Adiabatic response in open systems

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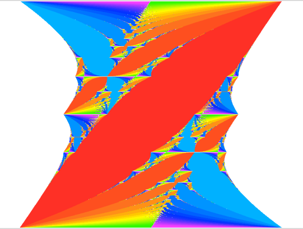


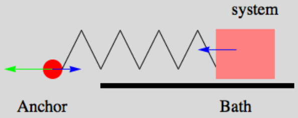
Thanks: Gal Yehoshua

Big picture

$$\begin{array}{ccc} |\psi\rangle_{s+b}(0) & \xrightarrow{Unitary} & |\psi\rangle_{s+b}(t) \\ Tr_b \downarrow & & \downarrow Tr_b \\ \rho_s(0) & \xrightarrow{Krauss} & \rho_s(t) \end{array}$$

Why?

- H_{s+b}: Complicated; Gapless
- ✓ Q-Optics, ✓ NMR; ¿Q-Transport?
- QI: Natural, Simple.



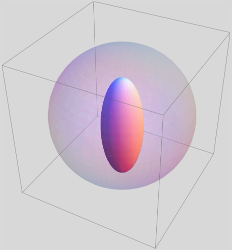
Linear response

- Friction: $F_{friction} = \nu_f \; \dot{X}$ ugly
- Anchor force: $F_{anchor} = \nu_a \; \dot{X}$ ugly
- Momentum rate: $\dot{P} = \nu_p \; \dot{X}$ simple

Unitary vs Krauss

$$\textit{Unitary}: \quad \rho \mapsto U\rho U^\dagger, \qquad U^\dagger U = \mathbb{1}$$

$$\textit{Krauss}: \quad \rho \mapsto \underbrace{K_0\rho K_0^\dagger + K_1\rho K_1^\dagger}_{\textit{positive}}, \quad \underbrace{K_0^\dagger K_0 + K_1^\dagger K_1 = 1}_{\textit{trace preserving}}$$



Infinitesimal

$$K_0 = \mathbb{1} + O(\delta t), \quad K_1 = O(\sqrt{\delta t})$$

$$\delta\rho = \mathcal{L}(\rho)\,\delta t$$

Lindblad: Generator of Krauss

Lindbladians (Davies)

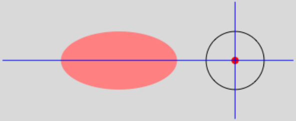
$$\dot{\rho} = \mathcal{L}(\rho)$$
$$\mathcal{L}(\rho) = \underbrace{-i[H,\rho]}_{unitary} + \underbrace{2\Gamma\rho\Gamma^\dagger - \Gamma^\dagger\Gamma\rho - \rho\Gamma^\dagger\Gamma}_{decoherence\ \&\ dissipa.}$$

Example (Decay)

$$H = a^\dagger a, \quad \Gamma = \sqrt{\gamma}a$$

Stationary states

$$\mathcal{L}(\sigma) = 0$$

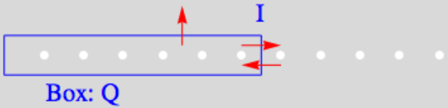


Spectrum($\mathcal{L}$) & Gap

Rates

Heisenberg picture

$$\begin{array}{ccc} Q \otimes \mathbb{1} & \xrightarrow{\textit{Hamiltonian}} & i[Q \otimes \mathbb{1}, H_{s+b}] \\ \downarrow & & \downarrow \\ Q & \xrightarrow{\textit{Lindbladian}} & \dot{Q} = \mathcal{L}^*(Q) \end{array}$$



Rate of Q (Gebauer & Car)

$$\dot{Q} = \mathcal{L}^*(Q) = \underbrace{i[H, Q]}_{\textit{unitary}} + \underbrace{\Gamma^\dagger[Q, \Gamma] + [\Gamma^\dagger, Q]\Gamma}_{\textit{decoherence \& dissipation}}$$

Velocity and current

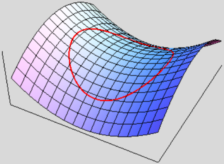
$$\dot{x} = \underbrace{\mathcal{L}^*(x)}_{Rate} \quad \text{vs} \quad v = \underbrace{i[H,x]}_{local} = p - A$$

Example (Landau)

- $H = \frac{1}{2} D^* D, \quad D = \underbrace{v_1 + i v_2}_{velocity}$
- $[v_1, v_2] = i \underbrace{B}_{Mag.\ field}$
- $\Gamma = \underbrace{\sqrt{\gamma}}_{decay} D$
- $2 j_\mu = \underbrace{\{\rho, v_\mu\}}_{Hamiltonian} + \underbrace{\gamma\, \partial_\mu \rho}_{diffusive} + \underbrace{\gamma \epsilon_{\mu\nu}\{\rho, v_\nu\}}_{Chiral\ \ dissipative}$

Parametrized Lindbladians

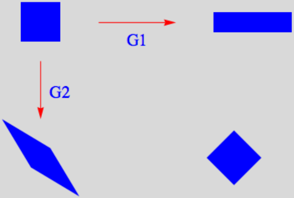
- $\phi \in$ Control space
- $H(\phi) = \underbrace{U(\phi) H U^\dagger(\phi)}_{iso-spectral}, \quad \Gamma(\phi) = U(\phi)\Gamma U^\dagger(\phi)$
- $U(\phi) = e^{iG\phi}$
- $\partial_\mu \sigma = i[G_\mu, \sigma]$



Control space

Example (Viscosity: Read Rezayi)

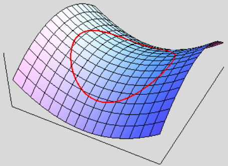
- $$G_1 = xp_y + yp_x,$$
  $$G_2 = xp_x - yp_y$$
  $$[G_1, G_2] = iL_z$$



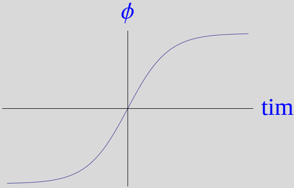
Controls: Deformations

Adiabatic

- adiabatic parameter: ϵ
- $\underbrace{s}_{slow\ \ time} = \underbrace{\epsilon}_{adiabaticity} t.$
- adiabatic evolution $\epsilon\dot{\rho} = \mathcal{L}_\phi(\rho)$
- ϵ in leading derivative: Singular perturbation:
- Initial data: Stationary $\rho(0) = \sigma(\phi_0)$

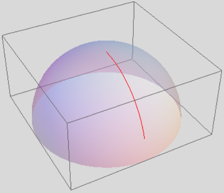


Closed adiabatic path in control space



Adiabatic expansion

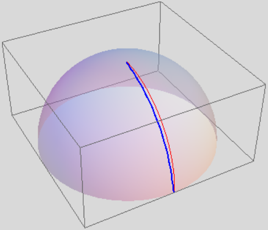
- $$\rho = \rho_0 + \epsilon\rho_1 + \dots$$
- $$\epsilon\dot\rho = \mathcal{L}(\rho)$$
- $$\underbrace{\epsilon\dot\rho_0}_{O(\epsilon)} + O(\epsilon^2) = \underbrace{\mathcal{L}(\rho_0)}_{O(1)} + \underbrace{\epsilon\mathcal{L}(\rho_1)}_{O(\epsilon)} + O(\epsilon^2)$$



Motion of σ: inst. stat.

Adiabatic expansion slaved to stationary states

- $$\underbrace{\mathcal{L}(\rho_0) = 0,}_{\textit{stationary}} \quad \dot\rho_0 = \underbrace{\mathcal{L}(\rho_1)}_{\textit{slave}}$$
- $$\rho = \underbrace{\sigma}_{\textit{stationary}} + \epsilon\mathcal{L}^{-1}(\dot\sigma) + \dots$$



adiabatic motion slaved to σ

Response theory

Theorem (Kubo for rates related to TKNN)

Unique $\sigma(\phi)$ + *Adiabaticity* $\Longrightarrow$ *Memoryless & slaved to stationary,*

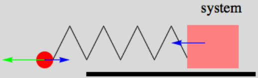
$$Tr(\dot{Q}\rho)(s) = \underbrace{Tr(Q\,\partial_\mu\sigma)}_{response\ \ coeff}\ \underbrace{\dot{\phi}_\mu}_{driving}(s), \qquad (\rho(0) = \sigma)$$

Isospectral: $\partial_\mu\sigma = i[G_\mu, \sigma]$

Theorem (No dissipation related to Berry Robbins)

Adiabaticity + *iso-spectral (generated by G)*

$$Tr(\dot{G}_\mu\rho) = -iTr(\underbrace{[G_\mu, G_\nu]}_{anti-symmetric}\ \sigma)\,\dot{\phi}_\mu$$



$F_{total} = \dot{P} = \nu_p\,\dot{X}, \quad \nu_p = 0,$

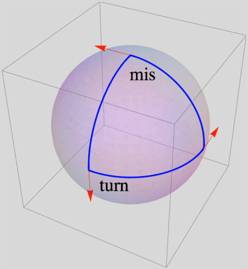
Response theory II

Projection: $\sigma^2 = \sigma \longrightarrow \sigma(d\sigma)\sigma = 0$

Theorem (Geometry)

Uniqueness+Adiabaticity + iso-spectral+projection

$$Tr(\dot{G}_\mu \rho) = \underbrace{-iTr(\sigma_\perp[\partial_\mu\sigma, \partial_\nu\sigma])}_{adiabatic\ \ curvature} \dot{\phi}_\mu$$



Curvature: Failure of parallel transport

Open

- Dictionary:
  Currents, Voltages $\Leftrightarrow$ Observables (von Neuman, POVM)
- Who pays? (Dissipation)
- Bloch+Lindblad+Fermions
- A prediction