Fragility in Money Market Funds: Sponsor Support and Regulation *

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Abstract

Money market funds (MMFs), which are crucial to short-term funding markets, rely on voluntary support of fund sponsors to maintain stable share values. I develop a general equilibrium model of MMFs to study how sponsor support affects the industry's fragility and regulation. Adverse asset-quality shocks lead MMFs to liquidate assets. When liquidity in asset markets is limited, asset prices are lower if more funds liquidate. Lower asset prices, in turn, make sponsor support costlier and even more liquidations occur. This feedback leads to complementarities in sponsors' support decisions. Based on the model's insights, I derive implications for the regulation of MMFs.

JEL classification: G01, G23, G28.

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1. Introduction

Money market funds (MMFs) account for a significant amount of plumbing in the financial system. They are among the most important suppliers of short-term liquidity to other financial institutions and, thus, flows into and out of MMFs can affect the financial system as a whole. By the end of 2013, US MMFs managed more than $2.6 trillion in assets, almost a quarter of all US mutual fund assets, and over 10% of mutual fund assets worldwide (see Investment Company Institute (2013).) In December of 2011, MMFs owned over 40% of U.S. dollar-denominated financial commercial paper and around a third of dollar-denominated negotiable certificates of deposit, and they were among the biggest category of repo lenders, with an estimated $460 billion in repos (see McCabe, Cipriani, Holscher, and Martin (2012), and Financial Times (2011)). Because of these features, the large outflows experienced by the MMF industry after the Reserve Primary Fund (RPF) broke the buck in mid-September 2008 contributed largely to the freezing of the short-term funding market. [1]

An important issue concerning the stability of MMFs is their lack of capital or precautionary liquid reserves to deter investor outflows. Instead, MMFs can receive support from their sponsors who, at their own discretion, can transfer outside funds to an MMF's balance sheet. In fact, one of the main ways in which outflows from MMFs can be prevented is through voluntary sponsor support. Outflows can be very costly for the companies that sponsor MMFs: there could be forgone returns and negative spillovers to other activities in which the sponsors participate. Therefore, to minimize outflows and prevent funds from breaking the buck and being liquidated, sponsors can choose to offer support to their funds by purchasing assets from them at a premium over their market value. As figure 1 shows, sponsor sup-

[1] See Board of Governors of the Federal Reserve System (2009), Securities and Exchange Commission (2009), President's Working Group on Financial Markets (2010), Financial Stability Oversight Council (2011) and Chernenko and Sunderam (2014).

port has been a common feature throughout the history of the MMF industry even prior to the 2007 − 08 financial crisis. Between 2007 and 2011, 78 MMFs (out of a total of 341 MMFs) received sponsor support in 123 instances for a total amount of at least $4.4 billion.

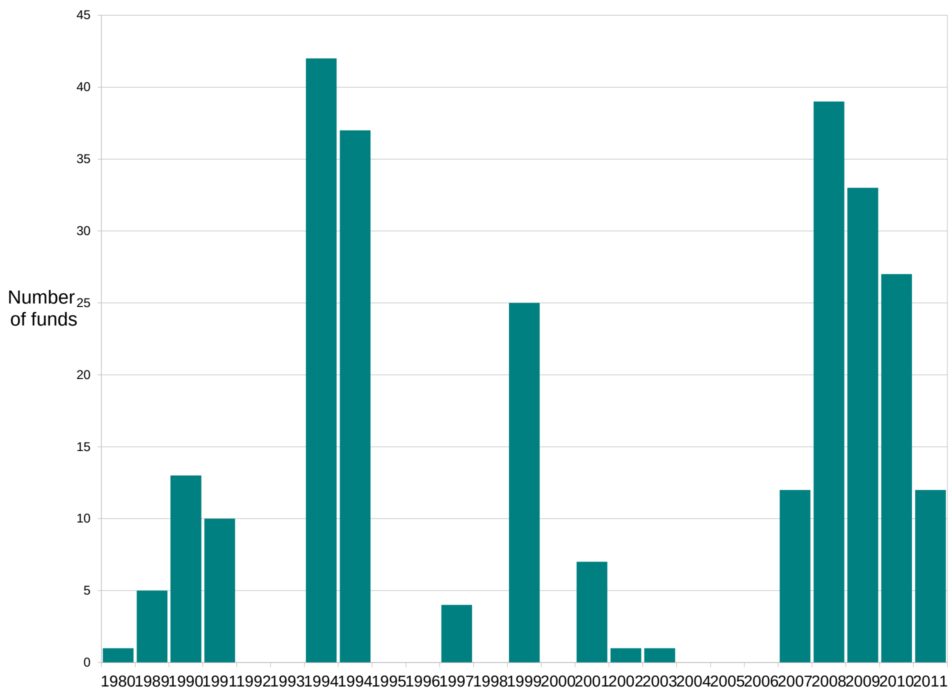

Fig. 1. Number of funds receiving support: 1980-2011. Data sources are Brady, Anadu and Cooper (2012) and Moody's Investors Service (2010).

This paper is the first to analyze theoretically sponsor support and its implications for the industry's stability and regulation. First, I build a general equilibrium model of the MMF industry to study the ability of sponsor support to provide stability to the industry. The main contribution of the paper is to show that sponsor support can be a source of fragility instead of mitigating it. Strategic complementarities in the sponsors' support decisions can lead to runs of the MMFs on the money market and amplify systemic shocks. I then analyze three policies that affect the amount of support offered by sponsors to MMFs: the prohibition of sponsor support; the adoption of a floating net asset value (NAV), which would decrease the sponsors' incentives to offer support; and the adoption of a capital buffer, which would force sponsors to offer a minimum amount of support.[2]

I build a three-period model of financial intermediation with two types of agents: risk-averse investors and risk-neutral fund managers. There are two assets, a short-term safe asset and a long-term risky asset, which are traded each period in competitive markets. Only managers can access the risky asset market directly. Therefore, investors can access the risky asset only through a manager. The intermediation contract between investors and managers captures three main features of MMFs: the demandable nature of the shares held by investors in MMFs, the eventual liquidation of a fund after it breaks the buck, and, through the possibility of voluntary sponsor support, the stability of the NAV.[3] To capture the fact that sponsors of different sizes have different incentives to support a MMF for the same realized NAV, the model introduces heterogeneity in the costs faced by fund managers after a breaking-the-buck event.

The quality of the risky asset is subject to shocks. At the time of issuance, uncertainty exists about the probability of default of the risky asset, which is resolved before the asset matures. Adverse asset-quality shocks, i.e., sufficiently high realizations of the probability of default, induce asset sales by MMFs and, absent sponsor support, can even lead to fund liquidations. However, when there is limited liquidity in the asset market, the sponsors'

[2]These policies are the main regulations that are being considered in the US and Europe to change the MMF industry.

[3]See the Appendix for a fuller description of the institutional features of MMFs.

support decisions determine the demand for the risky asset and its price. That is, asset prices are lower if more funds liquidate. Lower asset prices, in turn, make offering support costlier and even more liquidations occur. The interdependence between asset prices and support decisions gives rise to strategic complementarities in sponsor support decisions. These complementarities make the MMF industry vulnerable to runs that are different from the canonical bank runs in Diamond and Dybvig (1983). MMF runs are not runs of investors on the financial intermediaries but runs of financial intermediaries on the asset market. Thus, MMF runs are associated with a distinct form of pecuniary externality that arises from the interaction between coordination failures and asset prices.

Finally, I use the model to analyze the general equilibrium effects of three different policies: the prohibition of sponsor support, the adoption of a floating NAV, and the adoption of a capital buffer. These policies appear not to have been designed to target the equilibrium in the money market and, therefore, their general equilibrium effects seem largely ignored in the policy discussion. By affecting the sponsors' incentives to offer support and to supply liquidity, and given the relative size of MMFs in the money market, I show that these policies have crucial, albeit unforeseen, implications for asset prices.

Sponsor support has been instrumental in maintaining the stability of the NAV and preventing fund liquidations. Therefore, one could argue that forbidding sponsor support increases fund liquidations and the fragility of the MMF industry. However, if no sponsor support is allowed, individual managers have incentives to reduce the risk they take to decrease the probability of their fund being liquidated. In turn, this decrease in risk can lead to fewer asset liquidations, to more stable and higher asset prices and, thus, to an increase in overall stability.

When going from a stable to a floating NAV system, MMFs' investors lose the insurance provided by sponsor support. When everything else is equal, risk-averse investors reduce their exposure to MMFs in response to this loss of insurance. Thus, as argued by the industry, the supply of liquidity in the asset market decreases. However, in equilibrium, adopting a floating NAV for the whole industry also changes asset prices and, thus, the risk and the return of investing in an MMF change beyond the loss of insurance. In fact, the risk faced by investors in MMFs can decrease if asset prices become less volatile. Contrary to the partial equilibrium analysis, I show that the total supply of liquidity can increase with a floating NAV.

The adoption of a capital buffer in the MMF industry makes it costlier for managers to offer intermediation services and, thus, in principle, decreases the net return for investors and the liquidity provided by MMFs. Nevertheless, by forcing MMFs to absorb losses, a capital buffer also increases the investors' expected return and decreases the probability of liquidation. At the same time, capital buffers can make asset prices less volatile, decreasing the risk of investing in MMFs even more. These mechanisms have opposite effects on the investors' willingness to invest in MMFs. Yet, once all the general equilibrium effects are considered, the supply of liquidity by MMFs can increase even though the cost of intermediation increases.

The policies analyzed have countervailing effects on the risk and the return of intermediation for investors and MMF managers and, hence, on the provision of liquidity and on welfare. Indeed, the general equilibrium effects of the policies considered can go against the partial equilibrium predictions.[4] The main takeaway from these exercises is to emphasize that the effect of these policies on asset prices should not be ignored outright, especially given the relative size of the MMF industry on the money market.

[4]The results in the numerical examples depend mainly on the elasticity of the investors' demand for intermediation, i.e., on their attitude towards risk, and on the elasticity of the supply of the risky asset. This second elasticity depends on whether the money market can rely on market participants other than MMF to supply liquidity. The higher the relative importance of MMFs as suppliers of liquidity, the more important the general equilibrium effects would be.

2. Related Literature

Though a model of MMFs has not been developed in the literature, several papers have analyzed the MMF industry empirically.[5] Outflows from MMFs, and their correlation with the risk taken by the funds, have been shown in McCabe (2010), Kacperczyk and Schnabl (2013) and Schmidt, Timmermann, and Wermers (2014). Chernenko and Sunderam (2014) find large outflows from MMFs with exposures to Eurozone banks in 2011 and analyze the spillover effects of these runs on the provision of liquidity from MMFs to non-European firms. Chen, Goldstein, and Jiang (2010) show the presence of strategic complementarities in the redemption behavior of investors in all mutual funds, including MMFs. These strategic complementarities, which are not the focus of my paper, make MMFs prone to self-fulfilling runs such as those considered by Diamond and Dybvig (1983) and analyzed extensively in the banking literature.

Hanson, Scharfstein, and Sunderam (forthcoming) provide an extensive analysis of the reform proposals for MMFs focusing mainly on their ability to stop investor runs. McCabe, Cipriani, Holscher, and Martin (2012) propose a minimum balance at risk rule to reduce the investors' incentives to redeem MMF shares quickly when a fund is in distress and, therefore, mitigate the vulnerability of MMFs to investor runs. While these papers focus on investor runs, my paper offers a complementary analysis by focusing on intermediary runs on the money market triggered by strategic complementarities in sponsor support.

The frequency and magnitude of sponsor support in the MMF industry is considered by Moody's Investors Service (2010) and Brady, Anadu and Cooper (2012). Empirical evidence exists that differences in the sponsor's ability and incentives to offer support affect the portfolio choices of MMFs. McCabe (2010) finds that sponsor support was more likely for riskier MMFs

[5] Ennis (2012) analyzes MMFs as providers of liquidity (as banks) and as investment managers (as mutual funds).

and for funds with bank-affiliated sponsors. Kacperczyk and Schnabl (2013) show that, between 2007 and 2010, funds sponsored by financial intermediaries with more money market funds business took on more risk. Though the risk of investor runs on MMFs is closely related to the stability of the NAV, Gordon and Gandia (2014) find, consistent with Chen, Goldstein and Jiang (2010), that both constant and floating NAV MMFs in Europe experienced investor runs in September 2008 following the Lehman Brothers bankruptcy and that the sponsor's capacity to support the fund was relevant in explaining the outflows in both types of funds. The evidence in these papers suggests that sponsor support plays a very important role in the way in which MMFs work ex ante, by influencing the funds' portfolio choices, and ex post, as a mechanism to prevent their liquidation. Understanding these links, and their implications for the stability of the MMF industry, is the main goal of my paper.

Finally, the strategic complementarities highlighted in this paper are associated with pecuniary externalities. However, these externalities are different from the terms-of-trade and collateral externalities defined in Davila (2015). Terms-of-trade externalities are present when marginal rates of substitution between agents are not equalized across states in equilibrium as in, for example, Lorenzoni (2008) and He and Kondor (forthcoming). Collateral externalities appear when credit constraints depend on equilibrium asset prices as in Bianchi (2011) and Stein (2013). In contrast, the pecuniary externalities in this paper emerge from the interaction between coordination failures, in the spirit of Cooper and John (1988), and the equilibrium in the asset market.

In the next section, I introduce the model. In Section 3, I define and characterize the equilibrium. In Section 4, I discuss the forces at play in the model. Section 5 contains the policy analysis and I conclude in Section 6. The Appendix contains some detailed institutional features of MMFs and all the proofs.

3. Model

In this section, I present the model's environment and discuss its assumptions.

3.1. Environment

The model has three periods, $t = 0, 1, 2$, and one good. There are two types of agents in this economy: investors and managers. There is a continuum of measure 1 of each type of agent. Investors are risk-averse with preferences given by $\mathbb{E}\left[\log\left(W_2^I\right)\right]$, where W_2^I is the investor's wealth at $t = 2$. Investors are endowed with W_0^I units of the good at $t = 0$. Managers are risk-neutral with preferences given by $\mathbb{E}\left[W_2^M\right]$ where W_2^M is the manager's wealth at $t = 2$. Managers are endowed with W_0^M units of the good at $t = 0$ and $E > 0$ at $t = 1$.

There are two types of assets: a short-term safe asset and a long-term risky asset. The safe asset is a one-period bond supplied perfectly elastically at price q_t in periods $t = 0, 1$. One unit of the safe asset bought at t pays 1 unit of the good at $t + 1$. The risky asset is a two-period asset. Each unit of the asset bought at $t = 0$ has a random payoff d at $t = 2$. The payoff structure is

$$d = \begin{cases} \bar{d} > 0 & \text{with probability } \pi \\ 0 & \text{with probability } (1-\pi). \end{cases} \quad (1)$$

The probability π is a random variable whose realization is observed by everyone at $t = 1$. π is uniformly distributed over $[\underline{\pi}, \bar{\pi}]$ with $0 \leq \underline{\pi} < \bar{\pi} < 1$. This probability π can be interpreted as the quality of the risky asset.

The risky asset is traded in competitive markets at $t = 0, 1$, at prices p_0 and $p_1(\pi)$, respectively. At $t = 0$, the supply of the risky asset is given by $S(p_0)$, where $S'(p_0) > 0$. The supply of the asset in period 1 is fixed and equal to the amount of the risky asset traded at $t = 0$. Consistent with the regulation of MMFs, I assume that no short-selling is allowed.

To introduce motives for intermediation, I follow the limited market participation literature and assume that investors can invest directly only in the safe asset. As in He and Krishnamurthy (2012), I allow managers to invest in the risky asset on behalf of investors and act as financial intermediaries.

Managers can choose whether to become intermediaries. If they choose not to become intermediaries, they manage their own wealth. If they choose to offer intermediation services, they can manage assets on behalf of one investor, at most, by opening a fund. The intermediation relation is long-term. Once an investor chooses a manager with whom to invest at $t = 0$, he cannot invest with other managers at $t = 1$. Managers incur a fixed cost $C > 0$ if they manage funds for an investor. In each period t, a fund consists of the manager's wealth, W_t^M, and the amount the manager is managing for the investor, A_t^I. The manager makes all portfolio decisions on behalf of the fund and these decisions are non contractible. For simplicity, I assume that the manager has to invest his own wealth in the same way in which he invests the investor's wealth, i.e., managers can manage only one portfolio at a time.

The intermediation contract between a manager and his investor is the following. The manager charges a fraction f of the assets under management each period in return for intermediation services. For example, if a manager manages A_0^I for an investor in period 0 and A_1^I in period 1, he collects fees fA_0^I at $t = 0$ and fA_1^I at $t = 1$. By investing with a manager, an investor becomes a shareholder of the fund run by the manager owning $(1 - f)\, A_t^I$ shares out of a total of $\left(W_t^M + A_t^I\right)$ outstanding shares of the fund. At $t = 1$, managers and investors can readjust their portfolios. An investor chooses how much of his wealth, including his shares in the fund, to invest with his manager and how much to invest in the safe asset. A manager chooses how much of his assets under management to invest in the risky and safe assets. The value of a share, or NAV, at $t = 1$ is the value of the fund's assets at $t = 1$ divided by the total number of outstanding shares.

To capture the breaking-the-buck feature of MMFs, I assume that the fund is liquidated if the value of the investor's share goes below x in period 1 where $x < 1/q_0$. Upon early liquidation of the fund, the manager suffers a spillover loss $B\pi$, $B \in \left[0, \bar{B}\right]$, and he cannot offer intermediation services at $t = 1$. The magnitude of the spillover losses, B, is not known at $t = 0$. However, each manager has a probability $G\left(B_0\right)$ of facing a spillover loss smaller than $B_0\pi$ at $t = 1$. Because there is a continuum of managers, the law of large numbers implies that spillover losses across managers are distributed according to the cumulative distribution function $G\left(B\right)$.

Liquidation is costly for managers, both in terms of forgone fees and spillover losses $B\pi$. Therefore, a manager can choose to offer support to his investor to maintain the value of the investor's shares at x. Finally, I assume that the intermediation industry is competitive and that all the surplus of a match between an investor and a manger goes to the investor.

Regarding the timing of the model, at $t = 0$, each manager chooses whether to offer intermediation services. If a manager chooses to become an intermediary, he chooses the fund's portfolio. Otherwise, he chooses how to invest his own wealth. Each manager chooses which fraction of his assets under management to invest in the risky asset, a_0^M, and which to invest in the safe asset, $\left(1 - a_0^M\right)$. At the same time, investors make their portfolio decision. They choose a fraction a_0^I of their wealth to invest with their manager and a fraction $\left(1 - a_0^I\right)$ to invest in the safe asset. Simultaneously, the price of the risky asset, p_0, is determined in a centralized, competitive market.

At $t = 1$, the probability of success of the risky asset, π, is realized. After observing this probability, portfolio and support decisions are made simultaneously and, meanwhile, the price of the risky asset is determined in a competitive market. Each manager chooses a fraction a_1^M of his fund's assets to invest in the risky asset, a fraction $\left(1 - a_1^M\right)$ to invest in the safe asset, and the probability with which to offer support to investors, $s\left(\pi\right) \in [0, 1]$. If a fund is not liquidated early, the investor chooses a fraction a_1^I of his wealth to invest with his manager and a fraction $\left(1-a_1^I\right)$ to invest in the safe asset. Finally, at $t=2$, the payoff of the risky asset is realized and all funds are liquidated. Fig. 2 depicts this timeline.

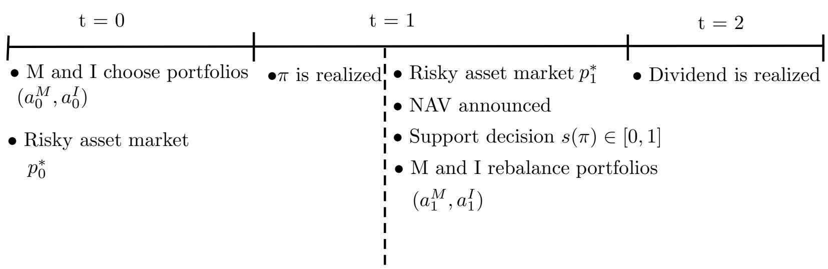

Fig. 2. Timeline. In the figure, m =manager; I =investor; NAV=net asset value.

3.2. *Discussion of assumptions*

Preferences The functional form for the utility functions makes the model tractable. The managers' risk neutrality makes the calculation of the support decision independent of wealth levels other than through the managers' ability to offer support. In the same way, investors' constant relative risk aversion implies that the portfolio choice in the interim period depends only on the quality of the risky asset, π, and on its price. This investment decision is independent of the investors' wealth and, therefore, of the support decision and the choices at $t=1$. These simplifications make it possible to have closed form solutions for liquidation prices and support thresholds.

Asset structure The asset structure is meant to capture the maturity mismatch problem inherent to MMFs and the trade-off between maturity, risk, and return. The short-term safe asset can be interpreted as a Treasury bill, and the long-term risky asset can be thought of as commercial paper. The uncertainty structure embedded in the payoff of the risky asset implies that there are two sources of risk for commercial paper. On one hand, the quality of commercial paper is subject to shocks before the maturity date (downgrades). On the other hand, at maturity, commercial paper is subject to default. The assumption of no short-selling is consistent with the regulation of MMFs.

Contract This model differs from other banking and delegated portfolio models in the intermediation contract considered, which incorporates key features of MMFs. This model captures the demandable nature of shares and the open-endedness of MMFs through the ability of investors to adjust their portfolio in the interim period. The liquidation threshold, x, introduced as part of the intermediation contract, gives managers incentives to keep the net asset value stable and captures the eventual liquidation of a fund after a breaking-the-buck event. The cost of early liquidation of a fund imposed on managers, $B\pi$, is meant to capture the spillover losses a sponsor could suffer if one of his MMFs breaks the buck. Finally, the possibility of sponsor support is introduced by allowing the managers, who fulfill both the role of fund managers and sponsor companies in the model, to transfer resources to investors to prevent the liquidation of the fund.

The MMF industry first appeared in the 1970s as an instrument to offer liquidity to investors while providing them a small return on their money. During this period, banks were subject to regulations that imposed caps on the interest rate that could be paid by checking accounts. This regulation applied only to banks, which made MMFs a convenient substitute to bank deposits.[6] Though the regulations that gave rise to the MMF industry have been lifted, banks face other restrictions, such as capital requirements, that could give MMFs a role from a regulatory arbitrage point of view. To capture the role of MMFs as a tool for regulatory arbitrage, throughout the paper I assume that the function of MMFs is to create safe assets for investors while providing them access to the money market. This is encompassed in the choice of intermediation contract.[7]

[6]For a discussion on the origin of MMFs, see chapter 1 in Carnell, Macey, and Miller (2008) and Rosen and Katz (1983).

[7]Given the origin of MMFs, the intermediation contract considered abstracts from opti-

Spillover losses. A sponsor's willingness to support an MMF depends on the size and scale of the other funds under the sponsor's scope at the time when support is needed. Sponsors of different sizes have different incentives to support an MMF for the same realized NAV. The heterogeneity in the magnitude of spillover losses, B, is meant to capture the heterogeneity across MMFs' sponsors.[8]

Liquidation threshold The assumption that $x < 1/q_0$ implies that a manager can always choose a risky portfolio that is still conservative enough to always avoid breaking the buck. If the only way for the manager to avoid breaking the buck in all states π was to invest in a safe portfolio, managers would have to choose between breaking the buck with positive probability or not opening a fund at all. Because investors can invest in the safe asset on their own, they would not pay fees to a manager who chose a safe portfolio.

Intermediation relation Investors must bear costs associated with the liquidation of an MMF, mainly due to the delay in the availability of their funds. For example, after the liquidation of the Reserve Primary Fund, it took investors up to 18 months to get their money out of the fund. During this time, the fraction of the investors' wealth that was held up in the fund earned almost no return. An investor's exclusion from the risky asset market after the liquidation of the fund is meant to capture this delay. This extreme assumption is made for ease of exposition. Qualitatively, all results in the model would hold qualitatively if investors were allowed to switch managers at $t = 1$ by incurring a positive real cost.

4. Equilibrium

The return of a unit invested with a manager depends on the asset prices and on the manager's portfolio and support decisions. Hence, to decide

mal compensation structures in portfolio management relations such as the ones analyzed by a large literature started by Bhattacharya and Pfleiderer (1985).

[8] I thank an anonymous referee for this suggestion.

how much to invest with a manager, investors need to anticipate not only the equilibrium asset prices, but also their managers' actions. In the same way, a manager's payoffs depend on the amount of resources he manages for investors. Therefore, managers need to anticipate asset prices and their investors' actions when making choices. This is captured in the definition of equilibrium.

Definition 1. *A symmetric equilibrium is a set of price functions* $\{p_0^*, p_1^*(\pi)\}$, *functions for portfolio choices* $\left\{a_0^{M*}, a_0^{I*}, a_1^{M*}(\pi), a_1^{I*}(\pi)\right\}$, *a support probability function* $s^*(\pi, B) \in [0,1]$, *and a fee* f^* *such that* $\left(a_0^{I*}, a_1^{I*}(\pi)\right)$ *solve each investor's problem, taking prices and his manager's decisions as given,* $\left(a_0^{M*}, a_1^{M*}(\pi), s^*(\pi, B)\right)$ *solve each manager's problem, taking prices and his investor's decisions as given, the risky asset market clears at* $t = 0, 1$, *and managers are indifferent between managing an investor's funds and not doing so.*

The equilibrium can be computed by backward induction. I divide the characterization of equilibrium in two parts: time 0 and time 1. I characterize the agents' problems at $t = 1$ for a given realization of π, given portfolio choices a_0^M and a_0^I and fees f. Then, I move to the decisions at $t = 0$, which take into account the equilibrium choices at $t = 1$.

4.1. t=1

All decisions at $t = 1$ depend on the choices and equilibrium objects determined at $t = 0$, that is, on a_0^M, a_0^I, f, and p_0. To simplify the notation, I mostly ignore this dependence when writing down equilibrium objects in the subgame at $t = 1$.

4.1.1. Manager's portfolio choice at t=1

At $t = 1$, a manager with wealth W_1^M who receives fees fA_1^I chooses a fraction a_1^M of his wealth to invest in the risky asset to maximize his expected wealth. He solves

$$\max_{a^M_{1,i}\in[0,1]} \mathbb{E}_d \left[\left(a_1^M \frac{d}{p_1} + \left(1 - a_1^M\right) \frac{1}{q_1} \right) \left(W_1^M + f A_1^I \right) \middle| \pi \right]. \tag{2}$$

The manager's risk neutrality implies that his portfolio choice depends only on the expected returns of the assets in which he can invest. If the expected return of the risky asset is larger than that of the safe asset, i.e., if $\pi\bar{d}/p_1 > 1/q_1$, he invests everything in the risky asset, i.e., $a_{1,i}^{M*} = 1$. Analogously, if the return of the safe asset is larger than the expected return of the risky asset, i.e., if $\pi\bar{d}/p_1 < 1/q_1$, a manager invests everything in the safe asset, i.e., $a_{1,i}^{M*} = 0$. If the safe and risky assets pay the same in expectation, i.e., if $\pi\bar{d}/p_1 = 1/q_1$, the manager is indifferent between any portfolio. The manager's portfolio choice at $t = 1$ is independent of his own wealth and of the amount investors invested with him. Moreover, the spillover losses are irrelevant for the portfolio decision at $t = 1$. However, the realized value of B affects the managers' support decision and, thus, the managers' funds under management.

4.1.2. Investor's problem at t=1

At $t = 1$, an investor chooses a fraction $a_{1,i}^I$ of his wealth to invest with his manager i to maximize his own expected wealth at $t = 2$. The investor's portfolio decision depends on the behavior he anticipates for his manager, which, in turn, depends only on the observable state π. Therefore, an investor with wealth W_1^I who anticipates that his manager will choose to invest a fraction $a_{1,i}^M$ in the risky asset solves

$$\max_{a^I_{1,i}\in[0,1]} \mathbb{E}_d \left[\log \left(\left(a_{1,i}^I (1-f) \left(a_{1,i}^M \frac{d}{p_1} + \left(1 - a_{1,i}^M\right) \frac{1}{q_1} \right) + \left(1 - a_{1,i}^I\right) \frac{1}{q_1} \right) W_1^I \right) \middle| \pi \right]. \tag{3}$$

An investor would never pay fees for something he could do himself. Therefore, investors choose not to invest with their manager if $\pi\bar{d}/p_1 \leq 1/q_1$. Furthermore, they invest with their manager only if the expected return of doing so is greater than the return of the safe asset, i.e., if $\pi(1-f)\bar{d}/p_1 > 1/q_1$. Therefore, the optimal portfolio choice for investors is

$$a_{1,i}^{I}(p_1,\pi) = \frac{\max\left\{\pi(1-f)\frac{\bar{d}}{p_1} - \frac{1}{q_1}, 0\right\}}{(1-f)\frac{\bar{d}}{p_1} - \frac{1}{q_1}}. \tag{4}$$

As it is usual with constant relative risk aversion (CRRA) preferences, the share of wealth invested with the manager is independent of the wealth level. This assumption makes the problem easier to track. To keep notation simple, I define $A_{0,i}^{i} \equiv a_{0,i}^{I} W_0^{I}$ and $A_{1,i}^{I}(p_1,\pi) \equiv a_{1,i}^{I}(p_1,\pi) W_{1,i}^{I}(p_1)$.

4.1.3. Demand for risky assets at t=1

The demand for the risky asset at $t=1$ of an individual manager i with wealth $W_{1,i}^{M}(p_1)$ is

$$D^{i}(p_1,\pi) = \begin{cases} a_{1,i}^{M}(p_1,\pi)\left(A_{1,i}^{I}(p_1,\pi) + W_{1,i}^{M}(p_1)\right)/p_1 & \text{if the fund continues} \\ a_{1,i}^{M}(p_1,\pi) W_{1,i}^{M}(p_1)/p_1 & \text{if the fund is liquidated,} \end{cases} \tag{5}$$

where $W_{1,i}^{M}(p_1)$ is the wealth level for manager i's investor when the price is p_1. If the fund continues at $t=1$, the manager manages his own wealth and whatever his investor gives him, $A_1^{I}(p_1,\pi)$. If the fund is liquidated, a manger will only manage his own wealth. The manager's wealth at time 1 will depend on the return of his portfolio at $t=1$, on the incoming fees, on his endowment E, and on whether he offers support to his fund.[9]

Given these individual demands for the risky asset, the equilibrium liquidation price $p_1^{*}(\pi)$ is such that

$$p_1^{*}(\pi)\int\left[D^{i}(p_1^{*}(\pi),\pi) - a_{0,i}^{M}\left(a_{0,i}^{I}W_0^{I} + W_0^{M}\right)/p_0\right]di = 0, \tag{6}$$

where the last term inside brackets is the supply of the risky asset, which is

[9]Expressions for these wealth levels can be found in the Appendix in Section **??**.

fixed and equal to the total amount traded at $t = 0$.

The wealth levels of managers and investors depends, not only on whether the fund continues, but also on whether sponsor support is offered. Therefore, to compute the equilibrium price as a function π one needs to consider four possible cases: π such that no sponsor support is needed, π such that all sponsors offer support if needed, π such that a fraction $\bar{s}^*(\pi)$ of sponsors offers support when it is needed, and π such that sponsor support is needed to prevent the liquidation of the funds, but it is not provided. Then, the equilibrium liquidation price of the risky asset is

$$p_1^*(\pi) = \begin{cases} p_1^{NS}(\pi) & \text{if no support is needed} \\ p_1^{S}(\pi) & \text{if all sponsors offer support always } \bar{s}^*(\pi) = 1 \\ p_1^{SS}(\pi) & \text{if only a fraction } \bar{s}^*(\pi) \in (0,1) \text{ offers support} \\ p_1^{L}(\pi) & \text{if support is needed but no sponsor offers it } \bar{s}^*(\pi) = 0. \end{cases} \tag{7}$$

The characterizations of the price functions $p_1^{NS}(\pi)$, $p_1^{S}(\pi)$, $p_1^{SS}(\pi)$, and $p_1^{L}(\pi)$ can be found in the Appendix.

Cash-in-the-market pricing and aggregate liquidity The extra endowment E that managers get at time 1 represents the aggregate liquidity in the market. For low values of E, the liquidity in the asset market at $t = 1$ is low. In this case, the marginal buyers of the asset are the risk-averse investors and the liquidation price of the risky asset involves a fire sale discount relative to its expected return. In this case, there is be cash-in-the-market pricing and the liquidation price of the asset depends on whether funds remain open or not.

For high values of E, the managers' demand for the risky asset in the interim period is high and risk-neutral managers are the marginal buyers of the asset in the secondary market. Therefore, the liquidation cost of the asset is equal to its expected return. In this case, the secondary market for the asset is perfectly liquid and risk-averse investors would rather redeem their shares at $t = 1$ than pay fees to managers for an additional period and have to bear the asset's dividend risk between $t = 1$ and $t = 2$. In this case, the managers are providing a service to the investors only from $t = 0$ to $t = 1$. From $t = 1$ to $t = 2$, managers are superfluous as all the liquidity that the investors need is provided by the market.

Formally, the cash-in-the-market pricing regime arises for parameter values such that

$$\min_{\pi \in [\underline{\pi}, \bar{\pi}]} \left\{ \pi \bar{d} q_1 - p_1^* (\pi) \right\} < 0. \tag{8}$$

The equilibrium liquidation price of the asset is an increasing function of the extra endowment managers get at $t = 1$. Moreover, $\lim_{E \to \infty} p_1^* (\pi) = \pi \bar{d} q_1$ for all π and $\lim_{E \to 0} p_1^* (\pi) < \pi \bar{d} q_1$. Thus, given the choices at $t = 0$, there exists a threshold $\bar{E}$ such that there is cash-in-the-market in the asset market at $t = 1$ for $E < \bar{E}$.

For the remainder of the paper I focus on the case in which liquidity is scarce and fire sales occur.

4.1.4. Equilibrium support decision

In a symmetric equilibrium, all managers choose the same portfolio at $t = 0$. However, their support strategies depend on their realized spillover losses $B\pi$. If no manager needs to offer support, the relevant price is p_1^{NS}; if all managers strictly prefer to offer support it is p^S; if only a fraction of managers chooses to offer support the price it is $p_1^{SS} (\pi)$; and if all funds are liquidated, it is p^L.

A manager's support decision depends on the equilibrium price through the market value of the shares or NAV. The NAV, $n \left(p_1, a_0^M \right)$, is the return on the portfolio of a manager who chooses to invest a fraction a_0^M of the fund in the risky asset when the liquidation price of the risky asset is p_1, i.e., $n \left(p_1, a_0^M \right) = a_0^M p_1 / p_0 + \left(1 - a_0^M \right) / q_0$. As long as $n \left(p_1, a_0^M \right) \geq x$, the manager does not have any need to offer support for his investor. This is the case whenever the risky asset's quality is high enough.

If support is needed for a fund to continue, given p_1, a manager with spillover losses $B\pi$ chooses his support decision to solve

$$\max_{s\in[0,1]} s\frac{\pi\bar{d}}{p_1}\left(n\left(p_1,a_0^M\right)\left(W_0^M+fA_0^I\right)+E+fA_1^I\left(p_1,\pi\right)-\left(x-n\left(p_1,a_0^M\right)\right)\left(1-f\right)A_0^I\right)$$
$$+\left(1-s\right)\left(\frac{\pi\bar{d}}{p_1}\left(n\left(p_1,a_0^M\right)\left(W_0^M+fA_0^I\right)+E\right)-B\pi\right)$$

subject to

$$n\left(p_1,a_0^M\right)\left(W_0^M+fA_0^I\right)+E+fA_1^I\left(p_1,\pi\right)\geq\left(x-n\left(p_1,a_0^M\right)\right)\left(1-f\right)A_0^I, \tag{9}$$

where s is the manager's support decision in state π. Because $p_1(\pi)\leq\pi\bar{d}/q_1$, the expected return of the manager's investment at $t=1$ is $\pi\bar{d}/p_1(\pi)$. If the manager offers support in state π, i.e., if $s=1$, he manages a portfolio made up of his shares in the fund, $n\left(p_1,a_0^M\right)\left(W_0^M+fA_0^I\right)$, his endowment in period 1, E, the incoming fees from keeping the fund open, fA_1^I, minus the cost of offering support, $\left(x-n\left(p_1,a_0^M\right)\right)(1-f)A_0^I$. If he does not offer support when support is needed to keep the fund open, he invests his shares of the fund and his endowment but he suffers a loss $B\pi$ from liquidating the fund early, the manager's support decision does not only depend on his willingness to offer support. The amount of support a manager is able to offer is determined by the amount of resources he has available at $t=1$. This is captured by constraint (9).

The maximization implies that the manager chooses to offer support to the investors if the benefit of offering support offsets the costs of doing so. That is, if support is needed and

$$\frac{\pi\bar{d}}{p_1(\pi)}\left(\left(x-n\left(p_1(\pi),a_0^M\right)\right)(1-f)A_0^I-fA_1^I\left(p_1(\pi),\pi\right)\right)$$
$$\leq\ \min\left\{B\pi,\frac{\pi\bar{d}}{p_1(\pi)}\left(n\left(p_1(\pi),a_0^M\right)\left(W_0^M+fA_0^I\right)+E\right)\right\}. \tag{10}$$

The left-hand side of this expression represents the opportunity cost of offering support to the investor, net of incoming fees. The right-hand side includes both the manager's willingness and ability to offer support. The first term inside the curly brackets captures the manager's willingness to offer support. Since the manager loses $B\pi$ if the fund is liquidated early, he is willing to offer support, on top of the incoming fees, to up to an amount equal to his loss, i.e., $B\pi$. The second term, captures the manager's ability to offer support. The manager is able to offer support only up to the amount of resources he owns. To offer support, the manager has to be both willing and able to offer support. This is captured by the minimum operator.

Eq. 10 characterizes the managers' support decision. The higher the spillover losses $B\pi$, the easier it is for Eq. 10 to be satisfied, i.e., the larger the managers' incentives to offer support.

Lemma 1. *Given a price function $p_1(\pi)$, for each realization of π there exists a threshold $\hat{B}(\pi, p_1(\pi))$ such that*

$$s(\pi, B) = \begin{cases} 0 & \text{if } B < \hat{B}(\pi, p_1(\pi)) \\ 1 & \text{if } B > \hat{B}(\pi, p_1(\pi)) \end{cases} \tag{11}$$

and the total fraction of managers offering support is given by $1 - G\left(\hat{B}(\pi, p_1(\pi))\right)$.

The proof of Lemma 1 follows directly from Eq. 10. Because the right-hand side of this inequality is increasing in B, if a manager with spillover losses of magnitude $\hat{B}$ chooses to offer support, all managers with $B > \hat{B}$ choose to offer support. Analogously, if a manager with spillover losses of magnitude $\hat{B}$ chooses not to offer support, all managers with $B < \hat{B}$ choose not to offer support.

Moreover, as can be seen from Eq. 10, the manager's support decision depends on the liquidation price of the risky asset and, through it, on the other managers' support decisions.

Proposition 1. *Given a portfolio choice for managers at $t = 0$, a_0^M, there exist thresholds $\pi_x\left(a_0^M\right)$, $\pi^*\left(a_0^M\right)$ and $\pi^{**}\left(a_0^M\right)$ such that in a symmetric* *equilibrium of the subgame at* $t = 1$

(i) all managers choose not to offer support and all funds remain open if $\pi_x\left(a_0^{M*}\right) \leq \pi \leq \bar{\pi}$*,*

(ii) all managers offer support and all funds remain open if $\pi^*\left(a_0^{M*}\right) \leq \pi < \pi_x\left(a_0^{M*}\right)$*, and*

(iii) all managers choose not to offer support and all funds are liquidated if $\underline{\pi} \leq \pi < \pi^{**}\left(a_0^{M}\right)$*.*

The proof of Proposition 1 relies on the monotonicity and affinity of $p_1^{NS}(\pi)$, $p_1^S(\pi)$, and $p_1^L(\pi)$ in π. From lemma 1, $\pi^*\left(a_0^{M*}\right)$ is such that $\hat{B}\left(\pi^*\left(a_0^{M*}\right), p_1^S\left(\pi^*\left(a_0^{M*}\right)\right)\right) = 0$. In other words, the threshold $\pi^*\left(a_0^{M*}\right)$ is the minimum value of π such that all managers have incentives to offer support if they expect all other managers to offer support and, hence, they expect the price to be $p_1^S(\pi)$. Analogously, $\pi^{**}\left(a_0^M\right)$ is such that $\hat{B}\left(\pi^{**}\left(a_0^M\right), p_1^L\left(\pi^{**}\left(a_0^M\right)\right)\right) = \bar{B}$. The threshold $\pi^{**}\left(a_0^M\right)$ is the maximum asset quality such that all managers choose not to offer support if they expect all other managers not to offer support and they expect the price to be $p_1^L(\pi)$. The Appendix characterizes the thresholds $\pi_x\left(a_0^M\right)$, $\pi^*\left(a_0^M\right)$, and $\pi^{**}\left(a_0^M\right)$ further.

Corollary 1. *(a) If* $\pi^{**}\left(a_0^M\right) < \pi^*\left(a_0^M\right)$*, there is a unique equilibrium in the subgame at* $t = 1$ *for each value of* π*. For* $\pi \in \left(\pi^{**}\left(a_0^M\right), \pi^*\left(a_0^M\right)\right)$*, only a fraction* $\bar{s}^*(\pi) = 1 - G\left(\hat{B}\left(\pi, p_1^{SS}(\pi)\right)\right)$ *of managers offers support when it is needed.*

(b) If $\pi^*\left(a_0^M\right) < \pi^{**}\left(a_0^M\right)$*, there are multiple equilibria in the subgame at* $t = 1$ *when* $\pi \in \left[\pi^*\left(a_0^M\right), \pi^{**}\left(a_0^M\right)\right]$*. There is an equilibrium in which all managers offer support, one in which all managers choose not to offer support, and one in which only a fraction* $\bar{s}^*(\pi) \in (0,1)$ *of managers offers support when it is needed*

Corollary 1 follows from Proposition 1. If there is a unique equilibrium for all values of π, i.e., if $\pi^{**} \leq \pi^*$, then some managers could choose to offer support while others choose not to do so In a symmetric equilibrium, all managers have the same resources to offer support because they invested in the same portfolio at $t = 0$. Therefore, differences in managers' support decisions are based purely on differences in their incentives to offer support. Managers with higher spillover losses face higher costs of liquidating the fund and, thus, have higher incentives to support their investors. Therefore, managers with high enough spillover losses, i.e, with $B > \hat{B}(\pi, p_1(\pi))$, offer support and those with $B < \hat{B}(\pi, p_1(\pi))$ are better of liquidating their fund. For $\pi \in (\pi^{**}, \pi^*)$, $\hat{B}\left(\pi, p_1^{SS}(\pi)\right) \in \left(0, \bar{B}\right)$ for all and, thus, $\bar{s}^* \in (0, 1)$.

Part (b) of Corollary 1 highlights the strategic complementarities that can arise in the sponsors' support decisions. If all managers choose (not) to offer support, the net demand for the risky asset is high (low) and the equilibrium liquidation price for the risky asset is high (low). Given this high (low) price, the cost of offering support is low (high) for an individual managers and, thus, he has more (less) incentive to offer support. This strategic complementarity between the managers' support decisions through the equilibrium liquidation price of the asset is present in the model only because support is voluntary. For the remainder of this paper, I assume that whenever there are multiple equilibria, managers will coordinate in the equilibrium in which all managers offer support.

Remark 1.*If the liquidity in the asset market is high enough and there is no cash-in-the-market, the price of the risky asset is independent of the fraction of funds that remain open. In this case, no complementarities exist in the sponsors' support decisions and the equilibrium in the subgame at $t = 1$ is unique.*

Runs on the risky asset market The managers' support decisions determine the demand for the risky asset in the interim period and, through it, the liquidation price of the risky asset. If a manager chooses not to offer support when support is needed to keep the fund open, his fund breaks the buck and is liquidated. Upon liquidation, the manager cannot offer intermediation services at $t = 1$ and his investor is excluded from the risky asset market. This implies that the manager's demand for the risky asset is going to be lower if he chooses not to offer support than if he keeps the fund open and manages funds for his investor.[10] If all managers choose not to offer support, the total demand for the risky asset in the interim period and, therefore, the liquidation price of the risky asset will be lower than if all funds remained open.

At the same time, the managers' support decisions depend on the liquidation price of the risky asset. This liquidation price determines the costs and the benefits of offering support [as can be seen from Eq. 10]. To illustrate this mechanism, suppose there are no spillover losses, i.e., $B = 0$ for all managers. On the one hand, a low liquidation price of the risky asset translates into a low NAV and a high cost of offering support. On the other hand, a low price of the risky asset implies a high expected return of investing in the risky asset and increases investors' incentives to invest with managers. The higher volume of intermediation increases the fees earned by the managers, as well as the benefits of offering support and avoiding the liquidation of the fund. If the increase in the cost dominates the increase in the benefit of offering support, there could be strategic complementarities in the managers' support decisions and multiple equilibria can arise. From Corollary 1, these complementarities arise when $\pi^* < \pi^{**}$.

When $\pi^* < \pi^{**}$, if an individual manager expects all other managers (not) to offer support and liquidate their funds, he expects a high (low) demand for the risky asset in the interim period, a high (low) liquidation price of the risky asset, and a low (high) cost of offering support, which increases (decreases) the manager's incentives to offer support. This source of complementarity gives rise to self-fulfilling equilibria that can lead to runs on the risky asset

[10] This assumes that the amount of support does not exceed the total amount investors invest with their managers if they receive support.

market. These runs are different from the canonical bank runs. They are not runs of investors on intermediaries, but runs of intermediaries on each other through fire sales in the risky asset market.[11]

4.1.5. Equilibrium price

Given the equilibrium support decision for the managers and the portfolio choice a_0^M, one can compute the equilibrium liquidation price of the risky asset.

Assumption 1. *$\bar{B}$ is such that the amount of support offered does not exceed the total amount investors invest with their manager when they receive support, i.e.,*

$$A_1^I < fA_1^I + \min\left\{\bar{B}\frac{p_1^L}{\bar{d}}, n\left(p_1^L, a_0^M\right)\left(W_0^M + fA_0^I\right) + E\right\}. \qquad (12)$$

This assumption holds for $\bar{B} = 0$ and for low enough values of $\bar{B}$. Furthermore, it implies that the demand for the risky asset is lower when sponsors decide not to offer support than when they do, and it guarantees the monotonicity of the price function.[12] For the remainder of the paper, I proceed as though this assumption holds.

Proposition 2. *Under Assumption 1, the price function $p_1^*\left(\pi, a_0^M\right)$ is non-decreasing π for all $\pi \in [\underline{\pi}, \bar{\pi}]$.*

The liquidation price of the risky asset is higher the higher the probability of success π. A higher realization of π implies a higher expected return of investing in the risky asset and a higher demand for it. Because the supply

[11]The possibility of fire sales arises due to the cash-in-the-market pricing in the risky asset market as in Allen and Gale (1994). See Shleifer and Vishny (2011) for a review of fire sales in the finance and macroeconomics literature.

[12]This assumption does not seem unrealistic. Between 2007 and 2010, the maximum amount of support offered by a sponsor accounted for 3% of the fund.

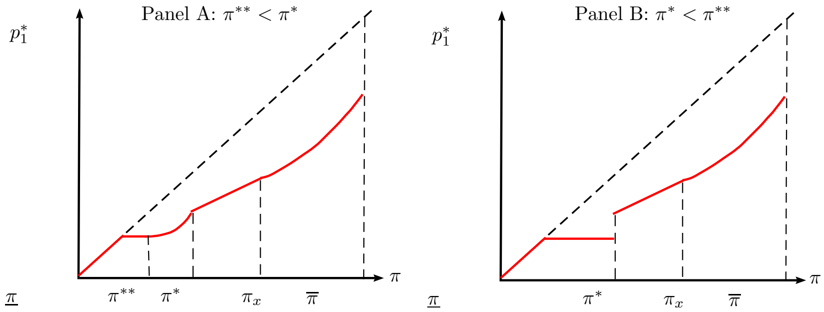

Fig. 3. Price function. The red solid line is the liquidation price of the risky asset at $t = 1$. The black dotted line is the expected discounted dividend paid by the risky asset. Panels (A) and (B) refer to the cases in which the subgame at $t = 1$ has a unique equilibrium or multiple equilibria, respectively.

of the risky asset at $t = 1$ is fixed, the equilibrium price has to increase with π for the market to clear.

Fig. 3 depicts the price function when (a) $\pi^{**} < \pi^*$ and when (b) $\pi^* < \pi^{**}$ (assuming that managers coordinate on the support equilibrium). When all funds are liquidated, i.e., $\pi < \min\{\pi^{**}, \pi^*\}$, only managers participate in the risky asset market. Since managers are risk-neutral, they either invest everything they have in the risky asset or are indifferent between investing in the risky asset and in the safe asset. If the demand for the risky asset is determined by the cash held by managers, i.e., by $\left(1 - a_0^M\right)\left(W_0^M + fA_0^I\right) + E$, the market clearing price is given by p_1^L, such that

$$\frac{\left(1 - a_0^M\right)\left(W_0^M + fA_0^I\right) + E}{p_1^L} = (1 - f)\, a_0^M A_0^I, \tag{13}$$

where the right-hand side is the amount of the risky asset held by investors indirectly through their shares. Moreover, in equilibrium, $p_1^*(\pi) \leq \bar{d}\pi q_1$. Otherwise, because the supply of the risky asset at $t = 1$ is fixed and positive, the demand for the risky asset at $t = 1$ would be zero and the market would not clear. Thus, if the funds are liquidated, the equilibrium liquidation price is $p_1^*(\pi) = \min\left\{\bar{d}\pi q_1, p_1^L\right\}$. If $p_1^*(\pi) = \bar{d}\pi q_1$ when liquidation occurs,

$a_1^M(p_1, \pi)$ adjusts to clear the market.

If at least some funds continue operating at $t = 1$, i.e., $\pi \geq \min\{\pi^{**}, \pi^*\}$, investors can invest in the risky asset, albeit indirectly. In this case, the demand for the risky asset also depends on the investors' propensity to invest with their managers, $a_1^I(p_1, \pi)$. Because this propensity is increasing in π the demand for the risky asset is increasing in π, and so is the equilibrium liquidation price of the risky asset.

When $\pi^* < \pi^{**}$, there could be a jump in the price function at π^*, which captures the sharp decrease in the demand for the risky asset. For $\pi > \pi^*$, all funds are open; for $\pi < \pi^*$, all funds are liquidated.

4.1.6. Individual support decisions

To characterize the manager's problem completely at $t = 0$, one needs to understand the individual manager's support decision and how this decision depends on the manager's portfolio choice in the initial period.

Proposition 3. *There exist unique thresholds $\pi_{x,i}\left(a_{0,i}^M, B\right)$ and $\pi_i^*\left(a_{0,i}^M, B\right)$ such that an individual managers with spillover losses $B\pi$ will strictly prefer to offer support, if and only if $\pi \in \left(\pi_i^*\left(a_{0,i}^M, B\right), \pi_{x,i}\left(a_{0,i}^M, B\right)\right)$.*

This proof of Proposition 3 follows from the monotonicity of the price function and can be found in the Appendix. The intuition for these results is analogous to the one presented for the aggregate thresholds π_x and π^*.

4.2. t=0

In this section, I characterize the equilibrium of the subgame at $t = 0$.

4.2.1. Investor's problem

An investor makes his portfolio choice at $t = 0$ to maximize his expected utility, anticipating the equilibrium that will be played at $t = 1$ for each realization of π and taking his manager's portfolio choices and prices as given.

An investor with manager i solves

$$\max_{a_0^I \in [0,1]} \mathbb{E}_\pi \left[\log \left(W_2^I \left(a_0^I; a_{0i}^M, \pi_i^*, \pi_{x,i}, \bar{s}_i (\pi), \pi \right) \right) \right], \tag{14}$$

where $\bar{s}_i (\pi) \in \{0, 1\}$ is the probability that the investor's manager offers support in state π, i.e., the probability that his managers has spillover losses $B > \hat{B} (\pi)$, and $W_2^I \left(a_0^I; \cdot \right)$ is his wealth at $t = 2$. This wealth depends on the investor's portfolio choice at $t = 0$, on the manager i's portfolio choice at $t = 0$ and support decisions at $t = 1$, and on the realized quality of the long-term asset, π. The investor's problem is concave in a_0^I and has a unique solution.

4.2.2. Manager's problem

At $t = 0$, a manager i chooses a portfolio $a_{0,i}^M$ to solve

$$\max_{a_{0,i}^M \in [0,1]} \mathbb{E}_{\pi,B} \left(W_2^M \left(a_{0,i}^M; \pi_{x,i} \left(a_{0,i}^M \right), \pi_i^* \left(a_{0,i}^M, B \right), a_0^I, \pi \right) \right) - \mathbb{E}_{\pi,B} \left(\mathbf{1} \left\{ \pi \leq \pi_i^* \left(a_{0,i}^M \right) \right\} B\pi \right) \tag{15}$$

where $\mathbf{1}$ is the indicator function, $W_2^M \left(a_{0,i}^M; \cdot \right)$ is the manager's wealth at $t = 2$, and I used that, for $\pi < \pi_i^* \left(a_{0,i}^M, B \right)$, the manager chooses not to offer support at $t = 1$ and the fund is liquidated. W_2^M depends on the manager's portfolio choice at $t = 0$ directly and through the support thresholds indirectly. Moreover, it depends on the investor's portfolio choice at $t = 0$ and on the quality of the risky asset π. The manager makes his portfolio choice taking his investor's decisions and prices as given. The manager's objective function is fully characterized in the Appendix as is W_2^M.

Remark 2.*As it is usual in the presence of threshold decisions, the manager's objective function is not well behaved. The liquidation rule and the possibility of offering support change the manager's attitude toward risk for different portfolio choices, $a_{0,i}^M$, from risk-neutral to risk-averse to risk lover.*

4.2.3. Risky asset market at t=0

The equilibrium price in the risky asset market at $t = 0$, p_0^*, is determined by

$$S\left(p_0^*\right) = \frac{a_0^M}{p_0^*}\left(W_0^M + A_0^I\right), \tag{16}$$

where the right-hand side is the total amount invested in the risky asset at $t = 0$, i.e., the fraction managers choose to invest in the risky asset, a_0^M, times the size of the funds, $W_0^M + a_0^I W_0^I$, divided by the price of the risky asset, p_0.

4.2.4. Intermediation fees

Finally, to close the model, since investors have all the bargaining power, the equilibrium fees are determined by the indifference condition for managers. If a manager chooses to open a fund, he incurs in operating costs $C > 0$ and gets utility $V_0^M\left(W_0^M; f^*\right)$. If he chooses not to open a fund, he invests his own endowment to maximize his expected wealth at $t = 2$. Therefore, in equilibrium, intermediation fees f^* are such that

$$V_0^M\left(W_0^M; f^*\right) - C = \max_{a\in[0,1]} \int_{\underline{\pi}}^{\bar{\pi}} \left(\frac{\pi \bar{d}}{p_1^*(\pi)}\left(\left(a\left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0}\right) + \frac{1}{q_0}\right) W_0^M + E\right)\right) d\pi. \tag{17}$$

5. Results

In this section, I focus on the special case in which there are no spillover losses, $\bar{B} = 0$. All omitted proofs are in the Appendix.

Proposition 4. *If $\bar{B} = 0$, $a_0^{M*} = 1$ in a symmetric equilibrium.*

Increasing the position in the risky asset has the following three effects for the manager: increases the expected return on the manager's wealth, increases the expected collected fees from intermediation at $t = 1$, and increases the probability of breaking the buck. Without any extra cost from liquidating the fund early other than the forgone fees, the benefits from taking risk offset the costs in equilibrium, and managers are better off taking as much risk as they can. This is shown by Proposition 4.

Proposition 5. *Suppose $\bar{B} = 0$ for all managers but for manager j. Then, in equilibrium, the risk taken by manager j at $t = 0$ will be decreasing in B_j and he will choose $a^M_{0,j} \in \left\{a^{MS}_0, 1\right\}$, where $a^{MS}_0 = \max a^M_{0,i}$ s.t. $\pi_{x,i}\left(a^M_{0,i}\right) = \underline{\pi}$.*

A larger B_j implies a higher cost of liquidating the fund early for the manager. Investing in the risky asset has two effects on the manager's expected wealth. On one hand, a higher exposure to the risky asset increases the manager's expected portfolio return, the expected intermediated amount at $t = 1$, and, thus, the expected fees collected from managing the investor's funds at $t = 1$. On the other hand, a higher $a^M_{0,j}$ increases the probability of early liquidation of the fund, which increases the expected losses suffered by the manager due to this early liquidation. When $B = 0$, the latter effect is not present and the manager is always better off by investing everything in the risky asset. If B_j is high enough, the loss from breaking the buck is too high and managers are better off choosing a portfolio such that the fund is never liquidated early. In this last case, managers prefer to forgo expected return to avoid losses from breaking the buck, which are too high.

This result is consistent with Kacperczyk and Schnabl (2013) who find that MMFs sponsored by companies that also offered non-money market mutual funds and other financial services took on less risk and that funds sponsored by financial intermediaries with limited financial resources took on less risk.

6. Policy analysis

Because of the systemic importance of MMFs, their regulation has been at the center of the policy discussion both in the US and in Europe. On July 23, 2014, after several years of debate, the Securities and Exchange Commission (SEC) voted to impose new regulations on the MMF industry. The European Commission is also in favor of MMF reform. The mutual fund industry opposes these changes and argues that further regulations would make the MMF industry less profitable and reduce the availability of short-term funding.[13] The regulations that are being considered target some key characteristics of MMFs, mainly the stable NAV and the support offered by sponsors to the funds. In the US, the SEC voted to impose redemption fees on investors in retail MMFs and to force institutional MMFs to abandon the stable NAV in favor of a floating NAV. Redemption fees increase investors' costs of redeeming shares and, therefore, reduce their incentives to run on the MMFs. By adopting a floating NAV, MMFs would not be subject to liquidation after breaking the buck. Alternatively, the European Commission proposed that stable NAV funds convert to variable NAV funds or hold a 3% capital buffer, and that sponsor support be prohibited unless approved by the appropriate regulator. These regulations aim to make MMFs more similar to other financial intermediaries: more like regular mutual funds in the case of adopting a floating NAV and more like banks in the case of a capital buffer.

In this section, I analyze this three policies (forbidding sponsor support, adopting a floating NAV and imposing capital buffers) focusing on their impact on sponsor support and their general equilibrium effects. The objective of this section is not to provide a comprehensive analysis of the proposed policies but rather to emphasize side effects of these policies, which have been avoided in the policy discussion. These side effects are general equilibrium effects that, given the relative size of the MMF in the money market, should not be ignored outright.

The parameters chosen for the numerical examples and all omitted proofs are in the Appendix.

In contrast to MMF in the US, European MMFs can be of two types: constant NAV (CNAV) funds and variable NAV (VNAV) funds. The main differences among these two types of funds reside in the way in which their

[13]See McCabe (2011), Mendelson and Hoerner (2011), Lacker (2011), Squam Lake Group (2011), Volcker (2011), Hanson, Scharfstein, and Sunderam (2014) , McCabe, Cipriani, Holscher, and Martin (2012) , and http://www.preservemoneymarketfunds.org/the-impact-on-you// (accessed November 25, 2012).

shares are priced and the timing of share redemptions. Like US MMFs, CNAV funds are priced to two decimal places (known as penny rounding) using amortized cost accounting and provide same-day liquidity.[14] In contrast, VNAV funds are priced to four decimal places on a mark-to-market basis and provide next-day liquidity.

Not all European funds are subject to an explicit breaking-the-buck rule. However, evidence exists suggesting that both types of funds also experienced runs during the 2007 − 08 financial crisis (see Gordon and Gandia (2014)). This fact seems to imply that, whether implicit or explicit, there is a share value below which investors choose to redeem their shares and, absent sponsor support, force funds to be liquidated. Moreover, it insinuates that the run risk in MMFs comes from investors' fear of a system-wide run and not on a run on an individual fund. This is exactly the focus of this paper.

The regulations proposed by the European Commission refer to CNAV funds.

6.1. No sponsor support allowed

I start by analyzing the case in which MMFs are stable NAV funds but have sponsors who are unable to offer support to the MMFs' investors. To emphasize the complexity of the general equilibrium analysis, I first consider the case in which one individual manager is not allowed to offer support. This highlights the partial equilibrium effects that come from the changes in the individual decisions. Then, in numerical examples, I consider the scenario in which all MMF managers are forbidden from offering support.

Suppose that all managers but one are allowed to offer support. Then, the problem solved by the manager who is not allowed to offer support is

[14]See the Appendix for a further discussion on penny rounding and amortized cost accounting and its implications for a fund's vulnerability to runs.

$$\max_{a_{0,i}^M \in [0,1]} \int \left(\frac{\pi \bar{d}}{p_1} \left(n \left(p_1^* (\pi), a_{0,i}^M \right) \left(W_0^M + f a_0^I W_0^I \right) + E \right) \right) d\pi + \int_{\pi_{x,i}\left(a_{0,i}^M\right)} \frac{\pi \bar{d}}{p_1} f A_1^{I*} (\pi) d\pi \quad (18)$$

$$- \int_0^{\bar{B}} \int^{\pi_{x,i}\left(a_{0,i}^M\right)} B \pi d\pi dG(B), \quad (19)$$

where the first term represents the expected return on his wealth at time 1 unconditionally on the liquidation of the fund, the second term represents the fees the manager collects if he keeps the fund open, and the third term represents the expected spillover losses that the manager suffers if his fund is liquidated.

Proposition 6. *Suppose, in an equilibrium, all managers are allowed to offer support. Given equilibrium fees f, asset prices p_0 and $p_1(\pi)$, and investors' choice a_0^I, an individual manager who cannot offer support will choose to take less risk than a manager who can offer support at $t = 1$.*

The only way in which the manager who cannot offer support can try to reduce the probability of his fund being liquidated is by decreasing the risk he takes. Managers who can offer support can either decrease the risk they take or offer support when a low quality of the asset is realized to prevent their fund's liquidation. Therefore, the manager who cannot offer support takes less risk than a manager who can insure himself against bad realizations of the asset quality by offering support.

The effect on risk taking when a manager looses the ability to offer support is intuitive. However, even taking fees and prices as given, the overall effect of not offering support on the probability of breaking the buck is ambiguous. Ex post, not being able to offer support increases the probability of a fund being liquidated. Ex ante, the decrease in the risk taken by the manager reduces the probability of needing support to keep the fund open. When the liquidity in the interim market is high enough, the ex ante effect dominates the ex post one and the manager who cannot offer support has a lower probability of breaking the buck than those managers who are allowed to support their investor.

Proposition 7. *Suppose, in an equilibrium, all managers are allowed to offer support. Given equilibrium fees f, asset prices p_0 and $p_1(\pi)$, and investors' choice a_0^I, a manager who is not allowed to offer support will be less likely to break the buck than a manager who can offer support if there is enough liquidity in the asset market in the interim period, i.e., if $E > \hat{E}$ for some $\hat{E}$.*

The higher the liquidity in the market, i.e., the higher E, the lower the fire sale discount in the risky asset market at $t = 1$ and the lower the return of investing with a manager at $t = 1$ for investors. This implies that when E is high, the benefit from keeping the fund open (the fees collected for offering intermediation services) is going to be small and, thus, the incentives to offer support will be low. Therefore, the manager's decrease in risk is enough to compensate for his inability to offer support and the probability of breaking the buck decreases.

Now suppose that all managers are forbidden from offering support to their investors. In this case, the risky asset's prices also change with respect to the case in which all funds offered support. How these prices change depends on the effects described above and on the investors' response to them. While a decrease in the risk taken by the managers reduces the risk faced by the investors, it also decreases the expected return of investing in a fund when the fund is not liquidated. Therefore, the overall effect on the amount intermediated is ambiguous and so is the effect on prices.

In numerical examples. I find that the MMF industry could be more stable if sponsor support were not allowed (see Fig. 4). These examples are consistent with the European Commission's recommendation to forbid sponsor support. At the same time, the examples reinforce the results in the previous sections that sponsor support can be destabilizing by leading to runs on the money market and, through them, amplifying systemic shocks.

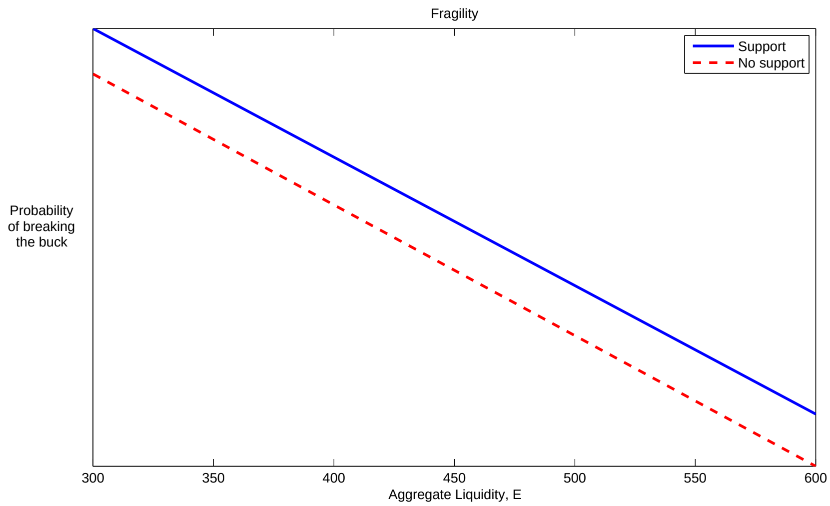

Fig. 4. Probability of breaking the buck. The figure depicts the probability of breaking the buck when support is allowed (blue solid line) and when no support is allowed (red dotted line).

6.2. Floating versus Stable NAV

The main regulation proposed by the SEC is to abandon the stable NAV in favor of a floating one. Adopting a floating NAV would diminish the risk of sudden redemptions by making MMFs like any other mutual fund and eliminating the possibility of breaking the buck. In the context of the model developed in this paper, going from a stable to a floating NAV system is equivalent to decreasing the liquidation threshold from $x > 0$ to $x = 0$. If $x = 0$, the funds managed by the managers become regular mutual funds and there is no breaking-the-buck rule. In this case, no support is ever offered. One unit transferred to investors represents $fa_1^I < 1$ worth of fees for the manager. The manager does not have any incentive to offer support.

The portfolio decisions at $t = 1$ presented in the benchmark model are independent of x and, thus, remain unchanged when going from $x > 0$ to $x = 0$. In terms of the thresholds presented in the Section 4, $\pi_{x,i}\left(a_{0,i}^M; a_0^M\right) =$

$\pi_i^* \left(a_{0,i}^M; a_0^M\right) = 0$ for all $\left(a_{0,i}^M; a_0^M\right) \in [0,1]^2$, when $x = 0$. The manager's problem at $t = 1$ now becomes

$$\max_{a_{0,i}^M \in [0,1]} \mathbb{E}_\pi \left[\frac{\pi \bar{d}}{p_1^* (\pi)} \left(n \left(p_1 (\pi), a_{0,i}^M \right) \left(W_0^M + f a_0^I W_0^I \right) + E \right) \right] \tag{20}$$

$$+\mathbb{E}_\pi \left[\frac{\pi \bar{d}}{p_1^* (\pi)} f a_1^{I*} (\pi) \left(n \left(p_1 (\pi), a_{0,i}^M \right) (1-f) + \left(1 - a_0^{I*}\right) \frac{1}{q_0} \right) W_0^I \right] \tag{21}$$

This implies that there is no downside for the risk-neutral managers to take risks and, therefore, $a_0^{M*f} = 1$.

The payoff for investors from investing with the manager is affected in many different ways when the liquidation threshold changes. If all decisions at $t = 0$ remained unchanged and the liquidation price of the risky asset was kept fixed, investors would lose insurance going from a stable NAV system to a floating one, i.e., they would lose the transfer they were receiving from their managers in all the states in which support was offered. Fig. 5 illustrates this argument. However, when all funds go from a stable to a floating NAV system, the liquidation price of the risky asset changes. Keeping all decisions at $t = 0$ unchanged, the equilibrium liquidation price of the risky asset is (weakly) higher when $x = 0$. To see this, first note that the demand for the risky asset increases in those states in which some support was offered when $x > 0$. When support is offered, for each unit managers transfer to investors, only a fraction a_1^I goes to the risky asset market. If managers kept this for themselves they would invest it all in the risky asset. Moreover, for those states in which the fund was liquidated but investors would still have liked to invest in the risky asset via the managers, demand goes up. When $x > 0$, they were prevented from investing because the funds were closed, but now they can do so. These effects drive the demand for the risky asset up and increase the equilibrium liquidation price. This change is illustrated in Fig. 6.

Therefore, when one considers the change in the liquidation price of the

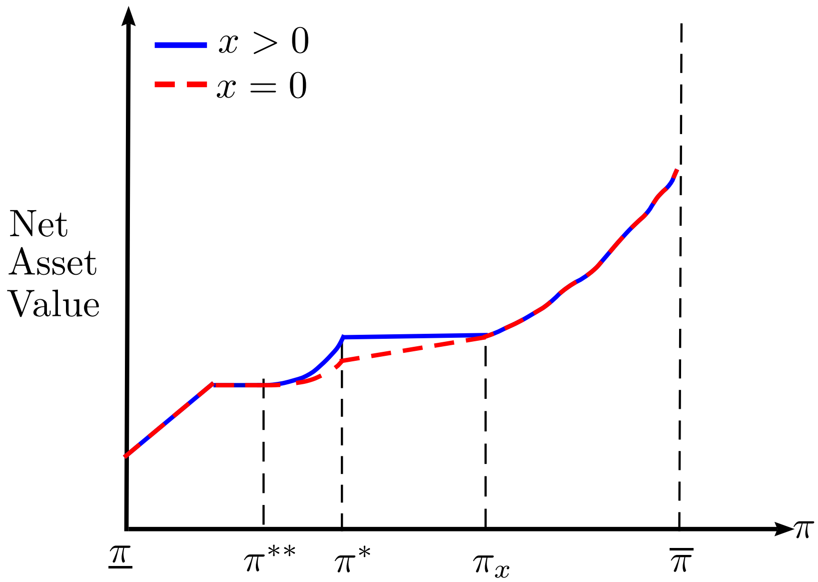

Fig. 5. Investors' payoff. The solid (dotted) line represents the payoff for an investor who invests with a manager who offers (does not offer) support when everyone else is offering support.

risky asset, one can see that investors prefer $x > 0$ in those states in which support is offered with high probability but are better off when $x = 0$ in the states in which the fund is very likely to be liquidated. Fig. 7 illustrates this trade-off.

Proposition 8. *Let $\widehat{NAV}(\pi)$ be the share value received by an investor in a fund when the aggregate state is π for given decisions at $t = 0$. Then, given time 0 decisions, fees and prices, if there is a unique equilibrium at $t = 1$, i.e., if $\pi^{**} < \pi^{*}$,*

$$\frac{\partial \mathbb{E}_{\pi}\left(\widehat{NAV}(\pi)\right)}{\partial x} \propto (\pi_x - \pi^{*}) + \int_{\pi^{**}}^{\pi*} \bar{s}(\pi)\, d\pi + \int_{\pi^{**}}^{\pi^{*}} \frac{\partial \bar{s}(\pi)}{\partial x}\left(x - n\left(p_1(\pi), a^{M}_{0,i}\right)\right) d\pi + \int_{\pi^{**}}^{\pi*} (1 - \bar{s}(\pi)) \frac{\partial n\left(p_1(\pi), a^{M}_{0,i}\right)}{\partial x} d\pi. \tag{22}$$

The proof follows directly from the definitions of $\widehat{NAV}(\pi)$ and support thresholds. From the expression in Proposition 8 above one can see how the

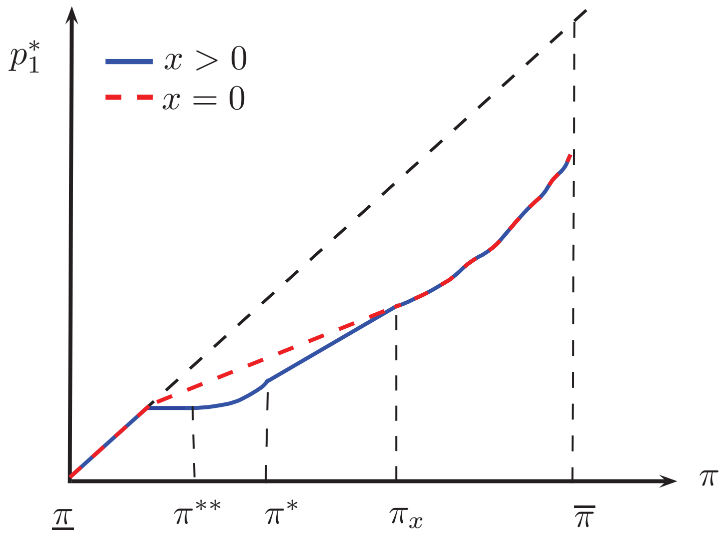

Fig. 6. Price functions for different policies. Liquidation price of the risky asset when going from a stable net asset value (NAV) ($x > 0$) to a floating NAV ($x = 0$) fixing all decisions at $t = 0$.

change in the liquidation threshold affects the expected payoff of investors. The first two terms represent the expected increase in the share value received by the investor if support is offered. A higher x implies that, if support is offered, investors will receive a higher share value. The third term captures the decrease in the expected share value that comes from a lower probability of support. The higher the liquidation threshold x, the costlier it becomes for a manager to offer support and, thus, the less likely it is he will support his investor. Finally, the fourth term captures the general equilibrium effects. It represents the decrease in the investor's share value if he does not receive support. This effect is coming from the change in the liquidation price of the asset when the liquidation threshold x changes for all funds in the economy.

The sign of the expression in Eq. 22 depends on the magnitude of the effects enumerated above. If the probability of a fund being liquidated rela-

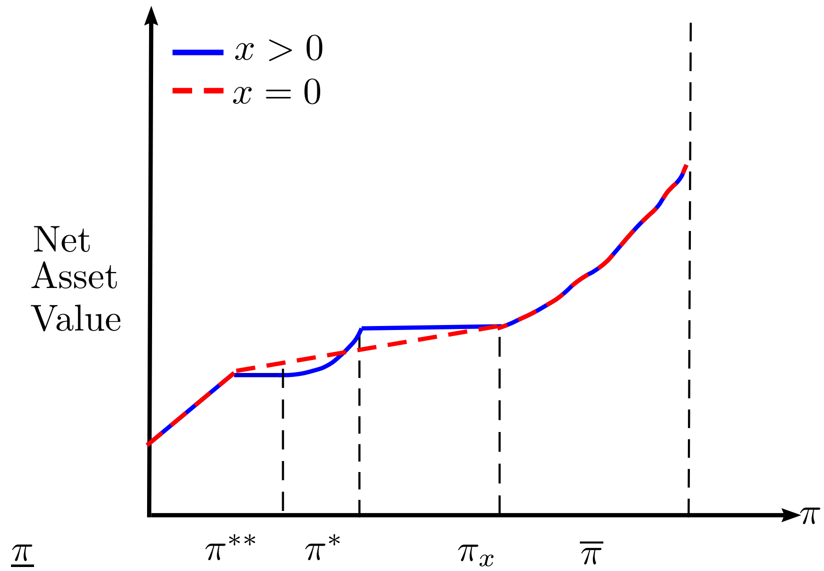

Fig. 7. Investors' payoffs for different policies. Payoff for investors from investing with a manager when going from a stable NAV ($x > 0$) to a floating NAV ($x = 0$) fixing all decisions at $t = 0$.

tive to the probability of getting support is small, an increase in x increases the expected share value for investors. However, if these two are comparable, whether the share value increases or decreases with the liquidation threshold x depends mostly on the elasticity of the support decision with respect to x and on the size of the support needed. Eq. 22 highlights the many forces involved in the policy analysis and emphasizes its complexity.

So far, the analysis kept all decisions at $t = 0$ fixed. Fig. 8 compares the equilibria for different values of aggregate liquidity E for $x > 0$ and $x = 0$. In the examples, when going from a stable to a floating NAV system, both the risk and the expected return of investing with a manager decrease for investors. Moreover, the intermediation level and the liquidity provided to the asset market at $t = 0$ increase when going from a stable to a floating NAV.

Finally, welfare levels for managers and investors are measured in con-

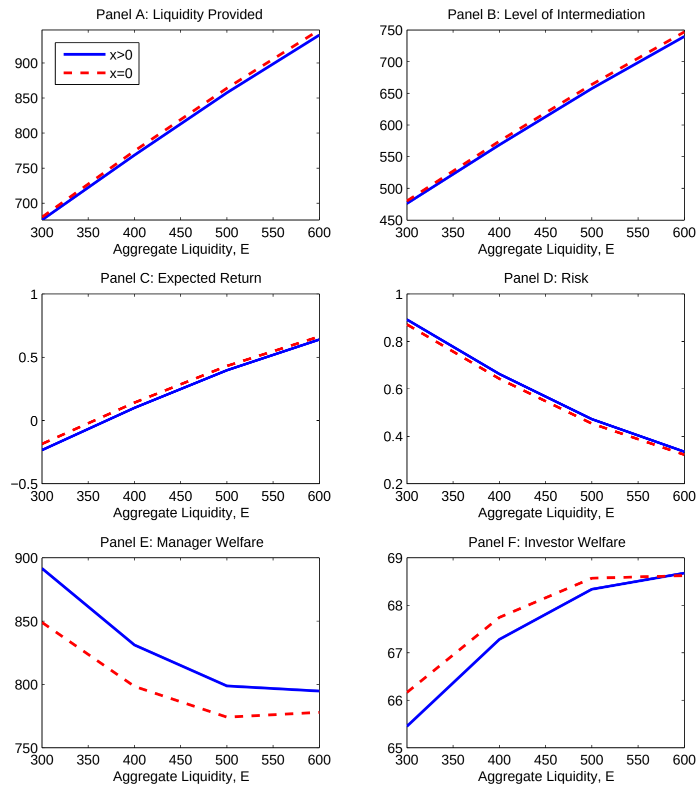

Fig. 8. Floating versus stable net asset values. The blue solid line represents the equilibrium in the benchmark economy with a sable NAV. The red dotted line represents the equilibrium in an economy with a floating NAV and no breaking the buck liquidation rule.

sumption equivalence. In this example, managers are worse off when adopting a floating NAV and investors are better off. This is consistent with the industry opposing abandoning the stable NAV and the SEC arguing for it.

6.3. *Capital buffer*

One of the main policies being considered by the European Commission is to impose a capital buffer on stable NAV MMFs. In the context of the model, a capital buffer requirement is equivalent to assuming that managers are required to hold capital equal to a fraction F of the funds they manage for investors to cover potential losses at $t = 1$. In this case, a fund consists of the manager's initial wealth minus the capital requirement plus the amount the investor chooses to invest with the manager, i.e, $W_0^M + (1 - Fq_0)\, A_0^I$.

The initial investment Fq_0 generates a capital buffer equal to F at $t = 1$, which the sponsor would be forced to use to cover any decrease of the NAV below x. Once this buffer has been exhausted, managers can still choose to keep the fund running at $t = 1$ by offering support beyond the amount required by the new regulation. The characterization of the new equilibrium is analogous to the one shown for the benchmark model, and it can be found in the Appendix.

Imposing a capital buffer on MMFs has countervailing effects on the asset prices. On one hand, it decreases the demand for the risky asset at $t = 0$ and, therefore, ceteris paribus, decreases the equilibrium price p_0 and increases the return of investing in the risky asset. On the other hand, keeping everything else equal, a higher capital buffer decreases the demand for the risky asset at $t = 1$ and leads to a lower liquidation price of the asset. These changes in asset prices affect the return and the risk of investing with a manager.

Proposition 9. *Let $\hat{\pi}_x$, $\hat{\pi}^*$, and $\hat{\pi}^{**}$ be the aggregate thresholds for the managers' support decisions when there is a capital buffer F in place and $\widehat{NAV}(\pi)$ is the share value received by an investor in a fund when the aggregate state is π for given decisions at $t = 0$.*[15] *Then, given time 0 decisions*

[15] A characterization of these thresholds can be found in the Appendix.

and fees, if there is a unique equilibrium at $t = 1$*, i.e., if* $\hat{\pi}^{**} < \hat{\pi}^{*}$*,*

$$\frac{d\mathbb{E}_{\pi}\left(\widehat{NAV}(\pi)\right)}{dF} \propto \frac{a_0^M}{p_0}\left[\int_{\hat{\pi}^{**}}^{\hat{\pi}^{*}} (1-\bar{s}(\pi))\frac{dp_1(\pi)}{dF}d\pi + \int_{\hat{\pi}_x}^{\bar{\pi}} \frac{dp_1(\pi)}{dF}d\pi\right] \tag{23}$$
$$+\int_{\hat{\pi}^{**}}^{\hat{\pi}^{*}} \frac{\partial \bar{s}(\pi)}{\partial p_1(\pi)}\frac{dp_1(\pi)}{dF}\left(\left(x - n\left(p_1(\pi), a_0^M\right)\right)\right)d\pi$$
$$-a_0^M\left[\int_{\hat{\pi}^{**}}^{\hat{\pi}^{*}} (1-\bar{s}(\pi))\left(p_1(\pi) - \frac{1}{q_0}\right)d\pi + \int_{\hat{\pi}_x}^{\bar{\pi}}\left(p_1(\pi) - \frac{1}{q_0}\right)d\pi\right]\frac{1}{p_0^2}\frac{\partial p_0}{\partial F}$$
$$+\int_{\hat{\pi}^{**}}^{\hat{\pi}^{*}} \frac{\partial \bar{s}(\pi)}{\partial p_0}\left(\left(x - n\left(p_1(\pi), a_0^M\right)\right)\right)d\pi\frac{\partial p_0}{\partial F},$$

where

$$\frac{dp_1(\pi)}{dF} = \frac{\partial p_1(\pi)}{\partial F} + \frac{\partial p_1(\pi)}{\partial p_0}\frac{\partial p_0}{\partial F}.$$

From Proposition 9 above, an increase in the capital buffer F affects the expected share value received by investors through asset prices at $t = 0$ and at $t = 1$. The first two terms in Eq. 23 take into account the total effect in the liquidation market which is reflected in a change in the liquidation price of the asset. The new liquidation price of the asset affects the return received by an investor when he is not receiving support from his fund manager, and it also affects the probability with which he receives support. The capital buffer F affects the asset market at $t = 1$ directly and indirectly through p_0. The third and fourth terms show the direct effect of the change in the initial price of the asset on the return received by investors. The first two terms are negative and the last two terms are positive. Therefore, the decrease in $p_1(\pi)$ and in p_0 have countervailing effects on the investors' return of investing with a manager.

On top of these changes, one should also consider the changes in the fees charged by the managers. Partial equilibrium analysis would suggest that fees would go up after a capital buffer is imposed. However, because the cost of intermediation depends on asset prices, the equilibrium fees can either increase or decrease depending on the magnitudes of all the changes mentioned above.

Fig. 9 shows the equilibrium fees, the level of intermediation, the expected return and risk of intermediation, and the managers' and investors' welfare for numerical examples with different values of aggregate liquidity E for the benchmark economy in which there is no capital buffer (blue solid line) and for the case in which a capital buffer of 3% is imposed on the funds (red dotted line). In these examples, capital requirements are successful in decreasing the fragility of the economy and they increase the level of intermediation period 0. Moreover, when capital requirements are imposed, managers need to be compensated for having to hold the capital buffer and intermediation fees are higher. In equilibrium, the changes in the liquidation price of the asset and in the managers' support decisions imply higher expected return and lower risk for investors investing with a manager. These effects offset the increase in fees and lead to a higher level of intermediation and higher investor welfare. The increase in the level of intermediation and in the intermediation fees are not enough to compensate the managers for having to hold the capital buffer. Therefore, their welfare is lower when a capital buffer is in place.

6.4. Discussion of numerical results

The numerical results illustrate how ignoring the general equilibrium effects of the policies can, in principle, lead to wrong conclusions.[16] The strength of these effects depends mainly on the elasticity of the demand for intermediation and the elasticity of the supply of the risky asset.

The demand for intermediation is given by the investors' attitude toward risk. The higher the risk aversion of investors, the higher the demand for stability and, thus, the stronger the response of the intermediation level to changes in the return and risk of investing with managers.

The elasticity of the supply of the risky asset can be interpreted as a measure of the importance of MMFs as liquidity suppliers in the money market. If there are other potential suppliers of liquidity, the price does

[16]See the Appendix for the parameters values.

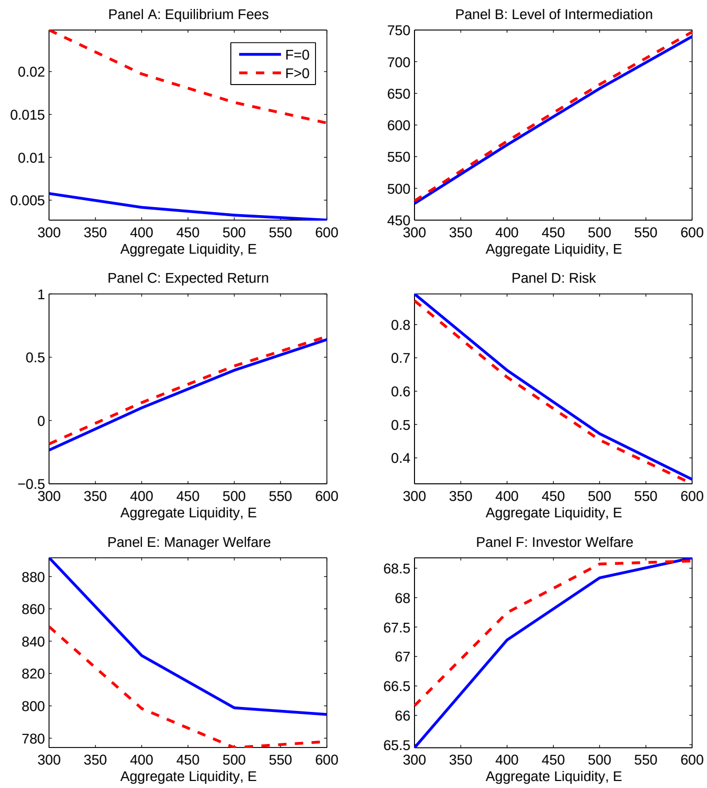

Fig. 9. Stable net asset value versus capital buffer. The blue solid lines represents the equilibrium for the benchmark economy in which there is a stable NAV and no capital buffer. The red dotted line represents the equilibrium in an economy with a stable NAV and a capital buffer of 3%.

not react much to changes in the amount of the risky asset demanded by the MMF industry. Alternatively, if the MMFs are the sole suppliers of liquidity in the money market one would expect the price of risky assets to react substantially to changes in the MMF industry. The more inelastic the supply of the risky asset, the more important the changes in the equilibrium prices are in determining the overall effect of the policy analyzed.

7. Conclusion

In this paper, I develop a novel model of MMFs to analyze the role of sponsor support on the industry's stability. The model incorporates several features that are characteristic of MMFs: the investors' ability to redeem their shares on demand, the stability of the NAV, the liquidation of the funds after breaking the buck, and, most important, the provision of voluntary sponsor support. The fluctuation in the value of the funds' assets is captured by shocks to the quality of risky assets that affect the equilibrium prices.

Even in the absence of investor runs, the MMF industry can be fragile. MMFs can be subject to a source of fragility that differs from the canonical bank runs: strategic complementarities in the sponsors' support decisions that could give rise to multiple equilibria and to runs of the MMFs on the asset market. Therefore, sponsor support, which is instrumental in providing stability to the MMFs after idiosyncratic shocks, might not be so effective when the shocks are systemic and it could even amplify them.

This model is consistent with stylized facts shown in the literature on MMFs. It captures the positive performance-flow sensitivity in the MMF industry and the difference in incentives for risk taking for funds with different sponsors. In particular, MMFs sponsored by companies that also offer non-money market mutual funds and other financial services tend to take on less risk.

I then use the model to analyze the trade-offs involved in three policies that affect the sponsors' incentives and ability to offer support: not allowing the sponsors to offer support, adopting a floating NAV, and imposing a capital buffer for MMFs. The consequences of the proposed regulations depend on the interaction between potentially countervailing effects. Changing the institutional setup of the MMF industry would affect the risks and returns of intermediation for investors and MMF managers not only directly, but also indirectly through the change in equilibrium outcomes such as intermediation fees, the sponsors' support decision, and asset prices. The model allows me to take into account these general equilibrium effects, which seem particularly important given the relative size of the MMF industry in the market for short-term financing. One of the key determinants of the overall effect of the policies is the elasticity of the supply of assets faced by MMFs. In light of this, the model suggests that a crucial piece in the policy analysis is whether other market participant would be able and willing to offer liquidity in the money market if MMFs were not there.

Appendix A.

A.1. Money market funds: institutional features

MMFs are open-ended mutual funds that offer individuals, corporations, and governments access to money market instruments, such as US Treasury bills and commercial paper. MMFs act as intermediaries between investors and borrowers who seek short-term financing. As required by regulation, all mutual funds, including MMFs, issue demandable shares, i.e., they provide same-day liquidity, allowing investors to redeem their shares at any time at the net asset value of the shares.[17] What makes these institutions, all MMFs in the US and stable NAV MMFs in Europe, special is that they seek to maintain a stable NAV, usually of $1. To prevent the NAV from going above $1, all positive investment returns are paid out entirely as dividends, with no capital gains or losses to track. If the NAV drops below $1, it is said that the fund broke the buck. In the event of a fund breaking the buck, US regulation states that "the board of directors shall promptly consider what action, if any, should be initiated by the board of directors." [Rule 2a-7(b), 17 C.F.R. § 270.2a-7(b)] In practice, in the only two cases in which a US MMF's NAV dropped below $1, the fund were eventually liquidated.[18]

Even though an MMF could become a floating NAV fund after breaking the buck, adopting a floating NAV does not reduce investors incentives to redeem their shares and it decreases the NAV even more. Because a fund would likely sell the more liquid assets to meet excess redemptions, investors who chose not to redeem their shares would be left holding a portfolio of less-liquid, longer-dated securities. This increases the incentives of investors to withdraw quickly, even at a reduced NAV, and drives the NAV even lower.[19] This downward spiral mechanism makes it impossible for an

[17]See Title 17 Commodity and Securities Exchanges - 17 CFR § 270.22c-1 (b)(c)

[18]See Fisch and Roiter (2011) for a detailed description of the current regulation of MMFs.

[19]See Hanson, Scharfstein, and Sunderam (2014).

MMF to transition to a floating NAV fund and makes liquidation the only viable option after breaking the buck. For example, on September 16, 2008, a day after Lehman Brothers declared bankruptcy, the Reserve Primary Fund delayed share redemptions for up to seven days, abandoned the stable NAV, and became a floating NAV fund. On September 18, 2008, RPF suspended the redemption of shares and started the orderly liquidation of the fund's assets after experiencing massive share redemptions. RPF, which had approximately $62 billion in assets under management on September 15, 2008, experienced redemptions of $60 billion between September 15 and September 18, 2008.[20]

To maintain a stable NAV, MMFs rely on two mechanisms: the computation of the NAV and sponsor support. The SEC, via Rule 2a−7, allows MMFs to use amortized cost valuation and penny-rounding pricing to compute the NAV.[21] European stable NAV funds are also allowed to use amortized cost accounting. Many of the assets held by MMFs lack market price quotations. This makes it difficult for MMFs to price their assets accurately. Amortized cost valuation allows MMFs to value their assets as if held to maturity. Penny-rounding pricing allows MMFs to report a NAV of $1 as long as the calculated value is between $0.995 and $1.005. Using amortized cost valuation and penny rounding makes MMFs prone to shareholder runs. For example, if the NAV is just below $1, shareholders who redeem their shares first get $1 and by doing so reduce the value of the fund's assets, imposing costs on non redeeming shareholders who might not get $1 for their shares. Analogously, if the NAV calculated using amortized cost valuation differs from the market value of the asset, investors could be better off redeeming their shares. Because the liquidation value of the assets differs from the

[20]See http://www.primary-yieldplus-inliquidation.com/pdf/PressRelease2008_0916.pdf and http://www.primary-yieldplus-inliquidation.com/pdf/PressReleasePrimGovt2008_0919.pdf (accessed February 15, 2013).

[21]Rule 2a-7 allows MMFs to use amortized cost valuation "only so long as the [fund] board of directors believes that it fairly reflects the market-based net asset value per share".

NAV computed by the MMF, investors who do not redeem their shares bear the cost of paying redeeming investors $1 for something that is worth less than $1 in the market. These phenomena resemble the mechanism behind the canonical bank runs described by Diamond and Dybvig (1983) and dealt with in a very large literature.

In my model, I abstract from the possibility of runs on MMFs by assuming that all assets are traded in frictionless competitive markets and that the shares in MMFs are priced-to-market. In my model, no need exists to use amortized cost accounting because price quotations are always available.

The features described above make MMF unique financial institutions. As illustrated in Fig. 10, the payoff received by investors in MMFs can be seen as a hybrid between that received by investors in other mutual funds and that received by depositors in banks. If no sponsor support is offered, investors in MMFs are true shareholders and, as for investors in other mutual funds, the value of their shares coincides with the market NAV. If sponsor support is offered, the value of the shares for investors in MMFs is the same independently of the value of the fund's assets. This flat portion of the investors' payoff makes shares in MMFs resemble debt. Nevertheless, sponsor support is voluntary, and, though it is anticipated by investors, it is not mandated by the intermediation contract. The intermediation contract considered in the model developed in this paper captures these features.

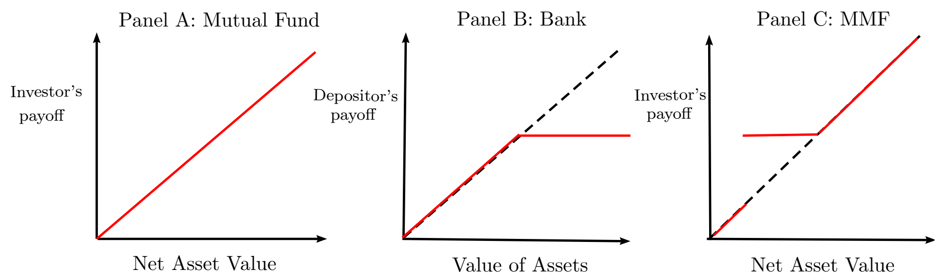

Fig. 10. Investors' payoff in different financial institutions. The net asset value for mutual funds and money market funds is calculated as the total value of the fund's assets divided by the total number of outstanding shares. The asset value of assets for banks is calculated as the total value of the bank's assets divided by the total amount of deposits.

A.2. Equilibrium price

When the return on the manager's portfolio is above x, no support is needed to keep the fund operating. In this case, if a_1^{I*} is interior, the equilibrium price is

$$p_1^{NS}(\pi) = \min\left\{ \frac{(1-f)\bar{d}\left(\pi\left(\left(1-a_0^M\right)(1-f)a_0^I+\left(1-a_0^I\right)\right)\frac{1}{q_0}W_0^I+\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M+fa_0^IW_0^I\right)+E\right)}{\frac{a_0^M}{p_0}a_0^I(1-f)^2(1-\pi)\bar{d}W_0^I+\frac{1}{q_1}\frac{1}{q_0}\left(\left(1-a_0^I\right)W_0^I+\left(1-a_0^M\right)\left(W_0^M+a_0^IW_0^I\right)\right)+\frac{E}{q_1}}, \bar{d}\pi q_1 \right\}. \tag{24}$$

If support is provided by the all managers, and a_1^{I*} is interior, the equilibrium price, $p_1^S(\pi)$, is determined by

$$\left(x(1-f)a_0^IW_0^I - \frac{\left(1-a_0^M\right)}{q_0}\left(W_0^M + a_0^IW_0^I\right) - E\right) = a_1^{I*}(p_1(\pi), \pi)\left(x(1-f)a_0^I + \frac{\left(1-a_0^I\right)}{q_0}\right)W_0^I. \tag{25}$$

If $x(1-f)a_0^IW_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + a_0^IW_0^I\right) - E < 0$, there is an excess demand for the risky asset at every price $p_1(\pi) < \bar{d}\pi q_1$. Therefore, in this case, $p_1(\pi) = \bar{d}\pi q_1$, $a_1^{I*}(p_1(\pi), \pi) = 0$, and the share invested by managers in the risky asset, $\bar{a}_1^M(\pi)$, is given by

$$\begin{aligned}&\left(1-\bar{a}_1^M(\pi)\right)\tfrac{\bar{d}\pi q_1}{p_0}a_0^M\left(W_0^M+a_0^IW_0^I\right)\\&+\bar{a}_1^M(\pi)\left(x(1-f)a_0^IW_0^I-\tfrac{\left(1-a_0^M\right)}{q_0}\left(W_0^M+a_0^IW_0^I\right)-E\right)=0.\end{aligned}\quad(26)$$

If $x(1-f)a_0^IW_0^I-\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M+a_0^IW_0^I\right)-E>0$, then

$$a_1^{I*}(p_1(\pi),\pi)=\frac{\left(x(1-f)a_0^IW_0^I-\frac{\left(1-a_0^M\right)}{q_0}\left(W_0^M+a_0^IW_0^I\right)-E\right)}{\left(x(1-f)a_0^I+\frac{\left(1-a_0^I\right)}{q_0}\right)W_0^I}=:a_1^{IS}.\quad(27)$$

This condition implies an affine equation in $\bar{d}/p_1^S(\pi)$, which gives the following equilibrium price

$$p_1^S(\pi)=\min\left\{\max\left\{\frac{\left(\pi-a_1^{IS}\right)}{\frac{1}{q_1}\left(1-a_1^{IS}\right)}(1-f)\bar{d},0\right\},\bar{d}\pi q_1\right\}.\quad(28)$$

When there is support, the share investors choose to invest with the manager in period 1 is independent of π.

If only a mass s of sponsors chooses to offer support and keep the fund open while a mass $(1-s)$ chooses not to offer support and liquidate the fund early, the net demand for the risky asset is

$$\begin{aligned}D(p_1,\pi)&=s\left(\frac{a_1^M(p_1,\pi)\left(A_1^I(p_1,\pi)+W_1^M(p_1,s=1)\right)}{p_1}-\frac{a_0^M\left(a_0^IW_0^I+W_0^M\right)}{p_0}\right)\\&\quad+(1-s)\left(\frac{a_1^M(p_1,\pi)W_1^M(p_1,s=0)}{p_1}-\frac{a_0^M\left(a_0^IW_0^I+W_0^M\right)}{p_0}\right).\end{aligned}\quad(29)$$

In this case, the equilibrium price if $A_1^I(p_1,\pi) > 0$ is given by

$$\begin{aligned} 0 &= s\left(\left(\pi\bar{d}(1-f)-\frac{p_1}{q_1}\right)\tfrac{\left(1-a_0^I\right)}{q_0}-(1-\pi)\,\bar{d}(1-f)\,x(1-f)\,a_0^I\right)W_0^I \\ &+\left(\tfrac{\left(1-a_0^M\right)}{q_0}\left(W_0^M+(f+(1-f)\,s)\,a_0^I W_0^I\right)+E\right)\left(\bar{d}(1-f)-\frac{p_1}{q_1}\right) \\ &-\frac{p_1}{p_0}a_0^M(1-f)(1-s)\,a_0^I W_0^I\left(\bar{d}(1-f)-\frac{p_1}{q_1}\right) \end{aligned} \tag{30}$$

and

$$\begin{aligned} 0 &= \bar{d}(1-f)\left(s\left(\pi\tfrac{\left(1-a_0^I\right)}{q_0}-(1-\pi)\,x(1-f)\,a_0^I\right)W_0^I+\tfrac{\left(1-a_0^M\right)\left(W_0^M+(f+(1-f)s)a_0^I W_0^I\right)}{q_0}+E\right) \\ &-\frac{p_1}{q_1}\left(s\tfrac{\left(1-a_0^I\right)}{q_0}W_0^I+\tfrac{\left(1-a_0^M\right)\left(W_0^M+(f+(1-f)s)a_0^I W_0^I\right)}{q_0}+E+\frac{q_1\bar{d}(1-f)\left(a_0^M(1-f)(1-s)\,a_0^I W_0^I\right)}{p_0}\right) \\ &+p_1^2\frac{a_0^M(1-f)(1-s)\,a_0^I W_0^I}{p_0 q_1}. \end{aligned} \tag{31}$$

This quadratic function is negative at $p_1 = \bar{d}(1-f)\,q_1$. Therefore, the equilibrium price is given by the smallest root

$$p_1^{SS}(\pi,s) = \frac{-c_1(\pi,s)-\sqrt{c_1(\pi,s)^2-4c_0(\pi,s)\,c_2(\pi,s)}}{2c_2(\pi,s)}, \tag{32}$$

if $s < 1$, where

$$\begin{aligned} c_0(\pi,s) &= \bar{d}(1-f)\left(s\left(\pi\tfrac{\left(1-a_0^I\right)}{q_0}-(1-\pi)\,x(1-f)\,a_0^I\right)W_0^I+\tfrac{\left(1-a_0^M\right)\left(W_0^M+(f+(1-f)s)a_0^I W_0^I\right)}{q_0}+E\right), \\ c_1(\pi,s) &= -\frac{1}{q_1}\left(s\tfrac{\left(1-a_0^I\right)}{q_0}W_0^I+\tfrac{\left(1-a_0^M\right)\left(W_0^M+(f+(1-f)s)a_0^I W_0^I\right)}{q_0}+E+\tfrac{q_1\bar{d}(1-f)a_0^M(1-f)(1-s)a_0^I W_0^I}{p_0}\right), \\ c_2(\pi,s) &= \frac{a_0^M(1-f)(1-s)\,a_0^I W_0^I}{p_0 q_1}. \end{aligned} \tag{33}$$

Otherwise, $A_1^I(p_1,\pi)$ would equal zero. Moreover, since the quadratic function is decreasing and positive at $p_1 = 0$, $p_1^{SS} > 0$.

In equilibrium, s is equal to $G\left(\hat{B}\right)$, where $\hat{B}$ is the solution to

$$\begin{aligned} &\left(\left(x-n\left(p_1^{SS}\left(\pi,G\left(\hat{B}\right)\right),a_0^M\right)\right)(1-f)\,A_0^I-fA_1^I\left(p_1^{SS}\left(\pi,G\left(\hat{B}\right)\right),\pi\right)\right) \\ = &\min\left\{\hat{B}\frac{p_1(\pi)}{\bar{d}},\left(n\left(p_1^{SS}\left(\pi,G\left(\hat{B}\right)\right),a_0^M\right)\left(W_0^M+fA_0^I\right)+E\right)\right\}. \end{aligned} \tag{34}$$

Finally, if all funds are liquidated, the equilibrium price, $p_1^L(\pi)$, is given by

$$p_1^L(\pi) = \min\left\{ \frac{\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + f a_0^I W_0^I\right) + E}{\frac{a_0^M}{p_0}(1-f)\, a_0^I W_0^I}, \bar{d}\pi q_1 \right\}, \tag{35}$$

and it is determined by the amount of resources managers have that do not come from their holdings of the risky asset, i.e., by the cash in the market.[22]

A.3. *Equilibrium threshold characterization*

This section characterizes the thresholds $\pi_x\left(a_0^M\right)$, $\pi^*\left(a_0^M\right)$, and $\pi_i^{**}\left(a_{0,i}^M\right)$ that determine the aggregate support decisions.

$\pi_x\left(a_0^M\right)$ is given by

$$\frac{p_1^{NS}\left(\pi_x\left(a_0^M\right)\right)}{p_0} a_0^M + \left(1-a_0^M\right)\frac{1}{q_0} = x \tag{36}$$

using that $p_1^{NS}\left(\pi_x\left(a_0^M\right)\right) = p_1^S\left(\pi_x\left(a_0^M\right)\right)$ and the definition of $p_1^S(\pi)$,

$$\pi_x\left(a_0^M\right) = \begin{cases} \left(\left(x-\frac{1}{q_0}\right) + a_0^M\frac{1}{q_0}\right)\frac{p_0}{a_0^M}\frac{\left(1-a_1^{IS}\right)}{\bar{d}(1-f)q_1} + a_1^{IS} & a_1^{IS} > 0 \\ \left(\left(x-\frac{1}{q_0}\right) + a_0^M\frac{1}{q_0}\right)\frac{p_0}{a_0^M}\frac{1}{\bar{d}q_1} & \text{else} \end{cases} \tag{37}$$

From lemma 1, $\pi^*\left(a_0^M\right)$ is such that $\hat{B}\left(\pi, p_1^S(\pi)\right) = 0$. Equivalently, it is determined by the equilibrium support threshold for a manager with spillover losses $B = 0$, i.e.,

$$\frac{\pi^*\bar{d}}{p_1^S(\pi^*)}\left(\left(x - n\left(p_1^S(\pi^*), a_0^M\right)\right)(1-f)A_0^I - f A_1^I\left(p_1^S(\pi^*), \pi^*\right)\right) = 0 \tag{38}$$

[22]See Allen and Gale (1994), (2005) for more on cash-in-the-market pricing.

Then, $\pi^*\left(a_0^M\right)$ is determined by

$$p_1^S\left(\pi^*\left(a_0^M\right)\right)=\frac{\left(\left(1-fa_1^{IS}\right)x\left(1-f\right)a_0^I-fa_1^{IS}\frac{\left(1-a_0^I\right)}{q_0}-\frac{\left(1-a_0^M\right)}{q_0}\left(1-f\right)a_0^I\right)W_0^I}{\frac{a_0^M}{P_0}\left(1-f\right)a_0^IW_0^I} \quad (39)$$

where $a_1^{IS}=0$ if $p_1^S\left(\pi^*\left(a_0^M\right)\right)=\bar{d}\pi^*\left(a_0^M\right)q_1$.

Analogously, $\pi^{**}\left(a_0^M\right)$ is given by

$$\begin{aligned}&\frac{\pi^{**}\bar{d}}{p_1^L\left(\pi^{**}\right)}\left(\left(x-n\left(p_1^L\left(\pi^{**}\right),a_0^M\right)\right)\left(1-f\right)A_0^I-fA_1^I\left(p_1^L\left(\pi^{**}\right),\pi^{**}\right)\right)\\ =\ &\min\left\{\bar{B}\pi^{**},\frac{\pi^{**}\bar{d}}{p_1^L\left(\pi^{**}\right)}n\left(p_1^L\left(\pi^{**}\right),a_0^M\right)\left(W_0^M+fA_0^I\right)+E\right\}. \quad (40)\end{aligned}$$

In other words, $\pi^{**}\left(a_0^M\right)$ is such that $\hat{B}\left(\pi,p_1^L\left(\pi\right)\right)=\bar{B}$.

Proposition 10. *The thresholds $\pi_x\left(a_0^M\right)$, $\pi^*\left(a_0^M\right)$, and $\pi^{**}\left(a_0^M\right)$ are increasing in a_0^M.*

The net asset value, $n\left(p_1,a_0^M\right)$, is decreasing in a_0^M for all $p_1<p_0/q_0$. Because $x<\frac{1}{q_0}$ by assumption, $p_1^S\left(\pi\right)<p_0/q_0$ for all $\pi<\pi_x\left(a_0^M\right)$, i.e., when support is needed. Therefore, the higher the exposure to the risky asset at $t=0$, the lower the net asset value when support is needed for any given price. This implies that a higher a_0^M requires a higher liquidation price for the manager not to need to offer support. Because the price function is increasing in π for $\pi\geq\pi_x\left(a_0^M\right)$, higher a_0^M requires higher realizations of π not to need to offer support, *i.e.*, a higher π_x. The same argument can be applied to the thresholds π^* and π^{**}.

Proposition 11 characterizes the support region and shows that managers will offer support in some states π as long as they have something to lose from closing the fund, either from forgone fees or from spillover losses.

Proposition 11. $\pi^*\left(a_0^M\right)\leq\pi_x\left(a_0^M\right)$ *and* $\pi^*\left(a_0^M\right)<\pi_x\left(a_0^M\right)$ *if and only if* $A_1^I\left(p_1\left(\pi_x\left(a_0^M\right)\right),\pi_x\left(a_0^M\right)\right)>0$*, or* $\bar{B}>0$.

Proof. From the definition of $\pi_x\left(a_0^M\right)$, it is easy to see that Eq. (10) always holds for $\pi_x\left(a_0^M\right)$. Therefore, $\pi_x\left(a_0^M\right) \geq \pi^*\left(a_0^M\right)$. Moreover, if $A_1^I\left(p_1\left(\pi_x\left(a_0^M\right)\right), \pi_x\left(a_0^M\right)\right)$ or $\bar{B} > 0$, Eq. (10) holds with strict inequality at $\pi_x\left(a_0^M\right)$, which implies that $\pi_x\left(a_0^M\right) > \pi^*\left(a_0^M\right)$. □

A.4. Equilibrium price function

Given these thresholds, the equilibrium price function can be further characterized.

Proposition 12. *Under Assumption* 1 *the equilibrium price function* $p_1^*\left(\pi, a_0^M\right)$ *is continuous and nondecreasing in* π *for all* $\pi \in \left[0, \pi^*\left(a_0^M\right)\right) \cup \left(\pi^*\left(a_0^M\right), 1\right]$ *for all* a_0^M.

Proof. The expressions for $p_1^{NS}\left(\pi, a_0^M\right)$, $p_1^{SS}\left(\pi, a_0^M\right)$, $p_1^S\left(\pi, a_0^M\right)$, and $p_1^L\left(\pi, a_0^M\right)$ are nondecreasing in π when Assumption 1 holds, $p_1^L\left(\pi^{**}\left(a_0^M\right), a_0^M\right) = p_1^{SS}\left(\pi^{**}\left(a_0^M\right), a_0^M\right)$, and $p_1^{NS}\left(\pi_x, a_0^M\right) = p_1^S\left(\pi_x, a_0^M\right)$. □

From the definition of $\pi_x\left(a_0^M\right)$ and $\pi^{**}\left(a_0^M\right)$, it is easy to see that

$$p_1^L\left(\pi^{**}\left(a_0^M\right), a_0^M\right) = p_1^{SS}\left(\pi^{**}\left(a_0^M\right), a_0^M\right) \tag{41}$$

and that $p_1^{NS}\left(\pi_x, a_0^M\right) = p_1^S\left(\pi_x, a_0^M\right)$. This follows from the continuity of the demand for the risky asset at $\pi_x\left(a_0^M\right)$ and $\pi^{**}\left(a_0^M\right)$. However, the demand for the risky asset can be discontinuous at $\pi^*\left(a_0^M\right)$. For $\pi \geq \pi^*\left(a_0^M\right)$, the demand for the risky asset is equal to the funds' size, which includes the managers' wealth and the investors' investment with their managers. For $\pi < \pi^*\left(a_0^M\right)$, all funds may be liquidated and the demand for the risky asset is conformed by the managers' wealth only. Then, if all funds are liquidated for $\pi < \pi^*\left(a_0\right)$, i.e., $\pi^*\left(a_0^M\right) < \pi^{**}\left(a_0^M\right)$, the demand function is discontinuous at $\pi^*\left(a_0^M\right)$ provided investors choose to invest with the manager when the realized quality of the risky asset is $\pi^*\left(a_0^M\right)$. This, in turn, implies that the equilibrium price can be discontinuous at $\pi^*\left(a_0^M\right)$.

Proposition 2 Under Assumption 1, the price function $p_1^*\left(\pi, a_0^M\right)$ is nondecreasing π for all $\pi \in [\underline{\pi}, \bar{\pi}]$.

Proof. From Proposition 12, the price function is nondecreasing when it is continuous. Therefore, to prove the proposition it is enough to show that

$$\lim_{\pi \to \pi^*\left(a_0^{M*}\right)^-} p_1^*\left(\pi, a_0^M\right) < \lim_{\pi \to \pi^*\left(a_0^{M*}\right)^+} p_1^*\left(\pi, a_0^M\right) \tag{42}$$

when the price function is discontinuous. The price is discontinuous only if $\pi^*\left(a_0^M\right) < \pi^{**}\left(a_0^M\right)$. In this case,

$$\lim_{\pi \to \pi^*\left(a_0^{M*}\right)^-} p_1^*\left(\pi, a_0^M\right) = p_L\left(\pi^*\left(a_0^{M*}\right), a_0^M\right) \tag{43}$$

and

$$\lim_{\pi \to \pi^*\left(a_0^{M*}\right)^+} p_1^*\left(\pi, a_0^M\right) = p_1^S\left(\pi^*\left(a_0^{M*}\right), a_0^M\right). \tag{44}$$

Suppose by contradiction that

$$p_L\left(\pi^*\left(a_0^{M*}\right), a_0^M\right) > p_1^S\left(\pi^*\left(a_0^{M*}\right), a_0^M\right). \tag{45}$$

Then, from the definition of $\pi^*\left(a_0^M\right)$, it follows that

$$\begin{aligned} &\left(x - n\left(p_1^L\left(\pi^*\left(a_0^M\right), a_0^M\right), a_0^M\right)\right)(1-f)\,A_0^I - fA_1^I\left(p_1^S\left(\pi^*\left(a_0^M\right), a_0^M\right), \pi^*\left(a_0^M\right)\right) \\ < \;& \min\left\{\frac{Bp_1^L\left(\pi^*\left(a_0^M\right), a_0^M\right)}{\bar{d}}, n\left(p_1^L\left(\pi^*\left(a_0^M\right), a_0^M\right), a_0^M\right)\left(W_0^M + fA_0^I\right) + E\right\}. \end{aligned} \tag{46}$$

Moreover, using that $\pi^*\left(a_0^M\right) < \pi^{**}\left(a_0^M\right)$ and the definition of $\pi^{**}\left(a_0^M\right)$

$$fA_1^I\left(p_1^L\left(\pi^{**}\left(a_0^M\right), a_0^M\right), \pi^{**}\left(a_0^M\right)\right) - fA_1^I\left(p_1^S\left(\pi^*\left(a_0^M\right), a_0^M\right), \pi^*\left(a_0^M\right)\right) < 0 \tag{47}$$

and

$$fA_1^I\left(p_1^S\left(\pi^{**}\left(a_0^M\right), a_0^M\right), \pi^{**}\left(a_0^M\right)\right) - fA_1^I\left(p_1^S\left(\pi^*\left(a_0^M\right), a_0^M\right), \pi^*\left(a_0^M\right)\right) < 0. \tag{48}$$

But this implies $\pi^*\left(a_0^M\right) > \pi^{**}\left(a_0^M\right)$ because $A_1^I\left(p_1^S\left(\pi, a_0^M\right), \pi\right)$ is increasing in π, which is a contradiction. □

A.5. Individual threshold characterization

This section characterizes the thresholds $\pi_{x,i}\left(a_{0,i}^{M}; B\right)$ and $\pi_i^*\left(a_{0,i}^{M}; B\right)$ that determine an individual manager's support decisions given his spillover losses $B\pi$. To simplify the notation, I omit the argument B in the individual threshold functions.

A.5.1. No support threshold

An individual manager who invested a fraction $a_{0,i}^{M}$ in the risky asset at $t = 0$ does not need to offer support at $t = 1$ if

$$n\left(p_1^*\left(\pi\right), a_{0,i}^{M}\right) \geq x. \tag{49}$$

Proposition 13. *There exists a unique threshold* $\pi_{x,i}\left(a_{0,i}^{M}\right)$ *such that support is not needed if* $\pi \geq \pi_{x,i}\left(a_{0,i}^{M}\right)$.

Proof. The proof is straightforward using that the price function $p_1^*\left(\pi\right)$ is nondecreasing in π. The left-hand side of this expression is always increasing in π.The right-hand side is constant in π. Therefore, there is a unique threshold $\pi_{x,i}\left(a_{0,i}^{M}\right)$ such that for all $\pi \geq \pi_{x,i}\left(a_{0,i}^{M}\right)$ Eq. (49) holds. □

As I show in Proposition 13, if the realized probability of success of the risky project is high enough, the manager does not need to offer support to his investor to keep the fund open. In this case, the realized net asset value is above the liquidation threshold x. Moreover, because the net asset value depends on the manager's portfolio choice at $t = 0$, how high a realization of π is needed not to need support depends on this portfolio choice. In Proposition 14, I show that the higher the risk incurred by the manager in the initial period, the higher the realization of π needed not to need to offer support. A full characterization of the threshold $\pi_{x,i}$ is provided in the

Appendix and shows that $\boldsymbol{\pi}_{x,i}$ is discontinuous in $a_{0,i}^{M}$ if the price function is constant for an interval $[\pi_a, \pi_b]$ where $\pi_a < \pi_b$.

Proposition 14. *$\pi_{x,i}\left(a_{0,i}^{M}\right)$ is increasing in $a_{0,i}^{M}$*

Proof. The left-hand side of (49) is constant in $a_{0,i}^{M}$ whereas the right-hand side is increasing in $a_{0,i}^{M}$ since $x \leq \frac{1}{q_0}$. Therefore, $\pi_{x,i}$ is increasing in $a_{0,i}^{M}$. □

If $\pi < \pi_{x,i}$, the manager cannot continue operating the fund unless he supports his investor. A manager who chooses to invest $a_{0,i}^{M}$ in the risky asset in period 0 whose realized spillover losses are $B\pi$ chooses to offer support if

$$H\left(\pi; a_{0,i}^{M}, B\right) \geq 0, \tag{50}$$

where

$$\begin{aligned} & H\left(\pi; a_{0,i}^{M}, B\right) \\ : \; & = \left(f a_1^{I*}\left(p_1^*\left(\pi\right), \pi\right) \left(x\left(1-f\right) a_0^I + \frac{\left(1-a_0^I\right)}{q_0} \right) - \left(x - n\left(p_1^*\left(\pi\right), a_{0,i}^{M}\right)\right) a_0^{I*}\left(1-f\right) \right) W_0^I \\ & + \min\left\{ \frac{Bp_1^*(\pi)}{d}, n\left(p_1^*\left(\pi\right), a_{0,i}^{M}\right)\left(W_0^M + f a_0^{I*} W_0^I\right) + E \right\}. \end{aligned} \tag{51}$$

This condition is analogous to the one presented to compute the aggregate support threshold π^*. Proposition 15 characterizes the support decision for an individual manager.

Proposition 15. *The characterization of $\pi_{x,i}$ is*

$$\pi_{x,i}\left(a_{0.i}^{M}; a_{\,0}^{\,M}\right) \begin{cases} = \underline{\pi} & \text{for } a_{0,i}^{M} \leq \widehat{a}_{0,i}^{M} \\ = \left(\frac{\left(x - \frac{1}{q_0}\right)}{a_{0.i}^{M}} + \frac{1}{q_0} \right) \frac{p_0}{dq_1} & \text{for } \widehat{a}_{0,i}^{M} < a_{0,i}^{M} \leq \underline{a}_{0,i}^{M}\left(a_0^M\right) \\ = \pi^*\left(a_0^M\right) & \text{for } \underline{a}_{0,i}^{M}\left(a_0^M\right) < a_{0,i}^{M} \leq \bar{a}_{0,i}^{M}\left(a_0^M\right) \\ > \pi^*\left(a_0^M\right) & \text{for } a_{0,i}^{M} \geq \bar{a}_{0,i}^{M}\left(a_0^M\right), \end{cases}$$

where

$$\widehat{a}_{0,i}^{M} = \frac{\left(x - \frac{1}{q_0}\right)}{\left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0}\right)},$$

$$\pi_1\left(a_0^M\right) = \frac{\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + f a_0^I W_0^I\right) + E}{\frac{a_0^M}{p_0}\left(1-f\right) a_0^I W_0^I \bar{d} q_1},$$

$$\underline{a}_{0,i}^M\left(a_0^M\right) = \frac{\left(x-\frac{1}{q_0}\right) a_0^M \left(1-f\right) a_0^I W_0^I}{\left(\left(1-a_0^M\right)\left(W_0^M + f a_0^I W_0^I\right) - a_0^M\left(1-f\right) a_0^I W_0^I\right)\frac{1}{q_0} + E},$$

and

$$\bar{a}_{0,i}^M\left(a_0^M\right) = \frac{\left(x-\frac{1}{q_0}\right)}{\bar{d}\left(1-f\right)\frac{\pi^*\left(a_0^M\right)-a_1^{IS}}{1-a_1^{IS}} - q_0}.$$

Proof. $\pi_{x,i}\left(a_{0.i}^M; a_0^M\right)$ is given by the minimum π such that

$$\frac{p_1^*\left(\pi; a_0^M\right)}{p_0} \geq \frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^M} + \frac{1}{q_0}. \tag{52}$$

If $a_{0,i}^M \leq \widehat{a}_{0,i}^M$, the right-hand side of this expression is ≤ 0, which implies that Eq. 52 holds for all $\pi \geq \underline{\pi}$ and therefore $\pi_{x,i}\left(a_{0.i}^M; a_0^M\right) = \underline{\pi}$. If $\underline{\pi} < \pi_1\left(a_0^M\right)$, for $\pi \in \left[\underline{\pi}, \pi_1\left(a_0^M\right)\right]$, $p_1^*\left(\pi; a_0^M\right) = \pi \bar{d} q_1$ and $\pi_{x.i}$ is given by

$$\frac{\pi_{x.i}\bar{d}q_1}{p_0} = \frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^M} + \frac{1}{q_0}. \tag{53}$$

But $\pi_{x.i}\left(a_{0.i}^M; a_0^M\right) \leq \pi_1\left(a_0^M\right)$ if and only if $a_{0,i}^M \leq \underline{a}_{0,i}^M\left(a_0^M\right)$. For $\pi \in \left(\pi_1\left(a_0^M\right), \pi^*\left(a_0^M\right)\right)$, $p_1^*\left(\pi\right) = p_L$. Therefore, $\pi_{x.i} \notin \left(\underline{\pi}\left(a_0^M\right), \pi^*\left(a_0^M\right)\right)$ since if Eq. 52 holds for one $\pi \in \left(\pi_1\left(a_0^M\right), \pi^*\left(a_0^M\right)\right)$, it holds for all $\pi_0 \in \left[\pi_1\left(a_0^M\right), \pi^*\left(a_0^M\right)\right)$. Moreover, $p_L = \pi_1\left(a_0^M\right)\bar{d}q_1$. Thus, $\pi_{x.i}\left(a_{0.i}^M; a_0^M\right) \geq \pi^*\left(a_0^M\right)$ for all $a_{0,i}^M > \underline{a}_{0,i}^M\left(a_0^M\right)$. Finally, for $\underline{a}_{0,i}^M\left(a_0^M\right) < a_{0,i}^M \leq \bar{a}_{0,i}^M\left(a_0^M\right)$,

$$\frac{p_1^*\left(\pi^*; a_0^M\right)}{p_0} \geq \frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^M} + \frac{1}{q_0} \tag{54}$$

and

$$\frac{\lim_{\pi\to\pi^*} p_1^*\left(\pi; a_0^M\right)}{p_0} < \frac{\left(x - \frac{1}{q_0}\right)}{a_{0.i}^M} + \frac{1}{q_0}, \tag{55}$$

which implies $\pi_{x,i}\left(a_{0.i}^M; a\,_0^M\right) = \pi^*\left(a_0^M\right)$. □

Lemma 2. $H\left(\pi; a_{0,i}^M, B\right)$ *is increasing in* π *whenever it is continuous.*

Proof. If the equilibrium price function is continuous in π, $H\left(\pi; a_{0,i}^M, B\right)$ is always increasing in π. $a_1^{I*}\left(p_1^*\left(\pi\right), \pi\right)$ is always increasing in π if the equilibrium price is continuous in π. □

A.5.2. Support threshold

Proposition 3 Given the equilibrium price and spillover losses $B\pi$, there exists a unique $\pi_i^*\left(a_{0,i}^M; B\right)$ such that for all $\pi_i^*\left(a_{0,i}^M; B\right) < \pi < \pi_{x,i}\left(a_{0,i}^M\right)$ a manager with spillover losses $B\pi$ strictly prefers to offer support.

Proof. An individual manager with spillover losses $B\pi$ offers support in all states π such that $H\left(\pi; a_{0,i}^M, B\right) \geq 0$. If the equilibrium price is continuous in π or if the equilibrium price is discontinuous at $\pi^*\left(a_0^M; B\right)$ but $\lim_{\pi\to\pi^*} H\left(\pi; a_{0,i}^M, B\right) < H\left(\pi^*; a_{0,i}^M, B\right)$, the proof follows from monotonicity of $H\left(\pi; a_{0,i}^M, B\right)$ in π, using Lemma 2. If $\lim_{\pi\to\pi^*} H\left(\pi; a_{0,i}^M, B\right) > H\left(\pi^*; a_{0,i}^M, B\right)$ and $\lim_{\pi\to\pi^*} H\left(\pi; a_{0,i}^M, B\right) > H\left(\pi^*; a_{0,i}^M, B\right) > 0$ or $0 > \lim_{\pi\to\pi^*} H\left(\pi; a_{0,i}^M, B\right) > H\left(\pi^*; a_{0,i}^M, B\right)$, $H\left(\pi; a_{0,i}^M, B\right)$ crosses zero only once and the proposition holds.

Suppose that $\lim_{\pi\to\pi^*} H\left(\pi; a_{0,i}^M, B\right) > 0 > H\left(\pi^*; a_{0,i}^M, B\right)$. Then, the set of realizations of π for which the manager offers support is given by $\left[\underline{\pi}_i^*\left(a_{0,i}^M; B\right), \pi^*\right] \cup \left[\bar{\pi}_i^*\left(a_{0,i}^M; B\right), 1\right]$, where

$$\begin{aligned} 0 \;=\; & \left(f a_1^{I*}\left(p_1^L\left(\underline{\pi}_i^*\right), \underline{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I + \frac{\left(1-a_0^I\right)}{q_0}\right) - \left(x - n\left(p_1^L\left(\underline{\pi}_i^*\right), a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I \\ & + \min\left\{\frac{B p_1^L\left(\underline{\pi}_i^*\right)}{\bar{d}}, n\left(p_1^L\left(\underline{\pi}_i^*\right), a_{0,i}^M\right)\left(W_0^M + f a_0^{I*} W_0^I\right)\right\} \end{aligned} \tag{56}$$

or $\underline{\pi}_i^*\left(a_{0,i}^M;B\right)=0$ and

$$0 = \left(fa_1^{I*}\left(p_1^S\left(\bar{\pi}_i^*\right),\bar{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I+\frac{\left(1-a_0^I\right)}{q_0}\right)-\left(x-n\left(p_1^S\left(\bar{\pi}_i^*\right),a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I + \min\left\{\frac{Bp_1^S\left(\bar{\pi}_i^*\right)}{\bar{d}},n\left(p_1^S\left(\bar{\pi}_i^*\right),a_{0,i}^M\right)\left(W_0^M+fa_0^{I*}W_0^I\right)\right\}. \quad (57)$$

Note that $\underline{\pi}_i^*\left(a_{0,i}^M;B\right)\leq\pi^*\left(a_0^M\right)\leq\bar{\pi}_i^*\left(a_{0,i}^M;B\right)$ and, therefore, the left hand side of the expressions above is decreasing in $a_{0,i}^M$, because $p_1^L\left(\underline{\pi}_i^*\right)<p_1^S\left(\bar{\pi}_i^*\left(a_{0,i}^M;B\right)\right)\leq\frac{1}{q_0}$ (if $\bar{\pi}_i^*\left(a_{0,i}^M;B\right)>\pi_x\left(a_{0,i}^M\right)$, left-hand side is greater than zero. Moreover, from the definition of $\pi^*\left(a_0^M\right)$ and using that $H\left(\pi;a_{0,i}^M;B\right)$ is increasing in π in $\left(\pi^*\left(a_0^M\right),1\right]$,

$$0 < \left(fa_1^{I*}\left(p_1^*\left(\bar{\pi}_i^*\right),\bar{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I+\frac{\left(1-a_0^I\right)}{q_0}\right)-\left(x-n\left(p_1^*\left(\bar{\pi}_i^*\right),a_0^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I + \min\left\{\frac{Bp_1^*\left(\bar{\pi}_i^*\right)}{\bar{d}},n\left(p_1^*\left(\bar{\pi}_i^*\right),a_0^M\right)\left(W_0^M+fa_0^{I*}W_0^I\right)\right\}, \quad (58)$$

which implies that $a_{0,i}^M\geq a_0^M$. From the definition of $\pi^*\left(a_0^M\right)$,

$$0 > \left(\begin{array}{c} fa_1^{I*}\left(p_1^L\left(\pi^*\left(a_0^M\right)\right),\pi^*\left(a_0^M\right)\right)\left(x\left(1-f\right)a_0^I+\frac{\left(1-a_0^I\right)}{q_0}\right) \\ -\left(x-n\left(p_1^L\left(\pi^*\left(a_0^M\right)\right),a_0^M\right)\right)a_0^{I*}\left(1-f\right)\end{array}\right)W_0^I + \min\left\{\frac{Bp_1^L\left(\pi^*\left(a_0^M\right)\right)}{\bar{d}},n\left(p_1^L\left(\pi^*\left(a_0^M\right)\right),a_0^M\right)\left(W_0^M+fa_0^{I*}W_0^I\right)\right\}, \quad (59)$$

and because the left-hand side is decreasing in a_0^M, this implies

$$0 > \left(fa_1^{I*}\left(p_1^L\left(\underline{\pi}_i^*\right),\underline{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I+\frac{\left(1-a_0^I\right)}{q_0}\right)-\left(x-n\left(p_1^L\left(\underline{\pi}_i^*\right),a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I + \min\left\{\frac{Bp_1^L\left(\underline{\pi}_i^*\right)}{\bar{d}},n\left(p_1^L\left(\underline{\pi}_i^*\right),a_{0,i}^M\right)\left(W_0^M+fa_0^{I*}W_0^I\right)\right\} \quad (60)$$

which is a contradiction.

Therefore, $H\left(\pi,a_{0,i}^M,B\right)$ crosses 0 at most once and the proposition holds.

□

Proposition 16. $\pi_i^*\left(a_{0.i}^M;a_0^M,B\right)$ *is increasing in* $a_{0,i}^M$.

Proof. Follows from the definition of H and the fact that $p_1^*\left(\pi\right)<\frac{1}{q_0}<x$ for

all $\pi \leq \pi_{x,i}\left(a_{0,i}^{M}\right)$. □

A.6. *Investors' problem*

In this section I characterize the investors' problem.

A.6.1. *Investors' wealth*

The $t = 2$ wealth of an investor with manager i is given by

$$
\begin{aligned}
&W_2^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}(\pi), \pi\right) \\
&= \begin{cases} \left(a_1^{I*}\left(p_1(\pi), \pi\right)\left((1-f)d - \frac{1}{q_1}\right) + \frac{1}{q_1}\right) W_1^I & \text{if} \quad \pi \geq \pi_i^* \\ \frac{1}{q_1} W_1^I \left(\bar{s}(\pi) a_1^{I*}\left(p_1(\pi), \pi\right)\left((1-f)d - \frac{1}{q_1}\right) + \frac{1}{q_1}\right) & \text{if} \quad \pi < \pi_i^*, \end{cases}
\end{aligned}
$$

where $W_1 = W_1\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}(\pi), \pi\right)$ and

$$
\begin{aligned}
&W_1^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}(\pi), \pi\right) \\
&= \begin{cases} \left(x(1-f)a_0^I + \left(1 - a_0^I\right)\frac{1}{q_0}\right) W_0^I \text{ if } \pi_{x,i} > \pi \geq \pi_i^* \\ \left(\bar{s}(\pi) x + (1 - \bar{s}(\pi))\, n\left(p_1(\pi), a_{0,i}^M\right)\right)(1-f) A_0^I + \left(1 - a_0^I\right)\frac{W_0^I}{q_0} \text{ otherwise.} \end{cases}
\end{aligned}
$$

A.6.2. *Optimization*

Because of log utility, the investor's problem can be rewritten as

$$
\begin{aligned}
&\max_{a_0^I \in [0,1]} \mathbb{E}_\pi s^*(\pi) \log W_1^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}(\pi), \pi\right) \\
&+\mathbb{E}_\pi\left(1 - s^*(\pi)\right) \log W_1^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}(\pi), \pi\right). \qquad (61)
\end{aligned}
$$

The first order condition for an interior solution is

$$\begin{aligned}
0 \;=\; & \int_{\pi_x\left(a_0^M\right)}^{\bar{\pi}} \frac{n\left(p_1(\pi), a_0^M\right)(1-f)-\frac{1}{q_0}}{\left(n\left(p_1(\pi), a_0^M\right)(1-f) a_0^I+\left(1-a_0^I\right) \frac{1}{q_0}\right)} dG(\pi) \\
& +\int_{\pi^*\left(a_0^M\right)}^{\pi_x\left(a_0^M\right)} \frac{x(1-f)-\frac{1}{q_0}}{x(1-f) a_0^I+\left(1-a_0^I\right) \frac{1}{q_0}} dG(\pi) \\
& +\int_0^{\pi^*\left(a_0^M\right)} \frac{n\left(p_1(\pi), a_0^M\right)(1-f)-\frac{1}{q_0}}{n\left(p_1(\pi), a_0^M\right)(1-f) a_0^I+\left(1-a_0^I\right) \frac{1}{q_0}} dG(\pi) .
\end{aligned} \tag{62}$$

The second order condition is negative.

$$\begin{aligned}
0 \;>\; & -\int_{\pi_x\left(a_0^M\right)}^{\bar{\pi}}\left(\frac{n\left(p_1(\pi), a_0^M\right)(1-f)-\frac{1}{q_0}}{n\left(p_1(\pi), a_0^M\right)(1-f) a_0^I+\left(1-a_0^I\right) \frac{1}{q_0}}\right)^2 dG(\pi) \\
& -\int_{\pi^*\left(a_0^M\right)}^{\pi_x\left(a_0^M\right)}\left(\frac{x(1-f)-\frac{1}{q_0}}{x(1-f) a_0^I+\left(1-a_0^I\right) \frac{1}{q_0}}\right)^2 dG(\pi) \\
& -\int_0^{\pi^*\left(a_0^M\right)}\left(\frac{\lambda n\left(p_1(\pi), a_0^M\right)(1-f)-\frac{1}{q_0}}{\lambda n\left(p_1(\pi), a_0^M\right)(1-f) a_0^I+\left(1-a_0^I\right) \frac{1}{q_0}}\right)^2 dG(\pi) .
\end{aligned} \tag{63}$$

A.7. *Managers' objective function*

In this subsection I describe and characterize the managers' wealth and objective function.

A.7.1. *Managers' wealth*

Manager i's wealth at $t=2$ is given by $\frac{\pi \bar{d}}{p_1} W_1^M(B)$, where

$$W_1^M(B)=W_1^M\left(a_{0,i}^M ; \pi_{x,i}\left(a_{0,i}^M, B\right), \pi_i^*\left(a_{0,i}^M, B\right), a_0^I, \pi\right) . \tag{64}$$

If no support is needed for the fund to continue at $t=1$, then

$$W_1^M(B)=n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_0^M+f a_0^I W_0^I\right)+E+f A_1^{I*}(\pi) . \tag{65}$$

If the manager offers support to his investor at $t = 1$, then

$$\begin{aligned} W_1^M(B) &= n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E + f A_1^{I*}(\pi) && (66) \\ &\quad - \left(x - n\left(p_1^*(\pi), a_{0,i}^M\right)\right) a_0^{I*}(1-f) W_0^I. && (67) \end{aligned}$$

Finally, if the fund is liquidated

$$W_1^M(B) = n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E. \tag{68}$$

A.7.2. Continuity

Because $\pi_{x,i}\left(a_{0,i}^M\right)$ can have a discontinuity at $\underline{a}_{0,i}^M\left(a_0^M\right)$ and $\pi_{x,i}\left(a_{0,i}^M; B\right) = \pi_{x,i}\left(a_{0,i}^M\right)$ for all B, the objective function can be discontinuous at $\underline{a}_{0,i}^M\left(a_0^M\right)$, too.

If $\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) = \pi_i^*\left(\underline{a}_{0,i}^M\left(a_0^M\right), B\right)$ for all B,

$$\begin{aligned} & Obj\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) - \lim_{a \to \underline{a}_{0,i}^M\left(a_0^M\right)^+} Obj(a) \\ = & \int_{\pi_{x,i}\left(\underline{a}_{0,i}^M\right)}^{\pi^*} \left(\frac{\pi \bar{d}}{p_1^*(\pi)} f a_1^{I*}(\pi)\left(a_0^{I*}(1-f)\left(\underline{a}_{0,i}^M \frac{p_1^*(\pi)}{p_0} + \frac{\left(1-a_{0,i}^M\right)}{q_0}\right) + \frac{\left(1-a_0^{I*}\right)}{q_0}\right) W_0^I\right) dF(\pi) \\ + & \int_{\pi_{x,i}\left(\underline{a}_{0,i}^M\right)}^{\pi^*} \int_0^{\bar{B}} B\pi dG(B)\, dF(\pi) \geq 0 \qquad (69) \end{aligned}$$

and the objective function jumps down and a maximum always exists.

For $\bar{B} = 0$, $p_1^*\left(\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)\right) = \bar{d}\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) q_1$, which implies that

$$a_1^I\left(p_1^*\left(\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)\right), \pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)\right) = 0 \tag{70}$$

and, hence, that $\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) = \pi_i^*\left(\underline{a}_{0,i}^M\left(a_0^M\right), 0\right)$.

If $\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) > \pi_i^*\left(\underline{a}_{0,i}^M\left(a_0^M\right), B\right)$,

$$Obj\left(\underline{a}_{0,i}^{M}\left(a_0^M\right)\right)-\lim_{a\to\underline{a}_{0,i}^{M}\left(a_0^M\right)^+}Obj\left(a\right) \tag{71}$$

$$=\int_{\pi_{x,i}\left(a_{0,i}^M\right)}^{\pi^*}\frac{\pi\bar{d}}{p_1^*\left(\pi\right)}\left(fa_1^{I*}\left(\pi\right)-1\right)\left(\left(a_{0,i}^M\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)+\frac{1}{q_0}\right)-x\right)a_0^{I*}\left(1-f\right)W_0^IdG\left(B\right)\leq 0,$$

and the objective function jumps up at $\underline{a}_{0,i}^{M}\left(a_0^M\right)$ and the existence of a maximum is not guaranteed. In a symmetric equilibrium, if $\pi_{x,i}\left(a_{0,i}^M\right)$ is discontinuous, $\underline{a}_{0,i}^{M}\left(a_0^M\right)<a_0^M$.

A.7.3. Differentiability

The first derivative of the manager's objective function with respect to $a_{0,i}^M$ when the objective function is differentiable for a given value of B is given by

$$\begin{aligned}&\int\frac{\pi\bar{d}}{p_1^*\left(\pi\right)}\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)\left(W_0^M+fa_0^IW_0^I\right)dF\left(\pi\right)\\&+\int_{\pi_i^*\left(a_{0,i}^M,B\right)}\frac{\pi\bar{d}}{p_1^*\left(\pi\right)}fa_1^{I*}\left(\pi\right)\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)a_0^{I*}\left(1-f\right)W_0^IdF\left(\pi\right)\\&+\int_{\pi_i^*\left(a_{0,i}^M,B\right)}^{\pi_{x,i}\left(a_{0,i}^M\right)}\frac{\pi\bar{d}}{p_1^*\left(\pi\right)}\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)a_0^{I*}\left(1-f\right)W_0^IdF\left(\pi\right)\\&-\frac{\partial\pi_i^*\left(a_{0,i}^M\right)}{\partial a_{0,i}^M}\frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)}\left(\begin{array}{c}fa_1^{I*}\left(\pi_i^*\left(a_{0,i}^M\right)\right)\left(a_0^{I*}\left(1-f\right)x+\left(1-a_0^{I*}\right)\frac{1}{q_0}\right)W_0^I\\-\left(x-n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right),a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)W_0^I\\+\frac{B}{d}p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)\end{array}\right)dF\left(\pi\right).\end{aligned} \tag{72}$$

Using the definition of π_i^* and the fact that if (condition) holds with strict inequality then $\frac{\partial\pi_i^*\left(a_{0,i}^M\right)}{a_{0,i}^M}=0$ (when it exists), the last term in the first derivative is equal to

$$\begin{aligned}&\frac{\partial\pi_i^*\left(a_{0,i}^M,B\right)}{a_{0,i}^M}\times\frac{\pi_i^*\left(a_{0,i}^M,B\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)}\\\times\quad&\min\left\{0,n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M,B\right)\right),a_{0,i}^M\right)\left(W_0^M+fa_0^IW_0^I\right)+E-\frac{B}{d}p_1^*\left(\pi_i^{*E}\left(a_{0,i}^M\right)\right)\right\}dF\left(\pi\right).\end{aligned} \tag{73}$$

Therefore, the first derivative of the objective function with respect to $a_{0,i}^M$ is

$\frac{\partial Obj}{\partial a_{0,i}^M} =$

$$\begin{aligned} & \int_{\underline{\pi}}^{\bar{\pi}} \frac{\pi \bar{d}}{p_1^*(\pi)} \left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0} \right) \left(W_0^M + f a_0^I W_0^I \right) dF(\pi) \\ + & \int_0^{\bar{B}} \int_{\pi_i^*\left(a_{0,i}^M, B\right)}^{\bar{\pi}} \frac{\pi \bar{d}}{p_1^*(\pi)} f a_1^{I*}(\pi) \left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0} \right) a_0^{I*} (1-f) W_0^I dF(\pi) dG(B) \\ + & \int_0^{\bar{B}} \int_{\pi_i^*\left(a_{0,i}^M, B\right)}^{\pi_{x,i}\left(a_{0,i}^M\right)} \frac{\pi \bar{d}}{p_1^*(\pi)} \left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0} \right) a_0^{I*} (1-f) W_0^I dF(\pi) dG(B) \qquad (74) \\ + & \int_0^{\bar{B}} \frac{\partial \pi_i^*\left(a_{0,i}^M, B\right)}{\partial a_{0,i}^M} \frac{\pi_i^*\left(a_{0,i}^M, B\right) \bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M, B\right)\right)} \qquad (75) \\ \times & \min\left\{0, n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M, B\right)\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E - \tfrac{B}{\bar{d}} p_1^*\left(\pi_i^*\left(a_{0,i}^M, B\right)\right)\right\} dF(\pi) dG(B). \end{aligned}$$

When $\bar{B} = 0$, this first derivative is continuous when the objective function is differentiable. In this case, when the first derivative is differentiable, the second derivative of the objective function is given by $\frac{\partial^2 Obj}{\partial a_{0,i}^{M2}} =$

$$\begin{aligned} & \left(1 - f a_1^{I*}\left(\pi_{x,i}\left(a_{0,i}^M\right)\right)\right) \frac{\pi_{x,i}\left(a_{0,i}^M\right) \bar{d}}{p_1^*\left(\pi_{x,i}\left(a_{0,i}^M\right)\right)} \left(\frac{p_1^*\left(\pi_{x,i}\left(a_{0,i}^M\right)\right)}{p_0} - \frac{1}{q_0} \right) a_0^{I*} (1-f) W_0^I dF(\pi) \frac{\partial \pi_{x,i}\left(a_{0,i}^M\right)}{a_{0,i}^M} \\ + & \int_0^{\bar{B}} \frac{\pi_i^*\left(a_{0,i}^M, B\right) \bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M, B\right)\right)} \left(\frac{p_1^*\left(\pi_i^*\left(a_{0,i}^M, B\right)\right)}{p_0} - \frac{1}{q_0} \right) a_0^{I*} (1-f) W_0^I dF(\pi) \frac{\partial \pi_i^*\left(a_{0,i}^M, B\right)}{a_{0,i}^M} dG(B) \\ + & \int_0^{\bar{B}} \frac{\partial^2 \pi_i^*\left(a_{0,i}^M, B\right)}{\partial a_{0,i}^{M2}} \frac{\pi_i^*\left(a_{0,i}^M, B\right) \bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M, B\right)\right)} \qquad (76) \\ \times & \min\left\{0, n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M, B\right)\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E - \tfrac{B}{\bar{d}} p_1^*\left(\pi_i^*\left(a_{0,i}^M, B\right)\right)\right\} dF(\pi) dG(B) \\ + & \int_0^{\bar{B}} \left(\frac{\partial \pi_i^*\left(a_{0,i}^M, B\right)}{\partial a_{0,i}^M} \right)^2 \\ \times & \frac{\partial \left(\frac{\pi_i^*\left(a_{0,i}^M, B\right) \bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M, B\right)\right)} \min\left\{0, n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M, B\right)\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E - \frac{B p_1^*\left(\pi_i^*\left(a_{0,i}^M, B\right)\right)}{\bar{d}}\right\} \right)}{\partial a_{0,i}^M} dF(\pi) dG(B). \end{aligned}$$

Thus, if $\bar{B} = 0$,

$$\frac{\partial^2 Obj}{\partial a_{0,i}^{M2}} \left(a_0^M\right) > 0 \qquad (77)$$

Therefore, there cannot be a symmetric equilibrium in which a_0^M is interior if $\bar{B} = 0$.

A.8. Results

In this section, I present the proofs of the results when $\bar{B} = 0$.

Proposition 4. If $\bar{B} = 0$, $a_0^{M*} = 1$ in a symmetric equilibrium.

Proof. Using the characterization of the objective function for the manager, this function is differentiable and convex at $a_{0,i}^M = a_0^M$ when $\bar{B} = 0$. Therefore, in a symmetric equilibrium, a_0^M cannot be an interior solution. Assuming that the supply function is such that there is always some risky asset bough at $t = 0$, a_0^M has to be one in a symmetric equilibrium. □

Lemma 3. *When $\bar{B} = 0$, the manager's support decision is a threshold decision for all $B_i > 0$ and all $W_{0,i}^M$ at the equilibrium price (given the equilibrium decisions for other managers).*

Proof. To show that the lemma holds it is enough to show that it cannot be the case that

$$\lim_{\pi \to \pi^*} H\left(\pi; a_{0,i}^M, B_i, W_{0,i}^M\right) \geq 0 > H\left(\pi^*; a_{0,i}^M, B_i, W_{0,i}^M\right) \tag{78}$$

Let $H\left(\pi; a_{0,i}^M, B_i, W_{0,i}^M\right) :=$

$$\left(f a_1^{I*}\left(p_1^*(\pi), \pi\right)\left(x(1-f)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) - \left(x - n\left(p_1^*(\pi), a_{0,i}^M\right)\right) a_0^{I*}(1-f)\right) W_0^I$$
$$+ \min\left\{\frac{B p_1^*(\pi)}{\bar{d}}, n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_{0,i}^M + f a_0^{I*} W_0^I\right) + E\right\}. \tag{79}$$

I know from the definition of the aggregate support decision threshold that

$$\begin{aligned} H(\pi^*; 1, 0, 0) &= f a_1^{I*}\left(p_1^S(\pi^*), \bar{\pi}_i^*\right)\left(x(1-f)a_0^I + \tfrac{\left(1-a_0^I\right)}{q_0}\right) W_0^I - \left(x - \tfrac{p_1^S(\pi^*)}{p_0}\right) a_0^{I*}(1-f) W_0^I \\ &= 0 \end{aligned} \tag{80}$$

and $\lim_{\pi \to \pi^*} H(\pi; 1, 0, 0)$

$$\begin{aligned} &= f a_1^{I*}\left(p_1^L(\pi^*), \bar{\pi}_i^*\right)\left(x(1-f)a_0^I + \tfrac{\left(1-a_0^I\right)}{q_0}\right) W_0^I - \left(x - \tfrac{p_1^L(\pi^*)}{p_0}\right) a_0^{I*}(1-f) W_0^I \\ &\leq 0. \end{aligned} \tag{81}$$

$H\left(\pi; a_{0,i}^{M}, B_i\right)$ is decreasing in $a_{0,i}^{M}$ and increasing in B_i and $W_{0,i}^{M}$. Therefore, it cannot be the case that for some triplet $\left(a_{0,i}^{M}, B_i, W_{0,i}^{M}\right)$

$$\lim_{\pi \to \pi^*} H\left(\pi; a_{0,i}^{M}, B_i; W_{0,i}^{M}\right) \geq 0 > H\left(\pi^*; a_{0,i}^{M}, B_i, W_{0,i}^{M}\right) \tag{82}$$

because $H\left(\pi^*; a_{0,i}^{M}, B_i, W_{0,i}^{M}\right) \geq 0$ always. □

Proposition 5. Suppose $B = 0$ for all managers but for manager j. Then, in equilibrium, the risk taken by manager j at $t = 0$ will be decreasing in B_j and he will choose $a_{0,j}^{M} \in \left\{a_0^{MS}, 1\right\}$, where $a_0^{MS} = \max a_{0,i}^{M}$ s.t. $\pi_{x,i}\left(a_{0,i}^{M}\right) = \underline{\pi}$.

Proof. Using lemma 3 and the characterization of the manager's objective function when $B = 0$, the two candidates for maxima are $V^M\left(W_0^M; a_0^{MS}\right)$ and $V^M\left(W_0^M; 1\right)$, where a_0^{MS} is the highest $a_{0,i}^{M}$ such that $\pi_{x,i}\left(a_{0,i}^{M}\right) = \underline{\pi}$. Then,

$$a_{0,j}^{M*} = \begin{cases} a_0^{MS} & \text{if } V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right) > 0 \\ 1 & \text{if } V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right) < 0. \end{cases} \tag{83}$$

When $B_j = 0$, $V^M\left(W_0^M; a_0^{MS}\right) < V^M\left(W_0^M; 1\right)$ (follows from Proposition 4). Moreover,

$$\lim_{B \to \infty} \left(V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right)\right) = \infty. \tag{84}$$

Let

$$\Theta = \mathbf{1}\left\{\frac{Bp_1^*\left(\pi^*\left(1\right)\right)}{\bar{d}} > \frac{p_1^*\left(\pi_i^*\left(1\right)\right)}{p_0}\left(W_{0,i}^M + fa_0^{I*}W_0^I\right) + E\right\} \tag{85}$$

Using the characterization of the objective function in Section $A.7$,

$$\frac{\partial\left(V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right)\right)}{\partial B_j} \tag{86}$$

$$\propto \begin{cases} \int_{\underline{\pi}}^{\pi_i^*(1)} B_i \pi d\pi + \frac{\partial \pi_i^*}{\partial B_i}(1)\left(\frac{\pi_i^*(1)\bar{d}}{p_1^*(\pi)}\left(\frac{p_1^*(\pi_i^*(1))}{p_0}\left(W_{0,i}^M + fa_0^{I*}W_0^I\right) + E\right) - B_i\pi_i^*(1)\right) & \text{if } \Theta = 1 \\ \int_{\underline{\pi}}^{\pi_i^*(1)} B_i \pi d\pi & \text{if } \Theta = 0 \end{cases}$$

and

$$0 < \frac{\partial\left(V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right)\right)}{\partial B_j}. \tag{87}$$

because $\frac{\partial \pi_i^*}{\partial B_i}(1) < 0$. □

A.9. Numerical examples

The values used for the numerical examples are chosen to illustrate how taking into account general equilibrium effects can change the results of the policy analysis. The supply of the risky asset assumed is $S(p_0) = kp_0$. The parameter values are the following: $k = 987.4167$, $C = 1.839$, $q_0 = 1$, $q_1 = 0.8$, $x = 0.995$, $\bar{d} = 3$, $W_0^M = 200$, $W_0^I = 800$, $\bar{\pi} = 0.85$, $\underline{\pi} = 0.25$, $F = 0.03$, $\bar{B} = 10$, and $G(B) = \frac{B}{10}$.

A.10. No support allowed

An individual manager i's wealth at $t = 1$ is

$$W_1^M = n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E + f A_1^{I*}(\pi) \tag{88}$$

if no support is needed for the fund to continue at $t = 1$ and

$$W_1^M = n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E \tag{89}$$

if the fund is liquidated.

The problem solved by a manager who is not allowed to offer support in the interim period is

$$\max_{a_{0,i}^M \in [0,1]} \int \left(\frac{\pi \bar{d}}{p_1}\left(n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E\right)\right) d\pi + \int_{\pi_{x,i}\left(a_{0,i}^M\right)} \frac{\pi \bar{d}}{p_1} f A_1^{I*}(\pi)\, d\pi - \int^{\pi_{x,i}\left(a_{0,i}^M\right)} B\pi\, d\pi, \tag{90}$$

where the first term represents the expected return on his wealth at time 1 unconditionally of the liquidation of the fund, the second term represents the fees the manager collects if he keeps the fund open, and the third term represents the spillover losses that the manager suffers if the fund is liquidated.

The first order condition with respect to the portfolio choice $a_{0,i}^{M}$ is

$$\int \frac{\pi \bar{d}}{p_1}\left(\frac{p_1^*(\pi)}{p_0}-\frac{1}{q_0}\right)\left(W_0^M+fa_0^IW_0^I\right)d\pi \tag{91}$$

$$+\int_{\pi_{x,i}\left(a_{0,i}^{M}\right)}\frac{\pi \bar{d}}{p_1}fa_1^{I*}(\pi)\left(\frac{p_1^*(\pi)}{p_0}-\frac{1}{q_0}\right)a_0^{I*}(1-f)W_0^Id\pi \tag{92}$$

$$-\frac{\partial \pi_{x,i}\left(a_{0,i}^{M}\right)}{\partial a_{0,i}^{M}}\left(\frac{\pi_{x,i}\left(a_{0,i}^{M}\right)\bar{d}}{p_1}fA_1^{I*}\left(\pi_{x,i}\left(a_{0,i}^{M}\right)\right)+B\pi_{x,i}\left(a_{0,i}^{M}\right)\right)\geq 0. \tag{93}$$

The first term represents the change in the expected portfolio return due to a change in the amount invested in the risky asset. The second term captures the positive flow-performance sensitivity due to an increase in the exposure to risk. A higher exposure to the risky asset implies that the return received by investors in the interim period is expected to be higher and, therefore, implies a higher amount of fees collected by the manager for keeping the fund open at $t=1$. Finally, the last term captures the decrease in value due to the increase in the probability of liquidation of fund.

Proposition 17. *The problem of an individual manager who cannot offer support is concave.*

Proof. The second order condition for this problem is

$$\begin{aligned}
SOC = &-\frac{\partial \pi_{x,i}\left(a_{0,i}^{M}\right)}{\partial a_{0,i}^{M}}\frac{\pi \bar{d}}{p_1}fa_1^{I*}(\pi)\left(\frac{p_1^*(\pi)}{p_0}-\frac{1}{q_0}\right)a_0^{I*}(1-f)W_0^I \\
&-\left(\frac{\partial \pi_{x,i}\left(a_{0,i}^{M}\right)}{\partial a_{0,i}^{M}}\right)^2\left(\frac{1}{p_1}fA_1^{I*}\left(\pi_{x,i}\left(a_{0,i}^{M}\right)\right)+\int_0^{\bar{B}}BdG(B)\right) \\
&-\frac{\partial^2 \pi_{x,i}\left(a_{0,i}^{M}\right)}{\partial a_{0,i}^{M2}}\left(\frac{\bar{d}}{p_1}fA_1^{I*}\left(\pi_{x,i}\left(a_{0,i}^{M}\right)\right)+\int_0^{\bar{B}}BdG(B)\,\pi_{x,i}\left(a_{0,i}^{M}\right)\right) \\
&-\frac{\partial \pi_{x,i}\left(a_{0,i}^{M}\right)}{\partial a_{0,i}^{M}}\pi_{x,i}\left(a_{0,i}^{M}\right)\left(\frac{\bar{d}}{p_1}fa_1^{I*}(\pi)\left(\frac{p_1^*(\pi)}{p_0}-\frac{1}{q_0}\right)a_0^{I*}(1-f)W_0^I\right).
\end{aligned} \tag{94}$$

$\pi_{x,i}$ is defined by

$$p_1\left(\pi_{x.i}\right) - \left(\left(x - \frac{1}{q_0}\right)\frac{1}{a_{0,i}^M} + \frac{1}{q_0}\right)p_0 = 0. \tag{95}$$

Therefore,

$$\frac{\partial \pi_{x,i}\left(a_{0,i}^M\right)}{\partial a_{0,i}^M} = -\frac{\left(x - \frac{1}{q_0}\right)}{p_1'\left(\pi_{x.i}\right)a_{0,i}^{M2}} > 0 \tag{96}$$

and

$$\frac{\partial^2 \pi_{x,i}\left(a_{0,i}^M\right)}{\partial a_{0,i}^{M2}} = -\frac{-\left(x - \frac{1}{q_0}\right)}{\left(p_1''\left(\pi_{x.i}\right)a_{0,i}^{M2} + 2p_1'\left(\pi_{x.i}\right)a_{0,i}^{M3}\right)}. \tag{97}$$

Because $p_1''\left(\pi_{x.i}\right) \geq 0$, $\frac{\partial^2 \pi_{x,i}\left(a_{0,i}^M\right)}{\partial a_{0,i}^{M2}} < 0$ and thus $SOC < 0$. □

Proposition 6. Suppose, in a equilibrium, all managers are allowed to offer support. Given equilibrium fees f, asset prices p_0 and $p_1(\pi)$, and investors' choice a_{0i}^I, an individual manager who cannot offer support will choose to take less risk than a manager who can offer support at $t = 1$.

Proof. The proof follows from Proposition 17 and the characterization of the manager's objective function in Section $A.7$. □

Proposition 7. Suppose, in equilibrium, all managers are allowed to offer support. Given equilibrium fees f, Asset prices p_0 and $p_1(\pi)$, and investors' choice a_{0i}^I, a manager who is not allowed to offer support will be less likely to break the buck than a manager who can offer support if there is enough liquidity in the asset market in the interim period, i.e., if $E > \bar{E}$ for some $\bar{E}$.

Proof. Let $a_0^{M,NS} < 1$ be the portfolio choice of the manager who is not allowed to offer support. From Proposition 4 we have that the managers who are allowed to offer support choose $a_0^M = 1$ when $\bar{B} = 0$. The probability of a fund not being liquidated at $t = 1$ is $1 - \frac{\pi_{x,i}\left(a_0^{M,NS}\right)}{\overline{\pi}}$ for the manager who cannot offer support and $1-\frac{\pi^*(1)}{\overline{\pi}}$ for the managers who can offer support, where

$$p_1\left(\pi_{x,i}\left(a_0^M\right)\right)=\left(\left(x-\frac{1}{q_0}\right)\frac{1}{a_0^M}+\frac{1}{q_0}\right)p_0, \tag{98}$$

$$p_1^S\left(\pi^*\left(a_0^M\right)\right)=\frac{\left(x\left(1-f\right)a_0^I-fa_1^{IS}\left(\frac{\left(1-a_0^I\right)}{q_0}+x\left(1-f\right)a_0^I\right)\right)}{\frac{1}{P_0}\left(1-f\right)a_0^I} \tag{99}$$

and

$$a_1^{IS}=\frac{\left(x\left(1-f\right)a_0^IW_0^I-E\right)}{\left(x\left(1-f\right)a_0^I+\frac{\left(1-a_0^I\right)}{q_0}\right)W_0^I}. \tag{100}$$

Because $p_1\left(\pi\right)$ is increasing in π, $\pi_{x,i}\left(a_0^{M,NS}\right)<\pi^*\left(1\right)$ as long as

$$\left(\left(x-\frac{1}{q_0}\right)\frac{1}{a_{0,i}^{M,NS}}+\frac{1}{q_0}\right)<\frac{\left(x\left(1-f\right)a_0^I-fa_1^{IS}\left(\frac{\left(1-a_0^I\right)}{q_0}+x\left(1-f\right)a_0^I\right)\right)}{\left(1-f\right)a_0^I}, \tag{101}$$

which holds if

$$E>\hat{E}:=\left(fx-\left(\frac{1}{q_0}-x\right)\left(\frac{1}{a_{0,i}^{M,NS}}-1\right)\right)\frac{\left(1-f\right)}{f}a_0^IW_0^I. \tag{102}$$

□

A.11. Capital buffer

In this subsection I compute the equilibrium when a capital buffer is imposed on managers.

A.11.1. *Price computation*

Let $\hat{W}_0^M = W_0^M - q_0 F (1-f) a_0^I W_0^I$. When a capital requirement F is in place, the demand for the risky asset at $t=1$ is given by

$$n\left(p_1, a_0^M\right)\left(\hat{W}_0^M + f a_0^I W_0^I\right) + a_1^I\left(p_1, \pi\right)\left(n\left(p_1, a_0^M\right)(1-f)\, a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) W_0^I$$
$$+E + F(1-f)\, a_0^I W_0^I \tag{103}$$

if the capital buffer is not needed to keep the fund open at $t=1$, by

$$n\left(p_1, a_0^M\right)\left(\hat{W}_0^M + a_0^I W_0^I\right) + a_1^I\left(p_1, \pi\right)\left(x(1-f)\, a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) W_0^I + E + F(1-f)\, a_0^I W_0^I \tag{104}$$

if the capital buffer and additional support are used to keep the fund open at $t=1$, and by

$$n\left(p_1, a_0^M\right)\left(\hat{W}_0^M + f a_0^I W_0^I\right) + E \tag{105}$$

if the fund is liquidated at $t=1$. Then, if no support is needed at all the equilibrium price is given by

$$\hat{p}_1^{NS}(\pi) = \tag{106}$$
$$\min\left\{\frac{(1-f)\bar{d}\left(\pi\left(\left(1-a_0^M\right)(1-f)a_0^I+\left(1-a_0^I\right)\right)\frac{1}{q_0}W_0^I+\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M+fa_0^IW_0^I\right)+E+F(1-f)a_0^IW_0^I\right)}{\frac{a_0^M}{p_0}a_0^I(1-f)^2(1-\pi)\bar{d}W_0^I+\frac{1}{q_1}\frac{1}{q_0}\left(\left(1-a_0^I\right)W_0^I+\left(1-a_0^M\right)\left(\hat{W}_0^M+a_0^IW_0^I\right)\right)+\frac{E+F(1-f)a_0^IW_0^I}{q_1}}, \bar{d}\pi q_1\right\}.$$

If support is provided using the capital buffer or if additional support is offered by the manager, and a_1^{I*} is interior, the equilibrium price, $p_1^S(\pi)$, is determined by

$$\frac{\left(x(1-f)\, a_0^I W_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + a_0^I W_0^I\right) - E - F(1-f)\, a_0^I W_0^I\right)}{\left(x(1-f)\, a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) W_0^I} = a_1^{I*}\left(p_1(\pi), \pi\right). \tag{107}$$

If $x(1-f)\, a_0^I W_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + a_0^I W_0^I\right) - E - F(1-f)\, a_0^I W_0^I < 0$, there is an excess demand for the risky asset at every price $p_1(\pi) < \bar{d}\pi q_1$. Therefore, in this case, $p_1(\pi) = \bar{d}\pi q_1$, $a_1^{I*}\left(p_1(\pi), \pi\right) = 0$ and the share invested by managers in the risky asset, $\bar{a}_1^M(\pi)$, is given by

$$\begin{aligned} & \left(1-\bar{a}_1^M(\pi)\right) \tfrac{\bar{d}\pi q_1}{p_0} a_0^M \left(\hat{W}_0^M + a_0^I W_0^I\right) \\ & + \bar{a}_1^M(\pi) \left(-\tfrac{\left(1-a_0^M\right)\left(\hat{W}_0^M + a_0^I W_0^I\right)}{q_0} - E - (F-x)(1-f)\, a_0^I W_0^I \right) = 0. \end{aligned} \tag{108}$$

If $x(1-f)\, a_0^I W_0^I - \left(1-a_0^M\right) \frac{1}{q_0} \left(\hat{W}_0^M + a_0^I W_0^I\right) - E - F(1-f)\, a_0^I W_0^I > 0$, the equilibrium price is given by

$$a_1^{I*}(p_1(\pi), \pi) = \frac{\left(x(1-f)a_0^I W_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + a_0^I W_0^I\right) - E - F(1-f)a_0^I W_0^I\right)}{\left(x(1-f)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) W_0^I} =: \hat{a}_1^{IS}\left(a_0^M\right). \tag{109}$$

This condition implies an affine equation in $\bar{d}/p_1^S(\varphi)$, which gives the equilibrium price

$$\hat{p}_1^S(\pi) = \min\left\{ \max\left\{ \frac{\left(\pi - \hat{a}_1^{IS}\right)}{\frac{1}{q_1}\left(1-\hat{a}_1^{IS}\right)} (1-f)\, \bar{d}, 0 \right\}, \bar{d}\pi q_1 \right\}. \tag{110}$$

As in the benchmark case, when there is support, the share investors choose to invest with the manager in period 1 is independent of π.

Finally, if all funds are liquidated the equilibrium price, $p_1^L(\pi)$, is given by

$$p_1^L(\pi) = \min\left\{ \frac{\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + f a_0^I W_0^I\right) + E}{\frac{a_0^M}{p_0}(1-f)\, a_0^I W_0^I}, \bar{d}\pi q_1 \right\}. \tag{111}$$

The payoff for investors at $t = 1$ from investing 1 unit with the manager at $t = 0$ is

$$\begin{cases} \mathrm{n}\big(p_1, a_0^M\big) & if f\mathrm{n}\big(p_1, a_0^M\big) > x, \\ \mathrm{x} & if f\mathrm{n}\big(p_1, a_0^M\big) + F > x \text{ or additional voluntary support is offered} \\ \mathrm{n}\big(p_1, a_0^M\big) & \text{if the fund is liquidated.} \end{cases}$$

Depending n the realization of π, there are five regimes: no support needed, $\pi \geq \hat{\pi}_x$; capital buffer is enough to cover losses, $\hat{\pi}_x > \pi \geq \hat{\pi}_B$; additional sponsor support is needed and offered by all funds, $\hat{\pi}_B > \pi \geq \hat{\pi}^*$; additional support is needed and funds offer support with positive probability but not for sure, $\hat{\pi}_B > \pi \geq \hat{\pi}^*$; and the fund is liquidated $\hat{\pi}^{**} > \pi$. This thresholds are determined by

$$\hat{p}_1^S\left(\hat{\pi}_x\right) = \left(\left(x - \frac{1}{q_0}\right)\frac{1}{a_0^M} + \frac{1}{q_0}\right)p_0, \tag{112}$$

$$\hat{p}_1^S\left(\hat{\pi}_{CB}\right) = \left(\left(x - F - \frac{1}{q_0}\right)\frac{1}{a_0^M} + \frac{1}{q_0}\right)p_0, \tag{113}$$

$$\begin{aligned} &\left(f\hat{a}_1^{IS}\left(a_0^M\right)\left(x\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) - \left(x - F - n\left(\hat{p}_1^S\left(\hat{\pi}^*\right), a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I \\ &+ \min\left\{\frac{B\hat{p}_1^S\left(\hat{\pi}^*\right)}{\bar{d}}, n\left(\hat{p}_1^S\left(\hat{\pi}^*\right), a_{0,i}^M\right)\left(\hat{W}_0^M + fa_0^{I*}W_0^I\right) + E\right\} = 0, \end{aligned} \tag{114}$$

and

$$\begin{aligned} &\left(fa_1^{I*}\left(\hat{p}_1^L\left(\hat{\pi}^{**}\right), \pi\right)\left(x\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) - \left(x - F - n\left(\hat{p}_1^L\left(\hat{\pi}^{**}\right), a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)\right)W_0^I \\ &+ \min\left\{\frac{B\hat{p}_1^L\left(\hat{\pi}^{**}\right)}{\bar{d}}, n\left(\hat{p}_1^L\left(\hat{\pi}^{**}\right), a_{0,i}^M\right)\left(\hat{W}_0^M + fa_0^{I*}W_0^I\right) + E\right\} = 0. \end{aligned} \tag{115}$$

The manager's utility when a capital requirement is in place is given by

$$V_0^{M,CR}\left(W_0^M; f\right) =$$

$$\begin{aligned}
&\sup_{a_{0,i}^M \in [0,1]} \int_{\underline{\pi}}^{\bar{\pi}} \frac{\pi \bar{d}}{p_1^*(\pi)} \left[\left(a_{0,i}^M \left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0} \right) + \frac{1}{q_0} \right) \left(\hat{W}_0^M + f a_0^I W_0^I \right) + E + F(1-f) a_0^I W_0^I \right] d\pi \\
&+ \int_{\hat{\pi}_{x,i}\left(a_{0,i}^M\right)}^{\bar{\pi}} \left(\frac{\pi \bar{d}}{p_1^*(\pi)} f a_1^{I*}(\pi) \left(a_0^{I*}(1-f) \left(a_{0,i}^M \left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0} \right) + \frac{1}{q_0} \right) + \left(1 - a_0^{I*}\right) \frac{1}{q_0} \right) W_0^I \right) d\pi \\
&+ \int_{\hat{\pi}_i^*\left(a_{0,i}^M\right)}^{\hat{\pi}_{x,i}\left(a_{0,i}^M\right)} \frac{\pi \bar{d}}{p_1^*(\pi)} f a_1^{I*}(\pi) \left(a_0^{I*}(1-f) x + \left(1 - a_0^{I*}\right) \frac{1}{q_0} \right) W_0^I d\pi \qquad (116) \\
&- \int_{\hat{\pi}_i^*\left(a_{0,i}^M\right)}^{\hat{\pi}_{x,i}\left(a_{0,i}^M\right)} \frac{\pi \bar{d}}{p_1^*(\pi)} \left(x - \left(a_{0,i}^M \left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0} \right) + \frac{1}{q_0} \right) \right) a_0^{I*}(1-f) W_0^I d\pi - \int_{\underline{\pi}}^{\hat{\pi}_i^*\left(a_{0,i}^M\right)} B \pi d\pi.
\end{aligned}$$

The free entry condition is

$$V_0^{M,CR}\left(W_0^M; f\right) - C = \max_{a_{0,i}^M \in [0,1]} \int_0^{\bar{\pi}} \left(\frac{\pi \bar{d}}{p_1^*(\pi)} \left(\left(a_{0,i}^M \left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0} \right) + \frac{1}{q_0} \right) W_0^M + E \right) \right) d\pi. \qquad (117)$$

A.11.2. Thresholds

If $p_1^S\left(\hat{\pi}_x\right) = \frac{\left(\hat{\pi}_x - \hat{a}_1^{IS}\right)}{\frac{1}{q_1}\left(1 - \hat{a}_1^{IS}\right)} (1-f) \bar{d}$,

$$\hat{\pi}_x^I = \left(\left(x - \frac{1}{q_0} \right) \frac{1}{a_0^M} + \frac{1}{q_0} \right) p_0 \frac{\left(\frac{1}{q_1} \left(1 - \hat{a}_1^{IS}\right) \right)}{(1-f) \bar{d}} + \hat{a}_1^{IS}. \qquad (118)$$

This happens as long as $\left(x - \frac{1}{q_0}\right) \frac{1}{a_0^M} + \frac{1}{q_0} > 0$ and $\hat{\pi}_x^I < \frac{\hat{a}_1^{IS}}{\left(\hat{a}_1^{IS} - f\right)}$.
If $\left(x - \frac{1}{q_0}\right) \frac{1}{a_0^M} + \frac{1}{q_0} < 0$, $\hat{\pi}_x$ is not well defined. If $\hat{\pi}_x^I > \frac{\hat{a}_1^{IS}}{\left(\hat{a}_1^{IS} - f\right)}$,

$$\hat{\pi}_x^C = \left(\left(x - \frac{1}{q_0} \right) \frac{1}{a_0^M} + \frac{1}{q_0} \right) \frac{p_0}{\bar{d} q_1}. \qquad (119)$$

Then,

$$\hat{\pi}_x = \begin{cases} \hat{\pi}_x^I & \text{if } \hat{\pi}_x^I < \frac{\hat{a}_1^{IS}}{\left(\hat{a}_1^{IS} - f\right)} \\ \hat{\pi}_x^C & \text{else.} \end{cases} \tag{120}$$

π_{CB} is such that

$$\begin{aligned} x - NAV &= F \\ x - \left(a_0^M \frac{p_1^S(\pi_{CB})}{p_0} + (1 - a_{0M}) \frac{1}{q_0}\right) &= F \\ \left(x - F - (1 - a_{0M}) \frac{1}{q_0}\right) \frac{p_0}{a_0^M} &= p_1^S(\pi_{CB}). \end{aligned} \tag{121}$$

Then, if $p_1^S(\pi_{CB}) = \frac{\left(\pi_{CB} - \hat{a}_1^{IS}\right)}{\frac{1}{q_1}\left(1 - \hat{a}_1^{IS}\right)} (1 - f) \bar{d}$

$$\pi_{CB}^I = \left(x - F - (1 - a_{0M}) \frac{1}{q_0}\right) \frac{p_0}{a_0^M} \frac{\frac{1}{q_1}\left(1 - \hat{a}_1^{IS}\right)}{(1 - f)\bar{d}} + \hat{a}_1^{IS} \tag{122}$$

If $\pi_{CB}^I > \frac{\hat{a}_1^{IS}}{\left(\hat{a}_1^{IS} - f\right)}$,

$$\pi_{CB}^C = \left(\left(x - F - \frac{1}{q_0}\right) \frac{1}{a_0^M} + \frac{1}{q_0}\right) \frac{p_0}{\bar{d} q_1}. \tag{123}$$

Then,

$$\hat{\pi}_x = \begin{cases} \pi_{CB}^I & \text{if } \pi_{CB}^I < \frac{\hat{a}_1^{IS}}{\left(\hat{a}_1^{IS} - f\right)} \\ \pi_{CB}^C = \left(\left(x - F - \frac{1}{q_0}\right) \frac{1}{a_0^M} + \frac{1}{q_0}\right) \frac{p_0}{\bar{d} q_1} & \text{else.} \end{cases} \tag{124}$$

π^* is such that

$$\begin{aligned} & \frac{\pi^* \bar{d}}{p_1^S(\pi^*)} \left(\left(x - F - n\left(p_1^S(\pi^*), a_0^M\right)\right)(1 - f) A_0^I - f A_1^I\left(p_1^S(\pi^*), \pi^*\right)\right) \\ = \; & \min\left\{B\pi^*, \frac{\pi^* \bar{d}}{p_1^S(\pi^*)} \left(n\left(p_1^S(\pi^*), a_0^M\right)\left(W_0^M - F(1 - f) a_0^I W_0^I q_0 + f A_0^I\right) + E\right)\right\}. \end{aligned} \tag{125}$$

If $A_1^I\left(p_1^S\left(\pi^*\right),\pi^*\right)>0$, i.e, if

$$x\left(1-f\right)a_0^IW_0^I-\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M+a_0^IW_0^I\right)-E-F\left(1-f\right)a_0^IW_0^I>0, \tag{126}$$

there are two possible cases.

If $p_1^S\left(\pi^*\right)=\frac{\left(\pi^*-\hat{a}_1^{IS}\right)}{\frac{1}{q_1}\left(1-\hat{a}_1^{IS}\right)}\left(1-f\right)\bar{d}$ and

$$B\pi^*<\frac{\pi^*\bar{d}}{p_1^S\left(\pi^*\right)}\left(n\left(p_1^S\left(\pi^*\right),a_0^M\right)\left(W_0^M-F\left(1-f\right)a_0^IW_0^Iq_0+fA_0^I\right)+E\right),$$

then

$$\frac{\left(\left(x-F-\frac{\left(1-a_0^M\right)}{q_0}\right)(1-f)a_0^IW_0^I-f\hat{a}_1^{IS}W_0^I\left(x(1-f)a_0^I+\frac{\left(1-a_0^I\right)}{q_0}\right)\right)}{\left(\frac{B}{\bar{d}}+\frac{a_0^M}{p_0}(1-f)a_0^IW_0^I\right)} = p_1^S\left(\pi^*\right) \tag{127}$$

and

$$\pi_{I1}^*=\hat{a}_1^{IS}+\frac{\left(\left(x-F-\frac{\left(1-a_0^M\right)}{q_0}\right)(1-f)a_0^IW_0^I-f\hat{a}_1^{IS}W_0^I\left(x(1-f)a_0^I+\frac{\left(1-a_0^I\right)}{q_0}\right)\right)\frac{1}{q_1}\left(1-\hat{a}_1^{IS}\right)}{\left(\frac{B}{\bar{d}}+\frac{a_0^M}{p_0}(1-f)a_0^IW_0^I\right)(1-f)\bar{d}}. \tag{128}$$

If $p_1^S\left(\pi^*\right)=\frac{\left(\pi^*-\hat{a}_1^{IS}\right)}{\frac{1}{q_1}\left(1-\hat{a}_1^{IS}\right)}\left(1-f\right)\bar{d}$ and

$$B\pi^*>\frac{\pi^*\bar{d}}{p_1^S\left(\pi^*\right)}n\left(p_1^S\left(\pi^*\right),a_0^M\right)\left(W_0^M-F\left(1-f\right)a_0^IW_0^Iq_0+fA_0^I\right)+E,$$

then

$$\frac{\left(x-F\right)\left(1-f\right)a_0^IW_0^I-f\hat{a}_1^{IS}\left(x\left(1-f\right)a_0^I+\frac{\left(1-a_0^I\right)}{q_0}\right)-E}{\left(W_0^M-F\left(1-f\right)a_0^IW_0^Iq_0+a_0^IW_0^I\right)}=n\left(p_1^S\left(\pi^*\right),a_0^M\right) \tag{129}$$

and

$$\pi_{I2}^{*}=\left(\frac{(x-F)(1-f)a_0^IW_0^I-f\hat{a}_1^{IS}\left(x(1-f)a_0^I+\frac{\left(1-a_0^I\right)}{q_0}\right)W_0^I-E}{\left(W_0^M-F(1-f)a_0^IW_0^Iq_0+a_0^IW_0^I\right)}-\frac{\left(1-a_0^M\right)}{q_0}\right)\frac{p_0}{a_0^M}\frac{1}{q_1\left(1-f\right)\bar{d}}\left(1-\hat{a}_1^{IS}\right)+\hat{a}_1^{IS} \tag{130}$$

Finally, π^{**} is such that

$$\begin{aligned}&\frac{\pi^{**}\bar{d}}{p_1^L\left(\pi^{**}\right)}\left(\left(x-F-n\left(p_1^L\left(\pi^{**}\right),a_0^M\right)\right)\left(1-f\right)A_0^I-fA_1^I\left(p_1^L\left(\pi^{**}\right),\pi^{**}\right)\right) \qquad (131)\\ =\ &\min\left\{B\pi^{**},\frac{\pi^{**}\bar{d}}{p_1^L\left(\pi^{*}\right)}n\left(p_1^L\left(\pi^{*}\right),a_0^M\right)\left(W_0^M-F\left(1-f\right)a_0^IW_0^Iq_0+fA_0^I\right)+E\right\}.\end{aligned}$$

If $p_1^L\left(\pi^{**}\right)<\bar{d}\pi q_1$ and

$$B\pi^{*}<\frac{\pi^{*}\bar{d}}{p_1^S\left(\pi^{*}\right)}n\left(p_1^L\left(\pi^{**}\right),a_0^M\right)\left(W_0^M-F\left(1-f\right)a_0^IW_0^Iq_0+fA_0^I\right)+E, \tag{132}$$

then

$$A_1^I\left(p_1^L\left(\pi^{**}\right),\pi^{**}\right)=\frac{\pi\bar{d}\frac{a_0^M}{p_0}(1-f)^2a_0^IW_0^I-\left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M+fa_0^IW_0^I\right)+E\right)\frac{1}{q_1}}{\bar{d}\frac{a_0^M}{p_0}(1-f)^2a_0^IW_0^I-\left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M+fa_0^IW_0^I\right)+E\right)\frac{1}{q_1}}\times \left(x\left(1-f\right)a_0^I+\left(1-a_0^I\right)\frac{1}{q_0}\right)W_0^I \tag{133}$$

and

$$\pi^{**}=\frac{\left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M+fa_0^IW_0^I\right)+E\right)\frac{1}{q_1}}{\bar{d}\frac{a_0^M}{p_0}\left(1-f\right)^2a_0^IW_0^I}+ \tag{134}$$

$$\frac{\left(\left(x-F-\left(\frac{a_0^M}{p_0}p_1^L+\left(1-a_0^M\right)\frac{1}{q_0}\right)\right)(1-f)A_0^I-\frac{B}{\bar{d}}p_1^L\right)\left(\bar{d}\frac{a_0^M}{p_0}(1-f)^2a_0^IW_0^I-\left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M+fa_0^IW_0^I\right)+E\right)\frac{1}{q_1}\right)}{\left(x(1-f)a_0^I+\left(1-a_0^I\right)\frac{1}{q_0}\right)fW_0^I\bar{d}\frac{a_0^M}{p_0}(1-f)^2a_0^IW_0^I}.$$

If $\pi\bar{d}\frac{a_0^M}{p_0}\left(1-f\right)^2a_0^IW_0^I-\left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M+fa_0^IW_0^I\right)+E\right)\frac{1}{q_1}<0$, then there is excess demand on behalf of the managers and $p_L\left(\pi\right)=\bar{d}\pi q_1$.

If $p_1^L(\pi^{**}) < \bar{d}\pi^{**}q_1$ and

$$Bp_1^L(\pi^{**})/\bar{d} > \left(n\left(p_1^L(\pi^{**}), a_0^M\right)\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + fA_0^I\right) + E\right),$$

then

$$\begin{aligned}\pi^{**} &= \frac{\left((x-F)(1-f)A_0^I - n\left(p_1^L(\pi^*), a_0^M\right)\left(\hat{W}_0^M + A_0^I\right) + E\right)}{\left(x(1-f)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right)fW_0^I} \\ &\quad \frac{\left(\left(\bar{d}\frac{a_0^M}{p_0}(1-f)^2 a_0^I W_0^I - \left(\frac{\left(1-a_0^M\right)}{q_0}\left(\hat{W}_0^M + fa_0^I W_0^I\right) + E\right)\frac{1}{q_1}\right) + \left(\left(1-a_0^M\right)\frac{1}{q_0}\left(\hat{W}_0^M + fa_0^I W_0^I\right) + E\right)\frac{1}{q_1}\right)}{\bar{d}\dfrac{a_0^M}{p_0}(1-f)^2 a_0^I W_0^I}.\end{aligned} \tag{135}$$

For the next two cases $\pi^* = \pi^{**}$.

If $p_1^S(\pi^*) = \bar{d}\pi^* q_1$ and $B\pi^* < \frac{\pi^*\bar{d}}{p_1^S(\pi^*)} n\left(p_1^S(\pi^*), a_0^M\right)\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + fA_0^I\right) + E$, then

$$\pi^* = \frac{\left(x - F - \frac{\left(1-a_0^M\right)}{q_0}\right)(1-f)a_0^I W_0^I}{\left(Bq_1 + \frac{\bar{d}q_1}{p_0}a_0^M(1-f)a_0^I W_0^I\right)}. \tag{136}$$

If $p_1^S(\pi^*) = \bar{d}\pi q_1$ and $B\pi^* > \frac{\pi^*\bar{d}}{p_1^S(\pi^*)} n\left(p_1^S(\pi^*), a_0^M\right)\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + fA_0^I\right) + E$, then

$$\pi^* = \left(\frac{(x-F)(1-f)a_0^I W_0^I - E}{\left(W_0^M - F(1-f)a_0^I W_0^I q_0 + a_0^I W_0^I\right)} - \frac{\left(1-a_0^M\right)}{q_0}\right)\frac{p_0}{\bar{d}a_0^M q_1}. \tag{137}$$

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