Goods-market Frictions and International Trade*

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Abstract

We present a tractable framework that embeds goods-market frictions in a general equilibrium dynamic model with heterogeneous exporters and identical importers. These frictions arise because it takes time and expense for exporters and importers to meet. We show that search frictions lead to an endogenous fraction of unmatched exporters, alter the gains from trade, endogenize entry costs, and imply that the competitive equilibrium does not generally result in the socially optimal number of searching firms. Finally, ignoring search frictions results in biased estimates of the effect of tariffs on trade flows.

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"MontGras [a Chilean Winery] has twice failed to penetrate the U.S. market because distributor relationships fell through and is deciding between two new potential partners."

Harvard Business Review - Arnold, Stevenson, and de Royere (2002)

1 Introduction

Locating and building connections with overseas buyers is a prevalent and costly barrier to exporting. Surveying firms in the U.K., Kneller and Pisu (2011) find that "identifying the first contact" and "establishing initial dialogue" are more common obstacles to exporting than "language barriers," "cultural differences," or "dealing with legal, financial and tax regulations overseas." Moreover, exporting firms pursue a number of costly activities to overcome these barriers. Eaton, Eslava, Jinkins, Krizan, and Tybout (2014) report that the four most expensive costs for Colombian exporters (in order) are maintaining foreign sales offices, sending sales representatives abroad, researching potential foreign buyers, and establishing a web presence. Likewise, Allen (2014) shows that it is costly for agricultural producers to gather information about market conditions across regions in the Philippines and that these information barriers alter trade flows. In this paper, we formalize this costly search process for international partners as a goods-market friction between importing retailers and exporting producers.

Our search theoretic framework is motivated by a literature that documents the important role these frictions play in input markets and most closely resembles Pissarides (2000, Ch. 1). We embed these frictions into a general equilibrium model with heterogeneous producers and identical retailers in the style of Hopenhayn (1992) and Melitz (2003). Combining these two canonical models provides a tractable framework that has rich implications for individual trading relationships and economic aggregates. This framework is also consistent with the empirical relevance of international intermediaries that move goods from producers to final consumers as documented by Bernard, Jensen, Redding, and Schott (2010). To obtain general results about the macroeconomy, we first analyze a closed version of the model. Our main application, however, considers an open economy as searching across international borders may be particularly expensive.

We begin by pointing out that any search model leads to a concept of unmatched agents in the aggregate because there always exists a mass of agents that are actively looking for partners but have yet to match. In our model, the "unmatched rate" of producers is the fraction of producers that are actively looking for retailers and is analogous to the unemployment rate of workers in labor economics.

The unmatched rate of producers is important because the associated product varieties cannot be consumed and are therefore absent from the indirect utility (welfare) function, price index, and other aggregates. In this way, we augment the results of Arkolakis, Costinot, and Rodríguez-Clare (2012) by showing that goods-market frictions have first-order implications for welfare responses; any shock that changes the unmatched rate will affect welfare through this new channel.

Crucially, the unmatched rate depends endogenously on a search market sufficient statistic called "market tightness," which is defined as the ratio of searching retailers to searching producers. Market tightness determines the expected number of times producers and retailers contact one another within each time period, also known as the "contact rate." In concurrent related work, Eaton et al. (2014) and Eaton, Jinkins, Tybout, and Xu (2016) also have endogenous contact rates, but in the context of a more complex search model that includes, among other features, many-to-many matches and learning about foreign markets. Our model presents a more stylized view of the search process and, with our simplifications, admits straightforward aggregation. The tractability this affords allows us to analytically show how changes to exogenous variables affect endogenous market tightness, contact rates, the unmatched rate of producers, and welfare. In particular, if unmatched rates are exogenous, then the response of welfare to any foreign shock remains the same as in Arkolakis et al. (2012).

We also find that, with endogenous market tightness, goods-market frictions change the elasticity of domestic consumption shares to iceberg trade costs. We argue that the elasticity in our model with market tightness is always at least as negative as the elasticity in a model without goods-market frictions because search frictions magnify the response of trade to changes in iceberg costs.

Unsurprisingly, goods-market frictions reduce aggregate trade flows relative to a model without them, but they do so through both prices and the fraction of unmatched producers. In particular, the equilibrium import price paid by the retailer and negotiated with the exporting producer is lower than the final sales price paid by consumers. This ensures that the importer can at least cover their search costs and is consistent with the empirical findings of Berger, Faust, Rogers, and Steverson (2012). This pricing approach, among other model features, is similar to Drozd and Nosal (2012) who use a trade model with search frictions to account for several pricing puzzles of international macroeconomics. These lower import prices in our framework reduce the value of imports relative to typical models of trade which provide aggregate imports evaluated at final sales prices.

The unmatched rate also reduces aggregate imports because a fraction of producers are not matched to importing retailers thereby reducing the number of varieties traded in equilibrium. Benguria (2015) has a similar feature and, as in that paper, we show that the goods-market friction provides a micro foundation for the cost associated with entering foreign markets. We point out that the quantity traded (intensive margin) between *matched* retailers and producers in our model is the same as in a model without search because the two parties still seek to maximize profits earned from consumers.

The gravity equation in our model captures precisely how goods-market frictions, which change import prices and the fraction of unmatched producers, affect aggregate trade flows. Consistent with our derived gravity equation, Rauch and Trindade (2002) estimate that a larger ethnic Chinese population within a country reduces goods-market frictions and increases bilateral trade flows between Southeast Asian countries. Our derivation also provides a search-theoretic justification, in addition to Rauch (1999), for including distance and measures of language similarity when estimating empirical gravity equations. Lastly, Portes and Rey (2005) find that bilateral telephone traffic and the number of bank subsidiaries, which may proxy for search costs, improve information flow between countries, significantly increasing trade flows in a standard gravity equation regression.

Finally, we find that the competitive equilibrium with goods-market frictions and endogenous market tightness is inefficient relative to the social optimum. The competitive equilibrium features two externalities that work in opposite directions. On the one hand, individual producers (retailers) ignore the positive market thickness externality they create for retailers (producers) when they search. On the other hand, individual producers (retailers) ignore the negative congestion externality they impose on other producers (retailers) by searching. As a result, the competitive equilibrium may have too many or two few producers (retailers) searching relative to the socially optimal level. To our knowledge, this observation has not been made before in the context of international trade. Moreover, the well-known Hosios (1990) condition from the labor search literature is, in general, insufficient to guarantee efficiency in the competitive equilibrium of our model.

We briefly mention other important empirical and theoretical work related to goods-market frictions. Specifically, Rauch (1996) uses a model of search to explain multiple phenomenon including the existence of large Japanese intermediate general trading companies which match producers and consumers without producing any goods themselves. Cadot, Iacovone, Pierola, and Rauch (2011), Egan and Mody (1992), Monarch and Schmidt-Eisenlohr (2015), and Rauch and Watson (2003) discuss search frictions as they relate to foreign-market entry and importer-exporter relationships. Also highlighting the importance of firm-to-firm relationships is Heise (2015a,b).

Goods-market frictions have also recently been used outside of international trade to help explain aggregate dynamics. For example, Petrosky-Nadeau and Wasmer (2015) investigate the ability of goods and credit market frictions to generate persistence in labor market responses to productivity shocks, while Bai, Ríos-Rull, and Storesletten (2012) develop a DSGE model that features search frictions between consumers and producers. Gourio and Rudanko (2014) study how goods-market frictions affect firm level variables like investment, profits and sales, and den Haan (2013) assesses how goods-market frictions can explain inventory dynamics over the business cycle. We think that goods-market frictions may be informative for the dynamics of aggregate trade, although this is a topic we leave to future research.

The next section introduces the closed-economy model. Section 3 presents comparative statics and analyzes the efficiency of the competitive equilibrium. Section 4 opens the economy to trade. Specifically, Section 4.3 analyzes the welfare implications and derives the gravity equation implied by our framework. The last section presents a discussion of further research.

2 Searching for business affiliates

2.1 Model introduction

The work on search and matching by Diamond (1982), Mortensen (1986), and Pissarides (1985) underpins the process by which producers and retailers find one another. Our model is formulated in continuous time and we focus on steady-state implications, thereby ignoring any transition dynamics. We are motivated by the facts summarized in Bartelsman and Doms (2000) and Syverson (2011) that, even within similar industries, firms exhibit persistent differences in measured productivity. We incorporate this firm heterogeneity as in Hopenhayn (1992) and Melitz (2003) and index producers by their productivity, φ. This permanent productivity is exogenously given and known to producers. Retailers are ex-ante homogeneous but ex-post differentiated depending on the productivity of the producing partner with whom they match.

Producers and retailers make optimal decisions to search by using backward induction knowing that upon meeting they will bargain continuously over the price, $n(\varphi)$, and the quantity, $q(\varphi)$, of the good to be exchanged. When bargaining, both parties know that retailers alone can access consumers and that they can sell the product to these consumers at a potentially different final price, $p(\varphi)$. Business relationships end at an exogenous rate, λ, that is the same for all matches, regardless of the producer's productivity level. In the remainder of this section we focus on a closed-economy setting to derive results under general conditions. In Section 4 we present an open-economy version of the model with particular functional form assumptions and derive closed-form expressions for key objects of interest like the gravity equation, trade elasticity and welfare responses to foreign shocks.

2.2 Matching function

The matching function, denoted by $m(uN^x, vN^m)$, gives the flow number of relationships formed at any moment in time as a function of the stock number of unmatched producers, uN^x, and unmatched retailers, vN^m. N^x and N^m represent the total mass of producing and retailing firms, respectively, that exist regardless of their match status. The fraction of producers looking for retailers is u while the fraction of producers that choose not to search, and therefore remain idle, is denoted by i. The fraction of retailers that are searching for producing firms, is v.

We assume that the matching function is homogeneous of degree one, strictly increasing in both arguments, and concave. Due to the homogeneity assumption market tightness,

$\kappa = vN^m/uN^x$, which is the ratio of the mass of searching retailers to the mass of producers, is sufficient to determine contact rates on both sides of the market. We use continuous time Poisson processes to model the random matching of retailers and producers. As such, the contact rate defines the average number of counterparty meetings during one unit of time. The rate at which retailers contact producers, $\chi(\kappa)$, is the number of matches in a period over the number of searching retailers:

$$\chi(\kappa) = \frac{m(uN^x, vN^m)}{vN^m} = m\left(\frac{uN^x}{vN^m}, 1\right) = m\left(\frac{1}{\kappa}, 1\right)$$

Similarly, the average rate at which producers contact retailers, $\kappa\chi(\kappa)$, is the number of matches in a period over the number of searching producers:

$$\kappa\chi(\kappa) = \frac{m(uN^x, vN^m)}{uN^x} = m\left(1, \frac{vN^m}{uN^x}\right) = m(1, \kappa)$$

Another feature of Poisson processes is that the inverse of the contact rate is the average units of time between counterparty meetings. For example, if the units of time in the model were years, then retailers would wait an average of $1/\chi(\kappa)$ years before meeting a producer. Appendix C.1 has more details.

The matching function is strictly increasing in both arguments which captures the fact that more matches are made if there are either more searching retailers or more searching producers. It also means that, due to congestion effects, retailers' contact rate falls with market tightness ($\chi'(\kappa) < 0$) and, due to market thickness effects, producers' contact rate rises with market tightness ($d\kappa\chi(\kappa)/d\kappa > 0$). We will refer to increasing κ as a "tighter" market and decreasing κ as a "looser" market. Market tightness is defined from the prospective of the producer, so when the market is tight, it is easy for a producer to find a retailer.

Lastly, it will be useful to denote the elasticity of the matching function with respect to the number of unmatched producers, uN^x, as $\frac{d\ln\left(m\left(uN^x, vN^m\right)\right)}{d\ln\left(uN^x\right)} = \eta\left(\kappa\right)$. Due to homogeneity of the matching function, this is also the (negative of the) elasticity of the retailers' finding rate with respect to market tightness $\eta\left(\kappa\right) \equiv -\frac{\kappa\chi'\left(\kappa\right)}{\chi\left(\kappa\right)}$. The elasticity captures the fact that when the number of searching retailers increases one percent more than the number of searching producers, the expected number of contacts that a retailer makes within each time period declines by η percent due to congestion.

2.3 Producers

The value of a producer with productivity φ being matched to a retailer, $X\left(\varphi\right)$, can be summarized by a value function in continuous time

$$rX\left(\varphi\right) = nq - t\left(q, \varphi, w, \tau\right) - f + \lambda\left(U\left(\varphi\right) - X\left(\varphi\right)\right) \tag{1}$$

This asset equation states that the flow return at the risk free rate, r, from the value of producing must equal the flow payoff plus the expected capital gain from operating as an exporting producer. Each producer is indexed by exogenous productivity, φ, with cumulative density function $G\left(\varphi\right)$, and probability density function $g\left(\varphi\right)$, that is defined over φ. The flow payoff consists of nq, the revenue obtained from selling q units of the good at price n to the retailer, less the variable, $t\left(q, \varphi, w, \tau\right)$, and fixed, f, cost of production. Here $t\left(\cdot\right)$ is an arbitrary variable cost function that depends on the level of output, q, exogenous productivity, φ, input prices, w, as well as an iceberg trade cost, τ. The last term in equation (1) captures the event of a dissolution of the match, which occurs at exogenous rate λ and leads to a capital loss of $U\left(\varphi\right) - X\left(\varphi\right)$. Here $U\left(\varphi\right)$ represents the producer's value of operating without a retailer and will be described in more detail in a moment. In writing equation (1), we explicitly write the value $X\left(\varphi\right)$ as a function of the producer's productivity, φ. We do this to remind the reader that each endogenous term is a function of this productivity but we conserve on notation by omitting this argument from the negotiated price, n, and traded quantity, q.

Although in principle producers could circumvent retailers and contact final consumers directly, we avoid this possibility by assuming that the net value of matching with a retailer is always greater than the net value of forming a relationship directly with a final consumer. This is similar to earlier work by Wong and Wright (2014) who assume that a middleman is necessary rather than deriving the conditions under which this is the case. There are a few potential ways to motivate this assumption. One is that retailers specialize in reaching and serving customers and therefore have lower costs of doing so. A second is that households maximize utility in a second stage and procure the quantity desired to consume in a searching first stage so that consumers are allowed to contact producers directly. With this approach, we suspect that many of our key results about the implications of search for international trade in Section 4 would go through.

The value that an unmatched producing firm gets from looking for a retail partner without being in a business relationship, $U(\varphi)$, satisfies

$$rU(\varphi) = -l + \kappa\chi(\kappa)(X(\varphi) - U(\varphi) - s) \tag{2}$$

The flow search cost, l, is what the producer pays when looking for a retailer. This captures the costs we highlighted in the introduction, namely, maintaining foreign sales offices, sending sales representatives abroad, researching potential foreign buyers, and establishing a web presence. The second term captures the expected capital gain, where $\kappa\chi(\kappa)$ is the endogenous rate at which producing firms contact retailers, and s is the sunk cost of starting up the relationship. As discussed above, market tightness is denoted by κ.

The producing firm also has the option of remaining idle and not expending resources to look for a retailer. For the producer, the value of not searching, $I(\varphi)$, satisfies

$$rI(\varphi) = h \tag{3}$$

Producers can always choose this outside option and not search for retailers. Idle firms in this context are analogous to workers who are out of the labor force. Choosing to remain idle provides the flow payoff, h. The value to a producer of remaining idle can be interpreted, for example, as the value of the stream of payments after liquidation or the flow payoff from home production if these firms are viewed as entrepreneurs.

Producing firms will choose to remain idle if the value of searching, $U(\varphi)$, is less than the value of remaining idle, $I(\varphi)$. Generally, producers desire to match with a retailer and reach consumers but doing so comes at the cost of searching. As such, some producers with low productivity, φ, may not find it optimal to engage in search. Importantly, if l is large enough relative to h, remaining idle may be optimal for many producers.

2.4 Retailers

All retailers are *ex-ante* identical but have values that vary *ex-post* only because producers are heterogeneous. The value of a retailing firm being in a business relationship with a producer of productivity φ, is defined by the asset equation

$$rM(\varphi) = p(q, Y, P)\, q - nq + \lambda (V - M(\varphi)) \tag{4}$$

The flow payoff from being in a relationship is the revenue generated by selling q units of the product to a representative consumer at a final sales price, p, determined by their inverse demand curve $p(q, Y, P)$, less the cost of acquiring these goods from producers at negotiated price n. Consumer demand is a function of income, Y, and all goods prices, P. To highlight other aspects of the search process, we assume the retailer does not use the product as an input in another stage of production but only facilitates the match between producers and consumers. When we solve the model in Section 2.5 (specifically, Section 2.5.4) we show that including an additional intermediate input does not substantively affect our main conclusions. In the event that the relationship undergoes an exogenous separation, the retailing firm loses the capital value of being matched, $V - M(\varphi)$.

The value of being an unmatched retailer, V, satisfies

$$rV = -c + \chi(\kappa) \int \left[\max\{V, M(\varphi)\} - V \right] dG(\varphi) \tag{5}$$

The retailer needs to pay a flow cost, c, to search for a producing affiliate. At Possion rate $\chi(\kappa)$, the retailing firm meets a producer of unknown productivity. Because the retailer does not know the productivity of the producer it will meet, it takes the expectation over all productivities it might encounter when computing the expected continuation value of searching. As a result, the value, V, is not a function of the producer's productivity, φ, but rather a function of the expected payoff. We assume that upon meeting, but before consummating a match, the retailer learns the productivity of the producer. In this sense, matches are inspection goods or search goods, as opposed to experience goods. Upon meeting, and depending on the producer's productivity, φ, the retailer chooses between matching with that producer, which generates value $M(\varphi)$, and continuing the search, which is associated with value V. Hence, the capital gain to a retailer from meeting a producer with productivity φ can be expressed as $\max\{V, M(\varphi)\} - V$ where the retailer maximizes over accepting a match and continuing to search.

We assume free entry into retailing so that, in equilibrium, the value of being unmatched, V, is driven to zero. The ability to expand retail shelf space or post a product online until it is no longer valuable to do so provides an intuitive basis for this assumption. Because retailers do not generate any revenue or incur any cost before entering the state of searching, V, we also assume that, unlike their producing counterparts, their value of remaining idle is zero. Hence, on the retailer's side of the market, searching and not searching both have zero value in equilibrium, allowing retailers simply to compare $M(\varphi)$ and V.

2.5 Solution to the search model

2.5.1 Overview

The retailing and producing firms use backward induction to maximize their value. The second stage is the solution that results from bargaining over price and quantity after a retailer and producer meet and decide to match. We solve this bargaining problem in Sections 2.5.2 through 2.5.4.

In the first stage, the retailer and the producer use the outcomes that will obtain in the bargaining stage to define the conditions under which they will consummate a match upon meeting and whether they will search for a business partner in the first place. Because producers are heterogeneous, the decision depends on productivity. Section 2.5.5 shows that there is a minimum productivity threshold, akin the entry condition defined in Melitz (2003), that makes searching worthwhile. In Sections 2.5.6 and 2.5.7 we present the two conditions which jointly characterize retailers' decision to search and define market-tightness and the negotiated price. Finally, in steady state there exists a set of unmatched producers that are actively looking for a retail partner and unmatched retailers that are actively looking for a producer. We define these concepts in Section 2.5.8.

2.5.2 Continuous Nash bargaining over price and quantity

Upon meeting, the retailer and producer bargain over the negotiated price and quantity simultaneously. We assume that these objects are determined by the generalized Nash bargaining solution which, as shown by Nash (1950) and Osborne and Rubinstein (1990), is equivalent to maximizing the following Nash product

$$\max_{q,n} \left[X(\varphi) - U(\varphi)\right]^{\beta} \left[M(\varphi) - V\right]^{1-\beta}, 0 \leq \beta < 1 \tag{6}$$

where β is the producer's bargaining power. The total surplus created by a match, which is the value of the relationship to the retailer and the producer less their outside options, is $S(\varphi) = M(\varphi) - V + X(\varphi) - U(\varphi)$. In Appendix A.1 we show this to be

$$S(\varphi) = \frac{pq - t(q) - f + l + s\kappa\chi(\kappa)}{r + \lambda + \beta\kappa\chi(\kappa)} \tag{7}$$

In Appendix A.1 we also derive the value of a relationship, $R(\varphi)$, in terms of model primitives.

2.5.3 Bargaining over price

Producers and retailers bargaining over the negotiated price, $n(\varphi)$, results in equations that divide the total surplus created by a match between the parties according to

$$\begin{aligned} X(\varphi) - U(\varphi) &= \beta S(\varphi) \\ M(\varphi) - V &= (1-\beta) S(\varphi) \end{aligned} \tag{8}$$

Here the producer receives β of the total surplus while the retailer receives the remainder. As such, we refer to this expression as the "surplus sharing rule." We have relegated the details regarding the derivation of equation (8) to Appendix A.2.1. We also point out that the reasoning behind the restriction that $\beta < 1$ in equation (6) is evident in equation (8). Retailing firms have no incentive to search if $\beta = 1$ because they get none of the resulting match surplus and therefore cannot recoup search costs $c > 0$. Any solution to the model with $c > 0$ and positive trade between retailers and producers also requires $\beta < 1$. Appendix A.2.1 provides greater detail.

2.5.4 Bargaining over quantity

In Appendix A.2.2 we show that bargaining over quantity, q, together with equation (8), yields

$$p\left(q,Y,P\right)+\frac{\partial}{\partial q}p\left(q,Y,P\right)q=\frac{\partial}{\partial q}t\left(q,\varphi,w,\tau\right) \tag{9}$$

The quantity exchanged *within* matches, q, equates marginal revenue obtained by the retailer with the marginal production cost. This is the same quantity we would get from a model without search frictions and therefore implies that adding search (as we do) does not change the quantity exchanged *within* matches. The quantity depends on consumers' demand curve $p\left(q,Y,P\right)$, the pricing power of retailers, and the production cost function $t\left(q,\varphi,w,\tau\right)$. We also show in Appendix A.3 that including an additional input in the retailer's production function does not change this result. Nevertheless, although the quantity exchanged does not depend on search frictions, these frictions do affect the mass of matches formed. We turn to this topic in the following section.

2.5.5 Productivity thresholds

Given the outcome of the bargaining stage above, we next define conditions when retailers and producers will consummate a match upon meeting and whether they will search for a business partner in the first place. Because producers differ by productivity, these conditions lead to two productivity thresholds.

The lower threshold, denoted by $\underline{\varphi}$, makes the producer and retailer indifferent between consummating a relationship upon contact and continuing to search, $X\left(\underline{\varphi}\right)-U\left(\underline{\varphi}\right)=0$. Equation (8) implies that an identical productivity threshold could be obtained by considering the lowest productivity that would leave a retailer indifferent between matching and continuing to search for a different producer, namely $M\left(\underline{\varphi}\right)-V=0$.

The upper threshold, denoted by $\bar{\varphi}$, makes the producer indifferent between searching and remaining idle so $U\left(\bar{\varphi}\right)-I\left(\bar{\varphi}\right)=0$. By using equations (2) and (3) this can be rewritten as

$$X(\bar{\varphi}) - U(\bar{\varphi}) = \frac{l+h}{\kappa\chi(\kappa)} + s \tag{10}$$

The equation states that the productivity necessary to induce a producer to search for a retailer equates the capital gain from forming a relationship with the expected cost of matching with a retailer. This cost includes the opportunity cost of remaining idle, h, the search cost, l, the rate $\kappa\chi(\kappa)$ at which producers meet retailers, and the sunk cost of forming a relationship, s.

We derive both thresholds in Appendix A.4 and show that $\bar{\varphi} > \underline{\varphi}$, if $l + h + \kappa\chi(\kappa)\, s > 0$. Intuitively, if a retailer and producer make contact, then search costs have already been paid and productivities $\varphi > \underline{\varphi}$ result in the match being consummated. Relative to producers who have already contacted retailers, producers that have not made contact with a retailer yet must incur additional search costs and this further discourages some lower productivity producers from searching in the first place. As a result, the productivity required to engage in search exceeds the productivity required to consummate a match, i.e. $\bar{\varphi} > \underline{\varphi}$.

We can translate equation (10) into an expression akin to the entry condition defined in Melitz (2003) even though the retailer earns pq revenue from the consumer and the producer pays the production cost. Our condition defines a threshold productivity that ensures that total flow profits cover what we call the "effective entry cost" which is the fixed cost of production, f, and the (appropriately discounted) flow cost of searching for a partner, l, the opportunity cost of remaining idle, h, and the sunk cost of starting up a business relationship, s.

Definition 1. *The effective entry cost, $F(\kappa)$, is defined as*

$$F(\kappa) \equiv f + \left(\frac{r+\lambda}{\beta\kappa\chi(\kappa)}\right) l + \left(1 + \frac{r+\lambda}{\beta\kappa\chi(\kappa)}\right) h + \left(\frac{r+\lambda}{\beta}\right) s$$

Proposition 1. *The binding threshold productivity, $\bar{\varphi}$, is determined by the implicit function*

$$\pi(\bar{\varphi}, Y, P) = F(\kappa) \tag{11}$$

where $\pi(\bar{\varphi}, Y, P) \equiv p(q(\bar{\varphi}), Y, P)\, q(\bar{\varphi}) - t(q(\bar{\varphi}), w, \bar{\varphi}, \tau)$.

Proof. See Appendix A.4.2. □

This proposition implies that the effective entry cost, and therefore the threshold productivity, depends endogenously on producers' finding rate $\kappa\chi(\kappa)$. Remember that $\kappa = vN^m/uN^x$ so that as the number of searching retailers increases (or searching producers decreases), it becomes easier for a producer to meet a retailer. Intuitively, higher κ reduces the time spent searching and, along with it, the effective entry cost. Related to this, if producers' finding rate is exogenous, Proposition 1 provides a novel micro level interpretation of the effective entry cost, but it remains a combination of exogenous parameters. Benguria (2015) makes a closely related point.

Another innovation of our model is that the opportunity cost of remaining idle, h, is an important determinant of the productivity threshold and the fraction of active producers. As pointed out by Armenter and Koren (2014), the fraction of exporting firms is an important moment for parameter identification and one that has been exploited by Eaton et al. (2014) and Eaton et al. (2016), among others. Allowing for the possibility that producers optimally choose not to search could change the estimates in these important papers. Another novel implication of our model is that the bargaining power, β, and the match destruction rate, λ, are determinants of the effective entry cost.

Finally, Proposition 1 nests the conditions defining the threshold productivity in many trade models. In particular, with $l = 0$, $h = -l$ and $s = 0$, we recover the equation defining the productivity threshold in Melitz (2003). We present evidence in Appendix A.4.3 on why, even though in this case it amounts to the same result, the standard trade model features $h = -l$ instead of what one might think is the intuitive value $h = 0$. In that appendix, we also relate Proposition 1 to expressions in other standard frameworks.

2.5.6 Retailer search

Here we specify the conditions under which unmatched retailers search in order to match with producers. Using equation (5) together with our assumption of free entry into the market of unmatched retailers, $V = 0$, implies that

$$\frac{c}{\chi(\kappa)} = \int_{\bar{\varphi}} M(\varphi)\, dG(\varphi) \tag{12}$$

This equation defines the equilibrium market tightness, κ, that equates the expected cost of being an unmatched retailer, on the left, with the expected benefit from matching, on the right. In defining equation (12), we removed the maximum over V and $M(\varphi)$ from equation (5) and simply integrated from the threshold productivity level defined by equation (10). This simplification is possible as long as $M(\varphi)$ is strictly increasing in φ so that the ex-post value of being matched is strictly increasing in the producer's productivity. In Appendix A.5 we provide further details. It is worth emphasizing that equation (12) does not inform the binding productivity threshold $\bar{\varphi}$, which is solely determined by Proposition 1. Details regarding this issue are in Section 2.5.5 above.

To get intuition from equation (12), notice that as the expected benefit (the right hand side) from retailing rises, free entry implies that retailers enter the search market. This raises market tightness, $\kappa = vN^m/uN^x$, reduces the rate at which searching retailers contact searching producers, $\chi(\kappa)$, and increases retailers expected cost of search (the left hand side). Hence, free entry ensures that $V = 0$ at all times and that κ always satisfies equation (12).

Equation (12) highlights that the retailers' cost of searching for producers, c, along with our assumption of free entry into retailing is at the heart of our model. If searching for producers was free ($c = 0$), but matching was associated with positive expected payoff, then free entry would lead to an infinite number of retailers in the economy driving the producer's finding rate to infinity and relieving the search friction.

Proposition 2. *With free entry into retailer search, market tightness, κ, is finite if and only if retailers' search cost, c, is positive.*

Proof. See Appendix A.6. □

Free entry also interacts with assumptions for how firms of both types come into existence. We describe in detail those assumptions in Appendix A.7 showing in Appendix A.7.1 that for retailers, free entry into search implies free entry into existence. We also consider the alternative assumption of free entry into search for producers and show that it yields additional restrictions on equilibrium market tightness. Appendix A.7.2 includes further details. We find our baseline approach of setting $V = 0$ to be a natural starting point, but other approaches lead to similar effects of search frictions and the major implications of our paper remain the same.

2.5.7 Negotiated price curve

Along with the condition for market tightness in equation (12), there is an expression that determines the negotiated price, n, at which producers and retailers exchange the good after a match has been consummated.

Proposition 3. *The negotiated price, n, at which the producer sells his good to the retailer satisfies*

$$n = [1 - \gamma]\, p\,(q, Y, P) + \gamma \frac{t\,(q, \varphi, w, \tau) + f - l - \kappa\chi\,(\kappa)\, s}{q} \tag{13}$$

where $\gamma \equiv \dfrac{(r + \lambda)\,(1 - \beta)}{r + \lambda + \beta\kappa\chi\,(\kappa)} \in [0, 1]$.

Proof. Use $V = 0$, equations (1) and (2) along with the surplus sharing rule defined by equation (8). See Appendix A.8 for a derivation of equation (13) and Appendix A.9 for a proof that $\gamma \in [0, 1]$. □

We remind the reader that equation (13) is a function of the producer's productivity, φ, but we have not written it as such in order to conserve on notation.

The equilibrium negotiated price is a convex combination of the final sales price and the average total production cost less the producer's search costs. A price outside of this range would be unsustainable. The highest negotiated price, n, that the retailer is willing to pay is the final sales price, $p(q, Y, P)$, and the lowest negotiated price that the producer is willing to accept is the average total production cost, $(t(q, \varphi, w, \tau) + f)/q$, net of the cost of looking for a retailer, l, and the expected sunk cost, $\kappa\chi(\kappa)s$. The search costs of the producer, l and s, enter negatively in equation (13) because they erode the producer's bargaining position and thereby allow the retailer to negotiate a lower transaction price.

The negotiated price also depends on the bargaining power and the finding rate of the producer. As the producer gains all the bargaining power ($\beta \to 1$), then $\gamma \to 0$ and $n \to p$ so the producer takes all the profits from the business relationship. Similarly, if producers find retailers immediately (no search friction) so that the finding rate $\kappa\chi(\kappa) \to \infty$ and the sunk cost, s, is set to zero, then the negotiated price also converges to the final sales price, $n \to p$. We provide details of this in Appendix A.8. Importantly, the case where $n \to p$ recovers the standard trade model as there is, in effect, no intermediate retailer; producers can be seen as selling their goods directly to the final consumer at the price $p(q, Y, P)$.

2.5.8 Matching in equilibrium

In steady state there exists a set of unmatched producers that are actively looking for a retail partner and unmatched retailers that are actively looking for a producer. These steady-state fractions of unmatched retailers and producers correspond to frictional unemployment and unfilled vacancies in the labor literature and will be positive as long as the finding rates are finite and the separation rate is non-zero. The mass of producers that are matched to retailers and selling their products is $(1-u-i)\,N^x$ where a fraction u are unmatched and actively searching for retailers and a fraction i choose not to search and therefore remain idle.

To determine the steady-state fraction of unmatched producers, it is useful to think about the flow into and out of the unmatched producer state. In particular, in any given instant, $(1-u-i)\,N^x$ matched producers separate exogenously at rate λ. Consequently, the inflow into the the unmatched state is $\lambda\,(1-u-i)\,N^x$. Flows out of this state are $\kappa\chi\,(\kappa)\,uN^x$ because uN^x producers find matches at rate $\kappa\chi\,(\kappa)$. In steady state, the inflows must equal the outflows and after re-arranging we get

$$\frac{u}{1-i} = \frac{\lambda}{\lambda+\kappa\chi\,(\kappa)} \tag{14}$$

The fraction of idle producers, i, that choose not to search is defined by the steady-state threshold, $\bar{\varphi}$, and the exogenous distribution of productivity:

$$i = \int_{1}^{\bar{\varphi}} dG\,(\varphi) = G\,(\bar{\varphi}) \tag{15}$$

where we have assumed that $G\,(\varphi)$ is defined over $[1,+\infty)$. The fraction of producers that are active, $1-i$, corresponds to the labor force participation rate in the labor literature. While u is the fraction of producers that are unmatched, $u/\,(1-i)$ is the fraction of active producers that are unmatched and is equivalent to the labor unemployment rate which is characterized as the fraction of the labor force that is actively searching for a job. From the definition of market tightness in this economy, $\kappa = vN^m/uN^x$, we can infer the fraction of unmatched retailers as $v = \kappa uN^x/N^m$.

We assume that every matched producer must have one, and only one, retailer as their counterpart. Doing so implies the mass of matched producers and retailers must be equal in steady state:

$$(1-u-i)\,N^x = (1-v)\,N^m \tag{16}$$

2.6 Aggregate resource constraint

The aggregate resource constraint in this economy can be expressed using either the income or expenditure approach to aggregate accounting as shown in Appendix A.10. Typically, models of international trade highlight the income perspective. We find it more natural to focus on the expenditure approach:

$$\begin{aligned} Y = &\underbrace{\left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} p(\varphi)\, q(\varphi)\, dG(\varphi)}_{\text{Aggregate consumption (C)}} \\ &+ \underbrace{\kappa u N^x c + u N^x (l + s\kappa\chi(\kappa)) + (1-u-i)\, N^x f + \kappa v^{-1} u N^x e_m + N^x e_x}_{\text{Aggregate investment (I)}} \\ &+ \underbrace{\left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} (\tau-1)\, t(\varphi)\, dG(\varphi)}_{\text{Government expenditure (G)}} \end{aligned} \tag{17}$$

Consumption expenditure, C, is total resources devoted to consumption, evaluated at final consumer prices. Investment expenditure, I, is resources devoted to creating retailer-producer relationships, paying for the per-period fixed costs of goods production, and funding the creation of firms. Here e_m and e_x are the sunk, one-time, fixed costs paid by retailers and producers, respectively, to come into existence. In Section 2.5.6 we mention that free entry into retailing implies that e_m must be zero (Appendix A.7).

The mass of producers that are matched to retailers and selling their products is $(1-u-i)\,N^x$. Producers that are idle or searching for retailers, but are currently not in a business relationship, do not contribute to aggregate output, consumption or prices. The integral term times $(1-i)^{-1}$ captures conditional average sales of producers that have productivity above the cutoff necessary to match. Another way to see that all aggregate variables must be scaled in this way is to compute the mass of matched producers $\left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}}^{\infty} dG(\varphi) = (1-u-i)\, N^x$.

To account for all resources in the economy, we assume the government spends the tax/tariff revenue, which it acquires by proportionally taxing variable production cost at rate $\tau \geq 1$. This tax will enter the final sales price, $p(\varphi)$, and the equilibrium quantity traded within a match, $q(\varphi)$. According to equation (17) the government buys labor from the household but does not use the labor to produce anything. As such, those resources are wasted. We make this assumption in the closed economy because it follows the international trade treatment of iceberg trade costs where resources simply "melt" away in transit.

Conversely, if instead of being wasted, tax revenues were refunded lump sum to the household, then the government expenditure term would be superfluous. In that case, taxes would still have a distortionary effect but would not reduce total resources. We could also, of course, set the tax rate to zero ($\tau = 1$) to ensure $G = 0$ and no tax distortions. Net exports, NX, do not appear in equation (17) because they are trivially zero in the closed economy and, when we turn to the open-economy setting in Section 4, we impose balanced trade.

We also treat total payments to idle producers, $(1-i)\, N^x h$, as balanced lump sum transfers. They enter negatively in the expenditure approach as a lump sum tax and enter positively as an additional lump sum expenditure. As such, these cancel out on the expenditure side of the accounting identity.

Total resources are given by the value of the labor endowment defined by

$$Y = wL \tag{18}$$

where L is the exogenous size of the economy and w is the equilibrium wage. It will often be convenient to normalize the nominal, $w = 1$, or the real wage, $w/P = 1$.

2.7 The representative consumer

We assume that a representative consumer maximizes utility by choosing quantities of the goods indexed by φ resulting in welfare (indirect utility) equal to

$$\begin{aligned} W(C,P) &= \arg\max_{q} U(q) \\ s.t. \quad & C = Y - I - G - NX \end{aligned} \tag{19}$$

where consumption expenditure C equals total resources Y minus non-consumption expenditures given by equation (17). We have used the (inverse) Marshallian demand that solves this utility maximization problem denoted as $p(q)$ extensively above. As long as preferences are homothetic, and can therefore be represented by a utility function that is homogeneous of degree one, welfare is the ratio of consumption expenditure to the price index, $W(C,P) = C/P = (Y - I - G - NX)/P$ as shown in Appendix C.2. The price index is defined as the minimum expenditure on consumption needed to obtain a utility level of one. Without explicit functional forms for preferences, we define it as

$$\begin{aligned} P &= \arg\min_{q} \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}}^{\infty} p(\varphi)\, q(\varphi)\, dG(\varphi) \\ s.t. \quad & U(q) = 1 \end{aligned} \tag{20}$$

3 Equilibrium and its properties

3.1 Defining the equilibrium in the closed economy

The equilibrium consists of 6 equations in 6 endogenous variables.

Definition 2. *A steady-state competitive equilibrium consists of 6 endogenous variables* q*,*

$\bar{\varphi}$, κ, n, p, and P, which jointly solve 6 equations given by (9)*,* (11)*,* (12)*,* (13)*,* (19)*, and* (20)*, given exogenous parameters β, c, f, h, l, λ, r, s, τ, exogenous productivity distribution, $G(\varphi)$, and exogenous resource endowment, L.*

We elaborate on this definition in more detail in Appendix A.11.

3.2 Closed-economy comparative statics

In this section we study how key equilibrium objects, like $\bar{\varphi}$ and κ, respond to changes in key exogenous parameters. Since we are particularly interested in the role of search frictions, we consider how endogenous variables respond to changes in the flow search costs of retailers, c, and the flow search costs of producers, l. To connect with the trade literature, we also analyze the equilibrium effects of a change in the iceberg trade cost, τ. To make definitive statements we assume that the flow payoff from being idle, is equal to (negative) the producer's search cost ($h = -l$) and that the marginal cost of production is constant (horizontal supply curve) so that the final sales price, p, does not move with exogenous parameters, except for τ which enters the cost function directly.

Proposition 4. *When the retailer's flow search cost, c, rises, the productivity threshold necessary to search, $\bar{\varphi}$, the price index, P, and quantity traded within a match, q, also rise. As c rises, market tightness, κ, and the negotiated price, n, fall.*

Proof. See Appendix A.12.1. □

These results are intuitive. For example, raising the cost of retailers' search, c, makes finding a producer more expensive and therefore reduces retailer entry relative to unmatched producers. This reduces market tightness, κ. Lower market tightness means that the effective entry cost in equation (11) rises, increasing the threshold productivity, $\bar{\varphi}$, so that only more productive firms decide to search in the first place. Due to lower market tightness, producers' finding rate, $\kappa\chi(\kappa)$, also falls, which means that producers' threat point in the bargaining game deteriorates, which reduces their negotiated price, n. Due to lower market tightness, and lower producer participation, the ideal price index, P, rises. Because we assume that goods are not perfect substitutes, this increase in the price index means that the quantity traded within a match, q, also rises. By assumption, the final sales price, p, does not move with c.

Proposition 5. *When the producer's flow search cost, l, rises, the productivity threshold necessary to search, $\bar{\varphi}$, and market tightness, κ, also rise.*

Proof. See Appendix A.12.2. □

From equation (7), we know that as producers' search costs, l, rise, match surplus increases. This also means that the value of retailing, $M(\varphi)$, rises which induces retailer entry, thereby raising market tightness, κ. Due to increased costs, the productivity threshold necessary for producers to search, $\bar{\varphi}$, rises. On the one hand, the higher market tightness serves to reduce the unmatched producer rate and lower the ideal price index, P, but on the other hand the higher threshold productivity, $\bar{\varphi}$, serves to raise the price index. In general, the response of the ideal price index to changes in l, therefore, cannot be signed. As such, the response of the quantity traded within a match, q, and the negotiated price, n, also cannot be signed. As before, by assumption the final sales price, p, does not respond to changes in l.

Proposition 6. *When the iceberg trade cost, τ, rises, the productivity threshold necessary to search, $\bar{\varphi}$, the price index, P, the final sales price, p, and the quantity traded within a match, q, also rise. As τ rises, market tightness, κ, and the negotiated price, n, fall.*

Proof. See Appendix A.12.3. □

Again, the results are intuitive. As the variable costs of production rise, the surplus from a match falls and therefore the value of retailing falls. This reduces retailer entry thereby reducing market tightness, κ. As before, this increases the threshold productivity, $\bar{\varphi}$, and reduces the negotiated price, n. Rising variable costs increase marginal costs and therefore increase the final sales price, p, which increases the aggregate price index, P. Since goods are not perfect substitutes, this increase in the ideal price index increases the quantity traded within a match, q.

3.3 Efficiency of the competitive equilibrium

The decision to search made by individual retailers and producers imposes externalities on other retailers and producers. The externalities arise because an additional producer (retailer) in the search market decreases (increases) market tightness, $\kappa = vN^m/uN^x$. Individual producers (retailers) ignore the positive "market thickness" externality they create for retailers (producers) when they search and affect κ. Likewise, individual producers (retailers) also ignore the negative "congestion" externality they impose on other producers (retailers) by searching. As a result, the competitive equilibrium may have too many or too few producers (retailers) searching. In a simpler setting than ours, Hosios (1990) shows that with continuous Nash bargaining, the "market thickness" and "congestion" externalities are just balanced in the competitive economy when the bargaining parameter, β, is equal to the elasticity of the matching function with respect to the number of unemployed workers, $\eta(\kappa)$. We characterize a similar condition in our model by finding the socially optimal market tightness, κ.

In general, the socially optimal market tightness maximizes welfare subject to the equations of motion for the number of producers, uN^x, the aggregate resource constraint, and the fact that u is constrained by an initial transversality condition to start at the steady-state value. In order to simplify the problem, we assume that the number of producers that take a draw from the productivity distribution is proportional to the size of the economy, Y. This implies that $\dot{N}^x = 0$. Our normalization that the real wage $w/P = 1$

ensures that real output is constant at $Y/P = wL/P$. We set $G = 0$ by assuming that there are no taxes, $\tau = 1$, or that tax revenue is rebated lump sum back to the consumer. After making these assumptions and substituting the resource constraint into the objective function, the welfare maximization problem becomes

$$\begin{aligned} &\max_{\kappa} \int_0^{\infty} \left[\frac{(1+\zeta)\, e_x}{\zeta} - \kappa u c - u\,(l + s\kappa\chi(\kappa)) - (1-u-i)\, f - e_x \right] e^{-rt} dt \\ &s.t. \\ &\dot{u} = \lambda\,(1-u) - u\kappa\chi(\kappa) \\ &u(0) = \frac{\lambda\,(1-i)}{\lambda + \kappa\chi(\kappa)} \end{aligned} \tag{21}$$

Solving this problem results in the socially optimal κ. The current value Hamiltonian, first order conditions, and steps used to derive the solution are included in Appendix A.13.

Proposition 7. *A sufficient condition for the competitive economy to obtain the social planner's solution is $\eta(\kappa) = \beta$ and $\beta = 1$, which is not a feasible value of β. More generally, if $\beta < 1$, in order for the competitive equilibrium and the social planner's problem to result in the same κ, it must be that $\eta(\kappa) = \beta$ and two expressions, which are in general not equal to each other, must be equivalent. As a result, in general, the competitive equilibrium is inefficient under the Nash bargaining protocol.*

Proof. See Appendix A.13. □

Given the work by Rogerson, Shimer, and Wright (2005), it is not surprising that the competitive equilibrium in a model with random search and Nash bargaining is not, in general, socially optimal. However, we find that the standard Hosios (1990) condition ($\eta(\kappa) = \beta$) is not sufficient for efficiency in our framework. The sufficient condition in our environment ensures that the producer receives the entire surplus from the match. We find this result intuitive because the retailer in our framework is merely an intermediary, not providing any additional services (utility) to the final consumer. Conversely, it is the consumption of producers' goods that determines final consumer welfare. Since the social planner does not need to take into account the incentives of the retailer, but rather just pays a fixed per-vacancy cost, to achieve efficiency in the competitive equilibrium we need to impose that the producer appropriates all the bargaining surplus. Aside from this amendment of the Hosios condition, we think the idea that the competitive equilibrium may be inefficient is novel in the trade context. Simply put, it implies that there can be too many (or too few) unmatched producers relative to unmatched retailers in the competitive equilibrium.

4 Open economy

4.1 Open economy introduction

Up to this point, the economy has been closed to international trade. Before opening to trade, we need to outline how markets interact. We assume searching or matching in one segmented market does not affect the costs of searching across markets. In particular, there are no economies of scale in a particular market for individual producers and retailers from being active in any other market. Opening to trade is relatively simple given this setup because we can consider the behavior of individual firms in each market separately. We define each segmented market using a *do* subscript to denote destination-origin country pairs. For example, product market tightness between destination country d and origin country o is denoted by κ_{do} and captures the number of producers and retailers in the *do* product market. Because every producer in every market is a monopolist in their variety, the only interaction among agents in our economy is via aggregate variables such as output, the price index, and search market tightness. Additionally, we assume all agents ignore their effect on these aggregate variables when making optimal decisions.

4.2 Functional form assumptions

In this section, for simplicity and clarity, we present our functional form assumptions which will allow us to obtain closed-form expressions for key objects of interest.

4.2.1 Consumer preferences

We assume the representative consumer in destination market d has Cobb-Douglas utility over a homogeneous and freely traded good, q_1, and a second good, q_2, that is a CES aggregate of differentiated varieties from all origins. The two goods are combined with exponents $1-\alpha$ and α according to

$$U_d(q) = q_1^{1-\alpha}\left[\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)N_k^x\int_{\bar{\varphi}_{dk}}^{\infty} q_{dk}^{\frac{\sigma}{\sigma-1}}(\varphi)\,dG(\varphi)\right]^{\frac{\alpha\sigma}{\sigma-1}} \tag{22}$$

The differentiated goods are substitutable with constant elasticity, $\sigma > 1$, across varieties and destinations.

4.2.2 The ideal price index

Given these preferences, the price index for differentiated goods from origin country o to destination country d is given by

$$P_{do} = \left[\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right)N_o^x\int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)^{1-\sigma}\,dG(\varphi)\right]^{\frac{1}{1-\sigma}} \tag{23}$$

where $p_{do}^{d}(\varphi)$ is the domestic final sales price for each variety. The total price index for differentiated goods in country d is given by

$$P_d = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma}\right]^{1/(1-\sigma)} = \left[\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)N_k^x\int_{\bar{\varphi}_{dk}}^{\infty} p_{dk}^{d}(\varphi)^{1-\sigma}\,dG(\varphi)\right]^{\frac{1}{1-\sigma}} \tag{24}$$

and the overall price index with both homogeneous and differentiated goods is $\Xi_d = \left(\frac{p_1}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_d}{\alpha}\right)^{\alpha}$. More details are provided in Appendix B.1.

4.2.3 Production structure

We will use the familiar variable cost function for producers with productivity φ given by $t\left(q_{do}, \varphi, w_o\right) = q_{do} w_o \tau_{do} \varphi^{-1}$. Here w_o is the competitive wage in the exporting (origin) country, $\tau_{do} \geq 1$ is a parameter capturing one plus the iceberg transport cost between destination d and origin o. Total production cost is $t\left(q_{do}, \varphi, w_o\right) + f_{do}$ where f_{do} is the fixed cost of production. This cost function implies a constant returns to scale production function where labor is the only input. The firm that produces variety q_{do} has efficiency φ and marginal cost equal to $w_o \tau_{do} \varphi^{-1}$.

4.2.4 Productivity distribution

Productivity is Pareto distributed with cumulative density function $G\left[\tilde{\varphi} < \varphi\right] = 1 - \varphi^{-\theta}$ over $[1, +\infty)$ so the probability density function is $g\left(\varphi\right) = \theta \varphi^{-\theta-1}$. We assume that $\theta > \sigma - 1$ so that aggregate variables determined by the integral $\int_{\tilde{\varphi}}^{\infty} z^{\sigma-1} dG\left(z\right)$ are bounded.

4.2.5 Optimal final sales price and quantity traded

Total resources in country d, as discussed in Section 2.6, are the value of the labor endowment, $Y_d = w_d L_d$, where w_d is the wage, and L_d is the endowment in labor units. Given this income and the ideal price index, P_d, consumer preferences imply the demand for a variety, φ, in the differentiated goods sector is $q_{do}\left(\varphi\right) = p_{do}\left(\varphi\right)^{-\sigma} \alpha Y_d P_d^{\sigma-1}$. The quantity traded is determined by negotiation between importers and exporters and ensures that marginal revenue equals marginal cost as determined by equation (9).

If retailers are monopolistic in their variety, equation (9), together with our functional form assumptions on the demand curve, q_{do}, and the cost function, $t(q_{do}, \varphi, w_o)$, imply that the final is $p_{do}(\varphi) = \mu w_o \tau_{do} \varphi^{-1}$ where $\mu = \sigma/(\sigma - 1)$. The price charged for the imported good in the domestic market takes the standard markup times marginal cost form. Using the demand curve and domestic optimal price implies the imported quantity $q_{do}(\varphi) = (\mu w_o \tau_{do} \varphi^{-1})^{-\sigma} \alpha Y_d P_d^{\sigma-1}$. We present the details of these derivations in Appendix B.2.

4.2.6 Productivity threshold

Flow profits from a match where the producer has productivity φ are $\pi_{do}(\varphi) = p_{do}(\varphi) q_{do}(\varphi) - t_{do}(\varphi)$. With these functional form assumptions, profits become $\pi_{do}(\varphi) = \frac{\alpha}{\sigma} \left(\frac{\mu w_o \tau_{do}}{P_d} \right)^{1-\sigma} Y_d \varphi^{\sigma-1} - f_{do}$, with details in Appendix B.2. Using this, the expression for the cutoff from equation (11) becomes

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha} \right)^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d} \right) Y_d^{\frac{1}{1-\sigma}} F_{do}^{\frac{1}{\sigma-1}} \tag{25}$$

where

$$F_{do} = f_{do} + \left(\frac{(r+\lambda)}{\beta \kappa_{do} \chi(\kappa_{do})} \right) l_{do} + \left(1 + \frac{(r+\lambda)}{\beta \kappa_{do} \chi(\kappa_{do})} \right) h_{do} + \frac{(r+\lambda)}{\beta} s_{do} \tag{26}$$

Notice that equation (25) is identical to equation (7) in Chaney (2008), except that our effective entry cost, F_{do}, differs from the fixed entry cost given there.

4.3 Analysis of the open economy

4.3.1 Real income

In this section we discuss how adding search frictions changes the response of real income to foreign shocks. We relate this to Arkolakis et al. (2012) who show that, in many different trade models, real income changes can be summarized by two sufficient statistics: the change in the domestic consumption share in response to a shock and the elasticity of trade with respect to variable trade costs. Our expression includes the change in the fraction of matched producers.

Definition 3. *Define a foreign shock in country d as a change from $(\mathbf{L}, \lambda, \mathbf{r}, \mathbf{f}, \mathbf{c}, \mathbf{l}, \mathbf{s}, \tau)$ to $(\mathbf{L}', \lambda', \mathbf{r}', \mathbf{f}', \mathbf{c}', \mathbf{l}', \mathbf{s}', \tau')$ such that $(L_d, \lambda_d, r_d, f_d, c_d, l_d, s_d, \tau_d) = (L'_d, \lambda'_d, r'_d, f'_d, c'_d, l'_d, s'_d, \tau'_d)$.*

Proposition 8. *Suppose the functional form restrictions in Section 4.2 hold and all consumption is devoted to differentiated goods so that $\alpha = 1$. Then the change in real income associated with any foreign shock in country d can be computed as*

$$\hat{W}_d \quad = \quad \hat{\lambda}_{dd}^{-\frac{1}{\theta}} \hat{F}_{dd}^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \left(\widehat{1 - \frac{u_{dd}}{1-i_{dd}}}\right)^{\frac{1}{\theta}} \tag{27}$$

where $\hat{x} \equiv x'/x$ denotes the change in any variable x between the initial and the new equilibrium, $\lambda_{do} \equiv C_{do}/Y_d$ is the share of country d's total expenditure that is devoted to goods from origin country o, θ governs the productivity distribution, and F_{dd} is the effective cost of entering the home market.

Proof. See Appendix B.3. □

Equation (27) states that the change in real income in country d, $\hat{W}_d$, is a function of the changes in the share of domestic expenditure, λ_{dd}, the effective entry cost, F_{dd}, and the matched rate, $\left(1 - \frac{u_{dd}}{1-i_{dd}}\right)$. As a result, unlike in Arkolakis et al. (2012), changes in the trade share are not sufficient to capture changes in real income. Moreover, if we think that trade liberalization increases the domestic matched rate of producers, our framework would predict larger gains from trade then a framework without goods-market frictions.

4.3.2 Trade elasticity

In their work, Arkolakis et al. (2012) show that in the Melitz (2003) model, the trade elasticity with respect to a change in the iceberg trade costs is given by

$$\frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \varepsilon_o^{ACRdd'} = \begin{cases} (1-\sigma) + \psi_{do} + \left(\psi_{do} - \psi_{dd}\right) \dfrac{\partial \ln\left(\bar{\varphi}_{dd}\right)}{\partial \ln\left(\tau_{do}\right)} & if\ d' = d \\ \\ \left(\psi_{do} - \psi_{dd}\right) \dfrac{\partial \ln\left(\bar{\varphi}_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} & if\ d' \neq d \end{cases} \tag{28}$$

where $\psi_{do} = \dfrac{\partial \ln\left(\Psi_{do}\right)}{\partial \ln\left(\bar{\varphi}_{do}\right)} \geq 0$ and $\Psi_{do} = \int_{\bar{\varphi}_{do}}^{\infty} \varphi^{\sigma-1} dG\left(\varphi\right)$. This is the same as the expression in Arkolakis et al. (2012) equation (21) page 110 except that $\psi_{do} \leq 0$ while $\gamma_{ij} \geq 0$ because we define our model in terms of productivity, φ, while they define theirs in terms of marginal cost.

In our context, the trade elasticity turns out to be similar to equation (28) but in general has many additional endogenous terms. We provide the full expression in Appendix B.4 and present a version under simplifying assumptions here.

Proposition 9. *The elasticity of consumption shares to iceberg trade costs in our model with goods-market frictions is given by*

$$\frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \begin{cases} \varepsilon_o^{ACRdd'} \\ + \left(\dfrac{u_{do}}{1-i_{do}}\right)\left(\dfrac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln\left(\tau_{do}\right)}\right) - \left(\dfrac{u_{dd}}{1-i_{dd}}\right)\left(\dfrac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln\left(\tau_{do}\right)}\right) & if\ d' = d \\ \\ \varepsilon_o^{ACRdd'} \\ + \left(\dfrac{u_{do}}{1-i_{do}}\right)\left(\dfrac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln\left(\tau_{d'o}\right)}\right) - \left(\dfrac{u_{dd}}{1-i_{dd}}\right)\left(\dfrac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)}\right) & if\ d' \neq d \end{cases} \tag{29}$$

where we assume that the number of producers in d and o do not change with tariff changes, $\partial \ln\left(N_d^x\right)/\partial \ln\left(\tau_{d'o}\right) = \partial \ln\left(N_o^x\right)/\partial \ln\left(\tau_{d'o}\right) = 0$ and we assume that $l = 0 = h$ so that F_{dd} and F_{do} are parameters.

Proof. See Appendix B.4. □

Our trade elasticity depends on the usual trade elasticity but also on the fraction of unmatched producers and the elasticity of producers' finding rate in the *do* product market. Using equation (12), we know that raising tariffs, τ_{do}, reduces the value of importing, $M(\varphi)$, and therefore reduces market tightness, κ_{do}, the producers' finding rate, $\kappa_{do}\chi(\kappa_{do})$. This implies that $\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})} < 0$. We suspect that, due to protectionism, $\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})} > 0$. In other words, raising tariffs in the *do* market makes the domestic market more attractive, encouraging domestic retailer entry and thus raising domestic market tightness, κ_{dd}, and domestic producers' finding rate. Since both *dd* and *do* unmatched rates of producers are weakly positive, this implies that the trade elasticity in our model is at least as negative as the trade elasticity in the model of Arkolakis et al. (2012).

4.3.3 The gravity equation

The gravity structure in our model, albeit more complicated, is similar to the gravity structure common to many trade models. To show this, begin with the definition of total imports (free on board) by destination d from origin o in the differentiated goods sector

$$I_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} n_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi) \tag{30}$$

Performing the required integration in equation (30) gives the following proposition.

Proposition 10. *The gravity equation in our model is:*

$$I_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right)(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))\, \alpha \left(\frac{Y_o Y_d}{Y}\right)\left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} \tag{31}$$

where the fraction of matched exporters $1 - \frac{u_{do}}{1 - i_{do}}$ *and the bundle of search frictions* $1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})$ *are each weakly in the unit interval. As such, in a model with search* *frictions, trade is reduced to a fraction of the value that would be obtained without search frictions.*

Proof. See Appendix B.5. □

The main message is clear: search frictions have a first order effect on the level of total imports. Although it is not surprising that adding search frictions reduces trade flows, it is important to realize that instead of simply raising the expected entry cost, F_{do}, faced by firms, they reduce imports to a fraction of the value that would be obtained without search frictions.

Investigating aggregate trade flows in greater detail shows that they reduce trade flows in three ways. First, search frictions give rise to a fraction of unmatched exporters, $\left(1 - \frac{u_{do}}{1 - i_{do}}\right)$. From equation (14) we know that this fraction is less than one, if we ignore the possibility of corner solutions such as $c = 0$ (as in Proposition 2) or $\lambda = 0$. Second, trade flows are diminished because imports are computed using import prices (as opposed to final sales prices) and these negotiated import prices are lower than final sales prices. This is consistent with work by Berger et al. (2012) who use prices for the same product to document that retail prices are 50 to 70 percent higher than at the dock. These lower import prices lead to the bundle of search friction parameters $1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)$ which is less than one as shown in Appendix B.5. Third, search frictions reduce imports due to the negative exponent on the effective entry cost, F_{do}, which is increasing in search frictions as shown in equation (26).

Even if imports are measured at final sales prices, as they are in the typical gravity equation, search frictions have a significant effect on trade flows. By evaluating equation (31) at $p_{do}(\varphi)$ instead of $n_{do}(\varphi)$ and using our functional form assumptions, we compute imports at final sales prices in Appendix B.5 as

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \alpha \left(\frac{Y_o Y_d}{Y}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} \tag{32}$$

By definition, this provides consumption expenditure in destination d on differentiated goods produced in origin o. Even without the wedge between final and import prices reflected in $b(\cdot)$, search frictions lead to a mass of inactive and searching producers $\left(1-\frac{u_{do}}{1-i_{do}}\right)$ which lowers imports. Search frictions also enter effective entry cost, F_{do} as before.

Consumption expenditure must equal imports plus the period profits of matched importers, $C_{do} = I_{do} + \Pi_{do}$. Combining equations (30) and (32) gives total period profits accruing to importers in matched relationships

$$\Pi_{do} = b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})\, \alpha \left(\frac{Y_o Y_d}{Y}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} \tag{33}$$

We could also obtain this quantity if we integrate profit to each variety $p_{do}(\varphi)\, q_{do}(\varphi) - n_{do}(\varphi)\, q_{do}(\varphi)$ over all imported varieties. Despite the value of posting a vacancy being driven to zero by free entry, flow profits are always positive as long as retailers' search costs, c ,are positive so that importers can recoup the costs expended while searching.

4.3.4 Implications for estimation

The fact that introducing search frictions into a model of trade results in a scalar times the typical gravity equation has a few interesting implications for estimation.

First, if the fraction of matched exporters and the bundle of search friction parameters do not vary by destination-origin pairs, then their impact on trade would be lost in the constant term of a gravity regression. In this case, while estimates of the other coefficients in the model would be unbiased, search frictions could be a pervasive feature of international trade but would not be identifiable using the gravity equation.

Second, if the fraction of unmatched producers and bundle of search frictions vary by importer-exporter pair, they may provide an additional rationale for why language, currency, common legal origin, historical colonial ties or other variables often included in gravity equations have an effect on aggregate trade flows. In particular, Rauch and Trindade (2002) argue populations of ethnic Chinese within a country facilitate the flow of information, provide matching and referral services, and otherwise reduce informal barriers to trade. Their empirical specification matches the gravity equation with search that we have derived here if the destination-origin search frictions are a function of the ethnically Chinese population.

Third, any gravity regression that does not include adequate proxies for search frictions would suffer from omitted variable bias. In particular, suppose that a researcher omits search frictions, as measured by the matched rate, and estimates the following equation:

$$\ln I_{do} = A_d + B_o + \beta_0 \ln \tau_{do} + \nu_{do} \tag{34}$$

where I_{do} are the imports from origin o to destination d, A_d is an importer-specific term, B_o is an exporter-specific term, β_0 is the partial elasticity of bilateral imports with respect to variable trade costs, and ν_{do} is an error term. Econometric theory suggests that the omitted variable, $Z_{do} = 1 - \dfrac{u_{do}}{1 - i_{do}}$, will introduce bias into the OLS estimate of $\hat{\beta}_0$ according to the well-known formula:

$$\mathbb{E}[\hat{\beta}_0|\mathbf{X}] = \beta_0 + \rho(\tau_{do}, Z_{do})\rho(I_{do}, Z_{do}) \tag{35}$$

where $\mathbf{X}$ is a vector of all right-hand-side variables and $\rho\,(X, Y)$ is the correlation between X and Y. We know that $\rho(\tau_{do}, Z_{do}) < 0$ (higher variable trade costs, τ_{do}, raise the threshold productivity, $\bar{\varphi}_{do}$, increasing the fraction of idle firms, i_{do}, and lowering the matched rate) and $\rho(I_{do}, Z_{do}) > 0$ (increasing the matched rate increases trade flows) so the sign of the bias is negative.

Proposition 11. *Omitting the matched rate from a standard gravity equation implies that* *the estimate of trade elasticity with respect to variable trade costs is more negative than if one included the matched rate in the estimating equation.*

Proof. This follows from the discussion in the text. □

5 Conclusion

The current framework presents a rich and tractable environment which we think has many interesting implications for considering search and trade in a combined framework. Among these are introducing a concept of unmatched producers and retailers in product markets, the typical inefficiency of the competitive equilibrium, the response of welfare to tariff adjustments, and the effects of search on the standard gravity equation.

Including search frictions at the micro level has significant implications for steady-state trade flows in the aggregate. Issues that seem of first-order importance for future work include: a) incorporating endogenous separations so that larger, more productive firms are in more stable trading relationships; b) modeling dynamics to include the transition path after a relevant exogenous shock; c) disciplining the model with data on trade relationships; d) modeling non-Nash bargaining protocols; and e) modeling the international transmission of shocks.

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A Search model appendix

A.1 The surplus, value and expected duration of a relationship

Denote the joint surplus accruing to both sides of a match as $S(\varphi)$. The bargain will divide this surplus such that the value of being a retailer equals $M(\varphi) - V = (1-\beta) S(\varphi)$ and the value of being a producer is $X(\varphi) - U(\varphi) = \beta S(\varphi)$ where β is the producer's bargaining power. Using the value functions presented in the main text (1), (2), (4), and (5) we can write the surplus equation as

$$S(\varphi) \quad = \quad \frac{pq - t(q) - f + l + s\kappa\chi(\kappa)}{r + \lambda + \beta\kappa\chi(\kappa)}$$

The surplus created by a match is the appropriately discounted flow profit, with the search cost l and the sunk cost s also entering the surplus equation because being matched avoids paying these costs. There are three things to notice here. First, the surplus from a match is a function of productivity. We show in Appendix A.5 that matches that include a more productive exporting firm lead to greater surplus, i.e. $S'(\varphi) > 0$. Second, the value of the relationship will fluctuate over the business cycle as shocks hit the economy and change the finding rate $\kappa\chi(\kappa)$. Finally, surplus is greater than or equal to zero when

$$pq - t(q) - f + l + s\kappa\chi(\kappa) \geq 0$$

Specifically, at the binding productivity cutoff we can use equation (10) and the surplus sharing rule to write

$$\beta S(\bar{\varphi}) = \frac{l+h}{\kappa\chi(\kappa)} + s$$

which, in order for surplus to be positive, puts a restriction on the parameter choices and the equilibrium value of market tightness, κ.

With the definition of surplus in hand, the value of a matched relationship, $R(\varphi) = X(\varphi) + M(\varphi)$, can be expressed as $R(\varphi) = S(\varphi)\left(\frac{r + \kappa\chi(\kappa)\beta}{r}\right) - \frac{l}{r}$. The value of the relationship to the producer is of course $X(\varphi)$ and to the retailer $M(\varphi)$. The value of a relationship in product markets has been of recent interest in Monarch and Schmidt-Eisenlohr (2015), Heise (2015a), and Heise (2015b).

Relationships are destroyed at Poisson rate λ in the model which implies the average duration of each match is $1/\lambda$. Since the destruction rate is exogenous and does not vary in our model, the average duration of each match is constant.

A.2 Solving the Nash bargain

For this section, it will be helpful to note that equations (1) and (4) imply that

$$X(\varphi) = \frac{nq - t(q) - f + \lambda U(\varphi)}{r + \lambda} \tag{36}$$

and

$$M(\varphi) = \frac{p(q)\,q - nq}{r + \lambda} \tag{37}$$

A.2.1 Bargaining over the price

Take equation (6), log and differentiate with respect to the price n and re-arrange to get

$$\beta \frac{q}{X(\varphi) - U(\varphi)} + (1 - \beta) \frac{-q}{M(\varphi) - V} = 0 \tag{38}$$

which implies the simple surplus sharing rule: the retailer receives β of the total surplus from the trading relationship, $S(\varphi) = M(\varphi) - V + X(\varphi) - U(\varphi)$. The producer receives the rest of the surplus, $(1 - \beta) S(\varphi)$.

In Section 2.5.3 of the main text, we point out the restriction that $\beta < 1$ in equation (6) is evident in equation (8) which results from equation (38). Retailing firms have no incentive to search if $\beta = 1$ because they get none of the resulting match surplus and therefore cannot recoup search costs $c > 0$. Any solution to the model with $c > 0$ and positive trade between retailers and producers also requires $\beta < 1$. This can be shown explicitly by using equations (4), (5), and (13) together with $\beta = 1$ to show that for productivity, φ, levels above the reservation productivity, $\bar{\varphi}$, (defined in Section 2.5.5), the retailing firm has no incentive to search.

Finally, we do not need to calculate the partial derivative with respect to $U(\varphi)$ or $V(\varphi)$ because the individual firms are too small to influence aggregate values. Hence, when they meet, the firms bargain over the negotiated price taking behavior in the rest of the economy as given. In particular, the outside option of the firms does not vary with the individual's bargaining problem.

A.2.2 Bargaining over the quantity

Take equation (6), log and differentiate with respect to the quantity q to get

$$\beta \frac{1}{X(\varphi) - U(\varphi)} (n - p'(q)) + (1 - \beta) \frac{1}{M(\varphi) - V} (p(q) + p'(q)\,q - n) = 0 \tag{39}$$

where we compute the partials of $X(\varphi)$ and $M(\varphi)$ using equations (36) and (37). Now, notice that equation (38) implies that $X(\varphi) - U(\varphi) = \frac{\beta}{1-\beta}(M(\varphi) - V)$, and plugging this into equation (39) and re-arranging slightly gives

$$p(q) + p'(q)\,q = t'(q) \tag{40}$$

This expression says that the quantity produced and traded is pinned down by equating marginal revenue in the domestic market with marginal production cost in the foreign country. This is the same restriction we get from a model without search and therefore implies that adding search does not change the quantity traded. The profit maximization implied by this equation is crucial: despite being separate entities, the retailer and the producer decide to set marginal revenue equal to marginal cost. The result follows because of the simple sharing rule, the maximization of joint surplus, and the trivial role of the retailer. In order to maximize surplus, the parties choose to equate marginal revenue and marginal cost.

A.3 Retailer production function

In this section we show that including another input for the retailer does not affect the conclusions of this paper under some weak additional assumptions. With an additional input, the value of being in a relationship for a retailer changes to

$$rM(\varphi) = p(f(q,k))f(q,k) - nq - n_k k - \lambda(M(\varphi) - V) \tag{41}$$

where retailer combines the input, denoted by k, with the input purchased from the producer, q, according to production function $f(q,k)$ for the final good sold to consumers. The price of the additional input, n_k, is determined outside of the search model and is taken as given by the retailer.

With this new Bellman equation, logging and differentiating the Nash product in equation (6) with respect to p gives the same surplus sharing (8) rule as before. The first order condition of equation (6) with respect to q, however, becomes

$$\beta \frac{n - t'(q)}{X(\varphi) - U(\varphi)} + (1-\beta) \frac{p'(f(q,h))f_q(q,h)f(q,h) + p(f(q,h))f_q(q,h) - n}{M(\varphi) - V} = 0 \tag{42}$$

Combining this with the surplus sharing rule (8) yields an expression similar to equation (9):

$$p'(f(q,h))f_q(q,h)f(q,h) + p(f(q,h))f_q(q,h) = t'(q) \tag{43}$$

This states that retailers and producers will negotiate to trade a quantity of q that ensures the marginal revenue equals marginal cost. Since the price of input k is taken as given, the firm chooses the optimal level of the input, k^*, so that $f_k(q,k^*) = n_k$. Strict concavity of the function $f(q,k)$ is sufficient to ensure that $f_k(q,k)$ is invertible. Making this assumption gives $f_k^{-1}(q,n) = k^*$ which can be substituted into equation (43) to get one equation in one unknown, q. The quantity traded depends on n_k, the price of the other input, but search frictions still do not enter the equation (43). The result in the main text, that optimal q is determined by the condition that ensures marginal revenue from q equals the marginal cost of producing q, remains intact.

A.4 Solving for the productivity thresholds

A.4.1 Solving for the lowest productivity threshold

First, let's solve for an expression for $X(\varphi) - U(\varphi)$, by plugging in equations (1) and (2):

$$\begin{aligned} rX(\varphi) - rU(\varphi) &= nq - t\left(q\right) - f - \lambda\left(X(\varphi) - U(\varphi)\right) + l - \kappa\chi(\kappa)(X(\varphi) - U(\varphi) - s) \\ &= nq - t\left(q\right) - f + l + \kappa\chi(\kappa)s - (\lambda + \kappa\chi(\kappa))(X(\varphi) - U(\varphi)) \\ &\Rightarrow (r + \lambda + \kappa\chi(\kappa))(X(\varphi) - U(\varphi)) = nq - t\left(q\right) - f + l + \kappa\chi(\kappa)s \qquad (44) \\ &\Rightarrow X(\varphi) - U(\varphi) = \frac{nq - t\left(q\right) - f + l + \kappa\chi(\kappa)s}{r + \lambda + \kappa\chi(\kappa)} \end{aligned}$$

Now plug this expression into the definition of $\underline{\varphi}$ from the main text to get

$$\begin{aligned} \frac{nq - t\left(q\right) - f + l + \kappa\chi(\kappa)s}{r + \lambda + \kappa\chi(\kappa)} &= 0 \\ \Rightarrow nq - t\left(q\right) - f + l + \kappa\chi(\kappa)s &= 0 \end{aligned}$$

By using the fact that $X'\left(\varphi\right) - U'\left(\varphi\right) > 0$ from above we can state that this threshold is unique.

We can be sure that for any positive cost of forming a relationship, $\frac{l+h}{\kappa\chi\left(\kappa\right)} + s$, (if and only if $l + h + \kappa\chi\left(\kappa\right)s > 0$) the expression $X\left(\bar{\varphi}\right) - U\left(\bar{\varphi}\right)$ exceeds $X\left(\underline{\varphi}\right) - U\left(\underline{\varphi}\right)$. As long as $X\left(\varphi\right) - U\left(\varphi\right)$ is increasing in φ, this implies that $\bar{\varphi} > \underline{\varphi}$. In Appendix A.5 we show the very general conditions under which $X\left(\varphi\right) - U\left(\varphi\right)$ is increasing in φ. The binding productivity threshold defining the mass of producers that have retail partners is the greater of these two and hence $\bar{\varphi}$. In other words, the productivity necessary to induce a producer to search for a retail partner is greater than the productivity necessary to consummate a match after meeting a retailer due to the costs that are incurred while searching. Similarly, the productivity necessary to form a match is greater than the productivity to maintain one already in place.

A.4.2 Proof of Proposition 1: Solving for the binding productivity threshold

Using equation (44) in the definition of $\bar{\varphi}$ in equation (10) yields

$$\begin{aligned}
\frac{nq - t(q) - f + l + \kappa\chi(\kappa)s}{r + \lambda + \kappa\chi(\kappa)} &= \frac{l+h}{\kappa\chi(\kappa)} + s \\
\Rightarrow nq - t(q) - f + l + \kappa\chi(\kappa)s &= (r + \lambda + \kappa\chi(\kappa))\frac{s\kappa\chi(\kappa) + l + h}{\kappa\chi(\kappa)} \\
\Rightarrow nq - t(q) - f + l + \kappa\chi(\kappa)s &= (r + \lambda)\frac{s\kappa\chi(\kappa) + l + h}{\kappa\chi(\kappa)} + s\kappa\chi(\kappa) + l + h \\
\Rightarrow nq - t(q) - f &= (r + \lambda)\frac{s\kappa\chi(\kappa) + l + h}{\kappa\chi(\kappa)} + h \\
\Rightarrow nq - t(q) - f - (r+\lambda)\frac{l+h}{\kappa\chi(\kappa)} - h &= (r+\lambda)s \\
\Rightarrow nq - t(q) - f &= (r+\lambda)s + (r+\lambda)\frac{l+h}{\kappa\chi(\kappa)} + h
\end{aligned}$$

Now, plug in for the equilibrium import price, n, from equation (13), to get

$$(1-\gamma)\,p(q)\,q + \gamma\,(t(q) - f - l - \kappa\chi(\kappa)s) - t(q) - f = (r+\lambda)\,s + \frac{(r+\lambda)}{\kappa\chi(\kappa)}l + \left(1 + \frac{(r+\lambda)}{\kappa\chi(\kappa)}\right)h$$

which can be re-arranged to obtain

$$p(q)\,q - t(q) - f = (1-\gamma)^{-1}\left[(r + \lambda + \gamma\kappa\chi(\kappa))\,s + \left(\gamma + \frac{(r+\lambda)}{\kappa\chi(\kappa)}\right)l + \left(1 + \frac{(r+\lambda)}{\kappa\chi(\kappa)}\right)h\right]$$

Further simplification of the terms with γ imply that

$$p(q)\,q - t(q) = f + \left(\frac{(r+\lambda)}{\beta\kappa\chi(\kappa)}\right)l + \left(1 + \frac{(r+\lambda)}{\beta\kappa\chi(\kappa)}\right)h + \frac{(r+\lambda)}{\beta}s$$

which is the expression in the main text.

A.4.3 Comparing our productivity threshold to previous models

Defining $F(\kappa) \equiv f + \left(\frac{(r+\lambda)}{\beta\kappa\chi(\kappa)}\right)l + \left(1 + \frac{(r+\lambda)}{\beta\kappa\chi(\kappa)}\right)h + \frac{(r+\lambda)}{\beta}s$ in this framework would allow us to replace the fixed cost in the standard models with $F(\kappa)$ from here. The key thing to remember when working with the other quantities of our model is we just want to work with them in terms of the cutoff and not in terms of these fundamental frictions just yet.

Another interesting comparison is to Eaton et al. (2014). That framework included a flow search cost, l, but did not have a sunk costs s or any idle state. If we set $h = 0$, we are implicitly including an idle state since the producer will have a zero value for being in the idle state but have a negative flow cost, $-l$, for being in the searching state since that state requires a payment each period of $l > 0$. In other words, since the producer cannot opt out of searching we must set the flow of the idle state to $h = -l$ instead of what one might think is the intuitive value of that state $h = 0$. Making this assumption together with $s = 0$ provides:

$$\begin{aligned} p(q)\,q - t(q) &= f + \left(\frac{(r+\lambda)}{\beta\kappa\chi(\kappa)}\right) l + \left(1 + \frac{(r+\lambda)}{\beta\kappa\chi(\kappa)}\right) h + \frac{(r+\lambda)}{\beta} s \\ &= f - l \end{aligned}$$

This is the very reason why Eaton et al. (2014) must have that $f > l$. Notice that we recover the standard model when we make these assumptions together with $l = 0$:

$$\begin{aligned} p(q)\,q - t(q) &= f + \left(\frac{(r+\lambda)}{\beta\kappa\chi(\kappa)}\right) l + \left(1 + \frac{(r+\lambda)}{\beta\kappa\chi(\kappa)}\right) h + \frac{(r+\lambda)}{\beta} s \\ &= f \end{aligned}$$

Another interesting way to remove just the search friction, l, from the model is to set the finding rate $\kappa\chi(\kappa) \to \infty$ so that $\dfrac{(r+\lambda)}{\beta\kappa\chi(\kappa)} \to 0$

$$\begin{aligned} p(q)\,q - t(q) &= f + \left(\frac{(r+\lambda)}{\beta\kappa\chi(\kappa)}\right) l + \left(1 + \frac{(r+\lambda)}{\beta\kappa\chi(\kappa)}\right) h + \frac{(r+\lambda)}{\beta} s \\ &= f + h + \frac{(r+\lambda)}{\beta} s \end{aligned}$$

The interpretation of the fixed cost includes the sunk cost, bargaining power of the producer, and the flow from the outside option even if one finds a partner immediately. If bargaining power differs by market, for example, the bundle of entry costs will as well.

A.5 The value of importing is strictly increasing in productivity

Here we show that the value of importing, $M(\varphi)$, is strictly increasing with the producer's productivity level, φ. This fact allows us to replace the integral of the max over V and $M(\varphi)$ (equation 5), with the integral of $M(\varphi)$ from the productivity threshold, $\bar{\varphi}$ (equation 12).

Starting with equation (4) and $V = 0$ obtain

$$\begin{aligned} (r+\lambda)\,M(\varphi) &= pq - nq \\ &= pq - [1-\gamma]\,pq - \gamma\left(t(q) + f - l - \kappa\chi(\kappa)\,s\right) \\ &= \gamma pq - \gamma\left(t(q) + f\right) + \gamma\left(l + \kappa\chi(\kappa)\,s\right) \\ &= \gamma\left(pq - t(q) - f\right) + \gamma\left(l + \kappa\chi(\kappa)\,s\right) \end{aligned}$$

Remember that $\gamma \equiv \frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa\chi(\kappa)}$. It is clear from the integral in the import relationship creation equation (12) that neither the finding rate for retailers, $\chi(\kappa)$, nor the tightness, κ, are functions of the productivity, φ. Given this, $M'(\varphi)$ and $\frac{\partial[p(q)q - t(q) - f]}{\partial\varphi}$ will have the same sign. As long as flow profits absent search frictions are strictly increasing in productivity, $M'(\varphi) > 0$. Using the specific functional forms for $t(q) + f$ used above, as well as the equilibrium values for n, p and q, we can derive this result explicitly. In this case

$$M(\varphi) = \gamma_{do}\left(\frac{1}{r+\lambda}\right)\left(\frac{\mu^{-\sigma}}{\sigma-1}\right)(w_o\tau_{do})^{1-\sigma}\alpha Y_d P^{\sigma-1}\varphi^{\sigma-1} - \gamma_{do}f_{do} + \gamma_{do}\left(l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do}\right)$$

Therefore the derivative is

$$\frac{\partial M(\varphi)}{\partial\varphi} = \gamma_{do}\left(\frac{1}{r+\lambda}\right)\mu^{-\sigma}(w_o\tau_{do})^{1-\sigma}\alpha Y_d P^{\sigma-1}\varphi^{\sigma-2}$$

which is always positive.

As long as $M'(\varphi) > 0$ we can demonstrate the way in which many other important quantities depend on the producer's productivity level, φ. From the surplus sharing rule (38) rewritten as

$$\beta M(\varphi) = (1-\beta)(X(\varphi) - U(\varphi)) \tag{45}$$

we know that in equilibrium since $M'(\varphi) > 0$ it must be that $X'(\varphi) - U'(\varphi) > 0$. Differentiating both sides of equation (2) gives $rU'(\varphi) = \kappa\chi(\kappa)(X'(\varphi) - U'(\varphi)) > 0$. We can combine these facts to show $X'(\varphi) > U'(\varphi) > 0$. Using the definition of the joint surplus of a match $S(\varphi) = X(\varphi) + M(\varphi) - U(\varphi) - V$ we get $S'(\varphi) > 0$. Likewise the value of a relationship, $R(\varphi) = X(\varphi) + M(\varphi)$, has $R'(\varphi) > 0$.

A.6 Proof of Proposition 2: Market tightness and the cost of search

Let's first prove that $\kappa < \infty$ if $c > 0$. To do this, let's prove the contrapositive: assume that $c = 0$ and show that $\kappa = \infty$. Re-arrange equation (12) slightly to get

$$0 = c = \chi(\kappa)\int_{\bar{\varphi}} M(\varphi)\,dG(\varphi)$$

We have shown than $M(\bar{\varphi}) \geq 0$ for any consummated match in equilibrium (Nash bargaining together with Appendix A.4) and $M'(\varphi) > 0$ (Appendix A.5). Therefore we know that $\int_{\bar{\varphi}} M(\varphi)\,dG(\varphi) > 0$. As such, $\chi(\kappa)$ must be zero. Since, $\chi'(\kappa) < 0$ this is true if and only if $\kappa = \infty$.

To prove that if $c > 0$ then $\kappa < \infty$, let's use equation (12) again. In particular, since $c > 0$ it must mean that $\chi(\kappa)\int_{\bar{\varphi}} M(\varphi)\,dG(\varphi) > 0$. As before, we know that

$\int_{\bar{\varphi}} M(\varphi)\, dG(\varphi) > 0$ so it must be that $\chi(\kappa) > 0$ as well, which is true if and only if $\kappa < \infty$.

A.7 Producer and retailer existence

A.7.1 Retailing firms

Free entry implies that the ex-ante expected value from entering for a potential retailer equals the expected cost of entering. Assume for a moment that the potential retailers consider the value of becoming a retailer as defined by E_m where

$$rE_m = -e_m + (V - E_m) \tag{46}$$

The potential retailer could sell the value E_m and invest the proceeds at the interest rate r getting flow payoff rE_m forever after. Alternatively, they could pay a cost e_m to become a retailer at which point they will begin in the state of having a vacancy with value V (with certainty) and give up the value of being a potential retailer E_m. Free entry into becoming a retailer implies that $E_m = 0$ in equilbrium so that

$$\begin{aligned} 0 &= -e_m + V \\ e_m &= V \end{aligned}$$

Hence, free entry into vacancies $V = 0$ implies $e_m = 0$ and we cannot have a sunk cost for entry into retailing. In other words, free entry into the search market along with assuming that one must post a vacancy before matching implies free entry into retailing.

Free entry into the search market subsumes free entry into retailing and so we only have one condition defined by free entry on the retailing side given by equation (12) and restated here

$$\frac{c}{\chi(\kappa)} = \int_{\bar{\varphi}} M(\varphi)\, dG(\varphi)$$

Remember this states that product vacancies continue being created until the expected cost of being an unmatched retailer, $c/\chi(\kappa)$, equals the expected benefit $\int_{\bar{\varphi}} M(\varphi)\, dG(\varphi)$. Because each potential retailer must post a product vacancy before forming a match, the expected cost of becoming a retailer (entering as a retailer) is the same as the expected cost of being an unmatched retailer. Likewise, the expected benefit of posting a vacancy and the expected benefit of becoming a retailer are also the same because we assume retailers must post a vacancy before matching.

A.7.2 Producing firms

Similar to the entry decision of retailers, the value of entry for producers, E_x, is defined by

$$\begin{aligned} rE_x &= -e_x + \int \max\{I(\varphi), U(\varphi)\} dG(\varphi) - E_x \\ &= -e_x + \int_1^{\bar{\varphi}} I(\varphi) dG(\varphi) + \int_{\bar{\varphi}}^{\infty} U(\varphi) dG(\varphi) - E_x \end{aligned}$$

We assume that the potential producer must transit through the unmatched state before forming a match. After paying e_x and taking a productivity draw φ, the potential producer loses the value E_x with certainty and, depending on the drawn productivity, chooses between searching for a retailer and getting value $U(\varphi)$ and remaining idle getting value $I(\varphi)$. If we assumed free entry into production, we would get $E_x = 0$ and that

$$e_x = \int_1^{\bar{\varphi}} I(\varphi) dG(\varphi) + \int_{\bar{\varphi}}^{\infty} U(\varphi) dG(\varphi) \tag{47}$$

which ensures the expected value of taking a productivity draw equals the expected cost.

Free entry into production, therefore, imposes another restriction on the equilibrium. We can use the facts that $X(\varphi) - U(\varphi) = (1-\beta) S(\varphi)$ and that $M(\varphi) = \beta S(\varphi)$ to write $X(\varphi) - U(\varphi) = \left(\frac{1-\beta}{\beta}\right) M(\varphi)$. Applying this to equation (2) gives

$$rU(\varphi) = -l + \kappa\chi(\kappa)\left(\left(\frac{1-\beta}{\beta}\right) M(\varphi) - s\right)$$

Computing the relevant integrals in equation (47)

$$\int_{\bar{\varphi}}^{\infty} U(\varphi) dG(\varphi) = -\left(\frac{l + s\kappa\chi(\kappa)}{r}\right)(1 - G(\bar{\varphi})) + \frac{\kappa\chi(\kappa)}{r}\left(\frac{1-\beta}{\beta}\right)\int_{\bar{\varphi}}^{\infty} M(\varphi) dG(\varphi)$$

Likewise from (3) we have

$$\int_1^{\bar{\varphi}} I(\varphi) dG(\varphi) = \frac{h}{r} G(\bar{\varphi})$$

Combining these with equation (47) gives

$$e_x = \frac{h}{r} G(\bar{\varphi}) - \left(\frac{l + s\kappa\chi(\kappa)}{r}\right)(1 - G(\bar{\varphi})) + \frac{\kappa\chi(\kappa)}{r}\left(\frac{1-\beta}{\beta}\right)\int_{\bar{\varphi}}^{\infty} M(\varphi) dG(\bar{\varphi}) \tag{48}$$

which is the restriction that free entry into production for producers would place on equilibrium market tightness κ.

From equation (47), we can see that free entry into search for producers would require

$\int_{\bar{\varphi}}^{\infty} U(\varphi)\, dG(\varphi) = 0$ in which case we would be left with

$$e_x = \frac{h}{r} G(\bar{\varphi}) \tag{49}$$

which still places a restriction on market tightness because $\bar{\varphi}$ from Proposition (11) includes κ.

Finally, we note that simultaneous combinations of free entry on both sides of the market are possible. Combining free entry into both existence and search for retailers from equation (12) with free entry into existence for producers from equation (48) gives

$$e_x = \frac{h}{r} G(\bar{\varphi}) - \left(\frac{l + s\kappa\chi(\kappa)}{r}\right)(1 - G(\bar{\varphi})) + \frac{c\kappa}{r}\left(\frac{1-\beta}{\beta}\right)$$

Likewise, allowing for free entry into both existence and search for retailers and producers would give the following system that defines κ

$$\begin{aligned} \frac{c}{\chi(\kappa)} &= \int_{\bar{\varphi}} M(\varphi)\, dG(\varphi) \\ e_x &= \frac{h}{r} G(\bar{\varphi}) \end{aligned}$$

A.8 Proof of Proposition 3: Solving for the equilibrium negotiated price

Start with equation (39) and re-arrange considerably:

$$\begin{aligned} \beta\frac{p(q)\, q - nq}{r + \lambda} &= (1-\beta)\frac{nq - t(q) - f + l + \kappa\chi(\kappa)\, s}{r + \lambda + \kappa\chi(\kappa)} \\ nq &= p(q)\, q(1-\gamma) + \gamma\left[t(q) + f - l - \kappa\chi(\kappa)\, s\right] \end{aligned}$$

where

$$\gamma = \frac{(r+\lambda)(1-\beta)}{r + \lambda + \beta\kappa\chi(\kappa)}$$

We show that $\gamma \in [0,1]$ in Appendix A.9.

The limit of γ when the finding rate $\kappa\chi(\kappa) \to \infty$ is simply $\gamma \equiv \dfrac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa\chi(\kappa)} \to 0$.

More complicated is the limit of $\gamma\kappa\chi(\kappa)$ as $\kappa\chi(\kappa) \to \infty$. First rewrite the expression as

$$\gamma\kappa\chi(\kappa) = \frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa\chi(\kappa)}\kappa\chi(\kappa)$$

Dividing the top and bottom of this expression by $\kappa\chi(\kappa)$ yields

$$\gamma\kappa\chi(\kappa) = \frac{(r+\lambda)(1-\beta)}{\dfrac{r+\lambda}{\kappa\chi(\kappa)}+\beta}$$

Now use this to derive the limit

$$\begin{aligned}
\lim_{\kappa\chi(\kappa)\to\infty} \gamma\kappa\chi(\kappa) &= \lim_{\kappa\chi(\kappa)\to\infty} \frac{(r+\lambda)(1-\beta)}{\dfrac{r+\lambda}{\kappa\chi(\kappa)}+\beta} \\
&= \frac{(r+\lambda)(1-\beta)}{\beta}
\end{aligned}$$

This can be used to derive limit of the negotiated price, n, as $\kappa\chi(\kappa)\to\infty$:

$$\begin{aligned}
\lim_{\kappa\chi(\kappa)\to\infty} n &= \lim_{\kappa\chi(\kappa)\to\infty}\left[[1-\gamma]\,p(q,Y,P)+\gamma\frac{t(q,\varphi,w,\tau)+f-l-\kappa\chi(\kappa)\,s}{q}\right] \\
&= \lim_{\kappa\chi(\kappa)\to\infty}\left[p(q,Y,P)-\gamma p(q,Y,P)+\gamma\left[\frac{t(q,\varphi,w,\tau)+f-l}{q}\right]-\frac{\gamma\kappa\chi(\kappa)\,s}{q}\right] \\
&= p(q,Y,P)-p(q,Y,P)\lim_{\kappa\chi(\kappa)\to\infty}\gamma+\left[\frac{t(q,\varphi,w,\tau)+f-l}{q}\right]\lim_{\kappa\chi(\kappa)\to\infty}\gamma-\frac{s}{q}\lim_{\kappa\chi(\kappa)\to\infty}\gamma\kappa\chi(\kappa) \\
&= p(q,Y,P)-p(q,Y,P)\cdot 0+\left[\frac{t(q,\varphi,w,\tau)+f-l}{q}\right]\cdot 0-\frac{s(r+\lambda)(1-\beta)}{q\beta} \\
&= p(q,Y,P)-\frac{s(r+\lambda)(1-\beta)}{q\beta}
\end{aligned}$$

The negotiated price is the final sales price, less the amount required to compensate the producer for the sunk cost to start up the business relationship. Notice that if $s=0$ then the negotiated price would be the final sales price as in standard trade models.

A.9 Bounding the search friction

Here we show that $\gamma\in[0,1]$. Starting with the definition

$$\gamma \equiv \frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa\chi(\kappa)}$$

Since all parameters are positive, $\gamma\geq 0$. The lower bound,$\gamma=0$, is reached only when $\beta=1$ and $c=0$ simultaneously. Next prove $\gamma\leq 1$ by contradiction. Assuming $\gamma>1$ implies that $0>\beta\kappa\chi(\kappa)$ which is a contradiction since $\beta\geq 0$ and $\kappa\chi(\kappa)\geq 0$.

A.10 Closed economy accounting

We first work through the expenditure approach and then turn to the income approach in the closed economy framework.

A.10.1 Expenditure approach

The expenditure approach to accounting allows total resources in the economy, Y, to be allocated to consumption, C, investing in searching and creating firms, I, and funding the government, G.

$$Y = C + I + G$$

Consumption expenditure is

$$C = \left(\frac{1-v}{1-i}\right) N^m \int_{\bar{\varphi}} p\left(\varphi\right) q\left(\varphi\right) dG\left(\varphi\right)$$

Investments in searching to create retailer-producer relationships, pay for period fixed goods production costs, and fund entry is

$$I = vN^m wc + uN^x \left(wl + ws\kappa\chi\left(\kappa\right)\right) + \left(1-u-i\right) N^x wf + N^m we_m + N^x we_x$$

Government is assumed to tax production cost at a proportional rate $\tau \geq 1$ and levy a lump sum tax on household's consumption to fund payments to idle firms.

$$\begin{aligned} G &= \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} \left(\tau - 1\right) t\left(q\right) dG\left(\varphi\right) + \left(1-i\right) N^x wh - \left(1-i\right) N^x wh \\ &= \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} \left(\tau - 1\right) t\left(q\right) dG\left(\varphi\right) \end{aligned}$$

To account for all resources in the economy, we assume the government spends the tax/tariff revenue raised by proportionally taxing variable production cost at rate $\tau \geq 1$, which enters equilibrium value $p(\varphi)q(\varphi)$. According to this equation the government buys labor from the household but does not use the labor to produce anything. As such, those resources are wasted. We make this assumption in the closed economy because it follows the international trade treatment of iceberg trade costs where resources simply "melt" away in transit. We could assume that tax revenues are refunded lump sum to the household in order to ensure that $G = 0$. In that case, taxes would still have a distortionary effect on production costs but would not reduce available resources.

Combining the terms defined above through the expenditure accounting identity gives

$$\begin{aligned} Y &= \left(\frac{1-v}{1-i}\right) N^m \int_{\bar{\varphi}} p(\varphi)\, q(\varphi)\, dG(\varphi) \\ &+ vN^m wc + uN^x \left(wl + ws\kappa\chi(\kappa)\right) + (1-u-i)\, N^x wf \\ &+ N^m we_m + N^x we_x \\ &+ \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} (\tau - 1)\, t(q)\, dG(\varphi) \end{aligned}$$

To simplify this expression, we assume that every matched producer must have one, and only one, retailer as their counterpart. Doing so implies the mass of matched producers and retailers must be equal, $(1-u-i)\, N^x = (1-v)\, N^m$. We also use the definition of κ to substitute for $N^m = \kappa v^{-1} u N^x$ leading to the expression in the main text

$$\begin{aligned} Y &= \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} p(\varphi)\, q(\varphi)\, dG(\varphi) \\ &+ \kappa u N^x wc + uN^x \left(wl + ws\kappa\chi(\kappa)\right) + (1-u-i)\, N^x wf \\ &+ \kappa v^{-1} u N^x we_m + N^x we_x \\ &+ \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} (\tau - 1)\, t(q)\, dG(\varphi) \end{aligned}$$

We will get the same expression if we instead use the income approach to national accounting.

A.10.2 Income approach

Total income in the economy is the sum of variable operating profits, Π, income earned by variable factors used to produce goods, Φ_p, payments made in the search for business relationships and to cover fixed production costs, Φ_i, and income to the factors that create retailing and producing firms Φ_e, along with government income, Φ_g:

$$Y = \Pi + \Phi_p + \Phi_i + \Phi_e + \Phi_g$$

We define aggregate operating profits as revenue, R, less variable input costs of production, Φ_p, so that $\Pi = R - \Phi_p$. Profits are zero when production is zero. Excluding costs that do not vary with output from aggregate profit implies the economy faces aggregate fixed costs. For example, factors in our economy must be paid income $\Phi_i + \Phi_e$ without production. If instead $\Pi = R - \Phi_p - \Phi_i - \Phi_e$ so that costs invariant to the level of output were included in operating profits, income to fixed factors would enter the accounting identity as a transfer instead of as a fixed cost.

Aggregate operating profits are the sum of retailer and producer profits, $\Pi = \Pi_m + \Pi_x$

where the sum is defined as:

$$\Pi = \left(\frac{1-v}{1-i}\right) N^m \int_{\bar{\varphi}} (p(q)\, q - nq)\, dG(\varphi) + \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} (nq - t(q))\, dG(\varphi)$$

Retailer profits include the revenue of selling the good to final consumers less the cost paid for the good to the producer. The latter is the producer's revenue and the producer pays $t(q)$ variable cost to produce q units of the good.

Variable factors of production are paid

$$\Phi_p = \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} t(q)\, dG(\varphi)$$

because each producer in a relationship pays variable production cost $t(q)$.

The search costs invested by unmatched retailers and producers in the hope of forming relationships and the investment made before production can occur is given by

$$\Phi_i = vN^m wc + uN^x (wl + ws\kappa\chi(\kappa)) + (1-u-i)\, N^x wf$$

In order to come into existence, potential retailers and producers must pay an entry cost, e_m, and cost, e_x, where both are measured in labor units

$$\Phi_e = N^m we_m + N^x we_x$$

Finally, the government levies an "iceberg" tariff on the variable production cost of each good that is exchanged between retailers and producers. The tariff means that $\tau \geq 1$ units must be produced in order to ensure one unit of the good is exchanged. Total government income is therefore

$$\Phi_g = \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} (\tau - 1)\, t(q)\, dG(\varphi)$$

Using the income approach our model yields

$$\begin{aligned} Y \;=\;& \left(\frac{1-v}{1-i}\right) N^m \int_{\bar{\varphi}} (p(q)\, q - nq)\, dG(\varphi) + \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} (nq - t(q))\, dG(\varphi) \\ &+ \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} t(q)\, dG(\varphi) \\ &+ vN^m wc + uN^x (wl + ws\kappa\chi(\kappa)) + (1-u-i)\, N^x wf \\ &+ N^m we_m + N^x we_x \\ &+ \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} (\tau - 1)\, t(q)\, dG(\varphi) \end{aligned}$$

As in the expenditure approach, in order to simplify this expression we must assume one-to-one matches so that $(1-u-i) N^x = (1-v) N^m$. We again use the definition of κ to substitute for $N^m = \kappa v^{-1} u N^x$. Using these two expressions above and canceling terms yields

$$\begin{aligned} Y &= \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} p(q)\, q dG(\varphi) \\ &+ \kappa u N^x w c + u N^x (wl + w s \kappa \chi(\kappa)) + (1-u-i) N^x w f \\ &+ \kappa v^{-1} u N^x w e_m + N^x w e_x \\ &+ \left(\frac{1-u-i}{1-i}\right) N^x \int_{\bar{\varphi}} (\tau - 1)\, t(q)\, dG(\varphi) \end{aligned}$$

This is precisely the expression we got with the expenditure approach. In summary, our model gives a resource constraint that is consistent with both income and expenditure approaches.

A.11 Equilibrium equations for closed economy

First consider the fully general definition of the equilibrium:

Definition 4. *A steady-state competitive equilibrium consists of 8 endogenous variables p, q, n, κ, $\bar{\varphi}$, N^x, N^m, w, which jointly solve 8 equations given by (9), (11), (12), (13), (16), (17), (18), (19) given exogenous parameters β, c, f, h, l, λ, r, s, τ, exogenous productivity distribution, $G(\varphi)$, and exogenous resource endowment, L.*

Definition 5. *The equilibrium is comprised of 3 prices p, n, w, and 5 quantities q, κ, $\bar{\varphi}$, N^x, N^m. These 8 endogenous variables jointly solve 8 equations.*

1. *Firms maximize profits*

   (a) *Equation (9) ensures retailers and producers maximize total profits within each match.*

   (b) *Equation (11) ensures that producers and retailers only form matches for varieties that will give positive profits.*

   (c) *Equation (12) ensures retailers maximize expected profits from creating vacancies by continuing to do so until the expected cost to create a vacancy equals the expected benefit.*

   (d) *Equation (13) ensures retailers and producers divide total profits within each match in a way that maximizes their own individual profits.*

2. *Markets clear*

   (a) *Equation (17) clears the market for creating firms.*

(b) *Equation (16) clears the matched relationship market by ensuring the mass of matched retailers equals the mass of matched producers.*

(c) *Equation (18) ensures the labor market clears.*

3. *Consumers maximize utility*

(a) *Equation (19) ensures consumers maximize utility.*

In the closed economy, a steady-state competitive equilibrium consists of 6 endogenous variables p, q, n, κ, $\bar{\varphi}$, and P, and 6 equations.

A representative consumer maximizes utility given aggregate income, Y, and the ideal price index, P, as in equation (19), which defines the demand curve $p(q)$. Bargaining over q between the producer and retailer yields equation (9) and defines q as a function of w, P, and Y

$$p\left(q\right)+p'\left(q\right)q=t'(q)$$

where P and Y enter consumer demand and w enters the cost function.

Three equations are needed to solve jointly for $\bar{\varphi}$, κ, and p. Equation (11) defines $\bar{\varphi}$, the resveration productivity for producer participation in search, as a function of $p\left(q\right)$, q, and κ

$$p\left(q\right)q-t\left(q\right)=f+\left(\frac{(r+\lambda)}{\beta\kappa\chi\left(\kappa\right)}\right)l+\left(1+\frac{(r+\lambda)}{\beta\kappa\chi\left(\kappa\right)}\right)h+\frac{(r+\lambda)}{\beta}s$$

As a reminder, $q=q\left(\bar{\varphi}\right)$ and $\kappa=\kappa\left(\bar{\varphi}\right)$, which highlights that this equation implicitly defines $\bar{\varphi}$. Equation (12), which comes from free entry into retailing, is an implicit function of κ and defines κ as a function of $p\left(q\right)$, q, n, and $\bar{\varphi}$.

$$\frac{c}{\chi\left(\kappa\right)}=\int_{\bar{\varphi}}M\left(\varphi\right)dG\left(\varphi\right)$$

Equation (13), a result of bargaining between retailers and producers over the trading price, defines n as a function of $p\left(q\right)$, q, and κ

$$n(\varphi)=\left[1-\gamma\right]p\left(q\right)+\gamma\frac{t\left(q\right)+f-l-\kappa\chi\left(\kappa\right)s}{q}$$

The aggregate price index is defined by

$$P=\left[\left(\frac{1-u-i}{1-i}\right)N^{x}\int_{\bar{\varphi}}^{\infty}p\left(\varphi\right)^{1-\sigma}dG\left(\varphi\right)\right]^{\frac{1}{1-\sigma}}$$

Notice that this is really just a function of κ, N^{x}, $\bar{\varphi}$, and $p\left(\varphi\right)$.

It should be clear that the rest of the equilibrium objects can be derived from these endogenous variables. Equation (15) defines i as a function of $\bar{\varphi}$

$$i = \int_1^{\bar{\varphi}} dG(\varphi) = G(\bar{\varphi})$$

Equation (14) defines u as a function of $\bar{\varphi}$, via i, and κ

$$u = \frac{\lambda(1-i)}{\lambda + \kappa\chi(\kappa)}$$

The vacancy rate is defined by

$$v = \frac{\kappa u N^x}{N^m}$$

which is a function of N^m, N^x, κ, and u.

We assume, as in Chaney (2008), that the number of producers is proportional to aggregate output and this pins down N^x. In particular, $N^x = Y/(1+\tilde{\pi})$ where $\tilde{\pi}$ are the share of profits from domestic retailers and producers rebated back to domestic consumers. In turn, the number of retailers, N^m, is given by one-to-one matches and the equilibrium condition $(1-v)N^m = (1-u-i)N^x$, which states that every retailer must have a producing partner. In the closed economy we normalize the price of labor, w, to one so that labor is the numeraire. The measure of workers in the economy, L, is given exogenously. Aggregate output, Y, is given by wL.

A.12 Comparative statics in the closed economy

A.12.1 Proof of Proposition 4

Here are our 6 equations:

$$F1 : \frac{c}{\chi(\kappa)} - \int_{\bar{\varphi}} M(\kappa, p, q, w, \varphi, Y; \Omega)\, dG(\varphi) = 0$$

$$F2 : \pi(q, p, P, w, \bar{\varphi}, Y; \Omega) - f - \left(\frac{r+\lambda}{\beta\kappa\chi(\kappa)}\right) l - \left(1 + \frac{r+\lambda}{\beta\kappa\chi(\kappa)}\right) h - \left(\frac{r+\lambda}{\beta}\right) s = 0$$

$$F3 : p - p(q, Y, P) = 0$$

$$F4 : -P + P(u, i, N_x, \bar{\varphi}, p(q, P, Y)) = 0$$

$$F5 : n(\kappa, p(q, P, Y), q, \gamma, w \mid \Omega)\, q(Y, P) - [1-\gamma]\, p(q, Y, P)\, q(Y, P) - \gamma\left(t(q, w \mid \varphi, \tau) + f - l - \kappa\chi(\kappa) s\right) = 0$$

$$F6 : q = q(Y, P)$$

Take the derivative of $F1$ with respect to c and group the partials to get

$$-\left[\frac{c}{\chi(\kappa)^2}\chi'(\kappa) + \int_{\bar{\varphi}} \frac{\partial M}{\partial \kappa} dG(\varphi)\right]\frac{\partial \kappa}{\partial c} + M(\kappa, p, q, w, \bar{\varphi}, Y; \Omega)\, g(\bar{\varphi})\frac{\partial \bar{\varphi}}{\partial c} - \frac{\partial P}{\partial c}\int_{\bar{\varphi}} \frac{\partial M}{\partial q}\frac{\partial q}{\partial P} dG(\varphi) = -\frac{1}{\chi(\kappa)}$$

where we assume that $\frac{\partial p}{\partial q} = 0$. This implies that the supply curve for the good is flat. This is true in our trade application as the final sales price is simply a markup over marginal cost and the marginal cost is constant.

Taking the derivative of $F2$ with respect to c:

$$\frac{\partial \pi\left(q,p,P,w,\bar{\varphi},Y;\Omega\right)}{\partial q}\frac{\partial q}{\partial c}+\frac{\partial \pi\left(q,p,P,w,\bar{\varphi},Y;\Omega\right)}{\partial p}\frac{\partial p}{\partial c}+\frac{\partial \pi\left(q,p,P,w,\bar{\varphi},Y;\Omega\right)}{\partial P}\frac{\partial P}{\partial c}+\frac{\partial \pi\left(q,p,P,w,\bar{\varphi},Y;\Omega\right)}{\partial \bar{\varphi}}\frac{\partial \bar{\varphi}}{\partial c}+\left(\frac{r+\lambda}{\beta\kappa^2\chi\left(\kappa\right)}\right)\left(1-\eta\left(\kappa\right)\right)\left(h+l\right)\frac{\partial \kappa}{\partial c} = 0$$

Taking the derivative of $F3$ with respect to c:

$$\frac{\partial p}{\partial c}-\frac{\partial p}{\partial q}\frac{\partial q}{\partial c}-\frac{\partial p}{\partial P}\frac{\partial P}{\partial c} = 0$$

and since in the simple case, equilibrium p is just markup over marginal cost, we have that $\frac{\partial p}{\partial q} = 0$ and $\frac{\partial p}{\partial P} = 0$ so we are left with

$$\frac{\partial p}{\partial c} = 0$$

Taking the derivative of $F4$ with respect to c:

$$-\frac{\partial P}{\partial c}+\frac{\partial P}{\partial u}\frac{\partial u}{\partial \kappa}\frac{\partial \kappa}{\partial c}+\frac{\partial P}{\partial i}\frac{\partial i}{\partial \bar{\varphi}}\frac{\partial \bar{\varphi}}{\partial c}+\frac{\partial P}{\partial \bar{\varphi}}\frac{\partial \bar{\varphi}}{\partial c}+\frac{\partial P}{\partial p}\frac{\partial p}{\partial q}\frac{\partial q}{\partial c}+\frac{\partial P}{\partial p}\frac{\partial p}{\partial c} = 0$$

and since the equilibrium price p is just a markup over marginal cost, we have that $\frac{\partial p}{\partial q} = 0$ and $\frac{\partial p}{\partial P} = 0$ so we end up with

$$\frac{\partial P}{\partial u}\frac{\partial u}{\partial \kappa}\frac{\partial \kappa}{\partial c}+\left(\frac{\partial P}{\partial i}\frac{\partial i}{\partial \bar{\varphi}}+\frac{\partial P}{\partial \bar{\varphi}}\right)\frac{\partial \bar{\varphi}}{\partial c}-\frac{\partial P}{\partial c} = 0$$

Taking the derivative of $F5$ with respect to c:

$$\begin{aligned}
\frac{\partial n}{\partial c}q+n\frac{\partial q}{\partial c}-\left(1-\gamma\right)\left[\frac{\partial p}{\partial c}q+p\frac{\partial q}{\partial c}\right]+\frac{\partial \gamma}{\partial \kappa}\frac{\partial \kappa}{\partial c}pq-\frac{\partial \gamma}{\partial \kappa}\frac{\partial \kappa}{\partial c}\left(t+f-l-\kappa\chi\left(\kappa\right)s\right)+\gamma\chi\left(\kappa\right)\left(1-\eta\left(\kappa\right)\right)s\frac{\partial \kappa}{\partial c} &= 0\\
\left(\frac{\partial \gamma}{\partial \kappa}pq-\frac{\partial \gamma}{\partial \kappa}\left(t+f-l-\kappa\chi\left(\kappa\right)s\right)+\gamma\chi\left(\kappa\right)\left(1-\eta\left(\kappa\right)\right)s\right)\frac{\partial \kappa}{\partial c}-\left(1-\gamma\right)q\frac{\partial p}{\partial c}+\frac{\partial n}{\partial c}q+\left(n-\left(1-\gamma\right)p\right)\frac{\partial q}{\partial c} &= 0\\
\left(\frac{\partial \gamma}{\partial \kappa}\left(pq-\left(t+f-l-\kappa\chi\left(\kappa\right)s\right)\right)+\gamma\chi\left(\kappa\right)\left(1-\eta\left(\kappa\right)\right)s\right)\frac{\partial \kappa}{\partial c}-\left(1-\gamma\right)q\frac{\partial p}{\partial c}+q\frac{\partial n}{\partial c}+\left(n-\left(1-\gamma\right)p\right)\frac{\partial q}{\partial c} &= 0
\end{aligned}$$

Taking the derivative of $F6$ with respect to c:

$$\frac{\partial q}{\partial c} - \frac{\partial q}{\partial P}\frac{\partial P}{\partial c} = 0$$

The system therefore looks like this

$$\begin{bmatrix} -\left[\frac{c}{\chi(\kappa)^2}\chi'(\kappa)+\int_{\bar{\varphi}}\frac{\partial M}{\partial \kappa}dG(\varphi)\right] & M(\kappa,p,q,w,\bar{\varphi},Y;\Omega)\,g(\bar{\varphi}) & 0 & \int_{\bar{\varphi}}\frac{\partial M(\kappa,p,q,w,\varphi,Y;\Omega)}{\partial q}\frac{\partial q}{\partial P}dG(\varphi) & 0 & 0 \\ \left(\frac{r+\lambda}{\beta\kappa^2\chi(\kappa)}\right)(1-\eta(\kappa))(h+l) & \frac{\partial \pi(q,p,P,w,\bar{\varphi},Y;\Omega)}{\partial \bar{\varphi}} & \frac{\partial \pi(q,p,P,w,\bar{\varphi},Y;\Omega)}{\partial p} & \frac{\partial \pi(q,p,P,w,\bar{\varphi},Y;\Omega)}{\partial P} & 0 & \frac{\partial \pi(q,p,P,w,\bar{\varphi},Y;\Omega)}{\partial q} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{\partial P}{\partial u}\frac{\partial u}{\partial \kappa} & \frac{\partial P}{\partial i}\frac{\partial i}{\partial \bar{\varphi}}+\frac{\partial P}{\partial \bar{\varphi}} & 0 & -1 & 0 & 0 \\ \frac{\partial \gamma}{\partial \kappa}(pq-(t+f-l-\kappa\chi(\kappa)s))+\gamma\chi(\kappa)(1-\eta(\kappa))s & 0 & -(1-\gamma)q & 0 & q & (n-(1-\gamma)p) \\ 0 & 0 & 0 & -\frac{\partial q}{\partial P} & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial \kappa}{\partial c} \\ \frac{\partial \bar{\varphi}}{\partial c} \\ \frac{\partial p}{\partial c} \\ \frac{\partial P}{\partial c} \\ \frac{\partial n}{\partial c} \\ \frac{\partial q}{\partial c} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\chi(\kappa)} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule we know that the ith element of the partials vector will be determined by the determinant of the left-hand-side (LHS) matrix and the determinant of the matrix formed by replacing the ith column of the LHS matrix with the right-hand-side vector. We omit many of the details in this appendix about how to obtain and sign these determinants but details are available from the authors upon request.

A.12.2 Proof of Proposition 5: Comparative statics with respect to producer search cost

The negotiated price does not interact with he four equations. Hence, let's just add the negotiated price to the system of equations and find it's derivative with respect to l without redoing the deriviatives for all the other equations.

$$F1: \frac{c}{\chi(\kappa)} - \int_{\bar{\varphi}} M(\kappa, p, q, w, \varphi, Y; \Omega)\, dG(\varphi) = 0$$

$$F2: \pi\left(q, p, P, w, \bar{\varphi}, \bar{Y}; \Omega\right) - f - \left(\frac{r+\lambda}{\beta\kappa\chi(\kappa)}\right) l - \left(1 + \frac{r+\lambda}{\beta\kappa\chi(\kappa)}\right) h - \left(\frac{r+\lambda}{\beta}\right) s = 0$$

$$F3: p - p(q, Y, P) = 0$$

$$F4: -P + P(u, i, N_x, \bar{\varphi}, p(q, P, Y)) = 0$$

$$F5: n(\kappa, p(q, P, Y), q, \gamma, w \mid \Omega)\, q - [1-\gamma]\, p(q, Y, P)\, q - \gamma\left(t(q, w \mid \varphi, \tau) + f - l - \kappa\chi(\kappa)\, s\right) = 0$$

$$F6: q = q(Y, P)$$

where $\gamma = \dfrac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa\chi(\kappa)}$ and $M(\kappa, p, q, w, \varphi, Y; \Omega) = (1-\beta)\dfrac{pq - t - f + l + s\kappa\chi(\kappa)}{r+\lambda+\beta\kappa\chi(\kappa)}$.

Differentiate F1 wrt to l:

$$-\frac{c}{\chi(\kappa)^2}\chi'(\kappa)\frac{\partial\kappa}{\partial l} - \left\{-\frac{\partial\bar{\varphi}}{\partial l} M(\kappa, p, q, w, \varphi, Y; \Omega)\, g(\bar{\varphi}) + \int_{\bar{\varphi}} \frac{\partial M}{\partial l} + \frac{\partial M}{\partial\kappa}\frac{\partial\kappa}{\partial l} + \frac{\partial M}{\partial p}\frac{\partial p}{\partial l} + \frac{\partial M}{\partial q}\frac{\partial q}{\partial P}\frac{\partial P}{\partial l} dG(\varphi)\right\} = 0$$

$$-\frac{c}{\chi(\kappa)^2}\chi'(\kappa)\frac{\partial\kappa}{\partial l} - \left\{-\frac{\partial\bar{\varphi}}{\partial l} M(\kappa, p, q, w, \varphi, Y; \Omega)\, g(\bar{\varphi}) + \int_{\bar{\varphi}} \frac{\partial M}{\partial\kappa}\frac{\partial\kappa}{\partial l} + \frac{\partial M}{\partial p} 0 + \frac{\partial M}{\partial q}\frac{\partial q}{\partial P}\frac{\partial P}{\partial l} dG(\varphi)\right\} = \int_{\bar{\varphi}} \frac{\partial M}{\partial l} dG(\varphi)$$

$$-\frac{c}{\chi(\kappa)^2}\chi'(\kappa)\frac{\partial\kappa}{\partial l} + \frac{\partial\bar{\varphi}}{\partial l} M(\kappa, p, q, w, \varphi, Y; \Omega)\, g(\bar{\varphi}) - \int_{\bar{\varphi}} \frac{\partial M}{\partial\kappa}\frac{\partial\kappa}{\partial l} + \frac{\partial M}{\partial q}\frac{\partial q}{\partial P}\frac{\partial P}{\partial l} dG(\varphi) = \int_{\bar{\varphi}} \frac{\partial M}{\partial l} dG(\varphi)$$

$$-\left[\frac{c}{\chi(\kappa)^2}\chi'(\kappa) + \int_{\bar{\varphi}} \frac{\partial M}{\partial\kappa} dG(\varphi)\right]\frac{\partial\kappa}{\partial l} + \frac{\partial\bar{\varphi}}{\partial l} M(\kappa, p, q, w, \bar{\varphi}, Y; \Omega)\, g(\bar{\varphi}) - \frac{\partial P}{\partial l}\int_{\bar{\varphi}} \frac{\partial M}{\partial q}\frac{\partial q}{\partial P} dG(\varphi) = \int_{\bar{\varphi}} \frac{\partial M}{\partial l} dG(\varphi)$$

Differentiate F2 wrt to l:

$$\frac{\partial\pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial q}\frac{\partial q}{\partial l} + \frac{\partial\pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial p}\frac{\partial p}{\partial l} + \frac{\partial\pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial P}\frac{\partial P}{\partial l} + \frac{\partial\pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial\bar{\varphi}}\frac{\partial\bar{\varphi}}{\partial l} + \left(\frac{r+\lambda}{\beta\kappa^2\chi(\kappa)}\right)(1-\eta(\kappa))(h+l)\frac{\partial\kappa}{\partial l} = \left(\frac{r+\lambda}{\beta\kappa\chi(\kappa}$$

$$\frac{\partial\pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial q}\frac{\partial q}{\partial l} + \frac{\partial\pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial p} 0 + \frac{\partial\pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial P}\frac{\partial P}{\partial l} + \frac{\partial\pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial\bar{\varphi}}\frac{\partial\bar{\varphi}}{\partial l} + \left(\frac{r+\lambda}{\beta\kappa^2\chi(\kappa)}\right)(1-\eta(\kappa))(h+l)\frac{\partial\kappa}{\partial l} = \left(\frac{r+\lambda}{\beta\kappa\chi(\kappa}$$

$$\frac{\partial\pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial q}\frac{\partial q}{\partial l} + \frac{\partial\pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial P}\frac{\partial P}{\partial l} + \frac{\partial\pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial\bar{\varphi}}\frac{\partial\bar{\varphi}}{\partial l} + \left(\frac{r+\lambda}{\beta\kappa^2\chi(\kappa)}\right)(1-\eta(\kappa))(h+l)\frac{\partial\kappa}{\partial l} = \left(\frac{r+\lambda}{\beta\kappa\chi(\kappa}$$

Differentiate F3 wrt to l:

$$\frac{\partial p}{\partial l} - \frac{\partial p}{\partial q}\frac{\partial q}{\partial l} - \frac{\partial p}{\partial P}\frac{\partial P}{\partial l} = 0$$

and since in the simple case, equilibrium p is just markup over marginal cost, we have that $\frac{\partial p}{\partial q} = 0$ and $\frac{\partial p}{\partial P} = 0$ so we are left with

$$\frac{\partial p}{\partial l} = 0$$

Differentiate F4 wrt to l:

$$-\frac{\partial P}{\partial l} + \frac{\partial P}{\partial u}\frac{\partial u}{\partial \kappa}\frac{\partial \kappa}{\partial l} + \frac{\partial P}{\partial i}\frac{\partial i}{\partial \bar{\varphi}}\frac{\partial \bar{\varphi}}{\partial l} + \frac{\partial P}{\partial \bar{\varphi}}\frac{\partial \bar{\varphi}}{\partial l} + \frac{\partial P}{\partial p}\frac{\partial p}{\partial q}\frac{\partial q}{\partial l} + \frac{\partial P}{\partial p}\frac{\partial p}{\partial l} = 0$$

and since the equilibrium price p is just a markup over marginal cost, we have that $\frac{\partial p}{\partial q} = 0$ and $\frac{\partial p}{\partial P} = 0$ so we end up with

$$\begin{aligned}
-\frac{\partial P}{\partial l} + \frac{\partial P}{\partial u}\frac{\partial u}{\partial \kappa}\frac{\partial \kappa}{\partial l} + \frac{\partial P}{\partial i}\frac{\partial i}{\partial \bar{\varphi}}\frac{\partial \bar{\varphi}}{\partial l} + \frac{\partial P}{\partial \bar{\varphi}}\frac{\partial \bar{\varphi}}{\partial l} &= 0 \\
\frac{\partial P}{\partial u}\frac{\partial u}{\partial \kappa}\frac{\partial \kappa}{\partial l} + \left(\frac{\partial P}{\partial i}\frac{\partial i}{\partial \bar{\varphi}} + \frac{\partial P}{\partial \bar{\varphi}}\right)\frac{\partial \bar{\varphi}}{\partial l} - \frac{\partial P}{\partial l} &= 0
\end{aligned}$$

Differentiate F5 wrt to l:

$$\begin{aligned}
\frac{\partial n}{\partial l}q + n\frac{\partial q}{\partial l} - (1-\gamma)\left[\frac{\partial p}{\partial l}q + p\frac{\partial q}{\partial l}\right] + \frac{\partial \gamma}{\partial \kappa}\frac{\partial \kappa}{\partial l}pq - \frac{\partial \gamma}{\partial \kappa}\frac{\partial \kappa}{\partial l}\left(t + f - l - \kappa\chi(\kappa)s\right) + \gamma\chi(\kappa)(1-\eta(\kappa))s\frac{\partial \kappa}{\partial l} + \gamma &= 0 \\
\left(\frac{\partial \gamma}{\partial \kappa}pq - \frac{\partial \gamma}{\partial \kappa}\left(t + f - l - \kappa\chi(\kappa)s\right) + \gamma\chi(\kappa)(1-\eta(\kappa))s\right)\frac{\partial \kappa}{\partial l} - (1-\gamma)q\frac{\partial p}{\partial l} + \frac{\partial n}{\partial l}q + (n - (1-\gamma)p)\frac{\partial q}{\partial l} &= -\gamma \\
\left(\frac{\partial \gamma}{\partial \kappa}\left(pq - (t + f - l - \kappa\chi(\kappa)s)\right) + \gamma\chi(\kappa)(1-\eta(\kappa))s\right)\frac{\partial \kappa}{\partial l} - (1-\gamma)q\frac{\partial p}{\partial l} + q\frac{\partial n}{\partial l} + (n - (1-\gamma)p)\frac{\partial q}{\partial l} &= -\gamma
\end{aligned}$$

Differentiate F6 wrt to l:

$$\frac{\partial q}{\partial l} - \frac{\partial q}{\partial P}\frac{\partial P}{\partial l} = 0$$

Hence, the system looks like

$$\begin{bmatrix} -\left[\frac{c}{\chi(\kappa)^2}\chi'(\kappa)+\int_{\bar{\varphi}}\frac{\partial M}{\partial\kappa}dG(\varphi)\right] & M(\kappa,p,q,w,\bar{\varphi},Y;\Omega)\,g(\bar{\varphi}) & 0 & \int_{\bar{\varphi}}\frac{\partial M}{\partial q}\frac{\partial q}{\partial P}dG(\varphi) & 0 & 0 \\ \left(\frac{r+\lambda}{\beta\kappa^2\chi(\kappa)}\right)(1-\eta(\kappa))(h+l) & \frac{\partial\pi(q,p,P,w,\bar{\varphi},Y;\Omega)}{\partial\bar{\varphi}} & 0 & \frac{\partial\pi(q,p,P,w,\bar{\varphi},Y;\Omega)}{\partial P} & 0 & \frac{\partial\pi(q,p,P,w,\bar{\varphi},Y;\Omega)}{\partial q} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{\partial P}{\partial u}\frac{\partial u}{\partial\kappa} & \left(\frac{\partial P}{\partial i}\frac{\partial i}{\partial\bar{\varphi}}+\frac{\partial P}{\partial\bar{\varphi}}\right) & 0 & -1 & 0 & 0 \\ \frac{\partial\gamma}{\partial\kappa}(pq-(t+f-l-\kappa\chi(\kappa)s))+\gamma\chi(\kappa)(1-\eta(\kappa))s & 0 & -(1-\gamma)q & 0 & q & (n-(1-\gamma)p) \\ 0 & 0 & 0 & -\frac{\partial q}{\partial P} & 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{\partial\kappa}{\partial l} \\ \frac{\partial\bar{\varphi}}{\partial l} \\ \frac{\partial p}{\partial l} \\ \frac{\partial P}{\partial l} \\ \frac{\partial n}{\partial l} \\ \frac{\partial q}{\partial l} \end{bmatrix} = \begin{bmatrix} \int_{\bar{\varphi}}\frac{\partial M}{\partial l}dG(\varphi) \\ \left(\frac{r+\lambda}{\beta\kappa\chi(\kappa)}\right) \\ 0 \\ 0 \\ -\gamma \\ 0 \end{bmatrix}$$

A.12.3 Proof of Proposition 6: Comparative statics with respect to iceberg trade cost

We start with the system of equations

$$F1 : \frac{c}{\chi(\kappa)} - \int_{\bar{\varphi}} M(\kappa, p, q, w, \varphi, Y; \Omega)\, dG(\varphi) = 0$$

$$F2 : \pi(q, p, P, w, \bar{\varphi}, \tau, Y; \Omega) - f - \left(\frac{r+\lambda}{\beta\kappa\chi(\kappa)}\right) l - \left(1 + \frac{r+\lambda}{\beta\kappa\chi(\kappa)}\right) h - \left(\frac{r+\lambda}{\beta}\right) s = 0$$

$$F3 : -P + P(u, i, N_x, \bar{\varphi}, p(q, P, Y)) = 0$$

$$F4 : n(\kappa, p(q, P, Y), q, \gamma, w \mid \Omega)\, q(Y, P) - [1-\gamma]\, p(q, Y, P)\, q(Y, P) - \gamma\left(t(q, w \mid \varphi, \tau) + f - l - \kappa\chi(\kappa) s\right) = 0$$

$$F5 : q = q(Y, P)$$

$$F6 : p - p(q, Y, P) = 0$$

where

$$M(\kappa, p, q, w, \varphi, Y; \Omega) = \frac{pq - t(\tau) - f + l + s\kappa\chi(\kappa)}{r + \lambda + \beta\kappa\chi(\kappa)}$$

The negotiated price, n, does not enter the other equilibrium equations so it largely does not change the analysis with three equations. Also, the final sales price is a markup over marginal cost so it does not interact with any of the other endogenous variables. We know, however, that $\frac{\partial p}{\partial \tau} > 0$. We will use this in the analysis below without using F6 above.

Take the derivative of F1 with respect to τ

$$-\frac{c}{\chi(\kappa)^2}\chi'(\kappa)\frac{\partial \kappa}{\partial \tau} - \left\{-\frac{\partial \bar{\varphi}}{\partial \tau} M(\kappa, p, q, w, \varphi, Y; \Omega)\, g(\bar{\varphi}) + \int_{\bar{\varphi}} \frac{\partial M}{\partial \tau} + \frac{\partial M}{\partial \kappa}\frac{\partial \kappa}{\partial \tau} + \frac{\partial M}{\partial q}\frac{\partial q}{\partial P}\frac{\partial P}{\partial \tau} + \frac{\partial M}{\partial p}\frac{\partial p}{\partial \tau} dG(\varphi)\right\} = 0$$

$$\therefore -\frac{c}{\chi(\kappa)^2}\chi'(\kappa)\frac{\partial \kappa}{\partial \tau} - \frac{\partial \kappa}{\partial \tau}\int_{\bar{\varphi}} \frac{\partial M}{\partial \kappa} dG(\varphi) - \frac{\partial P}{\partial \tau}\int_{\bar{\varphi}} \frac{\partial M}{\partial q}\frac{\partial q}{\partial P} dG(\varphi) + \frac{\partial \bar{\varphi}}{\partial \tau} M(\kappa, p, q, w, \varphi, Y; \Omega)\, g(\bar{\varphi}) = \int_{\bar{\varphi}} \frac{\partial M}{\partial \tau} + \frac{\partial M}{\partial p}\frac{\partial p}{\partial \tau} dG(\varphi)$$

$$-\left(\frac{c}{\chi(\kappa)^2}\chi'(\kappa) + \int_{\bar{\varphi}} \frac{\partial M}{\partial \kappa} dG(\varphi)\right)\frac{\partial \kappa}{\partial \tau} + M(\kappa, p, q, w, \varphi, Y; \Omega)\, g(\bar{\varphi})\frac{\partial \bar{\varphi}}{\partial \tau} - \frac{\partial P}{\partial \tau}\int_{\bar{\varphi}} \frac{\partial M}{\partial q}\frac{\partial q}{\partial P} dG(\varphi) = \int_{\bar{\varphi}} \frac{\partial M}{\partial \tau} + \frac{\partial M}{\partial p}\frac{\partial p}{\partial \tau} dG(\varphi)$$

Note: there is no $\frac{\partial M}{\partial P}$ term because M does not depend directly on P. Also, we cannot pull out $\frac{\partial p}{\partial \tau}$ out of the integral on the RHS because $p = p(w, \tau, \varphi)$ depends on productivity.

Taking the derivative of F2 with respect to τ

$$\frac{\partial \pi(q, p, P, w, \bar{\varphi}, \tau, Y; \Omega)}{\partial \tau} + \frac{\partial \pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial q}\frac{\partial q}{\partial \tau} + \frac{\partial \pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial p}\frac{\partial p}{\partial \tau} + \frac{\partial \pi(q, p, P, w, \bar{\varphi}, \tau, Y; \Omega)}{\partial \bar{\varphi}}\frac{\partial \bar{\varphi}}{\partial \tau} + \frac{\partial \pi(q, p, P, w, \bar{\varphi}, \tau, Y; \Omega)}{\partial P}\frac{\partial P}{\partial \tau} + \left(\frac{r+\lambda}{\beta\kappa^2\chi(\kappa)}\right)(1 - \eta$$

$$\frac{\partial \pi(q, p, P, w, \bar{\varphi}, \tau, Y; \Omega)}{\partial \bar{\varphi}}\frac{\partial \bar{\varphi}}{\partial \tau} + \frac{\partial \pi(q, p, P, w, \bar{\varphi}, \tau, Y; \Omega)}{\partial P}\frac{\partial P}{\partial \tau} + \left(\frac{r+\lambda}{\beta\kappa^2\chi(\kappa)}\right)(1 - \eta(\kappa))(h + l)\frac{\partial \kappa}{\partial \tau} + \frac{\partial \pi(q, p, P$$

and we have the $\frac{\partial \pi(q, p, P, w, \bar{\varphi}, Y; \Omega)}{\partial q}$ term because profits depend directly on τ through the cost function and we treat $\dfrac{\partial\pi\left(q,p,P,w,\bar{\varphi},Y;\Omega\right)}{\partial p}\dfrac{\partial p}{\partial\tau}$ as a constant in the system.

Taking the derivative of F3 with respect to τ

$$\begin{aligned}
-\frac{\partial P}{\partial\tau}+\frac{\partial P}{\partial u}\frac{\partial u}{\partial\kappa}\frac{\partial\kappa}{\partial\tau}+\frac{\partial P}{\partial i}\frac{\partial i}{\partial\bar{\varphi}}\frac{\partial\bar{\varphi}}{\partial\tau}+\frac{\partial P}{\partial\bar{\varphi}}\frac{\partial\bar{\varphi}}{\partial\tau}+\frac{\partial P}{\partial p}\frac{\partial p}{\partial\tau} &= 0\\
\frac{\partial P}{\partial u}\frac{\partial u}{\partial\kappa}\frac{\partial\kappa}{\partial\tau}+\left(\frac{\partial P}{\partial i}\frac{\partial i}{\partial\bar{\varphi}}+\frac{\partial P}{\partial\bar{\varphi}}\right)\frac{\partial\bar{\varphi}}{\partial\tau}-\frac{\partial P}{\partial\tau} &= -\frac{\partial P}{\partial p}\frac{\partial p}{\partial\tau}
\end{aligned}$$

Taking the derivative of F4 with respect to τ

$$\begin{aligned}
\frac{\partial n}{\partial\tau}q+n\frac{\partial q}{\partial\tau}-(1-\gamma)\left[\frac{\partial p}{\partial\tau}q+p\frac{\partial q}{\partial\tau}\right]+\frac{\partial\gamma}{\partial\kappa}\frac{\partial\kappa}{\partial\tau}pq-\frac{\partial\gamma}{\partial\kappa}\frac{\partial\kappa}{\partial\tau}\left(t+f-l-\kappa\chi\left(\kappa\right)s\right)+\gamma\chi\left(\kappa\right)\left(1-\eta\left(\kappa\right)\right)s\frac{\partial\kappa}{\partial\tau} &= 0\\
\left(\frac{\partial\gamma}{\partial\kappa}pq-\frac{\partial\gamma}{\partial\kappa}\left(t+f-l-\kappa\chi\left(\kappa\right)s\right)+\gamma\chi\left(\kappa\right)\left(1-\eta\left(\kappa\right)\right)s\right)\frac{\partial\kappa}{\partial\tau}-(1-\gamma)\,q\frac{\partial p}{\partial\tau}+\frac{\partial n}{\partial\tau}q+\left(n-(1-\gamma)\,p\right)\frac{\partial q}{\partial\tau} &= 0\\
\left(\frac{\partial\gamma}{\partial\kappa}\left(pq-\left(t+f-l-\kappa\chi\left(\kappa\right)s\right)\right)+\gamma\chi\left(\kappa\right)\left(1-\eta\left(\kappa\right)\right)s\right)\frac{\partial\kappa}{\partial\tau}+q\frac{\partial n}{\partial\tau}+\left(n-(1-\gamma)\,p\right)\frac{\partial q}{\partial\tau} &= (1-\gamma)\,q\frac{\partial p}{\partial\tau}
\end{aligned}$$

Taking the derivative of F5 with respect to τ

$$\frac{\partial q}{\partial\tau}-\frac{\partial q}{\partial P}\frac{\partial P}{\partial\tau} = 0$$

The system therefore looks like this

$$\begin{bmatrix} -\left(\frac{c}{\chi(\kappa)^2}\chi'(\kappa)+\int_{\bar{\varphi}}\frac{\partial M}{\partial \kappa}dG(\varphi)\right) & M(\kappa,p,q,w,\bar{\varphi},Y;\Omega)\,g(\bar{\varphi}) & \int_{\bar{\varphi}}\frac{\partial M}{\partial q}\frac{\partial q}{\partial P}dG(\varphi) & 0 & 0 \\ \left(\frac{r+\lambda}{\beta\kappa^2\chi(\kappa)}\right)(1-\eta(\kappa))(h+l) & \frac{\partial \pi(q,p,P,w,\bar{\varphi},\tau,Y;\Omega)}{\partial \bar{\varphi}} & \frac{\partial \pi(q,p,P,w,\bar{\varphi},\tau,Y;\Omega)}{\partial P} & 0 & \frac{\partial \pi(q,p,P,w,\bar{\varphi},Y;\Omega)}{\partial q} \\ \frac{\partial P}{\partial u}\frac{\partial u}{\partial \kappa} & \left(\frac{\partial P}{\partial i}\frac{\partial i}{\partial \bar{\varphi}}+\frac{\partial P}{\partial \bar{\varphi}}\right) & -1 & 0 & 0 \\ \left(\frac{\partial \gamma}{\partial \kappa}\left[pq-(t+f-l-\kappa\chi(\kappa)s)\right]+\gamma\chi(\kappa)(1-\eta(\kappa))s\right) & 0 & 0 & q & (n-(1-\gamma)p) \\ 0 & 0 & -\frac{\partial q}{\partial P} & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial \kappa}{\partial \tau} \\ \frac{\partial \bar{\varphi}}{\partial \tau} \\ \frac{\partial P}{\partial \tau} \\ \frac{\partial n}{\partial \tau} \\ \frac{\partial q}{\partial \tau} \end{bmatrix} = \begin{bmatrix} \int_{\bar{\varphi}}\frac{\partial M}{\partial \tau}+\frac{\partial M}{\partial p}\frac{\partial p}{\partial \tau}dG(\varphi) \\ -\frac{\partial \pi(q,p,P,w,\bar{\varphi},\tau,Y;\Omega)}{\partial \tau}-\frac{\partial \pi(q,p,P,w,\bar{\varphi},Y;\Omega)}{\partial p}\frac{\partial p}{\partial \tau} \\ -\frac{\partial P}{\partial p}\frac{\partial p}{\partial \tau} \\ (1-\gamma)\,q\frac{\partial p}{\partial \tau} \\ 0 \end{bmatrix}$$

By Cramer's rule we know that the ith element of the partials vector will be determined by the determinant of the left-hand-side (LHS) matrix and the determinant of the matrix formed by replacing the ith column of the LHS matrix with the right-hand-side vector. We omit many of the details in this appendix about how to obtain and sign these determinants but details are available from the authors upon request.

A.13 Proof of Proposition 7: Comparing the competitive economy and the social planner's problem

In general, the social planner (SP) maximizes welfare subject to the equations of motion for u and N^x and the aggregate resource constraint where we initiate the economy at the steady-state unemployment rate and the steady-state number of producers

$$\begin{aligned} \max_{\kappa} & \quad \int_0^\infty \frac{C}{P} e^{-rt} dt \\ s.t. & \\ (u\dot{N^x}) &= \dot{u}N^x + u\dot{N^x} \\ \dot{N^x} &= d\left(\dot{u}, u, \dot{N^x}, N^x, \kappa\right) \\ u(0) &= \frac{\lambda(1-i)}{\lambda + \kappa\chi(\kappa)} \\ N^x(0) &= N^x \\ Y &= C + I + G \end{aligned}$$

Here $d\left(\dot{u}, u, \dot{N^x}, N^x, \kappa\right)$ is some function characterizing the dynamics of N^x. The predetermined variables u and N^x are constrained by initial conditions to start at their steady-state values. We assume that the economy is endowed with a number of producers given by $N^x e_x = \zeta L$ where $\zeta \in (0, \infty)$ and L is given exogenously. This implies that $\dot{N^x} = 0$ and removes the equation of motion for N^x and the corresponding initial condition. We set $G = 0$ by assuming that there are no taxes, $\tau = 1$, or that tax revenue is rebated lump sum back to the consumer. Along with this assumption, the additional endowment means that

$$\frac{Y}{N^x} = \frac{w(1+\zeta)e_x}{\zeta}$$

As such, after re-arranging the resource constraint and dividing through by N^x and P, we can write

$$\begin{aligned} \frac{C}{N^x P} &= \frac{Y}{N^x P} - \frac{I}{N^x P} \\ &= \frac{w(1+\zeta)e_x}{\zeta P} - \frac{\kappa u w c - u(wl + ws\kappa\chi(\kappa)) - (1-u-i)wf - \kappa v^{-1} u w e_m - w e_x}{P} \end{aligned}$$

Here we have written the parameters c, l, s, f, e_m, and e_x in labor units instead of dollars as presented in the main text. There is no issue with this; we could have re-written the whole model in terms of labor units and we would obtain this expression for real consumption. We normalize the real wage to one so that $w/P = 1$ and this yields

$$\frac{C}{N^x P} = \frac{(1+\zeta)e_x}{\zeta} - \kappa u c - u(l + s\kappa\chi(\kappa)) - (1-u-i)f - \kappa v^{-1} u e_m - e_x$$

We show in Appendix A.7 that $e_m = 0$. Furthermore, scaling the Hamiltonian by a constant, N^x, makes no difference to the solution of the optimal control problem, so we can maximize $C/(N^x P)$ instead of C/P as long as we divide our constraint by N^x as well

$$\begin{aligned} (u\dot{N^x}) &= \dot{u}N^x + u\dot{N}^x \\ &= \dot{u}N^x \\ &= \lambda(1-u-i)N^x - \kappa\chi(\kappa)N^x \\ \frac{(u\dot{N^x})}{N^x} &= \dot{u} = \lambda(1-u-i) - \kappa\chi(\kappa) \end{aligned}$$

After making these assumptions and substituting the resource constraint into the objective function, the SP's problem becomes

$$\begin{aligned} \max_{\kappa} \quad & \int_0^\infty \left[\frac{(1+\zeta)e_x}{\zeta} - \kappa uc - u(l + s\kappa\chi(\kappa)) - (1-u-i)f - e_x\right] e^{-rt}dt \\ s.t. \quad & \\ \dot{u} &= \lambda(1-u) - u\kappa\chi(\kappa) \\ u(0) &= \frac{\lambda(1-i)}{\lambda + \kappa\chi(\kappa)} \end{aligned}$$

The current value Hamiltonian associated with this problem is

$$\mathcal{H} = \frac{(1+\zeta)e_x}{\zeta} - \kappa uc - u(l + s\kappa\chi(\kappa)) - (1-u-i)f - e_x + \nu[\lambda(1-u) - u\kappa\chi(\kappa)]$$

The first order conditions that solve this problem are

$$\mathcal{H}_\kappa : -uc - us(\kappa\chi'(\kappa) + \chi(\kappa)) + g(\bar{\varphi})\frac{\partial\bar{\varphi}}{\partial\kappa}f - \nu u(\chi(\kappa) + \kappa\chi'(\kappa)) = 0$$

$$\mathcal{H}_u : -\kappa c - (l + s\kappa\chi(\kappa)) + f + g(\bar{\varphi})\frac{\partial\bar{\varphi}}{\partial u}f - \nu\lambda - \nu\kappa\chi(\kappa) = -\dot{\nu} + r\nu$$

with transversality condition

$$\lim_{t\to\infty} e^{-rt}\nu u = 0$$

Solving $\mathcal{H}_\kappa = 0$ for ν gives

$$\nu = \frac{-c}{\chi(\kappa)(1-\eta(\kappa))} - s + \frac{g(\bar{\varphi})}{u\chi(\kappa)(1-\eta(\kappa))}\frac{\partial\bar{\varphi}}{\partial\kappa}f$$

where we define the elasticity of the matching function with respect to κ as $-\eta(\kappa) \equiv \dfrac{\kappa\chi'(\kappa)}{\chi(\kappa)}$ so that $\chi(\kappa) + \kappa\chi'(\kappa) = \chi(\kappa)(1-\eta(\kappa))$. Differentiating ν with respect to time gives

$$\dot{\nu} = -\frac{(\lambda (1-u) - u\kappa\chi(\kappa))}{u^2\chi(\kappa)(1-\eta(\kappa))} g(\bar{\varphi}) \frac{\partial \bar{\varphi}}{\partial \kappa} f$$

where $\dot{\kappa} = 0$ ensures that $\frac{\partial \bar{\varphi}}{\partial t} = \frac{\partial \bar{\varphi}}{\partial \kappa}\dot{\kappa} = 0$. Notice that we substituted the equation of motion for u in place of $\dot{u}$. Using ν and $\dot{\nu}$ and many simplifying steps, we can write the first order condition $\mathcal{H}_u = -\dot{\nu} + r\nu$ as

$$\frac{c}{\chi(\kappa)} = \frac{(1-\eta(\kappa))}{r+\lambda+\eta(\kappa)\kappa\chi(\kappa)} \left[\frac{[\lambda + ur] g(\bar{\varphi}) \frac{\partial \bar{\varphi}}{\partial \kappa} f}{u^2\chi(\kappa)(1-\eta(\kappa))} - s(r+\lambda+\kappa\chi(\kappa)) - g(\bar{\varphi}) \frac{\partial \bar{\varphi}}{\partial u} f - f + l + s\kappa\chi(\kappa) \right] \tag{50}$$

The comparable expression from the competitive equilibrium (CE) economy is

$$\frac{c}{\chi(\kappa)} = \frac{(1-\beta)}{r+\lambda+\beta\kappa\chi(\kappa)} \int_{\bar{\varphi}} (p(q)q - t(q) - f + l + s\kappa\chi(\kappa))\, dG(\varphi) \tag{51}$$

It is often the case that extreme constellations of parameters are required in order for the CE in search models to be efficient and Kudoh and Sasaki (2011) is one such example.

Using equations (50) and (51), when $c > 0$, it appears that sufficient conditions for the CE and SP to result in the same κ are $\beta = \eta(\kappa) = 1$. This imposes that the matching function exhibits constant elasticity equal to β and that foreign producers receive all match surplus. In this extreme case, however, the retailing importer has no incentive to be in a relationship with a foreign producer. A retailer must have some bargaining power, $\beta < 1$, to recoup search cost $c > 0$. These apparent sufficient conditions lead to a contradiction and are therefore not sufficient.

Turning to the case where a retailing importer has some bargaining power, $\beta < 1$, the CE and the SP's coincide when equation (50) equals equation (51). It is evident that they are not generally equal but we can consider a few special cases.

If $f = l = s = 0$, then setting equation (51) equal to (50) provides

$$\int_{\bar{\varphi}} (p(q)q - t(q))\, dG(\varphi) = 0$$

which is never true as long as variable profits are increasing in productivity.

If $f = s = 0$, then it would have to be that $\beta = \eta(\kappa)$ and

$$\int_{\bar{\varphi}} (p(q)q - t(q) + l)\, dG(\varphi) = l$$

which defines a knife-edge requirement for the endogenous quantities in the model.

If $f = 0$, then we need that $\beta = \eta(\kappa)$ and

$$\int_{\bar{\varphi}} (p(q)q - t(q) + l + s\kappa\chi(\kappa))\, dG(\varphi) = l - s(r+\lambda)$$

which also defines a knife-edge requirement for the endogenous quantities in the model.

If $l = h = 0$, then $\frac{\partial \bar{\varphi}}{\partial \kappa} = 0$. If it is also the case that $\frac{\partial \bar{\varphi}}{\partial u} = 0$ and $\beta = \eta(\kappa)$, then efficiency requires

$$\int_{\bar{\varphi}} (p(q)\, q - t(q) + f + s\kappa\chi(\kappa))\, dG(\varphi) = -f - s(r + \lambda)$$

which can never hold since $p(\bar{\varphi})\, q(\bar{\varphi}) - t(\bar{\varphi}) \geq 0$.

B Appendix for the open economy

B.1 The open economy ideal price index

The ideal price index for differentiated goods in the destination/domestic market will take a form similar to the index in Chaney (2008)

$$P_d = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where the price index for goods trade between destination d and origin o is given by

$$P_{do} = \left[\left(\frac{1 - u_{do} - i_{do}}{1 - i_{do}} \right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)^{1-\sigma} \, dG(\varphi) \right]^{\frac{1}{1-\sigma}}$$

Combining these two expression gives

$$P_d = \left[\sum_{k=1}^{O} \left(\frac{1 - u_{dk} - i_{dk}}{1 - i_{dk}} \right) N_k^x \int_{\bar{\varphi}_{dk}}^{\infty} p_k(\varphi)^{1-\sigma} \, dG(\varphi) \right]^{\frac{1}{1-\sigma}}$$

The utility function from equation (22) includes both a homogeneous and differentiated good so that the overall price index is given by $\Xi_d = \left(\frac{p_1}{1-\alpha} \right)^{1-\alpha} \left(\frac{P_d}{\alpha} \right)^{\alpha}$. The ideal price index relies on the final sales price and not the price negotiated between retailer and producers.

Following Chaney (2008), we will assume that the number of producers in the origin market that take a draw from the productivity distribution is proportional to the size of the economy, Y_o. The basic intuition behind this is that larger economies have a larger stock of potential entrepreneurs. To make this explicit, we denoted the total mass of potential entrants as $N_o^x = \xi_o Y_o$ where the proportionality constant $\xi_o \in [0, \infty)$ captures exogenous structural factors that affect the number of potential entrants in country k. Among others, these could include such factors as literacy levels and attitudes toward entrepreneurship. Because the number of producers is fixed, they make profits which are then split between retailers and producers. We assume that a global mutual fund collects profits from retailers and producers and redistributes them as π dividends per share to each worker who owns w_o shares. Total income in each country is labor income $w_o L_o$ plus dividends income, $Y_o = (1 + \pi) w_o L_o$. With these assumptions, the number of producers is $N_o^x = \xi_o (1 + \pi) w_o L_o$. It will be useful to write this as $N_o^x = \frac{Y}{(1+\pi)} \frac{Y_o}{Y}$ where we simplify by setting $\xi_o = 1$.

Another perspective on these conditions is to view the economy as being endowed with labor of value $w_o L_o$ and the number of producers the value of which is proportional to the labor endowment, $N_o^x e_o^x = \zeta_o w_o L_o$ where $\zeta_o \in [0, \infty)$ and $e_o^x \geq 0$ is the setup cost to create each producer. We can rewrite this expression as $N_o^x = \frac{\zeta_o}{e_o^x} w_o L_o$ which matches the discussion above when $\frac{\zeta_o}{e_o^x} = \xi_o (1 + \pi)$.

With our functional form assumptions, demand for a variety, φ, in the differentiated goods sector is $q_{do}(\varphi) = p_{do}(\varphi)^{-\sigma} \frac{\alpha Y_d}{P_d^{1-\sigma}}$. Given this demand, monopolistic competition and constant returns to scale production imply retailers set optimal final sales prices of $p_{do}(\varphi) = \mu w_o \tau_{do} \varphi^{-1}$. Using this and the number of producers allows us to compute the integral as

$$P_d = \left(\frac{Y}{(1+\pi)} \sum_{k=1}^{O} \left(\frac{1 - u_{dk} - i_{dk}}{1 - i_{dk}} \right) \frac{Y_o}{Y} (\mu w_k \tau_{dk})^{1-\sigma} \frac{\theta \bar{\varphi}_{dk}^{\sigma-\theta-1}}{\theta - \sigma + 1} \right)^{1/(1-\sigma)}$$

where $\int_{\bar{\varphi}_{dk}}^{\infty} z^{\sigma-1} dG(z) = \frac{\theta \bar{\varphi}^{\sigma-\theta-1}}{\theta - \sigma + 1}$. Now using the productivity thresholds, $\bar{\varphi}_{dk}$, defined above we can write the price index as

$$P_d = \lambda_2 \times Y_d^{\frac{1}{\theta} - \frac{1}{\sigma-1}} \times \rho_d$$

where

$$\rho_d \equiv \left(\sum_{k=1}^{O} \frac{Y_k}{Y} \left(\frac{\kappa_{dk} \chi(\kappa_{dk})}{\lambda_{dk} + \kappa_{dk} \chi(\kappa_{dk})} \right) (w_k \tau_{dk})^{-\theta} F_{dk}^{-\left[\frac{\theta}{\sigma-1} - 1\right]} \right)^{-\frac{1}{\theta}}$$

and

$$\lambda_2 \equiv \left(\frac{\theta}{\theta - (\sigma - 1)} \right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha} \right)^{\frac{1}{\sigma-1} - \frac{1}{\theta}} \mu \left(\frac{Y}{1+\pi} \right)^{-\frac{1}{\theta}}$$

B.2 Functional form implications

Total income to workers in country d is $Y_d = w_d L_d + w_d L_d \pi$ where w_d is the wage, L_d is labor endowment and π is the dividend per share paid by the global mutual fund. See Chaney (2008) for details regarding the global mutual fund. Given the consumer preferences in equation (22), income and the ideal price index P_d, the inverse demand for each variety within each sector is

Given these consumer preferences, income and the ideal price index P_d, the inverse demand for each variety within each sector is

$$p(q) = \left(\alpha Y_d P_d^{\sigma-1} \right)^{\frac{1}{\sigma}} q^{-\frac{1}{\sigma}}$$

The price charged in the domestic market is defined by the marginal cost equals marginal revenue expression (9) from above. Given the functional form assumptions we have made for the inverse demand curve and cost functions this becomes

$$p = \mu \frac{w_o \tau_{do}}{\varphi}$$

where $\mu = \dfrac{\sigma}{\sigma - 1}$. Notice the price charged for the imported good in the domestic market takes the standard markup over marginal cost form. Using the demand curve and domestic optimal price implies the imported quantity

$$q = \left(\mu \frac{w_o \tau_{do}}{\varphi} \right)^{-\sigma} \alpha Y_d P_d^{\sigma-1}$$

We will use a general form for the production cost function

$$t\left(q_{do}, \varphi, w_o, \mid \tau_{do}\right) - f_{do} = \frac{w_o \tau_{do}}{\varphi} q_{do} + f_{do}$$

where w_o is the wage in the exporting (origin) country, $\tau_{do} \geq 1$ is a parameter capturing one plus the iceberg transport cost between the domestic destination d and origin o, and f_{do} is the corresponding fixed cost of production for the export market in units of the numeraire. The firm that produces variety q has efficiency φ and marginal cost equal to $\dfrac{w_o \tau_{do}}{\varphi}$.

$$p\left(q\right) = \left(\alpha Y_d P_d^{\sigma-1}\right)^{\frac{1}{\sigma}} q^{-\frac{1}{\sigma}} \tag{52}$$

The price charged in the domestic market is defined by the marginal cost equals marginal revenue expression (9) from above. Given the functional form assumptions we have made for the inverse demand curve and cost functions this becomes

$$p\left(\varphi\right) = \mu \frac{w_o \tau_{do}}{\varphi} \tag{53}$$

where $\mu = \dfrac{\sigma}{\sigma - 1}$. Notice the price charged for the imported good in the domestic market takes the standard markup over marginal cost form. Using the demand curve and domestic optimal price implies the imported quantity

$$q\left(\varphi\right) = \left(\mu \frac{w_o \tau_{do}}{\varphi} \right)^{-\sigma} \alpha Y_d P_d^{\sigma-1}$$

Using the functional forms for the production cost function, consumer preferences, and the price index from equation (24) we know that final sales of each variety is

$$p\left(\varphi\right) q\left(\varphi\right) = \left(\mu \frac{w_o \tau_{do}}{\varphi} \right)^{1-\sigma} \alpha Y_d P_d^{\sigma-1}$$

Putting this together with the definition of flow profits, $\pi_{do}\left(\varphi\right) = p\left(\varphi\right) q\left(\varphi\right) - t_{do}\left(\varphi\right)$ we get that

$$\pi_{do}(\varphi) = \left(\mu \frac{w_o \tau_{do}}{\varphi}\right)^{1-\sigma} \alpha Y_d P_d^{\sigma-1} - \alpha w_o \tau_{do} (\mu w_o \tau_{do})^{-\sigma} Y_d P_d^{\sigma-1} \varphi^{\sigma-1} - f$$

which simplifies to

$$\pi_{do}(\varphi) = \frac{\alpha}{\sigma} \left(\frac{\mu w_o \tau_{do}}{P_d}\right)^{1-\sigma} Y_d \varphi^{\sigma-1} - f$$

B.3 Proof of proposition 8: Changes in real income

We prove proposition 8 assuming monopolistic competition and following steps similar to those used to prove proposition 1 in Arkolakis et al. (2012). Because, with the exception of the search frictions, the functional form assumptions detailed in Section 4, are the same as in Arkolakis et al. (2012), we can related the differentiated goods price index in our model from Section 4.2.2 to the price index equation (A22) in Arkolakis et al. (2012, p. 123)

$$P_{do}^{1-\sigma} = \left(\frac{1 - u_{do} - i_{do}}{1 - i_{do}}\right) \left(P_{do}^{ACR}\right)^{1-\sigma} \tag{54}$$

where $\left(P_{do}^{ACR}\right)^{1-\sigma} = N_o^x (\mu w_o \tau_{do})^{1-\sigma} \Psi_{do}$ and it will be useful to define the important one-sided moment $\Psi_{do} \equiv \int_{\bar{\varphi}_{do}}^{\infty} z^{\sigma-1} dG(z)$. Also define the elasticity of this integral with respect to the cutoff $\psi_{do} \equiv \frac{\partial \ln(\Psi_{do})}{\partial \ln(\bar{\varphi}_{do})}$. A sufficient condition for $\psi_{do} \leq 0$ is $\sigma > 1$. The overall price index with both homogeneous and differentiated goods is

$$\Xi_d = \left(\frac{p_1}{1-\alpha}\right)^{1-\alpha} \left(\frac{P_d}{\alpha}\right)^{\alpha}$$

where p_1 is the price of the freely trade homogeneous good and α is the share of consumption devoted to the differentiated goods bundle. Lastly, it will be very useful to denote the total derivative of the log of a variable x, as $d \ln x = \ln(x'/x) = \ln(\hat{x})$ and so $\exp(d \ln x) = \hat{x}$.

B.3.1 Step 1: Small changes in per capita real income satisfy

$$d \ln(W_d) \quad = \quad -\alpha d \ln(P_d) \tag{55}$$

Per capital real income in country d is given by $W_d \equiv \frac{w_d}{\Xi_d}$ where Ξ_d is the overall price index and w_d is the wage. Using the price index gives

$$W_d \quad = \quad w_d \left(\frac{p_1}{1-\alpha}\right)^{\alpha-1} \left(\frac{P_d}{\alpha}\right)^{-\alpha}$$

Normalizing $w_d p_1^{\alpha-1} = (1-\alpha)^{\alpha-1}$, then taking logs and totally differentiating gives equation (55). Throughout we will impose labor market clearing implies all labor is employed so that $Y_d = w_d L_d$.

B.3.2 Step 2: Small changes in the consumer price index satisfy

$$d\ln P_d = \sum_{k=1}^{O} \frac{\lambda_{dk}}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)} \left[d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) \right.$$
$$\left. +\ (1-\sigma+\psi_{do})\left(d\ln w_o + d\ln \tau_{do}\right) + d\ln N_o^x + \frac{\psi_{do} d\ln\left(F_{do}\right)}{\sigma-1} \right] \quad (56)$$

where ψ_{do} is defined above and $\psi_d = \sum_{k=1}^{O} \lambda_{dk}\psi_{dk}$.

Equation (56) is analogous to equation (A33) in Arkolakis et al. (2012, p. 125) other than a typo in the first multiplicative term where they should have γ_j instead of γ_{ij}. When the utility function has only differentiated goods ($\alpha = 1$) and there are no search frictions ($u_{do} = 0$), equations (A33) and (56) are the same. The sign on ψ_d and ψ_{do} differ between the models because ours is defined in terms of productivity while theirs is defined in terms of costs.

We derive equation (56) by starting with total consumption in destination country d for the differentiated goods bundle from origin country o by integrating over all varieties at final prices. Because CES preferences define the differentiated goods aggregate given in equation (22), this integral is the value of CES demand for the bundle of country o products

$$C_{do} = \left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi) = \alpha \frac{P_{do}^{1-\sigma} Y_d}{P_d^{1-\sigma}}$$

Define the consumption to income ratio (which in our model is different from the observed import share) as λ_{do}

$$\lambda_{do} \equiv \frac{C_{do}}{Y_d} = \alpha \frac{P_{do}^{1-\sigma} Y_d}{P_d^{1-\sigma} Y_d} = \alpha \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}$$

We will work with $P_d^{1-\sigma}$ using the definition of the price index for the differentiated good in the destination market d given by

$$P_d = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Take the log of this expression to get $(1-\sigma)\ln P_d = \ln \sum_{k=1}^{O} P_{dk}^{1-\sigma}$ and then totally differentiate to get

$$(1-\sigma)\, d\ln P_d = \sum_{k=1}^{O} \frac{P_{dk}^{1-\sigma}}{P_d^{1-\sigma}} \frac{dP_{dk}^{1-\sigma}}{P_{dk}^{1-\sigma}}$$

Rearranged λ_{do} to get that $\frac{\lambda_{do}}{\alpha P_{do}^{1-\sigma}} = \frac{1}{P_d^{1-\sigma}}$ and then use this to simplify

$$(1-\sigma)\, d\ln P_d = \sum_{k=1}^{O} \frac{\lambda_{dk}}{\alpha} d\ln P_{dk}^{1-\sigma}$$

Taking logs of equation (54) and totally differentiating gives

$$d\ln P_{do}^{1-\sigma} = d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) + d\ln\left(P_{do}^{ACR}\right)^{1-\sigma}$$

Employing our functional form assumptions which gives $\left(P_{do}^{ACR}\right)^{1-\sigma} = N_o^x \left(\mu w_o \tau_{do}\right)^{1-\sigma} \Psi_{do}$ we can derive

$$d\ln\left(P_{do}^{ACR}\right)^{1-\sigma} = d\ln N_o^x + (1-\sigma)\left(d\ln w_o + d\ln \tau_{do}\right) + \psi_{do} d\ln\left(\bar{\varphi}_{do}\right)$$

where we use the chain rule to get

$$d\ln \Psi_{do} = \psi_{do} d\ln\left(\bar{\varphi}_{do}\right)$$

Putting these parts together gives

$$(1-\sigma)\, d\ln P_d = \sum_{k=1}^{O} \frac{\lambda_{dk}}{\alpha}\left[d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) + (1-\sigma)\left(d\ln w_o + d\ln \tau_{do}\right) + d\ln N_o^x + \psi_{do} d\ln\left(\bar{\varphi}_{do}\right)\right] \quad (57)$$

If we set $\alpha = 1$ and $u_{do} = 0$, we match equation (A34) in Arkolakis et al. (2012, p. 125).

Next, take the log and total derivative of the cutoff expression from equation (25)

$$d\ln\left(\bar{\varphi}_{do}\right) = -d\ln\left(P_d\right) + d\ln\left(\tau_{do}\right) + \left(\frac{1}{\sigma-1}\right) d\ln\left(F_{do}\right) + d\ln\left(w_o\right) \quad (58)$$

which is the analog to equation (A36) in Arkolakis et al. (2012, p. 126). There are a few differences between equation (58) here and their equation (A36). First, the signs are reversed because we define everything in terms of productivity while they use costs. Second, their term ξ_{ij} captures the fixed cost of entry like our term F_{do} (see equation (A27) on page 124). And while their term ρ_{ij} allows for some foreign labor to be used to enter a foreign country, we do not. Making the same restriction in their model would require setting $h_{ij} = 1$ and hence $\rho_{ij} = 1$.

Combining equations (57) and (58) gives equation (56).

B.3.3 Step 3: Small changes in the consumer price index satisfy

$$
\begin{aligned}
d\ln P_d &= \sum_{k=1}^{O}\lambda_{dk}\left(\frac{d\ln(\lambda_{do})-d\ln(\lambda_{dd})}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)}\right)+\left(\frac{\psi_{dd}-\psi_d}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)}\right)d\ln(\bar{\varphi}_{dd}) \\
&+ \frac{d\ln N_d^x}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)} \\
&+ \frac{\psi_d d\ln(F_{dd})}{(\sigma-1)\,\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)} \\
&+ \frac{d\ln\left(\dfrac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)}
\end{aligned} \tag{59}
$$

If $\alpha=1$, $u_{do}=0$, and F_{dd} is a constant then (59) becomes equation (A37) of Arkolakis et al. (2012, p. 126).

Start again with the consumption to income ratio $\lambda_{do}=\alpha\frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}$ and form $\frac{\lambda_{do}}{\lambda_{dd}}=\frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}}$. Substitute into the ratio $\frac{\lambda_{do}}{\lambda_{dd}}$ our functional form assumptions for the price index, take logs, and then totally differentiate to get

$$
\begin{aligned}
d\ln(\lambda_{do})-d\ln(\lambda_{dd}) &= (1-\sigma)\left(d\ln w_o+d\ln\tau_{do}\right)+\psi_{do}d\ln(\bar{\varphi}_{do})-\psi_{dd}d\ln(\bar{\varphi}_{dd}) \\
&+ d\ln N_o^x-d\ln N_d^x \\
&+ d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right)-d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)
\end{aligned} \tag{60}
$$

We have simplified this by recalling that we are considering a foreign shock so that $d\ln\tau_{dd}=0$ and that we have made the normalization $w_d p_1^{\alpha-1}=(1-\alpha)^{\alpha-1}$ and will assume that the homogeneous good price remains fixed so that $d\ln w_d=0$. We have also ordered the terms as presented in Arkolakis et al. (2012, p. 126) to make the comparison easy.

We can derive two cutoff expressions

$$
d\ln(\bar{\varphi}_{do})=-d\ln(P_d)+d\ln(\tau_{do})+\frac{d\ln(F_{do})}{\sigma-1}+d\ln(w_o)
$$

and also

$$
d\ln(\bar{\varphi}_{dd})=-d\ln(P_d)+\frac{d\ln(F_{dd})}{\sigma-1}
$$

where we again impose that $\tau_{dd}=1$ and the normalization $w_d p_1^{\alpha-1}=(1-\alpha)^{\alpha-1}$. Combining these two cutoff expressions gives

$$
d\ln(\bar{\varphi}_{do}) = d\ln(\bar{\varphi}_{dd})+d\ln(w_o)+d\ln(\tau_{do})+\frac{d\ln(F_{do})}{\sigma-1}-\frac{d\ln(F_{dd})}{\sigma-1} \tag{61}
$$

which is akin to the last equation of Arkolakis et al. (2012, p. 126) with the exception that they have a typo where the equal sign should be a minus sign. In our model, it is not necessarily the case that $d\ln(F_{dd}) = 0$ in response to a foreign shock because the effective entry cost, F_{dd}, is an endogenous variable and not a parameter.

Combine expression (61) with $d\ln(\lambda_{do}) - d\ln(\lambda_{dd})$ to get

$$\begin{aligned} d\ln(\lambda_{do}) - d\ln(\lambda_{dd}) &= (1-\sigma+\psi_{do})(d\ln w_o + d\ln \tau_{do}) \\ &+ \psi_{do}\left(\frac{d\ln(F_{do})}{\sigma-1} - \frac{d\ln(F_{dd})}{\sigma-1}\right) \\ &+ (\psi_{do}-\psi_{dd})\, d\ln(\bar{\varphi}_{dd}) + d\ln N_o^x - d\ln N_d^x \\ &+ d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) - d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right) \end{aligned} \tag{62}$$

Equation (62) is analogous to equation (A38) in Arkolakis et al. (2012, p. 127) which has a typo because α^*_{ij} should be α^*_{jj}.

Substituting equation (62) into equation (56) and doing a lot of algebra gives equation (59).

B.3.4 Step 4: Small changes in the consumer price index satisfy

$$\begin{aligned} d\ln P_d &= \frac{-d\ln(\lambda_{dd})}{\alpha\left(1-\sigma+\alpha^{-1}(\sigma-1-\theta)\right)} + \frac{d\ln N_d^x}{\alpha\left(1-\sigma+\alpha^{-1}(\sigma-1-\theta)\right)} \\ &+ \frac{(\sigma-1-\theta)\, d\ln(F_{dd})}{(\sigma-1)\,\alpha\left(1-\sigma+\alpha^{-1}(\sigma-1-\theta)\right)} \\ &+ \frac{d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)}{\alpha\left(1-\sigma+\alpha^{-1}(\sigma-1-\theta)\right)} \end{aligned} \tag{63}$$

We depart somewhat from the approach taken in step 4 of Arkolakis et al. (2012, p. 127) in simplifying equation (59) to derive equation (63). They invoke macro-level restriction number 3, "R3: The import demand system is such that for any importer j and any pair of exporters $i \neq j$ and $i' \neq j$, $\varepsilon_j^{ii'} = \varepsilon < 0$ if $i = i'$, and zero otherwise." As they describe on page 103, this restriction imposes symmetry on the elasticity of the consumption ratio to changes in variable trade costs. That elasticity in our model in general is given by equation (69) and need not be symmetric across countries. A sufficient condition to derive equation (63), however, is that productivity distributions and consumer preferences are symmetric. For now we impose those restrictions in the following steps but could likely relax them in future work.

The term we need to consider from equation (59) is $\psi_d = \sum_{k=1}^{O} \lambda_{dk}\psi_{dk}$ which is the consumption share weighted average of the elasticity of the moment of the productivity distribution where $\psi_{do} = \dfrac{d\ln(\Psi_{do})}{d\ln(\bar{\varphi}_{do})}$. In Section 4.2, we present functional forms for preferences and productivity. In particular, productivity $\varphi \in [1, +\infty)$ is Pareto distributed with CDF $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$ and PDF $g(\varphi) = \theta\varphi^{-\theta-1}$ where, as usual, $\theta > \sigma - 1$ in order to close the model. With this distribution, the moment $\Psi_{do} = \frac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta - \sigma + 1}$ and the elasticity $\psi_{do} = -\frac{\bar{\varphi}_{do}^{\sigma}\theta\bar{\varphi}_{do}^{-\theta-1}}{\Psi_{do}} = -(\theta - \sigma + 1)$. Notice that the restriction that $\theta > \sigma - 1$ ensures $\psi_{do} < 0$ and $\psi_d < 0$. Also notice that $\psi_{do} = \psi_{dd}$ and the term we are actually interested in becomes

$$\psi_d = \sum_{k=1}^{O} \lambda_{dk}\psi_{do} = \sigma - 1 - \theta \tag{64}$$

because by definition consumption shares $1 = \sum_{k=1}^{O} \lambda_{dk}$ Substituting equation (64) into (59) and also using the fact that Euler's Homogeneous Function Theorem gives $\sum_{k=1}^{O} \lambda_{dk} d\ln(\lambda_{do}) = 0$ provides (63).

B.3.5 Step 5: Small changes in the number of exporters satisfy

$$\begin{aligned} d\ln P_d &= \frac{-d\ln(\lambda_{dd})}{\alpha\left(1 - \sigma + \alpha^{-1}(\sigma - 1 - \theta)\right)} \\ &+ \frac{(\sigma - 1 - \theta)\, d\ln(F_{dd})}{(\sigma - 1)\,\alpha\left(1 - \sigma + \alpha^{-1}(\sigma - 1 - \theta)\right)} \\ &+ \frac{d\ln\left(\dfrac{1 - u_{dd} - i_{dd}}{1 - i_{dd}}\right)}{\alpha\left(1 - \sigma + \alpha^{-1}(\sigma - 1 - \theta)\right)} \end{aligned} \tag{65}$$

Restricted entry immediately gives $d\ln N_d^x = 0$ and free entry with total profits proportional to output also gives $d\ln N_d^x = 0$.

B.3.6 Combining steps 1 to 5 into the final expression

Combining equation (55) with equation (65) provides the change in real income in response to a foreign shock in our model

$$\begin{aligned} d\ln(W_d) &= \frac{d\ln(\lambda_{dd})}{\left(1 - \sigma + \alpha^{-1}(\sigma - 1 - \theta)\right)} \\ &- \frac{(\sigma - 1 - \theta)\, d\ln(F_{dd})}{(\sigma - 1)\left(1 - \sigma + \alpha^{-1}(\sigma - 1 - \theta)\right)} \\ &- \frac{d\ln\left(\dfrac{1 - u_{dd} - i_{dd}}{1 - i_{dd}}\right)}{\left(1 - \sigma + \alpha^{-1}(\sigma - 1 - \theta)\right)} \end{aligned} \tag{66}$$

The terms $(1-\sigma+\alpha^{-1}(\sigma-1-\theta)) \leq 0$ and $1-\sigma+\theta \leq 0$. This implies that an increase (decrease) in the domestic consumption share lowers (raises) welfare, an increase (decrease) in the effective entry cost lowers (raises) welfare, and an increase (decrease) in the matched rate raises (lowers) welfare.

Assume we only have differentiated goods ($\alpha = 1$) so that $(1-\sigma+\alpha^{-1}(\sigma-1-\theta)) = -\theta$ then equation (66) becomes

$$d\ln W_d = -\frac{d\ln(\lambda_{dd})}{\theta} + \frac{(\sigma-1-\theta)\, d\ln(F_{dd})}{\theta(\sigma-1)} + \frac{d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)}{\theta}$$

Integrating this gives

$$\hat{W}_d = \hat{\lambda}_{dd}^{-\frac{1}{\theta}} \hat{F}_{dd}^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \left(1-\widehat{\frac{u_{dd}}{1-i_{dd}}}\right)^{\frac{1}{\theta}} \tag{67}$$

where $\frac{(\sigma-1-\theta)}{\theta(\sigma-1)} = \frac{1}{\theta} - \frac{1}{\sigma-1} \leq 0$. If effective entry costs are a parameter so that $d\ln(F_{dd}) = 0$ and we assume $1-\frac{u_{dd}}{1-i_{dd}} \equiv 1$, then

$$\hat{W}_d = \hat{\lambda}_{dd}^{-\frac{1}{\theta}}$$

This is the expression implied by the steps in Arkolakis et al. (2012) if we use the functional form assumptions we made in Section 4.2.

B.4 Proof of Proposition 9: Consumption elasticity

B.4.1 Relating price indexes

Start with the functional form assumptions detailed in Section 4. Because, with the exception of the search frictions, these functional form assumptions are the same as in Arkolakis et al. (2012) we can related the price index in our model given in Section 4.2.2 to the price index equation (A22) in Arkolakis et al. (2012, p. 123)

$$\left(P_{do}^{ACR}\right)^{1-\sigma} = N_o^x (\mu w_o \tau_{do})^{1-\sigma} \Psi_{do}$$

where it will be useful to define $\Psi_{do} = \int_{\bar{\varphi}_{do}}^{\infty} \varphi^{\sigma-1} dG(\varphi)$ and the elasticity of this integral with respect to the cutoff $\psi_{do} = \frac{\partial \ln(\Psi_{do})}{\partial \ln(\bar{\varphi}_{do})} \leq 0$ a sufficient condition for which is $\sigma > 1$.

B.4.2 Demand for a country's bundle of goods

We can derive total consumption in destination country d for the goods bundle from origin country o by integrating over all varieties at final prices. Because we have CES preferences, this integral is the value of CES demand for the bundle of country o products

$$C_{do} = \left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi) = \alpha \frac{P_{do}^{1-\sigma} Y_d}{P_d^{1-\sigma}}$$

Define the consumption to income ratio (which we note in our model is different from the observed trade share) as λ_{do}

$$\lambda_{do} = \frac{C_{do}}{Y_d} = \alpha \frac{P_{do}^{1-\sigma} Y_d}{P_d^{1-\sigma} Y_d} = \alpha \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}$$

We can also form relative consumption ratios which is equivalent to Arkolakis et al. (2012) equation (21), page 110 and is just the ratio of the price indexes raised to a power

$$\frac{\lambda_{do}}{\lambda_{dd}} = \frac{C_{do}}{C_{dd}} = \frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}}$$

Using the country specific price indexes given above we have

$$\frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}} = \frac{\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right)\left(P_{do}^{ACR}\right)^{1-\sigma}}{\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)\left(P_{dd}^{ACR}\right)^{1-\sigma}}$$

where we used the definition of P_{do}^{ACR} . Taking the log of relative consumption ratios therefore gives

$$\ln\left(\frac{C_{do}}{C_{dd}}\right) = \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) + \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) - \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right) \tag{68}$$

B.4.3 Derivative of consumption ratio with respect to tariffs

The goal is to derive two derivatives. The first is the direct effect of a change in the tariffs τ_{do} on the consumption ratio

$$\frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{do}\right)} = \varepsilon_o^{dd}$$

The second is the indirect effect, which documents how changing tariffs between a third country d' and the origin o changes relative consumption in country d

$$\frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \varepsilon_o^{dd'}$$

B.4.4 Direct effect of tariff changes ($d' = d$ case)

We begin by deriving $\frac{\partial \ln(C_{do}/C_{dd})}{\partial \ln(\tau_{do})} = \varepsilon_o^{dd}$ in the most general form and then apply the assumptions made by ACR. With that said, two assumptions that we will make from the start are that $\frac{\partial \ln(w_d)}{\partial \ln(\tau_{do})} = 0$ and $\frac{\partial \ln(w_o)}{\partial \ln(\tau_{do})} = 0$. The wage in the origin economy w_o is a function of the τ_{do} but only through general equilibrium. Another argument for how we can ignore how tariff changes affect the equilibrium wage is to normalize the relative wage $\frac{w_o}{w_d} = 1$. This would provide the elasticity subject to this normalization.

B.4.5 First and second terms of equation (68) ($d' = d$ case)

Differentiating and simplifying the first term of equation (68) gives

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} + (1-\sigma) + \psi_{do}\frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{do})}$$

and similarly

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{do})} + \psi_{dd}\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})}$$

Combining these gives

$$\begin{aligned}\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= (1-\sigma) + \psi_{do}\frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{do})} - \psi_{dd}\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})} \\ &+ \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{do})}\end{aligned}$$

The elasticities of the cutoffs $\bar{\varphi}_{do}$ and $\bar{\varphi}_{dd}$ are related because changing tariff τ_{do} changes the price index P_d which changes the cutoff $\bar{\varphi}_{dd}$. We can derive this relationship by differentiating the explicit expression for the cutoff given in equation (25)

$$\begin{aligned}\frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{do})} &= 1 + \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})} \\ &+ \left(\frac{1}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{do})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{do})}\right]\end{aligned}$$

Substituting this into the elasticity of the general expression for the ratio of relative price indexes and simplifying gives

$$\begin{aligned}\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})} \\ &+ \left(\frac{\psi_{do}}{\sigma - 1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{do})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{do})}\right] \\ &+ \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{do})}\end{aligned}$$

The first line is the same as equation (21) in ACR (2012) except that $\psi_{do} \leq 0$ in our case, while $\gamma_{ij} \geq 0$ in the ACR expressions because we define our model in terms of productivity, φ, while they define theirs in terms of marginal cost.

B.4.6 Elasticity of destination-origin market underutilization rate

Next we calculate the elasticity of destination-origin market underutilization rate. Because we are studying a steady state, use the definition $\frac{1-u_{do}-i_{do}}{1-i_{do}} = \frac{\kappa_{do}\chi(\kappa_{do})}{\lambda + \kappa_{do}\chi(\kappa_{do})}$ to derive

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})}\right)$$

where we used the chain rule to write

$$\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})} = \frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \kappa_{do}\chi(\kappa_{do})}\frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})} = \frac{1}{\kappa_{do}\chi(\kappa_{do})}\left(\frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})}\right)$$

B.4.7 Elasticity of destination-destination market underutilization rate

Calculating the elasticity of destination-destination market underutilization rate with respect to τ_{do} also relies on the definition of $\frac{1-u_{dd}-i_{dd}}{1-i_{dd}} = \frac{\kappa_{dd}\chi(\kappa_{dd})}{\lambda + \kappa_{dd}\chi(\kappa_{dd})}$. The steps to derive this will be identical to the ones we took in calculating the destination-origin market underutilization rate with only the subindexes changing. The final derivative is

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right) = \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})}\right)$$

where we used the chain rule again to calculate

$$\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})} = \frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \kappa_{dd}\chi(\kappa_{dd})}\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})} = \frac{1}{\kappa_{dd}\chi(\kappa_{dd})}\left(\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})}\right)$$

B.4.8 General expression for $d' = d$ case

Here we try to write the most general possible expression only assuming that $\frac{\partial \ln(w_d)}{\partial \ln(\tau_{do})} = 0$ and $\frac{\partial \ln(w_o)}{\partial \ln(\tau_{do})} = 0$. Combining the general ACR term expression with the elasticity of the finding rate with respect to tariffs gives

$$\begin{aligned}
\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\frac{C_{do}}{C_{dd}}\right) &= (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{do})} \\
&+ \left(\frac{u_{do}}{1 - i_{do}}\right)\left(\frac{\partial \ln \kappa_{do} \chi(\kappa_{do})}{\partial \ln(\tau_{do})}\right) - \left(\frac{u_{dd}}{1 - i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd} \chi(\kappa_{dd})}{\partial \ln(\tau_{do})}\right) \\
&+ \left(\frac{\psi_{do}}{\sigma - 1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))} \frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{do})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))} \frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{do})}\right]
\end{aligned}$$

B.4.9 Indirect effect of tariff changes ($d' \neq d$ case)

The second derivative is the indirect effect which documents how changing tariffs between a third country d' and the origin o changes relative consumption in country d

$$\frac{\partial \ln(C_{do}/C_{dd})}{\partial \ln(\tau_{d'o})} = \varepsilon_o^{dd'}$$

B.4.10 First and second terms of equation (68) ($d' \neq d$ case)

Following the general pattern above we first derive the change in the ACR price indexes as

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} + \psi_{do} \frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{d'o})}$$

and similarly

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} + \psi_{dd} \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})}$$

Combining these gives

$$\begin{aligned}
\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= \psi_{do} \frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{d'o})} - \psi_{dd} \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} \\
&+ \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})}
\end{aligned}$$

The elasticities of the cutoffs $\bar{\varphi}_{do}$ and $\bar{\varphi}_{dd}$ with respect to $\tau_{d'o}$ are also related because changing tariff $\tau_{d'o}$ changes the price index P which changes the cutoff $\bar{\varphi}_{dd}$. We can derive this relationship by differentiating the explicit expression for the cutoff given in equation (25)

$$\frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{d'o})} = -\frac{\partial \ln(P)}{\partial \ln(\tau_{d'o})} + \left(\frac{1}{\sigma-1}\right)\frac{\partial \ln(F_{do})}{\partial \ln(\tau_{d'o})}$$

and symmetrically

$$\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} = -\frac{\partial \ln(P_d)}{\partial \ln(\tau_{d'o})} + \left(\frac{1}{\sigma-1}\right)\frac{\partial \ln(F_{dd})}{\partial \ln(\tau_{d'o})}$$

So the relationship between the two cutoff elasticities is

$$\begin{aligned}\frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{d'o})} &= \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} \\ &+ \left(\frac{1}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right]\end{aligned}$$

and we use the chain rule to expand the derivatives with respect to the finding rate. Substituting the relationship between the cutoffs into the general expression for the ratio of relative prices and simplifying gives

$$\begin{aligned}\frac{\partial}{\partial \ln(\tau_{d'o})}\ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{d'o})}\ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= (\psi_{do}-\psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} \\ &+ \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right]\end{aligned}$$

B.4.11 Elasticity of destination-origin market underutilization rate

We continue to follow the pattern above and calculate the elasticity of destination-origin market underutilization rate. Because we are studying a steady state, use the definition $\frac{1-u_{do}-i_{do}}{1-i_{do}} = \frac{\kappa_{do}\chi(\kappa_{do})}{\lambda+\kappa_{do}\chi(\kappa_{do})}$ to derive

$$\frac{\partial}{\partial \ln(\tau_{d'o})}\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}\right)$$

where we used the chain rule to write

$$\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})} = \frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \kappa_{do}\chi(\kappa_{do})}\frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})} = \frac{1}{\kappa_{do}\chi(\kappa_{do})}\frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}$$

For the record, the elasticity of the third term boils down to

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}\right)$$

The thing that matters is the capacity utilization rate and the elasticity of the finding rate with respect to tariffs.

B.4.12 Elasticity of destination-destination market underutilization rate

The fourth term requires that we calculate

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)$$

The steps are identical to the ones we took in calculating the destination-origin market underutilization rate derivative with only the subindexes changing. The end result is

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right) = \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right)$$

where we again used the chain rule to write

$$\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})} = \frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \kappa_{dd}\chi(\kappa_{dd})}\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})} = \frac{1}{\kappa_{dd}\chi(\kappa_{dd})}\left(\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right)$$

B.4.13 General expression for $d' \neq d$ case

Here we try to write the most general possible expression only assuming that $\frac{\partial \ln(w_d)}{\partial \ln(\tau_{d'o})} = 0$ and $\frac{\partial \ln(w_o)}{\partial \ln(\tau_{d'o})} = 0$. The general ACR term expression was

$$\begin{aligned}
\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= (\psi_{do}-\psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} \\
&+ \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right]
\end{aligned}$$

Combining these with the elasticity of utilization rates gives

$$\begin{aligned}
\frac{\partial \ln(C_{do}/C_{dd})}{\partial \ln(\tau_{d'o})} &= (\psi_{do}-\psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} \\
&+ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right) \\
&+ \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right]
\end{aligned}$$

B.4.14 Final expression

The final expression is

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \varepsilon_o^{dd'} = \begin{cases} \begin{aligned} & (1-\sigma)+\psi_{do}+(\psi_{do}-\psi_{dd})\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}+\frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{do}\right)}-\frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{do}\right)} \\ + & \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{do}\right)}\right)-\left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}\right) \\ + & \left(\tfrac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}\frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{do}\right)}-\frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}\frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{do}\right)}\right] \end{aligned} & if\ d'=d \\ \\ \begin{aligned} & (\psi_{do}-\psi_{dd})\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}+\frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{d'o}\right)}-\frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{d'o}\right)} \\ + & \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right)-\left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) \\ + & \left(\tfrac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}\frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)}-\frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}\frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)}\right] \end{aligned} & if\ d'\neq d \end{cases} \tag{69}$$

B.4.15 Interesting restrictions on the general elasticity

We will make a few assumptions to show how the elasticity is related to models without search frictions. First assume that $\frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{do}\right)} = \frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{do}\right)}$ in which case the first additive term in each of the cases becomes the ACR elasticity denoted $\varepsilon_o^{ACR,dd'}$. Second, assume that $l = 0 = h$ so that F_{dd} and F_{do} are parameters making $\frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} = \frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\tau_{do}\right)} = 0$. Together these give

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \varepsilon_o^{dd'} = \begin{cases} \varepsilon_o^{ACRdd'}+\left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{do}\right)}\right)-\left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}\right) & if\ d'=d \\ \\ \varepsilon_o^{ACRdd'}+\left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right)-\left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) & if\ d'\neq d \end{cases}$$

Signing the effect of the addition of search depends on our work using the IFT.

B.5 Proof of Proposition 10: Deriving the gravity equation

The value of total imports will be

$$I_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right)N_o^x\int_{\bar{\varphi}_{do}}^{\infty} n\left(\varphi\right)q_{do}\left(\varphi\right)dG\left(\varphi\right)$$

We need to integrate over the varieties to get the total value of imports going into the domestic market. Demand for a variety, φ, in the differentiated goods sector is $q_{do}\left(\varphi\right) = p_{do}\left(\varphi\right)^{-\sigma}\frac{\alpha Y_d}{P_d^{1-\sigma}}$. Given this demand, monopolistic competition and constant returns to scale production imply producers set optimal prices of $p_{do}(\varphi) = \mu w_o \tau_{do} \varphi^{-1}$. For notational simplicity define $B_{do} \equiv \alpha (\mu w_o \tau_{do})^{-\sigma} Y_d P_d^{\sigma-1}$ and combine the optimal price with the demand curve to get $q_{do}(\varphi) = B_{do}\varphi^{\sigma}$. Evaluated at final prices, the value of sales of each variety is $p_{do}(\varphi) q_{do}(\varphi) = \mu w_o \tau_{do} B_{do} \varphi^{\sigma-1}$ and the cost to produce $q_{do}(\varphi)$ units of this variety is $t_{do}(\varphi) + f_{do} = w_o \tau_{do} B_{do} \varphi^{\sigma-1} + f_{do}$. This means that total profits generated by each variety are $p_{do}(\varphi) q_{do}(\varphi) - t_{do}(\varphi) = A_{do}\varphi^{\sigma-1}$ where it is also useful to define $A_{do} = w_o \tau_{do} B_{do} [\mu - 1]$. Using this profits expression, the productivity cutoff is $\bar{\varphi}_{do} = \left(\frac{F_{do}}{A_{do}}\right)^{\frac{1}{\sigma-1}}$ where F_{do} is given in equation (25). The value of total imports from the negotiated price curve in equation (13) is

$$n(\varphi) q_{do}(\varphi) = [1 - \gamma_{do}] p_{do}(\varphi) q_{do}(\varphi) + \gamma_{do} [t_{do}(\varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do}) s_{do}]$$

Using the functional forms assumptions from above this becomes

$$n(\varphi) q_{do}(\varphi) = (\sigma - \gamma_{do}) A_{do}\varphi^{\sigma-1} - \gamma_{do} [-f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do}) s_{do}]$$

Substituting the value of imports for a particular variety into the integral defining the value of total imports gives

$$I_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} (\sigma - \gamma_{do}) A_{do}\varphi^{\sigma-1} - \gamma_{do} [-f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do}) s_{do}] \, dG(\varphi)$$

We assume productivity, φ, has a Pareto distribution over $[1, +\infty)$ with cumulative density function $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$ and probability density function $g(\varphi) = \theta\varphi^{-\theta-1}$. The Pareto parameter and the elasticity of substitution are such that $\theta > \sigma - 1$ which ensures that the integral $\int_{\bar{\varphi}}^{\infty} z^{\sigma-1} dG(z)$ is bounded. Using these assumptions we can compute the integral to get

$$I_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \left[(\sigma - \gamma_{do}) A_{do} \frac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta - \sigma + 1} - \gamma_{do} [-f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do}) s_{do}] \bar{\varphi}_{do}^{-\theta}\right]$$

where we use the relevant moment of the productivity distribution $\int_{\bar{\varphi}_{do}}^{\infty} z^{\sigma-1} dG(z) = \frac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta - \sigma + 1}$. Define $\delta_{do} \equiv f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do}) s_{do}$ to conserve on notation, substitute the export productivity threshold into this expression, and simplify to get

$$I_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \left[(\sigma - \gamma_{do}) \frac{\theta}{\theta - \sigma + 1} + \gamma_{do} \frac{\delta_{do}}{F_{do}}\right] F_{do}^{-\left(\frac{\theta}{\sigma-1} - 1\right)} A_{do}^{\frac{\theta}{\sigma-1}}$$

Next, utilize the assumption that the number of producers in the origin market is proportional to output in that market $N_o^x = \left(\frac{Y}{1+\pi}\right) \frac{Y_o}{Y}$ and the definition for

$A_{do} = \mu^{-\sigma}\alpha\left(w_o\tau_{do}\right)^{1-\sigma} Y_d P_d^{\sigma-1}\left[\mu-1\right]$ to write

$$\begin{aligned} I_{do} &= \left(1-\frac{u_{do}}{1-i_{do}}\right)\left(\frac{Y}{1+\pi}\right)\frac{Y_o}{Y}\left[\left(\sigma-\gamma_{do}\right)\frac{\theta}{\theta-\sigma+1}+\gamma_{do}\frac{\delta_{do}}{F_{do}}\right] \\ &\times\ F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}\left(\mu^{-\sigma}\alpha\left(w_o\tau_{do}\right)^{1-\sigma} Y_d P_d^{\sigma-1}\left[\mu-1\right]\right)^{\frac{\theta}{\sigma-1}} \end{aligned}$$

We presented the price index above as

$$P_d = \lambda_2 \times Y_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d$$

substituting that in here and simplifying gives

$$\begin{aligned} I_{do} &= \left(1-\frac{u_{do}}{1-i_{do}}\right)\left[\left(\sigma-\gamma_{do}\right)\frac{\theta}{\theta-\sigma+1}+\gamma_{do}\frac{\delta_{do}}{F_{do}}\right] \\ &\times\ \left(\mu^{-\sigma}\alpha\left[\mu-1\right]\right)^{\frac{\theta}{\sigma-1}}\left(\frac{Y}{1+\pi}\right)\lambda_2^{\theta}\left(\frac{Y_oY_d}{Y}\right)\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} \end{aligned}$$

In the price index section above we also define λ_2 which we can now substitute in here and then simplify to get the final gravity equation

$$I_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-\frac{\gamma_{do}}{\sigma\theta}\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-\left(\sigma-1\right)\right)\right)\right)\alpha\left(\frac{Y_oY_d}{Y}\right)\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}$$

We could have also evaluated the integral at the beginning of this section at final sales prices $p_{do}\left(\varphi\right)$ instead of negotiated prices $n_{do}\left(\varphi\right)$. From equation (13) we can see $p_{do}\left(\varphi\right) = n_{do}\left(\varphi\right)$ if $\gamma_{do} = 0$. Setting $\gamma_{do} = 0$ then gives

$$C_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right)\alpha\left(\frac{Y_oY_d}{Y}\right)\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}$$

The utilization rate is bounded by

$$\left(1-\frac{u_{do}}{1-i_{do}}\right) = \left(\frac{\kappa_{do}\chi\left(\kappa_{do}\right)}{\lambda+\kappa_{do}\chi\left(\kappa_{do}\right)}\right) \in [0,1]$$

because the finding and destruction rates must be finite. Similarly, the bundle of search parameters

$$\left(1-\frac{\gamma_{do}}{\sigma\theta}\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-\left(\sigma-1\right)\right)\right)\right) \in [0,1]$$

Proving the second condition takes a few steps. First, prove that $\left(1-\frac{\gamma_{do}}{\sigma\theta}\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-\left(\sigma-1\right)\right)\right)\right) \leq 1$. We can prove this by noting that $\delta_{do} \equiv f_{do} - l_{do} - \kappa_{do}\chi\left(\kappa_{do}\right)s_{do}$ so it must be that $\delta_{do} \leq F_{do}$ and therefore $\delta_{do}F_{do}^{-1} \leq 1$. Also, the restriction that $\sigma > 1$, ensures $\theta - \left(\sigma-1\right) < \theta$. Together, these ensure

$\theta - \dfrac{\delta_{do}}{F_{do}}(\theta - (\sigma - 1)) \geq 0$. Combining this with the fact that $\gamma_{do} \in [0, 1]$ ensures that $1 - \dfrac{\gamma_{do}}{\sigma\theta}\left(\theta - \dfrac{\delta_{do}}{F_{do}}(\theta - (\sigma - 1))\right) \leq 1$.

Next, show that $\left(1 - \dfrac{\gamma_{do}}{\sigma\theta}\left(\theta - \dfrac{\delta_{do}}{F_{do}}(\theta - (\sigma - 1))\right)\right) \geq 0$ by showing that $1 \geq \dfrac{\gamma_{do}}{\sigma\theta}\left(\theta - \dfrac{\delta_{do}}{F_{do}}(\theta - (\sigma - 1))\right)$. Because $\gamma_{do} \in [0, 1]$ and $\sigma > 1$ we know that $\dfrac{\gamma_{do}}{\sigma} < 1$. Likewise, $\sigma > 1$ ensures $\theta - (\sigma - 1) < \theta$ so that $\dfrac{(\theta - (\sigma - 1))}{\theta} < 1$. We assume above that $\theta - (\sigma - 1) > 0$ in order to close the model. Together these imply that $\dfrac{(\theta - (\sigma - 1))}{\theta} \in [0, 1]$. Finally, because $\delta_{do}F_{do}^{-1} \leq 1$ we have that $\dfrac{\delta_{do}}{F_{do}}\dfrac{(\theta - (\sigma - 1))}{\theta} \leq 1$ and we have proved the result.

C Appendix for general mathematical results

C.1 Poisson process

Consider a continuous time Poisson process where the number of events, n, in any time interval of length t is Poisson distributed according to

$$P\{N(t+s) - N(s) = n\} = e^{-\lambda t}\frac{(\lambda t)^n}{n!} \quad n = 0, 1, \ldots$$

where $s, t \geq 0$, $N(0) = 0$, and the process has independent increments. The mean number of events that occur by time t is

$$E[N(t)] = \lambda t$$

Notice that λ is defined in units of time as λ events per t. For example, if producers in our model contact 9 retailers every six months on average, then we could recast our model measured in years with $t = 1$ and $\lambda = 4.5$ because $\lambda t = 9 \times 1/2 = 4.5$.

Using the Poisson process above, the probability that the first event occurs after time t equals the probability no event has happened before

$$P\{t_1 > t\} = P[N(t) = 0] = e^{-\lambda t}$$

where t_n denotes the time between the $(n-1)$st and nth events so t_1 is the time of the first event. The arrival time of the first event is an exponential random variables with parameter λ. Conversely, the probability the first event occurs between time 0 and time t is $P\{t_1 \leq t\} = 1 - e^{-\lambda t}$. Because the Poisson process has independent increments, the distribution of time between any two events, t_n, for $n = 1, 2, \ldots$ will also be an exponential random variables with parameter λ. The sequence of times between all events, $\{t_n, n \geq 1\}$, also known as the sequence of inter-arrival times, will be a sequence of *i.i.d.* exponential random variables with parameter λ. Given this distribution, the mean time between events is

$$E[t_n] = \frac{1}{\lambda}$$

For example, if producers in our model contact 9 retailers every six months on average, so that $\lambda t = 9/2$, then the average time between contacts is $1/\lambda = 2/9$ years (or about $365.25 \times 2/9 = 81.17$ days).

The arrival time of the nth event, S_n, also called the waiting time, is the sum of the time between preceding events

$$S_n = \sum_{i=1}^{n} t_i$$

Because S_n is the sum of n *i.i.d.* exponential random variables where each has parameter λ and the number of events n is an integer, S_n has an Erlang distribution with cumulative density function

$$P\left\{S_n \leq t\right\} = P\left[N\left(t\right) \geq n\right] = \sum_{i=n}^{\infty} e^{-\lambda t} \frac{\left(\lambda t\right)^i}{i!}$$

and probability density function

$$f\left(t\right) = \lambda e^{-\lambda t} \frac{\left(\lambda t\right)^{n-1}}{\left(n-1\right)!}$$

The Erlang distribution is a special case of the gamma distribution where the gamma allows the number of events n to be any positive real number while the Erlang restricts n to be an integer. The above discussion relies heavily on (Ross, 1995, Chapter 2).

C.2 Deriving aggregate welfare

Here we outline the steps to show that the indirect utility function (welfare) is C/P, where C is total consumption expenditure, n is the vector of prices for each good, and P is the ideal price index. Assume that preferences are homothetic, which is defined in MWG 3.B.6, page 45. This means that they can be represented by a utility function that is homogeneous of degree one in quantities and that the corresponding indirect utility function is linear in total consumption expenditure. We can begin with the indirect utility function and then manipulate it as follows

$$\begin{aligned} W\left(p, C\right) &= W\left(p, 1\right) C \\ W\left(p, e\left(p, u\right)\right) &= W\left(p, 1\right) e\left(p, u\right) \\ u &= W\left(p, 1\right) e\left(p, u\right) \\ 1 &= W\left(p, 1\right) e\left(p, 1\right) \\ \frac{1}{e\left(p, 1\right)} &= W\left(p, 1\right) \end{aligned}$$

where the first line comes from homothetic preferences; the second line follows by plugging in for consumption expenditure $C = e\left(p, u\right)$; the third line comes from equation (3.E.1) in MWG that says $W\left(p, e\left(p, u\right)\right) = u$ (also known as duality); and in the fourth line we plug in for utility level $u = 1$. The function $e\left(p, u\right)$ is the consumption expenditure function which solves the expenditure minimization problem. Using this result and the fact that the price index is defined as $e\left(p, 1\right) \equiv P$ we can show that

$$W\left(p, C\right) = W\left(p, 1\right) C = \frac{1}{e\left(p, 1\right)} C = \frac{C}{P}$$

Hence, as long as preferences are homothetic, we will always get welfare equal to consumption expenditure divided by the price index, $W\left(p, Y\right) = C/P$. The expenditure approach to accounting can be particularly useful for computing aggregate welfare in this setting because, $W\left(p, C\right) = \frac{C}{P} = \frac{Y - I - G - NX}{P}$.

C.3 Moving from an index to a distribution of goods

Melitz uses the following steps to move from index ω over a continuum of goods available to consume, Ω, to the distribution of productivity $G(\varphi)$ and the measure of goods available for consumption, N. Assume that the continuum of goods Ω has measure $M = |\Omega|$. In our model, this continuum of goods will have measure $(1 - u - h)N$. The following steps keep the notation in Melitz's original work.

Change of variables, also known as integration by substitution, states

$$\int_{i(a)}^{i(b)} f(\omega)d\omega = \int_{a}^{b} f(h(\varphi))h'(\varphi)d\varphi$$

Choose $\omega = h(\varphi) = G(\varphi)M$ where $G(\varphi)M$ is differentiable in φ then we can apply the rule from right to left to get

$$\int_{0}^{\infty} f(G(\varphi)M)\frac{\partial G(\varphi)M}{\partial \varphi}d\varphi = \int_{G(0)M}^{G(\infty)M} f(\omega)d\omega = \int_{0}^{M} f(\omega)d\omega = \int_{\omega\in\Omega} f(\omega)d\omega$$

Choosing $G(0) = 0$ allows us to start the continuous indexing such that the upper bound is the measure of Ω and is without loss of generality. Notice that in equation (22) we integrate from $\bar{\varphi}_s$ to ∞ and not 0 to ∞. This is because for all productivities below the cutoff,$\bar{\varphi}_s$, the quantity consumed is zero, so we can ignore that mass.

Remember that in our context $f(\cdot)$ is the indexing function so the final step

$$\int_{0}^{\infty} f(\varphi)\frac{\partial G(\varphi)M}{\partial \varphi}d\varphi = \int_{0}^{\infty} f(G(\varphi)M)\frac{\partial G(\varphi)M}{\partial \varphi}d\varphi$$

simply reassigns the indexing number $G(\varphi)M$ to φ.