# Misperceptions, Heterogeneity and Shared Knowledge

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**Abstract**

Agents use their knowledge on the history of the economy in order to choose what is the optimal action to take at any given moment of time, but each individual observes history with some noise. We present a dynamic model of social learning in which the returns of the different activities follow independently distributed stationary Markov processes. Each period the agents receive unbiased noisy signals on the payoff of each sector. The signals differ across agents, but all of them have the same variance, which depends on their aggregate behavior. The degree of heterogeneity across agents is then an endogenous variable, evolving across time, which determines, and is determined by, the amount of information disclosed. As long as both the level of social interaction and the underlying precision of the observations are relatively large agents behave in a very precise way. This behavior is unmodified for a huge range of informational parameters, and it is characterized by an excessive concentration of the investment in a few sectors. Periods of heterogeneity, learning, and reallocation of resources happen following recessions.

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# 1 Introduction

Economic agents do not observe *exactly* what happens in the economy; they have perceptions of it. An agent's perception of an event is shaped not only by her own experience, but also by other people's experiences, and, conceivably, by the way they communicate with each other. As long as temporal links exist, the accuracy of their forecasts on the future (and thus the agents' performance) is crucially determined by the accuracy of their *perceptions* of the past, even if they *know* the exact structure of the economy.

This paper studies the process of creation of beliefs when the agents face a changing environment that evolves according to a stationary Markovian process. Their information being related to the actions taken by other agents (social learning). We will focus on the bidirectional causality between the (ex-post) heterogeneity in the beliefs of the agents and the accuracy of their perceptions.

Our first point is that the accuracy of the perceptions of the past affects not only the accuracy of the forecasts of the future, but also the degree in which these forecasts will differ from one agent to another. If everybody is ex-ante equal, and have infinitely accurate perceptions (i.e., they observe *exactly* what happens), ex-post everybody is going to share the same beliefs: the "truth". In a stationary world the information depretiates as times passes. Thus, if there is an activity on which there is no information disclosed during some time, the beliefs of all agents will converge to the unconditional distribution of the activity's return, becoming very homogeneous. On the other hand if their perceptions have idiosyncratic noise, but are somewhat informative, their beliefs on the past evolution of the economy will differ. Their beliefs about the *future* evolution of the economy will also be different. They are ex-ante homogeneous, but ex-post heterogeneous. Consequently the accuracy of the perceptions and the stochastic structure of the economy jointly determine the degree of heterogeneity of the beliefs (and actions) of the agents.

Our second point is that the amount of noise in the perceptions on the return of a given activity is not independent from the number of agents involved in it. The more people invests in one activity, the more likely it is that I will receive information on its return. The dispersion of the agents' beliefs determines the concentration of their actions, consequently the heterogeneity of their beliefs affects the precision of their perceptions.

Both effects feed to each other, and tend to create an excessive concentration of the agents' actions in one activity, slowing down (or down right stopping) learning on the return of the less popular activities. If at time $t$ everybody holds the same beliefs on the payoff of two activities $a$ and $b$, and they all believe that action $a$ will pay better than action $b$; then everybody will take action $a$. Under these circumstances an informational externality may produce little or no disclosure of information on the return of $b$ at $t$, while very good (accurate) information on the return of $a$. The beliefs on the returns at $t+1$ will again be very homogeneous. *Knowing* the exact return of $a$ at $t$, and not having received any new information on $b$ they will keep investing in $a$ if its payoff at $t$ was high, and only if it was very low they will invest on $b$. It could happen that the return on $b$ suffers a positive shock and actually $b$ pays more than $a$, but nobody would see it because the homogeneity of their beliefs *and* the effects of the informational externality prevent any disclosure of information on $b$.

We present a model that allows to explicitly separate the effects due to increased heterogeneity and to the presence of informational externalities when the return of the different actions follow exogenous and independent Markov processes. By doing so we can clearly determine the cost that informational externalities have for this society.

Social learning models have attracted recently the attention of macroeconomist, as they seem suited to help explain "atypical" investment behaviour (thus, Chari and Kehoe [7] use it to explain lending behaviour to LDC). Our model radically differs from the standard social learning framework (Banerjee [2] or Bikchandani et al. [3]) , because players play simultanously and repeteadly, and the payoff of the different activities changes over time. Moscarini et al. [12] study the effects of informational externalities in a changing world. In their paper, as in ours, the return of the different activities have a Markovian structure, neverthless in order to simplify the analysis they mantain the sequential structure of the seminal papers of Banerjee [2]and Bikchandani et al. [3]. They prove that under Markovian returns only temporary informational cascades may arise, and that under suficiently unpredictable returns, no cascades at all. Nevertheless the artificiality of the sequential structure and single play makes imposible to study issues related to heterogeneity and dynamic generation of beliefs. Their results gain in sharpness, but their model looses generality. It can be understood as an adequate explanation of processes of social learning only under an unplausible set of circumstances.

The "word of mouth" learning literature scapes from the sequential play framework, allowing for the generation of heterogeneity ("diversity") or homogeneity ("conformity"). Nevertheless the payoff of the processes do not follow a markovian process, its distribution being constant across time. Thus in this framework past observations never depretiate, never loose value. Agents face a world that is identical in all periods. Information on what happened 10 periods ago is as good as information on what happened yesterday. Ellison and Funderberg [8] study the behaviour of boundedly rational agents in this context. Their results are imposible to translate to our model, because they study an inherently static world, while we study a specifically dynamic situation.

In our model we abstract from individual experimentation issues. To include them would make the model more realistic, but most certainly untractable. Keller and Rady [10] have develop a continuous time model of optimal experimentation in a changing world (see also Rustichini and Wolinsky [14]). In this literature there are no informational externalities, no learning from others; they study the optimal behaviour of a monopolist. They pressume the inexistence of social learning, we are going to make the opposite assumption, and pressume the inexistence of individual experimentation.

In section 2 we will set up the model. In the follwing two sections we will consider that the accuracy of the information that agents receive is an exogenous variable. We will then determine the heterogeneity of the agents' beliefs (section 3) and their investment activity (section 4) as a function of the accuracy of their perceptions. In section 5 we will endogenize the precision of the information received, at each period, by the agents. In sections 6 and 7 we will solve the complete model. Finally section 8 concludes by summarizing the results and offering some examples.

## 2 Set up of the model

There is a continuum of risk neutral agents in the interval $(0, 1)$. Each agent has an unit of investment good available at each period. They have to invest it in one of $K$ investment opportunities. At time t sector $k$ ($k = \{1, 2, ..., K\}$) pays an exogenous amount $R_t^k$ (its rate of return) per unit of investment plus some idiosyncratic noise independently distributed across sectors, individuals and time. We do not need to deal with intertemporal decision problems, because there is no accumulation (the good is perishable) and the informational structure rules out experimentation (see bellow).

The rates of return of each sector follow independent $AR(1)$ processes, all of them with the same autocorrelation and variance of the perturbations:

$$R_t^k = \rho \;\; R_{t-1}^k + e_t^k \qquad where \quad e_t^k \sim N\left(0, \sigma\right) \tag{1}$$

We will call $x_t^k$ to the amount of aggregate investment in sector $k$ at time $t$.

These processes are assumed to be stationary ($\rho < 1$). The model itself allows for non stationary processes, but its qualitative results would differ. To understand its solution in such a case would require further work.

Agents do not observe the true rate of return of the variables, they receive signals on their return. The signal that an agent receives is his *perception* of the return; it subsumes *all* the information that arrives to the agent on the mean return of each sector. The information that an agent receives (the signals) is independent of what he chooses to do. This rules out experimentation. Agents have no incentive to sacrifice some income (not maximizing expected income) in order to get information. By assuming this we will substantially simplify the model and make it tractable.

In section 5 we will study the origin of these perceptions, so far we will take them as given, and assume that after playing at any time $t$ each agent receives $K$ signals, one for each sector. Each signal being the summation of the true rate of return in the corresponding sector plus some noise that is independent across sectors, individuals and time.

Call this signal $S_t^k(i)$, then:

$$S_t^k(i) = R_t^k + \frac{1}{\sqrt{P_t^k}} \epsilon_t^k(i) \tag{2}$$

Where $\epsilon_t^k(i)$ comes from a standard normal independent across sectors, individuals and time. $\frac{1}{P_t^k}$ is the variance of the noise; $P_t^k$ the precision.

Before playing at time $t$, the information set of an individual consists of the perception that he has on the performance of each sector at all previous times. He has been collecting information since the beginning of time. This information consists in what he believes that each sector paid at each previous time, his perceptions coming in the form of signals. With all this information they establish priors on what is going to be the return on each sector at $t$.

## 3 Heterogeneity as a function of the accuracy of the perceptions.

In this section we will consider that the precision is exogenous[1]. We will determine how the agents build their beliefs. We will also determine how and why these beliefs are going to differ from agent to agent.

Each agent builds his belief about the future based on the information available to him: his perception of the history of the economy. This is, the collection of all the signals that he has received. When he receives a signal on the return of sector $k$ at time $t$ he updates his prior on it (using Bayes law) and determines his beliefs on the return at $t+1$ (knowing 1). We can then establish the following proposition.

**Proposition 1** *The priors of an individual on the rate of return at any time $t$ are always normally distributed. The mean of the priors may differ from agent to agent. The variance is equal for all agents. Calling $\mu_t^k(i)$ to its mean, and $V_t^k$ to its variance at $t$, then*

$$\mu_{t+1}^k(i) = \rho\left(\theta_t^k \mu_t^k(i) + \left(1-\theta_t^k\right) S_t^k(i)\right) \quad (3)$$

$$V_{t+1}^k = \rho^2 \theta_t^k V_t^k + \sigma \quad (4)$$

$$\text{with} \qquad \theta_t^k = \frac{1}{1+P_t^k V_t^k} \quad (5)$$

*Proof in appendix A.1*

The signals are normally distributed, as a consequence the priors are also normally distributed. $\theta_t^k$ is the weight that agents assign to the prior when updating their belief. If the signal is very noisy relative to the prior, Bayes updating implies that all the agents will give all the weight to the priors ($\theta_t^k$ close to one). If, on the contrary, the signals are very accurate (relative to the priors), the agents will put all the weight in the signal ($\theta_t^k$ close to zero).

The priors change *across agents* because they depend on the whole history of signals that the agents receive, and these signals are different for different agents. They have different perceptions of the past, and this induces different beliefs about the future.

On the other hand the variance of the beliefs depends only in the variance of the signals, something that the agents know and that is common for all of them. The precision of the signals ($P_t^k$) and so the variance of the beliefs ($V_t^k$) are common to all the agents. Note that the variance of the priors is bounded by above by $\frac{\sigma}{1-\rho^2}$ (unconditional variance) and by bellow by $\sigma$ (because always there is uncertainty on future events, even if you *know* the past).

[1] We will relax this assumption in section 5

Thus, the beliefs of agent $i$ may differ from the beliefs of agent $j$ only in their expectations, not in their variances. This allows us to identify the distribution of the beliefs of the agents at any moment of time by simply identifying the distribution of their expectations.

**Proposition 2** *The expectations of the beliefs of the agents are always normally distributed. Calling* $\bar{\mu}_t^k$ *to the mean of such a distribution at time* $t$*, and* $M_t^k$ *to its variance then:*

$$\mu_t^k(i) \sim N\left(\bar{\mu}_t^k, M_t^k\right) \qquad \textit{and} \qquad \mu_{t+1}^k(i) \sim N\left(\bar{\mu}_{t+1}^k, M_{t+1}^k\right) \tag{6}$$

$$\bar{\mu}_{t+1}^k = \rho \left(\theta_t^k \, \bar{\mu}_t^k + \left(1 - \theta_t^k\right) R_t^k\right) \tag{7}$$

$$M_{t+1}^k = \rho^2 \, \theta_t^k \left(\theta_t^k \, M_t^k + \left(1 - \theta_t^k\right) V_t^k\right) \tag{8}$$

*Proof in appendix A.2.*

$\bar{\mu}_t^k(i)$ is the average belief hold by the agents, and $M_t^k$ measures the dispersion of their beliefs. High values of $M_t^k$ imply heterogeneity, while low ones, consensus, homogeneity.

The state of nature is defined by the real aggregate shocks on the return of the sectors. These rates of return are the only exogenous variable. The noise in the signals is whipped out by the law of the large numbers, its only purpose being to generate heterogeneity in the beliefs of the agents. It has an effect through its variance, because it affects the weight that agents put on their signals, and so in the dispersion of the beliefs.

Note that the dispersion of the beliefs will tend to be very small both when the signals are very good *and* when they are very bad. If the signals are much more accurate than the priors[2] the agents weigh the signal much more than the prior ($\theta_t^k$ is close to zero), then the beliefs track the truth very well, ($\bar{\mu}_{t+1}^k \simeq \rho R_t^k = E(R_{t+1}^k \mid R_t^k)$) and the dispersion of the beliefs is minimal, because everybody trusts what it 'sees', and everybody 'sees' more or less the same signal. The very fact that the precision is high implies that all the signals will be quite similar, their value being close to what actually happened.

If the opposite happens (the priors are much more accurate than the signals[3]), $\theta_t^k$ is close to one and everybody moves its expected value towards zero (the long run mean), consequently the dispersion of beliefs decreases; both the average belief and the dispersion move towards zero independently of the realizations of $R_t^k$.

[Figure 1 about here.]

Equations 5, 4 and 8 account for a very non linear system of differential equations that determines the variances of the beliefs and their dispersion as a function of precision of the signals.

[2] Which will happen if the signals have large precision (low variance), because the lowest possible variance of the priors is $\sigma$

[3] Which will happen if the signals have a very low precision (high variance), because the maximum variance of the priors is $\frac{\sigma}{1-\rho^2}$

The solutions of $M$ as a function of the logarithm of precision of the signals (assuming that such a precision is always constant, $\rho = 0.75$ and $\sigma = 1$) appear in figure 1. There it is clear that when the precision of the observations is either very small or very large, the dispersion of the beliefs will be very low.

## 4 Aggregate Investment

Each individual has zero weight, so they are unable to change the variance of the signals by investing in one sector or another. This makes the individual decision problem quite trivial. They are risk neutral and they have no incentive to experiment, so they will invest everything in the sector from where they expect to get the highest return.

Thus, aggregate investment in sector $k$ ($x_t^k$) is the number of agents for whom:

$$\mu_t^k(i) > \mu_t^j(i) \quad ; \forall j \neq k \tag{9}$$

Given the distribution of the beliefs *across agents* and the fact that there is a continuum of agents, $x_t^k$ will be exactly the probability that the previous condition holds.

$\mu_t^k(i)$ depends on the history of signals that $i$ received and the true rate of return in sector $k$ at each different moment, but all this is independent across sectors, and the signals are also independent across agents. Thus $\mu_t^k(i)$ and $\mu_t^j(i)$ are independent random variables, both of them normally distributed.

**Proposition 3** *The probability that 9 holds ( $x_t^k$) is the cumulative distribution function of a standard multivariate normal with $K-1$ variables. Calling the variables $j = \{1, 2, ..., k-1, k+1, ...K\}$, the integration limit of variable $j$ and the correlation between variables $j$ and $h$ are respectively*

$$\frac{\bar{\mu}_t^k - \bar{\mu}_t^j}{\sqrt{M_t^k + M_t^j}} \tag{10}$$

$$\frac{M_t^k}{\sqrt{M_t^k + M_t^j}\ \sqrt{M_t^k + M_t^h}} \tag{11}$$

*Proof in appendix A.3*

Thus, we can postulate a function for each sector $k$ that maps the average belief and dispersion in all sectors into aggregate investment in sector $k$:

$$x_t^k = \Phi(\{\bar{\mu}_t^j, M_t^j\}\ \forall j) \tag{12}$$

If all the agents share very similar beliefs with respect to all the sectors (so that $M_t^k$ is close to zero for all $k$), then everybody does the same thing; everybody invests in the sector that offers the highest expected payoff[4]. This is so unless their beliefs are identical across sectors ($\bar{\mu}_t^k = \bar{\mu}_t \quad \forall k$), in which case the aggregate investment will also be identical across sectors.

On the other hand, if the beliefs are not homogeneous across agents, the distribution of investment will follow an extremely non linear function of their averages and dispersions, allowing for diversification.

## 5 Structure of the information; Externalities.

The accuracy with which individuals observe events is not independent from economic activity. If at time $t$ nobody invests in sector $k$, it is impossible to get any information on its return *at* $t$. The larger that $x_t^k$ is, the more likely that any individual receives information on $R_t^k$. Thus, the precision of the signals on sector $k$ ($P_t^k$) should be increasing in the investment share of $k$, ($x_t^k$).

It is much more problematic to determine *"how much"* should $x_t^k$ affect $P_t^k$. Thus, if agents get their information from randomly sampling among the returns of other agents, the relationship between investment share and precision would be linear (see bellow). On the other hand, if agents where oversampling popular activities, an increase of investment share should produce a more than proportional increase in precision.

In our formulation the signal captures an agent's perception, all the information to which she has access. The following assumption will help us solve the model for a very wide range of relationships between information disclosure and economic activity:

**Assumption 1** *The precision of the signals on the return of sector $k$ at time $t$ is:*

$$P_t^k = A \, \left(x_t^k\right)^p \tag{13}$$

*where $A$ and $p$ are informational parameters.*

$P_t^k$ is a constant elasticity function of $x_t^k$, "$p$" being the elasticity. The parameter "$A''$ (that we will call "underlaying precision") determines the maximum level of information that can be disclosed on the return of any activity. The parameter "$p$" (that we will call "level of social interaction") determines the sensitivity of precision to changes in the investment share.

Thus, $p = 0$ would correspond to a model where the amount of information disclosed is an exogenous variable, with no relationship whatsoever to economic activity; a model without any kind of informational externality, without social learning.

There are lots of circumstances in which such a model would be inappropriate. Imagine, for instance, that individuals get their information by randomly sampling among the returns obtained by other agents. Specifically, imagine that the sample size is always $N$, and that the variance in

[4] In such a case the denominator of 10 is always zero, and given that it is sufficient to have a single integration limit close to minus infinity to have a probability close to zero; then the probability will be almost zero in all the sectors where the numerator is not always positive, and it will be one in the sector where it is always positive.

the idiosyncratic noise in the realizations[5] is $\tilde{\sigma}$. Then the information that agent $i$ obtains on the realization of $R_t^k$ is the return obtained by all agents who invested in $k$ *and are in i's sample.* A sufficient statistic for this it is the mean of the realizations, that has as expected value the true $R_t^k$, and an expected precision of $\frac{N}{\tilde{\sigma}}\, x_t^k$.

Thus, $p = 1$ would correspond to a sampling model. The sampling size and the variance of the observations (together) determining $A$; larger values of $A$ corresponding to larger sample sizes (and so more information). All individuals have the same probability of being in my sample, *independently of their activity.* Thus the amount of information disclosed on the return of a sector increases proportionally with the sector's popularity. In a sampling model the relationship between precision and share is linear; the elasticity equal to one.

It is easy to imagine contexts in which an increase in a sector's share translates into a more than proportional increase in precision. This would be the case if the probability of being any sample were larger for individuals investing in popular activities (and consequently lower for individuals investing in less popular ones), or if some of the information that agents receive came from newspaper with a large degree of sensationalism, that focus its diffusion on the return of popular activities.

Our model captures these kind of circumstances by setting $p > 1$. The larger $p$ is, the more sensitive that $P_t^k$ is to $x_t^k$. A large $p$ implies that there is a large difference in the amount of information disclosed between sectors with large investment shares and sectors with low ones. In this case economic activity has a large incidence on the amount of information that agents receive.

On the other hand, by setting $p < 1$ we would be in a context in which a great deal of information is unrelated to social activity. The lower $p$ is, the smaller the effect of economic activity on information disclosure.

Acknowledging our ignorance on the way in which processes of information diffusion work in the real world (on if $p$ should be large or small), it is easy to see the advantages of our formulation. It allows us to represent widely different information structures by using a simple and very compact functional form. Certainly we would like to know the effects of different informational structures (different informational parameters $A$ and $p$) on the aggregate behavior of the economy, and to this task we devote the rest of the paper.

## 6 Dynamics

The dynamic structure of the model is the following:

Given the averages and dispersion of the beliefs referring to the returns at $t$, the investment in each sector is determined by $\Phi(.)$ (equation 12).

This level of investment induces the precision with which each agent will observe the returns at $t$ (equation 13). The precision of the signals and the variance of the priors at $t$ determines the share of the prior in the update for each sector, how much do they trust their priors (equation 5). With it we can calculate the variance of the priors the following period (equation 4). The only exogenous

[5] Or, alternatively, the noise in the communication process

variable is the rate of return at $t$, and this enters exclusively through the average belief at $t+1$ (equation 7). Finally, the variance of the priors and the dispersion at $t$ generate the dispersion of beliefs at t+1 (equation 8).

The function $\Phi$ has no closed form (there is no closed form for the CDF of a multivariate normal); consequently, it is not possible to find an analytical solution to the model. So we have to run simulations and see how the results change when the level of 'social interaction' $p$ and the 'underlying precision' $A$ change.

[Figure 2 about here.]

We saw that the agents have very homogeneous beliefs if in each sector the precision is either very high or very low, and that this implies that the investment will be very concentrated in one sector. The degree of precision in the signals of any sector $k$ depends on the underlying precision $A$, the level of social interaction $p$, and the aggregate investment in the sector. Even if the investment is very low (but positive) the precision level can be quite high, provided that either $A$ is very large and/or $p$ very small. Conversely, even if almost everybody is investing in a sector the precision might be very small. Thus an interesting experiment is to draw the solutions for $M$ of the system of differential equations defined by equations 5, 4 and 8 not as a function of precision, but of its components.

Figure 2(a) represents such a solution as a function of the level of underlaying precision ($A$, in the horizontal axis) and the level of social interaction ($p$, in the vertical axis) while we keep the share of investment constant at a value *"very close to zero"* [6]. There we draw both the 3-dimensional representation and its contour map. The shaded area in the contour map are pairs of $A$ and $p$ such that when investment is very small correspond to precision levels that generate substantial heterogeneity in the beliefs of the agents (precision levels that produce the "peak" in figure 1). That is, here the precision of the signals is of the same order of magnitude than the precision of the priors.

Points below the shaded area produce *relatively* high precision on the signals, even if almost nobody invests, and consequently very homogeneous beliefs. In points above the shaded area the agents have signals with a very low precision when there is very little investment on this sector, agents give large weight to the priors, the beliefs becoming very homogeneous.

Figure 2(b) represents the solution to the same system of equations when the investment share of the sector is $\frac{1}{4}$. If the number of sectors were four, this would be the share of each sector when the aggregate investment is diversified. Again points above the shaded area have a relatively low precision (generating convergence of the beliefs in the long run distribution) and points below it a relatively high one (generating convergence of the beliefs in the best forecast of the return).

Precision is increasing in the investment share, so all the points that fall below the shaded area when $x$ is very small are bellow the shaded area when $x$ is $\frac{1}{4}$. Drawing figure 2(b) on the top of

[6] Actually "very close to zero" in this context means the machine epsilon, the smallest number bigger than zero that the computer recognizes as different from zero. This is a *really* small number

2(a) allows us to divide the space $A \times p$ in three clearly defined regions (figure 3). We can predict (before doing simulations with the full stochastic model) what will be the evolution of the economy in each of them.

[Figure 3 about here.]

1. The area bellow the shaded area when $x$ is "very small" (marked as "weight always on the signals"), is such that that even if the number of agents investing in a sector is very small the precision of the signals is quite large. On the other hand the stochastic structure of the returns insures that the variance of the priors is always larger than $\sigma$, so the agents put all the weight always on the signals.

   In this case the dispersion of the beliefs will always be almost zero, *in all the sectors*. This implies that almost everybody will invest in the sector that in the immediate past produced the highest payoff.

   There will always be a few outliers that make 'mistakes'. They are very useful for the society because they generate signals with quite a high level of precision indicating the evolution of the non-optimal sectors. Thus almost all the members of the society are continuously keeping track of the evolution of the payoffs in all the sectors, and the aggregate investment is always close to one in the sector with the highest expected return (conditional on its previous realization).

2. In the area marked as "herding", between the two "arms", $A$ and $p$ are such that if the number of agents investing is very small, the precision is quite small (at the left of the peak in figure 1); and if the investment is larger than $\frac{1}{4}$, quite large (at the right of the peak in figure 1).

   Imagine that the informational parameters fall inside this region and people have flat priors on what is the best thing to do. They will diversify their investment. Doing so they will get very accurate signals, and in the next period they would concentrate their investment in the more promising sector. From then on, and until things change, they will get very accurate information about what is happening with that sector, but they will be getting no practical information on the evolution of the other sectors. The beliefs get very homogeneous, because all the weight is given to the signals in the 'chosen' sector (and the signals have low variance) and to the prior in the other sectors (and so the priors converge to zero because the payoffs are mean reverting). Homogeneity reinforces the situation, because the investment gets concentrated, and this produces homogeneous beliefs, et cetera.

   Almost everybody will invest in the chosen sector as long as it is paying above the long run average, even if other sectors are paying more; the concentration of the investment in one of the sectors excludes the possibility of learning. Only when the rate of return of the 'herd' is below the long run average will they diversify their investment and learn what is the best thing that they can do.

3. In points above the "upper arm" (the area marked as "weight always on priors") if aggregate investment is diversified agents do not use the signals when updating the priors. Thus, they hold very homogeneous beliefs that are equal to the unconditional distribution of equation 1. This implies that they expect the same return in all sectors, so they are indifferent on where to invest, and consequently aggregate investment will be diversified. But diversification implies that information disclosure is nil in all the sectors, and the priors always have all the weight in the updating, so aggregate investment is *always* diversified.

   They cannot see anything, they will never update their priors significatively. They start with a common prior (zero, the long run distribution of the returns), and they stay with it forever. The dispersion of the beliefs is almost zero, but the investment is distributed evenly across sectors because the expected return is zero in all of them.

# 7 Solution

As I said before, the solution of the model comes from the hand of simulations. All the simulations presented are done with parameters $\rho = .75$ and $\sigma = 1$, but changing these parameters does not change the qualitative results. Due to computational reasons all the simulations are done with $K = 4$, but as it will be clear this will provide us with enough information to adequately discuss how the solutions would be if there were more sectors. The simulations are the product of averaging the results for 25 different histories of 1000 observations each.

The results of these simulations are presented in figure 4. There each subfigure shows the results for an endogenous variable as a function of the logs of the informational parameters, $A$ and $p$[7]. The graphs of the top row are the 3-dimensional representation, while the figures in the bottom are the contour map of the respective surfaces. The columns represent the average production, the average Herfindal Index and the average dispersion of beliefs respectively.

[Figure 4 about here.]

To say that the model is non-linear is an understatement, a glimpse to the simulations is proof enough of it. Things change suddenly for small changes in the parameters and remain steady for very large ranges of them. In order to understand the results we have to take a careful look at the figures and think about them having in mind the arguments given in the previous section.

From a first look at the graphs we can divide the space $A \times p$ in three regions, defined by two "arms". In each of these three regions average output, average concentration of the investment and average dispersion of the beliefs are constant. Not surprisingly these regions correspond to the three polar cases exposed in the previous section.

1. The first one (the "weight on signals" region) corresponds to the area where the average payoff is maximum (see the first column of figure 4). In this area the precision of the signals

[7] Using logs allows to study the effects of large changes in the informational parameters

is relatively high even if almost nobody is investing in a sector. This induces very homogeneous beliefs (see the second column), and thus large concentration of the investment (see the third). If $A$ and $p$ fall in this region agents keep very good track of the *ranking* of the sectors from best to worst. There is always a very small number of agents making 'mistakes', because the returns are continuously changing and to learn takes time. If the rates of return were constant, eventually everybody would be investing in the 'right' stuff. The noisy structure of the model induces some people to take the "wrong" decisions, but doing so they generate a large amount of information that the society, as a whole, uses.

2. For values of A and $p$ in in the "herding" region (from figure 3) if almost nobody invests, the precision of the observations is very low relative to the priors[8]; while it is high relative to the priors[9] if the investment is diversified and even more so in a sector that concentrates most of the activity. As we saw this produces high concentration of the investment on one sector. Agents learn only on its evolution, and believe that all other sectors converge to their long run mean. Aggregate investment is diversified only when the return of the sector that they observe falls bellow the long run average. Upon diversification they will learn fast, and again concentrate their investment, so that the situation remains the same, perhaps with the investment concentrated in a different sector, but extremely concentrated nevertheless. So the average Herfindal Index (second column of figure 4) is very high, while the average dispersion of beliefs is very low (third column).

   The average output (see first column of figure 4) is substantially lower than in the first 'plateau' because the society only keeps track of what happens in one sector. Any of the other sectors could get a series of positive shocks and start paying above the one that carries almost all the investment. Actually this will happen most of the time, but the number of people that observes it is too small for the society as a whole to notice.

   If the world were not changing continuously, the information that these outliers generate would be used sooner or later. Little by little people would move to the sector with the highest payoff. A stationary stochastic world prevents this from happening because:

   (a) Mean reversion implies that the priors are continuously moved towards the long run mean, and the priors have a huge weight if the precision of the observations is low. The two effects work in the same direction, inducing the agents to believe that the sector is close to its unconditional expected value and minimizing heterogeneity.

   (b) The amount of information that the outliers generate is extremely low, so the learning process would be very slow in any case. The probability of the event '*the sector where everybody invest goes back to its mean before agents learn that another sector is paying more*' is close to one.

[8] We are at the left of the peak in figure 1, above it in figure 2(a)

[9] At the right of the peakin figure 1, bellow it in figure 2(a)

It is in this region where we can talk properly of herding. Agents 'follow the herd' because it is paying above the long run average, so they are not doing too badly, and they are unable to see what happens in the rest of the world. It is a conformist society, experimentation could produce higher returns for the economy as a whole, but nobody has the incentives to sacrifice expected income in order for everybody to learn.

Note that for low values of $p$ there is no value of $A$ such that the economy falls in the "herding" area. In order to be here you need the informational content of the signals to be sensitive enough to the aggregate behavior of the economy. You need the amount of information lost by concentration in one sector to be large, and individuals to be homogeneous enough how to concentrate their investment. In a sampling model ($p = 1$) for levels of $A$ that might be susceptible of generating "herds" the level of heterogeneity is still too large how to allow for excesive concentration of aggregate investment.

3. In the third region, at the diversification stage the precision of the signals is very low, the weight in the update falls overwhelmingly on the priors. The agents do not receive new information, and very soon all of them will use as a prior the long run distribution of the rates of return. In this area, beyond the 'upper arm', agents *never* learn. The beliefs are homogeneous, but the investment is equally distributed among all the sectors because they expect all of them to produce the same return (the numerator of the integration limits is zero).

The following subsections will clarify the role played by $A$ and $p$

### 7.1 Comparative Statics: $p$

Any increase of $p$ (the level of social interaction) produces a decrease in the precision of the signals. Nevertheless the decrease in precision is smaller the larger $x$ is:

$$\frac{\partial \frac{A\,x^p}{A\,y^p}}{\partial p} = \left(\frac{x}{y}\right)^p \log\left(\frac{x}{y}\right) > 0 \quad \Longleftrightarrow \quad x > y \tag{14}$$

The main direct effect of an increase of $p$ is that it increases the gap between the amount of information disclosed in popular sector and a less popular one, while decreases the information disclosed in both of them. This effect is compounded with the induced changes in homogeneity.

In points of the first region the precision is always far away and to the right of the peak in figure 1. If $p$ increases, the precision decreases, but we move towards the peak faster for small values of $x$. There are no substantial changes as long as the precision is at the right of the peak for small values of $x$, because the signals are always much more accurate than the priors. The investment keeps concentrated in the right stuff because almost everybody is able to establish the proper ranking on the return of the sectors.

If we keep increasing $p$ eventually the precision if $x$ is very small arrives to the peak of figure 1, while the precision for larger values of $x$ is still to the right of the peak (we are in the lower arm at any of the contour maps, in the area marked in figure 2(a)). Here if the investment were very concentrated in one sector, the heterogeneity of the beliefs in the other sectors would be quite high. However, this implies that the investment can not be very concentrated in any sector (the denominators of the integration limits in equation 12 are not close to zero). Thus, for this range of parameters we observe smaller concentration levels, more heterogeneity and a decrease in average output[10]. Most of the agents 'do the right thing', but there is a substantial number of agents investing in sectors that did not do well in the immediate past. Again, if there were no change in the returns, they would eventually learn and concentrate their investment; but the optimal action changes continuously, and this allows for heterogeneity in the beliefs.

Further increases of $p$ make the precision of the signals even more sensitive to the investment activity in the sectors. Thus, precision is at the left of the peak if $x$ is very small, while if it is larger than $\frac{1}{K}$, precision is at the right. Consequently the economy falls in the "herding" region. Here increases of $p$ have no effect as long as during diversification the precision of the signals is at the right of the peak.

If $p$ keeps increasing eventually we will arrive to the "upper arm" of any of the contour maps, the area marked in figure 2(b). Here during the diversification stage the learning process will be much slower and the beliefs will be heterogeneous for a longer period. This accounts for the sharp falls in average production and concentration, and the increase in heterogeneity observable in this area. The smaller the precision at the diversification stage, the slower the learning process. If we keep increasing $p$ the learning process becomes all too slow.

### 7.2 Comparative Statics: $A$

One can think of two effects due to an increase in the underlying precision ($A$):

1. The standard: people have better information, so they will do better.
2. The perverse: everybody has better information, so more people will do the right thing; but doing so the society becomes more homogeneous and increases the level of concentration of aggregate investment, not learning about the evolution of most sectors.

Notice that in order to observe any perverse effect it is necessary to have heterogeneity and informational externalities. It works by decreasing heterogeneity, so if the society is very homogeneous there is no heterogeneity to loose, no negative effect of an increase of the underlying precision. Additionally in order to appear it is necessary that the decrease in heterogeneity translates into a

[10] The 'peak' in figure 2(a) corresponds with the increase in average heterogeneity (see the second column of figure 4) and the 'valley' in the average concentration index (see third column). It also corresponds with the large fall in average output from the 'high' to the 'middle' plateaus. This can be seen more clearly in the contour 'maps', we are talking about the clearly defined lower 'arm' in all of them.

decrease in the information disclosed in the less popular sectors; it needs the amount of information disclosed to be sufficiently sensitive to the investment activity.

For low values of social interaction (low $p$) increases of $A$ have no negative effect on output (see first column of figure 4) because the informational externalities are not strong enough. The information disclosed by the signals is not suficiently sensitive to the agents' activity.

For larger values of $p$ average output might decrease as a consequence of increases of $A$. Observe that in the points where these happens *heterogeneity is large*. The perverse effect makes society more homogeneous, and so loosing the capability of keeping track of the evolution of less popular activities.

There is a clear limit to this effect: once heterogeneity is low it is impossible to push it further down. No negative effect might arise when the society is homogenous. Once this ha happened if increases of $A$ have any effect is by *increasing* the level of heterogeneity. With a large enough $A$ it is possible to get meaningful information on the evolution of the minoritarian sectors, and thus in the "lower arm" we observe an increase in both heterogeneity and average output.

In the herding region there is no possibility of having the perverse effect, because there is no heterogeneity to loose; and no positive effect as long as we are at the left of the "lower arm", because individuals already know everything that they need about the sector which concentrates the investment, and the gain in information in the minoritarian is order of magnitudes inferior *because the information is very sensitive to aggregate activity.*

## 8 Conclusions

In 1972 few, if any, companies were making research in energy-efficient bulbs. Everybody had a prior on what return such an investment would pay, and the commonly shared view considered it quite a foolish investment.

Nowadays all the major firms in the sector are producing new and every time more efficient lamps. This is so even if the price of energy is not that much different, in real terms, today than what it was before the oil crisis. This could be an indication that 25 years ago investment in energy efficient products would have been as wise as it is today. Nevertheless, the recession of the seventies was necessary in order for the companies to learn it.

The reason that we propose is that the priors that everybody had were not being tested. This kept the priors unchanged, which prevented them to be tested, et cetera...

The investment in research on low consumption bulbs *had* a very good rate of return, but this particular piece of information lied outside the information set of the agents. The rate of return that such an investment would have had during 1971 was, in 1972, part of the past evolution of the economy, but the information that the agents had on it was very inaccurate.

The interesting point is that this information was very inaccurate due to *endogenous* reasons. We can speculate that if a group of crazy outliers would have made the investment in 1971, assuming that this investment was successful, and that this success was publicized, *then* everybody would have tested their priors. In those circumstances most agents would have placed investments in such technologies before the '73 shock hit; but you need the outliers, their success and its publicity.

Under this view periods of recession produce experimentation, we may think of them as periods of "creative destruction". From the outside we observe a decline in average output, but they also bring experimentation, and a more efficient reallocation of resources.

We can now summarize the main results of our analysis:

- When the informational content of the perceptions is either bad or good, relatively to the priors, the agents will tend to share very similar views on the evolution of the return. If they are bad because agents expect the sector to go back to its long run average, and if they are good because all the agents observe the same thing, and so they all share basically the same information.

  It is only for a relatively small range of values in the precision of the signals that heterogeneity is generated; only when the precision of the signals is roughly of the same magnitude than the precision of the priors (which is bounded by bellow and by above).

- In general, under the presence of relatively high (but not too high) levels of "social interaction" (by which we refer to *"how"* and not to *"with how many agents"* do agents interact) and relatively high levels of "underlying precision"[11] we will observe that as long as a "popular" activity or sector is paying above the long run average agents are going to "herd" in such an activity, even if other sectors pay above it. In this environment herding is not a consequence of people thinking that other agents 'might-know-better'. Everybody does the same thing because different courses of action are not being explored , nobody has incentives to explore the unbeaten path. In our context herding takes the form of over-concentration of the investment, something that from the point of view of the society can be interpreted as under-investment in experimentation.

  The high level of $A$ allows the signals to be quite accurate when most people are investing in one sector, thus inducing a high degree of homogeneity in the perception that the members of the society have of that sector. Additionally, the high $p$ implies that, in order to be noticed, the number of agents investing in a sector has to be large. If only a few outliers invest in some sector, the beliefs of everybody on the evolution of that sector become more homogeneous because all the weight of the updating is given to the priors, and the processes that drive the rates of return are mean reversing. Thus one effect feeds the other and we end up with a very homogeneous society: everybody sees what happens in one sector and assumes that all the others are near their long run average. Not until the sector in which they are investing starts paying below the long run average do they diversify their investment, and really learn what happens in the rest of the world. Thus heterogeneity, diversity in beliefs, only appear in time

[11] In a sampling model we could interpret this conditions as oversampling of popular decisions and relatively big samples

of recession, when exogenous shocks prove beyond doubt that the commonly held beliefs are not true anymore.

- Across-the-board improvements in the accuracy of the information that agents receive may have quite perverse consequences. More information for everybody is not always good if we share that information.

  In a world of short-sighted people, individuals are going to be constantly insecure of what is the best thing to do, and their views on it are necessarily going to differ; as a consequence, new paths are continuously going to be explored, and new roads found. In a world where agents have almost $20 \times 20$ vision they are going to be overconfident on what is the best course to take. Nobody is going to explore off the beaten paths, and the society will be defenseless when exogenous shocks hit the chosen road.

That the results are not linear comes as no surprise, after all most papers in the literature restrict themselves to boundedly rational agents precisely because of this. That they are *so much* non linear it is probably more so. This is due to the large nonlinearity in the heterogeneity of the priors and the fact that what determines the output is the accuracy of the *ranking* that agents establish on the return of the different sectors.

I cannot deny the criticism that I have been using the mechanism of information diffusion as a black box. Undoubtedly the mechanism should be endogenous to the economy, and infinitely more complex than I have assumed. However reduced forms should be acceptable if they accurately represent a complex reality in a simplified way. By allowing for a very flexible functional form, and by not assuming any range of parameters, I have insured that whatever reality is, it is captured by some point in the space defined by $A$ and $p$. A sampling process is only a truly small subset of this space. The presence of news media, fashions and panics is, I think, an indication that people does not receive information by a simple sampling processes. The micro structure of such realistic information diffusion process is, still, unknown.

Most of the paper is a long exercise of comparative statics. It determines the effects of changes in the informational parameters. Nevertheless, whatever the values of the parameters, in the real world they are most likely to be constant. As long as we believe that both $A$ and $p$ are relatively large, we should expect agents to behave in a very precise way. This behavior is unmodified for a huge range of parameters and it is characterized by excessive concentration of the investment in a few sectors. Thus, it results in under-investment in experimentation for the society as a whole. At the same time beliefs are going to be very homogeneous most of the time. Heterogeneity is going to be patent only in times of crisis and recession, times when uncertainty is great and experimentation takes place.

# A Appendix

## A.1 Generating a prior about the future

At the origin of time an agent has no information other than the knowledge of the stochastic process driving the returns (equation 1), so his prior on the return in any sector will be normally distributed with mean zero and variance $\frac{\sigma}{1-\rho^2}$. After receiving a signal on what happened at $t = 1$, he updates his prior on the realization of the returns at $t = 1$, and after that, and based in his knowledge of equation 1, he will establish a prior on the value of the returns at $t = 2$.

We will now see that if the prior on the realization of the returns at any time $t$ that an agent has before playing at $t$ is normally distributed, then the prior on the returns at $t + 1$ after playing at $t$ will also be normally distributed. This implies that all the priors are going to be normally distributed at all times, because at $t = 1$ they already were so.

Assume then that at $t$, before investing, individual $i$ believes that $R_t^k$ is distributed[12] from a normal with a mean $\mu_t^k(i)$ and a variance $V_t^k$:

$$R_t^k(i) \quad \sim \quad N\left(\mu_t^k(i), V_t^k\right) \tag{15}$$

Independently of what he chooses to do, after playing he gets an *unbiased* signal on the rate of return in each sector with a (known) precision $P_t^k$.

When the prior is updated (using Bayes law) the posterior[13] on $R_t^k$ is also a normal distribution

$$R_t^{k+}(i) \quad \sim \quad N\left(\theta_t^k \mu_t^k(i) + \left(1 - \theta_t^k\right) S_t^k(i) \quad , \quad \theta_t^k V_t^k\right) \tag{16}$$

where :

$$\theta_t^k = \frac{1}{1 + P_t^k V_t^k} \tag{17}$$

is the weight that agents assign to the prior when updating their belief. If $\theta_t^k$ is close to one (because the signal is very noisy or the prior very accurate) the agents put all the weight on the priors. If it is close to zero, they put all the weight on the signal.

Now let's go back to equation 1. From the point of view of $i$, before playing at $t + 1$, $R_t^k$ is normally distributed, and he knows that $e_t^k$ is so too. Thus given his information he will perceive $R_{t+1}^k$ as a normally distributed random variable,

$$R_{t+1}^k(i) \sim N\left(\mu_{t+1}^k(i), V_{t+1}^k\right) \tag{18}$$

with mean:

$$\mu_{t+1}^k(i) = \rho \, \left(\theta_t^k \mu_t^k(i) + \left(1 - \theta_t^k\right) S_t^k(i)\right) \tag{19}$$

and variance:

$$V_{t+1}^k = \rho^2 \, \frac{V_t^k}{1 + P_t^k V_t^k} + \sigma = \rho^2 \theta_t^k V_t^k + \sigma \tag{20}$$

## A.2 Distribution of beliefs across agents

The strategy is equal that in the previous proof, first we show that at the origin of time the expected return is normally distributed across agents; then that if at any moment of time the distribution is a normal, it will always be so afterwards. Doing this we will also identify the stochastic differential equations that drive the beliefs of the agents.

Let's go back again to the first time that they played. They had common flat priors, and then they updated them using their signals, as a consequence the prior that an individual $i$ has on the value of the return in sector $k$ at $t = 2$ is a normal with mean:

$$\mu_2^k(i) = \rho \, \left(1 - \theta_1^k\right) S_1^k(i) \tag{21}$$

[12] With an small abuse of notation we will call $R_t^k(i)$ the prior of individual $i$ on the value of $R_t^k$

[13] We will make here the same abuse of notation

Across agents $S_1^k(i)$ is a normal with mean $R_1^k$ (what actually happened) and variance $\frac{1}{P_1^k}$. So, across agents $\mu_2^k(i)$ will also be a normal, with mean

$$\rho \left(1-\theta_1^k\right) R_1^k \tag{22}$$

and variance:

$$\rho^2 \left(1-\theta_1^k\right)^2 \frac{1}{P_1^k} \tag{23}$$

Now let's assume then that at some time $t$ (that is, immediately before playing at $t$) the expected return was normally distributed across agents:

$$\mu_t^k(i) \quad \sim \quad N\left(\bar{\mu}_t^k, M_t^k\right) \tag{24}$$

Where $\bar{\mu}_t^k$ is the average expectation on $R_t^k$, the 'consensus' expected return, and $M_t^k$ measures the degree of heterogeneity in the beliefs of $R_t^k$, the dispersion in these beliefs.

The expected return at $t+1$ for agent $i$ is

$$\mu_{t+1}^k(i) = \rho \left(\theta_t^k \, \mu_t^k(i) + \left(1-\theta_t^k\right) S_t^k(i)\right) \tag{25}$$

Given that $\mu_t^k(i)$ and $S_t^k(i)$ are independently distributed normals (across agents), then $\mu_{t+1}^k(i)$ will also be a normal,

$$\mu_{t+1}^k(i) \quad \sim \quad N\left(\bar{\mu}_{t+1}^k, M_{t+1}^k\right)$$

where its mean is

$$\bar{\mu}_{t+1}^k = \rho \left(\theta_t^k \, \bar{\mu}_t^k + \left(1-\theta_t^k\right) R_t^k\right) \tag{26}$$

and its variance

$$M_{t+1}^k = \rho^2 \, \theta_t^k \left(\theta_t^k \, M_t^k + \left(1-\theta_t^k\right) V_t^k\right) \tag{27}$$

Thus across agents the expected return is always distributed as a normal, because we know that this was already the case at time $t=2$.

## A.3 Aggregate Investment

Agents will invest in the sector that has the highest expected return.

Aggregate investment in sector $k$ is the number of agents for whom:

$$\mu_t^k(i) - \mu_t^j(i) > 0 \qquad \forall j \neq k \tag{28}$$

It is clear that

$$\begin{array}{c} x_t^k = \Pr\left\{\mu_t^k(i) - \mu_t^j(i) > 0 \quad \forall j \neq k\right\} = \\ \Pr\left\{\frac{\left(\mu_t^k(i)-\mu_t^j(i)\right)-\left(\bar{\mu}_t^k-\bar{\mu}_t^j\right)}{\sqrt{M_t^k+M_t^j}} > -\frac{\left(\bar{\mu}_t^k-\bar{\mu}_t^j\right)}{\sqrt{M_t^k+M_t^j}}\right\} = \\ \Pr\left\{\frac{\left(\mu_t^j(i)-\bar{\mu}_t^j\right)-\left(\mu_t^k(i)-\bar{\mu}_t^k\right)}{\sqrt{M_t^k+M_t^j}} < \frac{\left(\bar{\mu}_t^k-\bar{\mu}_t^j\right)}{\sqrt{M_t^k+M_t^j}}\right\} \end{array} \tag{29}$$

Both, $\mu_t^k(i)$ and $\mu_t^j(i)$ are independent random variables, both of them normally distributed:

$$\begin{array}{c} \mu_t^k(i) \sim N\left(\bar{\mu}_t^k, M_t^k\right) \\ \mu_t^j(i) \sim N\left(\bar{\mu}_t^j, M_t^j\right) \\ covariance\left(\mu_t^k(i), \mu_t^j(i),\right) = 0 \end{array} \tag{30}$$

so:

$$\mu_t^k(i) - \mu_t^j(i) \sim N\left(\bar{\mu}_t^k - \bar{\mu}_t^j \ , \ M_t^k + M_t^j\right) \tag{31}$$

Define $\nu^j$ as:

$$\nu^j = \frac{(\mu_t^j(i) - \bar{\mu}_t^j) - (\mu_t^k(i) - \bar{\mu}_t^k)}{\sqrt{M_t^k + M_t^j}} = \frac{\sqrt{M_t^j}}{\sqrt{M_t^k + M_t^j}} \frac{(\mu_t^j(i) - \bar{\mu}_t^j)}{\sqrt{M_t^j}} - \frac{\sqrt{M_t^k}}{\sqrt{M_t^k + M_t^j}} \frac{(\mu_t^k(i) - \bar{\mu}_t^k)}{\sqrt{M_t^k}} \tag{32}$$

Then:

$$\nu^j \sim N(0,1) \tag{33}$$

and, $\forall j, h \neq k$:

$$E\left(\nu^j \cdot \nu^h\right) = \frac{M_t^k}{\sqrt{M_t^k + M_t^j}\ \sqrt{M_t^k + M_t^h}} \tag{34}$$

Define now the vector $v$:

$$v = \left\{\nu^1, \nu^2, ..., \nu^{k-1}, \nu^{k+1}, ..., \nu^K\right\} \tag{35}$$

and the vector $l$ :

$$l = \left\{\frac{(\bar{\mu}_t^k - \bar{\mu}_t^j)}{\sqrt{M_t^k + M_t^j}}\right\} \qquad j = \{1, 2, ..., k-1, k+1, ...K\} \tag{36}$$

Then from 29:

$$x_t^k = \Pr\{v < l\} \tag{37}$$

The joint CDF of $v$ is a normal a standard multivariate normal with a matrix of variances defined by the correlations in equation 34.

# References


[1] Acemoglu, Daron. "*Learning about Other's Actions and the Investment Accelerator*". The Economic Journal, 1993.

[2] Banarjee, A.V. "*A Simple Model of Herd Behavior*". Mimeo, 1991.

[3] Bikchandani, Hirshleifer and Welch. "*A Theory of Fads, Fashions, Customs and Cultural Change as Informational Cascades*".JPE 1992.

[4] Burguet, Roberto and Xavier Vives. "*Social Learning and Costly Information Acquisition*". Mimeo IAE, 1996.

[5] Caplin, A. and Leahy, John. "*Business as Usual, Market Crashes and Wisdom after the Fact*". Mimeo, 1991.

[6] Caplin, A. and Leahy, John. "*Miracle on Sixth Avenue*". Mimeo, 1995.

[7] Chari,V.V. and Patrick J. Kehoe "*Hot Money*". Federal Reserve Bank of Minneapolis. Research Department Staff Report 228. 1996.

[8] Ellison, G. and Drew Fudenberg. "*Word of Mouth Communication and Social Learning*". QJE 1995.

[9] Hosking, Geoffrey. "*The First Socialist Society*". Harvard University Press, 1990.

[10] Keller, Godfrey and Sven Rady. *"Optimal Experimentation in c Changing Environment"* Research Paper, Graduate School of Business, Stanford University, 1997.

[11] Kirman, Alan. "*Ants, Rationality, and Recruitment* ". The Quarterly Journal of Economics, 1993.

[12] Moscarini, G., Marco Ottaviani and Lones Smith. *"Social learning in a changing world"*. Economic Theory, 1997.

[13] Rodríguez Mora, JV. "*Spirits, but not so Animal*". Thesis Chapter, MIT, 1995.

[14] Rustichini,A. and Wolinsky,A. *Learning about Variable Demand in the Long Run"*. Journal of Economic Dynamics and Control. 1995.

[15] Salisbury, Harrison E. ed. "*The Soviet Union: The Fifty Years*". Howard, Brace and World, NY 1967.

[16] Scharfstein, D.S. and J.C. Stein. "*Herd Behavior and investment*". AER, 1990.

[17] Vives, Xavier. "*Learning from others*".

[18] Vives, Xavier. "*The speed of Information Revelation in a Financial Market Mechanism*". JET 1995.

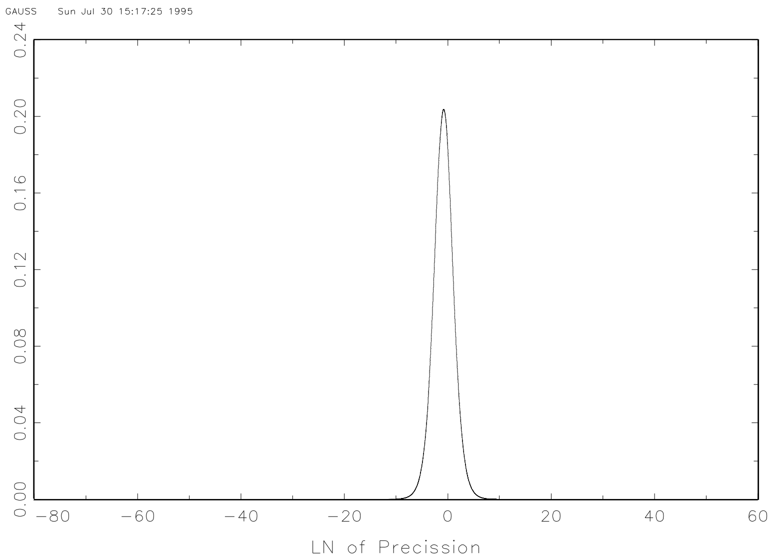
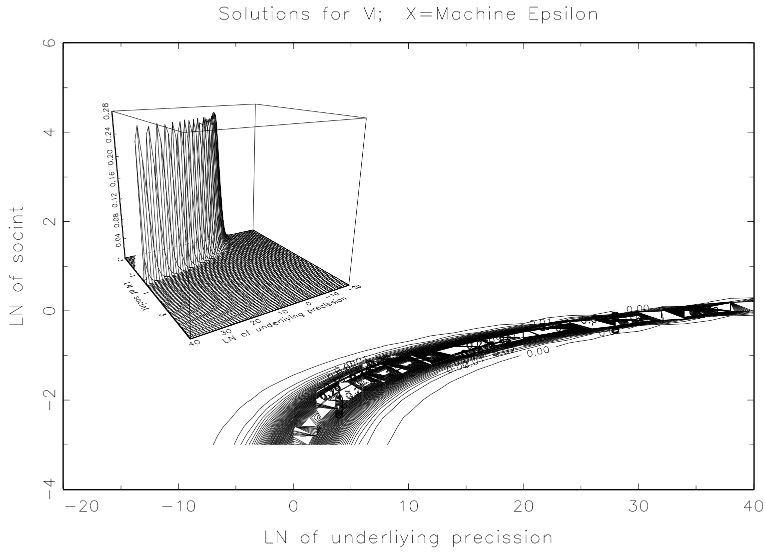
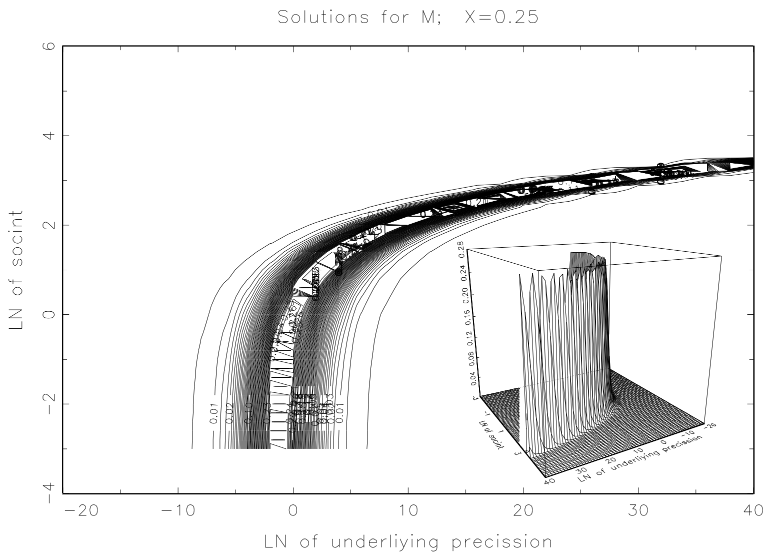



Figure 1: Solution for $M$ of equations 5, 4 and 8; assuming that the precision of the observations is fixed.




(a) $x_t^k$ is a very small positive number (machine epsilon)




(b) $x_t^k = 0.25$, (diversification)



Figure 2: Solution for $M$ of equations 5, 4 and 8; assuming different fixed values of $x_t^k$

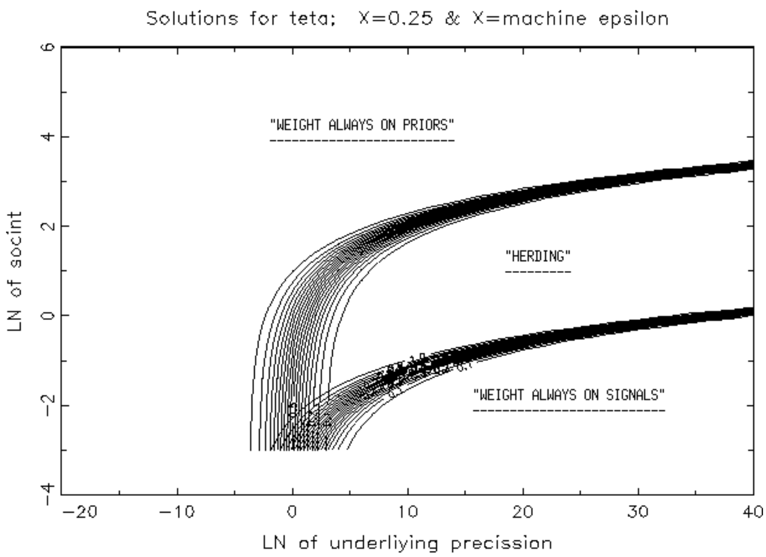



Figure 3: Area that produces relatively high heterogeneity when investment is close to zero and $\frac{1}{4}$

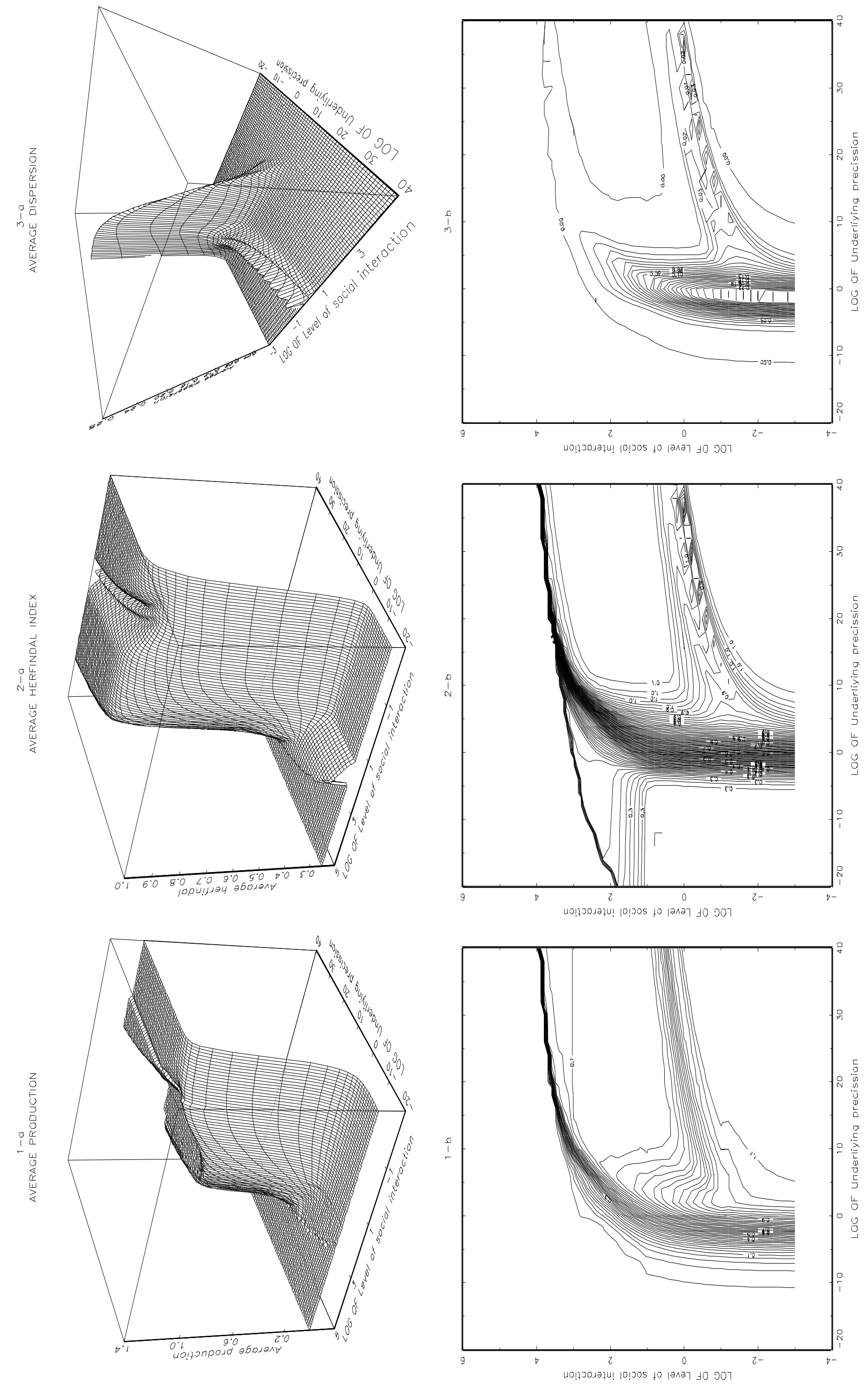





Figure 4: Averages from 25 simulations of 1000 observations each. $\sigma = 1$; $\rho = 0.75$