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THE OPTIMAL LEVEL OF INTERNATIONAL RESERVES FOR EMERGING MARKET COUNTRIES: A NEW FORMULA AND SOME APPLICATIONS*

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We present a model of the optimal level of international reserves for a small open economy seeking insurance against sudden stops in capital flows. We derive a formula for the optimal level of reserves and show that plausible calibrations can explain reserves of the order of magnitude observed in many emerging market countries. The buildup of reserves in emerging market Asia can be explained only if one assumes a large anticipated output cost of sudden stops and a high level of risk aversion.

The recent buildup in international reserves in emerging market countries has revived old debates about the appropriate amount of reserves for an open economy. It has been argued that many emerging market countries accumulated reserves as a form of self-insurance against capital flow volatility, the danger of which was learned the hard way in the international financial crises of the 1990s (Aizenman and Marion, 2003; Stiglitz, 2006).[1] Against this backdrop, there has been surprisingly little work trying to quantify the level of reserves that can be justified as an insurance against capital flow volatility.

This article contributes to filling this gap with a model and some calibrations. The model features a representative consumer in a small open economy who may lose access to external credit (a sudden stop). The consumer can smooth domestic consumption in sudden stops by entering insurance contracts with foreign investors, or equivalently, by financing a stock of liquid reserves with contingent debt. The model yields a closed-form expression for the welfare-maximising level of reserves. The optimal level of reserves depends in an intuitive way on the probability and the size of the sudden stop, the consumer's risk aversion and the opportunity cost of holding the reserves. We also present various extensions of the basic model, including one in which reserves have benefits in terms of prevention (they reduce the probability of a sudden stop).

With our formula in hand, we then explore the quantitative implications of the model by using data on a sample of sudden stops in emerging market countries. Our estimates of the optimal level of reserves are relatively sensitive to parameters that are

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[1] Another view is that the reserves buildup is the unintended consequence of policies that maintain large current account surpluses (Dooley *et al.*, 2004; Summers, 2006).

relatively difficult to measure, such as the opportunity cost of holding reserves or their benefits in terms of crisis prevention. However, we find that for plausible values of the parameters the model can explain reserves-to-gross domestic product (GDP) ratios of the order of magnitude observed in emerging market countries over the past decades. For a coefficient of constant relative risk aversion of 2 (a standard value in the real business cycle literature), our model predicts a reserves-to-GDP ratio of 9%, which is close to the average reserves-to-GDP ratio observed in a group of 34 middle-income countries over the period 1975–2003. However, it is more difficult for the model to account for the recent build-up of reserves in emerging market Asia. The high levels of reserves in Asia can be rationalised only if we assume that those countries anticipated crises with a high output cost and, in addition, had very high levels of risk aversion.

Our article contributes to a long line of literature on reserves adequacy. The first cost–benefit analyses of the optimal level of reserves were developed in the 1960s and the 1970s, when the focus was mainly on the current account (Heller, 1966). The main insights from that literature were later formalised in variants of the Baumol–Tobin inventory model in which the stock of reserves is being depleted by a stochastic current account deficit; see, e.g. Frenkel and Jovanovic (1981) and Flood and Marion (2002), for a review. The optimal level of reserves can be derived as a simple closed-form expression involving the volatility of the reserves-depleting process, the opportunity cost of holding reserves and the fixed costs of depleting and rebuilding the reserves stock. One problem with this framework is that it is a highly reduced form with no well-defined welfare criterion.

Following (with a substantial lag) a more general trend in macroeconomic theory, the recent literature on reserves adequacy has taken the welfare of the representative agent as the criterion to maximise. Two recent papers derive the optimal level of reserves in a welfare-based calibrated model, as we do here.[2] Durdu *et al.* (2009) present some estimates of the optimal level of precautionary savings accumulated by a small open economy in response to business cycle volatility, financial globalisation and the risk of sudden stop. They conclude that financial globalisation and the risk of sudden stop may be plausible explanations of the observed surge in reserves in emerging market countries.[3] Alfaro and Kanczuk (2009) present a model in which a small open economy can insure itself by defaulting on its external debt rather than holding reserves and find that the optimal level of reserves is zero.

The model presented here is one of insurance, rather than precautionary savings, and from this point of view is more directly comparable with that of Caballero and Panageas (2007). Those authors calibrated a dynamic general equilibrium model in which the country that is vulnerable to sudden stops can invest in conventional reserves (fixed income foreign assets) as well as more sophisticated assets whose payoffs are correlated with sudden stop arrivals. They find that the gains from the optimal hedging strategies can be substantial. Both Durdu *et al.* (2009) and Caballero and Panageas

[2] Other papers present stylised models that are useful to illustrate the basic trade-offs involved in the choice of optimal reserves but do not lend themselves to the kind of quantitative exercises that we present in this article (Aizenman and Marion, 2003; Miller and Zhang, 2006; Aizenman and Lee, 2007).

[3] They find that the risk of sudden stops can explain an increase in the country's foreign assets amounting to 20% of GDP. In an earlier contribution, Mendoza (2002) found that a shift from perfect credit markets to a world with sudden stops increases the average foreign assets-to-GDP ratio by 14 percentage points.

(2007) solve their models numerically, whereas we strive, in this article, to obtain closed-form expressions for the optimal level of reserves.

Policy analysts often assess reserves adequacy using simple rules of thumb, such as maintaining reserves equivalent to three months of imports, or the 'Greenspan–Guidotti rule' of full coverage of short-term external debt. The Greenspan–Guidotti rule is a natural benchmark of comparison for our estimates, which are also based on the idea that reserves help countries deal with a sudden stop in short-term debt inflows. We find that the optimal level of reserves suggested by our model may be close to the Greenspan–Guidotti rule for plausible calibrations of the model, although it could be significantly higher or lower.

The article is structured as follows. Section 1 presents a model yielding a simple formula for the optimal level of reserves. Section 2 calibrates the model and compares the model predictions and the data. Section 3 concludes.

1. An Insurance Model

We first present the assumptions of the model (Section 1.1), and derive a closed-form expression for the optimal level of reserves (Section 1.2). We then show that this model can be reinterpreted as one of balance sheet management in which the reserves are financed by contingent debt (Section 1.3). The last Subsection looks at an extension of the model in which reserves have a role in terms of crisis prevention.

1.1. *Assumptions*

We consider a small open economy in discrete infinite time $t = 0,1,2,\ldots$. There is one single good which is consumed domestically and abroad. The economy follows a deterministic path that may be disturbed by sudden stops in capital inflows. The only source of uncertainty in our model is the risk of sudden stop.

The country has a private sector and a government. The private sector is composed of a continuum of atomistic and identical infinitely lived consumers with an inter-temporal utility defined by

$$U_t = \mathrm{E}_t\left[\sum_{i=0,\ldots,+\infty}(1+r)^{-i}u(C_{t+i})\right], \tag{1}$$

where the flow utility function has a constant relative risk aversion $\sigma \geq 0$,

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}, \quad \sigma \neq 1 \tag{2}$$

and $u(C) = \log(C)$ for $\sigma = 1$. Consumers maximise their welfare subject to the budget constraint

$$C_t = Y_t + L_t - (1+r)L_{t-1} + Z_t, \tag{3}$$

where Y_t is domestic output, L_t is external debt and Z_t is a transfer from the government. The interest rate r is constant and the representative consumer does not default on her external debt.

We assume that there is a constraint on the quantity of output that can be pledged by the domestic private sector to foreign creditors. The debt is fully repaid in period $t+1$ only if

$$(1+r)L_t \leq \alpha_t Y_{t+1}^n, \tag{4}$$

where Y_{t+1}^n is trend output in period $t+1$ (to be defined shortly) and α_t is a time-varying parameter that captures the pledgeability of domestic output to foreign creditors.[4] We assume that both α_t and Y_{t+1}^n are known in period t, implying that debt issued in t is default-free if condition (4) is satisfied. The stringency of the private sector's external borrowing constraint can change over time, generating the possibility of sudden stops. The time variation in α_t could result, for example, from exogenous changes in expectations about the enforcement of creditor rights, or in the penalty that can be imposed on domestic defaulters.[5] For the purpose of this model we simply take this variable as exogenous.

The economy can be in two states: the normal – or non-crisis – state (denoted by n), or in a sudden stop (denoted by s). In normal times output grows at a constant rate g and the private sector can pledge a constant fraction of output

$$Y_t^n = (1+g)^t Y_0, \tag{5}$$

$$\alpha_t^n = \alpha. \tag{6}$$

We assume that if there is a sudden stop, output falls by a fraction γ below trend and pledgeable output falls to zero:

$$Y_t^s = (1-\gamma)Y_t^n, \tag{7}$$

$$\alpha_t^s = 0. \tag{8}$$

The assumption that pledgeable output falls to zero, rather than a positive level, is a matter of normalisation. The external debt that is rolled over does not contribute to the sudden stop and therefore plays no interesting role in our model. To ensure that the consumer can repay all her debt in a sudden stop we assume $\alpha + \gamma < 1$. We assume $r > g$ so as to keep the consumer's intertemporal income finite.

In the real world, it can take more than one year for capital to flow back to a country after a sudden stop. Thus, we assume that it takes a certain number of periods θ for the economy to go back to its trend path after a sudden stop. If a sudden stop occurs at time t, the economy is back in state n at time $t+\theta+1$. We define the time interval $[t, t+\theta]$ as a 'sudden stop episode'. Thus in a given period t the economy could be in one of $\theta+2$ states: the normal state, $s_t = n$, or in one of

[4] Constraint (4) can be justified by contractual enforcement problems or by agency problems; see Tirole (2005) for a review of the possible theoretical underpinnings in a corporate finance context. This type of constraint has been extensively used in international finance, in particular to model sudden stops in capital flows (see, e.g. Mendoza, 2002; Rancière *et al.*, 2008).

[5] See Guembel and Sussman (2009) for a model in which a country's willingness to repay foreign debt is determined by domestic political economy factors. Broner and Ventura (2011) present a model in which the sovereign's ability to borrow abroad is linked to the enforcement of domestic contracts.

the $\theta + 1$ substates corresponding to the different periods of a sudden stop episode, $s_t = s^0, s^1, \ldots, s^\theta$.

The dynamics of output and external credit in a sudden stop episode starting at date t are given by

$$Y^s_{t+\tau} = [1 - \gamma(\tau)] Y^n_{t+\tau}, \tag{9}$$

$$\alpha^s_{t+\tau} = \alpha(\tau), \tag{10}$$

where $\gamma(\cdot)$ and $\alpha(\cdot)$ are exogenous functions of $\tau = 0,1, \ldots ,\theta$. By (7) and (8), we have $\gamma(0) = \gamma$ and $\alpha(0) = 0$. We assume that the economy catches up with the trend path in a monotonic way, in the sense that $\gamma(\tau)$ and $\alpha(\tau)$ are both non-negative, and respectively decreasing and increasing in τ. We further assume that at the end of the sudden stop episode the consumer has regained the same level of access to external credit as before the sudden stop, $\alpha(\theta) = \alpha$.

Given our focus on insurance against sudden stops (rather than business cycle fluctuations), we streamline the model by assuming that the only source of uncertainty is the risk of a sudden stop. We denote by π the probability that a sudden stop occurs in the following period. At the end of a sudden stop episode the economy goes back to state n with certainty.

Sudden stops reduce the representative consumer's welfare in two ways. First, they perturb the consumption path around the trend level, which decreases the consumer's welfare if her elasticity of intertemporal substitution of consumption is finite. Second, sudden stops reduce the consumer's intertemporal income because of the fall in domestic output. This is illustrated in Figure 1, which shows the paths of output, external debt and domestic consumption in a sudden stop episode under the assumption that the borrowing constraint (4) is always binding and that there is no transfer from the government. Consumption falls sharply at the time of the sudden stop under the cumulative impact of the fall in output and of the capital outflow and then recovers as foreign capital flows back in.

The role of the government, in our model, is to insure the private sector against sudden stop shocks. We assume that the government can smooth domestic consumption against sudden stops by entering a 'reserves insurance contract' with foreign investors. A contract signed at time 0 stipulates that the government pays a premium X_t at time t to the foreign insurers until a sudden stop hits the country. In exchange, the government receives a payment R_t if a sudden stop occurs in period t. The occurence of a sudden stop ends the insurance contract. The government can enter a new contract at the end of the sudden stop episode. As the date of the sudden stop is not known, an insurance contract signed in period 0 must specify an infinite sequence of conditional payments $(X_t, R_t)_{t=1,\ldots,+\infty}$.

The government simply transfers the cash flows resulting from the contract to the domestic consumer, which implies

$$Z^n_t = -X_t, \tag{11}$$

as long as the economy stays in state n, and

$$Z^s_t = R_t - X_t, \tag{12}$$

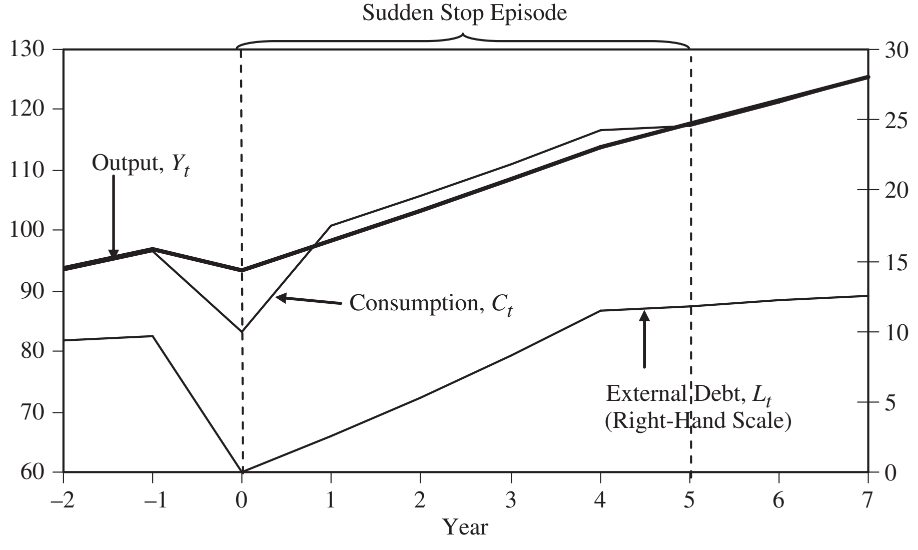

Fig. 1. *Output, External Debt and Consumption in a Sudden Stop*

Notes. The Figure shows the path of domestic output, consumption (left-hand scale) and external debt (right-hand scale) in a sudden stop episode starting in period 0 and lasting five periods. Trend output is normalised to 100 in the period of the sudden stop. The parameter values are those of the benchmark calibration given in Table 1; see Section 2.1.

Source. Authors' computations.

when the sudden stop occurs. We assume that the government pays the premium X_t for the last time in the period of the sudden stop, so that the net transfer is the difference $R_t - X_t$.

The 'reserves insurance contract' attempts to capture, in a stylised way, the trade-offs involved in reserves management. The government must sacrifice some resources (which can be interpreted as the cost of carrying the reserves) before the crisis in exchange of access to international liquidity at the time of the crisis. We show in Section 2.3 how this insurance can be achieved by holding a buffer stock of liquid assets financed by long-term liabilities.

Finally, we need to specify the price at which the foreign insurers provide the reserves. We denote by μ_t the marginal utility of funds (or pricing kernel) for the insurers at time t, and assume that it is higher if the domestic economy is in a sudden stop

$$\mu_t^s \geq \mu_t^n.$$

The insurers' marginal utility of funds could be higher during a sudden stop, for example, because sudden stops tend to be correlated with tight global liquidity conditions. The difference between μ_t^s and μ_t^n determines the cost of insurance for the small open economy. For simplicity, we assume that the price of a non-crisis dollar in terms of a crisis dollar is constant and denote it by

$$p = \frac{\mu_t^n}{\mu_t^s} \leq 1.$$

As we will see in Section 1.3, the level of p can be inferred from the pure risk premium on long-term bonds that default contingent on a sudden stop.

We assume that the foreign insurers are perfectly competitive and discount the future at the same rate as the domestic consumer. Thus, they are ready to provide any insurance contract $(X_t, R_t)_{t=1,\ldots,+\infty}$ whose present discounted value is non-negative, that is,

$$\sum_{t=1}^{+\infty} \beta^t (1-\pi)^{t-1} \left[(1-\pi) X_t \mu_t^n - \pi (R_t - X_t) \mu_t^s \right] \geq 0. \quad (13)$$

1.2. *A Formula for the Optimal Level of Reserves*

The government's insurance problem is fairly simple – and can be solved in closed form – if the borrowing constraint (4) is always binding.[6] We first derive a formula for the optimal level of reserves under the assumption that (4) is always binding, and then establish a set of conditions that are sufficient for this assumption to be satisfied in equilibrium. Note that if (4) is binding, in normal times, the country maintains a constant ratio of short-term debt to GDP, given by

$$\lambda = \frac{L_t^n}{Y_t^n} = \frac{1+g}{1+r} \alpha. \quad (14)$$

The optimal level of reserve insurance is characterised in the following Proposition.

Proposition 1. *If the external credit constraint* (4) *is always binding, the optimal level of reserves-to-GDP ratio* $\rho \equiv R_t / Y_t^n$ *is constant and given by,*

$$\rho^* = \frac{\lambda + \gamma - \left[1 - \dfrac{(r-g)}{1+g} \lambda \right] (1 - p^{1/\sigma})}{1 - \dfrac{\pi}{\pi + p(1-\pi)} (1 - p^{1/\sigma})}, \quad (15)$$

where λ is the ratio of short-term debt to GDP, γ is the output loss in the first period of the sudden stop, r is the interest rate, g is the growth rate, π is the probability of a sudden stop, σ is risk aversion and p is the price of a non-crisis dollar in terms of crisis dollar for global investors.

Proof. The government chooses the paths $(X_t, R_t)_{t=1,\ldots,+\infty}$ so as to maximise domestic welfare (1) subject to the budget constraints (3), (11), (12), the binding credit constraint (4) and the insurers' participation constraint (13). The Lagrangian can be written as:

$$\pounds = \sum_{t=1}^{+\infty} \beta^t (1-\pi)^t \left\{ (1-\pi) u(C_t^n) + \pi u(C_t^s) + v \left[(1-\pi) X_t \mu_t^n - \pi (R_t - X_t) \mu_t^s \right] \right\},$$

where v is the shadow cost of constraint (13), and the state-contingent levels of consumption are given by

[6] If the borrowing constraint is not always binding, one must solve for the consumer's optimal precautionary savings, a problem that has no closed-form solution. See Guimaraes (2010) for a model of sovereign debt and default that also has closed-form solutions because the borrowing contsraint is always binding in equilibrium.

$$\begin{aligned} C_t^n &= Y_t^n + \frac{\alpha}{1+r} Y_{t+1}^n - \alpha Y_t^n - X_t, \\ &= Y_t^n \left(1 - \frac{r-g}{1+g} \lambda \right) - X_t \end{aligned} \tag{16}$$

and

$$\begin{aligned} C_t^s &= Y_t^s - \alpha Y_t^n + R_t - X_t, \\ &= Y_t^n \left(1 - \gamma - \frac{1+r}{1+g} \lambda \right) + R_t - X_t. \end{aligned} \tag{17}$$

The first-order conditions then imply

$$u'(C_t^n) = pu'(C_t^s), \tag{18}$$

that is, the domestic consumer can substitute consumption between the two states at the same rate as global investors. Equation (15) then results from simple manipulations of this equation and of the foreign insurers' binding participation constraint

$$X_t = \frac{\pi}{\pi + p(1-\pi)} R_t.$$

Equation (15) is our formula for the optimal level of reserves. The optimal level of reserves responds in an intuitive way to changes in its determinants. It increases (more than one for one) with the level of short-term debt, λ, and with the output cost of a sudden stop, γ. The optimal level of reserves is also increasing with the probability of a sudden stop, π. On can see that $\rho^* \leq \lambda + \gamma$ by rewriting (15) as:

$$\lambda + \gamma - \rho^* = \frac{(1-p^{1/\sigma})}{1 - \frac{\pi}{\pi + p(1-\pi)}(1-p^{1/\sigma})} \left[1 - \alpha - \gamma + \frac{p(1-\pi)}{\pi + p(1-\pi)}(\lambda + \gamma) \right], \tag{19}$$

and noting that the right-hand side is positive because $p \leq 1$ and $\alpha + \gamma < 1$. The optimal level of reserves is equal to $\lambda + \gamma$ if $p = 1$, that is if foreign investors do not value liquidity more in a sudden stop. In this case, the reserves contract provides full insurance, in the sense that consumption is the same whether or not there is a sudden stop. Expression (19) can also be used to show that the level of reserves is increasing with risk aversion: an increase in σ raises $p^{1/\sigma}$ (given that $p \leq 1$) and reduces the right-hand side of (19).

How does our formula relate to the Greenspan–Guidotti rule? This rule says that the ratio of the reserves to short-term debt should be equal to 1, that is,

$$\rho = \lambda.$$

One can see from (19) that the Greenspan–Guidotti rule corresponds to full insurance ($p = 1$) if a sudden stop does not reduce output ($\gamma = 0$). In general, however, the optimal level of reserves could be larger or smaller than the Greenspan–Guidotti rule. It could be larger because the full-insurance level of reserves, $\rho^* = \lambda + \gamma$, must also cover the fall in output associated with a sudden stop. It could be smaller because insurance is costly so that the country will not, in general, fully insure.

We conclude this Section with a set of conditions that are sufficient for (4) to be always binding in equilibrium.

LEMMA 1. *The borrowing constraint* (4) *is binding at all times if the following inequalities are satisfied:*

$$(1+g)^{\sigma} \geq 1 + \pi\left(\frac{1}{p} - 1\right), \tag{20}$$

$$\forall\tau = 1, \ldots, \theta, \quad \alpha(\tau) - \alpha(\tau-1) \leq g\left(1 - \gamma - \frac{r-g}{1+r}\alpha\right), \tag{21}$$

$$\gamma(1) \leq \frac{1}{1+g}\left(g + \frac{r-g}{1+r}\alpha\right) + \frac{1+g}{1+r}\alpha(1). \tag{22}$$

Proof. See online Technical Appendix.

Condition (20) ensures that the credit constraint (4) is binding in normal times.[7] Conditions (21) and (22) ensure that the credit constraint (4) is also binding during sudden stop episodes. Condition (21) says that foreign capital should not flow back to the country too quickly after a sudden stop, so as to ensure that the consumer exhausts her borrowing capacity during the recovery. We show in Section 2 that the conditions of Lemma 1 are satisfied for plausible values of the parameters.

1.3. *The Opportunity Cost of Reserves*

The literature on international reserves generally defines the opportunity cost of reserves by reference to trade-offs involved in changing the composition of the country's balance sheet. Reserves can be used to repay external liabilities; the opportunity cost of reserves is defined as the difference between the interest rate paid on the country's liabilities and the lower return received on the reserves (Edwards, 1985; Garcia and Soto, 2004; Rodrik, 2006). We now show that our insurance model can be reinterpreted in those terms, conditional on certain assumptions about the menu of available assets and liabilities. This interpretation will be useful to calibrate the model in Section 2.

The reserves insurance contract is easy to replicate if the government can issue liabilities whose payoff is contingent on the occurrence of a sudden stop.[8] Let us assume that the government does not have access to the insurance contracts described before, but can issue a 'perpetuity' that yields an interest payment of 1 until a sudden stop occurs, and stops paying any interest after that. The unitary price of the perpetuity is the expected payoff discounted using the pricing kernel of foreign investors

[7] The fact that the constraint is binding means that there is no precautionary savings, in part because the reserves insurance contract provides a substitute to such savings.

[8] There are different ways a security could be made contingent on the occurrence of a sudden stop. The contingency could be put directly in the contract, like in a catastrophy bond, whose repayment is reduced contingent on the occurrence of a natural disaster. To limit moral hazard, one could make the bond contingent on a variable that is correlated with sudden stops but not significantly influenced by the country's policies, such as the VIX index (Caballero and Panageas, 2007). Finally, the contingency could take the form of a default on government debt (Grossman and Van Huyck, 1988).

$$q = \beta\left[(1-\pi)(1+q)\frac{\mu^n_{t+1}}{\mu^n_t} + \pi\frac{\mu^s_{t+1}}{\mu^n_t}\right].$$

Using the Euler condition for foreign investors, $\mu^n_t = \beta R[(1-\pi)\mu^n_{t+1} + \pi\mu^s_{t+1}]$, and $\mu^n_{t+1}/\mu^s_{t+1} = p$, we derive an expression for the unitary price of the perpetuity

$$q = \frac{1}{r+\pi+\delta}, \tag{23}$$

with

$$\delta = \frac{1-p}{p/\pi + 1/(1-\pi)}. \tag{24}$$

Expression (23) shows that the interest rate spread on the perpetuity is the sum of two terms: the probability of a sudden stop π, which is analogous to a default premium as the government stops servicing its debt after a sudden stop; and δ, a pure risk premium that comes from the fact that foreign investors must provide liquidity when it is more valuable to them. The pure risk premium is decreasing with p and is equal to 0 if $p = 1$.

Let us assume that the government issues a number R_t/q of perpetuities in period $t-1$, invests the proceeds in one-period bonds and buys back the perpetuities in period t if there is no sudden stop. The net payoff for the government in period t is:

$$(1+r)R_t - \frac{1+q}{q}R_t = -(\pi+\delta)R_t \quad \text{if there is no sudden stop in } t,$$

$$(1+r)R_t - \frac{1}{q}R_t = (1-\pi-\delta)R_t \quad \text{if there is a sudden stop in } t.$$

The payoffs are exactly the same as those of the insurance contract, given by (11) and (12), if one defines $X_t = (\pi+\delta)R_t$. Thus, the country can replicate the insurance contract by financing a buffer stock of reserves with a perpetuity that defaults contingent on a sudden stop. The opportunity cost of holding the reserves, δ, is the pure risk premium on this perpetuity.

This is reminiscent of Edwards' (1985) measure of the opportunity cost of reserves as the spread between the interest rate on the country's long-term external debt and the return on its reserves. The model suggests, however, that using the full interest rate spread overestimates the true opportunity cost of reserves. The full interest spread is the sum of π, the default premium, and δ, the pure risk premium. Only, the second variable, however, contributes to the opportunity cost of holding reserves. The cost of insurance, thus, should be measured by the pure risk premium δ, rather than the full spread $\pi+\delta$, which is generally used in the empirical literature on international reserves. The default risk premium π is a fair compensation for the risk that the government will not repay, and does not represent an opportunity cost of holding reserves in the same sense as δ.

1.4. *Crisis Prevention*

Our model has focused so far on the benefits of reserves in terms of crisis mitigation (reducing the welfare cost of a crisis). An additional benefit of reserves might be to instil confidence in the economy and thus reduce the probability of a sudden stop (Ben Bassat and Gottlieb, 1992; Garcia and Soto, 2004). We show in this Section how the model can be extended to incorporate the benefits of reserves in terms of crisis prevention.

The prevention benefits of reserves can be captured, in reduced form, by writing the probability of a sudden stop as a decreasing function $\pi(\rho)$ of the reserves ratio

$$\pi_t = \pi(\rho_t). \tag{25}$$

This is a generalisation of the previous model, which corresponds to the special case where function $\pi(\cdot)$ is constant.

One possible interpretation of the reduced form (25) is a model of self-fulfilling debt rollover crises *à la* Cole and Kehoe (2000). The key assumptions are that the country's pledgeable output is reduced by a debt rollover crisis, and that the lending decisions are taken by a large number of uncoordinated lenders.

More formally, let us assume that pledgeable output is an increasing function of the ratio of consumption to output, that is,

$$\alpha_t = \alpha(c_t),$$

where $c_t = C_t/Y_t^n$. For example, a justification for this reduced form could be that lower consumption undermines the political support for repaying foreign creditors or the fiscal receipts required to enforce repayment. Assuming that the credit constraint (4) is binding, and leaving reserves aside, one can divide both sides of the budget constraint (3) by Y_t^n to obtain

$$c_t = 1 + \frac{1+g}{1+r}\alpha(c_t) - \alpha(c_{t-1}).$$

Both sides of this equation are increasing with c_t so that there could be multiple equilibria if function $\alpha(\cdot)$ has the appropriate shape. A good equilibrium in which foreign lenders roll over their claims coexists with a bad equilibrium in which they reduce their lending. The strategic complementarity behind the equilibrium multiplicity is that an individual investor who fails to lend contributes to reduce the pledgeability of the country's output for all the other lenders. A self-fulfilling debt rollover crisis could be triggered by a sunspot variable that coordinates lenders on the bad equilibrium. As the stochastic process for the sunspot is essentially arbitrary – and could depend on the level of reserves – the shape of function $\pi(\cdot)$ is indeterminate and could be any function of ρ.[9]

Coming back to the general formulation of the problem with crisis prevention, the question is how the optimal level of reserves ρ^* changes in the benchmark model when the probability of a sudden stop is given by (25) rather than a fixed exogenous level π. We show in the rest of this Section that although closed-form expressions can no longer be obtained, ρ^* is the solution to a relatively simple fixed-point problem that can be solved numerically. We estimate the quantitative impact of this effect in a calibrated version of the model.

The optimal level of reserves can be computed as follows. We divide the value functions by $(Y^n)^{1-\sigma}$ to make the problem stationary, and denote the normalised value functions with tilde. We have,

[9] One could use a model with heterogeneous beliefs *à la* Morris and Shin (1998) to endogenise π as a function of a public signal on the level of reserves. See Kim (2008) for a model of reserves along those lines.

$$\rho^* = \arg\max\{\widetilde{V}(\rho) \equiv [1-\pi(\rho)]\widetilde{U}^n(\rho) + \pi(\rho)\widetilde{U}^s(\rho)\}, \tag{26}$$

where welfare in normal times is given by

$$\tilde{U}^n(\rho) \equiv u\left\{1 - \frac{r-g}{1+g}\lambda - [\pi(\rho)+\delta]\rho\right\} + \frac{(1+g)^{1-\sigma}}{1+r}\widetilde{V}(\rho^*), \tag{27}$$

and welfare in a sudden stop is:

$$\begin{aligned}\tilde{U}^s(\rho) = u\left\{1-\gamma-\frac{1+r}{1+g}\lambda + [1-\pi(\rho)-\delta]\rho\right\} + \sum_{\tau=1}^{\theta}\left[\frac{(1+g)^{1-\sigma}}{1+r}\right]^{\tau} \\ \times u\left[1-\gamma(\tau)+\lambda(\tau)-\frac{1+r}{1+g}\lambda(\tau-1)\right] + \left[\frac{(1+g)^{1-\sigma}}{1+r}\right]^{\theta+1}\widetilde{U}^n(\rho^*).\end{aligned} \tag{28}$$

This is a fixed-point problem in the pair $[\tilde{V}(\rho^*), \rho^*]$, which can be solved by iterating on the afore given equations.

2. Calibration

We now explore the quantitative implications of the model. We first construct a benchmark calibration by reference to the average sudden stop in our sample and present some sensitivity analysis (Subsections 2.1 and 2.2). We then discuss the extent to which our insurance model can account for the recent reserves buildup in emerging market countries (Subsection 2.3).

2.1. *Benchmark Calibration*

The behaviour of the model economy is determined by seven parameters: the ratio of short-term debt to GDP λ, the probability of a sudden stop π, the output loss γ, the growth rate g, the pure risk premium δ, the return on reserves r, and the risk-aversion parameter σ. Our benchmark calibration is given in Table 1.

Parameters π, γ and λ are calibrated by reference to a sample of sudden stops in 34 middle-income countries over 1975–2003. For this purpose we decompose domestic absorption as the sum of domestic output, the financial account, income from abroad and reserves decumulation:

$$A_t = Y_t + KA_t + IT_t - \Delta R_t, \tag{29}$$

where KA_t is the financial account, IT_t is the income and transfers from abroad and $\Delta R_t = R_t - R_{t-1}$ is the change in reserves.[10] A sudden stop is an abrupt fall in the financial account, KA_t, which, other things equal, reduces domestic absorption. The impact of the sudden stop on domestic absorption could be amplified by a concomitant fall in domestic output, Y_t, or mitigated by a fall in reserves, ΔR_t.

[10] The financial account (formerly called the capital account) is a measure of capital inflows. Domestic absorption is the sum of domestic (private and public) consumption and investment. Equation (29) results from the GDP identity $Y_t = A_t + TB_t$, where TB_t is the trade balance, and the balance of payments identity, $CA_t + KA_t = \Delta R_t$, where $CA_t = TB_t + IT_t$ is the current account balance.

Table 1

Calibration Parameters

Parameters	Baseline	Range of Variation
Size of sudden stop	$\lambda = 0.10$	[0, 0.3]
Probability of a sudden stop	$\pi = 0.10$	[0, 0.25]
Output loss	$\gamma = 0.065$	[0, 0.2]
Potential output growth	$g = 0.033$	
Term premium	$\delta = 0.015$	[0.0025, 0.05]
Risk-free rate	$r = 0.05$	
Risk aversion	$\sigma = 2$	[1, 10]

Source. Authors' calculations using data from International Financial Statistics and Federal Reserve Board.

To see the correspondence between the national accounting identity (29) and the model, note that the consumer's budget constraint (3) can be written in a sudden stop as:

$$\underbrace{C_t}_{A_t} = \underbrace{(1-\gamma)Y_t^n}_{Y_t} + \underbrace{(-L_{t-1})}_{KA_t} + \underbrace{[-rL_{t-1} - (\pi+\delta)R_t]}_{IT_t} - \underbrace{(-R_t)}_{\Delta R_t}.$$

Using this decomposition, it is possible to infer the size of the shocks to the economy in a sudden stop (λ and γ) from the empirical behaviour of the terms on the right-hand side of (29).[11]

In line with Guidotti *et al.* (2004), we identify a sudden stop in year t if the ratio of capital inflows to GDP, $k_t \equiv KA_t/Y_t$, falls by more than 5% relative to the previous year,

$$\text{sudden stop in year } t \Leftrightarrow k_t < k_{t-1} - 5\%.$$

The countries in our sample and the years in which they had a sudden stop are reported in Table 2.[12] Reassuringly, our criterion captures many well-known crises (Mexico, 1995; Korea, Thailand and the Philippines, 1997; Argentina, 2001).

Figure 2 shows the average behaviour of domestic absorption and the contribution of the various components on the right-hand side of (29) in a five-year event window centred around a sudden stop.[13] Real output is normalised to 100 in the year prior to the sudden stop. The income and transfers from abroad are not shown because they are small and do not vary much in a sudden stop.

[11] What matters for the optimal level of reserves is the size of the financial account reversal. In the model, this is the same as the level of short-term debt. For the calibration, however, we simply measure the size of the financial account reversal in the data without assuming that it is necessarily because of short-term debt. Long-term debt coming to maturity at the time of the sudden stop could also contribute to the sudden stop (although it is not likely to be a significant factor in practice relative to short-term debt).

[12] Our sample includes the countries classified as middle income by the World Bank, plus Korea. It excludes major oil-producing countries, for which a large change in the price of oil could be misinterpreted as a sudden stop. Capital inflows are measured as the deficit in the current account minus the accumulation of reserves and related items in the IMF's *International Financial Statistics* (IFS). Exceptional financing and IMF loans are counted as reserves rather than capital inflows.

[13] Figure 2 is based on the events that occurred after 1980, excluding the sudden stops that occurred inside the five-year window of a previous sudden stop. The data for the financial account, the change in reserves and the income and transfers come from the IFS database. They are converted from current US dollar to constant local currency units using the nominal exchange rate *vis-à-vis* the US dollar and the local GDP deflator index. The data for real GDP and the real GDP deflator come from the World Bank's *World Development Indicators*.

Table 2

Countries and Years of Sudden Stops

Country	Dates of Sudden Stops
Argentina	1989, 2001, 2002
Bolivia	1980, 1982, 1983, 1994
Botswana	1977, 1987, 1991, 1993
Brazil	1983
Bulgaria	1990, 1994, 1996, 1998, 2003
Chile	1982, 1983, 1985, 1991, 1995, 1998
China,P.R.: Mainland	
Colombia	
Costa Rica	
Czech Republic	1996, 2003
Dominican Republic	1981, 1993, 2003
Ecuador	1983, 1986, 1988, 1992, 1999, 2000
Egypt	1990, 1993
El Salvador	1979
Guatemala	
Honduras	1998, 2000
Hungary	1994, 1996
Jamaica	1983, 1985, 1986, 1988, 2002, 2003
Jordan	1976, 1979, 1980, 1984, 1989, 1992, 1993, 1998, 2001
Korea	1986, 1997
Malaysia	1984, 1987, 1994, 1999
Mexico	1982, 1995
Morocco	1978, 1995
Paraguay	1988, 1989, 1995, 2002
Peru	1983, 1984, 1998
Philippines	1983, 1997
Poland	1988, 1990
Romania	1988
South Africa	1985
Sri Lanka	
Thailand	1982, 1997, 1998
Tunisia	
Turkey	1994, 2001
Uruguay	1982, 2002, 2004

Notes. The sample includes the countries classified as middle-income by the World Bank, plus Korea, and minus major oil-producing countries. A country-year observation is identified as a sudden stop if the ratio of capital inflows to gross domestic product falls by more than 5%, where capital inflows are measured as the current account deficit minus reserves accumulation.
Source. International Financial Statistics.

We observe a large fall in capital inflows in the year of the sudden stop, amounting to about 10% of the previous year's output. This is not surprising as a large fall in the financial account is the criterion that was used to identify sudden stops. More interestingly, we see that most of the negative impact of the financial account reversal on domestic absorption is offset by a fall in reserve accumulation. Thus, domestic absorption falls by less than 3% of GDP on average in the year of the sudden stop – much less than capital inflows. This evidence is consistent with the view that emerging market countries accumulate reserves in good times so as to be able to decumulate them, thereby smoothing domestic absorption, in response to sudden stops.

Coming back to the calibration, the unconditional probability of a sudden stop is 10.2% per year, which is rounded to $\pi = 0.1$ in the calibration. Parameter λ was calibrated as the average level of $(k_{t-1} - k_t)$ over our sample of sudden stops, which is close

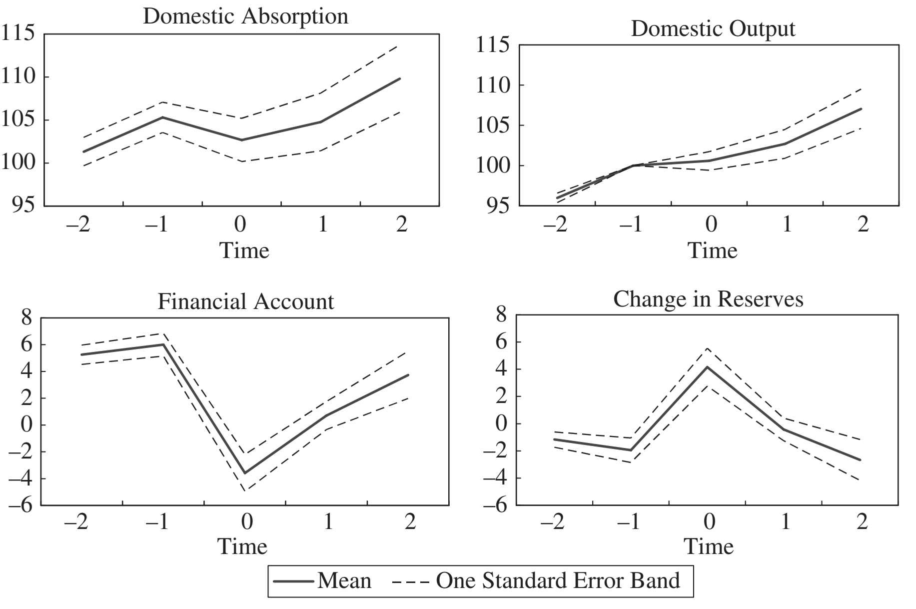

Fig. 2. *Domestic Absorption and International Reserves in Sudden Stops,* 1980–2003
Note. The five-year event window is centred around a sudden stop occurring at time 0. The list of countries and sudden-stop years is given in Table 1. The events that occurred before 1980 or inside the five-year window of the previous sudden stop were excluded. Domestic absorption and domestic ouput are expressed in percentage points of real gross domestic product (GDP) in the year before the sudden stop. The financial account and the change in reserves are expressed in percentage points of GDP. A positive level of 'change in reserves' corresponds to a loss of reserves.
Source. Authors' computations based on International Financial Statistics and World Development Indicators.

to 10%. Looking at the ratio of short-term external debt to GDP would give similar values. This ratio is equal to 8.2% on average in our sample according to the World Bank's *Global Development Finance* (GDF) data set and to 11.7% according to the Bank of International Settlements (BIS) database.[14]

We calibrated the output cost of a sudden stop by looking at the average difference between the GDP growth rate the year prior to the sudden stop and the growth rate in the first year of the sudden stop. We find that the GDP growth rate falls by 4% on average in the first year of a sudden stop and by 9% if we restrict the sample to the sudden stops in which output fell. We set γ to 6.5%, the average between the low and the high estimates. This is in the range of output costs of sudden stops that have been estimated in the literature.[15] The next Subsection discusses how the model predictions would be changed by assuming different values for γ.

[14] One source of discrepancy is that the definition of short-term debt is based on original maturity in the GDF data but on residual maturity in the BIS data. The two data sets also differ by their country coverage.

[15] The estimates in the literature tend to be somewhat larger but they refer to the cumulated output loss over several years. Hutchison and Noy (2006) find that the cumulative output loss in a sudden stop is around 13–15% of GDP over a three-year period. Becker and Mauro (2006) find an expected output cost of 16.5% of GDP.

The opportunity cost of holding reserves is often measured, in the literature, as the difference between the interest rate that the country pays on its long-term external debt and the return on its reserves. If one assumes for simplicity that the reserves are denominated in US dollars, the opportunity cost of reserves for country j in year t is given by

$$\delta_t(j) = r_t^l(j) - r_t^s(us),$$

where $r_t^l(j)$ is the interest rate on the country j's long-term dollar debt, and $r_t^s(us)$ is the US short-term interest rate. This can also be written as the sum of the US term premium plus the spread on the country's long-term debt

$$\delta_t(j) = \underbrace{r_t^l(us) - r_t^s(us)}_{\text{US term premium}} + \underbrace{r_t^l(j) - r_t^l(us)}_{\text{country spread}}.$$

Our model suggests a similar approach to the calibration of δ but with the caveat that the country spread should only include the *pure* risk premium and not the default risk premium.

The US term premium, measured as the differential between the yield on 10-year US Treasury bonds and the Federal Funds rate, was equal to approximately 1.5% on average over the period 1990–2005.[16] The second component (the pure risk premium on emerging market debt) has been found to be relatively small in the literature. Based on estimates of the average *ex post* returns on emerging market bonds and loans over the period 1970–2000, Klingen *et al.* (2004) find that the pure risk premium is approximately zero. Using a different approach, Broner *et al.* (2007) find risk premia on emerging markets bonds ranging from 0% to 1.5% in the period 1993–2003.[17] Based on this discussion, we set δ to 1.5% in the benchmark calibration of the model, and allow δ to vary in a relatively wide interval, from 0.25% to 5% in the sensitivity analysis. The resulting value of p is:

$$p = 1 - \frac{\delta}{(1-\pi)(\pi+\delta)} = 0.855.$$

Finally, the risk-free short-term dollar interest rate r is set at 5%. The growth rate g is set at 3.3%, the average real GDP growth rate in our sample of middle-income countries over 1975–2002 (excluding sudden-stop years). The benchmark risk-aversion and its range of variation are standard in the growth and real business cycle literature.

One can check that condition (20) is satisfied for the benchmark calibration. Condition (21) is also satisfied provided that debt does not flow back to the country too quickly after a sudden stop. If we assume a linear specification $\alpha(\tau) = \tau\alpha/\theta$, this condition is satisfied if a sudden stop episode lasts at least four years ($\theta \geq 4$). Finally, condition (22) is satisfied, for our benchmark calibration, provided that the output deviation from trend is lower than 6% of GDP after the first period of the sudden stop.

[16] This measure is not adjusted for fluctuations in the expected US rate of inflation over the sample period. See Rudebusch *et al.* (2007) for a review of the possible approaches to estimating the US term premium.

[17] Broner *et al.* (2007) find that the pure risk premium can increase to much higher levels in times of crisis. But the appropriate measure of δ is the level of the risk premium in non-crisis times (when the country insures itself against a crisis).

2.2. *Sensitivity Analysis*

Based on our formula for the optimal level of reserves, equation (15), the benchmark calibration implies an optimal level of reserves of 9.1% of GDP, or 91% of short-term external debt. This is close to the ratio of reserves to GDP observed in the data over 1975–2003 (11% on average, a level that can be explained by the model if the risk aversion parameter is raised from 2 to 2.75). However, this is significantly lower than the level observed in the most recent period, especially in Asia. It would be interesting to know what changes in the parameters would be required to increase the optimal level of reserves. The remainder of this Section explores the sensitivity of our results to parameter values.

Figure 3 shows how the optimal level of reserves depends on: the level of short-term debt (or size of sudden stop), λ; the output cost of a sudden stop, γ; the probability of sudden stop, π; the degree of risk aversion, σ; and the risk premium, δ. In each case, we contrast the level of reserves computed using our model with the one implied by the Greenspan–Guidotti rule. Several interesting results emerge. First, the Greenspan–Guidotti rule provides a good approximation to the variation of the optimal level of reserves with the level of short-term debt. The optimal ratio of reserves to short-term debt remains in the range of 90–100% if the size of the sudden stop exceeds 10% of GDP.[18]

Second, the optimal level of reserves is quite sensitive to the output cost of a sudden stop, γ, the probability of sudden stop, π, the premium δ and the risk aversion parameter, σ. This offers an interesting contrast with the Greenspan–Guidotti rule, which does not depend at all on these parameters. Doubling the probability of sudden stop from 5% to 10% more than doubles the optimal level of reserves, from 3.6% to 9.1% of GDP. Doubling δ from 1.5% to 3% reduces the optimal reserve-to-GDP ratio from 9.1% to 2.8%. A shift in risk aversion from 1 to 4 increases the optimal level of reserves from 2.1% to 12.7% of GDP. However, because the optimal level of reserves is a strongly concave function of σ, increasing risk aversion has a milder impact for σ larger than 4.

Next, we look at crisis prevention, based on the analysis in Section 1.4. The impact of crisis prevention crucially depends on the specification of function $\pi(\rho)$. First, we could assume (in line with the model of self-fulfilling crisis presented in Section 1.4), that the economy is vulnerable to sudden stops if and only if its reserves do not cover its short-term debt. That is, $\pi(\cdot)$ is a step function,

$$\pi(\rho) = \pi \quad \text{if } \rho < \lambda,$$

$$\pi(\rho) = 0 \quad \text{if } \rho \geq \lambda.$$

Then the country will never find it optimal to set reserves in excess of short-term debt as the extra reserves yield no benefit once the probability of sudden stop has been reduced to 0 (i.e. the Greenspan–Guidotti rule corresponds to full insurance in terms

[18] This is not true, however, for small sudden stops: the optimal level of reserves is equal to 0 if short-term debt amounts to less than 2.5% of GDP. This is because the marginal benefit of smoothing domestic absorption varies in proportion with the size of the sudden stop, whereas the marginal cost of holding reserves is constant.

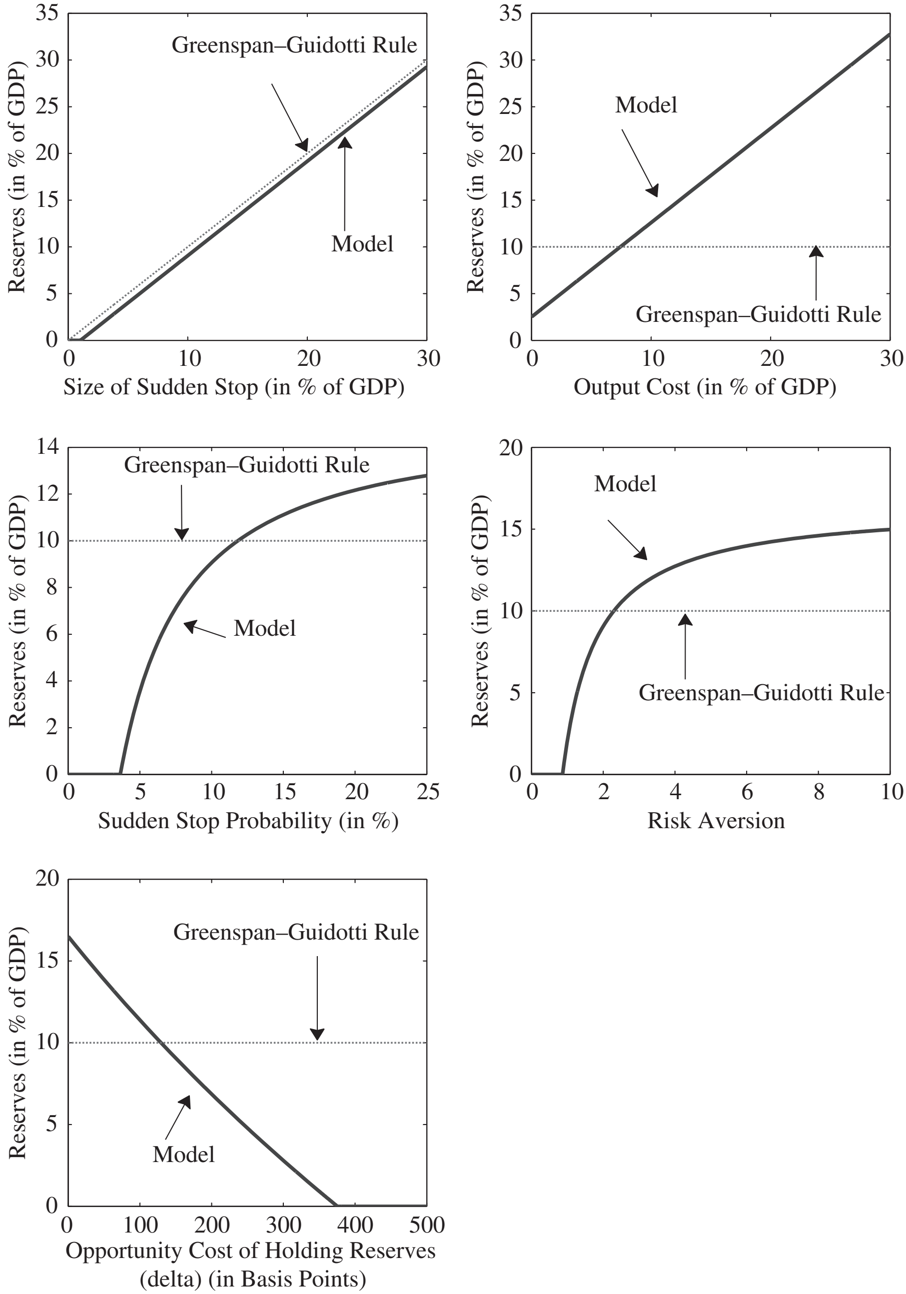

Fig. 3. *Optimal Ratio of Reserves to Gross Domestic Product: Basic Model*
Source. Authors' calculations.

of crisis prevention). A simple numerical exercise shows that if the other parameter values are set to their benchmark values, the Greenspan–Guidotti rule is indeed optimal provided that the probability of sudden stop π is larger than 1%.

Another approach to calibrating function $\pi(\cdot)$ relies on the empirical literature on early warning indicators of crisis, which has estimated probit equations of the type,

$$\pi = F\left(b - a\frac{\rho}{\phi}\right), \tag{30}$$

where ϕ is the denominator of the reserves-coverage ratio (e.g. if the denominator is short-term debt, $\phi = \lambda$), and $F(\cdot)$ is the cumulative distribution function of a standard normal law. In this specification, the probability of a crisis is a smoothly decreasing function of the reserve ratio.

Figure 4 shows the optimal level of reserves when the prevention benefits are specified as in (30). For the purpose of the sensitivity analysis, we used a range of [0,0.5] for the crisis prevention parameter a, which is consistent with probit regressions for the probability of currency crisis (Jeanne, 2007).[19] To illustrate, if the crisis prevention parameter is at $a = 0.3$, doubling the ratio of reserves to short-term debt from 1 to 2 can reduce the probability of a sudden stop from 10% to 6%.

The left panel of Figure 4 shows how the optimal level of reserves ρ^* varies with the crisis prevention parameter a.[20] Parameter b was set to $F^{-1}(0.1)$, so that $a = 0$ corresponds to the benchmark model. The optimal level of reserves increases markedly with a, and reaches a maximum of 34.4% of GDP for $a = 0.25$. The relationship between a and ρ^* is non-monotonic, as a low probability of sudden stop can be achieved with less reserves for higher levels of a.

The right panel of Figure 4 shows how the optimal level of reserves varies with parameter b, for $a = 0.15$. The range of variation of b makes π increase from 5% to 15% when reserves are equal to short-term debt. The Figure shows that more vulnerable countries (with higher b) tend to accumulate more reserves. The optimal level of reserves is always larger than 20% of GDP and can come close to 30% of GDP.

Finally, we looked at a number of variants of the model in previous versions of the paper. We added a real exchange rate to the model, and found that the model predicted a higher level of reserves (by about 4% of GDP relative to the benchmark) if there is a 10% real depreciation at the time of the sudden stop. We also explored the impact of assuming that reserves reduce the output cost of a sudden stop (making γ a decreasing function of ρ). This increases the optimal level of reserves by 1–5% of GDP, depending on the calibration.

2.3. *The Recent Buildup in Emerging Market Reserves*

There has been a large buildup in the reserves of emerging market countries, especially in Asia where the average ratio of reserves to GDP exceeded 28% in 2005 (Figure 5).[21] As noted in the introduction, this buildup has sometimes been interpreted in terms of

[19] The literature tends to find that reserves have prevention benefits for currency crises but not for sudden stops. See Berg *et al.* (2005) for a review on early warning indicators of currency crises. In contrast to currency crises, Calvo *et al.* (2004), Frankel and Cavallo (2004) and Jeanne (2007) do not find that reserves have a statistically significant effect of reducing the probability of sudden stop.

[20] The optimal level of reserves was computed using the numerical method presented in Section 1.4. We assumed linear paths: $\alpha_s(\tau) = (\tau/\theta)\alpha$ and $\gamma_s(\tau) = (1 - \tau/\theta)\gamma$, with $\theta = 5$.

[21] The sample of countries is the same as in the previous Section.

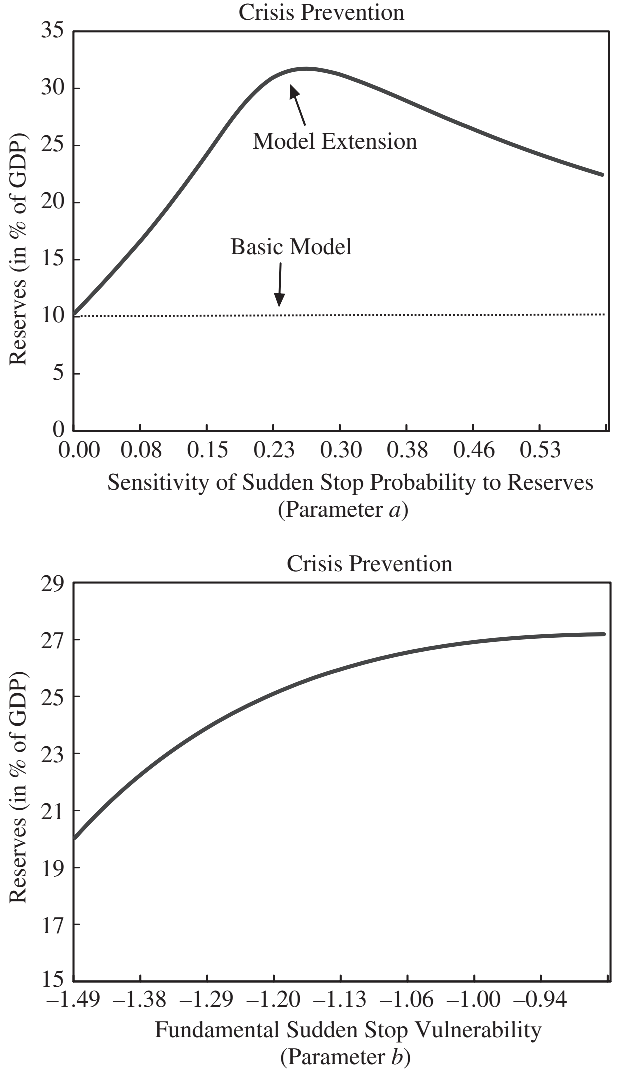

Fig. 4. *Optimal Ratio of Reserves to Gross Domestic Product: Model with Crisis Prevention*
Source. Authors' calculations.

self-insurance against the risk of sudden stops in capital flows, although it could be a result of other causes. We now explore how far one can go in explaining the recent buildup using our insurance framework.

Prima facie, the level of reserves in Latin America seems broadly consistent with the benchmark calibration of our model but the same is not true for Asian emerging market countries. In fact, since 2000 the average reserves-to-GDP in Asia has exceeded the full insurance level $\lambda + \gamma$, which is equal to 16.5% of GDP in our benchmark calibration. Can extensions or alternative calibrations of our model account for the Asian reserves buildup? To explore this question we first look at the extension of the model in which the probability of a sudden stop is decreasing with the level of reserves.

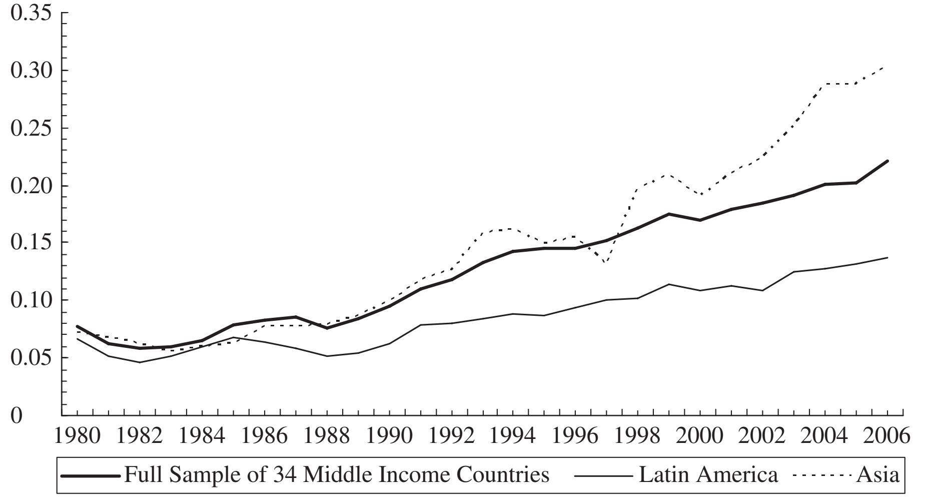

Fig. 5. *International Reserves as a Share of Gross Domestic Product,* 1980–2006
Notes. For each country group, the data refer to unweighted cross-country averages.
Source. Authors' computations based on International Financial Statistics and World Development Indicators.

Table 3
Probit Estimation of the Probability of a Sudden Stop, 1980–2004

	3.1	3.2	3.3	3.4
Real effective exchange rate				
Overvaluation (deviation from HP Trend)	2.33	2.29	2.42	2.32
	(2.78)**	(2.55)*	(2.77)**	(2.43)*
Public debt/GDP	0.65	0.66	0.8	0.93
	(2.81)**	(2.52)*	(2.04)*	−1.82
Financial openness (|Gross inflows|/GDP)	8.88	9.98	8.1	9.41
	(5.74)**	(5.82)**	(4.24)**	(4.29)**
Intercept	−2.17	−1.85	−2.03	−2
	(13.07)**	(5.66)**	(3.80)**	(2.60)**
Observations	706	706	543	543
Pseudo R^2	0.1	0.16	0.11	0.14
Time effects	No	Yes	No	Yes
Country-fixed effects	No	No	Yes	Yes

Notes. All explanatory variables are taken as average of first and second lags. Absolute value of *z*-statistics in parentheses. *means significant at 5% level; ** means significant at 1% level.

2.3.1. *Probability of sudden stop: cross-country estimates*

We estimate the probability of a sudden stop by running a probit regression in our sample of 34 middle-income countries over 1980–2004. Our preferred specification is reported in Table 3. The explanatory variables have been selected using a general-to-specific approach, starting from a set of more than 20 potential regressors.[22] All the

[22] The regressors are listed in Table A1 in online Technical Appendix.

explanatory variables are averages of the first and second lags, and are thus predetermined with respect to the sudden stop.

We find that the probability of a sudden stop increases with the level of real appreciation (measured as the deviation in the real exchange rate from a Hodrick–Prescott trend), the ratio of public debt to GDP and the country's openness to financial flows (measured by the absolute value of gross inflows as a share of GDP; regression 3.1). The last determinant suggests that the vulnerability to sudden stops rises with the degree of international financial integration. Interestingly, we found that trade openness did not significantly affect the probability of a sudden stop when financial openness was included as an explanatory variable. Our estimation remains robust when different combinations of time and country-fixed effects are introduced in the specification (regressions 3.2, 3.2 and 3.4). Finally, we tried various reserve adequacy ratios and did not find any significant negative impact on the probability of a sudden stop, in accordance with the results obtained previously in the literature.

Figure 6 shows the evolution of the probability of sudden stop in Latin America and in Asia according to our preferred specification (without fixed effects). Those estimates are based on regression 3.1, in which the probability of sudden stop is exogenous to the accumulation of reserves. Interestingly, the probability of sudden stop increased in Latin America from 1997 to 2003, which, according to our insurance model, could explain why reserves have increased too. In fact, the centre-left panel of Figure 3 shows that both the level and variations in the Latin American

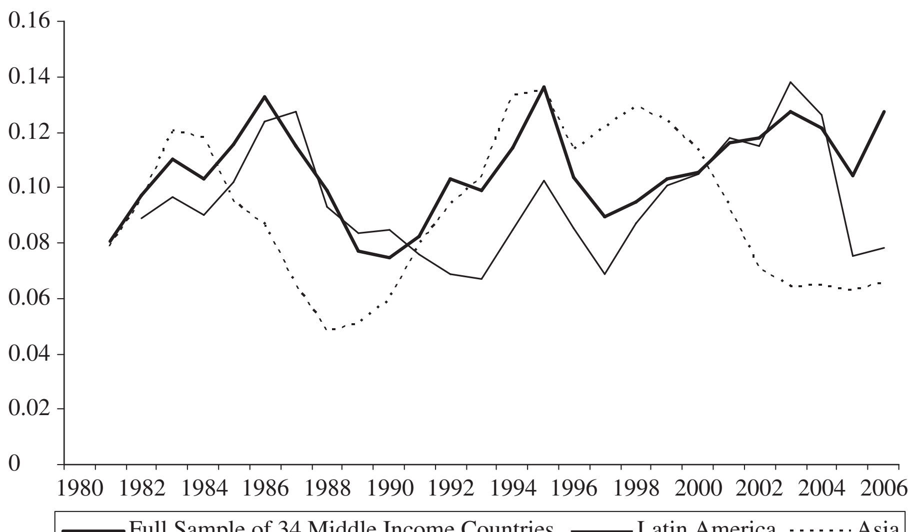

Fig. 6. *Estimated Probability of a Sudden Stop,* 1980–2006
Notes. The probability of a sudden stop for each country and each year is computed by using the estimates of the probit model presented in Table 3, regression 3.1. For each country group, the data refer to unweighted cross-country averages.
Source. Authors' computations based on International Financial Statistics and World Development Indicators.

reserves-to-GDP ratio between 1997 and 2003 can be explained by the benchmark model. In contrast, the probability of sudden stop declined in Asia after a peak that was reached at the time of the 1998 crisis. This probability also declined in Latin America after 2004.

2.3.2. *Asian reserves: puzzles and tentative solutions*

The large buildup in Asian reserves since 1998 (and the continuing buildup in Latin America since 2004) may seem puzzling from the point of view of our insurance model, as the probability of a sudden stop was going down during that period. Can variants of the benchmark model account for the reserves buildup?

First, the model with crisis prevention can explain levels of reserves of the order of magnitude recently observed in Asia (Figure 4). But there are several problems with this explanation. First, we do not find evidence that reserves have prevention benefits in our probit estimates. Second, even if crisis prevention benefits existed, the countries that are more vulnerable to sudden stops should hold more reserves (Figure 4), that is, reserves should be higher in Latin American than in Asia.

A possible hypothesis is that the severity of the 1998 crisis in Southeast Asia led to an upward revision of the output cost of a sudden stop. To test this, we re-calibrated γ by reference to the actual output costs for the four Asian countries that experienced a sudden stop in 1998: Korea, Malaysia, the Philippines and Thailand. We then re-computed the optimal level of reserves for each of the four countries for two different levels of risk aversion and compared the results with the actual level of reserves in 2005. The results are presented in Table 4. For three of the four countries affected the output cost exceeded 14% and was thus much larger than the baseline estimate (6.5%). As a consequence the new calibration brings the optimal level of reserves closer to the observations. If in addition we assume a higher risk aversion ($\sigma = 10$) than our baseline ($\sigma = 2$), the gap between the model predictions and the data is almost closed, except for Malaysia.

In sum, recalibrating the output cost of sudden stops by reference to the regional experience and assuming a higher level of risk aversion can help to reconcile the model with the accumulation of reserves in the four Asian countries that had a

Table 4
Output Cost of the 1997–1998 *Asian Crisis and the Optimal Level of Reserves in South East Asia*

Country	Outptut cost of 1997–8 sudden stop (in per cent of GDP)	Optimal level of reserves to GDP (risk aversion = 2)	Optimal level of reserves to GDP (risk aversion = 10)	Actual reserves to GDP (2005)
Korea	−14	0.16	0.22	0.25
Malaysia	−17	0.2	0.26	0.51
Philippines	−6	0.09	0.15	0.16
Thailand	−17	0.19	0.25	0.29

Notes. For each of the four Asian countries, the optimal level of reserves is computed using the output cost of the 1997–8 sudden stop for two levels of risk- aversion ($\sigma = 2$ and $\sigma = 10$). All the other parameters are identical to the baseline model calibration presented in Table 1. GDP, gross domestic product.

sudden stop in 1998. However, one needs to assume a very high level of risk aversion, and some countries that were affected little by the 1998 crisis (such as China) also increased their reserves.

Other factors may explain a large precautionary accumulation of reserves. Obstfeld *et al.* (2010) suggest that, in a financially integrated world, a financial crisis can result in a significant share of M2 being converted in foreign currency. In the context of our model this is equivalent to a higher size of capital outflows λ. Finally, one could argue that while the accumulation of reserves was excessive to cope with 'standard' sudden stops in East Asia, it was appropriate to insure against a large global financial crisis such as the one that started in 2007–8.[23]

3. Concluding Comments

This article derives a simple formula for the optimal level of international reserves, based on the assumption that reserves provide insurance allowing countries to smooth domestic absorption against the disruption induced by a sudden stop in capital flows associated with a fall in output.

We find that a plausible calibration of the model can account for the average level of reserves in emerging market countries since 1980 but not for the recent accumulation in Asia. There are, obviously, other explanations for this accumulation. For example, a number of authors argue that the reserves buildup in Asia is the unintended consequence of policies that maintain large current account surpluses (Dooley *et al.*, 2004; Summers, 2006). If one takes this view, our framework could help to assess the fraction of the public sector's foreign assets that should be held as liquid reserves for the sake of insurance against volatile capital flows, rather than invest with a longer-term perspective in 'sovereign wealth funds'.

The analysis was based on a stylised framework and there are several ways in which the model could be made more realistic. Adding productive capital and investment to the model would affect the benefits of reserves in an *a priori* ambiguous way. On one hand, investment offers a new margin to smooth consumption, which would tend to reduce the need for reserves. On the other hand, reserves also have a new benefit, which is to smooth domestic investment and output. It would be interesting to look which effect dominates by adding investment to the model presented in this article.

Our framework could be used to analyse issues related to the collective management of reserves. What would be the benefits of reserve-pooling arrangements between emerging market countries? What are the consequences, for reserve accumulation and domestic welfare, of an institution such as the IMF that relaxes the external credit constraint of emerging market countries in a crisis? These questions could be analysed using a multi-country extension of the framework presented in this article.

[23] As noted in the introductory paragraphs, a significant fraction of the reserves may also have been accumulated to resist currency appreciation.

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Additional Supporting information may be found in the online version of this article:

Appendix S1. Table A1 and Proof of Lemma.

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