g2o versus Toro: Cost Functions

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When comparing results from g2o [1] and Toro [2] it is important to realize that the two solvers use slightly different cost functions. The optimization problem solved in Toro is:

$$\min_{\boldsymbol{R}_i, \boldsymbol{t}_i} \sum_{(i,j)\in\mathcal{E}} \left\| \mathrm{Log}\left(\bar{\boldsymbol{R}}_{ij}^{\mathsf{T}} \boldsymbol{R}_i^{\mathsf{T}} \boldsymbol{R}_j \right) \right\|_{\Omega_{ij}^r}^2 + \left\| \bar{\boldsymbol{t}}_{ij} - \boldsymbol{R}_i^{\mathsf{T}} (\boldsymbol{t}_j - \boldsymbol{t}_i) \right\|_{\Omega_{ij}^t}^2 \tag{1}$$

where $\boldsymbol{R}_i$ and $\boldsymbol{t}_i$ are the (to-be-computed) rotation and translation at node i (for $i = 1, \ldots, n$), $\bar{\boldsymbol{R}}_{ij}^{\mathsf{T}}$ and $\bar{\boldsymbol{t}}_{ij}$ are the relative rotation and translation measurements between node i and j, Ω_{ij}^r is the rotation measurement information matrix, Ω_{ij}^t is the translation measurement information matrix, and $\mathcal{E}$ is the set of node pairs (i, j) for which a measurement is available. The symbol $\mathrm{Log}(\cdot)$ denotes the logarithm map for the rotation group, which, roughly speaking, converts a rotation to a vector (in 3D) or a scalar angle (in 2D).

The error definition in g2o is similar, with a small difference in the second summand:

$$\min_{\boldsymbol{R}_i, \boldsymbol{t}_i} \sum_{(i,j)\in\mathcal{E}} \left\| \mathrm{Log}\left(\bar{\boldsymbol{R}}_{ij}^{\mathsf{T}} \boldsymbol{R}_i^{\mathsf{T}} \boldsymbol{R}_j \right) \right\|_{\Omega_{ij}^r}^2 + \left\| \bar{\boldsymbol{R}}_{ij}^{\mathsf{T}} (\bar{\boldsymbol{t}}_{ij} - \boldsymbol{R}_i^{\mathsf{T}} (\boldsymbol{t}_j - \boldsymbol{t}_i)) \right\|_{\Omega_{ij}^t}^2 \tag{2}$$

Developing the Mahalanobis norms on the right, we get:

$$\min_{\boldsymbol{R}_i, \boldsymbol{t}_i} \sum_{(i,j)\in\mathcal{E}} \left\| \mathrm{Log}\left(\bar{\boldsymbol{R}}_{ij}^{\mathsf{T}} \boldsymbol{R}_i^{\mathsf{T}} \boldsymbol{R}_j \right) \right\|_{\Omega_{ij}^r}^2 + \left(\bar{\boldsymbol{R}}_{ij}^{\mathsf{T}} (\bar{\boldsymbol{t}}_{ij} - \boldsymbol{R}_i^{\mathsf{T}} (\boldsymbol{t}_j - \boldsymbol{t}_i)) \right)^{\mathsf{T}} \Omega_{ij}^t \left(\bar{\boldsymbol{R}}_{ij}^{\mathsf{T}} (\bar{\boldsymbol{t}}_{ij} - \boldsymbol{R}_i^{\mathsf{T}} (\boldsymbol{t}_j - \boldsymbol{t}_i)) \right)$$

$$\Updownarrow$$

$$\min_{\boldsymbol{R}_i, \boldsymbol{t}_i} \sum_{(i,j)\in\mathcal{E}} \left\| \mathrm{Log}\left(\bar{\boldsymbol{R}}_{ij}^{\mathsf{T}} \boldsymbol{R}_i^{\mathsf{T}} \boldsymbol{R}_j \right) \right\|_{\Omega_{ij}^r}^2 + \left(\bar{\boldsymbol{t}}_{ij} - \boldsymbol{R}_i^{\mathsf{T}} (\boldsymbol{t}_j - \boldsymbol{t}_i) \right)^{\mathsf{T}} \bar{\boldsymbol{R}}_{ij} \Omega_{ij}^t \bar{\boldsymbol{R}}_{ij}^{\mathsf{T}} \left(\bar{\boldsymbol{t}}_{ij} - \boldsymbol{R}_i^{\mathsf{T}} (\boldsymbol{t}_j - \boldsymbol{t}_i) \right)$$

$$\Updownarrow$$

$$\min_{\boldsymbol{R}_i, \boldsymbol{t}_i} \sum_{(i,j)\in\mathcal{E}} \left\| \mathrm{Log}\left(\bar{\boldsymbol{R}}_{ij}^{\mathsf{T}} \boldsymbol{R}_i^{\mathsf{T}} \boldsymbol{R}_j \right) \right\|_{\Omega_{ij}^r}^2 + \left\| \bar{\boldsymbol{t}}_{ij} - \boldsymbol{R}_i^{\mathsf{T}} (\boldsymbol{t}_j - \boldsymbol{t}_i) \right\|_{\bar{\boldsymbol{R}}_{ij} \Omega_{ij}^t \bar{\boldsymbol{R}}_{ij}^{\mathsf{T}}}^2 \tag{3}$$

which also shows that for a fair comparison between Toro and g2o we should use $\bar{\boldsymbol{R}}_{ij} \Omega_{ij}^t \bar{\boldsymbol{R}}_{ij}^{\mathsf{T}}$ as information matrices in g2o.

References

[1] R. Kümmerle, G. Grisetti, H. Strasdat, K. Konolige, and W. Burgard, "g2o: A general framework for graph optimization," in *Proc. of the IEEE Int. Conf. on Robotics and Automation (ICRA)*, Shanghai, China, May 2011.

[2] G. Grisetti, C. Stachniss, and W. Burgard, "Non-linear constraint network optimization for efficient map learning," *Trans. on Intelligent Transportation systems*, vol. 10, no. 3, pp. 428–439, 2009.

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