Firstly, I am honored to be re-invited to apply for the Giovanni Borrelli Fellowship, and I appreciate the generosity of Emeritus Professor Robert Borrelli for inspiring a new generation of mathematicians.

My current research interests include algebraic geometry, combinatorial optimization, and data mining. I also have a background in computation and would like to incorporate programming into my research. I have asked Professor Mohamed Omar to be my research mentor during summer 2014.

Proposed Project

The study of linear programming dates back at least as far as Leonid Kantorovich in 1939, for use during World War II to plan expenditures in order to reduce costs to the army and increase losses to the enemy. Today one prominent method to solve linear programs, called the simplex method, was selected as one of the “10 algorithms with the greatest influence on the development and practice of science and engineering in the 20th century” by the journal Computing in Science and Engineering [2]. In the context of simplex method, in 1957 Warren M. Hirsch stated a conjecture about properties of polytopes that affect the efficiency of the algorithm. The conjecture is generally false, as there is a counter-example proposed by Francisco Santos in 2010 [3], but the class of polytopes that satisfy the conjecture remains interesting.

In our research, we will study a continuous analog of the Hirsch conjecture. We first introduce some terminology. We consider the following standard linear programming problem in its primal and dual formulation

Maximize $c^T x$ subject to $Ax = b$, and $x \geq 0$; \hspace{1cm} (1)

Minimize $b^T y$ subject to $A^T y - s = c$, and $s \geq 0$, \hspace{1cm} (2)

where $A$ is a fixed matrix of rank $d$ having $n$ columns, and the vectors $c \in \mathbb{R}^n$ and $b \in \text{image}(A)$. The (primal) logarithmic barrier function for (1) is defined as

$$f_{\lambda}(x) := c^T x + \lambda \sum_{i=1}^{n} \log x_i.$$ \hspace{1cm} (3)

where $\lambda > 0$ is a real parameter. This gives a family of optimization problems:

Maximize $f_{\lambda}(x)$ subject to $Ax = b$, and $x \geq 0$. \hspace{1cm} (4)

Since the function $f$ is strictly concave, it attains a unique maximum $x^*$ in the interior of the feasible polytope $P = \{ x \in \mathbb{R}_{\geq 0}^n : Ax = b \}$. The primal central path is the curve $\{ x^*(\lambda) : \lambda > 0 \}$ inside the polytope $P$. 

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In our research, we want to prove or disprove a special case of the following conjecture.

**Conjecture (the continuous analog of Hirsch conjecture):**

> The order of the largest total curvature of the primal central path overall polytopes defined by $n$ inequalities in dimension $d$ is $n$.

The special case that we are interested in is when the polytopes are in the class of “transportation polytopes”. The 2-way transportation polytope, for example, is the set of all possible tables whose row and column sums equal the given margins, where each element in the table is nonnegative. The reason this problem would be interesting is that there is a natural way to see all convex polytopes as transportation polytopes of some kind. The tools we will use to approach this problem are new and algebraic, and can be found in [1].

Both positive or negative results would be interesting. If we prove the conjecture to be false for transportation polytopes, then it is false in general. If we proved it true for such polytopes, then it would have great implications in light of the continuous analogue of the Hirsch conjecture, as one could view this as a stepping stone to the full conjecture.

Moreover, this project is concrete. Transportation polytopes have a concrete description and this can then be used to obtain a concrete description of the central curve (up to Zariski closure), where the central curve is the Zariski closure of the central path in $(x, y, s)$-space.

**Funding**

Last year after I was awarded an honorable mention of the Giovanni Borrelli fellowship, I worked with Professor Michael Orrison and Jacob Brumbaugh-Smith (Pomona) during summer 2012 using the Giovanni Borrelli funding for accommodation expenses in Claremont. Please see a write-up of the project [here](#). In fall 2012, I continued to work with Professor Orrison, Professor Martonosi, and Jeremy Usatine on a slightly different topic. A write-up can be found [here](#).

This year, I would use the fellowship funding for expenditures of travelling to seminars and workshops. I would also use it for accommodation expenses during summer 2014.

**References**

