Topological Issues in (Loop) Quantum Gravity

FSU Nuclear Physics Seminar

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Outline of Talk

- Review of General Relativity (GR) and Motivation for Quantum Gravity
- Approaches: What do you/we mean by "Quantum Gravity"?
- Topology in Gravity
  - What is topology?
  - Topology change in GR
  - Topology in Quantum Gravity
  - Remarkable Mathematical Fact!
- Specific Approach: Loop Quantum Gravity
  - What is Loop Quantum Gravity?
  - Topology in LQG (me)
- Summary

Review of General Relativity

- Classical Gravity (General Relativity) constructed by Einstein in 1915 can be used (with great precision) to explain all measured gravitational effects[1].
- Basic variables are a 4 dimensional **manifold** ("smooth shape") M and a **metric** g_{ab} (rank 2 tensor).
- Background independent (*relational*)- "the theory constructs itself" Compare Electromagnetism or Quantum Field Theory (or string theory...) in which a background (like Minkowski space) is put in by hand.

[1] except Dark Matter...

Do We Even Need A Quantum Theory of Gravity?

"There are no data that suggests we need a quantum theory of gravity"[2]...

But there are things we cannot calculate with the standard model:

- Two-body gravitational scattering amplitude
- What happens when a black hole evaporates?
- Early universe cosmology
- Structure of spacetime at a very small scale

[2] DARK MATTER!

What Do We Mean When We Say "Quantum Gravity"?

Some Possible Approaches to Quantizing Gravity:

- "All the other fields are quantized and we can study them with QFT. Let's do that with gravity too! Gravitons!"
  $\eta_{ab} \longrightarrow \eta_{ab} + \delta\eta_{ab}$.
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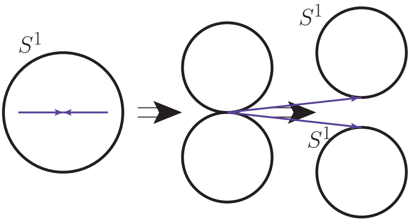
Topology in Quantum Gravity: What is Topology?

Topology is the study of manifolds (shapes) without reference to their size or other geometric properties.

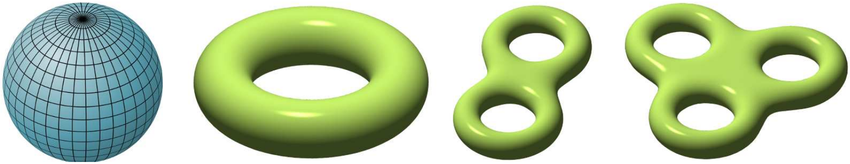
Topology is primarily concerned with **classification**, *i.e.* answering the questions

- Are two manifolds M, N topologically the same?
- Can we deform M into N?
- Can we find a continuous,invertable function $f : M \to N$?



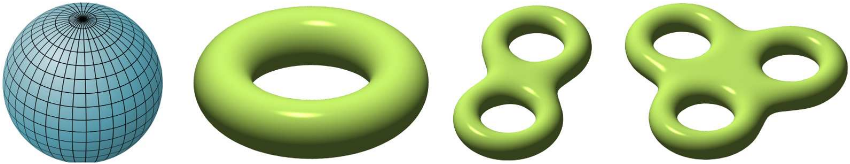
These questions answer the same thing. The number of holes in the manifold (**genus** g) is an **obstruction** to answering "yes":

- If $g(M) \neq g(N) \to$ answer "no"
- If $g(M) = g(N) \to$ answer "yes"



"Surfaces with a different genus $g =$ (number of holes) cannot be deformed into each other, and are therefore topologically inequivalent"

By generalizing this idea to higher dimensions, we have a set of topological invariants with names like **Euler Characteristic** $\chi(M)$, **Chern character** $c(M)$ which can tell us if a manifold is topologically distinct from any other one.



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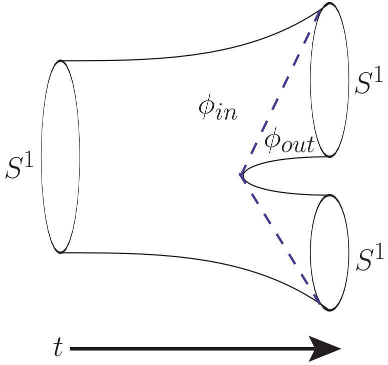
Does changing the topological class of a 4-manifold change the physics on the manifold?

Sidenote: Topology is not enough! Physics requires *geometric* information as well as topological information!

What Happens When Topology Changes? (Classical)[3]

Reduce the spacetime dimensions to 1+1 and consider the trousers topology:

- Consider modes of a massless scalar field ϕ propagating in this spacetime.
- These fields will be discontinuous at the point where the legs meet, and this discontinuity propagates into each leg.
- This discontinuity makes the expectation value of the stress-energy tensor infinite: "infinitely bright flashes".
- Can be extended to 4 dimensions, and appears to always accompany topology change.



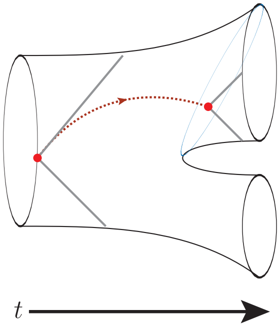
$$\langle T_{00} \rangle \sim \sum_i \partial_t \phi_i \partial_t \phi_i \sim \sum_i \delta(t - t_0)\delta(t - t_0)$$

[3] Anderson & DeWitt (1986)

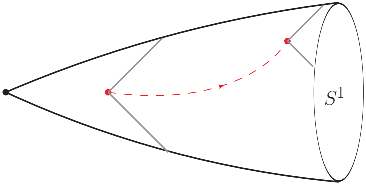
Good and Bad Topology Change (Classical)

A spacetime is **causally discontinuous** if the causal past (or future) of an observer changes discontinuously as the observer moves on it's worldline.

The trouser topology is causally discontinuous...



But note a "big bang" in 2 dimensions is not...



Borde-Sorkin Conjecture: No causally discontinuous spacetimes allowed! ("no saddles")

Should Quantum Gravity Include Topology Change?

...we have no idea. But it seems to follow from the "relational viewpoint" of fundamental physics[4]

1. There is no background
2. The fundamental properties of elementary entities consist entirely of relationships between them.
3. The relationships are not fixed, but evolve according to some physical law

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General Relativity is **partly relational**, since the geometry is relational but the topology, dimension, differentiable structure, and signature are absoute (background).

Can Quantum Gravity be "more relational" then classical GR?

Tracking topology with quantum gravity is a step to make gravity a completely relational theory.

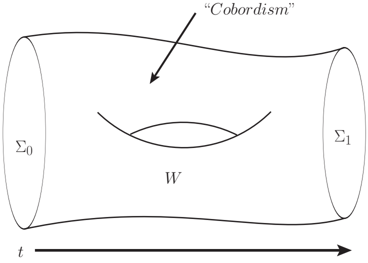
[4] Smolin (2005)

"The Proposal": In 3+1 gravity, consider an initial 3-manifold Σ_0 with metric h_0 going into a final 3-manifold Σ_1 with metric h_1. The transition amplitude might look something like

$$\langle \Sigma_0, h_0 | \Sigma_1, h_1 \rangle = \sum_W \int exp(iS[g]/\hbar)[dg].$$

Why is this hard?

Tracking topology of W can be done with topological invariants, but tracking *geometric* (smooth structure) of W is far more difficult.
Open problem for dimension 4.
(favorite topic of mathematicians...)



Remarkable Mathematical Fact[5]

[5] Denicola, Marcolli, al-Yasry (2010)

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Alexander's Theorem

Every orientable closed manifold of dimension $m \geq 2$ is a branched covering space of the m-sphere, branched along a $m-2$ dimensional submanifold.

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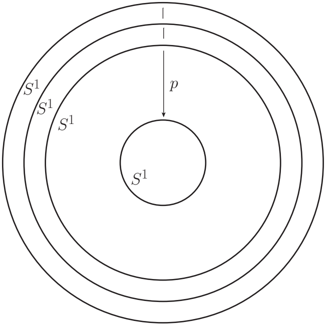
Covering Space: Disjoint union of n copies of a manifold which map into a single copy of that manifold with a map p.



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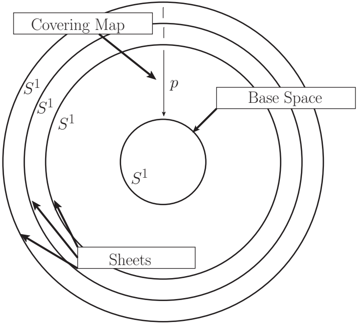

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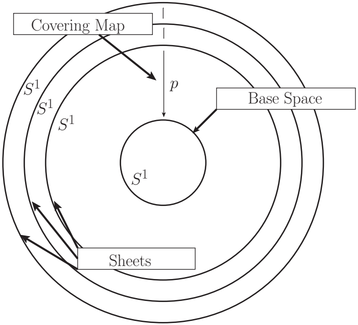
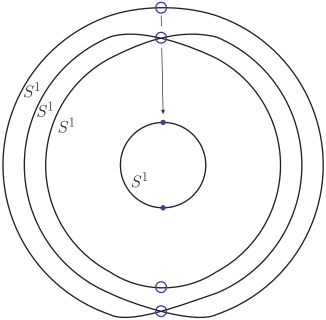


Branched Covering Space: A space which is everywhere a covering space except for a finite set called the branch locus.



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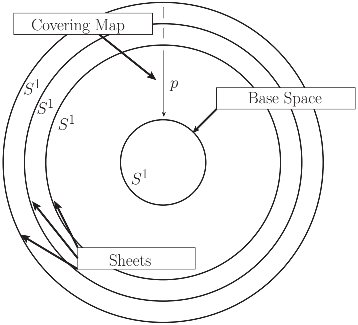

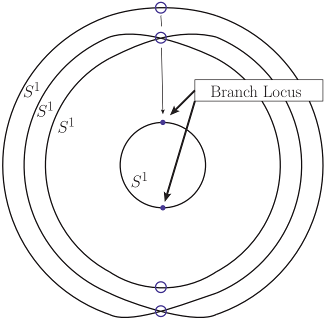

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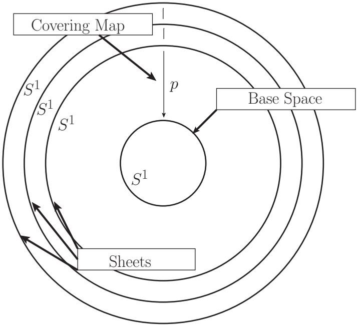
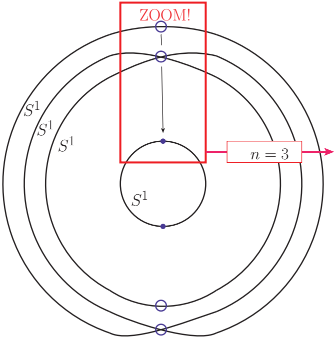
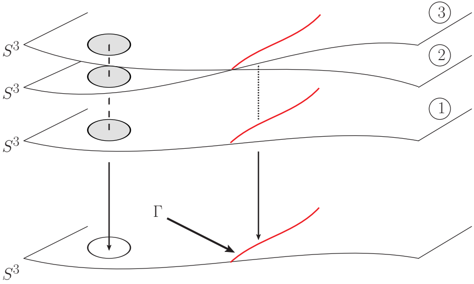
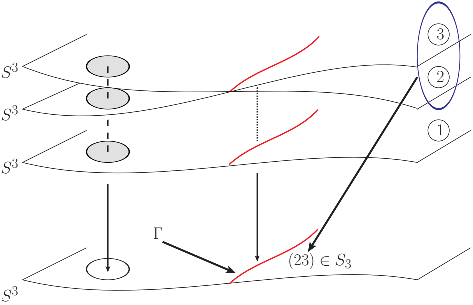




Now in 3 dimensions...with one suppressed...



Graph: 1-dimensional submanifold Γ



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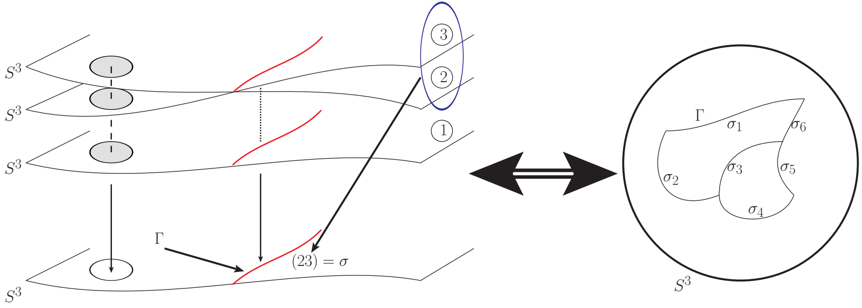
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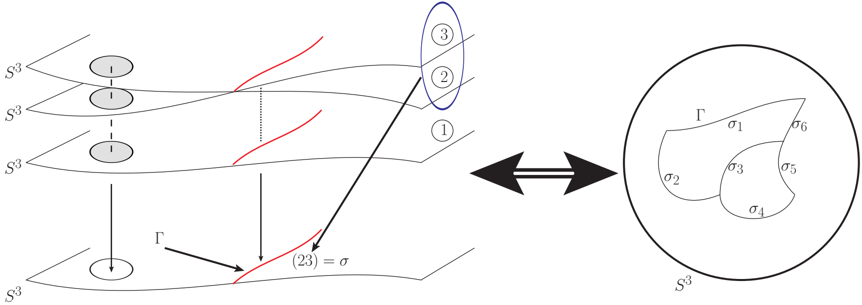


The Point:

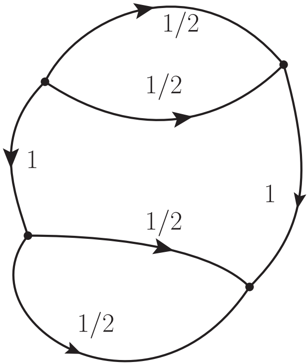
We can now track all relevant topological and geometric information using a specification of graph Γ and permutation labels $\vec{\sigma}$

$$\langle \Sigma_0, h_0 | \Sigma_1, h_1 \rangle = \sum_W \int exp(iS[g]/\hbar)[dg] = \sum_{\Gamma, \vec{\sigma}} \int exp(iS[g]/\hbar)[dg]$$

It'd be nice if these graphs had some kind of physical connection to gravity though...

They Do! (or might...): Loop Quantum Gravity

- Rewrite spatial metric using $SU(2)$ fields:
$$q_{ab} = \eta_{ij} e^i_a e^j_b.$$
- Gauge-invariant states are called *spin networks*, which are graphs Γ in spacetime with spin labels $\vec{j}$.



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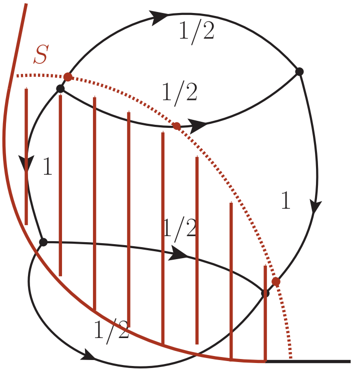
- The area operator for a surface S is the sum of the total spin over all the intersecting arcs with the surface:

$$A \sim \sum_{intersections} \hbar \sqrt{e^i_a e^i_a} = \hbar \sum_{k=1}^{3} \sqrt{j_k(j_k+1)}$$
$$\sim \hbar \left[\sqrt{\frac{3}{4}} + \sqrt{\frac{3}{4}} + \sqrt{2} \right]$$



($e^i e^i$ is the *Casimir* of the Lie algebra $\mathfrak{su}(2$

From Spin Networks to Topspin Networks[5]

A **Topspin Network** is a graph Γ with both spin labels $\vec{j}$ and permutation labels $\vec{\sigma}$.

[5] Duston,1111.1252

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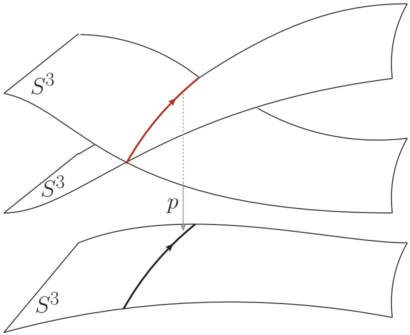
Promote Spin Networks to Topspin Networks...by adding a label!



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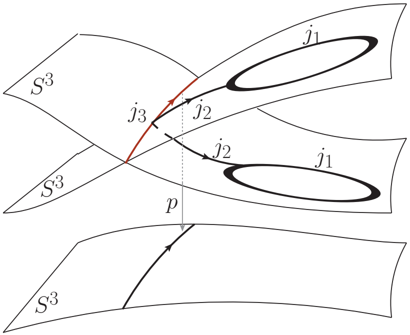

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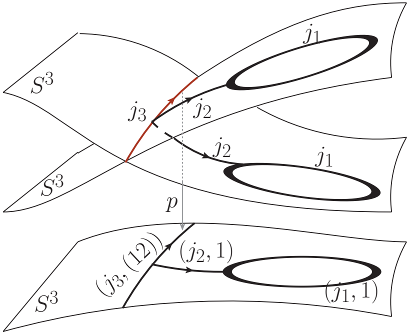

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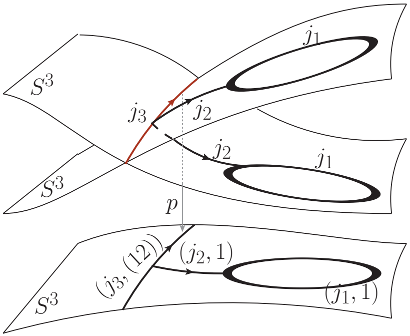


This adds an extra symmetry to the system, since some of the sheets of the cover need to be identified over the graph...

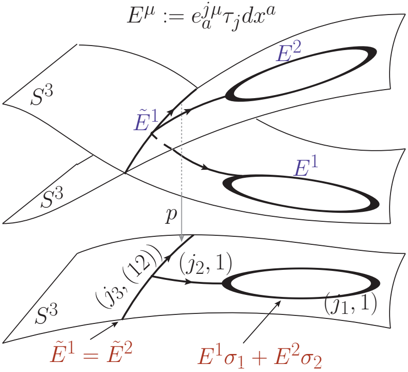
[5]Duston,1111.1252

Deck Transformation

The covering space is now the physical spatial manifold Σ, and the $SU(2)$ fields the branch locus has an extra S_n symmetry.

Thus the fields transform under the symmetry group $SU(2) \times S_n$, and carry an extra label for the S_n part:

$$\begin{aligned} E' &= (12)E \\ &= (12)(E^1\sigma_1 + E^2\sigma_2) \\ &= (E^2\sigma_1 + E^1\sigma_2) \\ &\to E^1 = E^2 \end{aligned}$$



(Specifically, $e_a \in \mathcal{U}(\mathfrak{su}(2)) \otimes \mathbb{C}S_n$.)

Area Operator in Topspin Networks

Need to calculate the Casimir for the algebra associated to $SU(2) \times S_n$.

$$A \sim \sum_{intersections} \hbar \sqrt{e_a^{i\mu} e_a^{i\mu}} \rightarrow \hbar \sum_k \sqrt{j_k(j_k+1)n_k^2}.$$

The integer n_k is the number of copies of each arc of Γ in the cover; in other words,

$$n_k = (\text{sheets} - \text{identifications}_k).$$

In fact, this is exactly the same result that one would get from calculating the area of a spin network with n_k identical edges...these are exactly the edges identified by the intersecting covers!

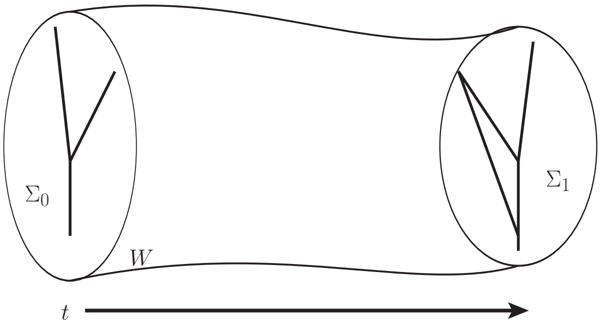
In this framework:

- Topology change is tracked by this n_k.
- Reproduce the area operator for the usual spin networks with degeneracies.
- Use (most) of the usual techniques from LQG.

Making Sense Of "The Proposal": State Sums

We can now make sense of our state sum:

$$\langle \Sigma_0, h_0 | \Sigma_1, h_1 \rangle = \sum_W \int exp(iS[g]/\hbar)[dg].$$



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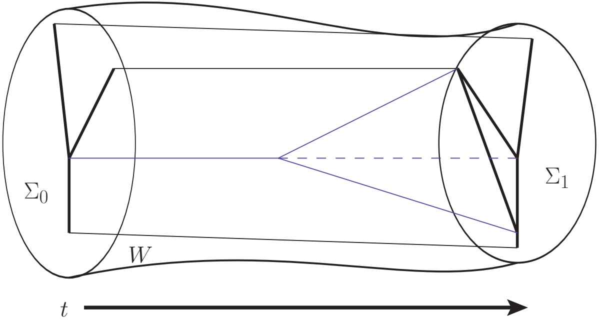

The cobordism W can be represented via Alexander's theorem as a branched cover of the 4-sphere over a 2-dimensional submanifold!

Summary

- Although classically topology change in GR is (kinda) forbidden, the same cannot be said of quantum gravity...
- ...and should probably not be true (relationalism).
- Alexander's theorem: counting manifolds = counting graphs + permutations.
- Topspin networks provide a framework to track the toplogy of spacetime in loop quantum gravity.
- Many of the usual techinques of LQG can be applied to a Topspin network.
- Since one can now track both the topological *and* the geometric structure of 3 and 4 manifolds, state sums which include both seem to be in our reach.
- (Aside: It turns out the action of the Hamiltonian has a much richer structure on topspin networks - how could this affect the dynamics used to find n_k?)