Probabilistic Mechanics of Quasibrittle Structures: Strength, Lifetime and Scaling

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Abstract: The size effect on structural strength and its statistical distribution is a complex problem for quasibrittle materials because their failure behavior transits from quasi-plastic at small sizes to brittle at large sizes. These are heterogeneous materials with brittle constituents in which the size of inhomogeneinty, or representative volume element (RVE), is not negligible compared to the structure size. Aside from concrete, the archetypical example, they include fiber composites, coarse-grained ceramics, rocks, sea ice, snow slabs, wood, bone, foam, stiff soil, dry snow, masonry, carton, etc., and on the micro- or nano-scale, all brittle materials become quasibritle. Since the break probability is known exactly only for interatomic bonds (being equal to frequency), Kramers'rule of transition rate theory is applied to nano-crack jumps. Based on proving the rules of multiscale transition of tail probabilities of break to material scale, the probability distribution of strength of one macro-scale representative volume element (RVE) is shown to have a Weibullian tail, calibrated to reach to probability 0.001, the rest being Gaussian. On the structure scale, only Type 1 failure is considered, i.e., the structure fails as soon as the first RVE fails. Hence the weakest-link model applies on the structure scale. But, crucially, the number of links is finite, because of non-negligible RVE. For increasing structure size, the Weibullian portion gradually spreads into the Gaussian core. Only in the infinite size limit the distribution becomes purely Weibull, but, importantly, with a zero threshold. Based on an atomistic derivation of the power law for subcritical macro-crack growth, a similar Gauss-Weibull transition is shown to apply to structure lifetime. The theory is then extended to the size dependence of Paris law and Basquin law for fatigue fracture, to statistics of fatigue lifetime, and to residual strength after a period of preload. The theory is shown to match the existing experimental results on the monotonic strength, residual strength after preload, static and fatigue crack growth rates, and static and fatigue lifetimes, including their distributions and size effects on the distributions. There are three essential consequences: 1) The safety factors must depend on structure size and shape; 2) To predict the pdf of strength, the size effect tests of mean strength suffice; 3) To predict the static and fatigue lifetimes, it suffices to add tests of initial subcritical crack growth rate. An interesting mathematical analogy predicting the lifetime of new nano-scale high-*k* dielectrics is also pointed out. Finally, an extension to failures after large stable crack growth is outlined and various practical implications are discussed.

Bio-Sketch: Born and educated in Prague (Ph.D. 1963), Bažant joined Northwestern in 1969, where he has been W.P. Murphy Professor since 1990 and simultaneously McCormick Institute Professor since 2002, and Director of Center for Geomaterials (1981-87). He was inducted to NAS, NAE, Am. Acad. of Arts & Sci., Royal Soc. London; to the academies of Italy (lincei), Austria, Spain, Czech Rep. and Lombardy; to Academia Europaea, Eur. Acad. of Sci. & Arts. Honorary Member of: ASCE, ASME, ACI, RILEM; received 7 honorary doctorates (Prague, Karlsruhe, Colorado, Milan, Lyon, Vienna, Ohio State); Austrian Cross of Honor for Science and Art 1st Class; ASME Timoshenko, Nadai and Warner Medals; ASCE von Karman, Newmark, Biot, Mindlin and Croes Medals and Lifetime Achievement Award; SES Prager Medal; RILEM L'Hermite Medal; Exner Medal (Austria); Torroja Medal (Madrid); etc. He authored seven books: Scaling of Structural Strength, Inelastic Analysis, Fracture & Size Effect, Stability of Structures, Concrete at High Temperatures, Concrete Creep and Probabilistic Quasibrittle Srength. H-index: 112, citations: 54,000 (on Google Dec.2016, incl. self-cit.), i10 index: 550. In 2015, ASCE established ZP Bažant Medal for Failure and Damage Prevention. He is one of the original top 100 ISI Highly Cited Scientists in Engrg. (www.ISIhighlycited.com). His 1959 mass-produced patent of safety ski binding is exhibited in New England Ski Museum.
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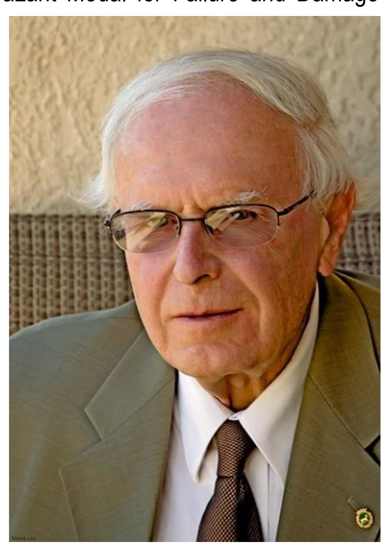

Selected References:

Z. P. Bažant and J.-L. Le (2017) Probabilistic Mechanics of Quasibrittle Structures: Strength, Lifetime and Size Effect, Cambridge U P

Bažant, Z.P. (2004). "Scaling theory for quasibrittle structural failure." Proc., Nat. Acad. of Sciences 101 (37), 14000—14007.

Bažant, Z.P., and Pang, S.-D. (2006). "Mechanics based statistics of failure risk of quasibrittle structures and size effect on safety factors." Proc. of the National Academy of Sciences 103(25), 9434-9439.

Bažant, Z.P., Le, J.-L., and Bazant, M.Z. (2008). "Scaling of strength and lifetime distributions based on atomistic fracture mechanics." Proc. of the Nat. Academy of Sciences, 106 (28), 11484-11489.

Le, J.-L., Bažant, Z.P., and Bazant, M.Z. (2011). "Unified Nano-Mechanics Based Probabilistic Theory of Quasibrittle and Brittle Structures. I. Strength, static crack growth, lifetime and scaling." J. of the Mechanics and Physics of Solids 59, 1291—1321.

Le, Jia-Liang, and Bažant, Z.P. (2014). "Finite weakest-link model of lifetime distribution of quasibrittle structures under fatigue loading." Mathematics and Mechanics of Solids 19(1), 56—70.