Consider the following game. We start with a population of 3 types of players: $K_{initial}$ killers, $R_{initial}$ revengers, and $X_{initial}$ peacers. Each player starts out not seeing anyone else, and has a probability p of detecting another player each turn. Once a player detects another player, they see them for the rest of the game. Players collect a certain amount of utility U at the end of each turn they are alive, and the game lasts for T turns.

Each type of player uses a different strategy. Killers will immediately bomb any other player they detect, killing them. Revengers, if killed, will send a counter-bomb to anyone that bombs them, so long as they have detected the bomber. Peacers never bomb anyone. Bombs B have a utility cost of C.

Given the above game, what is the expected payoff for the various strategies?

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Expected payoff for a killer E_K:

$$E_K = L_K U - B_K C$$

Expected payoff for a revenger E_R :

$$E_R = L_R U - B_R C$$

Expected payoff for a peacer E_X:

$$E_X = L_X U$$

Expected lifetime of a killer L_K:

$$L_K = T * P(\text{K survives turn } i) + \sum_{i=1}^{T} i * P(\text{K dies on turn } i) * P(\text{K survives turn } i-1)$$

$$P(\text{K survives turn } i-1) = \begin{cases} \prod_{j=1}^{i-1} 1 - P(\text{K dies on turn } j) & \text{if } i > 1; \\ 1 & \text{if } i = 1. \end{cases}$$

$$P(\text{K dies on turn } i) = 1 - [1 - P(\text{K kills K on turn } i)] * [1 - P(\text{R kills K on turn } i)]$$

$$P(\text{K kills K on turn } i) = 1 - (1-p)^{K_i - 1}$$

$$P(\text{R kills K on turn } i) = 1 - [1 - p * P(\text{K didn't see R by turn } i-1) * P(\text{R sees K by turn } i)]^{R_i}$$

$$P(\text{K didn't see R by turn } i-1) = (1-p)^{i-1}$$

$$P(\text{R sees K by turn } i) = 1 - (1-p)^i$$

Expected lifetime of a revenger L_B:

$$L_R = T * P(\text{R survives turn } i) + \sum_{i=1}^{T} i * P(\text{R dies on turn i}) * P(\text{R survives turn } i-1)$$

$$P(\text{R dies on turn } i) = 1 - (1-p)^{K_i}$$

$$P(\text{R survives turn } i-1) = \begin{cases} \prod_{j=1}^{i-1} (1-p)^{K_j} & \text{if } i > 1; \\ 1 & \text{if } i = 1. \end{cases}$$

Expected lifetime of a peacer L_X:

$$L_X = T * P(\text{X survives turn } i) + \sum_{i=1}^{T} i * P(\text{X dies on turn i}) * P(\text{X survives turn } i-1)$$

$$P(\text{X dies on turn } i) = 1 - (1-p)^{K_i}$$

$$P(\text{X survives turn } i-1) = \begin{cases} \prod_{j=1}^{i-1}(1-p)^{K_j} & \text{if } i > 1; \\ 1 & \text{if } i = 1. \end{cases}$$

Expected number of bombs a killer sends B_K:

$$B_K = \sum_{i=1}^{T} p(K_i - 1 + R_i + X_i) * P(\text{K survives turn } i-1)$$

Expected number of bombs a revenger sends B_R:

$$B_R = \sum_{i=1}^{T} (1 - [1 - p * P(\text{R sees K by turn i})]^{K_i}) * P(\text{R survives turn } i-1)$$

Expected population of killers at turn i K_i:

$$K_i = K_{i-1} - \text{Deaths}_{K_{i-1}}$$

$$K_0 = K_{initial}(\text{User Defined})$$

$$\text{Deaths}_{K_i} = K_i * [1 - (1 - P(\text{K kills K on turn } i)) * (1 - P(\text{R kills K on turn } i))]$$

Expected population of revengers at turn i R_i:

$$R_i = R_{i-1} - \text{Deaths}_{R_{i-1}}$$

$$R_0 = R_{initial}(\text{User Defined})$$

$$\text{Deaths}_{R_i} = R_i * P(\text{R dies on turn } i)$$