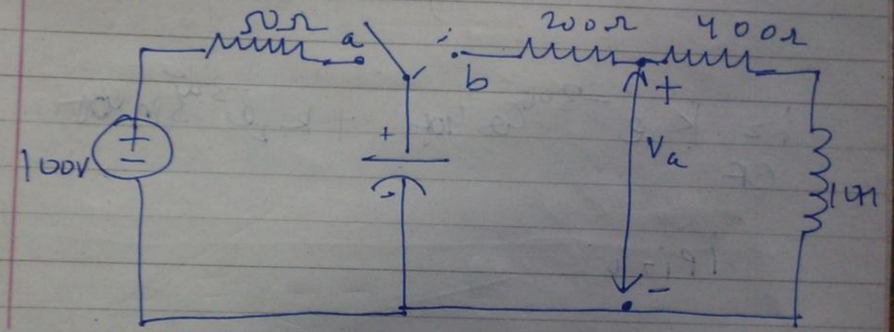
PAGE:
DATE: / /

III Semester Electrical circuit Analysis

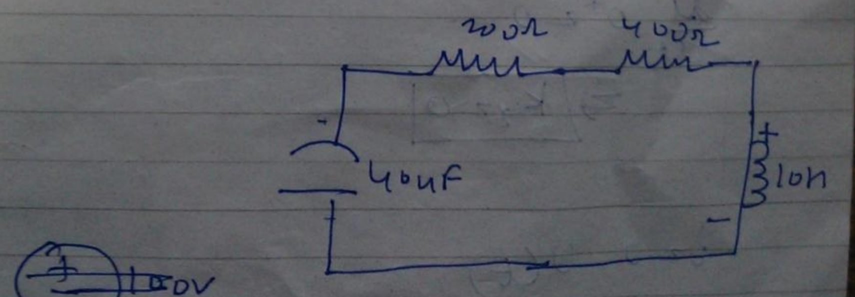
① The switch in the circuit shown in fig 1 has been in position 'a' for a long time. At t=0 the switch is moved instantaneously to position 'b'. Find V_a and dV_a/dt.



$V_C(0^-) = 100V = V_C(0^+) = 100$

$i_L(0^-) = i_L(0^+) = 0$.

$t > 0$



$\frac{1}{C}\int i\,dt + 10\frac{di}{dt} + 600i = 0$

DATE : / /

$$\frac{di_L}{dt}(0^+) = 0$$

$$2500\, i_s + ~~10~~\, 600 \frac{di}{dt} + 10 \frac{d^2 y}{dt^2} = 0$$

$$40p^2 + 600p + \frac{10^6}{40} = 0$$

$$i_{CF} = K_1 e^{-30t} \cos 40t + K_2 e^{-30t} \sin 40t$$

$i_{Pi} = 0$

$$i = K_1 e^{-30t} \cos 40t + K_2 e^{-30t} \sin 40t$$

$i(0^+) = 0 \Rightarrow \boxed{K_1 = 0}$

$\frac{di}{dt}\, 0^+ = 0$

$\Rightarrow \boxed{K_2 = 0}$

$i = 0\ U(t)$

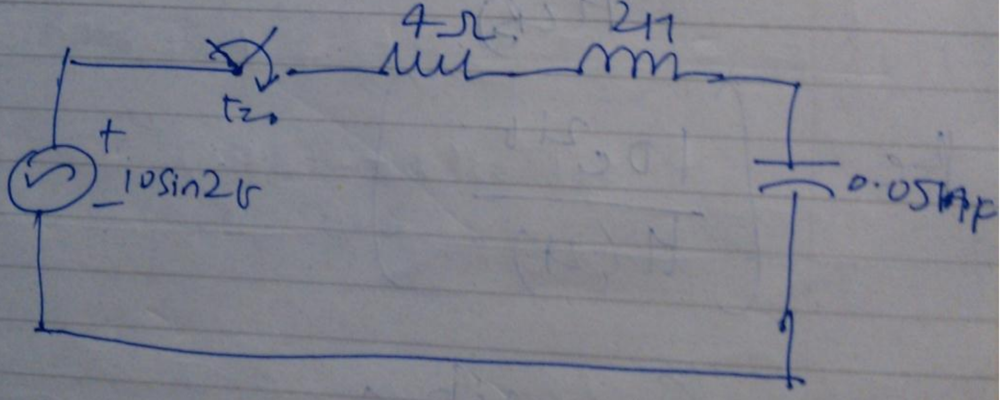
$\underline{i = 0\,A}$

$V_a(0^+) = V_C = 100V$

$V_{c}=0$

$\frac{dv_c}{dt}=0$

(2) The series RLC circuit shown in fig 2 is exited at t=0 by the sinusoidal source and the capacitor is initially uncharged. Determine the current i(t) for t>0 using time domain analysis.



$t=0^-$, $V_c=0$, $V_c(0^+)=0$

$i_L(0^-)=i_L(0^+)=0$

$10\sin 2t = 4i + \frac{2\,di}{dt} + 20\int dt$

$$\frac{di}{dt}(0^+) = 0$$

$$20\cos 4t = 4\frac{di}{dt} + 2\frac{d^2 i}{dt^2} + 20 i$$

$$2D^2 + 4D + 20 = \cancel{20}\,20\cos 2t$$

$$D^2 + 2D + 10 = 10\cos 2t$$

$$CF = K_1 e^{-t}\cos 3t + K_2 e^{-t}\sin 3t$$

$$A = \frac{K(x)}{H(B)}$$

$$Re\left(\frac{10e^{2it}}{H(2i)}\right)$$

$$Re\left(\frac{10\cancel{e^{2it}}}{6+2i}\right)$$

$$Re\left(\frac{\cancel{e^{2it}}(60-20i)}{40}\right)$$

$$\cancel{Re\ \frac{1}{40}\left[60e^{2it} - 20ie^{2it}\right]}$$

$$\cancel{\frac{60}{40}\cos 2t} \quad \cancel{+\frac{20}{60}\sin 2t}$$

$$1.58 - 0.5i$$

$$= 1.581\angle -18.4$$

$$Re\left[1.581e^{(2it - 18.43t)}\right]$$

$$= 1.581\cos(2t - 18.43) = P.I$$

$$i = k_1 e^{-t}\cos 3t + k_2 e^{-t}\sin 3t$$

$$+ 1.581\cos(2t - 18.43)$$

$$i(0^+) = k_1 + 1.581\cos(-18.43) = 0$$

$$k_1 \approx -1.5$$

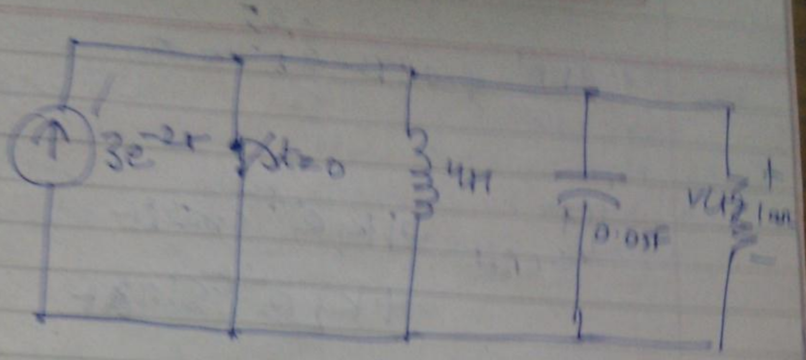
PAGE
DATE

$$\frac{di}{dt} = -K_1 e^{-t}\cos 3t + K_1 e^{-t}(3)(-\sin 3t)$$
$$-K_2 e^{-t}\sin 3t + K_2 e^{-t}(3)(\cos 3t)$$
$$+(1.581\times 2)(-\sin(2t-18.43)$$

$$0 = -K_1 + 3K_2 + 1 + 1$$
$$K_2 = -1.1$$

$$\boxed{i = -1.5e^{-t}\cos 3t + (-1.1)e^{-t}\sin(3t) + 1.581\cos(2t-18.43)}$$

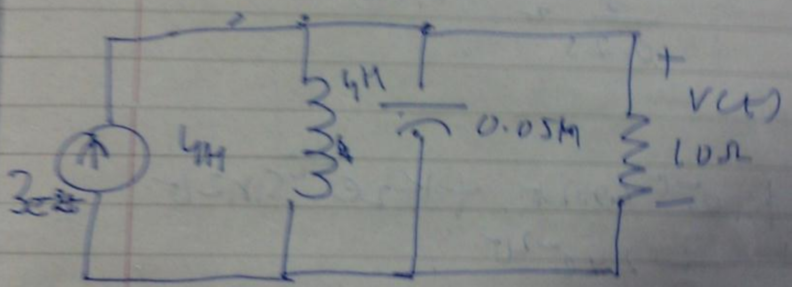
③ In the IIRLC circuit shown in fig 3, source is applied at t = 0. Determine the voltage v(t) for t > 0. Use time domain analysis.



$v_c(0^-) = v_c(0^+) = 0\,V$

$i_L(0^-) = i_L(0^+) = 0$

$t > 0$



$$0.05\frac{dv_c}{dt} + \frac{v_c}{10} + 0.25\int v_L\,dt$$
$$= 3e^{-2t}$$

$$\frac{dv_c}{dt}(0^+) = \frac{3}{0.05} = 60V$$

$$0.05\frac{d^2v_c}{dt^2} + \frac{1}{10}\times\frac{dv_c}{dt} + 0.25v_c = -6e^{-2t}$$

$\therefore 0.4D^2 + 0.4D + \cancel{2/5}\,0.25 = 0$

$V_{c(t)} = K_1 e^{-t}\cos 2t$

$+ K_2 e^{-t}\sin 2t$

$PI = \dfrac{6}{H(-2)}$

$= \dfrac{6}{0.25} = -24e^{-2t}$

$V_c = K_1 e^{-t}\cos 2t + K_2 e^{-t}\sin 2t$
$-24e^{-2t}$

$0 = K_1 - 24$
$K_1 = 24$. Differentiating again
$K_2 = 24$

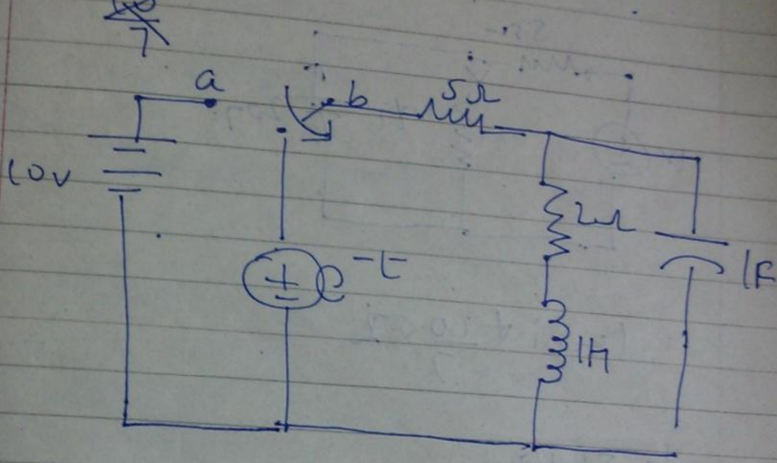
$24e^{-t}[\cos 2t + \sin 2t] - 24e^{-2t}$

$V_c = 24e^{-t}[\cos 2t + \sin 2t - e^{-t}]$

PAGE:
DATE:

4) In the network of fig 4, switch is changed from 'a' to 'b' at $t=0$, steady state being achieved before $t=0$. Find an expression for the current through the inductor for $t>0$ using time domain analysis.

$t=0^-$

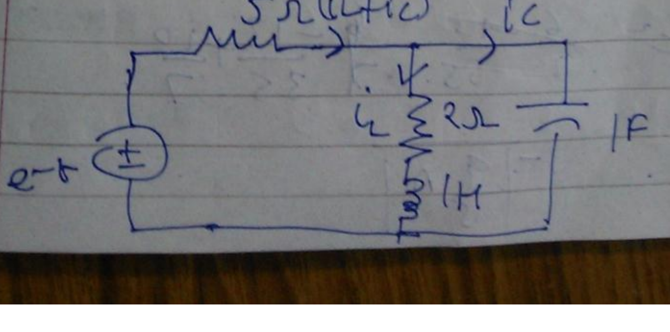


$t=0^-$

$\frac{10}{7}A = i_L(0^-) = i_L(0^+)$

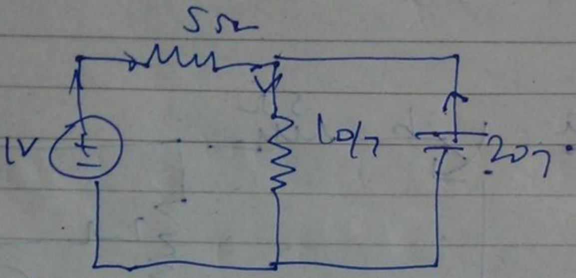
$V_C(0^-) = \frac{20}{7}A = V_C(0^+)$

$\underline{\underline{t>0}}$



$e^{-t} = 5(i_L + i_C) + 2i_L + L\frac{di_C}{dt} = 0$ (1)

$\Rightarrow i_C(0+)$



$1 = 5i + \frac{10}{7} \times 2$

$1 = 5i + \frac{20}{7}$

$5i = 1 - \frac{20}{7}$

$i = \frac{-13}{35}$

$\Rightarrow i_C = \frac{6}{7} - \frac{13}{35} = \cancel{\frac{6}{7}} \frac{-13}{35} + \frac{10}{7}$

$= \boxed{\frac{9}{5} A}$

$\int i_C\,dt = 2i_L + \frac{di_L}{dt}$

$\frac{2di_L}{dt} + \frac{d^2i_L}{dt^2} - i_C = 0 \quad (ii)$

$H(D) = \begin{vmatrix} D+7 & 5 \\ D^2+2D & -1 \end{vmatrix}$

$5D^2 + 11D - 17 = 0$

$-1.1 + 0.43i$

$i_L(t) = k_1 e^{-1.1t} \cos(0.43t)$
$+ k_2 e^{-1.1t} \sin(0.43t)$

$H(-1) = \begin{vmatrix} 6 & 5 \\ 1 & -1 \end{vmatrix}_2 \quad -6+5 = -1$

PAGE :
DATE : / /

$P = \frac{-1}{-1} = 1$

$i_L 0^+ = \frac{10}{7} A$

$\frac{di_L}{dt}(0+) = \frac{1\times 0}{\cancel{7}}\ 0\ A$

$i_L = K_1 e^{-1.1t}\cos(0.43t) + K_2 e^{-1.1t}\sin(0.43t)$
$+ e^{-t}$

$\frac{10}{7} = K_1 + 1$

$\boxed{K_1 = 3/7}$

$\frac{di_L}{dt} = -1.1 K_1 e^{-1.1t}\cos(0.43t)$
$+ K_2 e^{-1.1t} \times (0.43)(\sin - 0.43t)$
$- e^{-t}\ -1.1 K_2 e^{-1.1t}\sin(0.43t)$
$+ K_2 e^{-1.1t}(0.43t)(\cos)\ 0.43t$

$0.43K_2 - 1 - 1.1K_1 = 0$

$0.43K_2 = 1.1K_1 + 1$

PAGE :
DATE : / /

$P = \frac{-1}{-1} = 1$

$i_L(0^+) = \frac{10}{7} A$

$\frac{di_L}{dt}(0^+) = \cancel{\frac{1\times 0}{\lambda}}\ 0\ A$

$i_L = k_1 e^{-1.1t}\cos(0.43t) + k_2 e^{-1.1t}\sin(0.43t)$
$+ e^{-t}$

$\frac{10}{7} = k_1 + 1$

$\boxed{k_1 = 3/7}$

$\frac{di_L}{dt} = -1.1 k_1 e^{-1.1t}\cos(0.43t)$
$+ k_2 e^{-1.1t} \times (0.43)(\sin - 0.43t)$
$- e^{-t} - 1.1 k_2 e^{-1.1t}\sin(0.43t)$
$+ k_2 e^{-1.1t}(0.43t)(\cos)\ 0.43t$

$0.43k_2 - 1 - 1.1k_1 = 0$

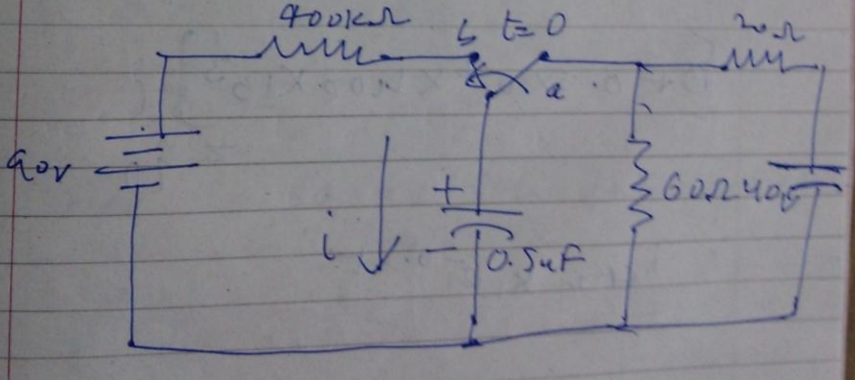
$0.43k_2 = 1.1k_1 + 1$

$\underline{\underline{K_2 = 3.42}}$

$$i_L = \left[\left(\frac{3}{7}\right) e^{-1.1t}\cos(0.43t) + 3.42 e^{-1.1t}\sin(0.43t) + e^{-t}\right] u(t)$$

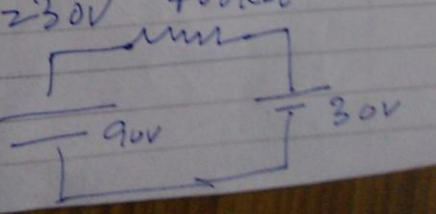
43t)

5. The switch in the circuit shown in fig 5 has been in position a for a long time. At t=0, the switch is moved to position 'b'. Obtain an expression for $V_c(t)$ & $i_L(t)$ for $t \geq 0$



$V_c(0^-) = V_c(0^+) = 30V$

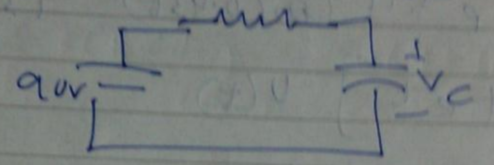
$t = 0^+$



$v_C(0^-) = v_C(0^+) = 30V$

$i_C(0^+) = 0.15mA$

$t > 0$



$90 = (400 \times 10^3) i_C + V_C$

$90 = V_C + C\frac{dV_C}{dt} \times 400 \times 10^3$

$[D + 0.5 \times 10^{-6} \times 400 \times 10^3] i = 90$

$i_{CF} = K_1 e^{-0.2t}$

$PI = \frac{90}{0.2} = 450$

$V_c = 450 + K_1 e^{-0.2t}$

$V_c(0^+)$

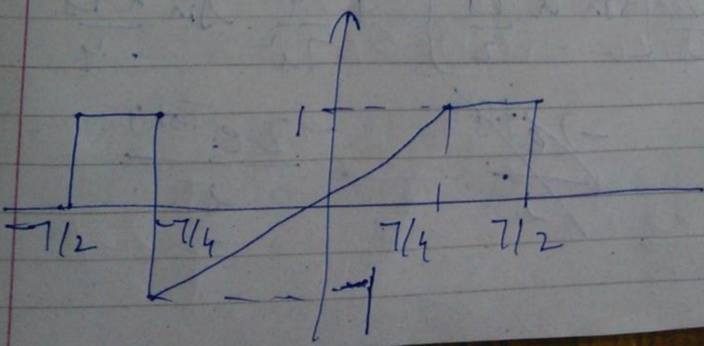
$= 30 = 450 + K_1$

$K_1 = -420$

$V_c = 450 - 420e^{-0.2t}$

$$\boxed{C\frac{dV_c}{dt} = 4.2\times10^{-5}\, e^{-0.2t}}$$

6 Find the laplace transform of waveform shown in fig 6a and 6b.



PAGE :
DATE :

$f_1(t) = u(t+T/2) - 2u(t+T/4)$

$\frac{t}{T}\,r(t+T/4) \times \frac{2(r)t}{T}$

$-\frac{2}{T}\,r(t-T/4) \quad -u(t-T/2) \Rightarrow$

$F_1(s) = \frac{e^{T/2s}}{s} \quad \frac{-2e^{T/4s}}{s} + \frac{2}{T}\left(\frac{1}{s^2}\right)e^{-T/4}$

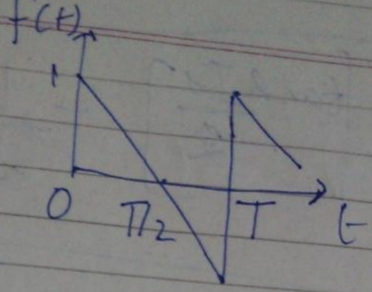
$-\frac{e^{-T/2}}{s} + \frac{2}{T}\left(\frac{1}{s^2}\right)e^{T/4}$

$\frac{1}{s}\left[e^{T/2} - e^{T/2}\right] + \frac{2}{T}\left(\frac{1}{s^2}\right)\left[e^{T/4} - e^{-T/4}\right]$

$= \frac{2}{s}\left[\sin h \frac{Ts}{2}\right] + \frac{4}{Ts^2}\sin h\frac{Ts}{4}$

$\cancel{\frac{-2e^{Ts/4}}{s}} \quad \frac{-2e^{Ts+4}}{s}$

6B



$$f(t) = u(t) - \frac{1}{T} r(t) + \frac{r}{T}(t-T) + u(t-T)$$

$$F_1(s) = \frac{1}{s} - \frac{1}{Ts^2} + \frac{e^{-Ts}}{Ts^2} + \frac{e^{-Ts}}{s}$$

$$F(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

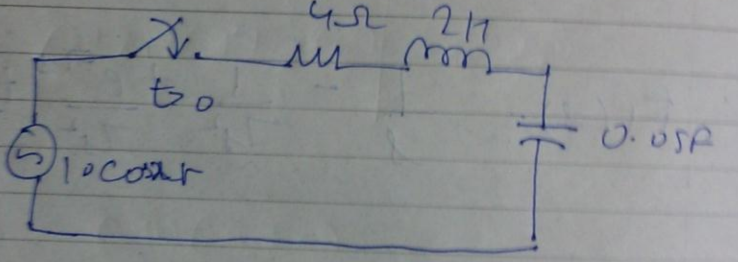
$$= \frac{\frac{1}{s}\left[1+e^{-Ts}\right] - \frac{1}{s^2}\left[1-e^{-Ts}\right]}{1-e^{-Ts}}$$

$$= \frac{1}{s}\left[\frac{1+e^{-Ts}}{1-e^{-Ts}}\right] - \frac{1}{s^2}\left[\frac{1-e^{-Ts}}{1-e^{-Ts}}\right]$$

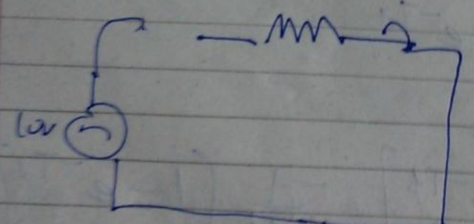
PAGE :
DATE : / /

$$\boxed{\frac{1}{s}\left[\tanh\frac{Ts}{2}\right] - \frac{1}{s^2}}$$

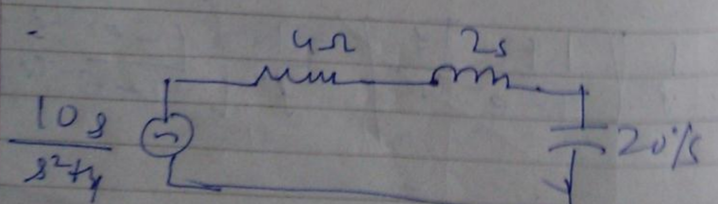
7 Determine $i(t)$ in the circuit shown in Fig7 use laplace transform method.



$t=0^-$

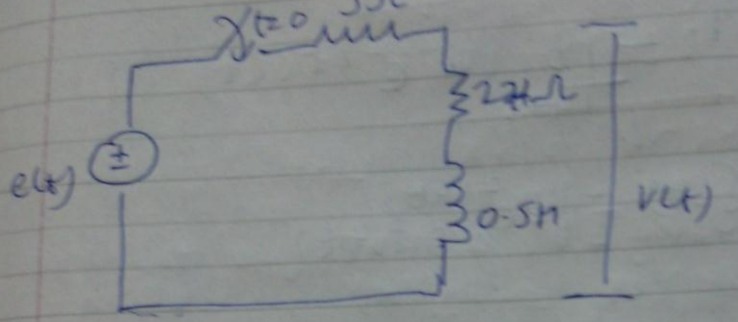


$V_C(0^-) = V_C(0^+) = 0V$
$i_L(0^-) = i_L(0^+) = 0A$

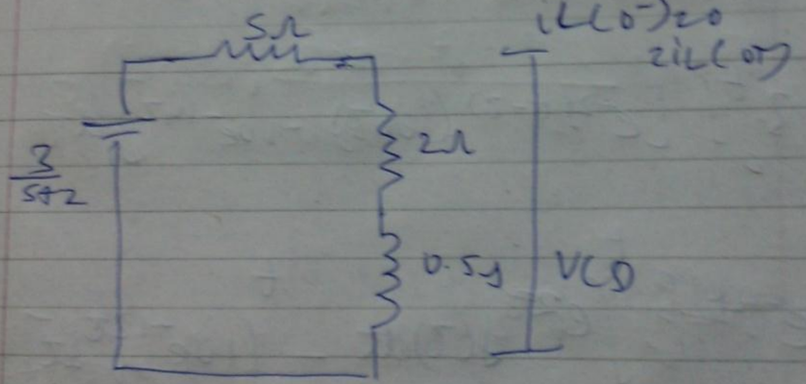


12 In the network shown, find v(t) using convolution integral theorem if

$e(t) = 3e^{-2t} u(t)$



in Laplace



$$V(s) = \left(\frac{\frac{3}{s+2}}{7+0.5s} \right) \times (2+0.5s)$$

$$= \frac{3(0.5s+2)}{(7+0.5s)(s+2)} = \frac{s+4}{(14+s)(s+2)}$$

$$3\left(\frac{s+4}{14+s}\right)^{\nearrow f_1(t)} \cdot \left(\frac{1}{s+2}\right)$$

$$\searrow e^{-2t} = f_2(t)$$

$$3\left(1 - \frac{10}{14+s}\right) \cdot e^{-2t}$$

$$\downarrow$$

$$\delta(t) - 10e^{-14t} = f_1(t)$$

By applying convolution theorem

$$3 \int_0^t \left[e^{-2(t-\tau)} \left(\delta(\tau) - 10e^{-14\tau}\right)\right] d\tau$$

$$3e^{-2t}\left[\int_0^t e^{2\tau}\delta(\tau)d\tau - \int_0^t 10e^{-12\tau}d\tau\right]$$

$$= 3e^{-2t}\left[1 + \frac{5}{6}e^{-12t} - \frac{5}{6}\right]$$

$$= 3e^{-2t}\left[\frac{1}{6} + 5e^{-12t}\right]$$

$$= \frac{3e^{-2t}}{6}\left[1 + 5e^{-12t}\right]$$

$$= \frac{3}{6}\left[e^{-2t} + 5e^{-14t}\right]$$

$$= \frac{1}{2}\left[e^{-2t} + 5e^{-14t}\right]$$

(13) The impulse response of a network is given by $h(t) = 2\delta(t) - 4e^{-3t}u(t)$. Find the current response of the network for a voltage excitation $v(t) = 7e^{-2t}u(t)$ using CIT.

$$h(t) = y(t) = 2\delta(t) - 4e^{-3t}$$

$$Y(s) \cdot V(s) = I(s)$$

By applying convolution integral theorem

$$i(t) = \int_0^t 7e^{-2(t-\tau)} u(t-\tau)\left[2\delta(\tau) - 4e^{-3\tau}u(\tau)\right] d\tau$$

$$= 7e^{-2t}\left[\int_0^t \left[e^{2\tau} \cdot 2\delta(t)\, d\tau\right] - \int_0^t 4e^{-\tau}\, d\tau\right]$$

$$= 7e^{-2t}\left[2 - \left[\frac{4e^{-t}}{-1} - \frac{4}{-1}\right]\right]$$

$$7e^{-2t}\left[2+4e^{-t}-4\right]$$

$$7e^{-2t}\left[4e^{-t}-2\right]$$

$$\boxed{28e^{-3t}-14e^{-2t}} = i(t)$$

14 Using IVT and FVT, find the values of $f(0)$ and $f(\infty)$, wherever applicable

(i) $$F(s) = \frac{2s^3 - s^2 - 3s - 5}{s^3 + 6s^2 + 10s}$$

$$f(0) = \lim_{s\to\infty} sF(s) \qquad f(\infty) = \lim_{s\to 0} sF(s)$$

$$\lim_{s\to 0} s\cdot F(s) = \frac{s\,(2s^3 - s^2 - 3s - 5)}{s(s^2+6s+10)}$$

$$= \frac{-5}{6} = \text{FVT}$$

IV

(ii) F(

IV

s

FV

s-

15 V

IVT = not applicable as degree is same

(ii) $F(s) = \frac{8s-2}{s^2+1}$

$$IVT = \lim_{s \to \infty} \frac{s^2\left[8 - \frac{2}{s}\right]}{s^2\left[1 + \frac{1}{s^2}\right]}$$

$= 8 = IVT$

$$FVT = \lim_{s \to 0} \frac{8s^2 - 2s}{s^2+1} = 0$$

15 $V(s) = \frac{3s(s+1)}{(s+2)(s^2+2s+2)}$

$$= \frac{3s(s+1)}{(s+2)(s-(-1+i))(s-(-1-i))}$$

$$k(-1+i) = \frac{3 \times \sqrt{2}\angle 135 \times 1\angle 90}{\sqrt{2}\angle 45 \times 2\angle 90}$$

$$= 1.5i$$

$\Rightarrow k(-1-i) = -1.5i$

$$k(-2) = 3 \times \left[\frac{1\angle 180 \times 2\angle 180}{2}\right]$$

$$= \underline{\underline{3}}$$

$$\Rightarrow v(t) = 3 \cdot e^{-2t} + 1.5i\,e^{(-1+i)t} - 1.5i\,e^{(-1-i)t}$$