Systemic Risk-Taking and the U.S. Financial Crisis.

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Abstract

The recent macroeconomic experience of the U.S. resembles the boom-bust cycles of emerging markets more so than the tame postwar US business cycles. We present a model in which a feebdack loop between credit and prices generates the boom and the bust, and accounts for several stylized facts that characterize of the U.S. experience. We then use this framework to analyze the dynamics of external imbalances and to evaluate post-crisis stabilization policy.

1 Introduction

The recent macroeconomic experience of the U.S. resembles the boom-bust cycles of emerging markets more so than the tame postwar U.S. business cycles. We present a model in which a feebdack loop between credit and prices generates the boom and the bust, and accounts for several stylized facts that characterize of the U.S. experience.

The U.S. exhibits a long period of prosperity from 1992 to 2007–characterized by high GDP growth and low macroeconomic volatility– that has been abruptly punctuated by a severe financial crisis in 2007-2008. Puzzingly, there is no large exogenous shock that one can point to as the cause of the crisis. A sharp asymmetric sectoral pattern seems to be at the root of this boom-bust pattern. The consensus view is that the boom was sustained by an abnormal growth in housing and sub-prime mortgages, and that financial institutions loaded on this insolvency risk factor. The crisis erupted when housing prices started to decline generating a deterioration in balance sheets that bankrupted many firms in the financial sector that had taken on insolvency risk.

There are several stylized facts associated with this sectoral assymetry that a model of the U.S. boom-bust cycle should account for. On the real side, the sectoral asymetry is reflected in the boom-bust cycle of unprecedented magnitude experienced by the housing sector. Real home prices have doubled between 1992 and 2006 before losing over 20 percent in 2007-2008, according to the Case-Shiller index (Figure 3, left panel). The quantity of new houses sold also doubled during the same period before falling back to the pre-boom level in 2008 (Figure 3, right panel). This housing boom has been associated with a sharp increase in the real estate services-to-GDP ratio over the last decade.

On the financial side, the sectoral asymetry is reflected in a pronounced credit boom and excessive increase in leverage that resulted in a fast growing credit-to-GDP ratio. This boom does not reflect a uniform expansion of credit across the economy. In the non-financial private sector, the boom reflects the rapid build-up of mortgage debt (Figure 1, left panel). This increase in leverage was amplified by the loading of mortgage assets by the financial sector, which was supported by a rapid increase in financial sector leverage (Figure 1, right panel). The rise of financial sector leverage occured mainly through the rapid development of non-deposit financial institutions, which now hold 2.5 times more assets and twice the amount of private credit stock than deposit banks (Figure 2).

The sharp sectoral financing asymmetry has been also reflected in asset prices. Figure 7 shows a dramatic boom-bust cycle in the S&P500 housing sub-index: it increaed by 400% between 2001 and 2006, but lost most of its gains between 2006 and 2008, This boom-bust pattern is in striking contrast with the smoother path of the general S&P500 index.

Putting together the high leverage of the non-financial sector and the high exposure of the financial sector to real estate generates "insolvency risk" for the combined housing/financial sector. If aggregate real home prices were to fall, the financial sector that had loaded on mortgage risk would default for the simple reason that its liabilities were denominated in nominal dollars, while its assets were indexed to housing prices. This insolvency risk taking is a form of currency mismatch typically observed in emerging markets.

To sum up, in order to account for the U.S. boom-bust cycle it is necessary to explain: (i)why did the financial sector take on so much insolvency risk; (ii)how did this risk taking interact with the housing sector to generate a positive correlation between housing prices and quantities; (iii) what is the amplifying mechanism that transformed the decline in house prices into a systemic meltdown of the financial sector and firesale prices over and beyond what is typically observed in business cycles.

In our model, there is a feedback loop that generates a positive correlation of prices and quantities. This correlation arises from the interaction of borrowing constraints and currency mismatch, which gives rise to a debt deflation mechanism. As home prices go up, net worth increases in the housing sector as the real value of mortgages declines relative to the value of housing assets. Higher net worth leads to more credit, which fuels higher demand for real estate services inducing an increase in prices and so on. In the meantime, since the profitability of housing investment depends on the price of real estate relative to the rest of the economy, financial fragility and insolvency risk can endogeously arise. This opens the possibility of a crisis during which prices, quantities and credit collapse simultaneously. If along this path agents' expectations about housing-price inflation changed, and the demand for housing stopped increasing, then many agents would be unable to repay their debts and so generalized banckruptcies would take place. This sudden shift in expectation would start a vicious feedback loop in which price declines lead to lower credit, which leads to lower prices and so on.

The question remains as to why would agents take on insolvency risky by choosing a currency mismatch in their balance sheets? In our model this occurs because there are systemic bailout guarantees that insure lenders against generalized defaults.

In contrast to standard business cycle models, our model is able to generate such an endogenous systemic risk-taking process and to replicate the asymmetric boom-bust cycle experienced by the U.S. economy. Our framework is then used to shed light on the two following issues: first, the pattern of external imbalances in tranquil times and in crisis times; second, the effect post-crisis stabilization policies on future growth performance. In particular, we argue that policies that transfer resources during the crisis to consumers and investors reduce the extent of firesales, save bankrupcy costs, and ameliorate the post-crisis peformance by mitigating the collapse of investment. However, by hoarding large liabilities, the government reduces its ability to borrow to cover future systemic bailouts and this has the unintended consequence of reducing the impetus for risk-taking and the resulting high leverage and high growht.

The rest of the paper is organized as follows. Section 2 presents the stylized facts and Section 3 presents the model. Section 4 discusses external imbalances and Section 5 stabilization policy.

2 Stylized Facts

In this section, we document a number of key stylized facts associated with the boom-bust episode experienced by the U.S. economy. In presenting those facts, we underline a number of asymmetries that we argue are at the core of the recent episode.

Fact 1: Asymmetric Financial Development and Insolvency Risk

Between 1960 and 2000, the portfolio of liabilities of the domestic financial sector remains roughly equally *balanced* between mortgage and non-mortgage debt (Figure 1, left panel). The situation changed dramatically in the last decade. The ratio of mortgage debt to GDP jumped from 70% to 105% of GDP, Meanwhile the ratio of non-mortgage debt cycled within a range of 65 to 70% of GDP. In fact, the trend increase in non-financial sector debt, in the last eight years, can be entirely explained by the rapid build-up of mortgage liabilities. In the same period, the financial sector became considerably more leveraged and loaded with mortgage related assets (Figure 2, right panel). The rapid build-up of financial firms'debts is paralleled on the asset side by a large hoarding of mortgage assets.

Another way to measure the sectoral asymmetry in financial exposure is to look at the share of mortgage-related assets in total financial assets. Figure 4 plots the partition of financial assets into different instruments for commercial banks (left panel) and finance companies (right panel). The share of mortgage-related assets in commercial banks' portfolios - mortgage loans, mortgage-backed securities and collateralized mortgage obligations - increases from 12% in 1960 to 20% and to 55% in 1984 . In the same period, the share of "safe assets" - U.S. treasuries, reserves at the Federal Reserve andcash - declines from 35% to 2%. Finance companies experienced a similar trend: the share of mortgage assets increases from 7.5% to 30% between 1960 and 2005. Moreover, in the last 7-8 years, banks and finance companies reduced their exposure to business loans to the same order that they increased their exposure to mortgage loans and mortgage-related assets.

Fact 2: Home Mortgage Boom-Bust Credit Cyle and Household Leverage

Figure 5, left panel, gives an account of the boom-bust credit cycle in mortgage credit by plotting the ratio to GDP of quarterly flows of net new borrowing by households in mortgage credit market between 1992 and 2008. The boom-bust cycle presents the standard asymmetric features that can be found in many emerging market countries. During the boom phase, the ratio of mortgage credit to GDP increased gradually from 2-3% to 9% in 2006 before collapsing to 2% in 2008. Figure 5, right panel, shows the corresponding change in leverage for homeowners by plotting the ratio of home equity to mortgage debt. In the same period, the ratio increased gradually from 0.65 to 1.2.

Fact 3: Housing Sector Profitability and the "Real Cost of Fund"

During the boom phase, higher leverage reflects both a higher loan to equity ratio for new homeowners and the withdrawal of equity from existing homeowners. Figure 6, left panel, plots the ratio of mortgage equity withdrawals as a fraction of disposable income. We can see that indeed mortgage equity withdrawals and new borrowing were comoving strongly during the boom-bust episode. Equity extraction is possible when the rate of home price appreciation exceeds interest payments on mortgage loans. Figure 6, right panel, plots the effective mortgage interest rate net of home price appreciation, the "real" interest rate expressed in terms of housing prices. Between 1992 and 2005, this "real" interest rate fell sharply and even became negative. The resulting capital gains that were cashed out through mortgage equity withdrawals. In 2007, as housing prices start falling sharply, the "real" interest jumped up dramatically. Capital losses ensued and home equity withdrawals shrank, making homeowners more likely to default on their mortgage payments. **I**n other words, the "currency mismatch" in homeowners' balance sheets reinforced the boom and precipitated the defaults.

Fact 4: Assymetric Asset Price Boom-Bust Cycle.

Figure 7 provides evidence of a very sharo sectoral asymmetry in asset prices: the stock of major U.S. home builders companies, included in the S&P500 Home Building Index, experienced a dramatic boom-bust cycle between 2001 and 2008 which make the fluctuations in the general S&P500 index look tamed. Between 2001 and 2006, the S&P Home Building Index was multiplied by 400% before falling back dramatically between 2006 and 2008 so as to eventually fully offset the previous gains. In the same period, the general S&P500 index first decreased between 2001 and 2003 in the aftermath of the dot-com crash before rising again, reachin in 2007 its previous peak of 2000.

Fact 5: Asymmetric Real Sector Development.

During the credit boom, the real estate sector of the economy grew significantly faster than the rest of the economy both in terms of investment and output. Figure 8, left panel, plots the ratio of an index of the stock of fixed assets in the construction sector over a similar index for the total economy. Over 1992-2007, the growth of the stock of fixed assets has been, on average, 50% percent larger in the construction sector than in the rest of the economy. Figure 8, right panel, plots the ratio of the value put in place of new construction over GDP. This ratio increased from 7% to 9% between 1992 and 2007. A similar pattern can be observed for residential upkeep investment which increased, in real terms, by 50% between 1992 and 2007.

Fact 6: Crisis and Defaults

In 2007, the insolvency risk materialized through a sharp increase in actual and anticipated defaults on mortgage loans and mortgage assets. Figure 9.1 plots the rate of foreclosure for all mortgage loans. Foreclosure rates slightly increased from 0.3 to 0.4 percentage points during the boom phase and thereafter jumps to one percent in 2008. Foreclosure rates only reflect *actual defaults*. In contrast, credit default swaps premiums on mortgage-backed securities reflect the anticipated increase in defaults. Figures 9.2 and 9.3 plot the default premium (expressed as spread over LIBOR) implied by two indexes of credit default swaps - CMBX (CDS on all mortgage backed securities) and ABX (CDS on subprime-backed mortgage-backed securities) for two credit ratings (AAA and A). For both indexes, the spreads that were low until 2007 exploded once the crisis started, the effect being much more pronounced for the ABX than the CMBX at the same credit rating.

Fact 7: Asymmetric Bust

As the boom phase was characterized by sector asymmetries, so was the bust phase. Figure 10 plots the CDS default premium for various sectors of the U.S. economy. From 2007, the default risk increased sharply in the home building industry but only very modestly in the basic industries sector. An interesting contrast emerges in the financial sectors. The CDS on banks, which reflect in large part large bank holding companies, increases in the initial phase of the crisis but then declines as bailout packages were announced. In contrast, the CDS on the rest of the financial sector, which includes non-bank finance companies, continue to rise sharply up to now.

3 Model

3.1 The Setup

There are two sectors: the H-sector that produces housing services, and the T-sector that produces a tradable good. Throughout the paper the T-good is the numeraire and the price of the H-good is denoted by p_t. The H-good (y^H_{t+1}) is produced using inputs from the H-sector (I_t) via a linear production technology.

$$y^H_{t+1} = \theta I_t,$$

The H-sector is populated by OLG of developers with two-period lives, who can be interpreted as conglomerates of housing service producers and housing finance specialists that intermediate outside debt financing for them.

In the first period of his life, a representative junior developer receives an endowment $\underline{w}$ of T-goods and works for a senior developer against a claim on a fraction $(1-c)$ of profits. He then uses all his income as internal funds to start a new business, borrow and invest. In the second period of his life, the now senior developer hires a junior developer, sells his output and uses his share c of the profits (if any) to consume the T-good. The economy has a finite horizon with T periods.

The amplitude of the cycles is a key difference between boom-bust episodes and standard business cycles. The former exhibit amplitudes with a higher order of magnitude than the latter, and are characterize by systemic bankruptcies. This difference indicates that insolvency risk-taking plays a key role in boom-bust cycles. To allow for the possibility insolvency risk-taking we assume that developers can finance themselves by issuing two types of one-period bonds: H-bonds that have an interest rate ρ^H_t and whose promised repayment is indexed to the price of H-goods, $p_{t+1}(1+\rho^H_t)b^H_t$, and T-bonds that have an interest rate ρ^H_t and whose promised repayment is not indexed, $(1+\rho_t)b_t$. That is, H-bonds allow developers to hedge their exposure to fluctuations in the price of housing services, while T-bonds expose developers to insolvency in case of a sharp fall in H-prices.[1] To simplify the menu of financing contracts, we assume that developers either are {stay } fully unhedged or are fully hedged.

The investable funds of a young developer equal his internal funds w_t plus the debt he issues. Thus, the budget constraint is

$$p_t I_t \leq w_t + b_t + b^H_t.$$

Since firms can go bust, profits next period are $\max\{\pi_{t+1}, 0\}$,with

$$\begin{aligned} \pi_t &= p_{t+1} q_{t+1} - L_{t+1}, \\ L_{t+1} &= (1+\rho_t)b_t + p_{t+1}(1+\rho^H_t)b^H_t \end{aligned}$$

[1] One way to implement an H-bond is to issue a T-bond and short a housing index (such as the Case-Shiller index). One other way is to issue *revenue bonds* whose interest payments are backed by sales of housing services.

The internal funds w_t of a young developer equal the endowment $\underline{w}$ plus either a share $1-\beta$ of profits under solvency or, if the firm is insolvent, a small government aid payment a_t

$$w_{t+1} = \begin{cases} \underline{w} + (1-c)[p_{t+1}q_{t+1} - L_{t+1}] & \text{if } \pi_{t+1} > 0 \\ \underline{w} + a_{t+1} & \text{if } \pi_{t+1} = 0 \end{cases}$$

Borrowing constraints are another necessary ingredient of a BBC. Otherwise, if borrowers could always borrow, a systemic financial crisis could never occur.[2] As it is standard in the literature, we generate borrowing constraints by introducing agency problems in credit markets. In particular, we assume that by incurring a non-pecuniary cost $h[w_t + b_t + b_t^H]$, a young developer can engineer a scam that will allow him to divert the revenues to himself and not repay any debt in the next period, provided the firm has positive notional profits.

There must be a reason that leads agents to take on insolvency risk, but that does not eliminate borrowing constraints. Here, the reason is that the government grants bailout guarantees if there is a systemic crisis, but not otherwise. We introduce 'systemic bailout guarantees' by assuming that in case a majority of H-firms defaults, the government pays lenders of non-diverting firms a share γ of the promised debt repayment ammount (L_t). However, in case of an isolated default the govenrment does not bail out lenders.

Bailouts are domestically funded by taxing the tradable sector and the profits of solvent H-sector developers. The government has access to international capital markets: during a crisis it can borrow any amount to finance the bailout as long as its intertemporal budget constraint is satisfied.

Tradables sector

The T-good is produced by a continuum of measure, one of perfectly competitive firms that use labor as an input with a linear production function:

$$y_t^T = A_t l_t$$

Workers live for one period and have Cobb-Douglas preferences over housing services and the tradable good: $u = c_T^{1-\alpha} c_N^{\alpha}$. Since the T-sector is perfectly competitive, in any equilibrium the wage equals A_t, and the T-sector demand for housing services is

$$C_{N,t} = \frac{\alpha A_t}{p_t}$$

[2] Except in case of a large economy-wide exogenous shock such as wars or natural disasters.

The parameter A_t summarizes the productivity of the economy outside of the H-sector. We assume that

$$A_t = A \text{ if } t < T \text{ and } A_T = \overline{A}$$

where $\overline{A}$ is a constant defined by (19) in the appendix. This constant is set high enough so that at time T the H-sector can repay all its debt.[3]

For notational convenience in the rest of the paper, we define the T-sector demand for housing services in tradeable units as:

$$d_t \equiv p_t C_{N,t}$$

There are no exogenous sources of risk, such as terms-of-trade or productivity shocks. The only source of risk is endogenous relative price risk. As we shall see, in the equilibrium we will characterize the price can take on two values

$$p_t = \begin{cases} \overline{p}_t & \text{with probability } u_t \\ \underline{p}_t & \text{with probability } 1 - u_t \end{cases}, \quad u_t = \{0, u\}$$

We will refer to $1 - u_t$ as the crisis probability.

The Credit Market.

During period t, given his internal funds w, the representative young developer borrows from international lenders. He then decides whether to implement a diversion scheme. Next period payoffs are as follows. If there is no diversion and no default, lenders receive their promised repayment L_{t+1}, the now old developer gets $c\pi_{t+1}$ and the young developer gets $(1 - c)\pi_{t+1}$. If there is no diversion, but the firm defaults lenders get γL_{t+1} if a bailout is granted and zero otherwise; the old developer gets zero and the young developer gets the aid a_{t+1}. Finally, if there is diversion, lenders get nothing, the old developer gets $cp_{t+1}q_{t+1}$ and the young developer gets $(1 - c)p_{t+1}q_{t+1}$.

3.2 Discussion of the Setup

In our model, the H-sector integrates housing services producers, such as real estate companies and home builders, and housing financial specialists such as mortgage financing companies. This is a simplification, but it has the advantage of making transparent the key balance sheet mechanism at play. Insolvency risk arises endogenously in our economy when the aggregate balance sheet of

[3] The process for A_t is meant to capture a fundamental long-run trend in the demand of housing. Such trend can reflect, for example, demographic factors such as aging or the divorce rate that influence the demand for housing in the very long-run.

the H-sector has assets backed by housing services and liabilities denominated in T-goods. In a more complex set-up, developers would borrow from mortgage companies, which in turn would sell bonds to external investors.[4]

Developers live for two periods and cash out profits when they sell housing services in the second period of their life. In the real world, the housing services sector is not restricted to professional developers, as it comprises all agents that invest in such services and resell them. It includes, for example, the "fixers and flippers" that buy run-down homes, fix them, and resell them. It also includes the homeowners that buy their homes, make residential upkeep investments, and eventually resell them.[5] In such cases, mortgage equity withdrawals allows agents that do not sell their house to make similar profits.

We model the production of housing services as a linear function of inputs from the same sector. The inclusion of other inputs, such as tradables, would not change qualitatively our results. It is key to our feedback mechanism, however, that developers that sell housing services also buy housing services. In input-output tables for the U.S. economy, the intermediate construction inputs and the intermediate real estate inputs represent 49% and 59% of the inputs used in those sectors, respectively.

An alternative to the linear H-good production function is to consider a production function of the form $y^N = Bh^{\beta}l^{1-\beta}$, where h is investment in housing services, l is the labor input of a junior developer, and $B = \theta\overline{h}^{1-\beta}$ is an external effect that depends on the average of housing investment in a community. This effect is related to the often mentioned positive neighborhood externalities in communities with a large share of homeowners and high residential upkeep investment. Under this alternative, ouput is $y^N = \theta h$ and he labor share is $(1-\beta)y^N$.

The diversion technology we have used is a simple way to introduce agency problems in lending and generate borrowing constraints. An alternative is to simply impose exogenous borrowing constraints. Such a shortcut is not possible in our setup, as it would assume away the mechanism by which insolvency risk-taking relaxes borrowing constraints and reduces the expected cost of external funds. The diversion technology we have used captures more complicated schemes that could be used to divert funds. For instance, developers could use a complex ownership structure to turn profits into dividends and tunnel them away from lenders. The large number of intermediaries involved in real estate transactions–realtors, mortgage brokers, loan servicers, appraisers, escrow representatives–opens many ways to set up fraudulent skims to defraud outside investors.

[4] Such a framework would exhibit a double agency problem, and would imply that the supply of loanable funds would be constrained by the internal funds of finance companies (e.g. Holmstrom and Tirole (1998)).

[5] Fix and flip activity surged during the recent housing boom and the share of loans to non-owner-occupied homes in all home-purchase loans increased from 4% in 1992 to 16% in 2005.

Developers are born with an endowment $\underline{w}$. This assumption is not necessary for the main results. Its only role is to ensure that in equilibrium there is an incentive to take on insolvency risk every period, even during a systemic crisis. This result allows us to make simple long-run comparisons of risky and safe equilibrium paths.

Developers can only invest in the production of N-goods. In particular, they do not have access to a standard storage technology that returns the world interest rate r. This assumption allows for the adoption of negative NPV projects, which is a property we want dthe equilibrium to have.

We will impose several parameters restrictions that ensure the existence of an internally consistent mechanism that generates the intuitive feedback-loop stories told by commentators, and that we summed up in the introduction. Specifically, we impose the following recursive set of parameter restrictions:

$$\begin{aligned} 1 &< h < 1+r && (1)\\ \frac{h}{1+r} &< c < \frac{1+(1+r)^{-1}}{1+h^{-1}} && (2)\\ \frac{h}{1+r} &< u < \frac{1}{c}\frac{h}{1+r} && (3)\\ \varepsilon &< \frac{1-\frac{hu^{-1}}{1+r}}{1-c} && (4) \end{aligned}$$

The first restriction says that diversion costs should be neither too high nor too low.[6] If they were too high, borrowing constraints would not arise in equilibrium. If they were too low, borrowing constraints would be too tight and so insolvency risk-taking would not be profitable. The second restriction imposes bounds on the payout rate to old developers. The third restriction says that the probability of a crisis, $1-u$, should be positive but small for risk-taking to occur in equilibrium. The last restriction implies that crisis costs are severe enough so as to generate a fall in the price of H-goods which is large enough to bankrupt H-sector producers with currency mismatch in their balance sheets.

3.3 Safe and Risky Equilibria

The expected payoff of a young developer is

$$E_t(u_{t+1}\pi_{t+1}c[1-\delta_t]) + E_t(cp_{t+1}q_{t+1} - h[w_t + b_t + b_t^n])\delta_t,$$

where $\delta_t = 1$ indicates that the developer adopts a diversion scheme.

[6] Note that imposing the constraint $h > 1$ simplifies the proof but is more stringent than is necessary for the risky equilibrium to exist.

A symmetric equilibrium is a sequence $\{b_t, b_t^N, \rho_t, \rho_\tau^n, I_t, C_t, p_t\}$ such that (i) given internal funds w_t, prices $(p_t, \overline{p}_{t+1}, \underline{p}_{t+1})$ and the likelihood of crisis u_{t+1} the young developer's plan $(b_t, b_t^N, \rho_t, \rho_\tau^n, I_t, \delta_t)$ maximizes his expected payoff and lenders break even; (ii) T-sector demand C_t maximizes workers utility; and (iii) the price p_t clears the H-sector market

$$C_t(p_t) + I_t(w_t, p_t) = q_t(I_{t-1})$$

We will characterize two symmetric equilibria: a risky one where all developers take on insolvency risk, and a safe one where they do not. We derive the equilibria in two steps. First, we take as given the price path and derive the allocation. We then derive the equilibrium price path.

In any equilibrium, lenders fund only plans that do not lead to diversion. Since they are risk neutral and perfectly competitive they set the interest rates so that they break even, and lend up to an amount so that developers don't divert. Along any equilibrium path a bailout next period will be granted only if a systemic crisis occurs. A crisis will occur only if a majority of developers denominate their debt in T-goods.

Since in a safe equilibrium neither bailouts nor crises occur, interest rates satisfy

$$1 + \rho_t = 1 + r \text{ and } (1 + \rho_t^n)\overline{p}_{t+1} = 1 + r$$

In contrast in a risky equilibrium

$$1 + \rho_t = \frac{1 + r}{u + (1 - u)\gamma} \text{ and } 1 + \rho_t^n = \frac{1 + r}{E_t p_{t+1}}$$

A developer will find it optimal not to set up a diversion scheme if and only if the diversion cost is no greater than the expected debt repayment.

$$\begin{aligned} (1 + r)b_t^n &\leq h[w_t + b_t^n] \quad \text{for a safe plan} \\ u\frac{1 + r}{u + (1 - u)\gamma} b_t &\leq h[w_t + b_t] \quad \text{for a risky plan} \end{aligned}$$

These no-diversion constraints generate borrowing constraints provided $h < 1+r$ and $h < \frac{1+r}{u+(1-u)\gamma}$, respectively. It follows that in a risky equilibrium "leverage" for risky and safe plans is, respectively,

$$\begin{aligned} \frac{b_t}{w_t} &= m^r - 1, \qquad m^r \equiv \frac{1}{1 - \beta h \frac{u+(1-u)\gamma}{u}}, \quad \beta \equiv \frac{1}{1 + r} \\ \frac{b_t^n}{w_t} &= m^s - 1, \qquad m^s \equiv \frac{1}{1 - \beta h} \end{aligned}$$

The advantage of risky debt over safe debt is twofold. First, risky debt is cheaper because it enjoys an implicit bailout guarantee (its expected repayment is $\frac{u}{u+(1-u)\gamma}(1+r)$, which is lower than the safe one $(1+r)$). Second, because it is cheaper, lenders are willing to extend a higher leverage to risky borrowers ($m^r > m^s$). The drawback of risky debt is that it can lead to bankruptcy if an unfavorable price drop occurs next period.

When is it rational for borrowers to load on insolvency risk? When the probability of crisis $1 - u_{t+1}$ is low but positive, and there is enough expected price volatility, so that (i) the high price is high enough to make it worth taking on the insolvency risk; and (ii) the low price is low enough so that a risky debtor actually goes bust (this is necessary to claim the implicit bailout subsidy). The condition is

$$\frac{\underline{p}_{t+1}\theta}{p_t} < \frac{h}{u} < \frac{\overline{p}_{t+1}\theta}{p_t} \tag{5}$$

3.4 Endogenous Price Risk

Here, we derive the conditions under which a risky price path (5) exists, and characterize a lending boom path. We will set the generosity of the bailout guarantee to $\gamma = 1$.

In a risky path there is a feedback loop that generates both increasing leverage and increasing asset price inflation: a higher price increases the value of internal funds, which relaxes borrowing constraints and increases leverage. Higher leverage, in turn, boosts the demand for housing services, increases their price, and so on.

Because H-sector developers use H-goods as inputs, a higher price might increase the demand for H-goods as developers can attain greater leverage and expect to sell H-goods at a much higher price tomorrow. Next, we make the precise conditions under which the mechanism is internally consistent.

In order to simplify the computation of growth rates, it will be convenient to impose a functional form on the aid payment. We assume that in a crisis the aid payment is a fraction of the profit a developer would have made had a crisis not occurred

$$a_t = \varepsilon(1-c)[\overline{p}_t q_t - L_t] \tag{6}$$

Proposition 3.1 (Risky Equilibrium) *A symmetric equilibrium with insolvency risk-taking exists if and only if*

1. $h > u(1+r)$ *so that borrowing constraints arise.*

2. $\eta \equiv (1-c)m^r > 1$ *and* $\varepsilon < \frac{1}{\eta}$ *so that in a crisis prices fall enough to bankrupt firms.*

3. *Internal funds are high enough* ($\underline{w} > \alpha A \left(\frac{h}{u} - 1\right)^{-1} (1 - \frac{h}{(1+r)u})$ *so that the feedback loop mechanism between credit and prices is at work.*

Proof. {See Appendix} ■

This proposition says that if parameters satisfy certain conditions, then borrowing constraints and insolvency risk taking coesxist in equilibrium. Furthermore, if the endowment is high enough, the feedback-loop mechanism between credit and prices will be operative.

The Boom

Along a no-crisis path internal funds evolve as

$$\begin{aligned} w_t &= (1-c)\left[[m_t w_t + d_t] - \frac{h}{u_{t-1}} m_{t-1} w_{t-1}\right] + \underline{w} \\ &= \frac{\eta_{t-1}}{\eta_t - 1} \frac{h}{u_{t-1}} w_{t-1} - \frac{1-c}{\eta_t - 1} d_t - \frac{1}{\eta_t - 1} \underline{w} \end{aligned} \tag{7}$$

If a risky equilibrium exists, internal funds necessarily increase at an increasing rate provided internal funds start from a high enough level w_0. This is because a crisis can occur only if $\eta > 1$ and borrowing constraints arise only if $h > u(1+r)$.

Since the demand for H-goods is $I_{t+1} + C_{t+1} = \frac{m^r w_{t+1}}{\overline{p}_{t+1}} + \frac{d_{t+1}}{\overline{p}_{t+1}}$ and the supply $q_{t+1} = \theta m^r w_t$, the no-crisis price is

$$\overline{p}_{t+1} = \frac{m^r w_{t+1} + d_{t+1}}{\theta m^r w_t} = \left[\frac{w_{t+1}}{w_t} + \frac{d_t}{m^r w_t}\right] \frac{1}{\theta} \tag{8}$$

On the one hand, the higher the price, the higher the H-profits and internal funds, which in turn generates higher investment demand. On the other hand, the higher the price, the lower the consumption demand (from the T-sector). The first effect dominates if w_t is high enough. Moreover, the first effect becomes stronger over time because investment demand increases at a higher rate than the supply–because of greater credit–and so the price grows at in increasing rate. This self-reinforcing mechanism ensures that developers will find it more and more profitable to take on insolvency risk over time as the return $\frac{\theta \overline{p}_{t+1}}{p_t}$ will increase along the equilibrium path.

- Along this no-crisis path there is a lending boom–i.e., credit grows at an increasing rate–and the H-sector grows faster than the rest of the economy. As a result, the credit-to-output ratio grows over time.

$$\frac{b_t}{b_{t-1}} = \frac{w_t}{w_{t-1}} = \frac{\eta_{t-1}}{\eta_t - 1} \frac{h}{u_{t-1}} - (w_{t-1})^{-1} \left(\frac{1-c}{\eta_t - 1} d_t + \frac{1}{\eta_t - 1} \underline{w}\right)$$

The "real cost of funds" declines over time and can even become negative, making it more and more profitable to take on insolvency risk

$$\frac{1+\rho_t}{\theta\overline{p}_{t+1}/p_t} = \frac{1+r}{\left[\frac{w_{t+1}}{w_t} + \frac{d}{m^r w_t}\right]\frac{1}{p_t}} \tag{9}$$

If we set H-productivity θ low enough, there is sort of a "debt deflation" that captures the by now maligned practice of no-income-documentation mortgages that allow borrowers buy houses they "cannot afford" with their projected income flows. The expected capital gains on the house will more than suffice to repay the loan either by selling the house or by refinancing.

In parallel to the boom, the H-sector investment demand captures an ever-increasing share of the H-sector in the economy. If we denote $I_t := \phi_t q_t$, then the equilibrium share commanded by the H-sector ϕ is

$$\phi_{t-1} = \frac{m_{t-1}w_{t-1}}{m_{t-1}w_{t-1} + d_{t-1}}$$

The greater the share ϕ, the greater the growth rate of H-output and the greater the H-to-T output share

$$\rho_{H/T} = \frac{p_t q_t}{p_t q_t + \frac{d_t}{\alpha}}$$

This H-boom **is** generated by an ever greater H-sector demand for its own goods and services is reminiscent of what went on in the housing and financial services sectors.

The Bust

In order for lenders to be willing to fund developers' strategies with insolvency risk at very low interest rates, it is necessary that they expect a bailout in case of crisis. In order for developers to be willing to take on insolvency risk it is necessary that insolvency can actually take place.

One could take a shortcut and assume an exogenous source of insolvency. In our case, this could be an exogenous productivity shock that would destroy part of output. But this shortcut erases what we want to explain. Namely, that agents do choose to take on insolvency risk, and that this choice in turn supports the boom.

For the mechanimsm to be internally consistent it is necessary that if the H-demand falls, the price fall is steep enough to bankrupt firms with unhedged debt: $\underline{p}_{t+1}q_{t+1} < L_{t+1}$. But can an H-demand fall induce such a firesale? Not always. The T-sector demand must be small enough and the feedback mechanism must work also in the opposite direction: lower prices lead to lower internal funds, which reduce credit and investment demand so much that prices fall even more and so on.

$$
\begin{aligned}
\underline{p}_{t+1} q_{t+1} &< L_{t+1} \\
m_{t+1}^{crisis} w_{t+1}^{crisis} + d_{t+1} &< \frac{h}{u_t} m_t w_t \\
\frac{1}{1-\varepsilon\eta^r}\left(\underline{w} + \frac{d_t}{m^r}\right) &< \frac{1}{1-\varepsilon\eta^r}\frac{h}{u} w_t
\end{aligned}
$$

This condition makes clear that a firesale price that bankrupts developers exists only if the cash flow multiplier η is greater than one and the $\varepsilon-$aid policy is not very generous: $\varepsilon < 1/\eta$. This condition implies that the fall in cash-flows is severe enough so as to ensure insolvency risk for any level of internal funds $w_t \geq \underline{w}$.

The longer the boom has been going on–and the greater the increase in internal funds–the greater the bust. Also, the less generous the aid policy the greater the bust.

$$
\frac{\underline{p}_{t+1}}{\overline{p}_t} = \frac{m^r w_{t+1}^{crisis} + d_{t+1}}{\theta m^r w_t} \frac{1}{p_t}
$$

Notice that agents choose risky debt every period, including a crisis. This is not an inherent property of the equilibrium. It derives from the assumption that the endowment $\underline{w}$ is high enough, and it will simplify computations of growth rates below. If $\underline{w}$ were small or zero, then a crisis would have to be followed by a safe phase during which agents do not take on insolvency risk, until w attains a high enough level for a risky phase to start again.

After a bust, internal funds and credit collapse. Thus, developers are able to command a much lower share of H-output and so most H-output is consumed by the T-sector. In the wake of crisis the price collapse is accompanied by a sharp fall in the H-to-T output ratio.

Figure 1 presents a set of plots summarizing the boom-bust cycle pattern of the risky economy. The top panel shows the rapid build-up of internal funds (left) and the growth of credit (right) during the boom and the sharp collapse during the bust. The middle panel plots the quantity and prices of housing services, which increase during the boom and fall during crises. The bottom panel plots the profits of the H-sector as a share of GDP and the real cost of funds outside crisis episodes (defined in (9)). During the boom phase, housing price inflation makes the real cost of funds negative, which in turn allows the H-sector to increase its profits as a share of GDP even if the ratio of debt to internal funds remains constant.

3.5 Growth

Here we compare the risky path we have characterized with a safe path where crises never occur.

In a safe equilibrium all debt is denominated in H-goods and so a crisis never occurs ($u_{t+1} = 1$). Thus, agents do not expect any bailout. As a result developers have no incentive to denominate debt in T-goods. It follows that credit and investment are given by

$$b_t^n = (1 + m^s)w_t, \qquad I_t = m^s w_t / p_t$$

Internal funds evolve according to

$$w_t^S = \frac{\eta_s h}{\eta_s - 1} w_{t-1}^S - \frac{1-c}{\eta_s - 1} d_t - \frac{1}{\eta_s - 1} \underline{w} \tag{10}$$

The credit multiplier in the safe equilibrium is smaller than the credit multiplier in the risky economy $m^s = \frac{1}{1-h(1+r)^{-1}} < \frac{1}{1-h[(1+r)u]^{-1}} = m^r$.

Growth in the Safe Equilibrium

Notice that $\eta^s < 1$ implies that the autoregressive term in (10) is strictly negative. Further if $\frac{\eta^s h}{\eta^s - 1} > -1, w$ converges to a steady state as T becomes large [7]:

$$\lim_{T\to\infty} \frac{w_T^S}{w_{T-1}^S} - 1 = 0 \quad \lim_{T\to\infty} w_T^S = \frac{(1-c)d + \underline{w}}{1 + \eta^s h - \eta^s} \tag{11}$$

Since $\frac{\eta^s h}{\eta^s - 1} \in (-1, 0)$, the economy in the safe equilibrium experiences convergent cycles around its long run steady state value. A high enough demand from the T sector insures that profits are positive in each period.

Growth in the Risky Equilibrium

In order to make a crisp comparison between safe and risky equilibria, we assume that crisis have arbirtrarily large costs ($\varepsilon \to 0^+$). In this extreme case, the average risky growth rate equals

$$\frac{E(w_{t+1})}{w_t} = \frac{\eta^r}{\eta^r - 1} h - (w_t)^{-1} \left(\frac{1-c}{\eta_t - 1} d_t + \left(\frac{1}{\eta_t - 1} - 1\right) \underline{w}\right) \tag{12}$$

If $h > 1$, the autoregressive term is larger than one, and so when T is large, the average growth rate converges to a positive number:

$$\lim_{T\to\infty} \left(\frac{E(w_T)}{w_{T-1}} - 1\right) = \frac{\eta^r}{\eta^r - 1} h - 1 > 0$$

This equilibrium is characterized by sequences of booms and busts: during lending booms internal funds, credit and investment will grow at an increasing rate until a crisis occurs. Since credit and

[7] One can show that if $\frac{\eta^s h}{\eta^s - 1} < -1$, a safe equilibrium does not exist so $-1 < \frac{\eta^s h}{\eta^s - 1} < 0$ is the relevant case to consider.

housing sector productions are linear in w_t, they grow at the same rate. The following proposition summarizes our findings.

Proposition 3.2 *There exists a set of parameters* $S = \{r, h, c, u, \varepsilon\}$, *recursively defined by* $\frac{1}{1+r} < 1$; $\frac{1+r}{(2-(1+r)^{-1})} < h < 1+r; h\beta < c < \frac{h+h\beta}{h+1}, u \in (h(1+r)^{-1}, u^{**}), \epsilon < \frac{1-\frac{\beta h}{u}}{1-c}$, *so that if* $\underline{w}$ *and* A *are large enough:*

1. *Both a safe and a risky equilibrium exist.*
2. *In the safe equilibrium, credit, housing sector production and internal funds converge to*

$$w^{ss} = \frac{(1-c)d + \underline{w}}{1 + \eta^s h - \eta^s}, \quad b^{ss} = m^s w^{ss}, \quad q^{ss} = \theta m^s w^{ss}$$

3. *In the risky equilibrium, the average growth of credit, housing sector production and internal funds converge to a positive rate*

$$\lim_{T\to\infty} \frac{E(w_T)}{w_{T-1}} = \frac{\eta}{\eta - 1} h - 1$$

Proof. see Appendix ■

The proposition above characterizes the co-existence of a stagnant safe equilibrium and a growing risky equilibrium punctuated by rare crises. Figure 2 compares the output and prices of the housing sector in the two equilibria (left panels) with their empirical counterparts (righ panels): the number of new houses sold and the real home price. As we can see, the safe equilibrium characterizes relatively well the behavior of housing prices and housing quantities before 1991. Meanwhile, the risky equilibrium seems a good characterization of the period 1992-2007.

3.6 The Financing of Bailout Guarantees

Systemic bailout guarantees in our model are financed out of domestic resources. Since we assume that the government has perfect access to capital markets, the financing of systemic bailout guarantees is subject to the following intertemporal solvency constraint:

$$\underbrace{E\sum_{t=0}^{T} \beta^t (\zeta_t [L_t - p_t q_t])}_{\text{net bailout payments}} - \underbrace{E\sum_{t=0}^{T} \beta^t T_t}_{\text{taxes revenues}} \leq 0 \tag{13}$$

where E denotes expectations at time $t = 0$, $\zeta_t = 1$ if a crisis occurs in period t and 0 otherwise, L_t are the outsanding debt liabilities of the H sector, T_t is the sequence of taxes levied on profits of solvent H-firms and on consumers in the T-sector, and β is equal to $1/(1+r)$. In pratical terms, a solvent government has access to an international contigent credit line: it draws on this facility in the case of a crisis and repays by using tax proceeds.[8]

For the solvency constraint (13) to be satsfied the expected discounted sum of profits in the H-sector and tradeable consumption by T-sector agents shall exceed the expected discounted sum of net bailout payments:

$$E\sum_{t=0}^{T}\beta^t\zeta_t(L_t - p_tq_t) \leq E\sum_{t=0}^{T}\beta^t(1-\zeta_t)(p_tq_t - L_t) + (1-\alpha)A_tl).$$

The expression above can be rearranged as

$$E\sum_{t=0}^{T}\beta^t\left[(p_tq_t - L_t) + (1-\alpha)A_tl\right] \geq 0,$$

and, using equation (12), as:

$$E\sum_{t=0}^{T}\beta^t m^r\left[(w_t - \frac{h}{u}w_{t-1})\right] + Al\frac{1-\beta^{T-1}}{1-\beta} + \beta^T A_T l \geq 0$$

Since $E_{t-1}(w_t)/w_{t-1} > \frac{\eta}{\eta-1}h - 1$ and $h > 1$, a sufficient condition for the later condition to hold is simply

$$h\left(\frac{\eta}{\eta-1} - \frac{1}{u}\right) > 1$$

4 External Imbalances

In this section, we analyze the consequences of the boom-bust cycle on the external balances of the country. In normal times, the country runs a large current account deficit that reflects (i) a trade deficit explained by the consumption of solvent H-sector entrepreneurs, and (ii) the interest payments on the borrowing of the H-sector. In the mean time, the financial account exhibits a large surplus as larger internal funds translate into larger capital inflows. This situation is dramatically reversed during crises times: (i) the trade balance reverts to a surplus as profits in the H-sector falls to zero, and (ii) the financial account exhibits a large deficit because the crisis is associated with large net capital outflows. During crisis times, the government draws upon its international contingent

[8] In this section and the next, we assume for simplicity that the government makes bailout payments but no aid paiment ($\varepsilon = 0$). We´ll come back to the issue of aid paiement in section ()

credit line, a financial transfer that is recorded as a disbursment of international reserves.[9]

We detail below the balance of paiement of our model economy as the sum of three component – (i) the current account (CA_t),(ii) the financial account (FA_t) (iii) net reserves disbursments Ω_t – that are linked by the following accounting identity:

$$CA_t + FA_t + \Omega_t = 0 \tag{14}$$

The Current Account

To compute the trade balance, we need to substract the consumption of T-agents and H-sector entrepreneurs to the production of tradeables goods by the T-sector

$$TB_t = (1-\alpha)A_t l_t - c\pi_t$$

By adding to the trade balace the net interest payments we derive the current account as:

$$CA_t = \alpha A_t l_t - c\pi_t - \frac{h}{u} m_{t-1} w_{t-1}$$

The *current account* in tranquil times and crisis times can be respectively expressed as:

$$\begin{aligned} CA_t &= \alpha(1-c)A_t l_t - m^r(cw_t - \tfrac{h}{u} w_{t-1}(c-1)) \\ CA_t^{crisis} &= \alpha A_t l_t - \tfrac{h}{u} m^r w_{t-1} \end{aligned}$$

The interest payments are the same in both tranquil times and crisis times since they are paid either by solvent borrowers or by the bailout agency. Thus, the reversal of the current account is entirely explained by the drop to zero of the consumption of H-sector entrepreneur during a crisis.

The Financial Account

The financial account records the change in the international investment position of the H-sector.

$$FA_t = m^r(w_t - w_{t-1})$$

In tranquil times since internal funds grow, net foreign borrowing is positive and the financial accounts exhibit a surplus. In crisis time, the collapse of internal fund leads to a large fall in new borrowing resulting in net outflows and a financial account deficit.

[9] It is convenient to classify the financial operations of the goverment as part of its international reserves account. However, the portion of the bailout that covers interest payment is included in the current account.

The Reserves Account.

We compute the reserves disbursement by combining the current account, the financial account, and the balance of payment accounting identy (14):

$$-\Omega_t = \alpha A_t l_t - c\pi_t - \frac{h}{u} m^r w_{t-1} + m^r (w_t - w_{t-1})$$

The variation of the stock of reserves is computed as:

$$R_t = (1+r)R_{t-1} + \Omega_t + T_t$$

R_t corresponds to the net asset position of the govermnent. Bailout paiements increase the liabilities of the government while thetransfer ot tax proceeds (T_t) reduce them. We can solve for the stock of reserves:

$$R_T = (1+r)^T R_0 + (1+r)^T \sum_{t=0}^{T} \beta^t (\Omega_t + T_t) + T_t)$$

Then in the case the government has zero reserves assets to start with, we can write the ex-ante solvency condition as:

$$\underbrace{E\{R_T\} \geq 0}_{\text{Expected Terminal Level of Reserves}} \Leftrightarrow \underbrace{E \sum_{t=0}^{T} \beta^t T_t}_{\text{tax revenues}} \geq \underset{\text{reserves disbursements}}{E \sum_{t=0}^{T} \beta^t (-\Omega_t)} \tag{15}$$

This condition indicates that taxes proceeds should covered reserves disbursements. Since reserves disbursements basically occur to cover bailout payments, the solvency condition (15) must be equivalent to (13).

Figure 13 plots the path of the current account, the financial account and the change in reserves for a simulation of the model economy.

5 Stabilization Policy

In our economy, the government makes two distinct payment in the case of a crisis: bailout payments to international creditors and aid payments to the new generation of domestic entrepreneurs. In section 3.6, we derive conditions the tax sequence must satisfy ex-ante so as to cover expected bailout payments. In this section, we explore the consequence of introducing aid payments to young entrepreneurs. The internal funds of a young entrepreneurs in the crisis period are:

$$w_t = \underbrace{\underline{w}}_{\text{endowment}} + \underbrace{\varepsilon(1-c)[\overline{p}_t q_t - L_t]}_{\text{aid payment}}$$

Suppose that in the aftermath of crisis the government implements a transfer policy that sets ε high enough so as to bring next period GDP back to its pre-crisis level. The transfer is financed by increasing its reserves liabilities, with the tax path left unchanged.[10] Although this is a simplistic policy, we believe it captures the stabilization policy that the U.S. is implementing.[11]

Under which circumstance will this policy work? The answer depends crucially on the long-term solvency of the government. If the transitory transfer is not too large, it will increase GDP and growth. However, if it is too large so that it increases so much goverment liabilities that a future bailout will not be financeable, then this policy will be ineffective in restarting the pre-crisis growth path and might lead to stagnation. This is because agents will correctly predict that if a crisis were to occur in the future, a bailout would not be granted. As a result, the risky equilibrium is not sustainable. Recall that in a risky equilibrium real interest rates are low and leverage is high because of an expectation that the the H-price will follow an increasing path. This price path is in turn supported by the implicit bailout guarantee.

Large transfers to entrepreneurs in the wake of a crisis can boost GDP in the short run. But if the size of transfers is such that it impedes the ability for the government to grant a systemic bailout in the future, then the short-run GDP gains will offset by long-run losses. The elimination of the implicit goverment insurance will induce a shift to a safe path under which leverage is low and growth is lower than under the alternative risky path. In other words, the stabilization policy has the unintended consequence of inducing a 'credit crunch' and hindering investment and future growth.

Tranfer policy

A simply way to study the effect of the aid policy is to assume that the tax policy is set so as to make the solvency constraint (13) binding in the case considered in section 3.6. That is, we consider the case of a government that commits to a full bailout to creditors but to zero aid payments to

[10] This policy might be implementes to smooth consumption fluctuations, though we will not model the reason for such policy.

[11] In our baseline model we have set up the endowment $\underline{w}$ large enough so that the economy can be in a risky equilibrium in the aftermath of crisis. If we relax this assumption and allows internal funds to collapse. If we relax this assumption and allow the internal funds to fall further during a crisis, then the economy would revert to a "safe" equilibrium where agents do not take on insolvency risk, leverage falls dramatically, and so does GDP. In this potentially more realistic environment, the objective of policy would be to bring back the economy to the risky equilibrium through a transfer that would increase intern funds to the required level.

entrepreneurs

$$E\sum_{t=0}^{T}\beta^t(\zeta_t[L_t - p_t q_t]) = E\sum_{t=0}^{T}\beta^t T_t \tag{16}$$

Without changing the tax policy, the only way to grant a transfer to young entrepreneurs is to *reduce the generosity of the bailout*. The implied solvency constraint can be expressed as:

$$E\sum_{t=0}^{T}\beta^t\zeta_t([\gamma L_t - p_t q_t] + \varepsilon(1-c)[\overline{p}_t q_t - L_t]) = E\sum_{t=0}^{T}\beta^t T_t, \tag{17}$$

which means that an increase in the generosity of the transfer payment must be associated with reduction of the generosity of the bailout. Futhermore, we can show, following Scheider and Tornell (2004), that there is a mimimum degree of bailout generosity $\gamma_{\min}$ beyond which an equilibrium with systemic risk-taking does not occur. Combining the constraint $\gamma \geq \gamma_{\min}$ so a risky equilibrium exists with the solvency constraint (17) defines an upper bound $\varepsilon_{\max}$ on the size of transfer payment:

$$\varepsilon_{\max} = \frac{E\sum_{t=0}^{T}\beta^t(T_t - p_t q_t) - \gamma_{\min}E\sum_{t=0}^{T}\beta^t\zeta_t L_t}{E\sum_{t=0}^{T}\beta^t(1-c)[\overline{p}_t q_t - L_t]} \tag{18}$$

Combining these results we have the following Corollary:

Corollary 5.1 *If the tax policy satisfies solvency constraint (16), there is a maximum generosity of the aid paiement $\varepsilon_{\max}$, defined in (18), such that a risky-equilibrium path in the aftermath of crisis exists only if $\varepsilon < \varepsilon_{\max}$.*

It follows that if in the aftermath of crisis the government sets the aid policy $\overline{\varepsilon} > \varepsilon_{\max}$, then the economy will revert to safe path as a risky path will not be possible to support. Since there is no long-run growth in the safe path, a very ambitious stabilization policy will lead stagnation.

Graphically Figure 14 shows the effects of policy. First, if there is a limited stabilization policy – labeled as "weak stimulus"–, the economy restarts in a risky path at a *lower starting level.* Second, with the stabilization policy that maintains GDP at its pre-crisis level - labeled as "strong stimulus", the economy reverts to a safe path forever, and so GDP remains transitorily at the high level and then falls and stagnates.

Although this conclusion seems counterintuitive, it is actually a form of Ricardian equivalence (somewhat non-linear form: it is a bifurcation type of effect that brings the economy from one equilibrium to another). Private agents will increase their saving if they foresee that the fiscal expansion will generate an implicit fiscal insolvency (conditional on a crisis happening in the future with a likelihood greater or equal to $1-u$).

6 Related Literature

to be added

7 Conclusion

to be added

APPENDIX (to be completed with additional proofs and derivations)

A Terminal Conditions

Along the risky path, the H-sector is accumulating debt. To satisfy the solvency constraint, at some point it must repay. A way to ensure solvency and keep the dynamics simple is to assume that the demand from the T-sector is constant and then experiences a jump at a terminal time T. So we set:

$$A_t = \begin{cases} A & \text{if } t \leq T-1 \\ \overline{A} & \text{if } t = T \end{cases}$$

Where $\overline{A}$ is large enough to allow developers to repay all debt at T.Since at T there is no H-investment, there is a unique price

$$p_T = \frac{\alpha \overline{A}}{q_T}$$

It follows that at $T-1$ all developers find optimal to only choose a safe plan. Thus

$$q_T = \theta m^s \frac{w_{T-1}}{p_{T-1}}$$

The net present value condition at period $T-1$, $E_{T-1}(\pi_T^s) \geq (1+r)w_{T-1}$,holds iff

$$\frac{p_T \theta}{p_{T-1}} \geq 1 + r \Leftrightarrow \alpha \overline{A} > (1+r) m^s w_{T-1}$$

Hence, there exists a risky equilibrium iff

$$\overline{A}_0 > \alpha^{-1}[1+r] m^s \cdot \overline{w}_{T-1}(w_0, a) \tag{19}$$

where $\overline{w}_{T-1}(w_0, a)$ is the internal funds that obtain if no crisis occurs on $[0, T-1]$. This completes the characterization of the risky equilibrium. The process for $\overline{A}$ with a jump is meant to capture some more fondamental long run trend in the demand of housing. Such trends can reflect, for example, some demographic factors such as aging or the rate of divorce that influence the demand for housing in the very long run.

B Proof of Proposition 3.1

Existence of a susnspot equilibrium

We prove that there exists a sunspot equilibrium by showing that if net worth w_t and the multiplier are high enough, then there are two equilbrium prices: a lucky one with positive profits $\pi(\overline{p}_{t+1}) > 0$ and a crisis one with negative profits $\pi(\underline{p}_{t+1}) < 0$. Then we show that along the no-crisis path net worth is increasing. Thus, if the initial endowment is large enough, every period there exist two market clearing prices, and so a risky equilibrium exists.

Recall that in a risky equilibrium the basic equations of the model are:

$$
\begin{array}{ll}
b_t = [m^r - 1]w_t, \quad m^r \equiv \frac{1}{1-\beta h/u} & \text{borrowing constraint} \\
L_{t+1} = b_t[1+r] = \frac{h}{u} m^r w_t & \text{promised debt repaymnet} \\
\pi_t = [p_t q_t - L_t] & \text{profits}
\end{array}
$$

Internal funds evolve as follows

$$
w_t = \begin{array}{ll} \underline{w} + (1-c)[p_t q_t - L_t] & \text{if } p_t q_t > L_t \\ \underline{w} + aid = w^{crisis} & \text{if } p_t q_t \leq L_t \end{array}
$$

where $\underline{w}$ is an endowment received by young investors. Also recall the following parametric restrictions

$$
u < h < \frac{u}{\beta}, \qquad \beta \equiv \frac{1}{1+r}
$$

Since equilibrium investment is $I_t = m^r w_t$, market clearing in the H-sector $q_t = I_t + d_t/p_t$ implies the following two prices:

$$
p_t = \left\{ \begin{array}{ll} \frac{m^r w_t + d_t}{q_t} \equiv \overline{p}_t & \text{no crisis} \\ \frac{m^r w_t^{crisis} + d_t}{q_t} \equiv \underline{p}_t & \text{crisis} \end{array} \right.
$$

For a risky equilibrium path to exist during a time interval we need to show that $\pi(\underline{p}_{t+1}) < 0$ and $\pi(\overline{p}_{t+1}) > 0$ hold simultaneously during that time interval. This is done in the next proposition.

To simplify notation we will set the aid to zero. In the appendix we consider the case where the government gives an aid payment to firms and show teh conditions for the existence of multiple equilibria.

We show in the proof below that there exists a risky equilibrium where every period there are two equilibrium prices $\{\underline{p}_{t+1}, \overline{p}_{t+1}\}$, such that profits are negative under the low price $\pi(\underline{p}_{t+1}) < 0$ and positive under the high price $\pi(\overline{p}_{t+1}) > 0$, if and only if the multiplier is large enough

$$\eta^r \equiv m^r(1-c) > 1, \tag{20}$$

and the endowment received by investors satisfies

$$\underline{w} > d\left[\frac{1-c}{\frac{h}{u}-1}\right] \tag{21}$$

Proof of proposition 3.1

The proof has two steps. First we show that $\pi(\underline{p}_{t+1}) < 0$ and $\pi(\overline{p}_{t+1}) > 0$ can hold simultaneously if (20) and (21) hold and w_t is high enough. Then we show that w_t is increasing over time if (20) and (21) hold. Consider $\pi(\underline{p}_{t+1}) < 0$. Since during a crisis net worth is $\underline{w}$, the default condition holds iff

$$\begin{aligned}
\underline{p}_{t+1} q_{t+1} &< L_{t+1} \\
m^r \underline{w} + d &< \frac{h}{u} m^r w_t \\
w_t &> \frac{u}{h}\left[\underline{w} + \frac{d}{m^r}\right]
\end{aligned} \tag{22}$$

It follows that $\pi(\underline{p}_{t+1}) < 0$ iff conditions (20) and (22) hold.

Next consider $\pi(\overline{p}_{t+1}) > 0$. After some algebra we get that internal funds are:

$$w_{t+1}^{lucky} = \frac{\eta^r}{\eta^r - 1}\frac{h}{u}w_t - \frac{1-c}{\eta^r - 1}d - \frac{1}{\eta^r - 1}\underline{w}$$

We thus have that positive profits require

$$
\begin{aligned}
\overline{p}_{t+1} q_{t+1} &> L_{t+1} \\
m^r w_{t+1} + d &> \frac{h}{u} m^r w_t \\
m^r \left[\frac{\eta^r}{\eta^r - 1} \frac{h}{u} w_t - \frac{1-c}{\eta^r - 1} d - \frac{1}{\eta^r - 1} \underline{w} \right] + d &> \frac{h}{u} m^r w_t \\
\left[-\frac{1}{\eta^r - 1} \underline{w} - \frac{1-c}{\eta^r - 1} d \right] + \frac{d}{m^r} &> \left[1 - \frac{\eta^r}{\eta^r - 1} \right] \frac{h}{u} w_t \\
\left[\frac{\eta^r}{\eta^r - 1} - 1 \right] \frac{h}{u} w_t &> \left[\frac{1}{\eta^r - 1} \underline{w} + \frac{1-c}{\eta^r - 1} d \right] - \frac{d}{m^r} \\
\frac{1}{\eta^r - 1} w_t &> \frac{1}{\eta^r - 1} \frac{u}{h} \left[\underline{w} + \frac{d}{m^r} \right] \qquad (23)
\end{aligned}
$$

Notice that conditions (22) and (23) are satisfied simultaneously if and only if (20) and (22) hold. In other words $\pi(\underline{p}_{t+1}) < 0$ and $\pi(\overline{p}_{t+1}) > 0$ hold simultaneously iff (20) and (22) hold.

The next step is to show that net worth follows an increasing path, so that we ensure condition (22) on w_t holds at all times. This increasing w_t path ensures that profits along the lucky (no crisis) path are always positive, and so a risky equilibrium path exists.

If $\eta^r > 1$, then

$$
\begin{aligned}
w_{t+1} &> w_t \\
\frac{\eta^r}{\eta^r - 1} \frac{h}{u} w_{t-1} - \frac{1-c}{\eta^r - 1} d - \frac{1}{\eta^r - 1} \underline{w} &> w_t \qquad (24) \\
\left[\frac{\eta^r}{\eta^r - 1} \frac{h}{u} - 1 \right] w_t &> \frac{1-c}{\eta^r - 1} d + \frac{1}{\eta^r - 1} \underline{w}. \qquad (25)
\end{aligned}
$$

That is, $w_{t+1} > w_t$ along a lucky path if and only if

$$
w_t > \frac{\frac{1-c}{\eta^r - 1} d + \frac{1}{\eta^r - 1} \underline{w}}{\left[\frac{\eta^r}{\eta^r - 1} \frac{h}{u} - 1 \right]} \qquad (26)
$$

This condition holds for all time periods provided $\underline{w}$ satisfies:

$$\begin{aligned}
\left[\left(\frac{\eta^r}{\eta^r-1}\frac{h}{u}\right)-1-\frac{1}{\eta^r-1}\right]\underline{w} &> \frac{1-c}{\eta^r-1}d \\
\left[\left(\frac{\eta^r}{\eta^r-1}\frac{h}{u}\right)-\frac{\eta^r}{\eta^r-1}\right]\underline{w} &> \frac{1-c}{\eta^r-1}d \\
\left(\frac{h}{u}-1\right)\underline{w} &> (1-c)d \\
\underline{w} &> \frac{(1-c)d}{\left(\frac{h}{u}-1\right)}
\end{aligned}$$

The last inequality is condition (21).

Finally we need to verify that Condition (22) holds for all time periods. This condition holds iff the endowment satisfies

$$\underline{w} > \frac{d}{m^r}\frac{1}{\left[\frac{h}{u}-1\right]}. \tag{27a}$$

Notice that if (21) holds then (27a) must also hold. This is because $\eta^r = (1-c)m^r > 1$, and so $(1-c) > 1/m^r$.

Appendix. Here we consider the case of a non-zero aid policy. Suppose that in a crisis, the government gives an aid payment to firms equal to a tiny fraction ε of the profit it would have made in no-crisis time. aca

Lemma B.1 *When $\eta^r > 1$ and $h > u$, under an $\epsilon-aid$ policy there exist risky equilibrtia on $[0,\overline{\epsilon}]$ only if*

$$\epsilon < \overline{\epsilon} := \frac{1}{\eta^r}$$

Proof. $w_t^{crisis} = \frac{\varepsilon\eta^r}{\varepsilon\eta^r-1}\frac{h}{u}w_{t-1} - \frac{(1-c)\epsilon}{\varepsilon\eta^r-1}d - \frac{1}{\varepsilon\eta^r-1}\underline{w}$

Given $\eta^r > 1$, the default cond holds iff

$$\underline{p}_{t+1}q_{t+1} < L_{t+1} \tag{28}$$

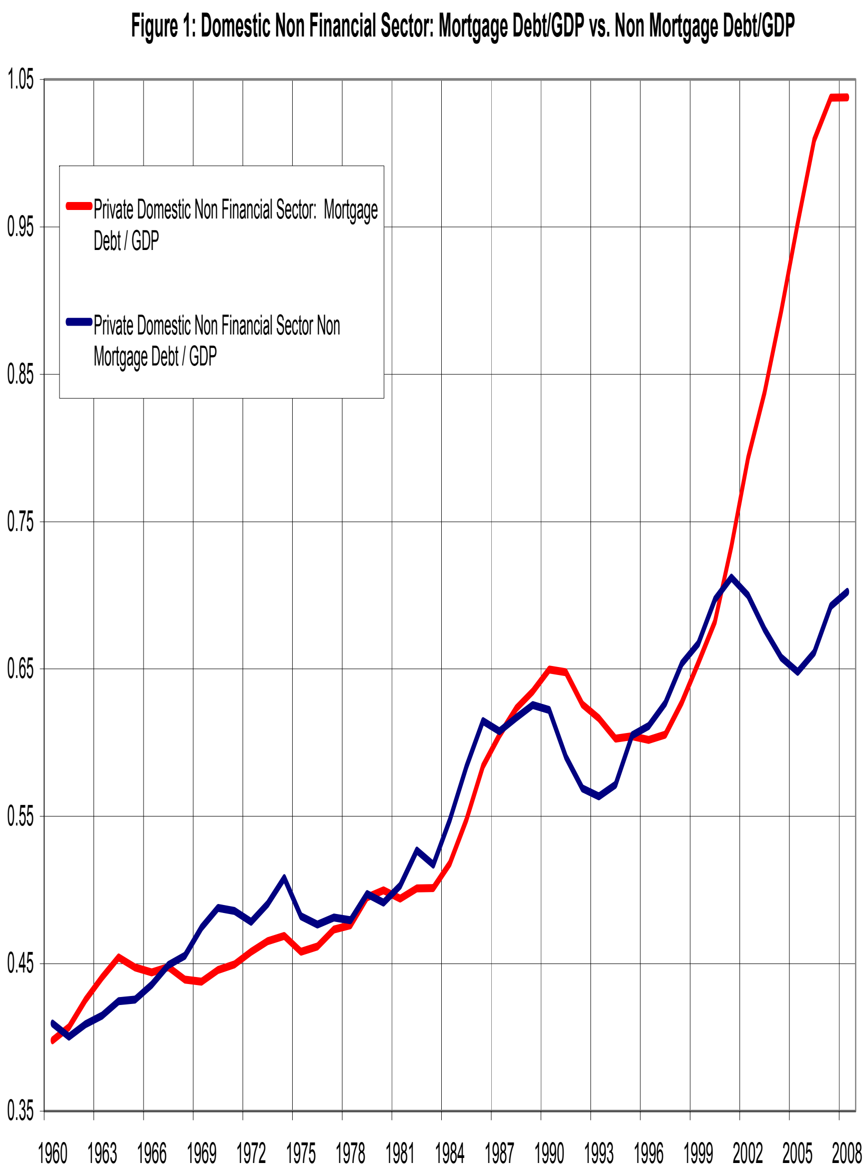
$$\begin{aligned}
m_{t+1}^{crisis}w_{t+1}^{crisis}+d_{t+1} &< \frac{h}{u_t}m_t w_t \\
m^r w_{t+1}^{crisis}+d &< \frac{h}{u}m^r w_t \\
m^r\left(\frac{\varepsilon\eta^r}{\varepsilon\eta^r-1}\frac{h}{u}w_{t-1}-\frac{(1-c)\,\epsilon}{\varepsilon\eta^r-1}d-\frac{1}{\varepsilon\eta^r-1}\underline{w}\right) &< \frac{h}{u}m^r w_t - d
\end{aligned}$$

$$\left(-\frac{(1-c)\,\epsilon}{\varepsilon\eta^r-1}d-\frac{1}{\varepsilon\eta^r-1}\underline{w}\right)+\frac{d}{m^r} < \left[1-\frac{\varepsilon\eta^r}{\varepsilon\eta^r-1}\right]\frac{h}{u}w_t \tag{29}$$

$$-\frac{1}{\varepsilon\eta^r-1}\left(\underline{w}+\frac{d}{m^r}\right) < -\frac{1}{\varepsilon\eta^r-1}\frac{h}{u}w_t \tag{30}$$

If $\varepsilon = 0$, we get condition (22). However, if $\varepsilon > \frac{1}{\eta^r}$ the condition above contradicts (23). Hence a risky equilibrium exists for any aid policy $\varepsilon \in [0, \frac{1}{\eta^r})$, but not for $\varepsilon \geq \frac{1}{\eta^r}$.

Figure 1. Asymmetric Financial Development



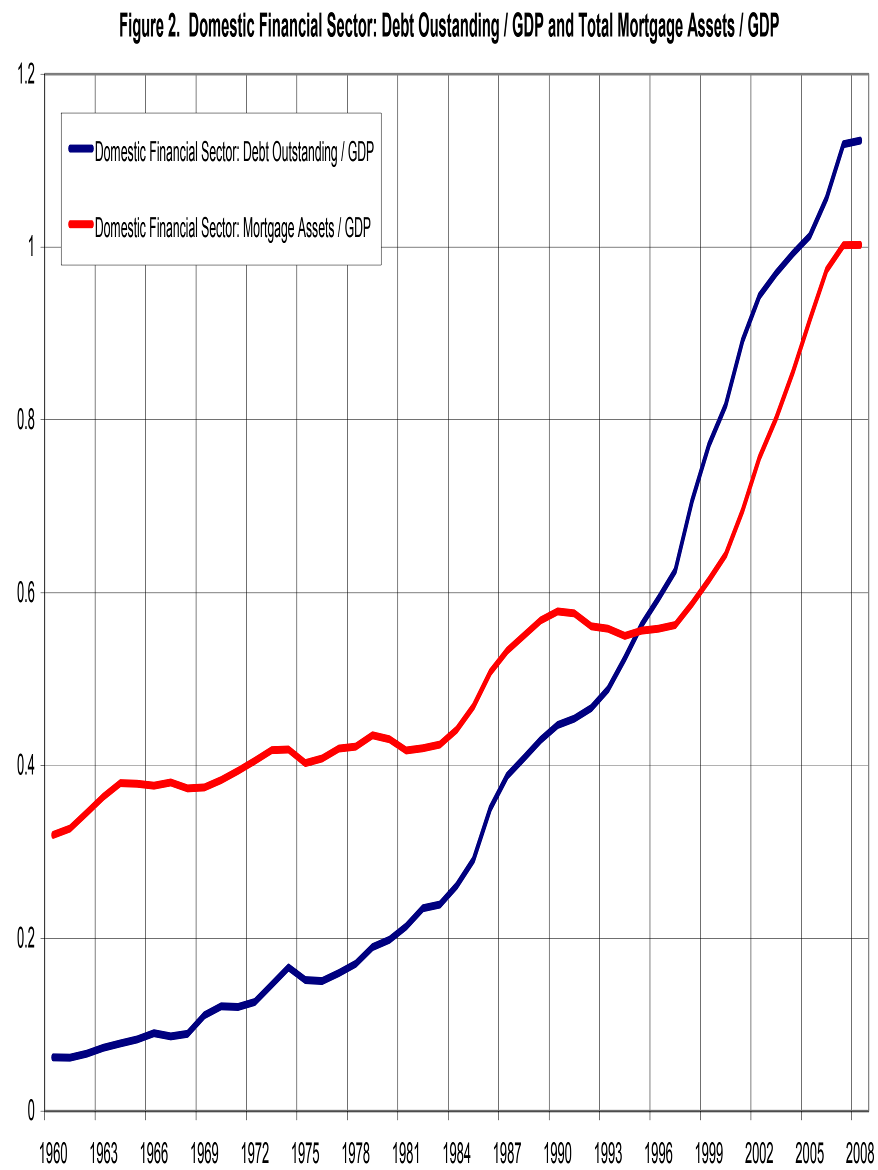
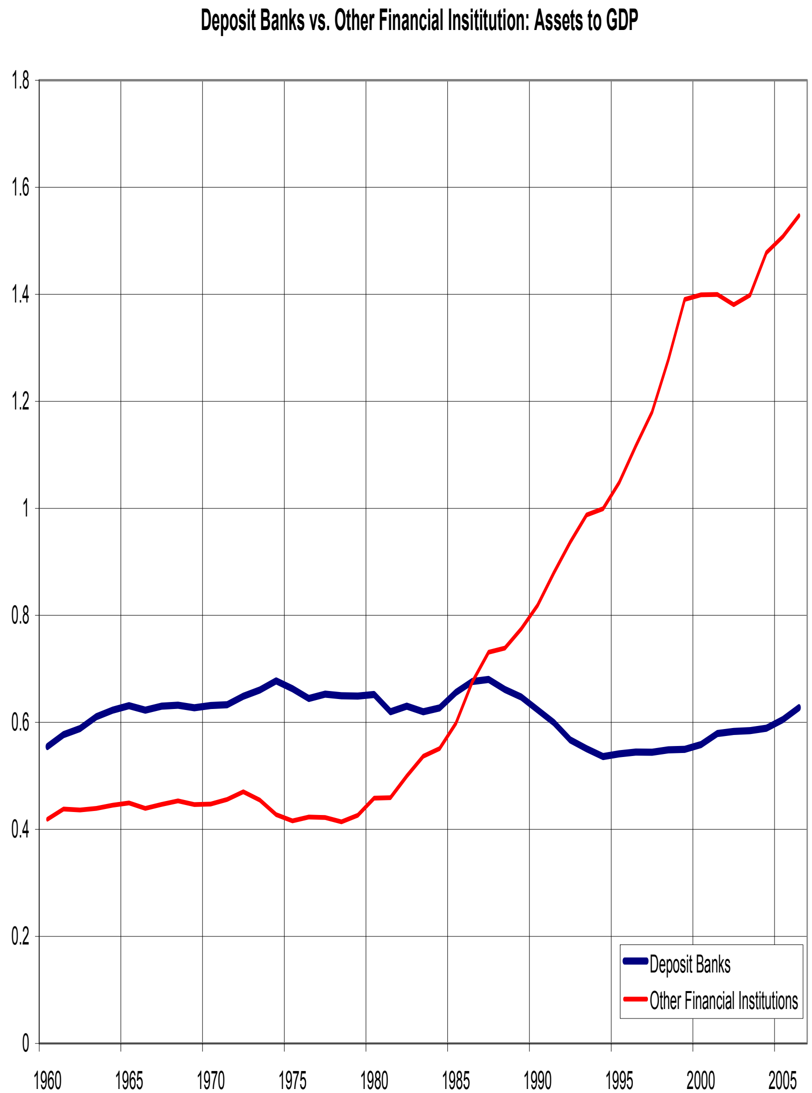


Figure 2. Asymmetric Financial Development (II)
Deposits Banks vs. Other Financial Institutions



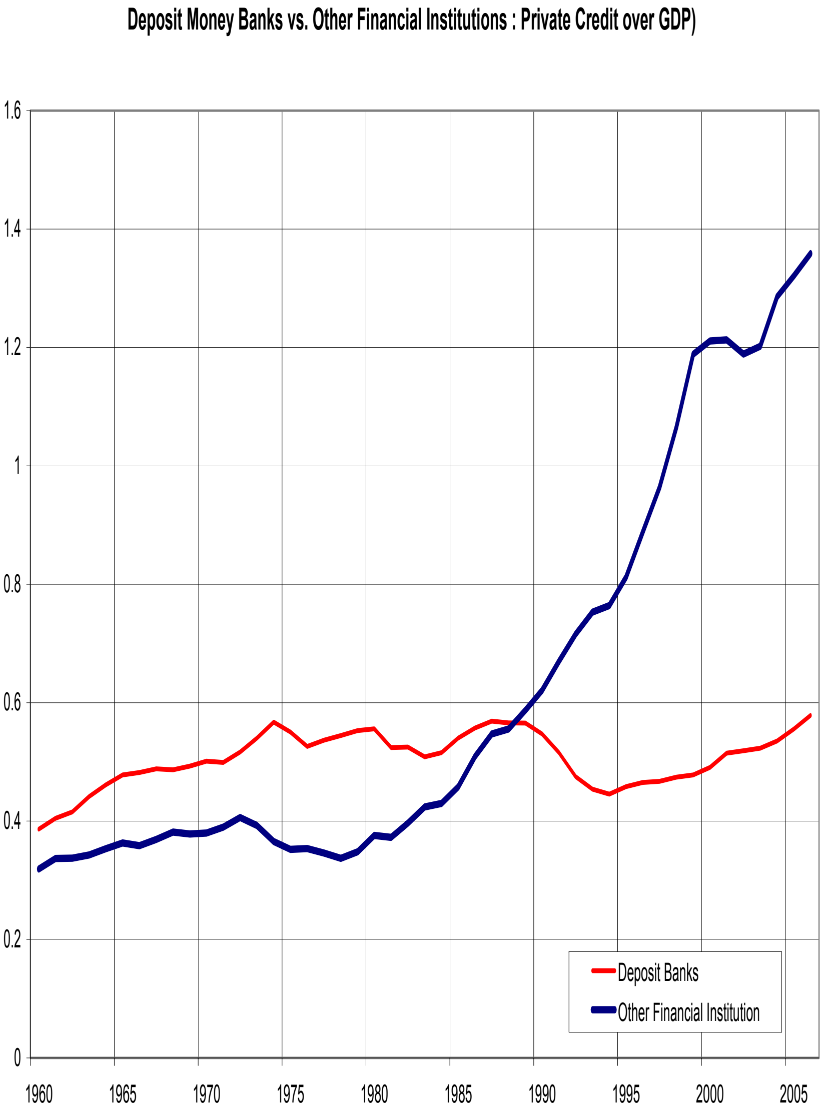


Figure 3 : Real Estate Boom-Bust: Prices and Quantities

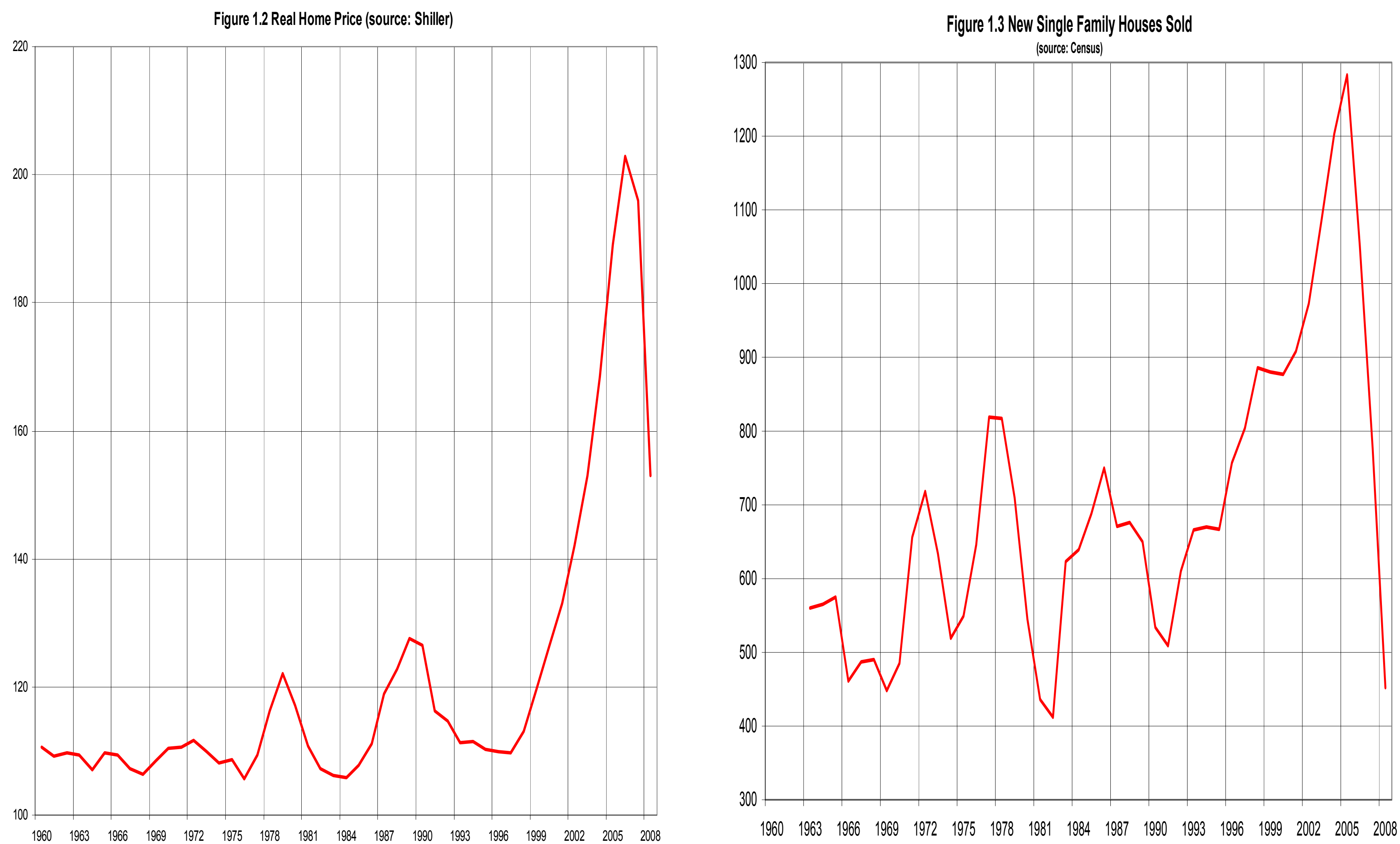
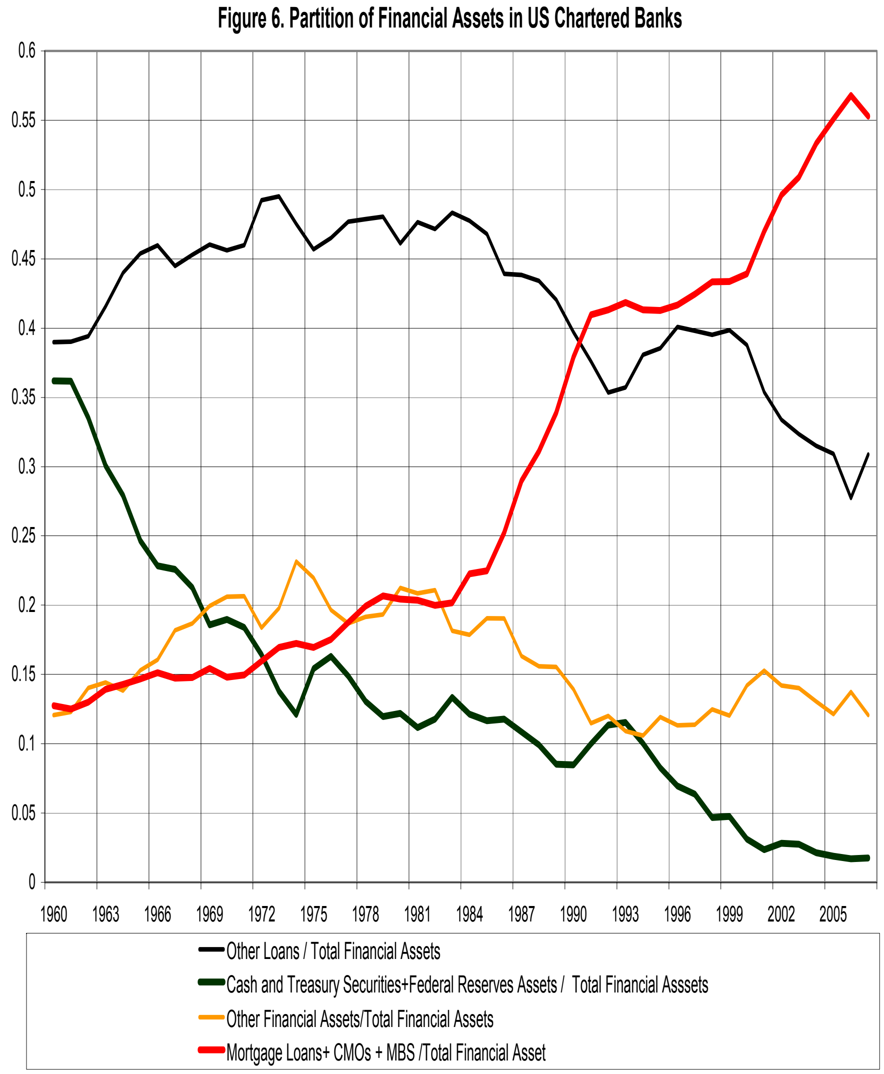


Figure 4: Financial Sector Real Estate Exposure



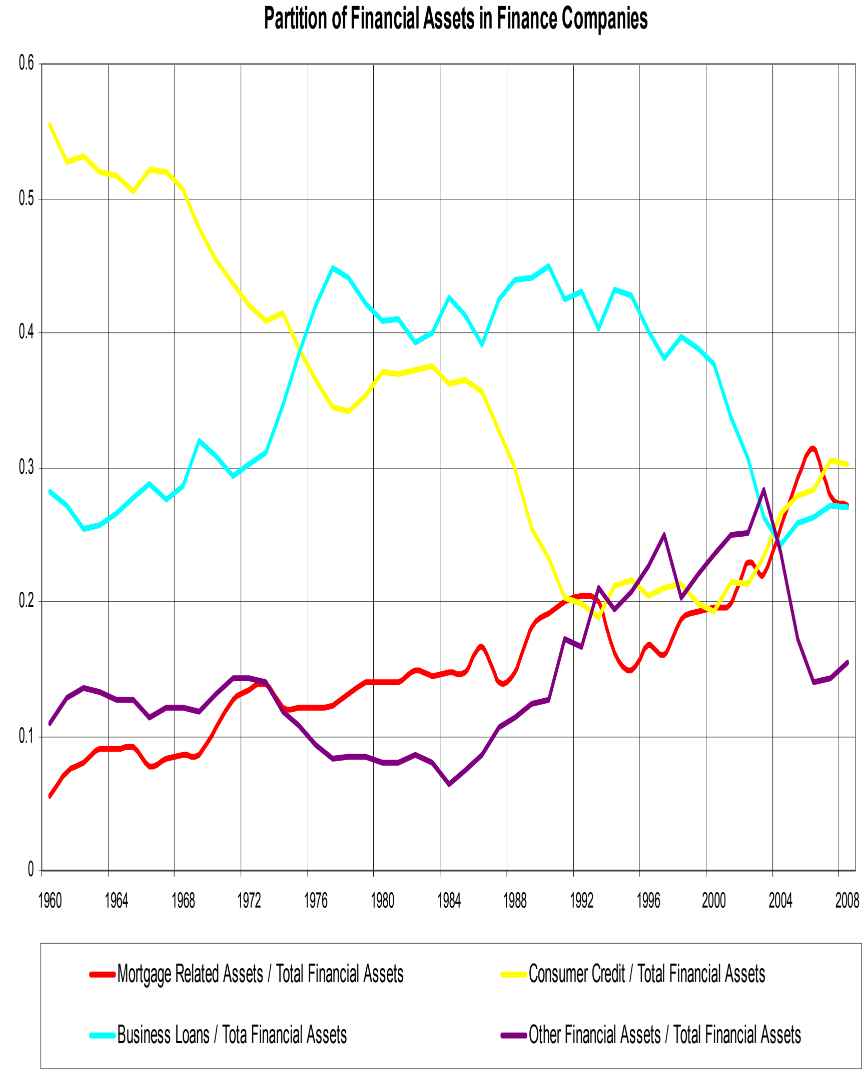
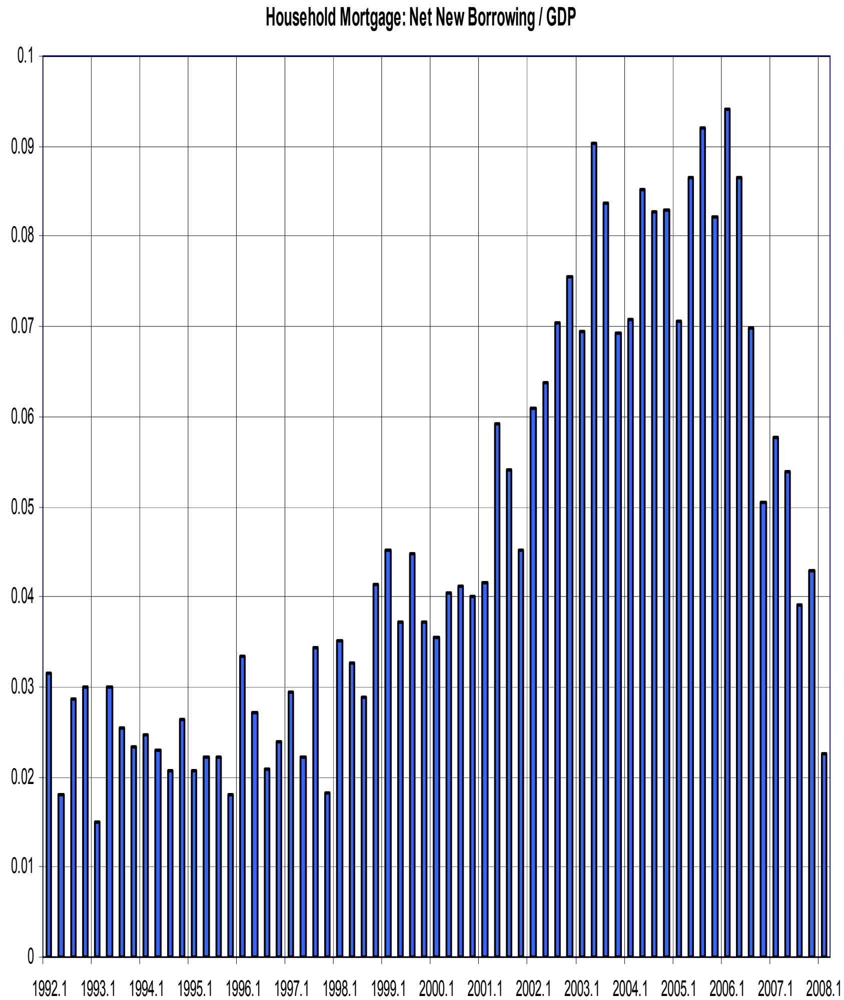


Figure 5 : Household Mortgage Credit and Home Owners Leverage



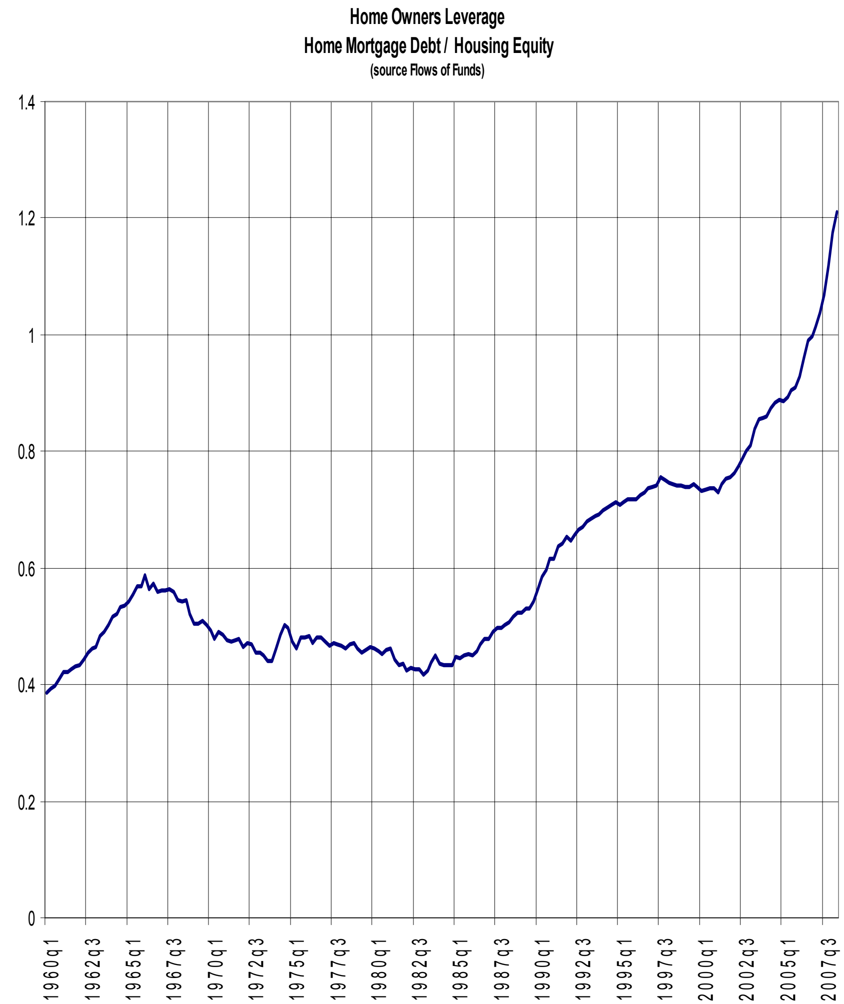
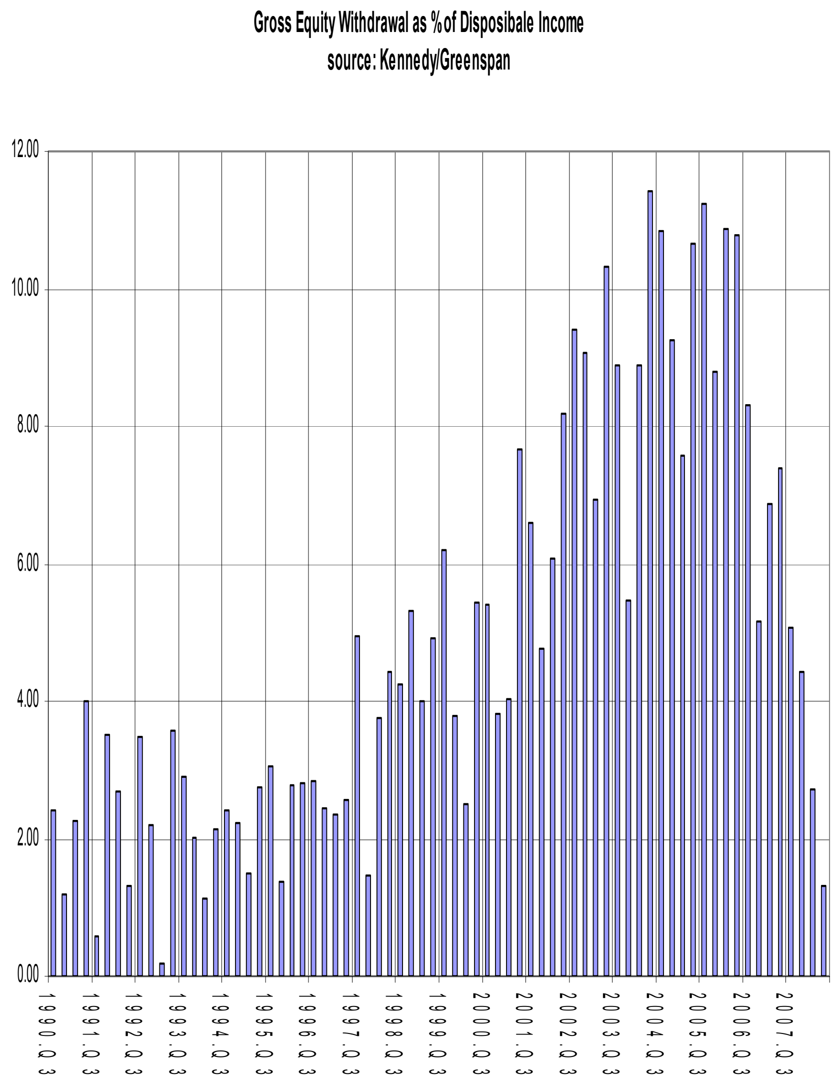


Figure 6 : Mortgage Equity Withdrawals and "Real" Mortgage Interest Rate



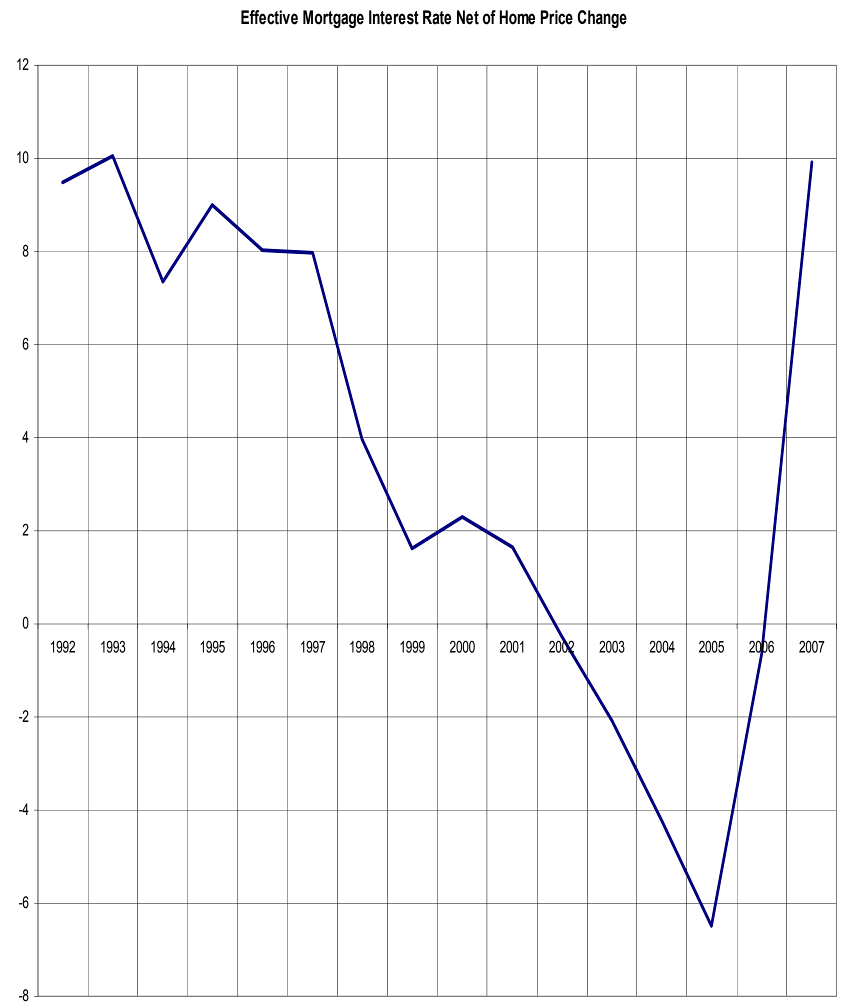


Figure 7. Asymmetric Asset Price Boom-Bust Cycle

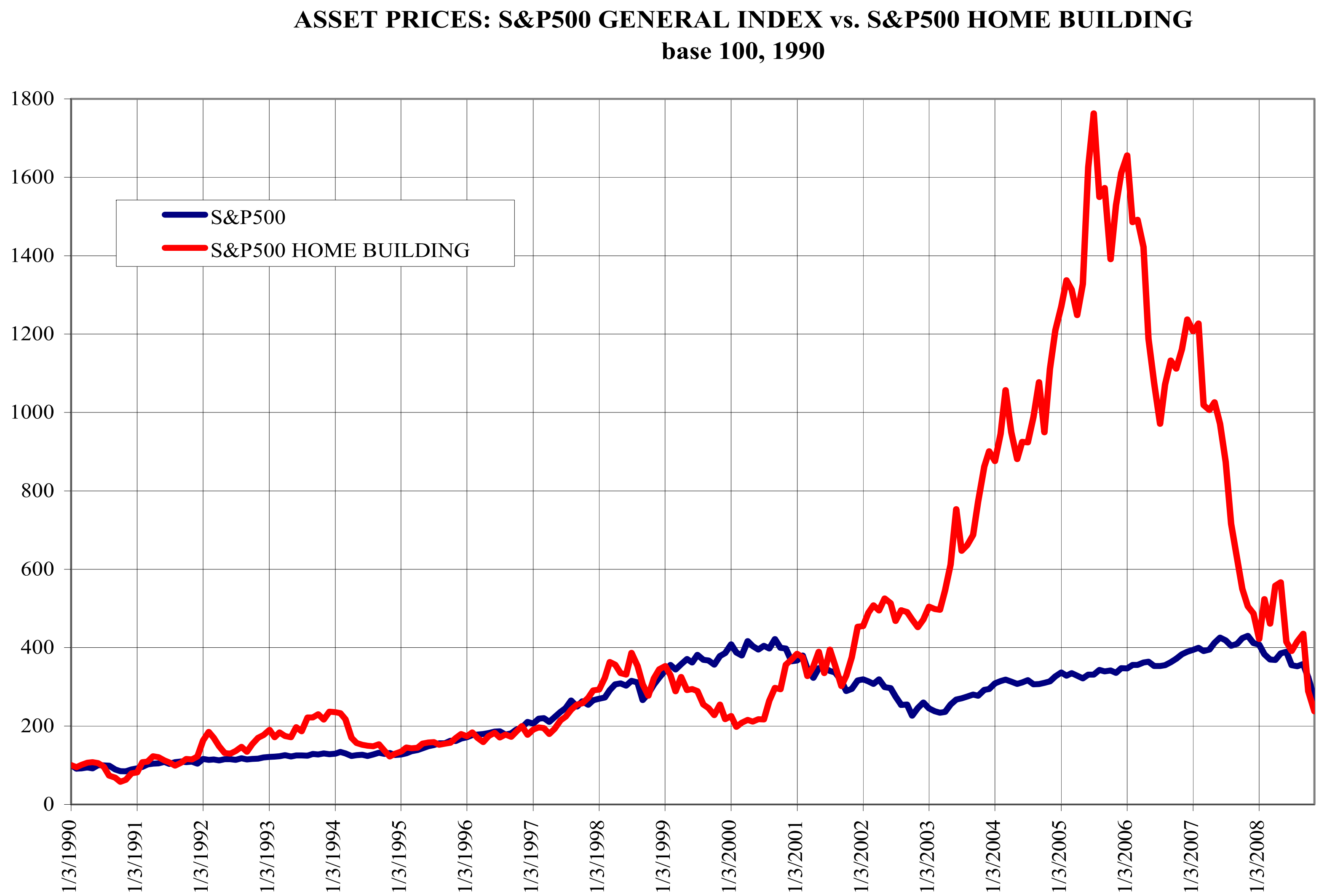
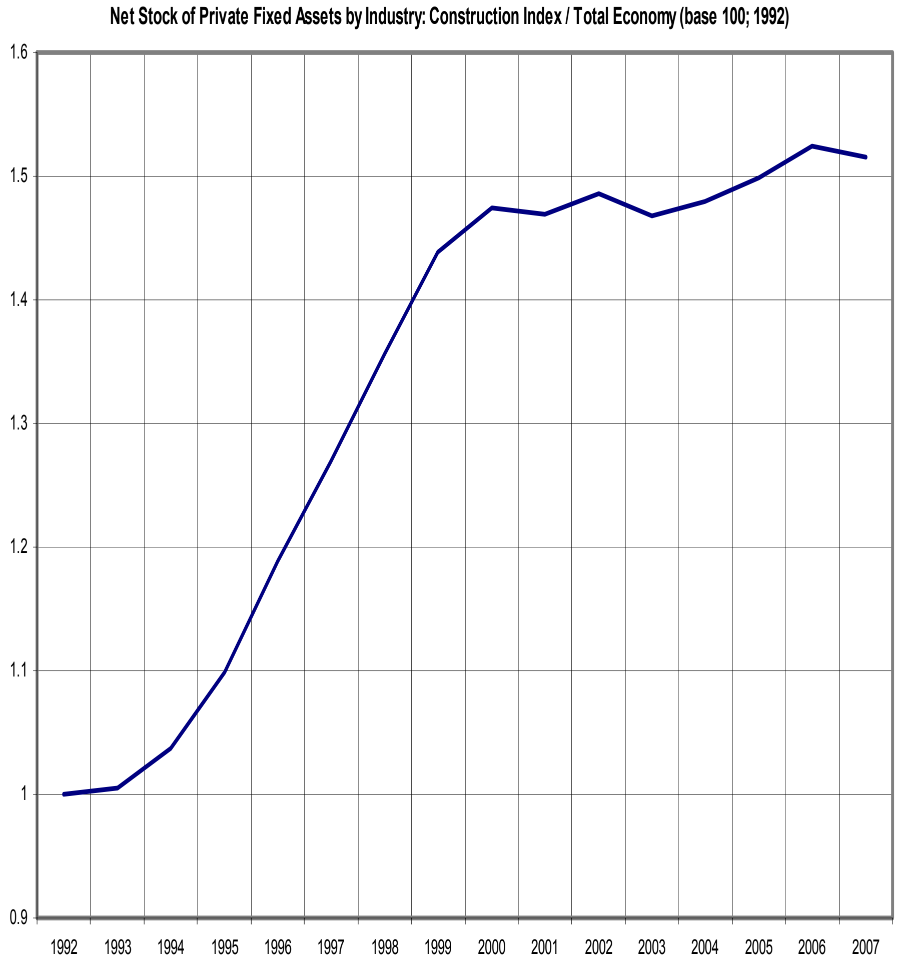


Figure 8: The Real Side of the Mortgage Boom



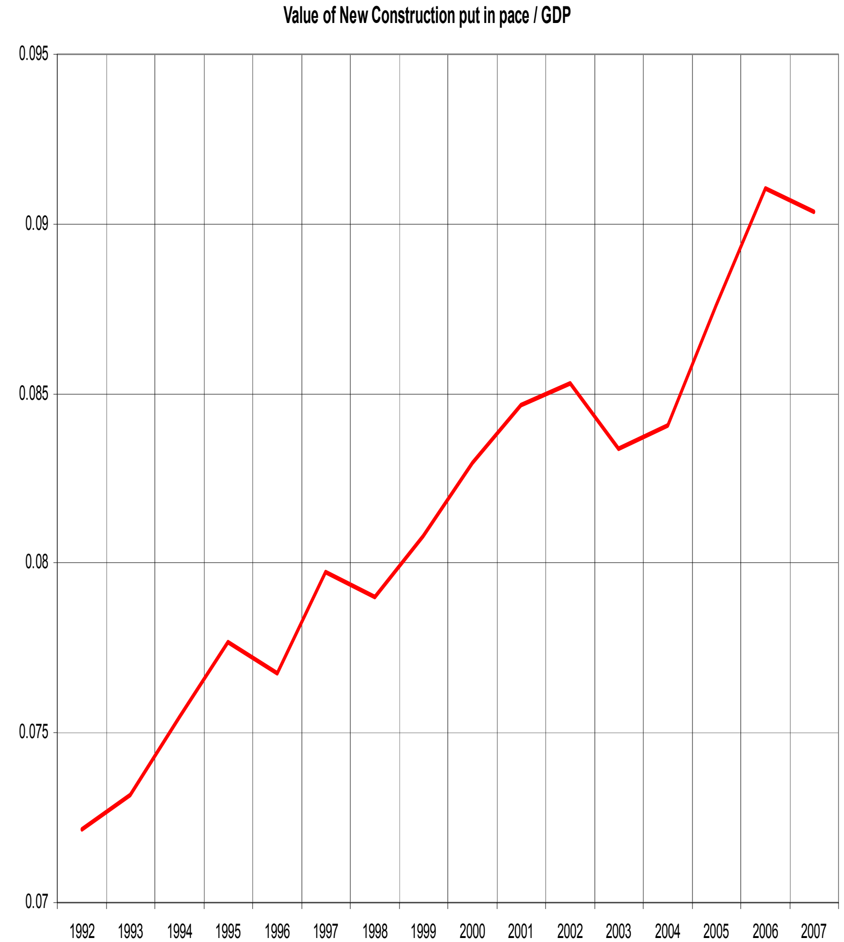
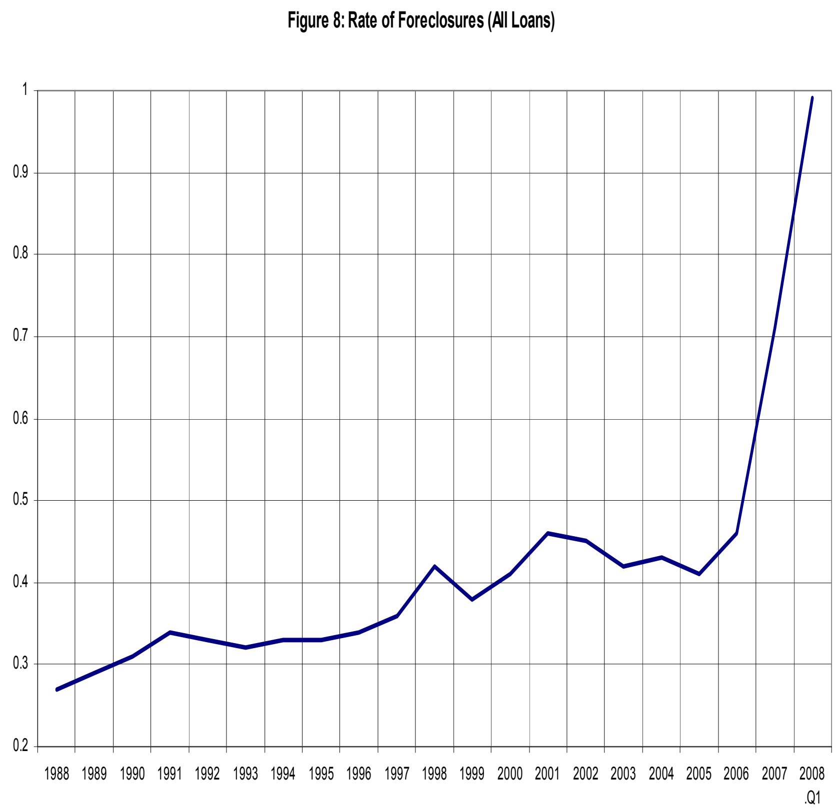
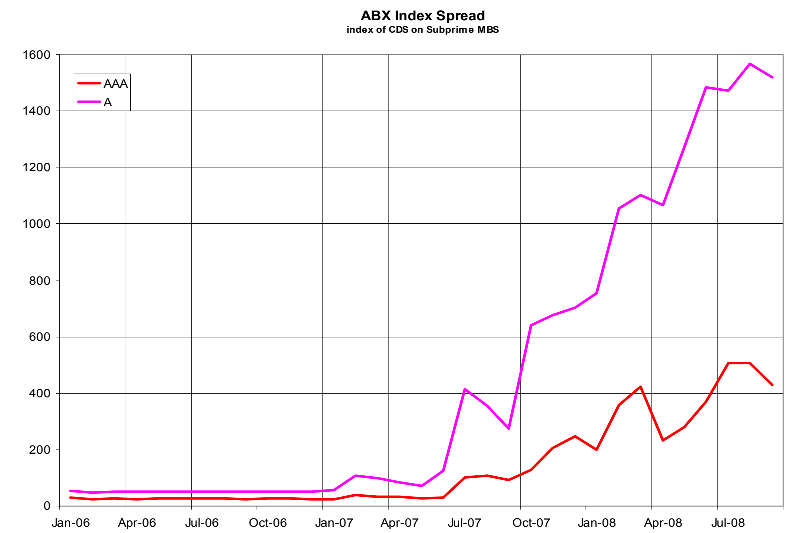


Figure 9: Crisis (I): Insolvency in Mortgage Related Assets





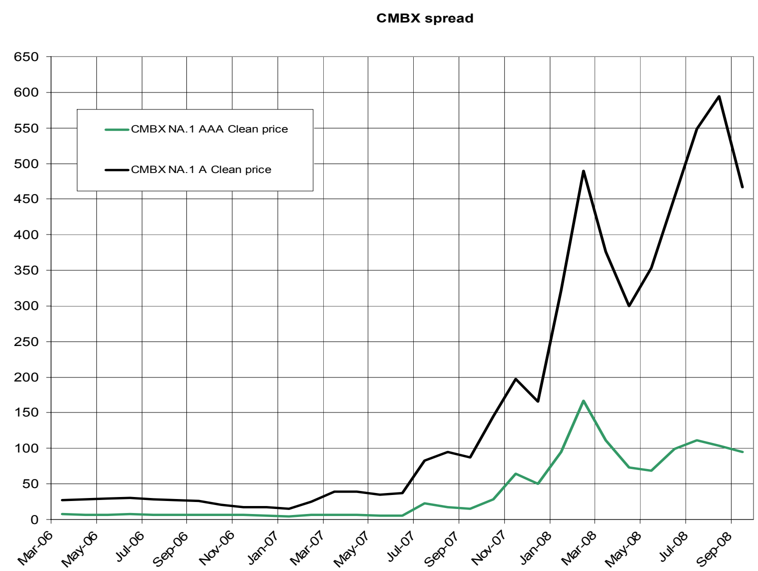


Figure 10: Crisis (II) CDS and The cross-section of insolvency risk

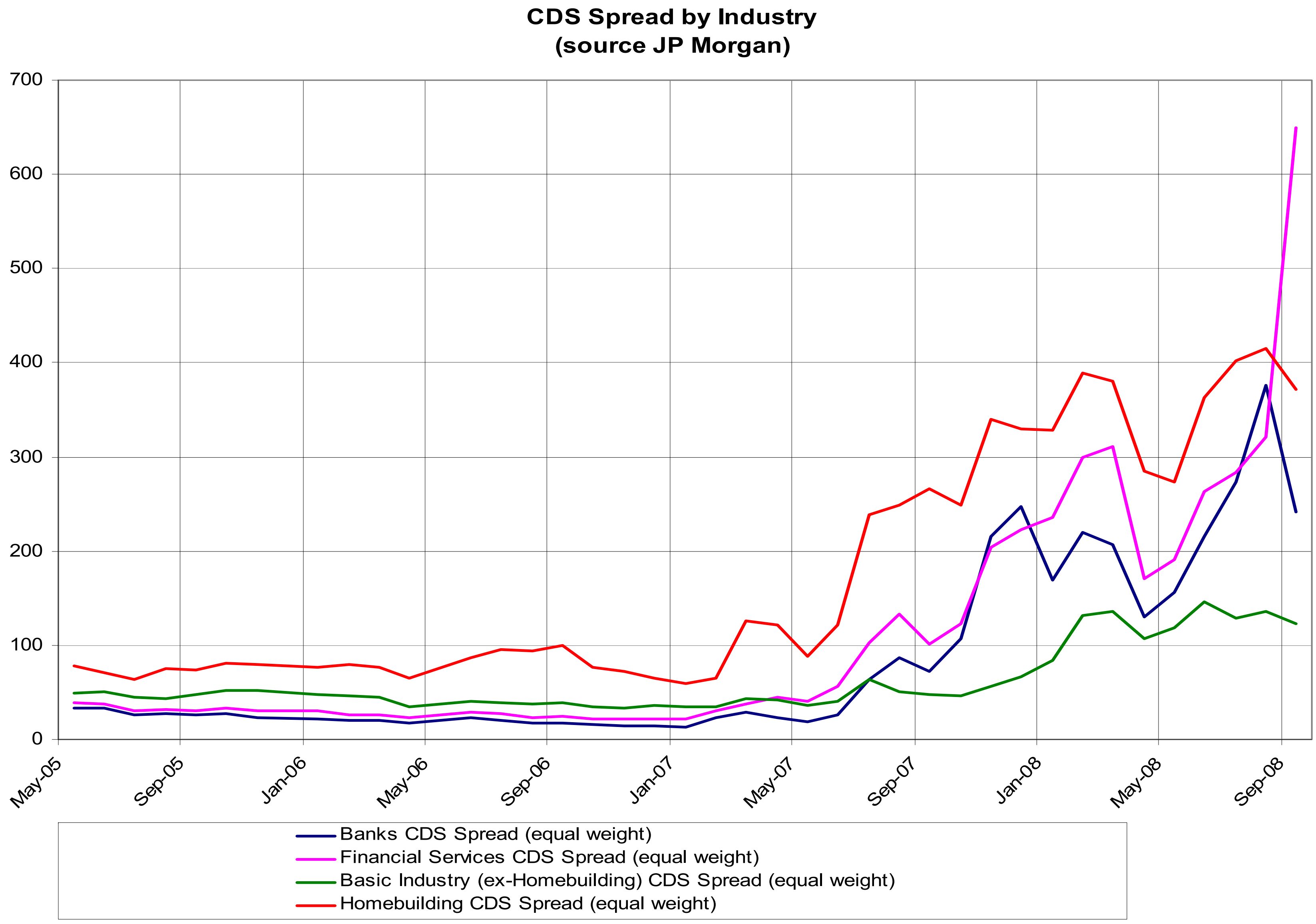
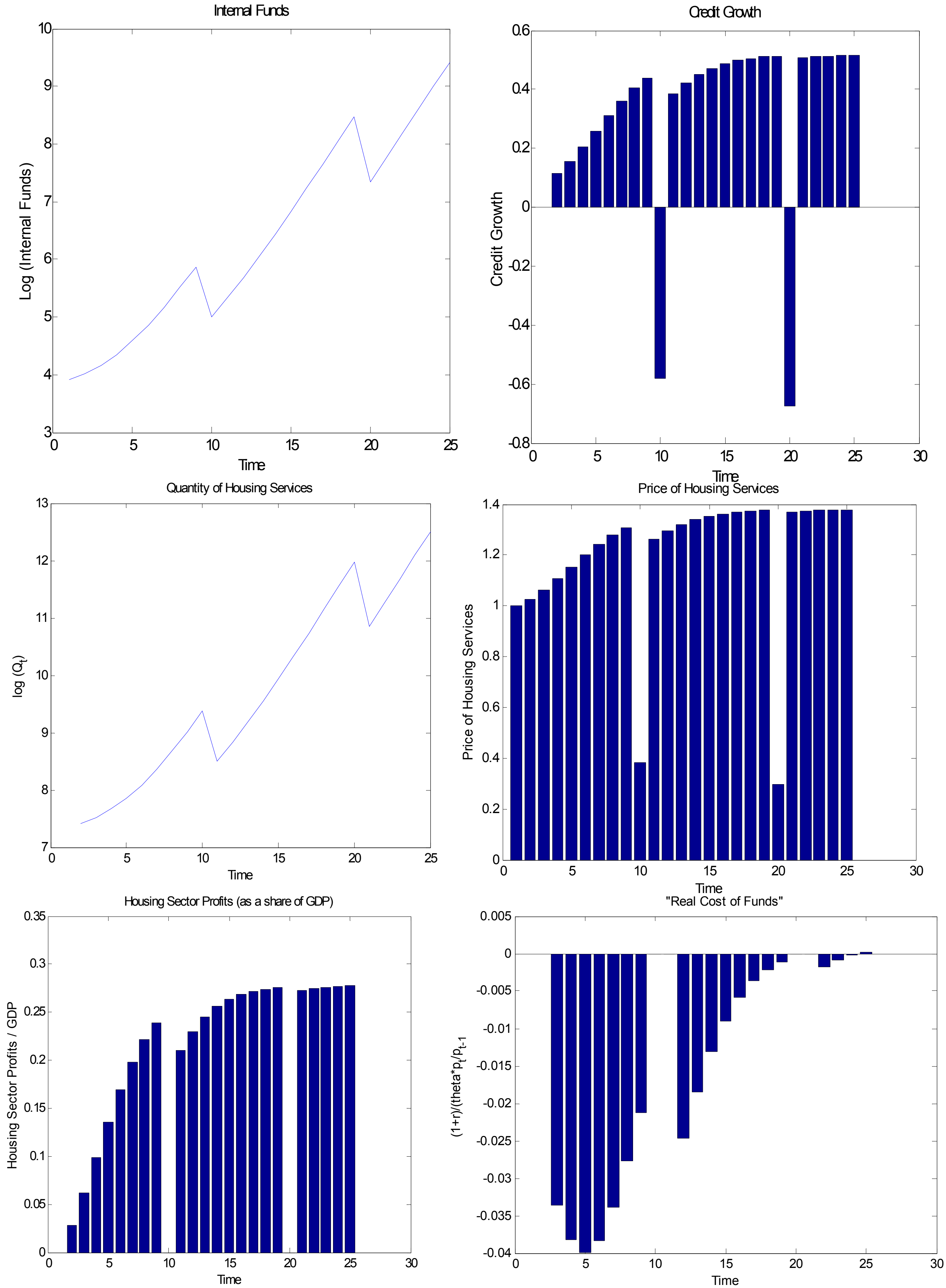
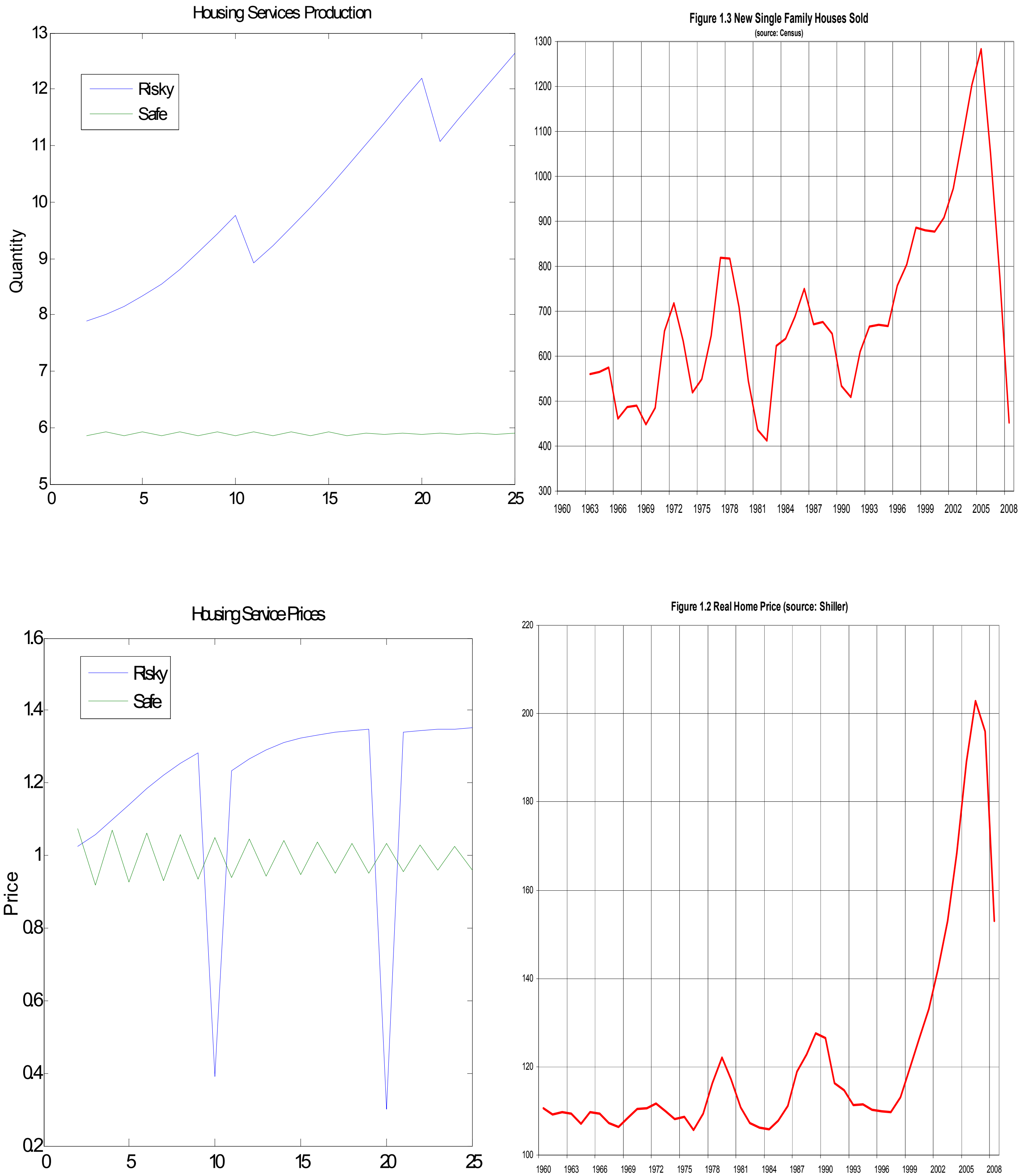


Figure 11: Risky Equilibrium



Paremeters: h=1; 1-u=0.0833; 1/(1+r)=0.9; theta=1.1; d=50; wbar=50; 1-c=0.05

Figure 12: Risky and Safe Equilibria: Model and Data



Paremeters: h=1; 1-u=0.0833; 1/(1+r)=0.9; theta=1.1; d=50; wbar=50; 1-c=0.05

Figure 13 External Imbalances

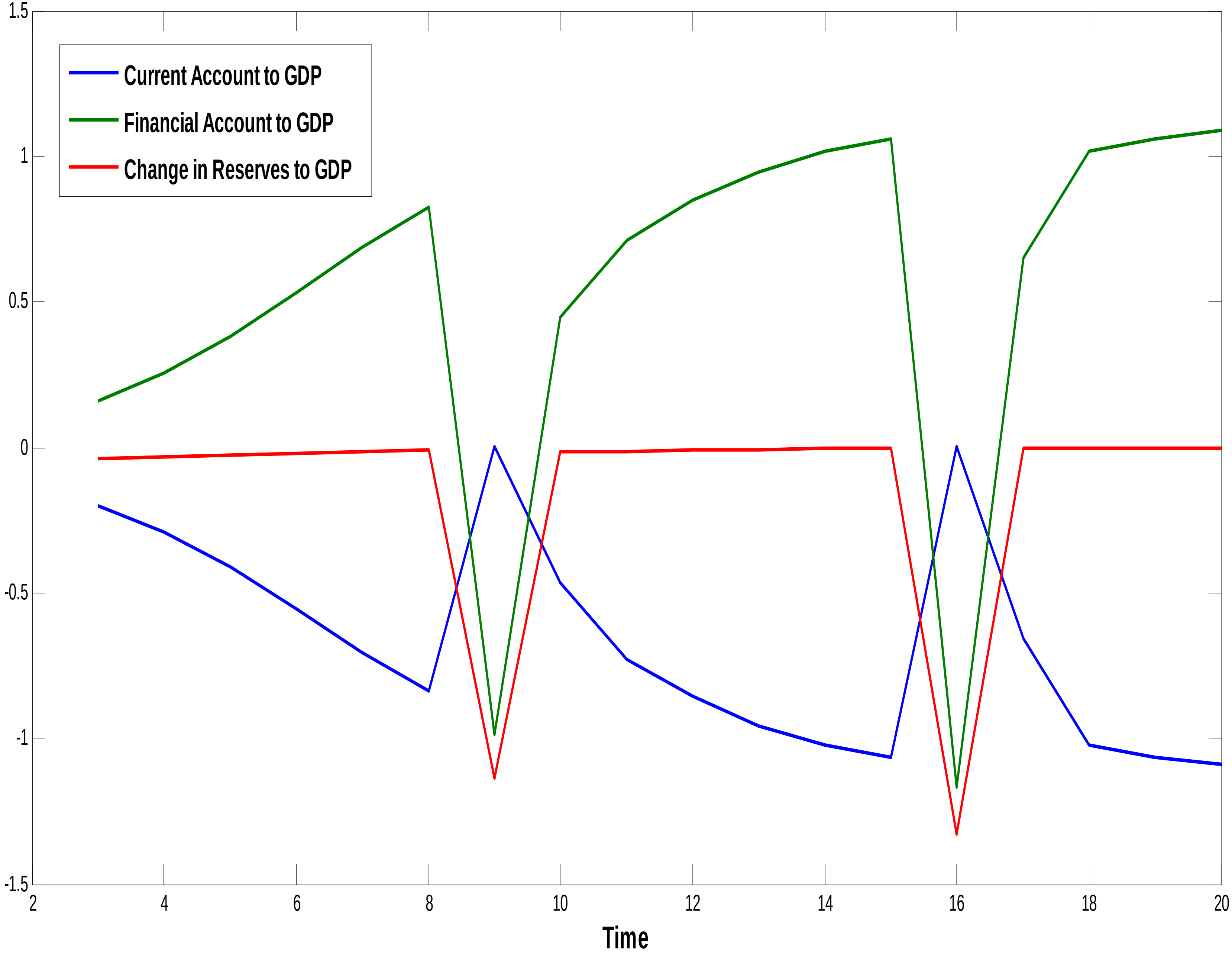


Figure 14. Stabilization Policy and GDP.