Spirits, but not so Animal

José V. Rodríguez Mora *
Universitat Pompeu Fabra
sevimora@upf.es

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Abstract

This paper stresses the importance of information diffusion problems in the business cycles. It shows that it is possible to modelize "animal spirits" in a context of perfectly rational and forward-looking agents whose only source of information is their perception on the past behaviour of the economy. In the animal spirits equilibrium the economy will move from booms to recessions according to changes in the optimism of the agents. These changes are due to what they *percieve* that was the past evolution of the economy *not to the presence of exogenous coordination devices or shocks.*

Agents use their perception of the past in order to forecast what will happen in the future. Each of them collects her information by observing the actions taken by a different sample of individuals. Consequently, the perceptions are noisy (no agent is ever sure that what she sees is what really happened) and may differ across agents. The amount of noise that agents face is in itself an endogenous variable, a product of the state of the economy. The stochastic dynamic structure of the aggregates is then endogenous and mimics business cycle behaviour.

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1 Introduction

In order to forecast future events agents make use of their knowledge of the workings of the economy and all the available information on past events. This information is not perfectly accurate. They do not know the exact realization of all the relevant economic variables during all the relevant past periods; nevertheless they have a *perception* of it.

These perceptions are based on their individual experiences, the experience of other agents with whom they communicate, and some information on aggregate variables that could be relevant for their decision; but even this information on aggregates is subject to a high degree of noise (see Rodríguez Mora and Schulstad (1995)).

Agents process all this different sources of information, and out of it they generate a perception of the state of the economy at any point of the past and, through it, a forecast on its future evolution.

To assume that agents have rational expectations it should mean that they process this information in a coherent way (that is, consistent with their capabilities for learning or, in a more extreme version, with the workings of the economy). Rational expectations imposes no conditions on the properties or accuracy of this perceptions, nevertheless it is a common practice to impose strong assumptions on the characteristics of this information. Usually it is assumed to be perfect, so that agents know the complete sequence of past relevant events, but even when this is not the case the *quality* of this information is assumed to be fixed and exogenous, independent of the activity of the agents or the evolution of the economy.

This paper shows that this is not an innocuous assumption. Its main objective is to get some insights on the way in which expectations are formed; its main result to give some theoretical support for models in which expectations lie ahead of events in the business cycle process.

Generalized waves of optimism and pessimism have been considered as a possible source of business cycles since very long ago (see Zarnowitz(1985)). The usual argument is that in a world with multiple equilibria changes in the expectations of the agents would produce changes in the aggregate variables by the way of "jumps" between the different equilibria. Traditionally such changes in the expectations were associated with irrational (or boundedly rational) behavior, thus Keynes explained them by *"Animal Spirits"*, but in recent times the literature on sunspots and multiple equilibria has attempted to include them in a rational, forward-looking context.

In essence sunspot and multiple equilibria models are coordination games. The expectations that agents have are self-fulfilling, and so rational. They change according to the realizations of a random, exogenous and commonly observable coordination device. Agents interpret the realizations of this device as *"it is good to invest"* or as *"it is not good to invest"*. As long as all the individuals interpret the realizations in the same manner this behavior produces a Nash equilibrium. They are optimistic or pessimistic depending in what the device tells them to be. The device becomes an oracle; an oracle that is believed because it proves to produce accurate forecasts, but it produces accurate forecasts because it is believed in the first place. This line of research has been explored by Azariadis (1981), Azariadis and Guesnerie (1985), Woodford (1988) and Howitt and McAfee (1992) among others.

Attractive as this approach might be, it has evident shortcomings. It looks as highly inaccurate as a description on how expectations are formed. Agents do not change their mind suddenly and simultaneously about the future evolution of the economy (a necessary implication of the sunspot approach), they seem to change their mind slowly, by talking with each other. They communicate, express their opinions to others. Each of them has different information and different perceptions, even if most individuals are optimistic about the future there are always some pessimistic agents. Even more important is the fact that people *do use* the past in order to forecast the future, at the very least time series econometricians do try hard. People makes use of their information on past events in order to forecast the future, they do not rely exclusively in the indications of some mysterious oracle.

The introduction of noisy information sets makes this approach more attractive, because it provides with a *"coordination device"* that comes out naturally from the structure of the model and is not subject to the criticisms expressed above. In our model each agent does her best to coordinate with the rest of the world knowing how each individual they will react to her (idiosyncratic) perception and using her own perception an her knowledge in the workings of the economy in order to establish priors on the perceptions (and so beliefs, and so actions) of others.

Probably the best way of explaining why this model is interesting is to first put it on the table, and locate it in the context of the modern literature on the topic. Therefore, I must ask for a leap of faith from the readers and immerse then in a sketch of the model before they have decided if it is or it is not worth the effort.

The basic set up of the model is the following:

- Agents want to coordinate with each other. It is worth to invest only if most contemporaneous agents do so. This is the point of most papers of the literature on multiple equilibria. Thus in Diamond (1982), Diamond and Fudenberg (1989), Bryant (1983), and Shleifer (1986) present static multiple equilibria models. Cooper and John (1988) show that the necessary and sufficient condition for obtaining multiple Pareto rankable equilibria in static models is the simultaneous presence of aggregate increased returns and strategic complementarities. In essence, these models are coordination games. Agents want to do whatever other agents are doing. They want to invest if everyone is investing; they do not want to if nobody is doing it. Given the objectives of this paper we will start by assuming that there exist an underlaying microstructure that results in the players being playing a coordination game, and we will not get into details.

- Agents have some private information regarding the past evolution of the economy. This information is not accurate, and its level of noise is dependent on the performance of the economy. The probability that an individual receives a signal saying that the past was a bad time to invest must be decreasing with the rate of return in the past. Thus when an individual quits investing it affects to the other individuals through two channels, by affecting their rate of return (we are in a coordination game), and by altering their information set, because now all other individuals receive signals subject to a higher degree of noise. Informational externalities are a fundamental component of the *"they may know better"*-herding literature on social learning (Banerjee(1992) and Bickchandani et al(1992) for instance), and consequently this papers shares something of its spirit, but it is most closely related to Kirman's Ants (in Kirman (1992)) because its emphasis in information *diffusion*, and to Rodríguez Mora (1995) because the externalities refer to the second order of the informational variables.

- Agents have a common prior on the state of the economy at any given time. In a rational expectations equilibrium this belief must be the unconditional distribution of aggregate investment. This is to say that we are going to look for Perfect Bayesian Equilibria.
- In an animal spirits equilibrium agents will coordinate using the past as a coordination device. This is, they will invest only if given their private information and common prior on past performance, they believe that the past was a good time to invest. This strategy is a rational expectations equilibrium if simultaneously happens that
  - The time series structure that produces in aggregate investment is consistent with the common prior of the agents.
  - It is optimal for each agent to follow the strategy if everybody else is doing it.

We will see that there exists an animal spirits equilibrium where all the agents invest only if their private signal tells them that yesterday was a good time to invest. This signal makes them believe that the future will be a good time to invest, it makes them optimistic. In this equilibrium aggregate investment will have a stochastic time series structure with the stylized features of business cycles. This is, it shows persistence (auto correlation) and its spectrum will assign high variances to the frequencies of the business cycle.

Imagine that the agents were able to see exactly what happened in the economy during the previous period. A possible equilibrium of the game would be the strategy "*Invest only if yesterday most people invested*", which is the same that "*Invest only if yesterday was a good time to invest*". In this equilibrium the past behavior of the economy is used as a coordination device. Imagine that at $t = 0$ the aggregate investment takes an exogenously given value y_0. At $t = 1$, and always thereafter, all the agents invest if y_0 was large, or nobody invest if it was low. The economy achieves a stationary state in one period, and never moves away from it. If the economy started from scratch a very large number of times, with random values for y_0, half of the realizations would finish with all the agents always investing and the other half with no agent ever doing it. In equilibrium there would be strong hysteresis, but not fluctuations.

In our model fluctuations will appear because *in equilibrium* agents will face uncertainty on what was the state of the economy in previous periods, and so in the future. To understand this result it is useful to make precise the exact stochastic structure of the model:

As I said before, in most rational expectations models uncertainty affects only the future realizations of the economy. The information set of the agents at time t includes all the realizations of all the variables up to that point. Uncertainty arises because future realizations of this variables might deviate from its deterministic path due to exogenous shocks. This is *not* the case in the present model.

In our model the unique source of noise lies on the private signals, and they refer to the *past* performance of the economy. There is neither aggregate productivity shocks nor mysterious stochastic coordination devices. We will use exclusively noise in the perception that agents have on the past realizations of the variables and the dynamic stochastic structure of this noise will be endogenous to the economy. In a way the economy in itself will generate the noise and uncertainty that agents face.

In the model we will need a degree of aggregate noise in this perceptions. We will need the number of agents that observe erroneous signals to be a random variable. We will discuss in further detail the significance of this hypothesis and how reasonable it is. So far it will be enough to say that this will happen if the number of agents is small, or if the idiosyncratic signals are correlated across agents.

This is the only source of aggregate noise that the economy will face. The main point of the paper is that the time series stochastic structure of this noise is determined in equilibrium; at each given moment in time its distribution function is determined by the performance of the economy. Imagine that almost everybody is investing, and so it is good to invest; the probability that an individual receives a signal saying that it is bad to invest must be very low. On the other hand assume that half of the agents are investing and half are not, it is as much likely for that his signal captures a majority of investors as it is that it captures a majority of agents that did not invest; the signal is much more noisy.

How much this noise affects aggregate investment depends on the equilibrium that the economy takes.

There will be equilibria where this noise does not affect either the information that agents have or the actions that they will take. If all the agents follow the strategies "*invest whatever your signal is*" (or " *never invest whatever your signal*"), the best response is clearly to follow the same strategy as everybody else. Additionally, the signal will be uninformative, because you know ex-ante that yesterday everybody invested (or nobody did).

On the other hand there will be an animal spirits equilibrium where the signals will affect the information that agents have, and the course of their actions. In this equilibrium agents will update their priors using their noisy signal. Suppose that an agent receives a signal that says that most people invested during the previous period (so the past was a good time for investing). We will see that it is rational for him to expect that most other agents would also observe similar samples. Therefore if he expects others to follow the rule *"Invest only if you believe that yesterday was a* *good time to invest"*, he should invest only if he sees a sample where most people invest. Thus the strategy is an equilibrium.

Business cycles follow a well determined pattern whose main characteristics are that presents persistence and that frequencies corresponding to events of a periodicity of several years have a big incidence on the total variance of the process.

Our equilibrium presents business cycles characteristics because when almost everybody is investing the variance of the signals is very low; they will almost always be accurate, and the economy will remain around there for quite a long time. Sooner or later some individuals will make mistakes [1] (because the signals are well defined random variables), in so doing they will move the economy away, towards the center. Now the signals are more noisy, and the economy more volatile.

In other words, aggregate investment will stay a long time in its extreme values, where the variance of the signals is small, and a short one in the center, where this variance is large. The economy may jump from high to low levels of investment every so often; in this way there will be persistence and the share on the total variance of the series due to things that repeat itself every several periods will be high; that is, the spectrum will assign high variance to the frequencies of the business cycle. The aggregate investment will follow a business cycle structure.

The rest of the paper is organized as follows. In section 2, I present the model and prove that the *"animal spirits"* strategy is an equilibrium. Section 3 discusses the dynamics of such an equilibrium. Section 4 adresses the issues derived from the existance of multiple equilibrium, and finally section 5 sumarizes and discusses the results.

2 Animal Spirits

There is a continuum of risk neutral agents, with mass 2π. Each of them will invest if and only if he expects half or more of the other agents to invest[2]. Given that the weight of any agent on the aggregate is zero, they will invest if and only if

$$E_t^i(y_t) \geq \frac{1}{2} \quad , \tag{1}$$

where the indexes i and t represent the information of individual i at period t.

In each period each agent makes an investment decision of 0 or 1, so:

$$y_t = \int_0^{2\pi} \frac{x_t(i)}{2\pi} di \qquad \Longrightarrow \qquad y_t \in [0,1] \tag{2}$$

[1] Agents never make mistakes in the sense that they always adopt the optimal policy. When I say mistake I mean ex-post mistake; even though they have reacted optimally to their information, this information was not accurate, they had a misperception, and so there was other action that offered a higher payoff.

[2] This is not an important assumption, the threshold for investment could be whatever value in (0,1) and the model would work fine, but it would complicate things unnecessarily.

2.1 Informational structure

Each agent will have at each moment two pieces of information:

- A prior on the distribution of the aggregate, $q(y)$. This is common for all agents.
- A privately observed signal $s_t(i)$ with information about the level of activity in the previous period. For the sake of simplicity we will assume that this signal takes only two values: 1 (or high, or "optimistic") or 0 (or low, or "pessimistic").

Each agent i receives a signal as information of the state of the world yesterday. The signal indicates, with noise, if yesterday had been a good time to invest; the better that yesterday was for investing, the more likely that the signal will say the truth. This signal (updating the prior) summarizes the perception that agent i has of the state of the world.

Each agent knows that if the aggregate had a value y_{t-1} yesterday, she has a probability $p(y_{t-1})$ of observing a high signal, and $1 - p(y_{t-1})$ of observing a low one. The function $p(y)$ is common knowledge and is required to have the following properties:

1. $p(y) = 1 - p(1 - y)$
2. $p(0) > 0$
3. $p(y)$ is not decreasing in all points of $[0, 1]$

The first requirement implies symmetry. The probability of receiving a high signal when the aggregate is at a 99% of its potential is the same that the one of receiving a low one when the aggregate is at a 1% of its potential. It implies that $p(\frac{1}{2}) = \frac{1}{2}$.

The second insures that there will always be people with misleading information.

The third implies that the higher the aggregate level is, the more likely it is to receive a high signal. Together with the first condition, it implies that the variance of the signals achieves a maximum at $y = \frac{1}{2}$, it is not decreasing for values of y in $[0, \frac{1}{2}]$ and symmetric around $\frac{1}{2}$.

2.2 Perceptions of reality

I like to think of the model as a set of producers of intermediate goods that sell to a common final good manufacturer. Each producer lives in his small market, his small island. In this circumstance it is likely that the information that arrives to a given producer is closely related to the one that arrives to a producer of a similar good. They are in relatively close markets, a shock in a given market is reflected in the surrounding ones, but not (or little) in faraway markets. In essence, the information of a producer will be correlated with the one of individuals close to him.

In our simple model the only relevant notion of proximity is the informational one. The correlation between $s_t(i)$ and $s_t(j)$ will be a given and known function of the distance between i and j.

So given that for all the individuals

$$Var(s_t(i)) = p(y) \cdot (1 - p(y)) \tag{3}$$

Then:

$$E\left[\left(s_t(i) - p(y)\right)\left(s_t(j) - p(y)\right)\right] = p(y) \cdot (1 - p(y)) \cdot \rho(i,j) \tag{4}$$

Where $\rho(i,j)$ is a function such that:

1. $\rho(i,i) = 1 \; \forall i$
2. $\rho(i,j) \geq 0 \; \forall i,j$
3. $\rho(i,j) = \rho(j,i) \; \forall i,j$
4. $\forall i,j : \rho(i,j) = \rho(i,j+2\pi)$
5. $\forall i$ there exist a set of positive measure $\Omega(i)$ such that $\forall j \in \Omega(i)$, $\rho(i,j) > 0$
6. $\forall i$ there exist a set of positive measure $\Phi(i)$ such that $\forall j \in \Phi(i)$, $\rho(i,j) < 1$

The first condition implies that each individual is perfectly correlated with herself. The second that nobody expects to have a perception of reality completely opposed (negatively correlated) to the one of other individuals. Everybody lives in the same world, if I am wrong this does not imply that others are right, and if I am right I should not expect others to be systematically wrong.

The third and fourth conditions imply symmetry. Once location is taken into account, everybody is ex ante identical to everybody else.

The fifth condition is the crucial one. By insuring that a relatively large number of individuals move together it will produce aggregate noise. This will allow us to fight back the law of the large numbers.

The sixth insures a degree of heterogeneity; without it a common aggregate shock would exist.

The average correlation is then:

$$R = \int_0^{2\pi} \frac{\rho(i,j)}{2\pi} dj \tag{5}$$

and clearly $R \in (0,1)$ and is independent of i.

I think of $\rho(i,j)$ as a sinusoidal function on j that achieves a maximum value of 1 at $j = i$ and a minimum of 0 at $j = i \pm \pi$, but much more general formulations are allowed.

Given this set up the conditional probabilities[3] that agents assign to the signals received by other agents are:

[3] See appendix A

- If i receives a signal $s_t(i) = 1$, the conditional probability of j receiving also a signal with value 1 is:

$$\Pr\left(s_t(j) = 1 \mid s_t(i) = 1\right) = 1 - (1 - p(y))(1 - \rho(i, j)) \tag{6}$$

- If i receives a signal $s_t(i) = 0$, the conditional probability of j receiving also a signal with value 1 is:

$$\Pr\left(s_t(j) = 1 \mid s_t(i) = 0\right) = p(y)(1 - \rho(i, j)) \tag{7}$$

2.3 Space of strategies

A strategy is a mapping from the set of private information to the set of possible actions, a function from {0,1} to {0,1}. We will study only stationary pure strategies, meaning pure strategies that are not indexed by time[4]. There are only four possible strategies of this type:

1. Never invest: $x(0) = 0$; $x(1) = 0$.
2. Always invest: $x(0) = 1$; $x(1) = 1$.
3. Invest if you get a high signal: $x(0) = 0$; $x(1) = 1$.
4. Invest if you get a low signal: $x(0) = 1$; $x(1) = 0$.

2.4 Equilibrium concept

In order to insure rational expectations we will require perfect Bayesian consistency to the relevant equilibrium concept. We will study only symmetric equilibria, in a strong sense; by this I mean equilibria where all the agents play the same strategy all the time. One could imagine these strategies as behavior rules that pass from parents to children.

Given a strategy and the process of information diffusion, a certain probability distribution on y_t will be generated for each possible level of y_{t-1}: $F(y \mid y_{t-1}) = \Pr(y_t < y \mid y_{t-1})$, with density function $f(y \mid y_{t-1})$.

This will generate an unconditional distribution $Q(y)$, whose density $q(y)$ will be such that:

$$q(y) = \int_0^1 f(y \mid z) q(z) dz \qquad . \tag{8}$$

This unconditional distribution will be the prior that all agents have each moment on the distribution of y_{t-1}.

[4]This will exclude strategies of the type *"when $t < 100$ invest only if s=1, when t=100 invest whatever you see, and when $t > 100$ never invest"*.

When an individual receives a signal, she will update her prior using it:

$$q\left(y_{t-1} \mid s_t(i)=1\right)=\frac{p(y_{t-1})\cdot q(y_{t-1})}{\int_0^1 p(y)\cdot q(y)\cdot dy} \tag{9}$$

$$q\left(y_{t-1} \mid s_t(i)=0\right)=\frac{(1-p(y_{t-1}))\cdot q(y_{t-1})}{\int_0^1 (1-p(y))\cdot q(y)\cdot dy} \tag{10}$$

The strategy is an equilibrium if and only if using this updated prior and the knowledge of the way that other people's information relates to their own:

$$\forall s_t\left(i\right) \quad x\left(s_t\left(i\right)\right)=1 \quad iff \quad E\left[y_t \mid s_t\left(i\right)\right]\geq \frac{1}{2} \tag{11}$$

2.5 Animal spirits equilibrium.

In order for the strategy "*invest if you observe high, do not invest if you observe low*" to be an equilibrium, it must be true that when an agent observes $s_t(i)=1$, she expects that most people are going to invest *today*, and when she observes $s_t(i)=0$, she expects that most people will not invest *today*, even though the signal in itself only carries information about the past, the signal depends only on the level of the aggregate *yesterday*. So the strategy is an equilibrium only if

$$E\left(y_t \mid s_t(i)=1\right)\geq \frac{1}{2}\geq E\left(y_t \mid s_t(i)=0\right) \quad . \tag{12}$$

The strategy is an equilibrium if the observation of $s_t(i)=1$ makes agents believe that most people will observe the same, and so most people will be optimistic about the future. Again, the belief in other people optimism makes you optimistic.

The expected value of the contemporaneous aggregate level is:

$$E\left(y_t \mid s_t(i)\right)=\int_0^{2\pi}\frac{E\left(x_t(j)\mid s_t(i)\right)}{2\pi}dj \tag{13}$$

and if everybody follows the strategy:

$$E\left(y_t \mid s_t(i)\right)=\int_0^{2\pi}\frac{E\left(s_t(j)\mid s_t(i)\right)}{2\pi}dj \tag{14}$$

Now, given equations (6), (7), (9) and (10):

$$\begin{aligned} E\left(s_t(j)\mid s_t(i)=1\right)= & \int_0^1\left(1-\left(1-p(y)\right)\left(1-\rho(i,j)\right)\right)\cdot q\left(y \mid s_t(i)=1\right)\cdot dy \\ & =1-\left(1-\rho(i,j)\right)\frac{\int_0^1 p(y)(1-p(y))q(y)dy}{\int_0^1 p(y)q(y)dy} \end{aligned} \tag{15}$$

and

$$E\left(s_t(j) \mid s_t(i)=0\right) = (1-\rho(i,j))\frac{\int_0^1 p(y)\,(1-p(y))\,q(y)dy}{\int_0^1 (1-p(y))\,q(y)dy} \tag{16}$$

So, when $s_t(i)=0$:

$$E\left(y_t \mid s_t(i)=0\right) = (1-R)\frac{\int_0^1 p(y)\,(1-p(y))\,q(y)dy}{\int_0^1 (1-p(y))\,q(y)dy} \tag{17}$$

and when $s_t(i)=1$:

$$E\left(y_t \mid s_t(i)=1\right) = 1-(1-R)\frac{\int_0^1 p(y)\,(1-p(y))\,q(y)dy}{\int_0^1 p(y)q(y)dy} \tag{18}$$

The unconditional distribution is symmetric[5]: $q(y)=q(1-y)$. In addition, the unconditional expected probability of receiving high or low signals is $\frac{1}{2}$[6].

Thus the strategy is an equilibrium (so (12) hold) if and only if :

$$(1-R)\cdot 2\cdot\int_0^1 p(y)\,(1-p(y))\,q(y)dy \leq \frac{1}{2} \tag{19}$$

We know that the variance of the signals is a number between $p(0)\,(1-p(0))$ and $\frac{1}{4}$. $\int_0^1 p(y)\,(1-p(y))\,q(y)dy$ is the expected unconditional variance of the signals because it is a weighted average of the variances:

$$p(0)\,(1-p(0)) \leq \int_0^1 p(y)\,(1-p(y))\,q(y)dy \leq \frac{1}{4} \tag{20}$$

With which we are almost done, because $(1-R)\in(0,1)$, so

$$(1-R)\cdot 2\cdot\int_0^1 p(y)\,(1-p(y))\,q(y)dy < 2\cdot\int_0^1 p(y)\,(1-p(y))\,q(y)dy \leq \frac{1}{2} \tag{21}$$

Which proves that the strategy "*invest if you think that yesterday was a good time for investing, do not invest if you do not think so*" can be sustained as an equilibrium.

3 Dynamics

Even though it is very difficult to get a closed form for $F(y \mid y_{t-1})$, we will be able to get a good amount of information on it by looking at the first two moments of this distribution.

[5] See appendix B.

[6] See appendix C .

3.1 Conditional mean and variance.

Aggregate output will follow a Markov process. If at $t-1$ the aggregate was y_{t-1}, then the expectation[7] at t:

$$E\left(y_t \mid y_{t-1}\right) = p(y_{t-1}) \tag{22}$$

And its variance:

$$\sigma^2\left[y_t \mid y_{t-1}\right] = p(y_{t-1}) \cdot (1 - p(y_{t-1})) \cdot R \tag{23}$$

As is clear, it is the fact that the signals are correlated across individuals what allows for aggregate noise. But the statement should be more general, as long as the law of the large numbers does not apply we will obtain aggregate fluctuations. We have used non independent random variables, but other schemes are plausible.

3.2 Persistence, volatility, and mean reversion.

We already know that $E\left[y_t \mid y_{t-1}\right] = p(y_{t-1})$. This means that the aggregate level of investment will *tend* to increase when $p(y_{t-1})$ is above the 45 degree line, and to decrease when is below.

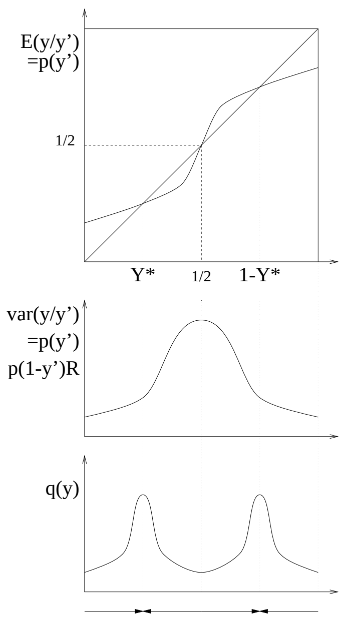

Figure 1: Bimodal Distribution

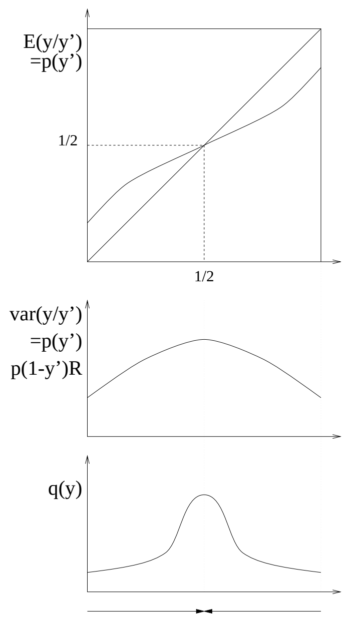

Figure 2: Single *"attraction point"*

If $p(y)$ is like the one drawn in figure 1 (recall that $p(y)$ is the process of information diffusion that we consider exogenous), there will be two points y* and 1-y*, that we will call *"attraction points"*, such that if the variance of the aggregate were 0 ($R = 0$) would be the only stable long

[7] See appendix D

run solutions of the differential equation that would drive the process of the economy: $y_t = p(y_{t-1})$. This economy would have a non-stable equilibria at $y = \frac{1}{2}$. For all initial values smaller than $\frac{1}{2}$ the economy would converge at y*, and for all initial points bigger than $\frac{1}{2}$ to 1-y*. The conditions that we have imposed on $p(y)$ insure that there exists at least one accumulation point (that would be $\frac{1}{2}$ if it were unique).

The interesting thing is that the economy will *tend* to move towards the closer attraction point, but *need not do so*. The economy will tend to move towards its expected value, but there exists a positive probability that it moves in the opposite direction. Here is where the effects of the informational externality are visible.

Imagine, in figure 1, that the economy is at or near the low attraction point y*. At this point the variance of the aggregate is positive, *and increasing*. Even though we would expect the economy to remain around y* (which it will do most of the time), it can happen that a relatively large number of individuals observe a high signal moving the aggregate towards the right, say to a point between y* and $\frac{1}{2}$. At this new point the variance is *higher* than before, the conditional distribution is flatter. That implies that new movements towards the right are more likely than before; even if we still would expect the economy to move towards y*, we will do it with less confidence . It is easier that a relatively large number of individuals observe a high signal.

This is the effect of the informational externality. When the economy is far away from the center, the probability of individuals making mistakes is relatively low because the unconditional variance of the signals is very low. So it is not very likely for the economy to move away, the aggregate level is persistent. But *if* it moves towards the center, and sooner or later it will, the probability that individuals receive the wrong signal increases substantially. Aggregate uncertainty increases accordingly. When a small group of individuals makes a mistake increases the volatility of the signal, and makes more likely for others individuals to make mistakes. Prediction is more difficult, and it is possible that we cross $\frac{1}{2}$. Once in high levels of output we will tend to stay there, but again every so often we will move to the left, and the process will go on, with long periods staying around y* or 1-y*, but with jumps every so often between high and low states, in between the jumps a kind-of-mean reverting process will take place; things tend to move toward the attractor, but this is only a "local mean", not the mean of the process (clearly 1/2). In other words, the unconditional distribution will have maxima in y* and 1-y*, and a local minimum at $\frac{1}{2}$. A good deal of the total variance of the time series of the output will come from things that repeat itself every so often (the time between jumps will have an expected duration), one of the characteristics of the business cycles.

Even if the only attraction point is $\frac{1}{2}$ (figure 2), we will have some degree of persistence. At the attraction point the variance achieves a maximum, so the aggregate is particularly unstable. Movements towards the extremes will go back to the center (mean reversion), but relatively slowly, because the distribution in the extremes is more concentrated around its conditional expectation.

4 Comments on the multiplicity of equilibria

The animal spirits equilibrium is not unique. Actually all four strategies can be sustained as an equilibrium[8]. I do not know up to what point we should be worried about it. After all in coordination games with a dynamic setting is inevitable the existence of multiple equilibria. In a sunspot equilibrium is as much an equilibrium to invest when the sun is covered with spots as to invest when it is spotless, as long as everybody agrees to do so. There are, though, several good reasons to make the case for the animal spirits equilibrium in the present model.

The basic intuition of the animal spirits equilibrium is that if the reduced form of the world has persistence, you will like to follow your perception of the immediate past, because it is likely to be close to what will happen in the future. In the model as has been stated the only ex-ante source of persistence is the fact that the we centered our study on stationary strategies. We will see bellow that given this the only equilibrium robust to the presence of uninformed agents is the animal spirits one.

We would like the equilibrium to be robust to the presence of "*uninformed*" agents; to be such that if an agent starts playing, being the world in an equilibrium, she would play the equilibrium strategy. In our model if an agent have to play knowing that the world is in an equilibrium, but not which one, she will play the animal spirits strategy.

The intuition is that the signal does not only carry information about what physically happened in the past, it also carries information about which was the equilibrium in place yesterday, and given that we concentrate in stationary strategies, on which one will be the equilibrium tomorrow. We will see that for a Martian who falls on earth with complete ignorance of what strategies agents are playing, her best bet is to follow what the signal tells.

More formally, let $U(k, s, x)$ be the utility that an individual playing x expects to get if she observes s and everybody else is playing strategy k . There are K equilibrium strategies of the form $X_k(s)$, the kth of them giving an expected payoff of $U(k, s, x)$ when s is observed and an action x is taken[9]. The equilibrium strategy l is robust to the presence of uninformed agents if and only if :

$$\forall s \quad X_l(s) = Arg\max_x \left(\sum_{k=1}^{K} \frac{1}{K} U(k, s, x) \right)$$

So, if you are uninformed about which is the equilibrium strategy that everybody else is playing, and thus you assign equal priors to all equilibrium strategies; when you see a signal s, you will follow the action indicated by the robust strategy. It is as if you were playing that equilibrium

[8] The strategies "always invest" and "never invest" produce priors that assign probability one to $y = 1$ and $y = 0$ respectively. The "fourth" strategy, invest iff $s_t(i) = 0$, is shown in appendix E to be an equilibrium.

[9] Because k is an equilibrium,

$$U(k, s, X_k(s)) \geq U(k, s, x)$$

strategy.

In appendix F it is shown that if we assign to the agents a payoff function such that they get $y_t - \frac{1}{2}$ if they invest and 0 if they do not, the animal spirits equilibrium is the only one robust in the sense expressed above.

The point is that if you assign an equal prior to all the possible equilibria and then you see $s_t(i) = 1$, your posterior that the equilibrium is "*never invest*" is very low, so you have incentives to invest, and vice versa when you see $s_t(i) = 0$. The best bet is to always follow the animal spirits strategy, to always follow the past.

If there were other sources of persistence, as it should be expected in the real world, the animal spirits equilibrium would get reinforced. The more reasons an agent has to believe that tomorrow will be similar to yesterday, the more reasons she has to follow her perception of the past. If there are externalities in capital, past investment affecting positively present returns, the higher you believe that past investment was, the more you would like to invest independently of what other agents do today.

In the economy that we have studied the only source of persistence was the derived from the animal spirits equilibrium. It is only natural to expect that persistence is also derived for other, more physical, causes. For instance, the return on investment may depend on aggregate capital, and not aggregate investment. Introducing exogenous sources of persistence "reinforces" the animal spirits equilibrium.

Assume that the return to present investment is $y_{t-1} + y_t - 1$, so that there is an inter temporal externality due to past investment as in Acemoglu(1993). In this case if an agent has flat priors on the strategies that other agents may take, and so flat priors on both y_{t-1} and y_t , she will follow the animal spirits strategy. Note that now we do not require the world to be in an equilibrium. It is not that there are dominant strategies, because in an equilibrium how much we trust the signal depends on the equilibrium strategy, but an individual very uncertain on other people's strategies will play the animal spirits strategies.

There is yet another reason that should focus our attention in the Animal Spirits strategy: it is adaptative. In a context with correlated aggregate shocks on productivity where the return on investment it depends on both the aggregate shock and aggregate investment, but where it is difficult to discern one effect from the other, the adaptative strategy presents obvious long run advantages. Agents that use the strategy *"always invest"* would suffer a lot in a recession caused by the aggregate productivity shock, while the ones using the Animal Spirits strategy would adapt to the recession quitting investment. This suggest that from an evolutionary point of view the Animal Spirits strategy has much more chances of survival. This is one of the lines in which presently I am extending the model.

5 Conclusion

We have seen that to follow one's perception of the past can be sustained as an equilibrium, and that given some particular schemes of distribution of the information, that seem to be pretty plausible, this equilibrium would reproduce some of the stylized facts of the business cycles.

The past is used as a coordination mechanism, the equilibrium is a sunspot, with the particularity that the transition matrix is endogenous, and it is such that produces the kind of stochastic processes that we were looking for.

One could question why individuals do not coordinate in other, possibly exogenous, variables. Why is the (not perfectly observable) past chosen instead of other variables? Why is the past chosen as a focal point?

On this I have a double line of defense. On one hand people can learn to know, learn to believe in sunspots. As in Howitt and McAfee (1992) the equilibrium could be such that people can learn to believe in it, if the rest of the agents are coordinating in the same way. Probably an even better line of defense is the most obvious one: in all coordination games agents will look for a focal point; a focal point must be in one way or another observed by all individuals, and there is nothing more observable than the past. Everybody has a feeling about what happens with the economy, and everybody is *likely* to be right, so it is difficult to think of a more plausible focal point that the perception that agents have of the past.

There is even more; if the past affects directly to present returns, agents will have an additional, and direct, incentive to herd and doing so to coordinate with the past. We have build the model without stocks, but the existence of state variables would by itself generate persistence. The more persistence there is, the more reasons agents have to do something close to what they did in previous periods, and so the more natural it becomes to coordinate using the past as reference.

In the real world the externalities are local, a shoe manufacturer in Boston does not care that much about the potato harvest in Idaho. The coordination process will be local too, this has been explored by Durlauf in a series of papers. My model tries to capture this with a simple set up, to be a simplification. That is why the perception of the past is assumed to be correlated, but not perfectly, across agents and why I do not believe that the assumption of the unobservability of past aggregate investment is too restrictive.

Agents may know the true value of aggregate investment, but not the statistic that they care about (an average of investment weighted by the distance to the agent), to learn it would be way too expensive. They have nevertheless a perception of the state of the economy; each one thinks that was good or bad to invest in the previous period, and most likely they are right.

The machinery developed in this paper could prove itself useful, but in order to make it truly realistic it would be necessary to specify models where the externality is local, and so individuals want to coordinate only with the people around (the aggregate rates could be meaningless for them), and where individuals are subject to uncertainty about other people's reaction functions (probably generated by stochastic growth). The most difficult problem that this context would require to solve is that individuals want to predict not only a number, but a distribution, whose functional form does not need to be closed.

This model does not pretend to say that there are no real sources of business cycles, but makes the point that it is possible to conceptualize them without exogenous productivity shocks. Local productivity shocks could well be at the source of the informational noise. The model would be identical and would create cycles out of white noise shocks.

Models are simplifications of reality and this model is more so than most of them, but tries to make the point that the distribution of information across the economy and the willingness of agents to coordinate their investments could in itself generate a behavior of the aggregate with a business cycle structure. The point of the model is to show that it is possible to generate animal spirits equilibrium using only informational noise on the state of the economy, a noise that in itself depends on the performance of the economy. Surely this is not the only source of recessions and booms, but its contribution to the process could be important and should be determined empirically[10].

A Conditional probabilities

The covariance of the signals is:

$$\begin{array}{c} E\left[\left(s_t(i)-p(y)\right)\left(s_t(j)-p(y)\right)\right]= \\ \Pr\left[s(i)=1,s(j)=1\right]\left(1-p(y)\right)^2+\Pr\left[s(i)=1,s(j)=0\right]\left(1-p(y)\right)\left(-p(y)\right) \\ +\Pr\left[s(i)=0,s(j)=1\right]\left(1-p(y)\right)\left(-p(y)\right)+\Pr\left[s(i)=0,s(j)=0\right]\left(p(y)\right)^2 \end{array} \tag{24}$$

Assuming further symmetry:

$$\Pr\left[s(i)=1,s(j)=0\right]=\Pr\left[s(i)=0,s(j)=1\right] \tag{25}$$

Then:

$$\begin{array}{c} E\left[\left(s_t(i)-p(y)\right)\left(s_t(j)-p(y)\right)\right]=Pr\left[s(i)=1,s(j)=1\right]\left(1-p(y)\right)^2 \\ -2\Pr\left[s(i)=0,s(j)=1\right]\left(1-p(y)\right)p(y)+\Pr\left[s(i)=0,s(j)=0\right]\left(p(y)\right)^2 \end{array} \tag{26}$$

Given that:

$$\Pr\left[s(i)=1,s(j)=1\right]+\Pr\left[s(i)=1,s(j)=0\right]=p(y) \tag{27}$$

and

$$\Pr\left[s(i)=0,s(j)=0\right]+\Pr\left[s(i)=0,s(j)=1\right]=1-p(y) \tag{28}$$

Then, using 4

$$p(y)\cdot\left(1-p(y)\right)\cdot\rho(i,j)=p(y)\cdot\left(1-p(y)\right)-\Pr\left[s(i)=1,s(j)=0\right] \tag{29}$$

and so, the probability that individuals i,j observe different signals is:

$$\Pr\left[s(i)=1,s(j)=0\right]=p(y)\cdot\left(1-p(y)\right)\cdot\left(1-\rho(i,j)\right) \tag{30}$$

[10] On this line Rodriguez Mora and Schulstad (1995) show that once (noisy) information on the past behaviour of GNP is taken into account the real value of past GNP has no predective power on its future realizations

The probability that both observe 1 is:

$$\Pr\left[s(i)=1,s(j)=1\right]=p(y)\cdot\left[1-(1-p(y))\cdot(1-\rho(i,j))\right] \tag{31}$$

And the probability that they observe both 0 is:

$$\Pr\left[s(i)=0,s(j)=0\right]=(1-p(y))\cdot\left[1-p(y)\cdot(1-\rho(i,j))\right] \tag{32}$$

For each value of y then, the conditional probability of j to observe 1, given that i has observed one:

$$\begin{aligned}\Pr\left[s(j)=1 \mid s(i)=1\right] &= \tfrac{\Pr[s(i)=1,s(j)=1]}{\Pr[s(i)=1]} \\ &= (1-(1-p(y))\cdot(1-\rho(i,j)))\end{aligned} \tag{33}$$

And the probability that j observes 1 given that i observed 0:

$$\Pr\left[s(j)=1 \mid s(i)=0\right]=\frac{\Pr\left[s(i)=0,s(j)=1\right]}{\Pr\left[s(i)=0\right]}=p(y)\cdot(1-\rho(i,j)) \tag{34}$$

B Symmetry of the unconditional distribution

Now let's look at the unconditional distribution. Given that our strategy is perfectly symmetric and that the process of information distribution is also perfectly symmetric, it must be the case that the conditional distributions must be symmetric, that is:

$$F(y \mid y_{t-1})=1-F(1-y \mid 1-y_{t-1}) \tag{35}$$

and differentiating with respect to y:

$$f(y \mid y_{t-1})=f(1-y \mid 1-y_{t-1}) \tag{36}$$

Now the unconditional distribution:

$$q(y)=\int_0^1 f(y \mid x)q(x)dx=\int_0^1 f(y \mid x)\left(\int_0^1 f(x \mid z)q(z)dz\right)dx \tag{37}$$

Now lets define a succession of infinite variables $\{x_1,x_2,x_3,...\}$, then always that $q(y)$ exists, must be the case that:

$$q(y)=\int_0^1\int_0^1\int_0^1\int_0^1 ...f(y \mid x_1)f(x_1 \mid x_2)f(x_2 \mid x_3)...dx_1dx_2dx_3... \tag{38}$$

And:

$$q(y)=\int_0^1\int_0^1\int_0^1\int_0^1 ...f(1-y \mid 1-x_1)f(1-x_1 \mid 1-x_2)f(1-x_2 \mid 1-x_3)...dx_1dx_2dx_3... \tag{39}$$

Now, defining $z_n=1-x_n$,

$$\begin{aligned}q(y) &= \textstyle\int_1^0\int_1^0\int_1^0\int_1^0 ...f(1-y \mid z_1)f(z_1 \mid z_2)f(z_2 \mid z_3)...(-dz_1)(-dz_2)(-dz_3)... \\ &= \textstyle\int_0^1\int_0^1\int_0^1\int_0^1 ...f(1-y \mid z_1)f(z_1 \mid z_2)f(z_2 \mid z_3)...dz_1dz_2dz_3...=q(1-y)\end{aligned} \tag{40}$$

So the unconditional distribution is symmetric.

C Expected unconditional probabilities.

We know that

$$p(y)=(1-p(1-y)) \tag{41}$$

and

$$q(y)=q(1-y) \tag{42}$$

In such a case

$$\int_0^1 p(y)q(y)dy = 1 - \int_0^1 p(1-y)q(y)dy = 1 - \int_0^1 p(1-y)q(1-y)dy \tag{43}$$

Defining $z = 1 - y$:

$$\int_0^1 p(y)q(y)dy = 1 - \int_0^1 p(z)q(z)dz \tag{44}$$

$$\int_0^1 p(y)q(y)dy = \frac{1}{2} \tag{45}$$

So the unconditional expected probability of receiving a high signal is $\frac{1}{2}$. The same argument shows that the unconditional expected probability of receiving a low one is also $\frac{1}{2}$.

D Moments of the conditional distribution.

The conditional expectation:

$$\begin{aligned} E\left(y_t \mid y_{t-1}\right) &= E\left[\int_0^{2\pi} \frac{x_t(j)}{2\pi} dj \mid y_{t-1}\right] \\ &= \int_0^{2\pi} \frac{E\left[x_t(j) | y_{t-1}\right]}{2\pi} dj \\ &= \int_0^{2\pi} \frac{E\left[s_t(j) | y_{t-1}\right]}{2\pi} dj = p(y_{t-1}) \end{aligned} \tag{46}$$

And its variance:

$$\begin{aligned} \sigma^2\left[y_t \mid y_{t-1}\right] &= E\left[\left(y_t - p(y_{t-1})\right)^2 \mid y_{t-1}\right] \\ &= E\left[\left(\int_0^{2\pi} \frac{s_t(j) - p(y_{t-1})}{2\pi} dj\right)^2 \mid y_{t-1}\right] \\ &= E\left[\left(\int_0^{2\pi}\int_0^{2\pi} \frac{\left(s_t(i) - p(y_{t-1})\right)\left(s_t(j) - p(y_{t-1})\right)}{(2\pi)^2} didj\right) \mid y_{t-1}\right] \\ &= \int_0^{2\pi}\int_0^{2\pi} \frac{p(y_{t-1})\cdot\left(1 - p(y_{t-1})\right)\cdot\rho(i,j)}{(2\pi)^2} didj \\ &= \int_0^{2\pi} \frac{p(y_{t-1})\cdot\left(1 - p(y_{t-1})\right)}{2\pi}\left[\int_0^{2\pi} \frac{\rho(i,j)}{2\pi} dj\right] di \\ &= p(y_{t-1})\cdot\left(1 - p(y_{t-1})\right)\cdot R \end{aligned} \tag{47}$$

E The fourth strategy is an equilibrium.

E.1 Symmetry of the Unconditional Distribution with the fourth strategy.

Given a level of output at $t-1$ agents are as likely to receive the wrong signals under strategy 3 or under strategy 4 (this is so because the process of information distribution is exogenous and does not depend on the strategy). Calling $f_4(\cdot)$ to the conditional probability under the " *jumping*" strategy:

$$f(x \mid y) = f_4(1 - x \mid y)$$

This is so because $f(x \mid y)$ is the probability that in the animal spirits equilibrium a x percent of the agents receive the signal $s_t(i) = 1$, if the equilibrium is the fourth one, when a x percent of the agents receive this signal, a $1 - x$ percent invest.

In this circumstances:

$$f_4(1 - x \mid 1 - y) = f(x \mid 1 - y) = f(1 - x \mid y) = f_4(x \mid y) \tag{48}$$

and it follows from above that q_4 is symmetric and

$$\int_0^1 p(y)q_4(y)dy = \int_0^1 (1 - p(y))q_4(y)dy = \frac{1}{2} \tag{49}$$

E.2 Equilibrium

If everybody is following the fourth strategy $x_t(j) = 1 - s_t(j)$. Thus:

$$E(y_t \mid s_t(i)) = \int_0^{2\pi} \frac{E(x_t(j) \mid s_t(i))}{2\pi} dj = \int_0^{2\pi} \frac{E\left(\Pr\left(s_t\left(j\right) = 0 \mid s_t\left(i\right)\right)\right)}{2\pi} \tag{50}$$

When $s_t\left(i\right) = 1$:

$$\Pr\left(s_t\left(j\right) = 0 \mid s_t\left(i\right) = 1\right) = \left(1 - p\left(y\right)\right)\left(1 - \rho\left(i, j\right)\right) \tag{51}$$

And when $s_t\left(i\right) = 0$:

$$\Pr\left(s_t\left(j\right) = 0 \mid s_t\left(i\right) = 0\right) = 1 - p\left(y\right)\left(1 - \rho\left(i, j\right)\right) \tag{52}$$

So:

$$\begin{array}{c} E\left(y_t \mid s_t(i) = 1\right) = (1 - R)\frac{\int_0^1 p(y)(1-p(y))q_4(y)dy}{\int_0^1 p(y)q_4(y)dy} = \\ = (1 - R) \cdot 2 \cdot \int_0^1 p(y)\left(1 - p(y)\right) q_4(y)dy \end{array} \tag{53}$$

and

$$\begin{array}{c} E\left(y_t \mid s_t(i) = 0\right) = 1 - (1 - R)\frac{\int_0^1 p(y)(1-p(y))q_4(y)dy}{\int_0^1 (1-p(y))q_4(y)dy} = \\ = 1 - (1 - R) \cdot 2 \cdot \int_0^1 p(y)\left(1 - p(y)\right) q_4(y)dy \end{array} \tag{54}$$

As we saw $0 < \int_0^1 p(y)\left(1 - p(y)\right) q_4(y)dy < \frac{1}{4}$ (this is true for whatever density function q), which implies:

$$E\left(y_t \mid s_t(i) = 1\right) < \frac{1}{2} \tag{55}$$

and

$$E\left(y_t \mid s_t(i) = 0\right) > \frac{1}{2} \tag{56}$$

So the strategy is an equilibrium.

F Robustness of the Animal Spirits equilibrium to the presence of uninformed agents.

We have four equilibria:

- e_1:always invest.
- e_2:never invest.
- e_3:animal spirits.
- e_4:invest iff $s_t(i) = 0$.

All of them generate equilibria, but only the Animal Spirits one is robust to the presence of uninformed agents; this is, in the hypothetical case that there appears a Martian who does not know in which equilibrium the economy is, after observing her signal she would play the Animal Spirits strategy. We can consider that everybody is a Martian that knows that everybody else is also a Martian, then they will know that everybody is playing the Animal Spirits strategy, and so the best that they can do is to play it too.

The point is that if you assign a equal prior to all the possible equilibria and then you see $s_t(i) = 1$, your posterior that the equilibrium is "*never invest*" is very low, so you have incentives to invest, and vice versa when you see $s_t(i) = 0$. The best bet is to always follow the animal spirits strategy, to always follow the past.

F.1 Priors and posteriors.

If an agent knows that the world is at an equilibrium, but is completely ignorant about which on it is, we should consider that she would assign flat priors across the equilibria.

Calling ϵ to the equilibrium in place (that from the point of view of the Martian is a random variable),

$$\Pr(\epsilon = e_k) = \frac{1}{4} \quad \forall i$$

Before playing she receives a signal about what happened yesterday, and she knows that the equilibrium is atemporal. Whatever the equilibrium was yesterday, today we will have the same one.

So she will update her prior on which the equilibrium is by using the signal:

$$\Pr(\epsilon = e_k \mid s_t(i)) = \frac{\Pr(s_t(i) \mid \epsilon = e_k)}{\sum_{k=1}^{4} \Pr(s_t(i) \mid \epsilon = e_k)}$$

The probabilities of receiving one or the other signal given the equilibrium are:

$$\begin{array}{ll} \Pr(s_t(i) = 1 \mid \epsilon = e_1) = (1 - p_0) & Pr(s_t(i) = 0 \mid \epsilon = e_1) = p_0 \\ \Pr(s_t(i) = 1 \mid \epsilon = e_2) = p_0 & \Pr(s_t(i) = 0 \mid \epsilon = e_2) = (1 - p_0) \\ \Pr(s_t(i) = 1 \mid \epsilon = e_3) = \frac{1}{2} & \Pr(s_t(i) = 0 \mid \epsilon = e_3) = \frac{1}{2} \\ \Pr(s_t(i) = 1 \mid \epsilon = e_4) = \frac{1}{2} & \Pr(s_t(i) = 0 \mid \epsilon = e_4) = \frac{1}{2} \end{array} \tag{57}$$

So:

$$\begin{array}{ll} \Pr(\epsilon = e_1 \mid s_t(i) = 1) = \frac{1-p_0}{2} & \Pr(\epsilon = e_1 \mid s_t(i) = 0) = \frac{p_0}{2} \\ \Pr(\epsilon = e_2 \mid s_t(i) = 1) = \frac{p_0}{2} & \Pr(\epsilon = e_2 \mid s_t(i) = 0) = \frac{1-p_0}{2} \\ \Pr(\epsilon = e_3 \mid s_t(i) = 1) = \frac{1}{4} & \Pr(\epsilon = e_3 \mid s_t(i) = 0) = \frac{1}{4} \\ \Pr(\epsilon = e_4 \mid s_t(i) = 1) = \frac{1}{4} & \Pr(\epsilon = e_4 \mid s_t(i) = 0) = \frac{1}{4} \end{array} \tag{58}$$

F.2 Payoffs.

The utility that the Martian would expect to get, conditional to the equilibrium in place and the signal that she receives:

$$\begin{array}{ll} EU(s_t(i) = 1,\ \epsilon = e_1) = \frac{1}{2} & EU(s_t(i) = 0,\ \epsilon = e_1) = \frac{1}{2} \\ EU(s_t(i) = 1,\ \epsilon = e_2) = -\frac{1}{2} & EU(s_t(i) = 0,\ \epsilon = e_2) = -\frac{1}{2} \\ EU(s_t(i) = 1,\ \epsilon = e_3) = \frac{1}{2} - A_3 & EU(s_t(i) = 0,\ \epsilon = e_3) = A_3 - \frac{1}{2} \\ EU(s_t(i) = 1,\ \epsilon = e_4) = A_4 - \frac{1}{2} & EU(s_t(i) = 0,\ \epsilon = e_4) = \frac{1}{2} - A_4 \end{array} \tag{59}$$

Where

$$A_3 = 2 \cdot (1 - R) \cdot \int_0^1 p(y)\ (1 - p(y))\ q(y)\, dy < \frac{1}{2} \tag{60}$$

and

$$A_4 = 2 \cdot (1 - R) \cdot \int_0^1 p(y)\ (1 - p(y))\ q_4(y)\, dy < \frac{1}{2} \tag{61}$$

When the Martian observes $s_t(i) = 1$ if she invest, she expects to get:

$$\begin{array}{c} U(x_t(i) = 1 \mid s_t(i) = 1) = \\ \frac{1-p_0}{2}\frac{1}{2} + \frac{p_0}{2}\left(-\frac{1}{2}\right) + \frac{1}{4}\left(\frac{1}{2} - A_3\right) + \frac{1}{4}\left(A_4 - \frac{1}{2}\right) \end{array} \tag{62}$$

And if she observes $s_t(i) = 0$:

$$\begin{array}{c} U(x_t(i) = 1 \mid s_t(i) = 0) = \\ \frac{p_0}{2}\frac{1}{2} + \frac{1-p_0}{2}\left(-\frac{1}{2}\right) + \frac{1}{4}\left(A_3 - \frac{1}{2}\right) + \frac{1}{4}\left(\frac{1}{2} - A_4\right) \end{array} \tag{63}$$

The Martian would invest whatever she sees (strategy 1) only if both 62 and 63 are non-negative. She will never invest (strategy 2) only if both 62 and 63 are non-positive. She will follow the Animal Spirits strategy if and only if 62 is non-negative and 63 is non-positive. She will follow the fourth strategy only if 62 is non-positive and 63 is non-negative.

F.3 "Martian Proof" equilibrium.

If our Martian invest after observing $s_t(i) = 1$, she gets:

$$\begin{array}{c} U\left(x_t(i) = 1 \mid s_t(i) = 1\right) = \\ \frac{1-p_0}{2}\frac{1}{2} + \frac{p_0}{2}\left(-\frac{1}{2}\right) + \frac{1}{4}\left(\frac{1}{2} - A_3\right) + \frac{1}{4}\left(A_4 - \frac{1}{2}\right) = \\ \frac{1}{2}\left[\frac{1}{2} - p_0 + (1-R)\left(\int_0^1 p(y)\,(1-p(y))\,q_4(y)\,dy - \int_0^1 p(y)\,(1-p(y))\,q(y)\,dy\right)\right] \end{array} \tag{64}$$

and after observing $s_t(i) = 0$,

$$\begin{array}{c} U\left(x_t(i) = 1 \mid s_t(i) = 0\right) = \\ \frac{p_0}{2}\frac{1}{2} + \frac{1-p_0}{2}\left(-\frac{1}{2}\right) + \frac{1}{4}\left(A_3 - \frac{1}{2}\right) + \frac{1}{4}\left(\frac{1}{2} - A_4\right) = \\ \frac{1}{2}\left[p_0 - \frac{1}{2} + (1-R)\left(\int_0^1 p(y)\,(1-p(y))\,q(y)\,dy - \int_0^1 p(y)\,(1-p(y))\,q_4(y)\,dy\right)\right] = \\ -U\left(x_t(i) = 1 \mid s_t(i) = 1\right) \end{array} \tag{65}$$

So the necessary and sufficient condition for the animal spirits strategy to be a robust equilibrium is:

$$\frac{1}{2} - p_0 + (1-R)\left(\int_0^1 p(y)\,(1-p(y))\,q_4(y)\,dy - \int_0^1 p(y)\,(1-p(y))\,q(y)\,dy\right) > 0 \tag{66}$$

The variance of the signals is bounded by $p_0\,(1-p_0)$ and $\frac{1}{4}$, and so is a weighted average of all this possible variances. From this it follows that

$$\int_0^1 p(y)\,(1-p(y))\,q_4(y)\,dy - \int_0^1 p(y)\,(1-p(y))\,q(y)\,dy \geq p_0\,(1-p_0) - \frac{1}{4} \tag{67}$$

Thus, given that $p_0 < \frac{1}{2}$ (and so $p_0\,(1-p_0) - \frac{1}{4} < 0$):

$$\begin{array}{c} \frac{1}{2} - p_0 + (1-R)\left(\int_0^1 p(y)\,(1-p(y))\,q_4(y)\,dy - \int_0^1 p(y)\,(1-p(y))\,q(y)\,dy\right) \geq \\ \frac{1}{2} - p_0 + (1-R)\left(p_0\,(1-p_0) - \frac{1}{4}\right) = \\ \frac{1}{2} - p_0 + p_0\,(1-p_0) - \frac{1}{4} - R\left(p_0\,(1-p_0) - \frac{1}{4}\right) > \\ \frac{1}{4} - (p_0)^2 > 0 \end{array}$$

Which proves that the Animal Spirits equilibrium is the only one that is robust to the presence of uninformed agents.

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