

MAKING SHORT WORK OF LONG DIVISION

by Greg Tang

Of the four “standard algorithms” in arithmetic, division is typically the most difficult for students to master. There are several reasons for this. First, it requires proficiency with the other three operations – addition, subtraction and multiplication. Second, for many students, putting parts together is easier than taking them apart. As a result, adding is easier than subtracting and multiplying is easier than dividing.

But there is a third important reason and it is the focus of this article. Division, unlike the other three operations, requires students to do multi-digit computations in their head. With addition, subtraction and multiplication, computations involving large numbers can be done as a series of single-digit operations. No matter how large the numbers, they can be computed one digit at a time.

For example, when adding $234 + 345$, we can simply add 4 ones plus 5 ones, 3 tens plus 4 tens, and 2 hundreds plus 3 hundreds to get the answer 579. Even though we are adding two, triple-digit numbers, we never have to do more than a single-digit calculation.

With subtraction and multiplication, computations work the same way. Even with multi-digit numbers, any problem can be done as a series of single-digit computations. It may be necessary to regroup from one place value to another, but we never have to subtract or multiply anything except single-digit numbers. This is an incredibly important benefit of place value and the reason standard algorithms are so powerful.

Unfortunately, this is only partially true with division. Place value can help make the dividend more manageable, but it offers no help for the divisor. For example, suppose we want to divide 1,072 by 16. The first step is to divide 107 tens by 16, which typically means figuring out how many 16s are in 107 or how many times 16 “goes into” 107. We have no choice but to work with the double-digit divisor 16.

Consider another example. To solve the problem $2,184 \div 28$, the first step is to divide 218 tens by 28. For even a strong student, figuring out how many 28s are in 218 takes some thought. For a struggling student, it can be overwhelming. Fortunately, there is a simple solution that combines the intuition of partial quotients with the efficiency of place value. My students have dubbed it “Tang Division!”

Let’s revisit our first example. In Figure 1, I start again by asking, “How many 16s are in 107?” But instead of trying to figure out how many in one step, I simply do it in two steps. I know there are at least five 16s because $5 \times 16 = 80$, so I record a 5 in the ten’s place, subtract 80 tens and have 27 tens left.

Now I figure out how many 16s are in 27. That’s easy – there is just one. I record a 1 in the ten’s place, subtract 16 tens and have 11 tens left. I cannot divide 11 tens by 16, so I think of them as 110 ones, “bring down” or add the original 2 ones and now have 112 ones.

How many 16s are in 112? Again, we do not need to figure this out in one step. There are at least five, so I record a 5 in the one's place, subtract 80 ones and have 32 ones left. How many 16s are in 32? There are two, so I record a 2 in the one's place, subtract 32 ones and have zero ones left. I am done dividing, so I add up my tens and ones and get an answer of 67.

$$\begin{array}{r}
 12 \\
 55 \Rightarrow 67 \\
 16 \overline{) 1072} \\
 \underline{- 80} \\
 27 \\
 \underline{- 16} \\
 112 \\
 \underline{- 80} \\
 32 \\
 \underline{- 32} \\
 0
 \end{array}$$

Figure 1

Let's do one more. In Figure 2 below, I am dividing 2,184 by 28. I start by figuring out how many 28s are in 218, but I do it in two steps. I know there are at least five, so I record a 5 in the ten's place, subtract 140 tens and have 78 tens left. How many 28s are in 78? There are at least 2, so I record a 2 in the ten's place, subtract 56 tens and have 22 tens left. I cannot divide 22 tens by 28, so I regroup the tens and ones and have 224 ones all together.

$$\begin{array}{r}
 1 \\
 22 \\
 55 \Rightarrow 78 \\
 28 \overline{) 2184} \\
 \underline{- 140} \\
 78 \\
 \underline{- 56} \\
 224 \\
 \underline{- 140} \\
 84 \\
 \underline{- 56} \\
 28 \\
 \underline{- 28} \\
 0
 \end{array}$$

Figure 2

Moving to the ones, I need to figure out how many 28s are in 224. Once again, I know there are at least 5, so I record a 5 in the one's place, subtract 140 ones and have 84 ones left. How many 28s are in 84? There are at least two, so I record a 2 in the one's place, subtract 56 ones and have 28 ones left. How many 28s are in 28? There is one, so I record a 1 in the one's place, subtract 28 ones and have zero ones left. I add up my tens and ones and get an answer of 78.

This algorithm combines the ease of partial quotients with the efficiency of place value. It brings together important division concepts including the relationship between multiplication and division, partial quotients, remainders and the standard algorithm. The best part is that it makes dividing faster and easier and gives struggling students a chance to enjoy math. I hope you will give it a try!