A parametric analysis of magnetospheric energy budgets of non-stormtime substorms

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Abstract
Magnetospheric substorms are global phenomena in which energy, transported from the solar wind into the magnetosphere where it is stored, is released and dissipated in the ionosphere and through other sinks. Since there are currently no continuous measures of flow of energy between the solar wind, magnetosphere, and ionosphere many empirical relations have been created to describe the transfer of energy between these different regions. From these, energy budgets have been created and studied to determine the relative roles of different energy sinks as well as determining the extent to which substorms are directly driven or a loading-unloading process. We will assess the dependence of the final energy budget on each of the inputs into these relations, and the variability in the estimate of total energy stored in the magnetosphere due to the choice of free parameters and empirical relations used to form the energy budget.

Keywords: Magnetospheric substorms; Energy sources and sinks; Ring current; Joule heating; Particle precipitation

Introduction
Energy budgets have been applied to closed physical systems to learn more about the path energy takes in relationship to different phenomena within the system. If the system is closed, an energy budget can determine whether all sinks and sources have been included and can aid in determining if any sinks and sources have been overlooked or wrongly accounted for. A magnetospheric substorm is a system-scale energy release event, and one of the many questions surrounding substorms is where in the magnetosphere the energy is released. Many authors have attempted to answer this question [e.g. Akasofu, 1981; Rosenqvist *et al.*, 2006]. Perrault and Akasofu [1978] proposed that the major sinks for energy dissipation in the magnetosphere during a substorm are the ring current and the ionosphere, giving an energy balance equation for the inner magnetosphere as

$$W_T = W_{in} - W_{RC} - W_{Iono} + W_{other} \tag{1}$$

where W_T is the total energy in the magnetotail, W_{in} is the energy input to the magnetosphere, W_{RC} and W_{Iono} are the energy dissipated through the ring current and ionosphere respectively, and W_{other} are other sinks not considered in this study. None of these sources or sinks can be monitored continuously, thus empirical relations have been developed to create estimates of the flow of energy from one region to the next. In this paper we will explore the effects of changing the inputs to equation 1.

Method
Many empirical relations have been developed to describe the transfer of energy through different regions of the geospace environment (magnetosheath, magnetosphere, ionosphere). As this is a coupled system it is hard to distinguish the effects of changes in the individual terms on the overall energy budget. In this study we test the effects of varying the different parameters commonly used in estimating the energy transfer by using idealized inputs that can be changed independently, using values representative of non-stormtime magnetospheric conditions.

When Perreault and Akasofu [1978] first proposed a substorm energy budget for the inner magnetosphere, they used estimates of the energy flow into the ring current and the ionosphere to find a correction factor (L_o) to be multiplied by a solar wind-magnetosphere coupling function. This function, ϵ, has since been proposed to physically represent the solar wind Poynting flux into the magnetosphere and is defined as

$$\epsilon = \frac{4\pi}{\mu_o} L_o^2 v B^2 \sin^4\left(\frac{\theta_c}{2}\right) \qquad [\mathrm{W}] \tag{2}$$

where v is the magnitude of the solar wind velocity, B is the magnitude of the IMF, μ_o is the permeability of free space, θ_c is the IMF clock angle, and L_o is scale length (given as $7\,R_E$ for the inner magnetosphere). Many studies have attempted to better estimate the energy flow into the ring current and the ionosphere, but very few have altered ϵ itself thus for this study we will use ϵ with $L_o = 7\,R_E$.

The two primary magnetospheric energy sinks during a substorm are thought to be the ring current and the ionosphere where Joule heating and particle precipitation are the major ionospheric loss mechanisms [Akasofu, 1981]. The empirical relations used to estimate the energy dissipated through these sinks have been re-evaluated recently using

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both satellite- and ground-based data to improve the estimates [e.g. Østgaard *et al.*, 2002a; Rosenqvist *et al.*, 2006]. Other sinks include plasmoids, plasma sheet heating and plasma wave generation, but we neglect these in our study for two reasons: 1. To our knowledge these sinks have not been parameterized and hence are difficult to estimate routinely; and 2. the purpose of this paper is to determine the sensitivity of the energy budget calculation to the parameters and proxies most commonly discussed in the literature.

The power dissipated in the ring current is given as [see Akasofu, 1981]

$$U_{RC} = 4 \times 10^4 \left(\frac{d\mathrm{D}_{st}}{dt} + \frac{\mathrm{D}_{st}}{\tau} \right) \qquad \text{[GW]} \tag{3}$$

where D_{st} [nT] is the geomagnetic index used as a proxy for ring current energy and τ [s] is the ring current decay time. Akasofu [1981] notes that equation 3 describes the ring current injection rate and as such is only meaningful when $U_R > 0$. Many empirical estimates are used for τ: Lu *et al.* [1998] determined decay times for fixed ranges of D_{st}; O'Brien and McPherron [2000] found the decay time of the ring current to depend on the solar wind conditions such that $\tau = 2.40 \exp((9.74)/(4.69 + VB_s))$ [hr] where $VB_s = 0\,[\mathrm{mV\,m^{-1}}]$ for $B_z > 0$ and $VB_s = |VB_z|\,[\mathrm{mV\,m^{-1}}]$ otherwise. Another parameterization of τ was given by Valdivia et al. [1996] where $\tau = (12.5)/(1 - 0.0012D_{st})$ [hr]. Both give a faster decay time for more disturbed magnetospheric conditions in agreement with the observations of Lu et al. [1998]. For this study we will use $\tau = 20\,\mathrm{hr}$ [Lu et al., 1998] as a reference value, as this was determined for quiet magnetospheric conditions.

The primary dissipation mechanisms in the ionosphere are Joule heating and particle precipitation. A number of empirical relations are used to estimate the energy dissipated via these mechanisms [Østgaard *et al.* 2002b]. The most commonly used of these formulae give the energy dissipated as a function of an auroral index (typically AE or AL) based on a linear fit to an estimate of the height-integrated energy dissipation see, e.g., Rosenqvist et al. [2006] for an overview. Here we use the empirical relations of Østgaard et al. [2002a, 2002b] because of their high correlation coefficients. The power dissipated by Joule heating, U_J, and by particle precipitation, U_{pp}, are estimated using

$$U_J = 5.4 \times 10^8 AE + 1.8 \times 10^9 \qquad \text{[GW]} \tag{4}$$

$$U_{pp} = 4.4 \times 10^9 \sqrt{AL} - 7.6 \times 10^9 \qquad \text{[GW]} \tag{5}$$

where AL and AE are given in nT.

Inserting equations 2, 3, 4 and 5 into equation 1 we can construct an energy budget. Later in the paper we will vary some of the inputs into these equations. We adopt a window half-width of two hours centered on a substorm onset,

$$W_T(t) = \int_{t_0-h}^{t_0+h} (U_{in} - U_{RC} - (U_J + U_{pp}))\, dt \qquad \text{[J]} \tag{6}$$

where W_T is the energy stored. By controlling the inputs into the energy budget and how these are varied we can see the sensitivity of the energy budget to the parameters in equation 6.

Unlike previous studies, we take the empirically determined equations, form equation 6 and use idealized inputs to see how these changes affect the energy budget. Constant values were chosen for the solar wind parameters, consistent with typical solar wind bulk parameters. To determine the idealized inputs to be used we examined 135 non-stormtime ($\mathrm{D}_{st} > -30\,\mathrm{nT}$) substorms identified over 2001-2005 using the IMAGE satellite [Frey and Mende, 2006] and found the means and trends of the solar wind parameters and geophysical indices within a 4 hour window centred on substorm onset. Figure 1 shows selected input parameters (upper panels) and the reference energy budget (bottom panel). The black lines give the reference inputs, the blue and green lines give the variations. The top panel shows the B_z [nT] used in estimating solar wind power input, [0, -2, -4]. The second panel shows the values of D_{st} [nT] used, [-2, 10, -20]. For the initial parameter study $(d\mathrm{D}_{st})/(dt)$ was set to zero (solid lines), in accord with the ensemble average of the 135 quiet-time substorms. We also investigate the effect of $(d\mathrm{D}_{st})/(dt)$ [nT/hr] on W_{RC}, using [0, 1.5, -1.5] in accord with observation. The third panel shows the model AL bays; these were generated using the Weimer [1994] model which gives the AL bay as a function of time expressed as $\mathrm{AL}(t) = \mathrm{C}_0 + \mathrm{C}_1 t \exp(pt)$ where C_0, C_1, p are constants. C_1 [nT/s] determines the bay magnitude and small, medium and large bays have C_1 [-794, -1878, -3232]. Since our focus is on non-stormtime substorms, we use the small substorm as our reference input. Using these inputs with equation 6 we find that we estimate a net loss of energy from the magnetosphere (bottom panel of figure 1). To determine how changing parameters can affect the stored energy we vary each parameter in turn, holding all others constant. The final parameter we vary is τ [hr], [20, 14.15, 12.35].

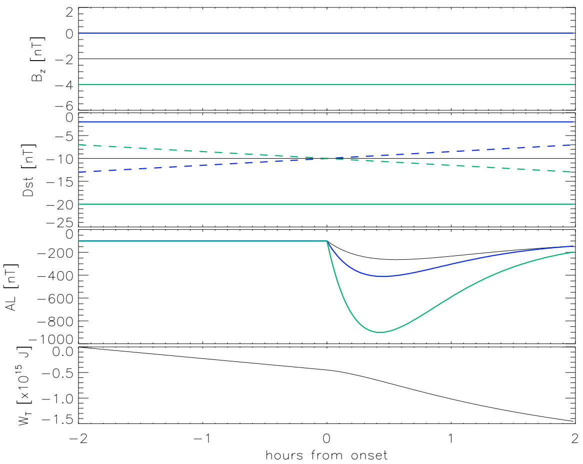

Figure 1: Selected input parameters (upper panels) and the reference energy budget (bottom panel). The black lines give the reference inputs, the blue and green lines give the variations. The panels show the B_z (top), D_{st} (second) and AL bays (third) described in the text. The bottom panel shows the total energy stored using the reference inputs given by the black lines.

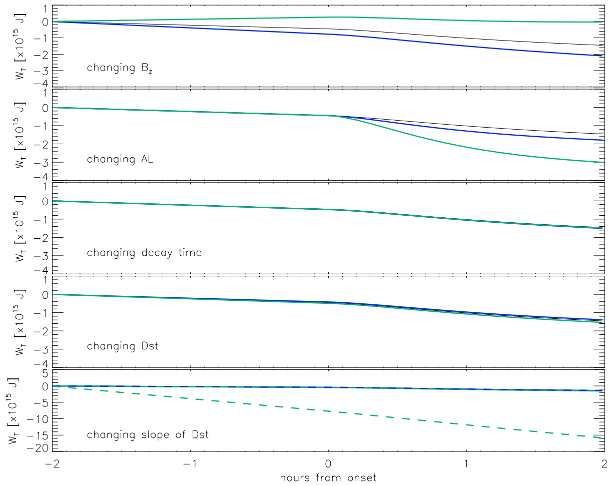

Figure 2: Estimates of the total energy stored for variations of the input parameters. In each panel the black line corresponds to the energy budget using the reference values of all parameters. The blue and green lines use reference values for all quantities except that named in the plot annotation, e.g. the blue line in the top panel uses the blue curve in panel 3 of figure 1 for the AL index.

Results and Discussion

Figure 2 shows estimates of the total energy stored for variations of each input parameter. In each panel the black line corresponds to the energy budget using the reference values of all parameters. The blue and green lines use reference values for all quantities except that named in the plot annotation. The panels show (top to bottom) energy budgets for variations of B_z, AL, τ, D_{st} and dD_{st}/dt. Note the change of scale on the bottom panel. Increasing B_z by 2 nT, thus increasing ϵ, for the 4 hr window we see a decrease in the energy input of 4.5×10^{14} J. Decreasing B_z from -2 nT to -4 nT gives almost an order of magnitude more energy input. Changing B_z from 0 nT to -4 nT gives a two order of magnitude difference in W_T.We estimate that a small substorm will dissipate about 2.3×10^{15} J in the ionosphere over our four hour integration time; this almost doubles with a large substorm. This gives a range of estimates of total energy stored of $-3.0 \times 10^{15} < W_T < -1.8 \times 10^{15}$ J.

For non-storm times we expect τ ~10-20 hr and hence $0 < W_{RC} < 1.8 \times 10^{14}$ W. Using the three decay times, τ, we find that the resulting changes in W_T are negligible. The energy storage using the given D_{st} values is $-5.8 \times 10^{15} < W_T < -1.3 \times 10^{15}$ J and the energy loss through the ring current is an order of magnitude less than the estimates of energy dissipated in the ionosphere, or transferred into the magnetosphere from the solar wind. Figure 2 (bottom panel) shows that the estimate of dissipation into the ring current increases by two orders of magnitude when there is a negative slope in D_{st}. When there is a positive slope in equation 3, U_R becomes positive and is set to zero, thus less energy is dissipated in the ring current.

If the estimates for the magnetospheric sinks are accurate, we can directly determine that the energy transfer from the solar wind into the magnetosphere is underestimated. Due to the lack of understanding and direct measurements of the energy transfer between the solar wind and the magnetosphere, it is likely that the magnetospheric sinks are uncertain, and the scaling of ϵ is not correct.

Many papers have studied the decay time of the ring current, τ. This study shows that, when looking at the energy budget of the magnetosphere during quiet times, the exact value of τ is relatively unimportant. Of all the variables tested in this study, changing the decay time changed the final energy budget the least. This result is not necessarily expected to hold for storm times and requires further study. Next to the decay time, the magnitude of D_{st} least affects the energy budget. There are many studies that have looked at correcting the D_{st} for other current systems [e.g. O'Brien and McPherron, 2000]. Typically a 'corrected' D_{st}, called D^*_{st}, is used which has a correction based on the solar wind dynamic pressure as well as a constant offset. We have shown that for the quiet-time energy budget this offset makes little difference as long as $D_{st} < 0$, while the term based on the pressure correction can be important to the slope. If D_{st} becomes positive, which occurs when the magnetopause currents dominate, it would change the sign of equation 3 and the function must be set to zero as it no longer represents the ring current energy. The slope term in the ring current power equation dominates the energy budget. Whilst the magnitude of D_{st} must be corrected for the contributions of other current systems, it is more important to the energy budget to capture the variation of D_{st} and variations due to other contributing currents. The approach to pressure correcting D_{st} can greatly affect the slope of D_{st} and hence the estimate of ring current power. This may affect energy estimates if we were to use the Sym-H index as a high-resolution version of D_{st}. Some work has been done to estimate the contribution to D_{st} of the tail current [Turner *et al.*, 2001], however there has been little quantitative work done to estimate the effect on the variability of the D_{st} from other magnetospheric currents.

The Joule heating, as expected, dominates over the particle precipitation. Since both are estimated using auroral indices, a change in the substorm bay size can greatly affect the energy budget. Hence it is important that we correctly estimate the magnitude of the substorm. If the substorm does not occur over a standard AL magnetic observatory, we will underestimate the energy dissipated in the ionosphere.

Conclusions

- We have examined the energy budgets for idealized, isolated non-stormtime substorms. The energy budget is more sensitive to variations in D_{st} than to the magnitude of D_{st}. This means that when correcting D_{st} for contributions from other current systems such as the tail current, it is more important to capture the variations due to these currents than the offset.
- Although the ring current plays a significant role in the energy budget, the decay time τ of the ring current only minimally affects the resultant energy budget.
- Given the uncertainty in the energy dissipation through the magnetospheric sinks, it is not currently possible to use an energy budget to estimate the solar wind power input.

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