi^h(r_2)$:

$$\bar{u} \ \ = \mathscr{X}(r_1,0,\phi)=u(\omega-D^{1h})+\beta\phi u((1+r_1)D^{1h}), \tag{46}$$

and as $D^{2h}<\omega$ implies the consumption plan $(\omega-D^{2h},D^{2h})$ is feasible according to the budget constraint (2) for $a=De$ and $j=h$, and D^{1h} is the equilibrium deposit:

$$\begin{aligned}
&\geq u(\omega-D^{2h})+\beta\phi u((1+r_1)D^{2h})\\
&> u(\omega-D^{2h})+\beta\phi u((1+r_2)D^{2h}) \qquad (r_1>r_2)\\
&= \mathscr{X}(r_2,0,\phi).
\end{aligned}$$

Then $\phi\notin\Phi^h(r_2)$, which is a contradiction. This proves the first statement. Similarly with the first case, suppose that $r_1\leq r_2$. Then:

$$\begin{aligned}
\bar{u} \ \ &= \mathscr{X}(r_1,0,\phi)=u(\omega-D^{1l})+\beta\phi u((1+r_1)D^{1l})\\
&\leq u(\omega-D^{1l})+\beta\phi u((1+r_2)D^{1l}):
\end{aligned} \tag{47}$$

and given the consumption plan $(\omega-D^{1l},D^{1l})$ is feasible for the budget constraint (2) for $a=De$ and $j=l$, but does not constitute the equilibrium consumption:

$$\leq \mathscr{X}(r_2,0,\phi). \tag{48}$$

Thus, we obtain $\bar{u}\leq\mathscr{X}(r_2,0,\phi)$, which implies $\phi\notin\Phi^l(r_2)$ and this is a contradiction. □

[9] Similar results to Proposition 7 can be proved by taking finite differences of two distinct interest rates rather than taking a derivative. Proposition 8 shows how this technique can be used.

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