"WRONG, BUT STILL RIGHT" – TEACHERS REFLECTING ON MKT ITEMS

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The mathematical knowledge for teaching (MKT) measures have become widely used among researchers both within and outside the U.S. Despite the apparent success, the MKT measures and underlying framework have been subject to criticism. The multiple-choice format of the items has been criticized, and some critics have suggested that opening up the items might be an option. One way of opening up the items is to include commentary boxes that allow teachers to explain their thinking. This paper reports on a Norwegian study where commentary boxes were added to MKT items in order to investigate the connection between teachers' responses to the items and their written reflections. The results indicate that there is a mismatch between the answers given by the teachers on the MKT items and their written reflections. Teachers' written reflections do not always support their responses to the MKT items.

Keywords: Mathematical Knowledge for Teaching; Teacher Knowledge; Teacher Reflections

Introduction

Knowledge about mathematical topics and teaching tasks with which teachers struggle is useful when preparing professional development (PD) programs (Hill, 2010). Various methods have been used to study and assess different aspects of teachers' knowledge (e.g., Hill, Sleep, Lewis, & Ball, 2007). The work by Ball and colleagues at the University of Michigan (e.g., Ball, Thames, & Phelps, 2008) is well known, and they have developed the concept of "mathematical knowledge for teaching" (MKT). MKT is defined as "the mathematical knowledge used to carry out the work of teaching mathematics" (Hill, Rowan, & Ball, 2005, p. 373). They have also developed sets of multiple-choice (MC) items to measure MKT. These MKT measures were designed from studies of the work of teaching mathematics in the U.S. (e.g., Hill, 2010). The results from these researchers' efforts are encouraging. MKT appears to make a difference to the mathematical quality of instruction (Hill et al., 2008), as well as to students' achievements in mathematics (Hill, et al., 2005). Morris et al. (2009, p. 492) have described MKT as: "the most promising current answer to the longstanding question of what kind of content knowledge is needed to teach mathematics well."

Many researchers have built upon the efforts of Ball and colleagues, and the MKT measures have been widely used both within and outside the U.S. (e.g., Ng, 2012). Despite the apparent success of this research, there have also been critics (e.g., Schoenfeld, 2007). It is suggested, for example, that the knowledge required for teaching may be more culturally based than simply pertaining to mathematical knowledge (Stiegler & Hiebert, 1999; Stylianides & Delaney, 2011), and that cultural aspects have not been taken into consideration in the development and application of the MKT measures. There have been efforts to study the challenges of translating and adapting the items into a different cultural context (e.g., Fauskanger, Jakobsen, Mosvold, & Bjuland, in press; Ng, 2012) and to compare some of these challenges (Ng, Mosvold, & Fauskanger, 2012). Substantial additional investigation is needed to learn more about the cultural issues related to the translation, adaptation and use of MKT items in different cultural contexts. Another criticism relates to the MC format of the items. Schoenfeld (2007) claimed that the MC format has the potential to complicate the items for test-takers. This claim was supported by findings from a Norwegian study (Fauskanger, Mosvold, Bjuland, & Jakobsen, 2011). In the Norwegian study, teachers suggested that the items include commentary boxes to enable the teachers to explain their thinking, which has been proposed as one way to open up the items. Thus, an extended discussion of the validity of the items appears to be necessary.

The present paper contains a discussion of the criticism as addressed by adding commentary boxes to enable the inclusion of teachers' written reflections to the MKT items. One expectation would be that teachers responding correctly to an item would also provide reflections that support their responses, but to our knowledge no previous attempts have been made to investigate opening up of MKT items in this way. We address herein the following research question:

> What is the connection between teachers' responses to MKT items and their written reflections concerning the content of the items?

Given the importance of developing students' fluency in multi-digit arithmetic as a foundation on which to build a proper understanding of the decimal number system (e.g., Verschaffel, Greer, & De Corte, 2007), we have chosen to analyze data from a MKT testlet, including four items where different methods of decomposing a three-digit number are presented.

Methods

The research reported in this paper is part of a larger project focusing on teachers' MKT and their corresponding beliefs about MKT (e.g., Fauskanger, 2012). For the purpose of this paper, data from 30 teachers' responses to MKT items and their written reflections will be analyzed, taken from one testlet including four MC items. This testlet has not been released for publication,[1] therefore we are only able to provide a description of it. The stem presents a context dealing with groups of students who have decomposed a three-digit number (e.g., 456) into hundreds, tens, ones and tenths in different ways. The question posed is which answer the teacher should accept as correct. In the first item (1a), the students have answered incorrectly (e.g., 456 decomposed into 4 hundreds, 50 tens and 6 ones). The remaining three items represent correct decompositions including hundreds, tens and ones (1b), hundreds, tens and tenths (1c) and tens and ones (1d). The decomposition that strictly follows the positions (e.g., 456 divided into 4 hundreds, 5 tens and 6 ones) is not present in any of the items.

Although MKT items are normally used in a testing situation, other alternative uses have been applied. In our study, the teachers responded to the items at home. It is important to consider the advantages as well as disadvantages of allowing teachers to work with the MKT items at home (see e.g., Hill, 2011). One obvious consequence is that the teachers in our study had the opportunity to discuss the items with others. Their written reflections, however, were individual. They were asked to reflect on the following questions in the commentary boxes: (1) What do the students responding as in items (a) to (d) know? (2) What, if anything, do they need to learn more about? (3) Do the items in this testlet reflect a content that is relevant for the grade(s) you teach? (Why?/Why not? Please provide an illustrating example from your classroom). The reflections were provided for the entire testlet, not for each individual item.

The 30 teachers (8 men and 22 women) were participating in a one and one-half year PD course, and the written work reported on was given as an assignment after their first day in this PD. Sixteen of these teachers worked in grades 1–4, nine in grades 5–7 and five in grades 8–10. Their working experience as teachers varied from less than 5 years to more than 20 years, and their formal education in mathematics (education) varied from 0 to 60 ECTS.[2]

The analysis was conducted with the aid of the computer software NVivo9 (QSR International). The teachers' written reflections were first divided into two groups. One group contained reflections from teachers who had identified the correct key for all four items in the testlet (main group 1, table 1), whereas the second group contained reflections from teachers who had responded incorrectly to one or more of the items (main group 2, Table 1). In the next phase of the analysis, a grounded theory approach (Strauss & Corbin, 1998) was applied in order to analyze the reflections from these two groups of teachers according to how they had commented on the items. Based on several cycles of reading and re-reading the data, the teachers' reflections were refined into codes that were revised several times to establish consistency. The codes were based on well-established findings from the literature concerning place value (e.g., Jones, Thornton, Putt, et al., 1996; Verschaffel, Greer, & DeCorte, 2007). Two sub-categories (a and b in table 1) were discovered for each of the two main groups as a result of this analysis.

Results

In this section, we present the results from our analysis of data regarding the connection between teachers' responses to the four MKT items and their written reflections on the content of the items. Item 1a presents a context where a group of students had given an incorrect response to the MKT item. All 30 teachers in our study identified the key in this item, but 10 teachers had incorrect responses to at least one of the other items in the testlet. If the teachers selected the alternative "I am not sure," their answer would then be coded as incorrect (see Table 1).

Table 1: Teachers' Reflections Regarding Multiple Decompositions

	Think multiple decompositions are correct (1)	Think multiple decompositions are incorrect (2)
All correct (a)	7	13
At least one incorrect (b)	1	9

It is important to notice that it was not always evident in which groups the teachers should be placed; when placing the teachers in groups we allowed them the benefit of doubt if their reflections were ambiguous (e.g. Gerd's reflection below). Two among the seven teachers in the first group could have been placed in the second group, but were placed in the first group although some thoughts in their written reflections were incomplete.

Correct Responses with Supporting Reflections (Group 1a, Table 1)

Although 20 teachers identified all four correct keys in this testlet, only seven displayed supporting thinking in their written reflections. Oda[3] was one of the seven teachers who gave the correct answer to all four items, and she wrote this in her reflections on item 1c:

> I think we have before us an advanced solution in relation to the place value system (in this item). This student has a well-developed number concept and is able to use his fantasy when replacing the one with tenths. In this way, his knowledge about tenths is displayed.

Two of the seven teachers indicate that the students display a very good understanding when they make a decomposition of numbers that differs from the standard decomposition. Tor writes:

> They could have given a more simple solution by using 4 hundreds, 5 tens and 6 ones, but we can say that some (students) are clever in the way they don't necessarily use the correct decomposition but still get the right answer.

The reflections of some teachers were less clear, and as a result it was difficult to evaluate whether or not they displayed a correct manner of thinking. In relation to item 1b, Gerd writes:

> This is, consciously or unconsciously, written in a more advanced way. It might be that he wants to show that he has complete mastery of this, or it might just be a coincident.

These reflections indicate that Gerd believes that the students might have a more advanced understanding, but she is not certain about whether or not their responses are conscious. Later in her reflections, however, she writes that: "none of these solutions are perfect according to the place value system." This last assertion is not explained further.

Correct Responses with Non-Supporting Reflections (Group 2a, Table 1)

Of the 20 teachers who had correct responses to all four MC items in this testlet, 13 provided reflections that did not entirely support their responses. Dina writes:

> The students need to learn about the standard place value system and the proper exchange between digits.

Brit writes:

> The students need to learn more about exchange, learn to fill up the ones, tens, hundreds, etc. Know that each position has its (distinct) value. When the value exceeds 9, they should shift position.

In relation to item 1d, Frida writes that the students:

> ... lack an understanding of the place value system, and the student only understands the tens place and ones place in the place value system. The total sum is still 456, so the student obviously understands decomposition and the value of the number (...) All students are on their way towards an understanding of how a number can be written in extended form. They have to learn more about the place value system. They need to reach an understanding of which number belongs where, one number in each position.

The suggested solutions in items (b) through (d) are all mathematically correct, and these teachers have identified the correct solutions in the MC items. In their reflections, however, they seem to insist on the mathematical convention: "When the value exceeds 9, they should shift position." Although they recognize that the students' solutions were mathematically correct, they do not regard them as "the answer the teacher is seeking," and, therefore, their reflections do not support their responses to the MC items. The written reflections of these teachers are in line with those given by the nine teachers in the last group (2b in Table 1), which consists of teachers who have identified one or more incorrect keys and who have written reflections supporting their responses to the MC items. For example, Erna identified three correct keys and writes this in her reflections:

> Student b) is wrong, but still right. Incorrect decomposition, but the correct total (amount). The student has understood how to decompose the number so that it doesn't increase or decrease in value, but still hasn't placed it correctly according to the place value system.

Eli identified the correct key for item 1a only, and she writes this in connection with items 1b-d:

> b) This student manages to decompose the number 456, but apparently hasn't completely understood the value of the digits in the place value system. It is indeed correct that you can decompose 456 into 3 hundreds, 15 tens and 6 ones, but this is not the answer the teacher is seeking.
>
> c) This student has understood the value of the numbers 4 and 5, but is mixing up the ones. It is a little bit funny to see that the student makes it so hard on himself. This student knows that there are 10 tenths in 1 one.
>
> d) This student knows how many tens there are in 456, but hasn't understood the value of the digits.
>
> Which of these answers you should approve as correct depends on how long the students have been working on this topic. If the students have been working with this for an extended period of time, I wouldn't have approved any of the answers. If, however, this is the introduction to decomposition of numbers, I would have approved b) through d).

In their reflections, the teachers in the fourth group appear to insist on the same mathematical convention as the teachers in the second group do.

Incorrect Responses but "Correct" Reflections (Group 1b, Table 1)

Out of the ten teachers who gave incorrect responses to one, two or three items, or who gave the response "I am not sure" to some of these items, one teacher showed an understanding of the MKT being measured in her reflections. Laura marked all three items with "I am not sure" (which is coded as an incorrect response), but she argued in her reflections that the testlet stem could be interpreted in different ways and that the key for each item would be dependent on how the stem was understood. The following is an excerpt from Laura's reflections:

> Item a) is wrong by all means. Items b), c) and d) are wrong if it (the problem presented in the stem) is a closed problem, but they are correct if it is an open problem.

By "closed problem" this teacher means using the positions given (e.g., 456 = 4 hundreds, 5 tens and 6 ones) and by "open problem" the teacher means open to other ways of decomposing three-digit numbers. This teacher's written reflections are in line with some of those from the seven teachers in the first group.

Discussion and Conclusions

Four groups (as presented in Table 1) emerged in our analysis, and the results from our study indicate that there is not always a clear connection between the teachers' responses to the MKT items and their written reflections. Researchers who use the MKT items would probably expect, or at least hope, that teachers who answer the MC items in the measures correctly also have an appropriate understanding of the content, and the other way around. In our study, there are some teachers who follow this pattern. We have, however, identified an apparent mismatch between the responses to the MC items and the written reflections of several teachers. We have seen an example of one teacher (group 1b) who provided incorrect responses to the MC items but who displayed a high level of understanding in her written reflections. She appears to know the mathematics but she is still unable to determine for what answer the test-makers are looking. Another group of teachers (group 2a), were interesting as they were able to select the correct answer, but appeared to still believe that there is a particular name for the number that is better. These teachers are able to see that "4 hundreds, 5 tens and 6 ones" is the same value as "4 hundreds, 15 tens and 6 ones," but still believe that the name that matches the place values is better. If we consider the example of using the standard Norwegian algorithm for calculating 456 minus 37 (Figure 1), we have an example where the "place value name" is clearly not the best name for the number. As a result, teachers who hold such beliefs could be seen as in transition along a continuum. First, we have the teachers who are not able to understand non-standard decompositions of numbers. Second, there are teachers who can understand multiple decompositions of numbers, but who still believe that the standard decomposition is best. Third, there are teachers who understand and value multiple decompositions of numbers. Fourth and finally, there is a possibility that some teachers understand, value and can explain the use of alternate decompositions.

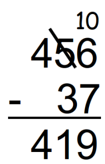

Figure 1: Standard Norwegian algorithm for subtracting 37 from 456

The findings may be explained in a variety of ways. We present four possible explanations as follows.

1. The findings are incidental. This might be connected with our study being limited by the number of participants as well as the limited focus of the items. More research is necessary in order to investigate whether or not the same tendencies can be found in a larger number of participating teachers.
2. This apparent mismatch is specific to this particular topic. It would be pertinent to also investigate whether or not, or if the same pattern can be found for all sets of MKT items.

3. There are cultural differences involved in how teachers reflect upon these items. One such difference might be related to how the decimal number system is taught. If such cultural differences are involved, it would be of great interest to conduct additional research to investigate this further. Researchers have already adapted and used MKT items in different countries (e.g., Ng, 2012), and if there are cultural differences related to the connection between teachers' responses to items and their reflections, great care should be taken when it comes to how the results from such studies are interpreted. We suggest that efforts should be made to investigate these issues both inside and across cultures to learn more about the connection between teachers' MKT, their corresponding beliefs and the educational culture(s).
4. A final possible explanation of the differences between these Norwegian teachers' responses to the MKT items and their written reflections is that there are indeed, as Schoenfeld (2007) argued, constraints resulting from the MC format of the items. Such possible difficulties with this format might be specific to culture, as Fauskanger and colleagues (2011) suggest. From the results of the present study, however, it does not appear that the MC format itself makes it more difficult for the teachers. The complicating connection between the teachers' responses to the MKT items and their written reflections only indicates that the MC items are harder to interpret than they might appear. The inclusion of commentary boxes along with the items, or other ways of opening up the items, should be investigated further. It would also be relevant to include interviews with teachers to further investigate teachers' reflections as well as the connection between these reflections and their MKT as measured by their responses to the items.

Our study indicates that researchers have to be careful concerning the conclusions they draw when measuring teachers' MKT. Particular care should be taken when using these measures in other cultural settings and more research is needed in this area. We argue that it is important to include the teachers' reflections in order to learn more about their MKT, and more research is needed to investigating teachers' epistemic beliefs (Fives & Buehl, 2008) related to the different aspects of MKT. Analyses of teachers' reflections concerning MKT items can be particularly useful in this regard. Follow-up interviews with teachers in groups 1b and 2a would also be relevant for future studies.

Endnotes

[1] The numbers have been changed in our descriptions of the item in order not to reveal the entire item, and these details in the teachers' reflections have been changed accordingly.

[2] ECTS stands for European Credit Transfer and Accumulation System. One year of full-time studies in Norway gives 60 ECTS.

[3] The teachers' names have been changed to ensure anonymity.

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