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Physics Assignment

7. The wave function for a quantum particle is given by $\psi(x) = Ax$ between $x = 0$ and $x = 1$ and $\psi(x) = 0$ elsewhere. Find (a) The value of the normalisation constant A (b) The probability that the particle will be found between $x = 0.3$ and $x = 0.4$ and (c) the expectation value of the particle's position.

Ans $\psi(x) = \cancel{A(x)}\, Ax$

Using normalisation condition,

$$\int_0^1 (\psi)^2_{(x)}\, dx = 1$$

$$= \int_0^1 A^2 x^2\, dx = 1 \qquad \cancel{\frac{A^2 x^3}{3} = 1}$$

$$\frac{A^2}{3} = 1 \qquad \Rightarrow A = \sqrt{3}$$

The probability of finding the electron is

$$\int_{0.3}^{0.4} A^2 x^2\, dx = \int_{0.3}^{0.4} 3x^2\, dx$$

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0.037

The expectation value

$$\langle x\rangle = \int_0^1 \Psi(x)(x)\,\Psi^*(x)\,dx$$

$$= \int_0^1 A^2 x^3\,dx = 3\left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{3}{4}$$

(8) A proton confined in an infinitely high ~~wall~~ square well of length 10.0 fm, a typical nuclear diameter. Assuming the proton makes a transition from the $n=2$ state to the ground state, calculate

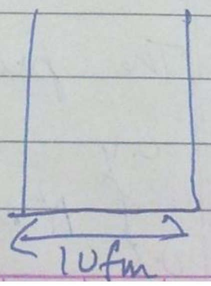
a) The energy

b) The wavelength of the emmitted photon

c) ~~The~~ Identify the region of the electromagnetic spectrum to which this wavelength belongs.

Length $= 10\,\text{fm}$

$= 10\times10^{-15}\,\text{m}$



In 1d system

$$\text{Energy} = \frac{n^2h^2}{8mL^2}$$

$$\frac{h^2}{8mL^2} = \frac{(6.626\times10^{-34})^2}{8\times 1.67\times10^{-27}\times(10\times10^{-15})^2}$$

$$= 3.2862\times10^{-13}\,J$$

$\therefore$ Energy at $n=2$ state $= 3.2862\times10^{-13}\times2^2\,J$

Energy at $n=1$ state $= 3.2862\times10^{-13}\times1^2\,J$

$\therefore$ Energy radiated $= 9.8587\times10^{-13}\,J$

$$\lambda = \frac{hc}{E} = \frac{6.626\times10^{-34}\times3\times10^{8}}{9.8587\times10^{-13}}$$

$$= \cancel{2.016\times10^{-13}\,m}$$

$$2.016\times10^{-13}\,m$$

$\Rightarrow$ This corresponds to gamma rays.

PAGE NO.
DATE: / /

(9) An electron has a kinetic energy of 12.0eV. The electron is incident upon a rectangular barrier of height 20.0eV and thickness 1.0nm. By what factor would the electron's probability of ~~tunneling~~ tunneling through the barrier increase assuming that the e⁻s absorb all the energy of a photon with wavelength 546nm.

Ans In case I

$U = 20eV$

$K = 12eV$

$$c = \frac{2\pi}{h}\sqrt{2m(U-K)}$$

$$= \frac{2\times\pi}{6.626\times10^{-34}}\sqrt{2\times9.11\times10^{-31}\times8\times1.6\times10^{-19}}$$

$$= 1.448\times10^{10}$$

$$\therefore \text{Transmissibility} = e^{-2cl}$$

$$= e^{-2\times c\times10^{-9}}$$

$$= 2.647\times10^{-13}$$

When the electron absorbs all the ~~ener~~ energy;

its new energy becomes

$$\left(12 + \frac{6.626\times10^{-34}\times3\times10^{8}}{546\times10^{-9}\times1.6\times10^{-19}}\right) eV$$

$$= 14.274 eV$$

$$\therefore c = \frac{2\pi}{h}\sqrt{2m\times5.726\times e} = 1.225\times10^{10}$$

$$T' = e^{-2cl}$$

$$= e^{-2\times1.225\times10^{10}\times10^{-9}}$$

$$= 2.2897\times10^{-11}$$

$$\Rightarrow \text{Ratio} = \frac{T'}{T} = \frac{2.2897\times10^{-11}}{2.67\times10^{-13}} = 86.47$$

(10) A quantum simple harmonic oscillator consists of an electron bound by a restoring force proportional to its position relative to a certain equilibrium point. The proportionality constant is 8.99 N/m. What is the longest wavelength that oscillate + excite the oscillator

Ans $F = -kx$

$k = 8.99\, N/m$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\frac{1}{\lambda} = \frac{1}{2\pi c}\sqrt{\frac{k}{m}}$$

$$= \frac{1}{2\pi \times 3\times10^{8}}\sqrt{\frac{8.99}{9.1\times10^{-31}}}$$

$$\Rightarrow \lambda = 600\,nm$$

∴ The maximum wavelength corresponding to minimum energy to excite one in the oscillator is 600 nm.

(11) An atom in an excited state 1.80eV above the ground state remains in that state 2.00μs before moving to the ground state. Find a) The frequency (b) the wavelength of the emitted photon. (C) Approximate uncertainty in energy of the photon.

Ans $hf = 1.8 \times 1.6 \times 10^{-19}\ J$

(a) $f = \dfrac{1.8 \times 1.6 \times 10^{-19}}{6.626 \times 10^{-34}} = 4.34 \times 10^{14}\ Hz$

(b) $\lambda = \dfrac{c}{f} = \dfrac{3 \times 10^{8}}{4.34 \times 10^{14}} = 690.208\ nm$

(C) $\Delta E \Delta T \geq \dfrac{h}{4\pi}$

$\Delta E \geq \dfrac{h}{4\pi \Delta t} = \dfrac{6.626 \times 10^{-34}}{4\pi \times 2 \times 10^{-6}}$

$\Delta E \geq 2.636 \times 10^{-29}\ J$

$\Delta E \geq 1.65 \times 10^{-10}\ eV$

(12) A hydrogen atom is in its second excited state corresponding to n=3. Find (a) the ~~ele~~ radius of the electron's bohr orbit and (b) the ~~de~~ de Brogile wavelength of the electron in this orbit.

Ans ~~(a)~~ radius $= \cancel{0.6}\ 0.529 \frac{n^2}{Z} A^\circ$

$$= 0.529 \times 9 = 4.761\ \text{Å}$$

$$mvr = \frac{nh}{2\pi} \Rightarrow mv = \frac{nh}{2\pi r} = P$$

$$\lambda = \frac{h}{P} \Rightarrow \frac{2\pi r}{n} = \frac{2\pi \times 4.761 \times 10^{-10}}{3}$$

$$= 9.971\ \text{Å}$$

(13) A hydrogen atom is in its fifth excited state, with principal quantum number 6. The atom emits a photon with a wavelength of 1090nm. Determine the maximum possible orbital angular momentum of the electron after emission.

Ans $E = \frac{-13.6 Z^2}{n^2}$

$\Rightarrow \frac{-13.6 \times 1^2}{36}$

$= -0.37 eV$

Energy of emmited photon

$= \left(\frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1090 \times 10^{-9} \times 1.6 \times 10^{-19}} \right) eV$

$= 1.139 eV$

∴ New energy of electron

$x + 1.139 = -0.37$

$x = -0.37 - 1.139$

$= -1.509 eV$

$\frac{1.509}{13.6} = \frac{1}{n^2}$

$\Rightarrow \cancel{n=6}$ $\boxed{n=3}$

~~Orbital angular momentum~~

~~$= \frac{nh}{2\pi} = \frac{3h}{2\pi}$~~

~~$= 3.16 \times 10^{-34}$ J/s~~

$n = 3$

$\Rightarrow l = 0, 1, 2 \Rightarrow$ maximum orbital angular momentum $= \sqrt{2(2+1)}\ \hbar$

$= \sqrt{6}\ \hbar$

14) The K series of the discrete x-rays spectrum of tungsten contains wavelengths of 0.0185 nm, 0.0209 nm and 0.0215 nm. The K shell ionisation energy is 69.5 keV. a) Determine Ionisation energies of the L, M and N shell (b) Draw a diagram of the transition

$E_{K shell} = 69.5$ keV

for $\lambda = 0.0185$ nm

$\Delta E = \frac{hc}{\lambda} = 67.135$ keV

$\therefore E_1 = 2.365\,keV$

for 0.0209 nm, $\Delta E = 59.42\,keV$

$\therefore E_2 = 10.08\,keV$

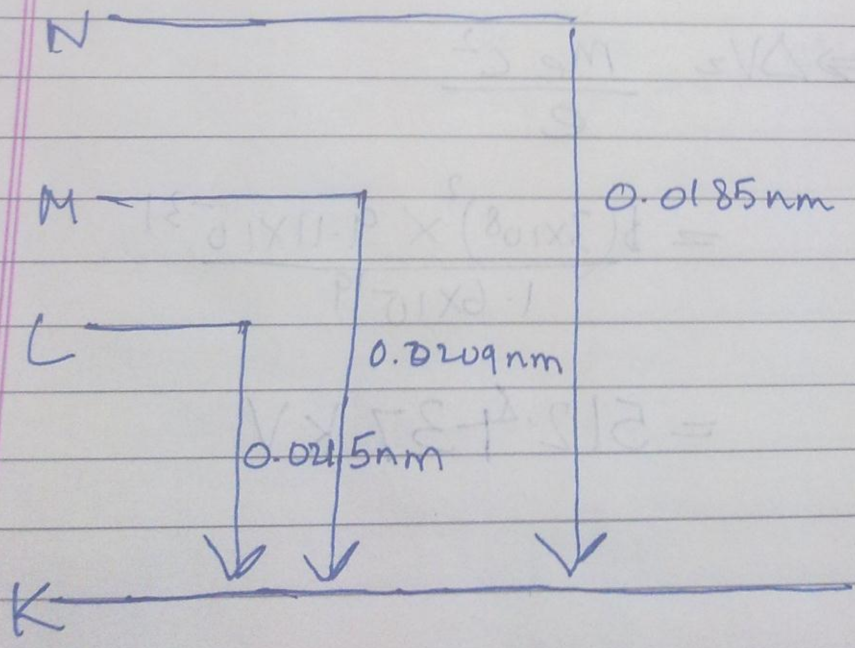
for 0.0215 nm $\Delta E = 57.76\,keV$

$\therefore E_3 = 11.75\,keV$

So for L shell ionisation energy $= 11.75\,keV$

" " M shell " " $= 10.08\,keV$

" " N shell " " $= 2.365\,keV$



(15) Find the minimum potential difference that must be applied to an x-ray tube to produce X-rays with a wavelength equal to the compton wavelength

Ans $\lambda_c = \frac{h}{m_e c}$

$e\Delta V = \frac{hc}{\lambda_c}$

$e\Delta V = \frac{hc}{\frac{h}{m_e c}}$

$\Rightarrow \Delta V = \frac{m_e c^2}{e}$

$= \frac{(3 \times 10^8)^2 \times 9.11 \times 10^{-31}}{1.6 \times 10^{-19}}$

$= 512.437 \text{ kV}$

(16) A hypothetical atom has energy evenly spaced by 1.2eV in energy. For a temperature of ~~200K~~ 2000K, calculate the ratio of the number of atoms in the 13th excited state to the number in the 11th excited state.

$\Delta E = E_{13} - E_{11}$

$= E_{13} - E_{12} + E_{12} - E_{11}$

$= 1.2 + 1.2 = 2.4 eV$

$= 3.84 \times 10^{-19} J$

We know that

$$\frac{N_2}{N_1} = B e^{-\left(\frac{E_2 - E_1}{kT}\right)}$$

the ratio $= 9.13 \times 10^{-7}$

Hence the ratio of no. of atoms in the 13th excited state to the no. of atoms in the 11th excited state is 9.13×10^{-7}

PAGE NO.
DATE: / /

(17) A N D:YAG laser used in eye surgery emits a 3mJ pulse in 1.00ns, focused to a spot 30.0um in diameter on the retina. (a) Find the power per unit area at the retina. (b) What energy is delivered by the pulse to an area of molecular size, taken as a circular area 0.60nm in diameter.

Ans Power $= \frac{E}{A}$

$$= \frac{3\times10^{-3}}{\pi\times(30\times10^{-6})^2}\times 4$$

$$= 4.224\times10^{6}\ W/m^2$$

(b) Energy delivered by the pulse

$$\text{Power} = \frac{\text{Energy}}{\text{time}}$$

$$\text{Power/area} = \frac{\text{Energy}}{\text{time}\times\text{area}}$$

= Intensity

$$= \frac{3\times10^{-3}}{10^{-9}\times\pi\times\left(\frac{30\times10^{-6}}{4}\right)^2}$$

$$= 4.244\times10^{15}\ W/m^2$$

(b) ~~Energy => P×t~~

(b) Energy delivered by the pulse
= ~~P × area × time~~ Intensity × Area × time

$$= \frac{4.244\times10^{5}\times\pi\times(0.60\times10^{-9})^{2}}{4}\times10^{-9}$$

$$= 1.19\times10^{-12}\,\text{J}$$