Gravitational microlensing and the search for extrasolar planets

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Abstract

A summary of gravitational lensing is given, covering the basic theory and history of the effect. Focus is placed on microlensing, in particular use of microlensing as a means for discovering new extrasolar planets. Microlensing is compared with other methods of planet-detection. Recent developments are discussed.

1 Introduction – Gravitational lensing from theory to practice

Gravitational lensing is the name given to a wide range of astronomical effects caused by massive objects bending light trajectories. In its simplest form (shown in Figure 1), a light ray leaving a distant star is curved around a massive object, such as a star or a galaxy. This object is known as the *lens.* As this light reaches a terrestrial telescope, the image of the lensed object appears to be in a different place in the sky than where it actually is.

The effect lives up to its name in that many well-known optical effects and distortions are present in gravitational lensing (with the notable exception of chromatic aberration). As described above, distant objects appear in different places across the sky, and can often be cloned into multiple distinct images. Similarly, most lensing exhibits a magnification effect.

Gravitational lensing provides another rich set of tools for astronomers and physicists to probe our universe.

1.1 Prediction of the effect [1]

Long before gravitational lensing was observed, the effect was theorized by several prominent scientists. Newton considered the possible particle-like nature of light and its repercussions. If light particles had mass, they would be affected by gravity like all other massive particles. In 1796, Laplace wrote that extremely massive objects may be able to affect light. None of these concepts were verifiable using then-available lab equipment or telescopes.

Einstein's 1915 formulation of his theory of General Relativity [2] incorporated an accurate derivation of gravitational deformation of light, including predictions of star lensing by the sun's mass. This treatment introduced a factor of two over that of the Newtonian theory, which caused doubt in the scientific community. However, experiments performed during the 1919 solar eclipse proved Einstein's new calculations.

Einstein continued to develop more specific models of the effect, notably in his short 1936 article [3]. This paper approximated lensing using a point source and a point lens, similar to the contemporary model known as "microlensing".

1.2 First uses

The first recorded use of gravitational lensing was in 1979, with the interpretation of two extremely similar quasars (Q0957+561 A and B) as the same object, whose image appears twice [4]. The lensing mass in this case is known to be a galaxy (YGKOW G1).

As of writing, there are about one hundred separate instances of quasar images appearing multiple times in our skies [5]. Finding these lensed quasars involved detailed spectral analysis of the images suspected of being lensed. Closely-matching spectra and red-shift usually indicate a high possibility of a single gravitationally-lensed object. The widest-lensed quasar is the QSO pair 2QZ J1435+0008, identified as a single object in 2003 [6]. The angular separation is 33″.

Perhaps the most famous example of a multiple-imaged quasar is known as the "Einstein Cross" (Figure 2). Discovered in 1985 [7], a striking cross-like pattern is easily discernible, clearly showing the optical lens-like properties of these massive intermediate objects. The cross is formed by the galaxy ZW 2237+030.

1.3 Classifications of the effect [9]

Gravitational lensing is classified according to the degree of the effect as observed. In general terms, the larger the mass of the lens is, the wider the separation visible from Earth.

The smallest masses available for a gravitational lens is a single body, such as a star or a planet. On the gravitational lensing scale, these are very small masses, and the images created are typically beneath the resolving power of today's terrestrial telescopes (with separation on the order of $10^{-6} - 10^{-3}\,''$, depending on distance to the lens and source). Both the images and the source are indiscernible, and the effect manifests itself as

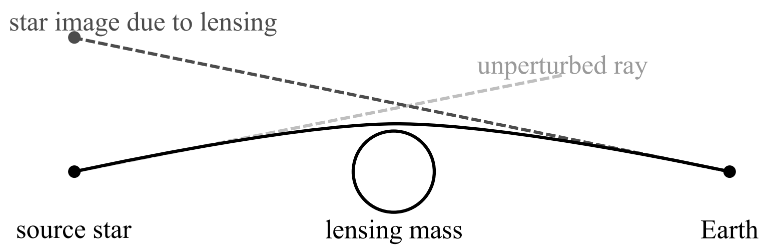

Figure 1: Simplest form of gravitational lensing

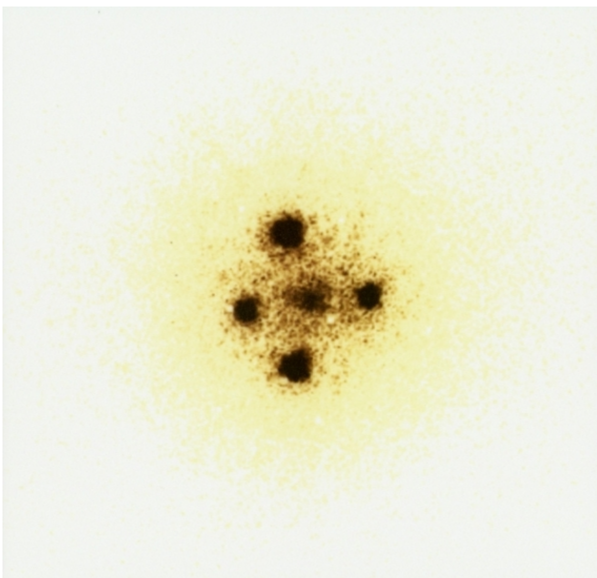

Figure 2: The Einstein Cross (negative). Image credit: [8].

a brightening in flux amplitude as the lens and source align. This is known as *microlensing*. Because of the small masses involved, a high degree of alignment is required between the two bodies and Earth. Microlensing events are discovered by observing large star populations in bulk, letting statistics provide the required alignment.

Galaxies can act as a cohesive mass, able to lens distant objects. Separation can be on order of single arcseconds.

The most dramatic effect gravitational lensing has to offer is when galactic clusters bend light emitted from high red-shift sources, often entire galaxies, reaching tens of arcseconds of separation.

2 Microlensing theory and applications

Microlensing is the simplest and most intuitive form of gravitational lensing. It is also the easiest to model computationally. Using a mathematical point as both the source and the lens lead us to simplified equations describing the light path.

When Einstein showed calculations of point-point gravitational lensing in his 1936 note [3], he claimed it would never be detectable on Earth. His reasoning was the vast improbability of such precise alignment ever occurring. Today, several programs perform constant monitoring of millions of stars on a daily basis. Every year, hundreds of microlensing events are detected, tracked, and analyzed. The stellar density of the galactic bulge provides a microlensing optical depth of about 10^{-6} [10].

2.1 Mathematical model [11]

In order to calculate the behavior of observed microlensing events, we approximate a point source and a small, uniformly-distributed spherical lens. Given an incoming photon from the source star (see Figure 3), approximately parallel to the optical axis, we can determine the angular deflection using General Relativity:

$$\alpha = \frac{4GM}{c^2 \xi} \tag{1}$$

Note that this is an approximation when the impact parameter, ξ, is much larger than the Schwarzschild radius of the lensing mass.

We now assume D, the distance from Earth to the lens, to be much larger than R_0, the radius of

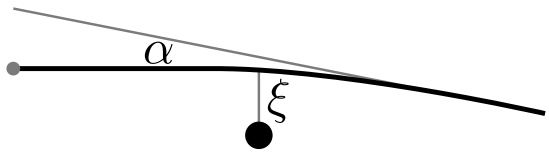

Figure 3: Approximately parallel photon with impact parameter ξ, deflected by star with mass M.

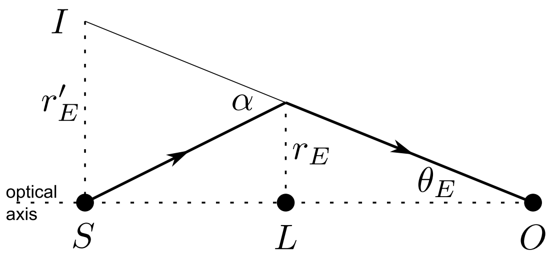

Figure 4: Diagram showing Einstein radius, in terms of physical distances and observed angular radius.

the lensing mass. For an observer perfectly aligned with the source star and the lensing mass, Einstein used simple geometric analysis [3] to determine that the source would now appear as a *ring* with angular radius θ_E, known as the *Einstein radius*:

$$\theta_E = \sqrt{\alpha \frac{R_0}{D}} \tag{2}$$

This ring corresponds to light being bent at a specific angle α, and therefore also corresponds to light approaching at a specific impact parameter ξ. We set $\xi \equiv r_E$, the physical Einstein radius distance from the lensing mass. We define $OL \equiv D_l$ (distance from Earth to the lens), $OS \equiv D_s$ (distance from Earth to the source), and $LS \equiv D_{rel}$. Using similar triangles (see Figure 4), we can now write the relation between the various distances in the problem and the Einstein radius observed on Earth:

$$\theta_E D_s = \frac{4GM}{D_l \theta_E c^2} \tag{3}$$

And the Einstein radius is:

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{rel}}{D_l D_s}} \tag{4}$$

The Einstein radius is important because it represents the alignment angle from the viewpoint of Earth. For a solar-mass lens, situated in the galactic bulge (≈ 8kpc from Earth), and a source twice that distance away, we get $\theta_E \approx 7.14 \times 10^{-4}\,''$. As the distance to this system increases, the radius decreases proportional to $D^{-1/2}$.

We note that it is a manifestation of a multitude of independent variables (the various distances and the mass of the lens in question), and as such, is not indicative of any one variable. This is known as *lens degeneracy.* Microlensing must be coupled with other methods of astronomical measurements in order to provide quantitative information about the source-lens system.

We now turn to the more interesting case where the three bodies (Earth, the lens, and the source) do not necessarily share a common optical axis. If the alignment is close, we expect to see two images along the symmetry axis, one on either side of the lens (also predicted by Einstein [3]). For an angular deviation β of the source from the Earth-lens axis, we arrive at the new angular locations of the source's light:

$$\theta_{\pm} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right) \tag{5}$$

When $\beta = 0$, we recover the original symmetric solution for a the Einstein ring.

As these angles are too small to resolve using terrestrial imaging, an *amplification* is registered instead of separate images. By defining the unitless factor $u \equiv \frac{\beta}{\theta_E}$, we arrive at the amplification of the source:

$$a_{rel} = \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \tag{6}$$

The amplification a_{rel} is the *increase* of observed flux during the microlensing event (as lensing cannot reduce the amplification of the source). When $u \to \infty$, no amplification occurs and $a_{rel} \to 1$. As u goes to 0, signifying perfect alignment, the amplification becomes infinite. The source is not a point source, so in practice amplification will also be finite. According to OGLE data [12], amplifications are routinely an order of magnitude above the

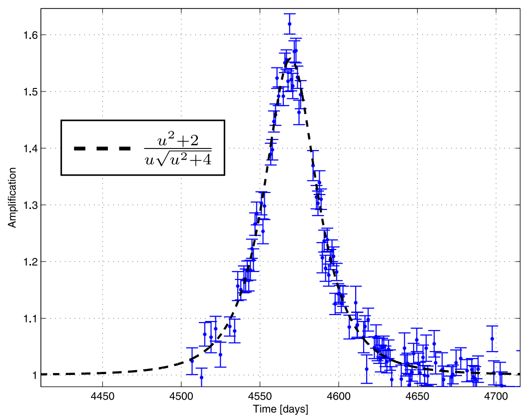

Figure 5: OGLE data and fit for microlensing event 2008-BLG-089 [12]

source's original flux. When $u = 1$, and the lens is exactly on the Einstein radius, $a_{rel} \approx 1.34$. Indeed, most of the events registered by OGLE consist of amplifications of at least 1.34.

Finally, we take into account the transient nature of the effect. Due to the amplification's high sensitivity to alignment, even small velocities of the lensing mass causes the deviation from alignment to become time-dependent ($\beta = \beta(t)$). If the lens moves at velocity v, then the time it takes to cross the Einstein ring is $t_E \equiv \frac{\theta_E D_l}{v}$. The closest alignment during a given event (assuming near-linear motion of the lens) will give $u_{min} \equiv \frac{\beta_{min}}{\theta_E}$, and so:

$$u(t) = \sqrt{u_{min}^2 + \frac{(t - t_0)^2}{t_E^2}} \tag{7}$$

The Einstein-crossing timescale t_E for events observed in the Galactic bulge can range widely from days to several months. As can be seen from the definitions of t_E and θ_E, the time-scale is proportional to $\sqrt{M}$, as well as the velocity of the lensing star.

From data gathered during microlensing events (an example of which appears in Figure 5), these parameters can be discovered.

3 Discovering extrasolar planets with microlensing

3.1 Planet detection techniques

Distant stars have been observed and studied from Earth for thousands of years. However, until recently, the only planets observed by humans were those in our solar system. This changed in 1992, when planets were detected orbiting the pulsar PSR B1257+12 (the initial report was confirmed in 1994 [13]).

The pulses generated by lone neutron stars are extremely uniform over time, not counting the slow spin-down of the neutron star itself. Orbiting planets cause a reflex motion in the host star according to Keplerian mechanics, resulting in a delay in the pulse arriving on Earth. This delay is extremely small, and changes as the reflex motion goes in and out of phase with the rotation of the host. By conducting continuous observations for a number of years, an orbital model can be reconstructed. Using this accurate method, planets of down to $1/10$ Earth's mass could be discovered. Unfortunately, pulsar timing can only be used when the host is a neutron star.

The challenge of observing a planet on its own is formidable, as a planet generally emits far less radiation than a star (though direct imaging has been successful [14]). Therefore, standard planet-searching methods involve the effect the planet has on its host star. Like the pulsar timing method described above, most techniques take advantage of the induced Keplerian motion orbiting planets have on the host. Directly observing this reflex motion is one technique, relevant when the orbital plane is orthogonal to Earth. Another related technique is Doppler spectrometry, which measures the red- and blue-shift as the host moves away from and towards Earth – most effective when Earth is on or near the orbital plane of the distant system. Similarly, changes in flux of the host star can be measured from Earth as planets traverse their surface.

Despite having found thousands of planets using the above methods or combinations thereof, each one has its weaknesses. As of writing, direct observation of host motion has not provided any confirmed planet discoveries, due to lack of measurement accuracy and sky coverage. Furthermore, all methods mentioned are highly dependent on the orientation of the system in regards to Earth, and demand long, focused observations, the duration of which depend on the orbital period of the planets involved and the accuracy of the measurements.

Another consideration is the planetary parameters favorable to detection using each technique. The transit method is biased towards small orbits (< 0.5AU) . Similarly, the Doppler technique works best with planets of large mass and a short orbital period [15]. These limitations may cause studies to overlook systems where planets are present, or fail to represent the actual planetary population of visible stars.

3.2 Exoplanet detection using gravitational microlensing

Simple inspection of Equation 6 shows the amplification curve to be perfectly symmetrical, as the microlensing event occurs. This is clearly visible in observed events, as shown in Figure 5. However, we recall that this perfect symmetry is a product of the assumption of a point lens. This single-point model is good enough for most cases, where the lensing mass is a discrete object. In contrast, often the lens is a binary star system, which complicates the form of the microlensing peak.

In 1991 [16], calculations were made showing how different configurations of the binary system would affect the resulting amplification curve. Furthermore, the authors suggested that even planets could be discovered in this fashion. Instead of a binary system, the lens would be made up of a host star and one or more planets in orbit. While the caustics created by planets are not as extreme as those formed by a true binary star system, analysis and simulations allow information about the planets to be reconstructed. Later that year, Gould and Loeb gave explicit instructions [17] on how to find planets using gravitational microlensing. The authors proposed daily large-volume observation of galactic-bulge stars, and high-resolution follow-up observations of the events as they take place. This scheme has largely become a reality today with programs such as OGLE [12], μFUN [18], and PLANET [19].

3.3 Why use gravitational microlensing?

The characteristics of planetary detection using microlensing differs substantially from other methods. As explained in Section 3.1, most methods demand long, focused observations in order to provide results. Conversely, microlensing provides a nearly-instantaneous snapshot of the system's properties during the lensing event. While Earth-based instruments have only one chance to observe the event, the high magnification can reveal much about a system, which can then be paired with other conventional methods. These events can be as short as a few days, during which all measurements must be taken. Even shorter is the caustic showing evidence of a planet: $t_p \approx 1\text{day}\sqrt{\frac{M_P}{M_J}}$.

One advantage of this method is that it is less sensitive to orbital orientation of the lensing system. Planets near the Einstein radius of the lens star can form caustics on the registered amplification, even if they have very small mass. At galactic distances, the Einstein radius can be $1 - 10$AU away from the star. This opens a window for microlensing as a means to discover the outer planets of a solar-system analog. Indeed, microlensing has proven to be more effective in finding cold, distant planets (beyond the so-called "snowline") than other methods. Figure 6 makes it clear that lower-mass planets found outside the snowline are almost entirely due to microlensing.

Because the effect is governed only by mass, there is no need to be able to measure flux originating at the planet or even its host star. Depending on the degree of alignment between the planet and the source star, magnifications caused by planets

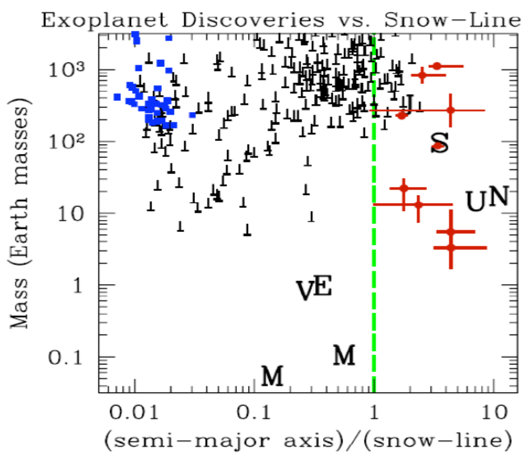

Figure 6: Exoplanets found outside the snowline are dominated by microlensing discoveries (shown in red).

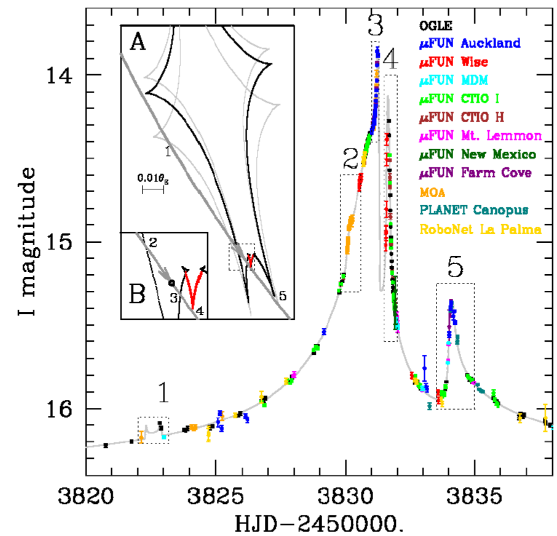

Figure 7: Two-planet system discovered in 2008. Features 1, 2, 3, 5 are due to the outer planet (roughly Saturn-mass), feature 4 is caused by the Jupiter analog.

can be significant. The mass of the planet dictates the timescale of the caustic. Therefore, even small masses can be detected, and theoretically can be improved greatly, as the accuracy and frequency of sampling per event are improved.

3.4 Recent discoveries

So far, the smallest planet found with microlensing has about 5.5 Earth masses, discovered in 2005 [20].

In 2008, a two-planet Jupiter/Saturn analog was discovered [21]. The resulting light amplification curve can be seen in Figure 7, showing caustics thought to correspond with planets in the lens system. One planet is $\sim 0.27 M_J \approx M_{Saturn}$, the other is $\sim 0.7 M_J$. Note the wide collaboration of global follow-up networks. The entire event lasted $\sim$ 20 days, and as discussed above, the planetary caustics are on the order of $t_p \sim 1$day.

Another important study was performed that same year. By using statistical analysis and a Monte-Carlo simulation, planets discovered using microlensing were compared with those *expected* to have been discovered, had every system been a scaled version of our solar system [22]. The conclusion was that our solar system is about three times richer in planets than those discovered so far.

Indeed, microlensing is an extremely useful statistical tool, teaching us not only of mass densities in our galactic bulge, but also about existence of planets, perhaps even helping us learn more about the genesis of our own solar system [15].

A recent report [23] used data mining to search the large catalog of microlensing events for specific cases in which the lens was a single planet. The report's conclusion was that loosely-bound or completely unbound planetary bodies are about twice as abundant in our galaxy as main-sequence stars. Detecting these planetary populations would not be possible without microlensing.

3.5 Future improvements

A first priority in improving our ability to observe microlensing events is to increase coverage. As we have seen, events occur often, it is only a matter of finding them. Often, the successful observation of a single event depends on the followup tracking after the onset of a microlensing event is detected. Development increasing the automation and accuracy of such followup observations could be highly beneficial.

Microlensing events most likely occur among the stars of distant galaxies. Most of today's microlensing tracking arrays concentrate on the galactic bulge [24], due to its large density of stars and proximity to Earth. However, as technology advances, we may be able to record microlensing events as they occur in these neighboring galaxies.

Another viable opportunity is using the parallax effect. By observing the same microlensing effect from two locations (for instance, two points on Earth or one on Earth and one in orbit), the velocity of the lens could be determined as well. While this has been performed occasionally, having dedicated equipment could improve measurements and reduce lens degeneracy.

Following this, another important improvement would be to increase our resolution. As mentioned above, microlensing can detect only planets with relatively large masses. As our amplification resolution increases, and more reliable samples are taken during monitored events, so increases the ability to detect smaller and smaller planets. Today, most microlensing telescopes are situated on Earth. In the future, it may be worth-while to set up a space-borne array of microlensing telescopes, in Earth orbit. There is hope that even planetary moons could be detected (to $\sim 1.6 M_L$), as resolution improves and signals are increased.

4 Conclusions

As we have seen, microlensing can provide a much-needed tool for discovering planets. Existing methods generally have trouble detecting low-mass planets, especially distant ones. Free or extremely distant planets have no known method of discovery besides direct observation and microlensing. While still a statistical tool, much is being learned about the distribution of mass in our galactic bulge, as well as emerging studies of planetary populations, which grow every year. With improving technology, we will probe for smaller and smaller planets and planet-analogs, learning more about our own solar system in the process.

References

[1] R. Narayan and M. Bartelmann. Lectures on gravitational lensing. In *Formation of Structure in the Universe*. Cambridge University Press, 1996.

[2] A. Einstein. Grundgedanken der allgemeinen relativitatstheorie und anwendung dieser theorie in der astronomie. Preussische Akademie der Wissenschaften, Berlin, April 1915.

[3] A. Einstein. Lens-like action of a star by the deviation of light in the gravitational field. *Science*, 84:506, December 1936.

[4] K. Chang and S. Refsdal. Flux variations of QSO Q0957+561 A,B and image splitting by stars near the light path. *Nature*, 282:561, December 1979.

[5] C.S. Kochanek, E.E. Falco, C. Impey, J. Lehar, B. McLeod, and H.W. Rix. CASTLES Survey. `http://www.cfa.harvard.edu/castles/`.

[6] A. R. Marble, C. D. Impey, K. A. Eriksen, L. Bai, and L. Miller. The Most Widely Separated Lensed QSO. In *American Astronomical Society Meeting Abstracts*, volume 35 of *Bulletin of the American Astronomical Society*, December 2003.

[7] J. Huchra, M. Gorenstein, S. Kent, I. Shapiro, G. Smith, E. Horine, and R. Perley. 2237 + 0305: A new and unusual gravitational lens. *Astronomical Journal*, 90:691–696, May 1985.

[8] NASA. `http://hubblesite.org/newscenter/archive/releases/1990/20/image/a/`.

[9] Schneider P. Wambsganss J. Kochanek, C.S. Gravitational lensing: Strong, weak & micro, proceedings of the 33rd Saas-Fee advanced course. Springer-Verlag: Berlin, 2004.

[10] B. S. Gaudi. Exoplanetary Microlensing. *ArXiv e-prints*, February 2010.

[11] A. Gould. A Natural Formalism for Microlensing. *The Astrophysical Journal*, 542:785–788, October 2000.

[12] A. Udalski. The Optical Gravitational Lensing Experiment. Real Time Data Analysis Systems in the OGLE-III Survey. *Acta Astronomica*, 53(4):291–305, December 2003.

[13] A. Wolszczan. Confirmation of Earth-Mass Planets Orbiting the Millisecond Pulsar PSR B1257+12. *Science*, 264:538–542, April 1994.

[14] D. Lafreniére, R. Jayawardhana, and M.H. van Kerkwijk. The Directly Imaged Planet Around the Young Solar Analog 1RXS J160929.1. *The Astrophysical Journal*, 719(1):497, 2010.

[15] A.W. Mann, E. Gaidos, and S. Gaudi. The Invisible Majority? Evolution and Detection of Outer Planetary Systems without Gas Giants. *The Astrophysical Journal*, 719(2):1454, 2010.

[16] Mao, S. and Paczynski, B. Gravitational microlensing by double stars and planetary systems. *Astrophysical Journal Letters*, 374:L37–L40, June 1991.

[17] A. Gould and A. Loeb. Discovering planetary systems through gravitational microlenses. *The Astrophysical Journal*, 396:104–114, September 1992.

[18] μFUN project. `http://www.astronomy.ohio-state.edu/~microfun/`.

[19] PLANET consortium. `http://planet.iap.fr/`.

[20] Beaulieu, J.-P. et al. Discovery of a cool planet of 5.5 Earth masses through gravitational microlensing. 439:437–440, January 2006.

[21] B. S. et al Gaudi. Discovery of a Jupiter/Saturn Analog with Gravitational Microlensing. *Science*, 319(5865):927–930, 2008.

[22] Gould, A. et al. Frequency of Solar-like Systems and of Ice and Gas Giants Beyond the Snow Line from High-magnification Microlensing Events in 2005-2008. *The Astrophysical Journal*, 720:1073–1089, September 2010.

[23] Unbound or distant planetary mass population detected by gravitational microlensing. *Nature*, 473:349–352, May 2011.

[24] S. Mao. Gravitational Microlensing: Past, Present, and Future. In T. G. Brainerd & C. S. Kochanek, editor, *Gravitational Lensing: Recent Progress and Future Go*, volume 237 of *Astronomical Society of the Pacific Conference Series*, pages 215–+, January 2001.