

# Project Euler<sup>1</sup> Problems<sup>2</sup>

## Expressions and Sums

**(Problem 1)** If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Find the sum of all the multiples of 3 or 5 below one thousand.

**(Problem 6)** Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum (that is calculate

$$\left( \sum_{n=1}^{100} n \right)^2 - \sum_{n=1}^{100} n^2).$$

**(Problem 48)** The series,  $1^1 + 2^2 + 3^3 + \dots + 10^{10} = 10,405,071,317$ . Find the last ten digits of the series,  $1^1 + 2^2 + 3^3 + \dots + 1,000^{1,000}$ .

**(Problem 97)** Find the last ten digits of  $28433 \cdot 2^{7830457} + 1$ , a massive prime number found in 2004.

## If statements and loops

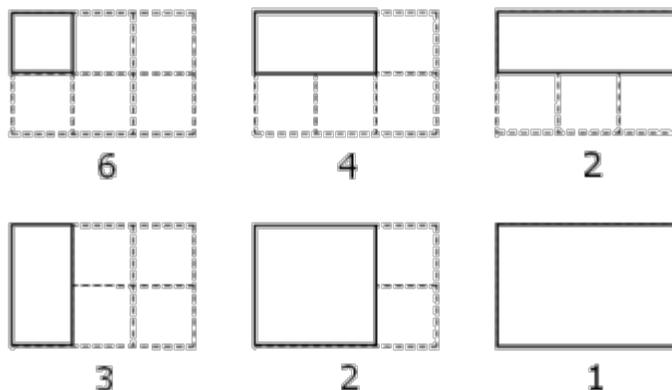
**(Problem 2)** Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ... By considering the terms in the Fibonacci sequence whose values do not exceed four million, find the sum of the even-valued terms.

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<sup>1</sup> <http://projecteuler.net>

<sup>2</sup> Some problems have been edited for brevity or clarity. All problems on this sheet are unanswered, but they are all approachable in the same manner as other problems you've seen.

**(Problem 85)** By counting carefully it can be seen that a rectangular grid measuring 3 by 2 contains eighteen rectangles:



Although there exists no rectangular grid that contains exactly two million rectangles, find the area of the grid with the nearest solution.

**(Problem 92)** A number chain is created by continuously adding the square of the digits in a number to form a new number until it has been seen before. For example:

$$44 \rightarrow 32 \rightarrow 13 \rightarrow 10 \rightarrow \underline{1} \rightarrow \underline{1}$$

$$85 \rightarrow \underline{89} \rightarrow 145 \rightarrow 42 \rightarrow 20 \rightarrow 4 \rightarrow 16 \rightarrow 37 \rightarrow 58 \rightarrow \underline{89}$$

Therefore any chain that arrives at 1 or 89 will become stuck in an endless loop. What is most amazing is that EVERY starting number will eventually arrive at 1 or 89.

How many starting numbers below ten million ( $10^7$ ) will arrive at 89?

**(Problem 193)** A positive integer  $n$  is called squarefree if no square of a prime divides  $n$ . For instance, 1, 2, 3, 5, 6, 7, 10, and 11 are squarefree, but 4, 8, 9, and 12 are not. How many squarefree numbers are less than  $2^{50}$ ?

## Functions

**(Problem 55)** If we take 47, reverse it and add the reversed number, we get 121 ( $47 + 74 = 121$ ) which is palindromic (It is the same number written forwards and backwards).

Not all numbers produce palindromes so quickly. For example,  $349 \rightarrow 1292 \rightarrow 4213 \rightarrow 7337$ , so 349 takes three iterations to arrive at a palindrome. It is conjectured, however, that some numbers, like 196, never produce a palindrome through this reverse and add process. Such numbers are referred to as “Lychrel numbers”. Because the existence of Lychrel numbers is unproven, assume that a number is a Lychrel number if it completes at least 50 iterations without becoming a palindrome.

By this measure, how numbers below 10,000 are Lychrel numbers?

