Technical Appendix

Intégration bancaire et conjoncture macroéconomique dans une union monétaire hétérogène

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Contents

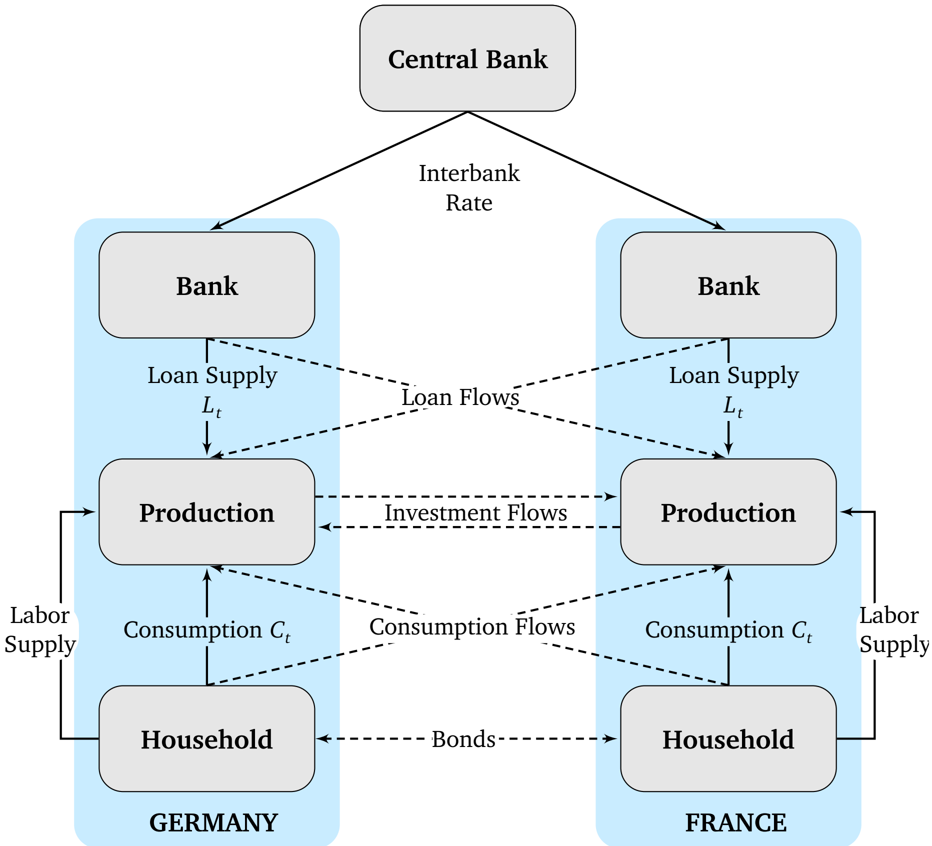

Figure 1: A monetary union model with cross border loans, investment and consumption goods, bonds.

As shown in Figure 1, the two countries share a common currency and have the same size. Each country $i \in \{h, f\}$ (where h is for home and f for foreign) is populated by households that consume save and supply differentiated labor types to labor unions. Monopolistically competitive labor unions negotiate with intermediate firms the real wage in staggered contracts with timing like Calvo. Intermediate firms supply differentiated goods in a monopolistically competitive market and set prices in a staggered basis. Final producers are CES packers, they aggregate the differentiated goods from intermediate firms and sell it to households and capital producers. Capital producers recycle the used capital stock and invest new capitals. Entrepreneurs buy capitals from capital producers and sell it to intermediate firms. Entrepreneurs are credit constrained, they borrow funds to the banking system. Banks provide loans to entrepreneurs in a monopolistically competitive market and set interest rate in staggered contracts.

1 Decision Problems

1.1 Households

1.1.1 The utility maximization problem

In each economy there is a continuum of identical households who consume, save and work in intermediate firms. The total number of households is normalized to 1. The representative household $j \in [0,1]$ maximizes the welfare index,

$$\max_{\{C_{i,t}(j),H_{i,t}(j),B_{i,t}(j)\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} e^{\varepsilon^{\beta}_{i,t+\tau}} \left[\frac{\left(C_{i,t+\tau}(j) - h^c_i C_{i,t-1+\tau}\right)^{1-\sigma^c_i}}{1-\sigma^c_i} - \chi_i \frac{H_{i,t+\tau}(j)^{1+1/\sigma^L_i}}{1+1/\sigma^L_i} \right], \tag{1}$$

subject to,

$$\frac{W^h_{i,t}}{P^c_{i,t}} H_{i,t}(j) + R_{t-1} \frac{B_{i,t}(j)}{P^c_{i,t}} + \frac{\Pi_{i,t}(j)}{P^c_{i,t}} = C_{i,t}(j) + \frac{B_{i,t+1}(j)}{P^c_{i,t}} + \frac{T_{i,t}(j)}{P^c_{i,t}} + \frac{P_{i,t}}{P^c_{i,t}} AC^B_{i,t}(j) \tag{2}$$

Here, $C_{i,t}(j)$ is the consumption index, $h^c_i \in [0;1]$ is a parameter that accounts for consumption habits, $H_{i,t}(j)$ is labour effort, $\epsilon^{\beta}_{i,t}$ is an exogenous $AR(1)$ shock to household preferences. The income of the representative household is made of labour income (with the nominal wage $W_{i,t}$ and the consumption price index $P^c_{i,t}$), interest payments for bond holdings, (where $B_{i,t}(j)$ stands for the bonds subscribed in period $(t-1)$ and R_{t-1} is the gross nominal rate of interest between period $t-1$ an period t), and earnings from shareholdings (where $\Pi^y_{i,t}(j)$, $\Pi^w_{i,t}(j)$ and $\Pi^b_{i,t}(j)$ are the nominal amount of dividends he receives from final good producers, unions and banks). The representative household spends this income on consumption, bond subscription and tax payments (for a nominal amount of $T_{i,t}(j)$). Finally, we assume that the household has to pay quadratic adjustment costs to buy new bonds, according to the function, $AC^B_{i,t}(j) = \frac{\chi^B}{2}\left(B_{i,t+1}(j) - B_i(j)\right)^2$, where $B_i(j)$ is the steady state level of bonds.

The Lagrangian problem for the household j writes,

$$\mathcal{L}_t = \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} e^{\varepsilon^{\beta}_{i,t+\tau}} \left[\begin{array}{c} \frac{\left(C_{i,t+\tau}(j) - h^c_i C_{i,t-1+\tau}(j)\right)^{1-\sigma^c_i}}{1-\sigma^c_i} - \chi_i \frac{H_{i,t+\tau}(j)^{1+1/\sigma^L_i}}{1+1/\sigma^L_i} \\ +\lambda^c_{i,t+\tau} \left[\frac{W^h_{i,t+\tau}}{P^c_{i,t+\tau}} H_{i,t+\tau}(j) + R_{t-1+\tau} \frac{B_{i,t+\tau}(j)}{P^c_{i,t+\tau}} + \frac{\Pi_{i,t+\tau}(j)}{P^c_{i,t+\tau}} \right] \\ -\lambda^c_{i,t+\tau} \left[C_{i,t+\tau}(j) + \frac{B_{i,t+1}(j)}{P^c_{i,t+\tau}} + \frac{T_{i,t+\tau}(j)}{P^c_{i,t+\tau}} + \frac{P_{i,t+\tau}}{P^c_{i,t+\tau}} AC^B_{i,t+\tau}(j) \right] \end{array} \right]$$

where $\lambda^c_{i,t}$ is the Lagrangian multiplier of the household budget balance sheet and can be understood as the shadow value of one unit of consumption, or the marginal utility of one unit of consumption. FOC. in $C_{i,t}(j)$, $H_{i,t}(j)$ and $B_{i,t}(j)$,

$$
\begin{aligned}
(\partial C_{i,t}(j)) &: \lambda^c_{i,t} = \left(C_{i,t}(j) - h^c_i C_{i,t-1}\right)^{-\sigma^c_i} \\
(\partial H_{i,t}(j)) &: \chi_i H_{i,t}(j)^{\sigma^L_i} = \lambda^c_{i,t} \frac{W^h_{i,t}}{P^c_{i,t}} \\
(\partial B_{i,t+1}(j)) &: \mathbb{E}_t \left\{ \frac{e^{\epsilon^\beta_{i,t}} \lambda^c_{i,t}}{e^{\epsilon^\beta_{i,t+1}} \lambda^c_{i,t+1}} \frac{P^c_{i,t+1}}{P^c_{i,t}} \right\} = \frac{\beta R_t}{1 + P_{i,t} \mathcal{X}^B \left(B_{i,t+1}(j) - B_i(j)\right)}
\end{aligned}
$$

The first order conditions that solve this problem can be summarized with a Euler bond condition,

$$
\frac{\beta R_t}{1 + P_{i,t} \mathcal{X}^B \left(B_{i,t+1}(j) - B_i(j)\right)} = \mathbb{E}_t \left\{ \frac{e^{\epsilon^\beta_{i,t}}}{e^{\epsilon^\beta_{i,t+1}}} \frac{P^c_{i,t+1}}{P^c_{i,t}} \left(\frac{\left(C_{i,t+1}(j) - h^c_i C_{i,t}\right)}{\left(C_{i,t}(j) - h^c_i C_{i,t-1}\right)} \right)^{\sigma^c_i} \right\}, \tag{3}
$$

and a labour supply function,

$$
\frac{W^h_{i,t}}{P^c_{i,t}} = \chi_i H_{i,t}(j)^{\sigma^L_i} \left(C_{i,t}(j) - h^c_i C_{i,t-1}\right)^{\sigma^c_i} \tag{4}
$$

1.1.2 Labor Unions and Packers

Households provide differentiated labor types j, sold by labor unions to perfectly competitive labor packers who assemble them in a CES aggregator and sell the homogenous labor to intermediate firms. Labor packers are perfectly competitive and maximize profits subject to the supply curve,

$$
\begin{aligned}
&\max_{H_{i,t}(j)} W_{i,t} H^d_{i,t} - \textstyle\int_0^1 W_{i,t}(j) H_{i,t}(j) \, \mathrm{d}j \\
&s.t. H^d_{i,t} = \left(\textstyle\int_0^1 H_{i,t}(j)^{(\epsilon_w - 1)/\epsilon_w} \, \mathrm{d}j \right)^{\epsilon_w/(\epsilon_w - 1)}
\end{aligned}
$$

where $H^d_{i,t}$ is the aggregate demand from intermediate firms. Thus relative labour demand writes,

$$
H_{i,t}(j) = \left(\frac{W_{i,t}(j)}{W_{i,t}} \right)^{-\epsilon_p} H^d_{i,t}, \qquad \forall j \tag{5}
$$

Concerning the representative unions $j \in [0;1]$, they are an intermediate between households and the labor packers. The wage evolves according to,

$$
W_{i,t}(j) = \begin{cases} W^*_{i,t}(j) \text{ with probability } 1 - \theta^w_i \\ \left(\pi^c_{i,t-1}\right)^{\xi^w_i} W_{i,t-1}(j) \text{ with probability } \theta^w_i \end{cases}
$$

Trade unions are not allowed to renegotiate the wage with probability θ^w_i, then the wage is partially indexed on previous inflation of consumption goods at a degree $\xi^w_i \in [0;1]$. Assuming that the trade union is able to modify its wage with a probability $1 - \theta^w_i$, it chooses the optimal wage $W^*_{i,t}(i)$ to maximize its expected sum of profits by solving,

$$
\max_{\left\{W^*_{i,t}(j)\right\}} \mathbb{E}_t \left\{ \sum\nolimits_{\tau=0}^{\infty} \left(\theta^w_i \beta\right)^\tau \frac{\lambda^c_{i,t+\tau}}{\lambda^c_{i,t}} \left[(1 - \tau^w) \frac{W^*_{i,t}(j)}{P^c_{i,t+\tau}} \prod_{k=1}^{\tau} \left(\pi^c_{i,t+k-1}\right)^{\xi^w_i} - \frac{W^h_{i,t+\tau}(j)}{P^c_{i,t+\tau}} \right] H_{i,t+\tau}(j) \right\},
$$

subject to the downgrade sloping demand constraint from labor packers,

$$H_{i,t+\tau}(j) = \left(\frac{W^*_{i,t}(j)}{W_{i,t+\tau}} \prod_{k=1}^{\tau} \left(\pi^c_{i,t+k-1} \right)^{\xi^w_i} \right)^{-\epsilon_w} H^d_{i,t+\tau}, \forall \tau > 0$$

where $H_{i,t}(j)$ denotes the quantity of differentiated labor type j that is used in labor packer production with substitutability ϵ_w between labor varieties and τ^w is a proportional tax. The first order condition is given by,

$$\frac{W^*_{i,t}(j)}{P^c_{i,t}} = \frac{\epsilon_w}{(1-\tau^w)(\epsilon_w - 1)} \frac{\mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta^w_i \beta)^\tau \frac{\lambda^c_{i,t+\tau}}{\lambda^c_{i,t}} \frac{W^h_{i,t+\tau}(j)}{P^c_{i,t+\tau}} H_{i,t+\tau}(j)}{\mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta^w_i \beta)^\tau \frac{\lambda^c_{i,t+\tau}}{\lambda^c_{i,t}} \prod_{k=1}^{\tau} \frac{\left(\pi^c_{i,t+k-1}\right)^{\xi^w_i}}{\pi^c_{i,t+k}} H_{i,t+\tau}(j)} \tag{6}$$

The markup of the aggregate wage over the wage received by the households is distributed to the households in the form of dividends. Using the zero profit condition, the aggregate wage index of all labor varieties in the economy writes,

$$W_{i,t} = \left(\int_0^1 W_{i,t}(j)^{1-\epsilon_w} \mathrm{d}j \right)^{\frac{1}{1-\epsilon_w}} \tag{7}$$

1.2 The Final Goods Sector

This sector is populated by two groups of agents: intermediate firms and final firms. Intermediate firms produce differentiated goods i, decide on labour and capital inputs, and set prices according to the Calvo model. Final goods producers act as a consumption bundler by combining national intermediate goods to produce the homogenous final good.

Final good producers are perfectly competitive and maximize profits subject to the supply curve,

$$\begin{aligned} &\max_{Y_{i,t}(i)} P_{i,t} Y^d_{i,t} - \int_0^1 P_{i,t}(i) Y_{i,t}(i) \mathrm{d}i \\ &s.t. Y^d_{i,t} = \left(\int_0^1 Y_{i,t}(i)^{(\epsilon_p - 1)/\epsilon_p} \mathrm{d}i \right)^{\epsilon_p/(\epsilon_p - 1)} \end{aligned}$$

We find the intermediate demand functions associated with this problem are,

$$Y_{i,t}(i) = \left(\frac{P_{i,t}(i)}{P_{i,t}} \right)^{-\epsilon_p} Y^d_{i,t}, \qquad \forall i \tag{8}$$

where $Y^d_{i,t}$ is the aggregate demand. Thus the aggregate price index of all varieties in the economy writes,

$$P_{i,t} = \left(\int_0^1 P_{i,t}(i)^{1-\epsilon_p} \mathrm{d}i \right)^{\frac{1}{1-\epsilon_p}} \tag{9}$$

1.3 The Intermediate Goods Sector

The representative intermediate firm $i \in [0,1]$ has the following technology,

$$Y_{i,t}(i) = e^{\varepsilon^A_{i,t}} K^u_{i,t}(i)^\alpha H^d_{i,t}(i)^{1-\alpha} \tag{10}$$

where $Y_{i,t}(i)$ is the production function of the intermediate good that combines capital $K^u_{i,t}(i)$, labour demand $H^d_{i,t}(i)$ to household and technology $e^{\varepsilon^A_{i,t}}$. Here, $e^{\varepsilon^A_{i,t}}$ is an $AR(1)$ productivity shock.

Intermediate goods producers solve a two-stage problem. In the first stage, taken the input prices $W_{i,t}$ and $Z_{i,t}$ as given, firms rent inputs $H^d_{i,t}(i)$ and $K^u_{i,t}(i)$ in a perfectly competitive factor markets in order to minimize costs subject to the production constraint (10). Put differently, maximizing profits subject to the supply curve gives the same results. Thus the Lagrangian problem for the firm i writes,

$$\mathcal{L}_t = \frac{MC_{i,t}(i)}{P^c_{i,t}} Y_{i,t}(i) - \frac{Z_{i,t}}{P^c_{i,t}} K^u_{i,t}(i) - \frac{W_{i,t}}{P^c_{i,t}} H^d_{i,t}(i) + \frac{\lambda_{i,t}}{P^c_{i,t}} \left[e^{\varepsilon^A_{i,t}} K^u_{i,t}(i)^{\alpha} H^d_{i,t}(i)^{1-\alpha} - Y_{i,t}(i) \right]$$

where $\lambda_{i,t}/P^c_{i,t}$ is the lagrange multiplier and is interpreted as the real marginal profit to input movements. Deriving in $Y_{i,t}(i)$, $K^u_{i,t}(i)$ and $H^d_{i,t}(i)$ for profits maximization,

$$\begin{aligned}
(\partial Y_{i,t}(i)) &: \frac{\lambda_{i,t}}{P^c_{i,t}} = \frac{MC_{i,t}(i)}{P^c_{i,t}} \\
(\partial K^u_{i,t}(i)) &: \frac{Z_{i,t}}{P^c_{i,t}} = \alpha \frac{\lambda_{i,t}}{P^c_{i,t}} \frac{Y_{i,t}(i)}{K^u_{i,t}(i)} \\
(\partial H^d_{i,t}(i)) &: \frac{W_{i,t}}{P^c_{i,t}} = (1-\alpha) \frac{\lambda_{i,t}}{P^c_{i,t}} \frac{Y_{i,t}(i)}{H^d_{i,t}(i)}
\end{aligned}$$

The first order condition leads to marginal cost expression,

$$\frac{MC_{i,t}(i)}{P^c_{i,t}} = \frac{MC_{i,t}}{P^c_{i,t}} = \frac{1}{e^{\varepsilon^A_{i,t}}} \left(\frac{Z_{i,t}}{P^c_{i,t}} \right)^{\alpha} \left(\frac{W_{i,t}}{P^c_{i,t}} \right)^{(1-\alpha)} (1-\alpha)^{-(1-\alpha)} \alpha^{\alpha} \tag{11}$$

inputs must also satisfy,

$$\alpha \frac{W_{i,t}}{P^c_{i,t}} H^d_{i,t}(i) = (1-\alpha) \frac{Z_{i,t}}{P^c_{i,t}} K^u_{i,t}(i) \tag{12}$$

In the second-stage, firm i set prices according to a Calvo mechanism, each period firm i is not allowed to reoptimize its price with probability θ^p_i but price increases of $\xi^p_i \in [0;1]$ at last period's rate of price inflation, $P_{i,t}(i) = \pi^{\xi^p_i}_{i,t-1} P_{i,t-1}(i)$. The final firm allowed to modify its selling price with a probability $1-\theta^p_i$ chooses $\left\{P^*_{i,t}(i)\right\}$ to maximize its expected sum of profits,

$$\max_{\left\{P^*_{i,t}(i)\right\}} \mathbb{E}_t \left\{ \sum\nolimits_{\tau=0}^{\infty} (\theta^p_i \beta)^{\tau} \frac{\lambda^c_{i,t+\tau}}{\lambda^c_{i,t}} \left[(1-\tau^y) P^*_{i,t}(i) \prod_{k=1}^{\tau} \pi^{\xi^p_i}_{i,t+k-1} - MC_{i,t+k} \right] Y_{i,t+\tau}(i) \right\},$$

under the demand constraint from final goods producers,

$$Y_{i,t+\tau}(i) = \left(\frac{P^*_{i,t}(i)}{P_{i,t+\tau}} \prod\nolimits_{k=1}^{\tau} \pi^{\xi^p_i}_{i,t+k-1} \right)^{-\epsilon_p} Y^d_{i,t+\tau}, \forall\ \tau > 0$$

where $Y^d_{i,t}$ represents the quantity of the goods demanded in country i, τ^y is a proportional tax income on final goods producers' profits which removes the steady state price distortion caused by monopolistic competition, $\lambda^c_{i,t}$ is the household marginal utility of consumption and $\beta^{\tau} \frac{\lambda^c_{i,t+\tau}}{\lambda^c_{i,t}}$ is the household stochastic discount factor. The first order condition that defines the price of the representative firm i is,

$$P^*_{i,t}(i) = \frac{\epsilon_p}{(\epsilon_p - 1)(1-\tau^y)} \frac{\mathbb{E}_t \left\{ \sum\limits_{\tau=0}^{\infty} (\theta^p_i \beta)^{\tau} \frac{\lambda^c_{i,t+\tau}}{\lambda^c_{i,t}} MC_{i,t+k} Y_{i,t+\tau}(i) \right\}}{\mathbb{E}_t \left\{ \sum\limits_{\tau=0}^{\infty} (\theta^p_i \beta)^{\tau} \frac{\lambda^c_{i,t+\tau}}{\lambda^c_{i,t}} \prod\limits_{k=1}^{\tau} \pi^{\xi^p_i}_{i,t+k-1} Y_{i,t+\tau}(i) \right\}}. \tag{13}$$

1.4 Entrepreneurs

Each intermediate firm hires labour freely, but requires funds to finance the renting of capital needed to produce the intermediate good. The amount of capital to be financed by the representative entrepreneur is equal to $Q_{i,t}K_{i,t+1}(e)$, where $Q_{i,t}$ is the price of capital. This quantity is financed by two means: the net wealth of entrepreneur e, $N_{i,t}(e)$, and the amount that is borrowed by the entrepreneur from the banking system, $L^{d}_{i,t+1}(e)$.

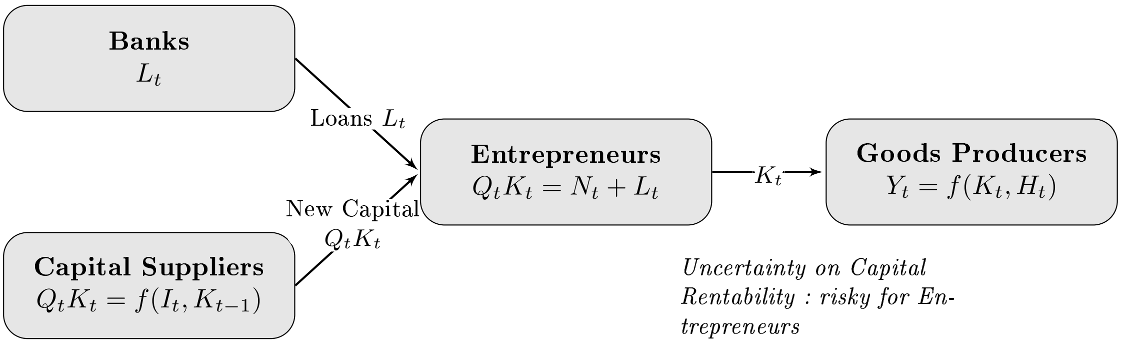

Figure 2: Implementing the financial accelerator in a real business cycles model

1.4.1 The Balance Sheet

The entrepreneur balance sheet writes,

$$Q_{i,t}K_{i,t+1}(e) = L^{\mathcal{H}}_{i,t+1}(e) + P^{c}_{i,t}N_{i,t+1}(e) \tag{14}$$

where $L^{\mathcal{H}}_{i,t+1}(e)$ stands for lending demand with exogenous habits h^{k}_{i} to fit the data implying that,

$$L^{\mathcal{H}}_{i,t+1}(e) = L^{d}_{i,t+1}(e) - h^{k}_{i}\left(L^{d}_{i,t} - \bar{L}^{d}_{i}\right)$$

Concerning $L^{\mathcal{H}}_{i,t+1}(e)$, when $h^{k}_{i} = 0$ then $L^{\mathcal{H}}_{i,t+1}(e) = L^{d}_{i,t+1}(e)$, thus in steady state $L^{\mathcal{H}}_{i}(e) = L^{d}_{i}(e)$. These lending demand habits are deemed necessary to replicate the dynamic of loans. In the estimation, we use the outstanding amount of loans, they are of different maturities implying a strong autocorrelation. Simply by introducing loan demand habits, taking into account the autocorrelation of loans becomes tractable easily and does not change the steady state of the model.

1.4.2 The Distribution of Risky Investment Projects

We assume that each entrepreneur $e \in [0,1]$ conducts a mass $\omega \in [\omega_{\min}, +\infty)$ of heterogeneous investment projects, they are risky so that some of the projects will have negative profits. To model individual riskiness, we assume that the aggregate return of investment projects $R^{k}_{i,t}$ is multiplied by a random value ω, so that the net return of its individual project is, $\omega R^{k}_{i,t}$. Since he must repay to the bank $L^{\mathcal{H}}_{i,t+1}(e)$ given a borrowing rate $P^{L}_{i,t}(e)$, the net profit of the project ω is $\omega R^{k}_{i,t}Q_{i,t-1}K_{i,t}(e,\omega) - P^{L}_{i,t-1}(e)L^{\mathcal{H}}_{i,t}(e,\omega)$. To separate profitable investment project from non-profitable ones, there exists a critical value (a cutoff point) defined as $\omega^{C}_{i,t}(e)$ such that the project just breaks even. Thereby the threshold is computed by,

$$\omega^{C}_{i,t}(e)R^{k}_{i,t}Q_{i,t-1}K_{i,t}\left(e,\omega^{C}_{i,t}\right) = P^{L}_{i,t-1}(e)L^{\mathcal{H}}_{i,t}\left(e,\omega^{C}_{i,t}\right) \tag{15}$$

We assume that the level of the individual profitability affects the survival of the entrepreneur:

- for a high realization of the ω, namely $\omega \geq \omega_{i,t}^{C}(e)$, the entrepreneur's ω^{th} project is profitable;
- for a low realization of ω, namely $\omega < \omega_{i,t}^{C}$, the entrepreneur's ω^{th} project is not gainful, and he does not make any repayment to the banking system.

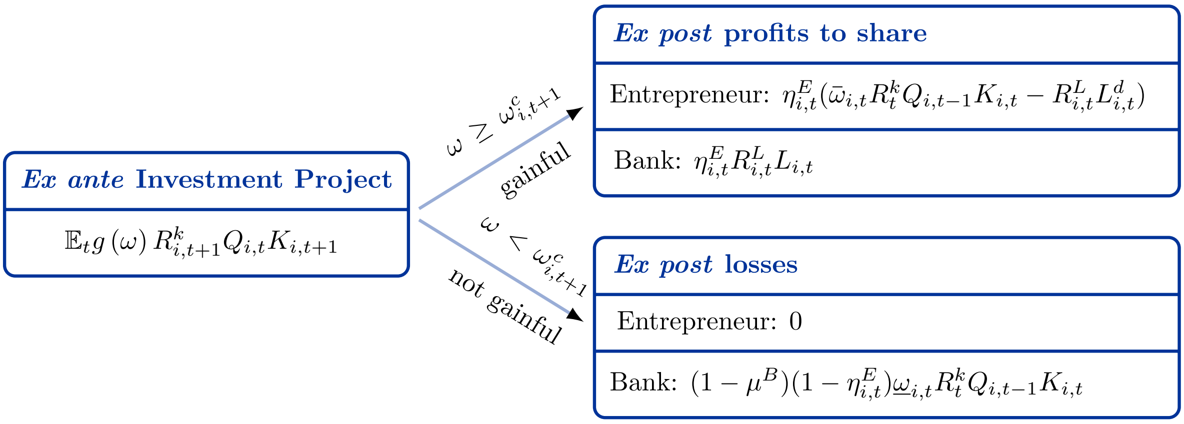

Figure 3: Profit sharing between the entrepreneur and the bank

By assuming that entrepreneurs projects are drawn from a Pareto distribution, then $\omega \sim \mathcal{P}(\kappa, \omega_{\min})$ where $\omega \in [\omega_{\min}; +\infty[$, κ is the shape parameter and $\omega_{\min}$ is the minimum bound of ω. Aggregating the financial contracts, we define the conditional expectation of ω when entrepreneur's project is gainful by, $\eta^\pi \bar{\omega} = \int_{\omega^C}^{\infty} \omega f(\omega)\, d\omega$, while the conditional expectation of ω when entrepreneur's project is insolvent by, $(1-\eta^\pi)\underline{\omega} = \int_{\omega_{\min}}^{\omega^C} \omega f(\omega)\, d\omega$. The share of profitable projects is computed as, $\eta^\pi = \Pr\left[\omega \geq \omega^C\right] = \int_{\omega^C}^{\infty} f(\omega)\, d\omega = \left(\omega_{\min}/\omega^C\right)^k$. The conditional expectations is computed via, $\bar{\omega} = E\left[\omega | \omega \geq \omega^C\right] = \int_{\omega^C}^{\infty} \omega f(\omega)\, d\omega / \int_{\omega^C}^{\infty} f(\omega)\, d\omega = \frac{\kappa}{\kappa-1}\omega^C$. Since $E[\omega] = \eta^\pi E\left[\omega|\omega \geq \omega^C\right] + (1-\eta^\pi) E\left[\omega | \omega < \omega^C\right] = 1$, then $\underline{\omega} = (1-\eta^\pi \bar{\omega}) / (1-\eta^\pi)$.

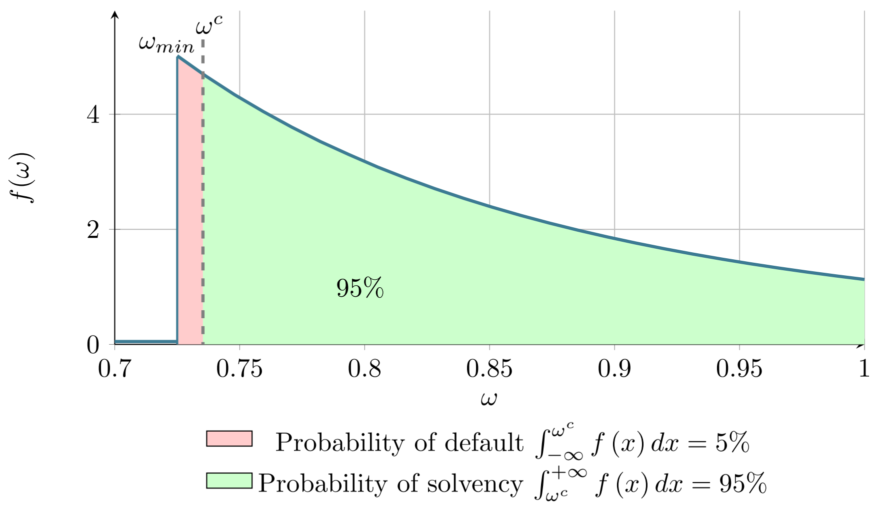

Figure 4: Pareto distribution of heterogenous entrepreneurs' projects $\omega \in [\omega_{\min}, +\infty[$

The aggregation of financial projects ω, $\int_{\bar{\omega}_{i,t+1}}^{+\infty} \omega R_{i,t+1}^{k} Q_{i,t} K_{i,t+1}(e,\omega) \mathrm{d}F(\omega)$, leads to the following expression of the expected profitability,

$$\Pi_{i,t+1}^{E}(e) = \mathbb{E}_t \begin{cases} \bar{\omega}_{i,t+1} R_{i,t+1}^{k} Q_{i,t} K_{i,t+1}(e) - P_{i,t}^{L}(e) L_{i,t+1}^{\mathcal{H}}(e) \text{ with probability } \eta_{i,t+1}^{\pi} \\ 0 \text{ with probability } 1 - \eta_{i,t+1}^{\pi} \end{cases} \tag{16}$$

1.4.3 Profit Maximization and the Financial Accelerator

Following De Grauwe (2010), we assume that because agents cannot screen the value of $\bar{\omega}_{i,t+1}(e)$ *ex ante*, their forecasts are biased. The representative entrepreneur is pessimistic and systematically bias the expected value of its profitable projects $\bar{\omega}_{i,t}(e)$ downwards. In the previous expression (16), we assume that the perceived *ex ante* value of profitable projects is defined by the CES function,

$$g\left(\bar{\omega}_{i,t+1}, \varepsilon_{i,t}^{Q}\right) = \gamma_i \left(\bar{\omega}_{i,t+1}\right)^{\frac{\varkappa_i}{(\varkappa_i - 1)}} \left(e^{\varepsilon_{i,t}^{Q}}\right)^{\frac{1}{(\varkappa_i - 1)}}$$

where $\varepsilon_{i,t}^{Q}$ represents an exogenous uncertainty shock following an $AR(1)$ process[1], $\varkappa_i$ denotes the elasticity of the external finance premium and γ_i is a scale parameter in order to have a steady state independent of $\varkappa_i$ such that $\gamma_i = \bar{\omega}^{1/(1-\varkappa_i)}$. In this expression, the exogenous shock is affected by exponent $1/(\varkappa_i - 1)$ to normalize to unity the impact of the financial shock $\varepsilon_{i,t}^{Q}$ in the log deviation form of the model.

In our setting, the elasticity of the external finance premium expresses the degree of bias in estimating the expected rentability of entrepreneurs' projects such that if $\bar{\omega} > 1$ and $\varkappa_i > 0$ then $\bar{\omega} > g(\bar{\omega})$. Expressed *à la* De Grauwe (2010) such that $\mathbb{E}_t^{pes}$ is the pessimistic expectation operator, $\mathbb{E}_t^{pes} \bar{\omega}_{i,t+1} = \mathbb{E}_t \gamma_i \left(\bar{\omega}_{i,t+1}\right)^{\varkappa_i/(\varkappa_i - 1)}$. Assuming that the entrepreneur is pessimistic, he maximizes $\mathbb{E}_t^{pes} \Pi_{i,t+1}^{E}(e)$ before the realization of ω,

$$\max_{\{K_{i,t+1}(e)\}} \mathbb{E}_t \left\{ \eta_{i,t+1}^{\pi} \left[g\left(\bar{\omega}_{i,t+1}, \varepsilon_{i,t}^{Q}\right) R_{i,t+1}^{k} Q_{i,t} K_{i,t+1}(e) - P_{i,t}^{L}(e) L_{i,t+1}^{\mathcal{H}}(e) \right] \right\} \tag{17}$$

The first order solution of the entrepreneur optimizing program defines the expected external finance premium ($\mathbb{E}_t \left\{ R_{i,t+1}^{k} / P_{i,t}^{L} \right\}$) as,

$$\frac{\mathbb{E}_t R_{i,t+1}^{k}}{P_{i,t}^{L}(e)} = \frac{1}{\gamma_i \left[E_t \bar{\omega}_{i,t+1}(e) \left[e^{\varepsilon_{i,t}^{Q}} \right]^{1/\varkappa_i} \right]^{\frac{\varkappa_i}{(\varkappa_i - 1)}}}. \tag{18}$$

This premium captures the extra remuneration needed by the entrepreneur to undertake the decision to finance the investment of the intermediate firm. The interest rate spread and the accelerator phenomenon disappear if $\varkappa_i = 0$. We assume that the entrepreneur cannot make an arbitrage with a riskless asset. Thus the net wealth of the entrepreneur in the next period is equal to,

$$N_{i,t+1}(e) = \left(1 - \tau^{E}\right) \frac{\Pi_{i,t}^{E}(e)}{e^{\varepsilon_{i,t}^{N}}} \tag{19}$$

[1] This shock affects the expected profitability of financial projects by rising in exogeneously the risk premium implying an increase in the cost of capital and hence a reduction in investment as underlined by Gilchrist et al. (2009) for the US economy.

where $\varepsilon_{i,t}^{N}$ is an exogenous process of net wealth destruction and τ^{E} is a proportional tax.

Recall that $\bar{\omega}_{i,t+1} = \frac{\kappa}{\kappa-1}\omega_{i,t+1}^{c}$, replacing $\bar{\omega}_{i,t+1}$ and $\omega_{i,t+1}^{c}$ using equation (15) from equation (18), we obtain the financial accelerator equation,

$$S_{i,t}(e) = \frac{\mathbb{E}_t R_{i,t+1}^{k}}{P_{i,t}^{L}(e)} = \gamma_i^{\varkappa_i - 1}\left[\frac{\kappa}{\kappa-1}\left(1 - \frac{P_{i,t}^{c} N_{i,t+1}(e)}{Q_{i,t} K_{i,t+1}(e)}\right)\right]^{\varkappa_i} e^{\varepsilon_{i,t}^{Q}} \tag{20}$$

$S_{i,t}(e)$ is the external finance premium, further details are in appendix A about the relationship between the external finance premium and the leverage ratio.

1.5 Capital Goods Producers

The capital supplier is an alternative decentralization scheme in which a new type of firms, capital producers, make the capital supply and utilization decisions.

1.5.1 Capital Supply Decisions

The suppliers of capitals lend capital to the intermediate firms, once it is financed by the entrepreneurs. Capital suppliers are homogeneous and distributed over a continuum normalized to one. The representative capital supplier $k \in [0;1]$ acts competitively to supply a quantity of capital $K_{i,t+1}(k)$ to intermediate firms and invest a quantity of final goods $I_{i,t}(k)$ to keep it productive. We assume that it is is costly to invest, *i.e.* it has to pay an adjustment cost on investment,

$$AC_{i,t}^{I}(k) = \chi_i^{I}\left(\frac{I_{i,t}(k)}{I_{i,t-1}(k)} - 1\right)^2 \tag{21}$$

where χ_i^{I} is the adjustment cost. Thus the capital stock of the representative capital supplier evolves according to,

$$K_{i,t+1}(k) = \left(1 - AC_{i,t}^{I}(k)\right) I_{i,t}(k) + (1-\delta) K_{i,t}(k) \tag{22}$$

The capital producer produces the new capital stock $Q_{i,t}K_{i,t+1}(k)$ by buying the deprecated capital and investment goods. The project of the representative supplier thus writes,

$$\Pi_{i,t}^{k}(k) = Q_{i,t}K_{i,t+1}(k) - (1-\delta) Q_{i,t}K_{i,t}(k) - P_{i,t}^{I} I_{i,t}(k), \tag{23}$$

Replacing (22) in (23), the representative capital supplier chooses $I_{i,t}(k)$ to maximize profits,

$$\max_{\{I_{i,t}(k)\}} \mathbb{E}_t \left\{\sum\nolimits_{\tau=0}^{\infty} \beta^{\tau} \frac{\lambda_{i,t+\tau}^{c}}{\lambda_{i,t}^{c}} \Pi_{t+\tau}^{k}(k)\right\},$$

the Lagrangian problem is,where $\beta^{\tau}\frac{\lambda_{i,t+\tau}^{c}}{\lambda_{i,t}^{c}}$ is the household subjective discount factor.

$$\mathcal{L}_t = \mathbb{E}_t \sum\nolimits_{\tau=0}^{\infty} \beta^{\tau} \frac{\lambda_{i,t+\tau}^{c}}{\lambda_{i,t}^{c}} \left[Q_{i,t}\left(1 - AC_{i,t}^{I}(k)\right) I_{i,t}(k) - P_{i,t}^{I} I_{i,t}(k)\right]$$

The price of capital renting thus solves,

$$Q_{i,t} = P_{i,t}^{I} + Q_{i,t}\frac{\partial\left(I_{i,t}(k)\, AC_{i,t}^{I}(k)\right)}{\partial I_{i,t}(k)} + \beta\mathbb{E}_t \frac{\lambda_{i,t+\tau}^{c}}{\lambda_{i,t}^{c}} Q_{i,t+1}\frac{\partial\left(I_{i,t+1}(k)\, AC_{i,t+1}^{I}(k)\right)}{\partial I_{i,t}(k)}. \tag{24}$$

Ignoring investment adjustment costs in this last expression (i.e. imposing $\chi^I = 0$), we simply get, $Q_{i,t} = P^I_{i,t}$. $Q_{i,t}$ stands for the asset price given the adjustment costs on investment production function. The derivatives of $AC^I_{i,t}$ in $I_{i,t}$ are,

$$\begin{aligned}\frac{\partial \left[I_{i,t}AC^I_{i,t}\right]}{\partial I_{i,t}} &= \frac{\mathcal{X}^I_i}{2}\left(3\left(\frac{I_{i,t}}{I_{i,t-1}}\right)^2+1-4\frac{I_{i,t}}{I_{i,t-1}}\right)\\ \frac{\partial \left[I_{i,t+1}AC^I_{i,t+1}\right]}{\partial I_{i,t}} &= \mathcal{X}^I_i\left(\left(\frac{I_{i,t+1}}{I_{i,t}}\right)^2-\left(\frac{I_{i,t+1}}{I_{i,t+1}}\right)^3\right)\end{aligned}$$

the first order condition taken in real terms writes,

$$\begin{aligned}\frac{Q_{i,t}}{P^c_{i,t}} &= \frac{P^I_{i,t}}{P^c_{i,t}}+\frac{Q_{i,t}}{P^c_{i,t}}\frac{\mathcal{X}^I_i}{2}\left(3\left(\frac{I_{i,t}(k)}{I_{i,t-1}(k)}\right)^2+1-4\frac{I_{i,t}(k)}{I_{i,t-1}(k)}\right)\\ &+\beta\mathbb{E}_t\frac{\lambda^c_{i,t+1}}{\lambda^c_{i,t}}\frac{Q_{i,t+1}}{P^c_{i,t+1}}\frac{P^c_{i,t+1}}{P^c_{i,t}}\mathcal{X}^I_i\left(\left(\frac{I_{i,t+1}(k)}{I_{i,t}(k)}\right)^2-\left(\frac{I_{i,t+1}(k)}{I_{i,t+1}(k)}\right)^3\right)\end{aligned}$$

1.5.2 Capital Utilization Decisions

As in Smets and Wouters (2003, 2007), we assume that capital requires one period to be settled so that,

$$K^u_{i,t+1} = u_{i,t}K_{i,t} \tag{25}$$

given a level of capital utilization of capital $u_{i,t}$. The total amount of capital services in the production function is $u_{i,t}K_{i,t}$, total revenues from renting capital are $Z_{i,t}u_{i,t}K_{i,t}$. The benefit of increased utilization must be weighted against utilization costs, expressed by $\Phi(u_{i,t})K_{i,t}$. Following Christiano et al. (2001), the capital utilization is defined by

$$\Phi(u_{i,t}) \simeq \phi_{i,1}(u_{i,t}-1)+\frac{\phi_{i,2}}{2}(u_{i,t}-1)^2$$

and we estimate the ratio of $\phi_{i,2}/\phi_{i,1}$. This relationship is determined by,

$$\max_{u_{i,t}} Z_{i,t}u_{i,t}K_{i,t}-\Phi(u_{i,t})K_{i,t}$$

The first order condition writes,

$$Z_{i,t} = \Phi'(u_{i,t})$$

taken in logs,

$$\hat{z}_{i,t} \simeq \frac{\psi_i}{1-\psi_i}\hat{u}_{i,t} \tag{26}$$

where $\psi_i \in [0;1]$ is the elasticity of utilization costs with respect to capital inputs.

1.5.3 The Rentability of one Unit of Capital

Equation (27) takes into account the assumption that households make the capital accumulation and utilization decisions. The capital supplier is actually an alternative decentralization scheme in which firms make the capital supply decisions. To get the equation of the *ex post* rentability of one unit of capital, we must suppose that the household supplies capital by investing $P^I_{i,t}I_{i,t}(j)$, paying utilization costs $P_{i,t}\Phi(u_{i,t})K_{i,t}(j)$ and earning revenues from renting capital $Z_{i,t}u_{i,t}K_{i,t}(j)$. Maximizing utility under capital accumulation constraint (22), the FOCs write,

$$\begin{aligned}(\partial B_{i,t+1}(j)) &: \mathbb{E}_t\left\{\frac{e^{\varepsilon^{\beta}_{i,t}}\lambda^c_{i,t}}{e^{\varepsilon^{\beta}_{i,t+1}}\lambda^c_{i,t+1}}\frac{P^c_{i,t+1}}{P^c_{i,t}}\right\} = \frac{\beta R_{t+1}}{1+\mathcal{X}^B\left(B_{i,t+1}(j)-B_i(j)\right)}\\ (\partial K_{i,t+1}(j)) &: \lambda^k_{i,t}+\beta\mathbb{E}_t\lambda^c_{i,t+1}\left[P_{i,t+1}\Phi\left(u_{i,t+1}\right)-Z_{i,t+1}u_{i,t+1}\right]-\beta(1-\delta)\mathbb{E}_t\lambda^k_{i,t+1}\end{aligned}$$

where $\lambda^c_{i,t}$ ($\lambda^k_{i,t}$) is the Lagrangian multiplier on the budget constraint (capital accumulation constraint). Tobin's Q is defined by $Q_t = \lambda^k_{i,t}/\lambda^c_{i,t}$, then replacing in the FOC $(\partial K_{i,t+1}(j))$,

$$Q_{i,t} = \beta\frac{\lambda^c_{i,t+1}}{\lambda^c_{i,t}}\left[Z_{i,t+1}u_{i,t+1}-P_{i,t+1}\Phi\left(u_{i,t+1}\right)+Q_{i,t+1}(1-\delta)\right]$$

Combining the previous equation with FOC $(\partial B_{i,t+1}(j))$ leads to the following expression,

$$\frac{R_{t+1}}{1+\mathcal{X}^B\left(B_{i,t+1}(j)-B_i(j)\right)}\frac{e^{\varepsilon^{\beta}_{i,t+1}}}{e^{\varepsilon^{\beta}_{i,t}}} = \frac{\left[Z_{i,t+1}u_{i,t+1}-P_{i,t+1}\Phi\left(u_{i,t+1}\right)+Q_{i,t+1}(1-\delta)\right]}{Q_{i,t}}$$

Following Bernanke et al. (1999) we lag the previous equation and replace R_{t+1} by $R^k_{i,t+1}$ to get the *ex post return* of capital. Since capital supply decisions are decentralized, we suppose that they are independent of preference shocks. Finally, the return of holding one unit of capital from $t-1$ to t is determined by,

$$\frac{\mathbb{E}_{t-1}R^k_{i,t}}{\left(1+\mathcal{X}^B\left(B_{i,t}(j)-B_i(j)\right)\right)} = \frac{Z_{i,t}u_{i,t}-P_{i,t}\Phi\left(u_{i,t}\right)+(1-\delta)Q_{i,t}}{Q_{i,t-1}} \tag{27}$$

1.6 The Banking Sector

1.6.1 The Retail Branch

There is one type of retail loan produced using wholesale loans with the following production function,

$$L^d_{i,t+1} = \left(\int_0^1 L^s_{i,t+1}(b)^{\frac{1}{\mu^b_{i,t}}}\,\mathrm{d}b\right)^{\mu^b_{i,t}} \tag{28}$$

where $\mu^b_{i,t} = \mu^b+\varepsilon^b_{i,t} = \frac{\varepsilon_b}{\varepsilon_b-1}+\varepsilon^b_{i,t}$ is the time-varying markup of banks, and $L^s_{i,t+1}(b)$ denotes the quantity of domestic wholesale loan of type b.

$$\max_{L^s_{i,t+1}(b)} R^L_{i,t}L^d_{i,t}-\int_0^1 R^L_{i,t}(b)\,L^s_{i,t+1}(b)\,\mathrm{d}b$$

maximizing the previous program and equation (28),

$$L^s_{i,t+1}(b) = \left(\frac{R^L_{i,t}(b)}{R^L_{i,t}}\right)^{-\frac{\mu^b_{i,t}}{\mu^b_{i,t}-1}}L^d_{i,t+1} \tag{29}$$

where $R^L_{i,t}(b)$ is the interest rate of the wholesale good b and $R^L_{i,t}$ the retail interest rate. The aggregation of retail interest rates on loans writes,

$$R^L_{i,t} = \left(\int_0^1 R^L_{i,t}(b)^{\frac{1}{1-\mu^b_{i,t}}}\,\mathrm{d}b\right)^{1-\mu^b_{i,t}}$$

1.6.2 The Wholesale Branch

The total number of homogenous banks is normalized to one. The representative wholesale bank $b \in [0;1]$ in country i operates under monopolistic competition to provide a quantity of loans $L^s_{i,t+1}(b)$ that is financed by loans from the central bank (with a one period maturity) at the interbank interest rate R_t. The representative bank sets the rate of interest that has to be charged to the entrepreneur loan. We assume that banks ignore the individual *ex ante* viability of borrowers. However we assume that banks know the distribution of individual projects in terms of ω so that they can compute the expected value of earnings of the next period, depending on the state of nature of its customers. Thus, the expected profit is defined as,

$$\Pi^b_{i,t+1}(b) = \mathbb{E}_t \begin{cases} R^L_{i,t}(b) L^s_{i,t+1}(b) - R_t L^s_{i,t+1}(b) & \text{with probability } \bar{\eta}^{\pi}_{i,t+1} \\ -R_t L^s_{i,t+1}(b) & \text{with probability } 1-\bar{\eta}^{\pi}_{i,t+1} \end{cases} \tag{30}$$

In this setting we assume that there is no discrimination between borrowers, so that the representative and risk-neutral bank serves both domestic and foreign entrepreneur without taking into account specificities regarding the national viability of projects. Bank default expectation regarding entrepreneurs' projects is defined as, $\bar{\eta}^{\pi}_{i,t+1} = \left(1-\alpha^L_i\right)\eta^{\pi}_{h,t+1} + \alpha^L_i \eta^{\pi}_{f,t+1}$. The marginal cost of one unit of loan $MC^b_{i,t}(b)$ is the solution of the problem,

$$\max_{L^s_{i,t+1}(b)} \bar{\eta}^{\pi}_{i,t+1} MC^b_{i,t}(b) L^s_{i,t+1}(b) - R_t L^s_{i,t+1}(b)$$

The marginal cost of one unit of loan is the same for all wholesale branches and equal to,

$$MC^b_{i,t}(b) = MC^b_{i,t} = \frac{R_t}{E_t \bar{\eta}^{\pi}_{i,t+1}} \tag{31}$$

so that each bank decides the size of the spread depending on the expected failure rate of its customers.

Under Calvo pricing with partial indexation, banks sets the interest rate on loans contracted by entrepreneurs on a staggered basis as in Pariès et al. (2011). A fraction θ^b_i of banks is not allowed to optimally set the credit rate and index it by ξ^b_i percent of the past credit rate growth, $R^L_{i,t}(b) = \left(R^L_{i,t-1}/R^L_{i,t-2}\right)^{\xi^b_i} R^L_{i,t-1}(b)$. Assuming that it is able to modify its loan interest rate with a constant probability $1-\theta^b_i$, it chooses $R^{L*}_{i,t}(b)$ to maximize its expected sum of profits,

$$\max_{\left\{R^{L*}_{i,t}(b)\right\}} \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \left(\theta^b_i \beta\right)^{\tau} \frac{\lambda^c_{i,t+\tau}}{\lambda^c_{i,t}} \left[\left(1-\tau^b\right) R^{L*}_{i,t}(b)\, \Xi^b_{i,t,\tau} - MC^b_{i,t+\tau}\right] L^s_{i,t+1+\tau}(b) \right\}, \tag{32}$$

subject to the demand constraint,

$$L^s_{i,t+1+\tau}(b) = \left(\frac{R^{L*}_{i,t}(b)}{R^L_{i,t+\tau}} \Xi^b_{i,t,\tau} \right)^{-\mu^b_{i,t}/\left(\mu^b_{i,t}-1\right)} L^s_{i,t+1+\tau}, \qquad \tau > 0, \tag{33}$$

where $L^s_{i,t+1}$ represents the quantity of the loans produced in country i, $\lambda^c_{i,t}$ is the household marginal utility of consumption and the sum of previous prices related to the indexation is,

$$\Xi^b_{i,t,\tau} = \begin{cases} \prod_{j=1}^{\tau} \left(\frac{R^L_{i,t-1+j}}{R^L_{i,t-2}}\right)^{\xi^b_i}, & j > 0 \\ 1, & j = 0 \end{cases}$$

where $\xi_i^b \in [0;1]$ stands for the level of indexation, this indexation catches some imperfect interest rate pass-though.

Finally, the first order condition writes,

$$\sum_{\tau=0}^{\infty} \left(\theta_i^b \beta\right)^{\tau} \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} \frac{1}{\left(\mu_{i,t}^b - 1\right)} \left[R_{i,t}^{L*}(b)\, \Xi_{i,t,\tau}^b - \frac{\mu_{i,t}^b}{\left(1-\tau^b\right)} MC_{i,t+\tau}^b\right] L_{i,t+1+\tau}(b) = 0 \quad (34)$$

1.7 Authorities

National governments finance public spending by charging a proportional taxes on the revenues of final producers τ^y, unions τ^w, retail banks τ^b, entrepreneurs and by receiving a total value of taxes $\int_0^1 T_{i,t}(j)\, dj$ from households. The budget constraint of the national government writes,

$$\int_0^1 T_{i,t}(j)\, \mathrm{d}j + \mathcal{T}_{i,t} = P_{i,t} G_{i,t} = P_{i,t} \left(\int_0^1 G_{i,t}(i)^{\frac{\epsilon_p - 1}{\epsilon_p}}\, \mathrm{d}i\right)^{\frac{\epsilon_p}{\epsilon_p - 1}} = P_{i,t} G \varepsilon_{i,t}^G$$

$G_{i,t}$ is the total amount of public spending in the i^{th} economy that follows and AR(1) shock process. Thus $\mathcal{T}_{i,t}$ is the amount of proportional tax writes,

$$\begin{aligned}\mathcal{T}_{i,t} &= \tau^y \int_0^1 P_{i,t}(i)\, Y_{i,t}(i)\, \mathrm{d}i + \tau^w \int_0^1 W_{i,t}(j)\, H_{i,t}(j)\, \mathrm{d}j + \tau^b \int_0^1 L_{i,t+1}^s(b)\, R_{i,t}^L(b)\, \mathrm{d}b \\ &\quad + \tau^E \int_0^1 P_{i,t} N_{i,t}^E(e)\, \mathrm{d}e\end{aligned}$$

where $\tau^y = (1-\epsilon_p)^{-1}$, $\tau^w = (1-\epsilon_p)^{-1}$ and $\tau^b = 1 - \mu^b$ are taxes that mitigate the negative effects of monopolistic competition in steady states. The government demand for home goods writes, $G_{i,t}(i) = \left(\frac{P_{i,t}(i)}{P_{i,t}}\right)^{-\epsilon_p} G_{i,t}$.

Concerning federal monetary policy, the general expression of the interest rule implemented by the monetary union central bank writes,

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho^R} \left[\left(\pi_{h,t}^c \pi_{f,t}^c\right)^{\phi^\pi} \left(\frac{Y_{h,t} Y_{f,t}}{Y_{h,t-1} Y_{f,t-1}}\right)^{\phi^{\Delta y}}\right]^{\frac{1}{2}(1-\rho^r)} e^{\varepsilon_t^R} \quad (35)$$

where ε_t^R is a monetary policy shock common to the monetary union members, ϕ^π is the inflation target parameter, $\phi^{\Delta y}$ is the GDP growth target.

2 Aggregation and Market Equilibrium

The model is solved under the hypothesis that, $\alpha_h^s = \alpha^s \Leftrightarrow \alpha_f^s = (1-\alpha^s)$, $\forall s = C, I, L$, *i.e.* market openness is the same across countries and countries are of equal size. For each quantity denoted X_t, we aggregate households $X_t = \int_0^1 X_t(j)\mathrm{d}j$, firms $X_t = \int_0^1 X_t(i)\mathrm{d}i$, entrepreneurs $X_t = \int_0^1 X_t(e)\mathrm{d}e$ and banks $X_t = \int_0^1 X_t(b)\mathrm{d}b$.

2.1 Goods Market

From equation (9), the aggregate price index of the national goods evolves according to,

$$P_{i,t}^{1-\epsilon_p} = \theta_i^p \left[P_{i,t-1} \left(\frac{P_{i,t-1}}{P_{i,t-2}} \right)^{\xi_i^p} \right]^{1-\epsilon_p} + (1-\theta_i^p) \left(P_{i,t}^* \right)^{1-\epsilon_p} \tag{36}$$

The equilibrium condition on the final goods market writes is defined by the aggregation of equation (8),

$$\int_0^1 Y_{i,t}(i)\,\mathrm{d}i = Y_{i,t}^d \int_0^1 \left(\frac{P_{i,t}(i)}{P_{i,t}} \right)^{-\epsilon_p} \mathrm{d}i$$

where $\int_0^1 Y_{i,t}(i)\mathrm{d}i = e^{\varepsilon_{i,t}^A} \int_0^1 K_{i,t}^u(i)^\alpha H_{i,t}^d(i)^{1-\alpha}\mathrm{d}i$ is the aggregation of intermediate goods suppliers and $Y_{i,t}^d$ is the resources constraint,

$$Y_{i,t}^d = \begin{array}{l} C_{h,i,t} + C_{f,i,t} + \left(1 + AC_{h,i,t}^I\right) I_{h,i,t} + \left(1 + AC_{f,i,t}^I\right) I_{f,i,t} \\ +G_{i,t} + AC_{i,t}^B + \left(1 - \eta_{i,t}^\pi\right) \underline{\omega}_{i,t} Q_{i,t-1} K_{i,t} + \Phi(u_{i,t}) K_{i,t} \end{array}$$

Thus, replacing the demand functions of foreign and home goods (consumption and investment) we finally obtain the home final goods market equilibrium,

$$\begin{aligned} \frac{Y_{h,t}}{\Delta_{h,t}^p} &= \left(1-\alpha^C\right) \left(\frac{P_{h,t}}{P_{h,t}^c} \right)^{-\mu} C_{h,t} + \alpha^C \left(\frac{P_{h,t}}{P_{f,t}^c} \right)^{-\mu} C_{f,t} \\ &+ \left(1-\alpha^I\right) \left(\frac{P_{h,t}}{P_{h,t}^I} \right)^{-\mu} \left(1 + AC_{h,t}^I\right) I_{h,t} + \alpha^I \left(\frac{P_{h,t}}{P_{f,t}^I} \right)^{-\mu} \left(1 + AC_{h,t}^I\right) I_{h,t} \\ &+ G_{h,t} + AC_{h,t}^B + \left(1 - \eta_{h,t}^s\right) \underline{\omega}_{h,t} Q_{h,t-1} K_{h,t} + \Phi(u_{h,t}) K_{h,t-1} \end{aligned} \tag{37}$$

where $\Delta_{i,t}^p = \int_0^1 \left(\frac{P_{i,t}(i)}{P_{i,t}} \right)^{-\varepsilon_p} \mathrm{d}i$ denotes the price dispersion term, which is induced by the assumed nature of price stickiness, is inefficient and entails output loss. Since we perform a first approximation of the model, the price dispersion terms disappears. For the foreign economy,

$$\begin{aligned} \frac{Y_{f,t}}{\Delta_{f,t}^p} &= \alpha^C \left(\frac{P_{f,t}}{P_{h,t}^c} \right)^{-\mu} C_{h,t} + \left(1-\alpha^C\right) \left(\frac{P_{f,t}}{P_{f,t}^c} \right)^{-\mu} C_{f,t} \\ &+ \alpha^I \left(\frac{P_{f,t}}{P_{h,t}^I} \right)^{-\mu} \left(1 - AC_{h,t}^I\right) I_{h,t} + \left(1-\alpha^I\right) \left(\frac{P_{f,t}}{P_{f,t}^I} \right)^{-\mu} \left(1 + AC_{f,t}^I\right) I_{f,t} \\ &+ G_{f,t} + AC_{f,t}^B + \left(1 - \eta_{f,t}^s\right) \underline{\omega}_{f,t} Q_{f,t-1} K_{f,t} + \Phi(u_{f,t}) K_{f,t-1} \end{aligned} \tag{38}$$

To close the model, additional costs are entirely home biased, *i.e.* adjustment costs on bonds $AC_{i,t}^B = \left(\int_0^1 AC_{i,t}^B(i)^{(\epsilon_p-1)/\epsilon_p} \mathrm{d}i \right)^{\epsilon_p/(\epsilon_p-1)}$, insolvent investment projects of entrepreneurs and capital utilization costs from capital suppliers $K_{i,t} = \left(\int_0^1 K_{i,t}(i)^{(\epsilon_p-1)/\epsilon_p} \mathrm{d}i \right)^{\epsilon_p/(\epsilon_p-1)}$. The demands associated with the previous costs are, $AC_{i,t}^B(i) = \left(P_{i,t}(i)/P_{i,t}\right)^{-\epsilon_p} AC_{i,t}^B$, $K_{i,t}(i) = \left(P_{i,t}(i)/P_{i,t}\right)^{-\epsilon_p} K_{i,t}$.

2.2 Labour Market

The market equilibrium between labor demand from firms and supply from households (or unions) is defined by the aggregate demand curve (5),

$$H_{i,t} = \int_0^1 H_{i,t}(j)\,\mathrm{d}j = \Delta_{i,t}^w \int_0^1 H_{i,t}^d(i)\,\mathrm{d}i$$

where $\Delta_{i,t}^w = \int_0^1 \left(\frac{W_{i,t}(j)}{W_{i,t}}\right)^{-\epsilon_w} \mathrm{d}j$ is the wage dispersion term. Taking into account staggered price in the aggregate wage equation (7) leads to the following expression,

$$W_{i,t}^{1-\epsilon_w} = \theta_i^w \left[W_{i,t-1}(\pi_{i,t-1})^{\xi_i^w}\right]^{1-\epsilon_w} + (1-\theta_i^w)\left(W_{i,t}^*\right)^{1-\epsilon_w}$$

2.3 Loan Market

The equilibrium on loan market is defined by the aggregate demand function from loans packers (29),

$$\int_0^1 L_{i,t+1}^s(b)\,\mathrm{d}b = \Delta_{i,t}^b L_{i,t+1}^d$$

where $\Delta_{i,t}^b = \int_0^1 \left(\frac{R_{i,t}^L(b)}{R_{i,t}^L}\right)^{-\mu_{i,t}^b/\left(\mu_{i,t}^b-1\right)} \mathrm{d}b$ is the interest rate dispersion term and $L_{i,t+1}^d$ is the aggregate demand defined by,

$$L_{i,t+1}^d = \int_0^1 L_{h,i,t+1}^d(e)\,\mathrm{d}e + \int_0^1 L_{f,i,t+1}^d(e)\,\mathrm{d}e$$

Recalling that entrepreneurs e borrow to domestic and foreign banks with varieties b produced by wholesale branches, the equilibrium finally writes,

$$L_{i,t+1}^s = \left(\left(1-\alpha^L\right)\left[\frac{R_{i,t}^L}{P_{i,t}^L}\right]^{-\nu} L_{i,t+1}^d + \alpha^L \left[\frac{R_{i,t}^L}{P_{-i,t}^L}\right]^{-\nu} L_{-i,t+1}^d\right)\Delta_{i,t}^b \tag{39}$$

Aggregate loan rate index evolves according to,

$$\left(R_{i,t}^L\right)^{\frac{1}{1-\mu_{i,t}^b}} = \theta_i^b \left(R_{i,t-1}^L \left(\frac{R_{i,t-1}^L}{R_{i,t-2}^L}\right)^{\xi_i^b}\right)^{\frac{1}{1-\mu_{i,t}^b}} + \left(1-\theta_i^b\right)\left(R_{i,t}^L\right)^{\frac{1}{1-\mu_{i,t}^b}} \tag{40}$$

2.4 General Equilibrium

Given the realization of 13 shocks $\{S_t\}_{t=0}^{\infty}$,

$$\{S_t\}_{t=0}^{\infty} = \left\{\begin{array}{c} \eta_{h,t}^{\beta}, \eta_{h,t}^{A}, \eta_{h,t}^{G}, \eta_{h,t}^{I}, \eta_{h,t}^{Q}, \eta_{h,t}^{N}, \\ \eta_{f,t}^{\beta}, \eta_{f,t}^{A}, \eta_{f,t}^{G}, \eta_{f,t}^{I}, \eta_{f,t}^{Q}, \eta_{f,t}^{N}, \eta_{t}^{R} \end{array}\right\}_{t=0}^{\infty}$$

a competitive equilibrium is defined as a sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty}$,

$$\{\mathcal{Q}_t\}_{t=0}^{\infty} = \left\{\begin{array}{c} Y_{h,t}, Y_{f,t}, C_{h,t}, C_{f,t}, X_{h,t}, X_{f,t}, H_{h,t}, H_{f,t}, \\ K_{h,t}^u, K_{f,t}^u, K_{h,t}, K_{f,t}, u_{h,t}, u_{f,t}, \\ I_{h,t}, I_{f,t}, L_{h,t}^s, L_{f,t}^s, L_{h,t}^d, L_{f,t}^d, B_{h,t}, B_{f,t}, N_{h,t}, N_{f,t}, \end{array}\right\}_{t=0}^{\infty},$$

and a sequence of prices $\{\mathcal{P}_t\}_{t=0}^{\infty}$,

$$\{\mathcal{P}_t\}_{t=0}^{\infty} = \left\{ \begin{array}{c} P_{h,t}^x, P_{f,t,}^x P_{h,t}, P_{f,t,} P_{h,t}^c, P_{f,t,}^c, \pi_{h,t}, \pi_{f,t}, \pi_{h,t}^c, \pi_{f,t}^c, \\ W_{h,t}, W_{f,t,}, W_{h,t}^h, W_{f,t}^h, Z_{h,t}, Z_{f,t}, P_{h,t}^x, P_{f,t}^x \\ R_{h,t}^k, R_{f,t}^k, R_{h,t}^L, R_{f,t}^L, P_{h,t}^L, P_{f,t}^L, R_t, \omega_{h,t}^C, \omega_{f,t}^C \\ Q_{h,t}, Q_{f,t}, ToT_{h,t}, CA_{h,t} \end{array} \right\}_{t=0}^{\infty},$$

such that for a given sequence of prices $\{\mathcal{P}_t\}_{t=0}^{\infty}$, the realization of shocks $\{\mathcal{S}_t\}_{t=0}^{\infty}$, the sequence $\{\mathcal{Q}_t\}_{t=0}^{\infty}$ respects first order conditions for households and maximizes firms, bank... profits and for a given sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty}$, the realization of shocks $\{\mathcal{S}_t\}_{t=0}^{\infty}$, the sequence $\{\mathcal{P}_t\}_{t=0}^{\infty}$, guarantees markets equilibriums.

3 The Linear Model

3.1 Households

Euler equation (3) taken in logs:

$$\hat{c}_{h,t} = \frac{1}{(1+h_h^c)} E_t \hat{c}_{h,t+1} + \frac{h_h^c}{(1+h_h^c)} \hat{c}_{h,t-1} - \frac{(1-h_h^c)}{(1+h_h^c)\,\sigma_h^c} \left(\hat{r}_t - \mathbb{E}_t \hat{\pi}_{h,t+1}^c + \varepsilon_{h,t+1}^{\beta} - \varepsilon_{h,t}^{\beta} - C\mathcal{X}^B \hat{b}_{h,t} \right) \quad (41)$$

$$\hat{c}_{f,t} = \frac{1}{(1+h_f^c)} E_t \hat{c}_{f,t+1} + \frac{h_f^c}{(1+h_f^c)} \hat{c}_{f,t-1} - \frac{(1-h_f^c)}{(1+h_f^c)\,\sigma_f^c} \left(\hat{r}_t - \mathbb{E}_t \hat{\pi}_{f,t+1}^c + \varepsilon_{f,t+1}^{\beta} - \varepsilon_{f,t}^{\beta} + C\mathcal{X}^B \hat{b}_{h,t} \right) \quad (42)$$

hours supply equation (4) implies the following households' desired wage $\hat{w}_{i,t}^h$ before wage negociation:

$$\hat{w}_{i,t}^h = \frac{1}{\sigma_i^L} \hat{h}_{i,t} + \frac{\sigma_i^c}{(1-h_i^c)} \left(\hat{c}_{i,t} - h_i^c \hat{c}_{i,t-1} \right) \quad (43)$$

3.2 Unions

The staggered real wage equation (6) combined with aggregate wage index equation (7) give rise to the new Keynesian Phillips curve for wages,

$$\begin{aligned} \hat{w}_{i,t} = {} & \frac{\xi_i^w}{(1+\beta)} \hat{\pi}_{i,t-1}^c + \frac{1}{(1+\beta)} \hat{w}_{i,t-1} - \frac{(1+\beta\xi_i^w)}{(1+\beta)} \hat{\pi}_{i,t}^c + \frac{\beta}{(1+\beta)} \mathbb{E}_t \left(\hat{w}_{i,t+1} + \hat{\pi}_{i,t+1}^c \right) \quad (44) \\ & + \frac{(1-\beta\theta_i^w)(1-\theta_i^w)}{(1+\beta)\,\theta_i^w} \left(\hat{w}_{i,t}^h - \hat{w}_{i,t} \right) \end{aligned}$$

3.3 Firms

The production (10) writes:

$$\hat{y}_{i,t} = \varepsilon_{i,t}^A + \alpha \hat{k}_{i,t}^u + (1-\alpha)\,\hat{h}_{i,t}. \quad (45)$$

The marginal cost (11) is:

$$\widehat{mc}_{i,t} = \alpha \hat{z}_{i,t} + (1-\alpha)\,\hat{w}_{i,t} - \varepsilon_{i,t}^A, \quad (46)$$

and inputs are linked by (12):

$$\hat{w}_{i,t} + \hat{h}_{i,t} = \hat{k}_{i,t}^u + \hat{z}_{i,t}. \quad (47)$$

Taking in logs equation (36) and (13) give rise to the new Keynesian Phillips curve for prices:

$$\hat{\pi}^c_{h,t} = \frac{\xi^p_h}{(1+\beta\xi^p_h)}\hat{\pi}^c_{h,t-1} + \frac{\beta}{(1+\beta\xi^p_h)}\mathbb{E}_t\hat{\pi}^c_{h,t+1} + \frac{(1-\beta\theta^p_h)(1-\theta^p_h)}{(1+\beta\xi^p_h)\theta^p_h}\left(\widehat{mc}_{h,t} + \alpha^C\widehat{ToT}_t\right) \quad (48)$$

$$\hat{\pi}^c_{f,t} = \frac{\xi^p_f}{\left(1+\beta\xi^p_f\right)}\hat{\pi}^c_{f,t-1} + \frac{\beta}{\left(1+\beta\xi^p_f\right)}\mathbb{E}_t\hat{\pi}^c_{f,t+1} + \frac{\left(1-\beta\theta^p_f\right)\left(1-\theta^p_f\right)}{\left(1+\beta\xi^p_f\right)\theta^p_f}\left(\widehat{mc}_{f,t} - \alpha^C\widehat{ToT}_t\right) \quad (49)$$

The home final demand (37) is:

$$\begin{aligned}\hat{y}_{h,t} = {} & \frac{\bar{C}}{\bar{Y}}\left(\left(1-\alpha^C\right)\hat{c}_{h,t} + \alpha^C\hat{c}_{f,t}\right) + \frac{\bar{I}}{\bar{Y}}\left(\left(1-\alpha^I\right)\hat{\imath}_{h,t} + \alpha^I\hat{\imath}_{f,t}\right) + \frac{\bar{G}}{\bar{Y}}\varepsilon^G_{h,t} + \bar{Z}\bar{K}\hat{u}_{h,t} \quad (50)\\ & +2\mu\left(\bar{C}\alpha^C\left(1-\alpha^C\right) + \bar{I}\alpha^I\left(1-\alpha^I\right)\right)\widehat{ToT}_t\\ & +(1-\eta^\pi)\underline{\omega}\left(\kappa\hat{\omega}^c_{h,t} + \widehat{r}^k_{h,t} + \widehat{q}_{h,t-1} + \widehat{k}_{h,t-1}\right),\end{aligned}$$

and foreign (38),

$$\begin{aligned}\hat{y}_{f,t} = {} & \frac{\bar{C}}{\bar{Y}}\left(\left(1-\alpha^C\right)\hat{c}_{f,t} + \alpha^C\hat{c}_{h,t}\right) + \frac{\bar{I}}{Y}\left(\left(1-\alpha^I\right)\hat{\imath}_{f,t} + \alpha^I\hat{\imath}_{h,t}\right) + \frac{\bar{G}}{\bar{Y}}\varepsilon^G_{f,t} + ZK\hat{u}_{f,t} \quad (51)\\ & -2\mu\left(\bar{C}\alpha^C\left(1-\alpha^C\right) + \bar{I}\alpha^I\left(1-\alpha^I\right)\right)\widehat{ToT}_t\\ & +(1-\eta^\pi)\underline{\omega}\left(\kappa\hat{\omega}^c_{f,t} + \widehat{r}^k_{f,t} + \widehat{q}_{f,t-1} + \widehat{k}_{f,t-1}\right).\end{aligned}$$

3.4 Entrepreneurs

The net wealth law of motion (19) writes:

$$\hat{n}_{i,t} = \left(1-\tau^E\right)\frac{\bar{V}}{\bar{N}}\hat{v}_{i,t} - \varepsilon^N_{i,t}. \quad (52)$$

The one-period-profit equation of entrepreneurs reads:

$$\hat{v}_{i,t} = (1-\kappa)(1-\varkappa_i)\frac{\bar{N}/\bar{K}}{1-\bar{N}/\bar{K}}\widehat{kn}_{i,t-1} + \hat{r}^k_{i,t} + \hat{q}_{i,t-1} + \hat{k}_{i,t-1}. \quad (53)$$

From the balance sheet of entrepreneurs (14) is, the demand for credit is determined by:

$$\hat{l}^d_{i,t} = h^k_i\hat{l}^d_{i,t-1} + \frac{\bar{K}}{\bar{K}-\bar{N}}\left(\hat{q}_{i,t} + \hat{k}_{i,t} - \frac{\bar{N}}{\bar{K}}\hat{n}_{i,t}\right).$$

The external premium (or the firm spread) (20) is:

$$\hat{s}_{i,t} = \varkappa_i\frac{\bar{N}/\bar{K}}{1-\bar{N}/\bar{K}}\widehat{kn}_{i,t} + \varepsilon^Q_{i,t},$$

where

$$\hat{s}_{i,t} = \mathbb{E}_t\hat{r}^k_{i,t+1} - \hat{p}^L_{i,t}.$$

Finally, the *ex post* threshold (15) is determined by:

$$\hat{\omega}^c_{i,t} = \hat{p}^L_{i,t-1} + \hat{l}^d_{i,t-1} - h^k_i\hat{l}^d_{i,t-2} - \hat{r}^k_{i,t} - \hat{q}_{i,t-1} - \hat{k}_{i,t-1},$$

and the capital-to-net-wealth ratio is defined by:

$$\widehat{kn}_{i,t} = \hat{q}_{i,t} + \hat{k}_{i,t} - \hat{n}_{i,t}.$$

The expected rate of insolvent investment projects is defined by,

$$\hat{\eta}^\pi_{i,t} = -\kappa\hat{\omega}^c_{i,t}. \quad (54)$$

3.5 Banks

From the credit market equilibrium in equation (39), we obtain the dynamic of the loan supply:

$$\hat{l}^s_{h,t} = \left(1-\alpha^L\right)\hat{l}^d_{h,t} + \alpha^L \hat{l}^d_{f,t} + \alpha^L 2\nu\left(1-\alpha^L\right)\left[\hat{r}^L_{f,t} - \hat{r}^L_{h,t}\right] \tag{55}$$

$$\hat{l}^s_{f,t} = \left(1-\alpha^L\right)\hat{l}^d_{f,t} + \alpha^L \hat{l}^d_{h,t} - \alpha^L 2\nu\left(1-\alpha^L\right)\left[\hat{r}^L_{f,t} - \hat{r}^L_{h,t}\right] \tag{56}$$

the real marginal cost of one unit of loan (31),

$$mc^b_{i,t} = \kappa\frac{\eta^k}{\eta^k-1}\left(\left(1-\alpha^L\right)\left(1-\varkappa_h\right)\widehat{kn}_{h,t} + \alpha^L\left(1-\varkappa_f\right)\widehat{kn}_{f,t}\right) + \left(\hat{r}_t - \mathbb{E}_t\hat{\pi}^c_{i,t+1}\right). \tag{57}$$

Mixing equations (34) and (40) gives rise to the real credit rate dynamic:

$$\hat{r}^L_{i,t} = \frac{1}{1+\beta\left(1+\xi^b_i\right)}\left(\begin{array}{c}\left(1+\xi^b_i\left(1+\beta\right)\right)\hat{r}^L_{i,t-1} - \xi^b_i\hat{r}^L_{i,t-2} + \beta\mathbb{E}_t\hat{r}^L_{i,t+1} \\ +\beta\theta^b_i\mathbb{E}_t\pi^c_{i,t+2} - \left(1+\beta\theta^b_i\right)\mathbb{E}_t\pi^c_{i,t+1} + \pi^c_{i,t} \\ +\frac{\left(1-\theta^b_i\right)\left(1-\theta^b_i\beta\right)}{\theta^b_i}\left[\widehat{mc}^b_{i,t} - \hat{r}^L_{i,t}\right]\end{array}\right) + \varepsilon^{rL}_{i,t.} \tag{58}$$

and the banking spread is:

$$\hat{s}^b_{i,t} = \hat{r}^L_{i,t} - \left(\hat{r}_t - \mathbb{E}_t\hat{\pi}^c_{i,t+1}\right). \tag{59}$$

3.6 Capital Supply Decisions

The *ex post* return on capital (27) is:

$$\hat{r}^k_{h,t} = \frac{Z}{R^k}\hat{z}_{h,t} + \frac{(1-\delta)}{R^k}\hat{q}_{h,t} - \hat{q}_{h,t-1} - \varepsilon^\beta_{h,t+1} + \varepsilon^\beta_{h,t} + \frac{C}{R^k}\mathcal{X}^B\hat{b}_{h,t-1}, \tag{60}$$

$$\hat{r}^k_{f,t} = \frac{Z}{R^k}\hat{z}_{f,t} + \frac{(1-\delta)}{R^k}\hat{q}_{f,t} - \hat{q}_{f,t-1} - \varepsilon^\beta_{f,t+1} + \varepsilon^\beta_{f,t} - \frac{C}{R^k}\mathcal{X}^B\hat{b}_{h,t-1}. \tag{61}$$

The law of motion of capital is standard (22), then we obtain the dynamic of capital:

$$\hat{k}_{i,t} = \delta\hat{\imath}_{i,t} + (1-\delta)\hat{k}_{i,t-1}, \tag{62}$$

and the capital utilized by the intermediate sector (25) is:

$$\hat{k}^u_{i,t} = \hat{u}_{i,t} + \hat{k}_{i,t-1}. \tag{63}$$

The capital utilization rate (26) is:

$$\hat{u}_{i,t} = \frac{1-\psi_i}{\psi_i}\hat{z}_{i,t}. \tag{64}$$

The first order condition of capital producers in real terms (24) writes,

$$\hat{q}_{h,t} + \left(\alpha^C - \alpha^I\right)\widehat{ToT}_t = \mathcal{X}^I_h\left(\hat{\imath}_{h,t} - \hat{\imath}_{h,t-1}\right) - \beta\mathcal{X}^I_h\left(\mathbb{E}_t\hat{\imath}_{h,t+1} - \hat{\imath}_{h,t}\right) \tag{65}$$

$$\hat{q}_{f,t} - \left(\alpha^C - \alpha^I\right)\widehat{ToT}_t = \mathcal{X}^I_f\left(\hat{\imath}_{f,t} - \hat{\imath}_{f,t-1}\right) - \beta\mathcal{X}^I_f\left(\mathbb{E}_t\hat{\imath}_{f,t+1} - \hat{\imath}_{f,t}\right) \tag{66}$$

3.7 International Macroeconomics Definitions

Home and foreign consumption price inflation indexes are:

$$\hat{\pi}^c_{h,t} = \hat{\pi}_{h,t} + \alpha^C \left(\widehat{ToT}_t - \widehat{ToT}_{t-1} \right) \tag{67}$$

$$\hat{\pi}^c_{f,t} = \hat{\pi}_{f,t} - \alpha^C \left(\widehat{ToT}_t - \widehat{ToT}_{t-1} \right) \tag{68}$$

the credit rate index in real terms is:

$$\hat{p}^L_{h,t} = \left(1 - \alpha^L\right) \hat{r}^L_{h,t} + \alpha^L \hat{r}^L_{f,t} + \alpha^L \mathbb{E}_t \left(\hat{\pi}^c_{f,t+1} - \hat{\pi}^c_{h,t+1} \right) \tag{69}$$

From the home country perspective, the terms of trade is:

$$\widehat{ToT}_t = \hat{\pi}_{f,t} - \hat{\pi}_{h,t} + \widehat{ToT}_{t-1} \tag{70}$$

The home bonds law of motion:

$$\begin{aligned} \hat{b}_{h,t} - \hat{b}_{h,t-1} = & \frac{C}{Y} \alpha^C \left(\hat{c}_{f,t} - \hat{c}_{h,t} + \left(2\mu \left(1 - \alpha^I\right) - 1\right) \widehat{ToT}_t \right) \\ & + \frac{I}{Y} \alpha^I \left(\hat{\imath}_{f,t} - \hat{\imath}_{h,t} + \left(2\mu \left(1 - \alpha^C\right) - 1\right) \widehat{ToT}_t \right) \end{aligned} \tag{71}$$

the home current account dynamic,

$$\begin{aligned} \widehat{ca}_{h,t} = & C \left(\hat{b}_{h,t} - \hat{b}_{h,t-1} \right) + L\alpha^L \left(R^L - 1\right) \left(\hat{l}^d_{f,t} - \hat{l}^d_{h,t} + \left(1 - 2\alpha^C\right) \widehat{ToT}_t \right) \\ & + L\alpha^L \left(R^L - 1\right) \left(2\iota \left(1 - \alpha^L\right) - 1\right) \left(\hat{r}^L_{f,t} + \hat{\pi}^c_{f,t} - \hat{r}^L_{h,t} - \hat{\pi}^c_{h,t} \right) \end{aligned} \tag{72}$$

and finally the Taylor rules follows the law of motion (35),

$$\hat{r}_t = \rho^R \hat{r}_{t-1} + \frac{1}{2} \left(1 - \rho^R\right) \left[\phi^\pi \left(\hat{\pi}^c_{h,t} + \hat{\pi}^c_{f,t} \right) + \phi^{\Delta y} \left(\hat{y}_{h,t} - \hat{y}_{h,t-1} + \hat{y}_{f,t} - \hat{y}_{f,t-1} \right) \right] + \eta^R_t \tag{73}$$

4 Shocks Processes

Shock processes are defined by:

$$\varepsilon^A_{i,t} = \rho^A_i \varepsilon^A_{i,t-1} + \eta^A_{i,t}, \tag{74}$$

$$\varepsilon^G_{i,t} = \rho^G_i \varepsilon^G_{i,t-1} + \eta^G_{i,t}, \tag{75}$$

$$\varepsilon^\beta_{i,t} = \rho^\beta_i \varepsilon^\beta_{i,t-1} + \eta^\beta_{i,t}, \tag{76}$$

$$\varepsilon^Q_{i,t} = \rho^Q_i \varepsilon^Q_{i,t-1} + \eta^Q_{i,t}, \tag{77}$$

$$\varepsilon^N_{i,t} = \rho^N_i \varepsilon^N_{i,t-1} + \eta^N_{i,t}, \tag{78}$$

$$\varepsilon^{rL}_{i,t} = \rho^{rL}_i \varepsilon^{rL}_{i,t-1} + \eta^{rL}_{i,t}. \tag{79}$$

5 Measurement equations

Finally, measurement equations are termined by:

$$
\begin{bmatrix} y_{i,t}^{obs} \\ c_{i,t}^{obs} \\ i_{i,t}^{obs} \\ \pi_{i,t}^{obs} \\ l_{i,t}^{obs} \\ r_{i,t}^{Lobs} \\ r_{t}^{obs} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{y}_{i,t} - \hat{y}_{i,t-1} \\ \hat{c}_{i,t} - \hat{c}_{i,t-1} \\ \hat{\imath}_{i,t} - \hat{\imath}_{i,t-1} \\ \hat{\pi}_{i,t}^{c} \\ \hat{l}_{i,t}^{s} - \hat{l}_{i,t-1}^{s} \\ 4 \times \hat{r}_{i,t}^{L} \\ 4 \times \hat{r}_{t} \end{bmatrix}. \tag{80}
$$

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A The cost-of-funds in the economy

Figure 5 represents the cost of funds in the economy in a partial equilibrium.

The left panel is drawn from equation (20) and displays the cost-of-funds in the production sector. As underlined by Bernanke et al. (1999), the external finance premium is decreasing in the leverage ratio of entrepreneurs. Another important feature of this curve is the role of the elasticity of the external finance premium $\varkappa_i$. When the elasticity of the external finance premium is 0, the cost-of-funds for the production sector is independent of the leverage of the economy. In the other hand, when the elasticity of the external finance premium is high ($\varkappa_i = 0.1$), the response is more sensitive to financial distress.

Finally, we discuss the bank spread from equation (31). We study the case in which the expected share of insolvent investment projects η^{π} is higher. The banking spread is increasing with the expected insolvency rate. As the augmenting share of insolvent projects is costly for banks, they rise their interest rate to maintain their expected payoffs.

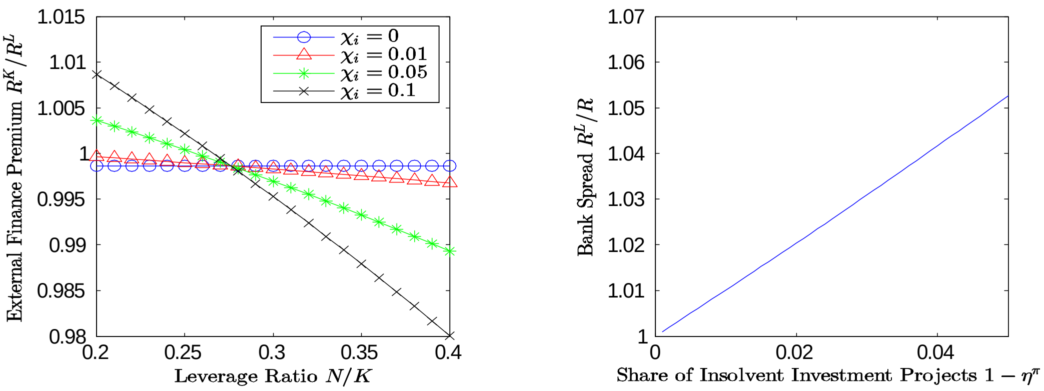

Figure 5: Spreads determinants in the economy: effect of the entrepreneur net worth on the external finance premium (left panel), effect of the expected share of defaulting entrepreneur on the banking spread (right panel).