ENVIRONMETRICS
Environmetrics 2001; **12**:103–116

Modelling the diffuse fraction of global solar radiation on a horizontal surface

John Boland[1]*, Lynne Scott[1] and Mark Luther[2]

[1] *CIAM, School of Mathematics, University of South Australia, Mawson Lakes, SA 5095, Australia*
[2] *School of Architecture and Building, Deakin University, Geelong Waterfront Campus, Geelong, Victoria 3217, Australia*

SUMMARY

For various applications it is necessary to know not only global solar radiation values, but also the diffuse and beam components. Because often only global values are available, there have been several models developed to establish correlations between the diffuse fraction and various predictors. These typically include the clearness index, but also may include the solar angle, temperature and humidity. The clearness index is the proportion of extraterrestrial radiation reaching a location, where the extraterrestrial value used in the calculation varies with latitude and time of year. These correlations have been developed using data principally from latitudes greater than 40°, often using only data from a few locations and with few exceptions have not used solar altitude as a predictor. Generally the data consist of hourly integrated values. A model has been developed using hourly data from a weather station set up at Deakin University, Geelong. Another model has also been developed for 15 minute data values in order to ascertain if the smoothing generated by using hourly data makes a significant difference to overall results. The construction of such models has been investigated, enabling an extension to the research, inclusive of other stations, to be performed systematically. A final investigation was carried out, using data from other Australian locations, to explain some of the considerable scatter by adding apparent solar time as a predictor, which proved to be significantly better than solar altitude. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: diffuse solar radiation; multivariate regression; mathematical modelling

1. INTRODUCTION

The evaluation of the performance of a solar collector such as a solar hot water heater or photovoltaic cell requires knowledge of the amount of solar radiation incident upon it. Solar radiation measurements are usually for global radiation on a horizontal surface (Reindl *et al.*, 1990; Skartveit and Olseth, 1987). These global values comprise two components, the direct and the diffuse. 'I_{DN}, the direct normal irradiance, is the energy of the direct solar beam falling on a unit area perpendicular to the beam at the Earth's surface. To obtain the global irradiance the

* Correspondence to: J. Boland, CIAM, School of Mathematics, University of South Australia, Mawson Lakes, SA 5095, Australia.

Received 22 February 1999
Accepted 6 May 2000

additional irradiance reflected from the clouds and the clear sky must be included' (Lunde, 1979, p. 69). This additional irradiance is the diffuse component.

Typically, solar collectors are not mounted on a horizontal surface but tilted at some angle to it. Thus, it is necessary to calculate values of total solar radiation on a tilted surface given values for a horizontal surface. It is not possible to merely employ trigonometric relationships to calculate the solar radiation on a tilted collector. This is because the diffuse radiation is anisotropic over the sky dome and the 'radiative configuration factor from the sky to the tilted solar collector is not only a function of the collector orientation, but is also sensitive to the assumed distribution of the diffuse solar radiation across the sky' (Brunger, 1989). There are two different approaches to calculating the diffuse radiation on a tilted surface, an analytic model (Brunger, 1989) and empirical models such as that of Perez *et al.* (1990). Each rely on knowledge of the diffuse radiation on a horizontal surface. The diffuse component is not generally measured. Consequently, a method must be derived to estimate the diffuse radiation on a horizontal surface from the global radiation upon that surface.

Numerous researchers have studied this problem and have been successful to varying degrees. Liu and Jordan (1960) developed a relationship between daily diffuse and global radiation, but it has also been used to predict hourly diffuse values. The predictor normally used in studies is not precisely the global radiation but the 'hourly clearness index k_t, the ratio of hourly global horizontal radiation to hourly extraterrestrial radiation' (Reindl *et al.*, 1990). Orgill and Hollands (1977) and Erbs (1980) correlate the hourly diffuse radiation with k_t, but Iqbal (1980) extended the work of Bugler (1977) to develop a correlation with two predictors, k_t and the solar altitude. Skartveit and Olseth (1987) also use these two predictors in their correlations. Reindl *et al.* (1990) use stepwise regression to 'reduce a set of 28 potential predictor variables down to four significant predictors: the clearness index, solar altitude, ambient temperature and relative humidity'. A further reduction was made to two variables, k_t and the solar altitude, because the other two variables are not always readily available. Another possible reason was that some combinations of predictors may produce unreasonable values of the diffuse fraction, e.g. greater than 1.0 (Reindl *et al.*, 1990).

Weather data were collected from a station at Deakin University in Geelong, Victoria (latitude 38.09 S, longitude 144.34 E). There are several motivating factors for undertaking a study of this data. Spencer's (1982) adaptation of the Orgill and Hollands (1977) model was the only one developed with a significant number of stations with latitude less than 40°. Even with this model, a number of possible improvements could be envisaged. Could the addition of more predictors as found by Skartveit and Olseth (1987) or Reindl *et al.* (1990) enhance the predictability? Is the calculation of piecewise fits necessary or is one function adequate for all ranges of the predictors? Are the models for different time scales significantly different? In this paper we give preliminary results based on the Deakin weather station data which show that the inclusion of solar altitude as a predictor does not significantly improve the predictability. The model was constructed with a single function of k_t for all values of $k_t \leqslant 0.9$, beyond which the data are essentially random and the mean diffuse fraction in that range is the most reasonable predictor. We also demonstrate that the model for a 15 minute time interval is essentially the same as the model for an hourly time interval. There is also one improvement on previous work in that coefficients of determination are calculated in the comparison of models, whereas previous work has included only relative measures of fit.

A final investigation demonstrates that apparent solar time is a valuable second predictor. This variable has the advantage over solar altitude in that it reflects the asymmetry of meteorological processes about solar noon (Satyamurty and Lahiri, 1992; Zelenka, 1988; Bivona *et al.*, 1991).

DAY OF YEAR CALENDAR

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
JAN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
FEB	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60		
MAR	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
APR	91	92	93	49	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	
MAY	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151
JUN	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	
JUL	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212
AUG	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243
SEP	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	
OCT	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304
NOV	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	
DEC	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365

Figure 1. Days used for the study (the days in black were used for model construction and those in grey for validation).

2. DESCRIPTION OF DATA

Planning for a Deakin University weather station at the Woolstores camput began at the end of 1995 with the inception of the Building Environmental Performance Program in conjunction with the department of Building and Grounds. It was suggested that components of weather information could provide useful information when implemented with building control systems. Furthermore, the B.E.P.P. project was interested in developing on-site weather data sets for building thermal simulation. In situ experimental studies on daylighting control integration with artificial lighting, as well as other studies on shading devices, glazing systems, solar collectors all depend on some form of external weather information. Aside from experimental studies, a Website was initially established to report 15 minute updates of Geelong's weather to the public. The development of the Website prompted an educational purpose to report graphically aspects of weather information such as clearness index, solar position and instrumentation measurement.

The data for the modelling comprised 67 days taken from various seasons of the year. A sample of 88 days not employed in the model building was used for validating the model. These days are shown in Figure 1. A number of days were selected under the criterion of clear days ($k_t > 0.6$) to ascertain whether there was any significant difference in the fit for hourly and 15 minute values for those days as compared with non-clear days. For the calculation of extraterrestrial irradiance, the hour angle was taken to be the apparent local solar time halfway through the time interval.

Single year data values are also used for the locations of Adelaide, Alice Springs, Brisbane and Melbourne in the examination of extensions of the modelling procedure.

3. MODELLING THE DIFFUSE FRACTION

We focus on the hourly values first as this is the time interval used for other studies and this will allow a direct comparison of the models. Scatter diagrams were prepared for the diffuse fraction

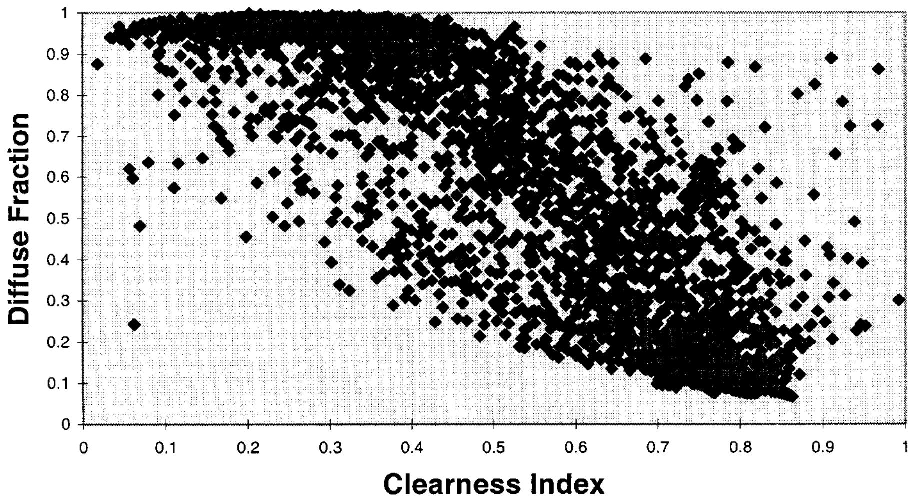

Figure 2. The diffuse fraction and clearness index for 15 minute data.

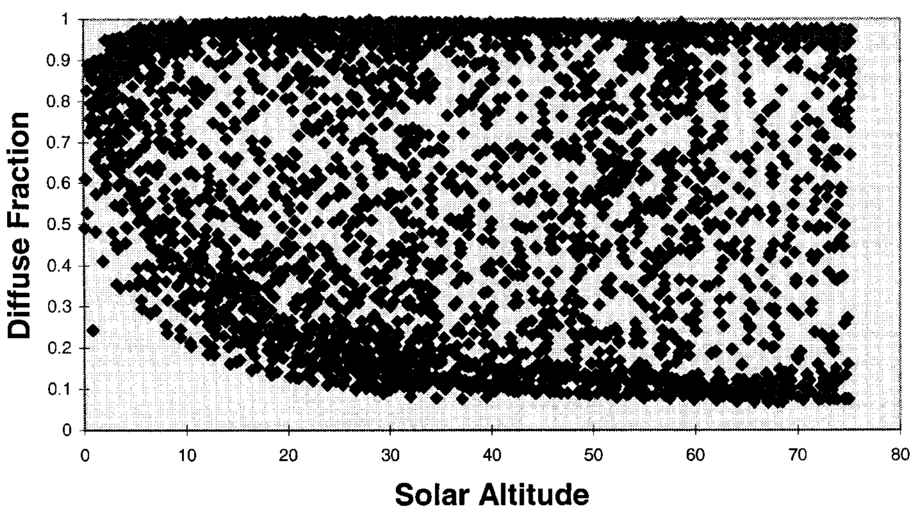

Figure 3. The diffuse fraction and solar altitude for 15 minute data.

versus the clearness index (Figure 2) and versus the solar altitude (Figure 3) for 15 minute data. Some dependence is apparent between the diffuse fraction and the clearness index, but no relationship is apparent between the diffuse fraction and the solar altitude. Figures 4 and 5 give similar scattergrams but for hourly data.

Before trying to construct a model for the connection between the diffuse fraction and clearness index, the points which are in the region defined by $\{(k_t, d)\colon k_t \leqslant 0.2, d \leqslant 0.7\}$ where d is the *diffuse fraction* in Figure 4, are examined to see if they can be regarded as outliers. The significant point is that the solar altitude for all points in this region is less than 5°. There is a case for eliminating readings for solar altitude angles of this magnitude. Skartveit and Olseth (1987)

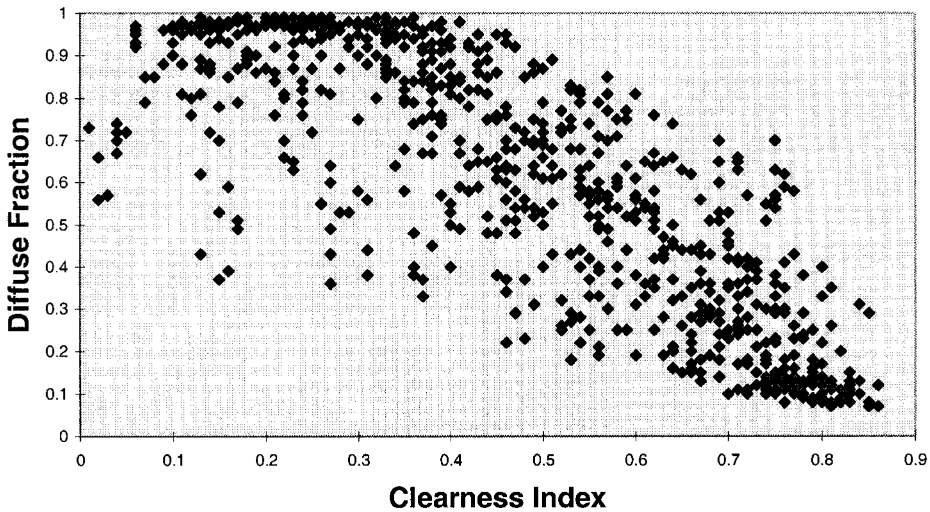

Figure 4. The diffuse fraction and clearness index for hourly data.

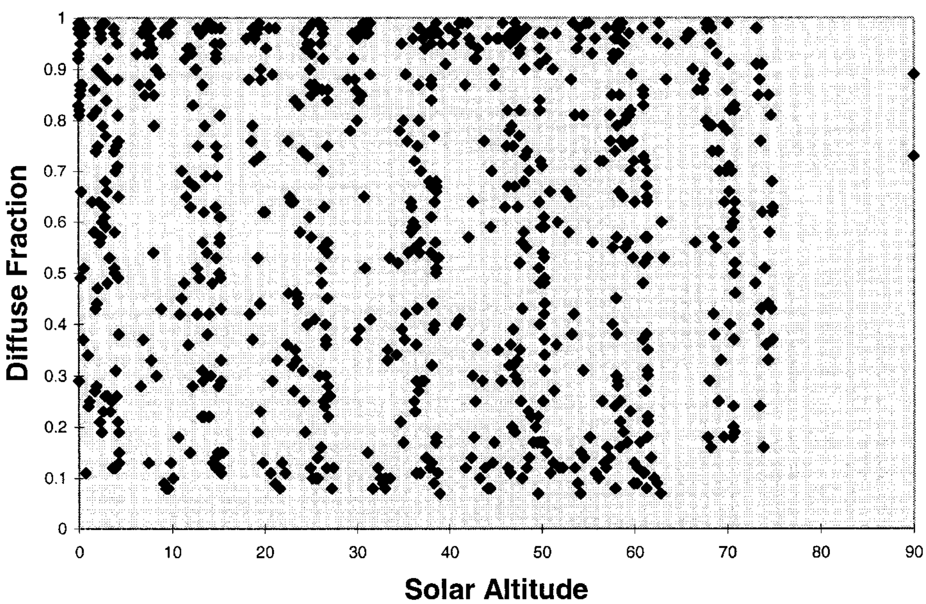

Figure 5. The diffuse fraction and solar altitude for hourly data.

neglect readings if the average solar altitude for the hour (they only analyse hourly data) is less than 10°. The calculations for the clearness index assume that the terrain surrounding the site is a perfect horizontal plane – in other words, the incoming solar radiation is unimpeded by topographical structures. If this is not the case then at low solar altitudes the k_t values would be questionable. This decision is further supported by the accuracy of the instrumentation at high zenith angles and its capability to register a correct value. The cosine correction factor varies among instrument manufacturers and independently within the instrument itself. Most manufacturers indicate that their instruments are unreliable below 10°. It should also be recognised that the solar intensity on a horizontal surface and its breakdown into direct and diffuse components at high zenith angles is relatively insignificant.

Note that the equation for extraterrestrial radiation includes the day of year and the solar zenith angle. These variables would inherently be considered in the development of any statistical model of the diffuse fraction. As a result the authors of this paper have found little significance when including it as a separate independent predictor for the diffuse fraction. The seasonal and location (latitude) differences were accounted for in the equation of solar zenith and the extraterrestrial component. It is accepted that seasonal and location differences can influence the diffuse fraction. It is also known that winter months exhibit clearer skies than summer months for most locations. The causes of such conditions are related to colder temperatures, moisture content and even a seasonal difference in air mass. Variables such as these (including solar zenith) affect the *turbidity* of the atmosphere (Robinson, 1966). As a result of the sky turbidity and its variation in contents of CO_2, H_2O, O_2, pollution and aerosols, a scattering of the direct beam component will occur. Scattering and variation of atmospheric turbidity occur not only on a seasonal but a daily basis. Horizon brightening during hours of sunrise and sunset is a typical result of beam scattering. Perhaps this is the foundation to further investigation of a diffuse fraction model which is inclusive of air mass, temperature, turbidity, etc. which will explain the variation in beam scattering throughout the day. The authors believe that the causes of a variable and reasons for its inclusion in a statistical model should be justified (not only by statistical significance).

Figures 6 and 7 give the scattergrams for diffuse fraction versus clearness index for 15 minute and hourly data, respectively, after the quality control has been undertaken. The task now is to try and find a model for the relationship between diffuse fraction and clearness index in each case. Reindl *et al.* (1990), Skartveit and Olseth (1987), Spencer (1982), and Orgill and Hollands

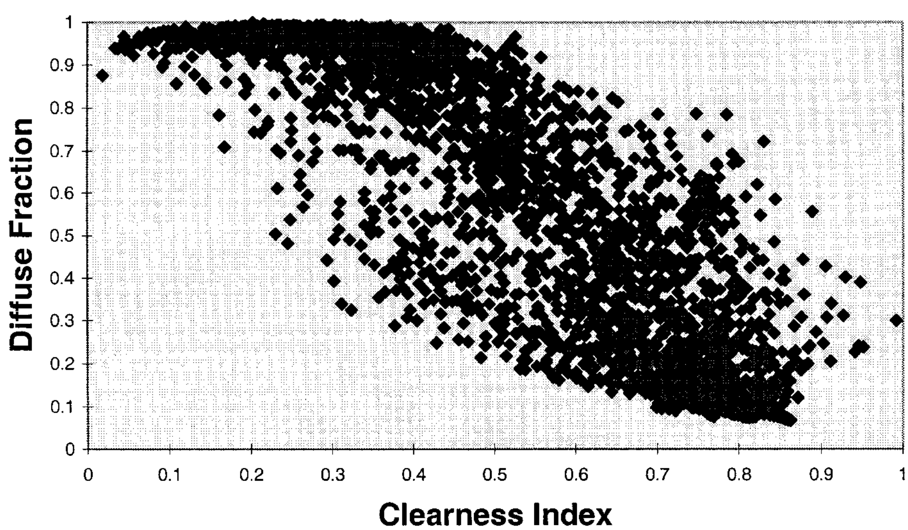

Figure 6. Diffuse fraction and clearness index for 15 minute data after quality control.

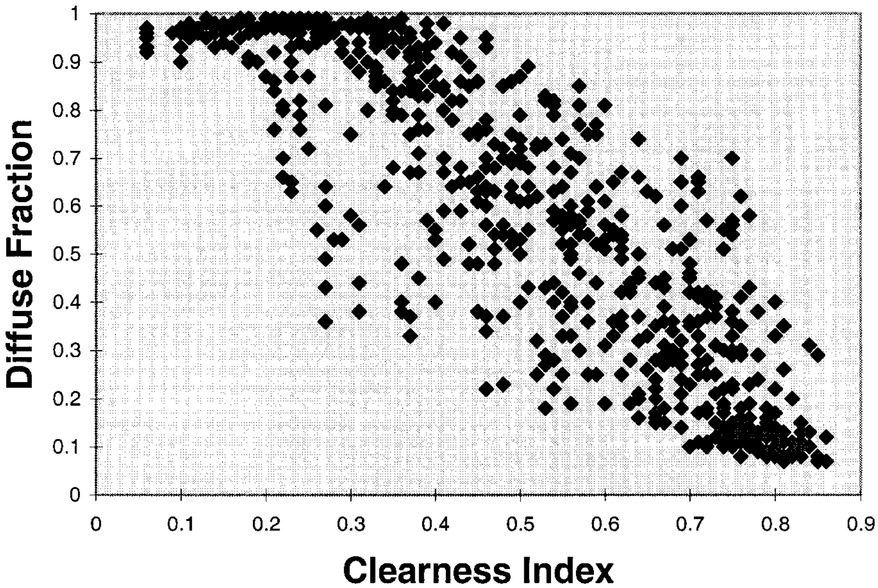

Figure 7. Diffuse fraction and clearness index for hourly data after quality control.

(1977) all split the data into regions defined for different values of clearness index, for instance $0 \leqslant k_t \leqslant 0.3$, $0.3 < k_t < 0.78$, $0.78 \leqslant k_t \leqslant 1$ before attempting any regression. The present authors attempted to construct a model for the whole range of clearness index since it appeared that a logistic function might fit the data adequately. The curve fitting was performed using Tablecurve™ and the fitted curves are given in Equations (1) and (2) for the 15 minute data and hourly data, respectively.

$$d = \frac{1}{1 + e^{8.645(k_t - 0.613)}} \tag{1}$$

$$d = \frac{1}{1 + e^{7.997(k_t - 0.586)}} \tag{2}$$

4. VALIDATION OF THE MODELS

4.1. Hourly model

Validation was undertaken via two means. Comparisons were made using statistical measures between the results of the fit using the present model and the Reindl *et al.* (1990) model. Reindl was chosen since there are extensive comparisons undertaken between this model and all the accepted alternatives. Predictions using the Reindl model were compared to data values for Geelong not used in the model construction. The inter-model comparison could only be performed using the hourly data since no correlations were performed by Reindl *et al.* for other

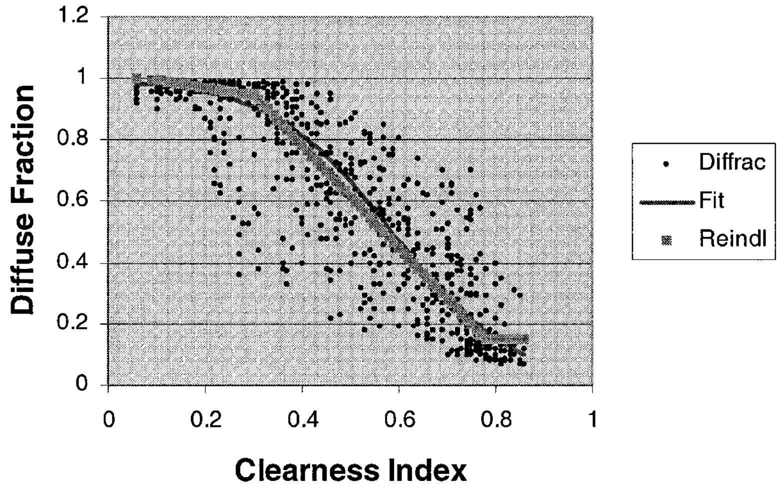

Figure 8. The model fit for hourly data.

time intervals. The fitted curves plus original data are depicted in Figure 8. As can be seen, the Reindl correlations are performed for three different intervals for k_t and are given in equation (3).

$$
\begin{aligned}
d &= 1.02 - 0.248k_t, && 0 \leqslant k_t \leqslant 0.3\\
d &= 1.45 - 1.67k_t, && 0.3 < k_t < 0.78\\
d &= 0.147, && 0.78 \leqslant k_t
\end{aligned}
\tag{3}
$$

Two statistical measures were used for comparison. The composite residual sum of squares (CRSS) was calculated, since Reindl *et al.* (1990) use this measure, and also the coefficient of determination R^2 was calculated. The results are given in Table I. As can be seen from the results, there is very little difference between the predictions of the two models. The only difference is the Reindl model requires a constraint since there are values for the diffuse fraction greater than unity calculated at times and these have to be changed to unity. More significantly, the present model does not require the clearness index interval to be split into three sub-intervals. This allows for greater flexibility for altering the relationship to cater for location differences.

It is possible that the addition of another predictor may enhance the predictability. Reindl *et al.* (1990) have a separate correlation where in each sub-interval there is a solar altitude depen-

Table I. Comparison of results using the Reindl and the Boland and Luther models.

	Reindl	Boland and Luther
CRSS	15.87	15.75
r^2	0.761	0.761

dence added via a $\sin\alpha$ term, where α is the solar altitude. Unfortunately, they changed the form of the original k_t dependence in that for the interval $k_t \geqslant 0.78$, they used the model $d = ak_t + b\sin(\alpha)$ rather than having it independent of k_t. Thus, no direct comparison can be made when the two models were applied to the current data. When the model including the solar altitude dependence was applied, the CRSS was 18.95 and the R^2 was 0.722. The most that can be said is that the addition of solar altitude as a predictor did not enhance the fit.

When the solar altitude was added as a predictor to the present model, the CRSS rose to 15.87 and the R^2 rose marginally to 0.762. The relationship was not found to be significant and coupled with only minor improvement to R^2 a reasonable conclusion is to omit it as a predictor. It should be noted that we also attempted to use the air mass as a possible predictor and found that there was no significant influence of this variable on the prediction. Thus, modelling using the one predictor is obviously preferable.

The coefficient of determination was calculated when predictions were made for hourly values for a sample of days which were not used in the model construction. For this validation test, $R^2 = 0.80$ which supports the model construction method.

4.2. *The 15 minute model*

For 15 minute data, no comparison was possible with the results using the Reindl *et al*. model since it was constructed for hourly data. However, during model construction the r^2 value was calculated to be 0.761. Examination of the similarity of the relationships (1) and (2) led us to assume that one relationship would probably suffice for both 15 minute data and hourly data. Accordingly, the relationship (2) for hourly data has been adopted for both. If this is tested, the r^2 value actually rises slightly to 0.767. When the model was used to predict the diffuse fraction for a sample of days not used in the model construction, once again a coefficient of determination was calculated. For this data, the r^2 was equal to 0.806. This result once again supports the model building methodology.

5. CLEAR DAYS

Another area of investigation was the question of whether there is a necessity for the construction of a separate model for either hourly or 15 minute data for clear days, i.e. those with $k_t > 0.6$. When the relationship is used to predict diffuse fraction values for the clear day data and subsequently r^2 values are calculated there is no significant difference between these values and the ones obtained for all days together. Intuitively, one might assume there might be less random scatter on clear days, but Figures 9 and 10 demonstrate that this is not the case. Thus, there would appear to be no need to construct a separate model for these days. It is still an open question as to whether this might be necessary for other locations – the fact that Geelong is a coastal location may influence this.

6. USING TIME OF DAY AS A PREDICTOR

The method of least squares was employed to fit the model with different parameters to some Adelaide data. The values of the parameters were only slightly changed, i.e. $\alpha = 6.952$ and

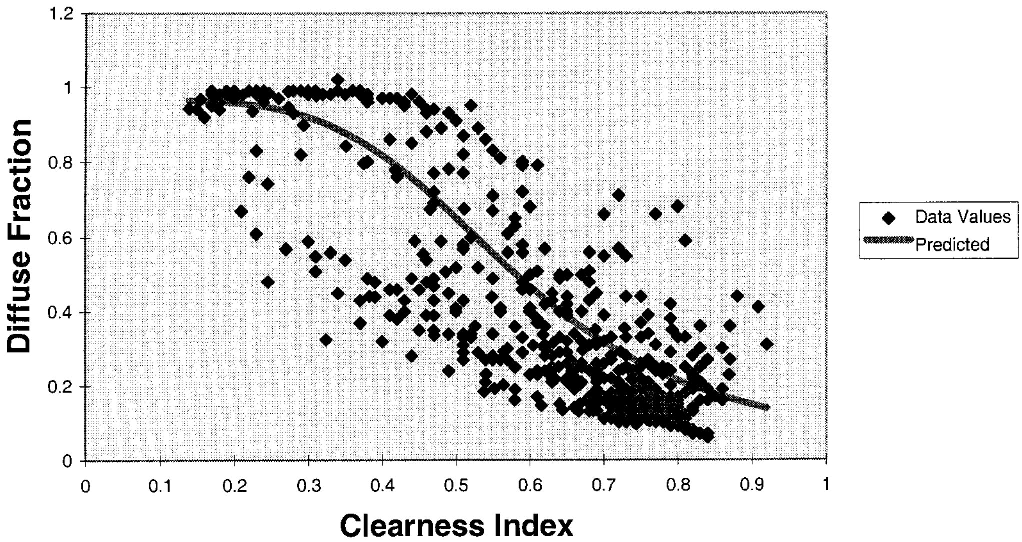

Figure 9. Data values and the predicted values for a sample of clear days (15 minute data).

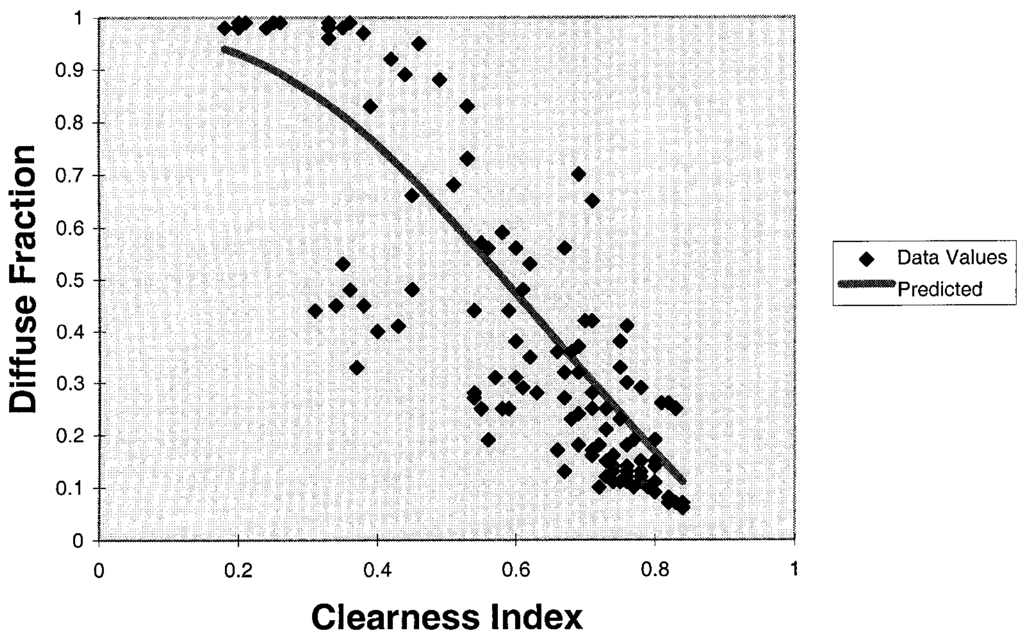

Figure 10. Data values and the predicted values for a sample of clear days (hourly data).

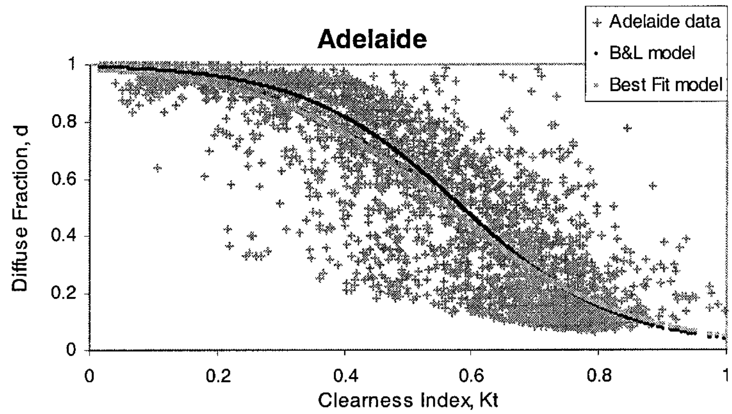

Figure 11. The Adelaide 1995 diffuse fraction plotted against the clearness index. The Boland and Luther model for Geelong and the best fit model for Adelaide are shown on the plot.

$\beta = 0.564$ with $R^2 = 0.760$. The coefficient of determination was slightly improved, but not sufficiently to warrant replacement of the Geelong model parameters. Figure 11 shows the Adelaide data, the Geelong model and the model fitted to the data using the method of least squares.

It is evident from Figure 11 that even though the coefficient of determination is relatively high, there remains a significant amount of scatter about the fitted curve (which is a feature common to all previously proposed models). As has previously been stated, numerous attempts have been made to add other predictors to explain the scatter. They have met with limited success and in particular we have found that for the locations we have studied, this strategy has met with no success. One noticeable feature of the scatter plot of the Adelaide data, evident in Figure 11, is the skewness of the residuals about the model. While investigating this, it was discovered that the skewness appeared dependent on the time of day. This led the authors to consider the possibility that the *time of day* might be an important predictor in the diffuse fraction model. There is considerable asymmetry about solar noon in terms of climatic factors, an aspect not considered by most other authors in the construction of diffuse fraction predictor models. Satyamurty and Lahiri (1992) point out this asymmetry in their work on similar diffuse fraction models, Zelenka (1988) presents work on monthly direct beam radiation which refers to the asymmetry about solar noon and Bivona *et al.* (1991) also allude to this phenomenon.

The inclusion of time of day, or more correctly apparent solar time (AST), in the model produced much better correlations. Again we shall use Adelaide to demonstrate the results. We retained the same parameters used for the Geelong data and added a linear time component to the model. Thus, the new model has the form

$$d = c + \frac{1-c}{1+\exp(\beta_0 + \beta_1 k_t + \beta_2 t)} \tag{4}$$

where $t = $ AST and β_0, β_1, β_2 and c are coefficients to be determined.

The coefficients were determined using optimisation techniques on Microsoft Excel and are

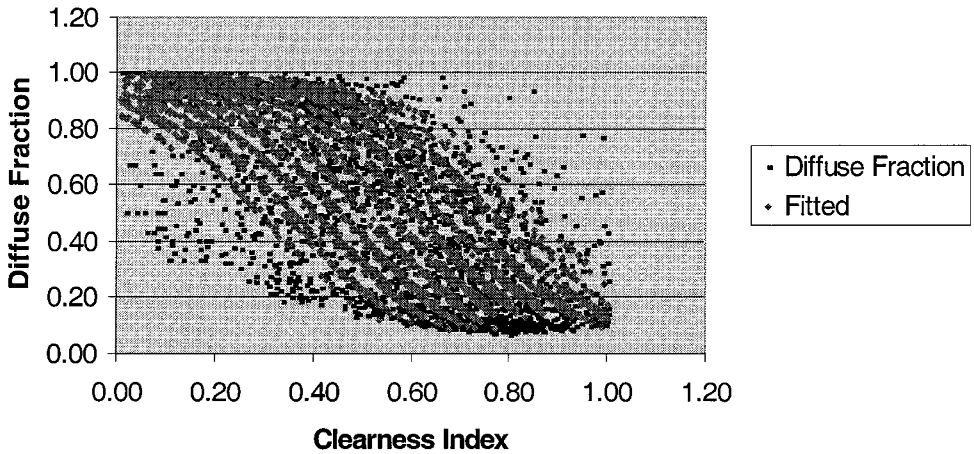

Figure 12. The time dependent model, Equation (2), for the Adelaide data.

$$\beta_0 = -8.769$$
$$\beta_1 = 7.325$$
$$\beta_2 = 0.377$$
$$c = -0.039$$

The coefficient of determination for this time dependent model is $R^2 = 0.798$, which is an improvement on the previous model. Significantly, the fitted values cover the variation in the diffuse fraction versus the clearness index in a fashion which depicts the difference in scatter over the day. Figure 12 shows this model for Adelaide. A partial *F*-test statistic was calculated to determine if the addition of the second predictor (time of day) is significant. This statistic is the difference in the residual sum of squares between the two models divided by the mean sum of squares for the second model, that with two predictors. If this statistic is sufficiently large, the addition of the second predictor is deemed to have resulted in significantly better predictive ability. The critical value for the test at the 1 per cent level of significance is 4.60, but the test statistic value was very much greater than this, allowing us to conclude that the second predictor is indeed significant. Similar results were obtained for all locations studied so far. Results of the addition of the time dependence for various locations appear in Table II. Also included is a comparison with results obtained using the Reindl *et al.* (1990) model.

7. CONCLUSION

A model has been constructed relating the diffuse fraction of solar radiation on a horizontal surface to the clearness index. A significant finding is that the same model can be used for both 15 minute and hourly data. A number of issues arise from this study. The model has to be extended to be able to be used for at least any Australian location and preferably any other location. In this regard, if there is any adjustment to be made for latitude, then since the model

Table II. Comparison of the present models with those of Reindl *et al*. Note that in some cases the R^2 value for two predictors in the Reindl models is lower than for one predictor. This is because they recalculated all coefficients when the second predictor was added to the model.

Location	R^2			
	Present paper models		Reindl	
	Time independent model	Time dependent model	One predictor	Two predictors
Adelaide	0.757	0.798	0.763	0.759
Alice Springs	0.354	0.649	0.437	0.440
Brisbane	0.698	0.781	0.570	0.564
Melbourne	0.742	0.834	0.756	0.758

consists of a single function over all values of k_t, there is the possibility of adjusting a single parameter to cater for this. Solar elevation, air mass and time of day were tested as possible extra predictors, but only time of day added to the predictive abilities of the model. This aspect would also have to be tested for other locations, especially since Reindl *et al*. (1990) have found a contribution from the solar altitude.

This research will be extended to other Australian locations, as well as locations in other parts of the world. In particular, it is to be established whether one can change the methodology slightly and perform the curvilinear regression for various values of the AST, but only for a relatively small number of locations. Ideally, each location would be representative of a climate type of the Koppen climate classification and the results could be used for any location of the climate type.

ACKNOWLEDGEMENT

We wish to thank SECV-International Solar Monitoring Network for the generous donation of the solar tracking station, so that the logging of solar direct, global and diffuse radiation could be established.

REFERENCES

Bivona S, Burton R, Leone C. 1991. Instantaneous distribution of global and diffuse radiation on horizontal surfaces. *Solar Energy* **46**(4):249–254.

Brunger AP. 1989. Application of an anisotropic sky model to the calculation of the solar radiation absorbed by a flat plate collector. *Proceedings Solar World Congress*, Kobe. Biennial Meeting International Solar Energy Society, Kobe, Japan, September.

Bugler JM. 1977. The determination of hourly insolation on an inclined plane using a diffuse irradiance model based on hourly measured global horizontal insolation. *Solar Energy* **19**: 477–491.

Erbs DG. 1980. *Methods for estimating the diffuse fraction of hourly, daily and monthly average global solar radiation*, MS Thesis, Mechanical Engineering, University of Wisconsin-Madison.

Iqbal M. 1980. Prediction of hourly diffuse solar radiation from measured hourly global solar radiation on a horizontal surface. *Solar Energy* **24**: 491–503.

Liu BYH, Jordan RC. 1960. The interrelationship and characteristic distribution of direct, diffuse and total solar radiation. *Solar Energy* **4**(3):1–19.

Lunde PJ. 1979. *Solar Thermal Engineering*. Wiley: New York.

Orgill JF, Hollands KGT. 1977. Correlation equation for hourly diffuse radiation on a horizontal surface. *Solar Energy* **19**:357–359.

Perez R, Ineichen P, Seals R, Michalsky J, Stewart R. 1990. Modeling daylight availability and irradiance components from direct and global irradiance. *Solar Energy* **44**(5):271–289.

Reindl DT, Beckman WA, Duffie JA. 1990. Diffuse fraction correlations. *Solar Energy* **45**(1):1–7.

Robinson N. 1966. *Solar Radiation*. Elsevier: Amsterdam.

Satyamurty VV, Lahiri PK. 1992. Estimation of symmetric and asymmetric hourly global and diffuse radiation from daily values. *Solar Energy* **48**(1):7–14.

Skartveit A, Olseth JA. 1987. A model for the diffuse fraction of hourly global radiation. *Solar Energy* **38**(4):271–274.

Spencer JW. 1982. A comparison of methods for estimating hourly diffuse solar radiation from global solar radiation. *Solar Energy* **29**(1):19–32.

Zelenka A. 1988. Asymmetrical analytically weighted R_b factors. *Solar Energy* **41**(5):405–415.

