Introduction to Probability and Information Theory
Homework #6

1. For each of the following, compute the entropy of the given distribution, in bits.

   (a) $[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}]$

   (b) $[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, 0]$

   (c) $[\frac{1}{4}, \frac{1}{4}, \frac{1.1}{4}, \frac{0.9}{4}]$

   (d) $[\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}]$

2. (a) Let X be the result of a roll of a fair six-sided die. What is $H(X)$ in bits?

   (b) Let Y be the result of a roll of four fair eight-sided dice. What is $H(Y)$ in bits?

3. Show that $I(X;Y) = D(P(X,Y)||P(X)\,P(Y))$

4. Let X be the number of flips of a fair coin required until the first head comes up. What is $H(X)$ in bits? This one is hard, but if you try it you might find these helpful:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$
$$\sum_{n=0}^{\infty} n r^n = \frac{r}{(1-r)^2}$$