Financial Distress and Endogenous Uncertainty

preliminary

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Abstract

What is the macroeconomic effect of having a substantial number of firms close to default? This paper studies the effects of financial distress in a model where customers, suppliers and workers suffer losses if their employer goes bankrupt. I show that this mechanism generates amplification of fundamental shocks by creating a cyclical TFP and labor macroeconomic wedges. Because the strength of this amplification depends on the share of firms that are in financial distress, it operates mostly in recessions, when equity values are low. This leads macroeconomic volatility to be endogenously countercyclical. Moreover, the cross-sectional dispersion of firms' outcomes is also endogenously countercyclical. Empirical evidence consistent with the model is provided.

JEL: E32, E44, G12.

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1 Introduction

Economic recessions and financial crises sometimes lead a large number of firms close to default. This arises either because of contraction in credit supply, or because equity values fall and become more volatile, reducing the cushion protecting solvency.[1] Intuitively, the fact that a nontrivial share of the nonfinancial corporate sector is close to default would seem to have significant negative macroeconomic effects. This paper is concerned with the modeling and measuring the macroeconomic costs of such financial distress.

In particular, the paper focuses on a specific channel: when a firm becomes close to default, it becomes harder to find and retain employees, suppliers and customers, as they worry that they would suffer losses if the firm goes under. The anticipation of these losses make it more costly for firms to operate, and

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[1]See Atkeson, Eisfeldt and Weil (2013) for a recent empirical analysis of the business cycle variation in the distance to insolvency.

hence leads directly to lower production and employment. The losses include direct missed payments,[2] but also capture more broadly the costs of the loss of a working relationship - employees, customers and suppliers must look for another match, and they may lose some relationship-specific capital. Hence, the ex-ante perception of a risk of bankruptcy leads to a "default wedge" that disconnects marginal revenue from marginal cost. This wedge is endogenous, depend on the distribution of default risk in the economy, and is time-varying, becoming larger when more firms are close to default.

This financial distress mechanism stands in contrast to alternative mechanisms where default risk affects the economy chiefly by creating investment wedges. For instance, well-studied mechanisms include limited pledgeability -a smaller distance to default makes it more costly to raise external finance- or debt-overhang -a smaller distance to default reduces incentives for investment on the part of equity-holders as debtholders stand to reap the gains of investment. The investment wedge, however, mostly affects investment. A key challenge in macroeconomic models is to generate large variation in output and employment, making the "default wedge" more appealing than the investment wedge. In particular, I show that it can generate the appareance of a labor wedge, so that an econometrician that looks at the data through the lens of the neoclassical model would find an excessive employment contraction, making it look as if labor income taxes were countercyclical. As argued by Mulligan (1997), Hall (1997), and Chari, Kehoe and McGrattan (2005), this is a desirable feature of the data. Furthermore, the mechanism also reduces aggregate total factor productivity by reducing input usage, and by allocating labor and inputs across firms in part based on their default likelihood rather than their productivity. As a result of these two wedges, financial distress amplifies macroeconomic fluctuations.

Moreover, this amplification effect operates only under certain circumstances, leading to nonlinearities. In goods times, when productivity or equity values are high, a small aggregate shock does not change substantially the likelihood of default of most firms, which remains low. But in bad times, the same-size shock could be sufficient to increase significantly the likelihood of default of many firms, with larger aggregate effects. The time-varying elasticity implies time-varying macroeconomic volatility even if the underlying, fundamental shock is homoskedastic. As a result, financial distress can generate endogenously time-varying second moments. This is of interest in light of the large recent literature on uncertainty shocks, which for the most part takes the variation of uncertainty as exogenous.[3] For the same reason, this mechanism also generates a countercyclical dispersion of firm-level sales or employment, consistent with the data. Finally, I show that this model generates moderate negative skewness and positive kurtosis of the *level* of GDP and employment (rather than their growth rates), also in line with the data.

[2]For instance, in the United States, employee wages have a priority in bankruptcy up to 10,000$, if they were earned in the past six months. The excess over 10,000$ is treated as a regular unsecured claim, payable in proportion to the assets recovered, after administrative expenses are paid, and possibly with a substantial delay. Furthermore, bonuses, sick and vacation days are lost if they were earned over 180 days ago, and may be hard to recover otherwise. Finally, while company pension plans benefit from the backstop of the Pension Benefit Guarantee Corporation, this guarantee is incomplete (e.g. capped).

[3]For some models with shocks to aggregate uncertainy, see Bloom (2009), Fernandez-Villaverde et al. (2010, 2013), or Gourio (2012). Studies that attempt to endogenize volatility include, among others, Bachmann and Moscarini, Berger and Vavra, Brunnermeier and Sannikov, Bianchi and Mendoza.

There is a lot of anecdotal evidence that customer/supplier and employee relationships are deteriorated when firms become close to default. For instance, in November 2008, Circuit City, the second largest electronics retailer in the U.S., was forced to file for bankruptcy when news of its deteriorating financial position led its suppliers (such as Samsung, Sony, and other big electronic manufacturers) to refuse extending credit for deliveries.[4] These phenomena are likely frequent and important, because trade credit is large: in Compustat, the median ratio of account receivables to total assets is around 0.15, versus 0.21 for debt to total assets. The impact of bankruptcy risk on customers was also an important element for Chrysler and GM - there was widespread concern that consumers would stop buying these manufacturers' cars once they realized the companies were likely to file for bankruptcy, since product warranties and auto parts might not be available later. This motivated the U.S. Federal government to introduce the warranty guarantee program (see Hortacsu et al. (2012)). More systematic evidence has been provided by the corporate finance literature. For instance, Brown and Matsa (2012) find that (financial) firms in financial distress during the recent recession received fewer applications to job openings than did financially healthy firms. Moreover, the quality of the applicants was worse.

A key challenge for the mechanism is to generate significant macroeconomic impact in spite of the fairly low observed default rates. Two observations are important here. First, this mechanism applies not only to firms that are *actually* in default or restructuring their debt, but rather to all firms that have some significant likelihood of default. Second, for some purposes, the relevant default rate is the exit rate. For instance, an employee who is let go faces a loss from the firm discontinuing its operations, even if the firm exits without financial default. For instance, even if his wages and benefits are paid, the worker will spend time searching for a new job, which might not be as good, and there is some lost firm-specific human capital.

To illustrate the cyclicality of default and bankruptcies, figure 1 presents the time series of business chapter 7 and chapter 11 filings in the United States.[5] Figure 2 presents the S&P corporate bond default rate.

1.1 Related Literature (tba)

A large literature in corporate finance studies the costs of financial distress (e.g., Opler and Titman (2004)). A motivation for this literature is that it is difficult to understand the low observed leverage of most nonfinancial firms in light the large tax savings that firms could generate, and the relatively small likelihood of default or loss in the event of default. Or to put it a different way, most models require a loss upon default of nearly 50% to be roughly consistent with observed leverage choices and default probabilities. This deadweight loss seems too large in light of estimates of actual bankruptcy costs of 10% or less. This suggests there must be some other costs to being in financial distress. A natural interpretation is that some costs arise prior to default, in the form of higher costs of financing, inefficiencies driven by debt overhang, or the "default wedge" studied in this paper (and which is not original to this paper). Despite the large empirical work in corporate finance, there is less work in the

[4] See for instance http://online.wsj.com/news/articles/SB122632305224313513

[5] Unfortunately, these are the raw numbers of filings, i.e. they are not weighted by the firm size or the debt in bankruptcy. These data do not appear to exist.

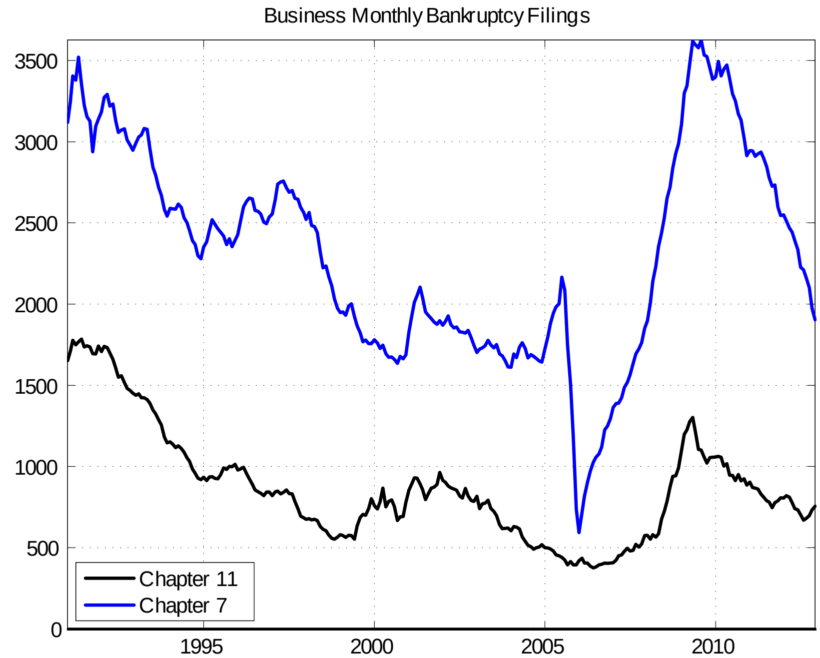

Figure 1: US business bankruptcy filings. Note: series are smoothed using a 6-month moving average. The 2005 spike due to the bankruptcy reform is removed.

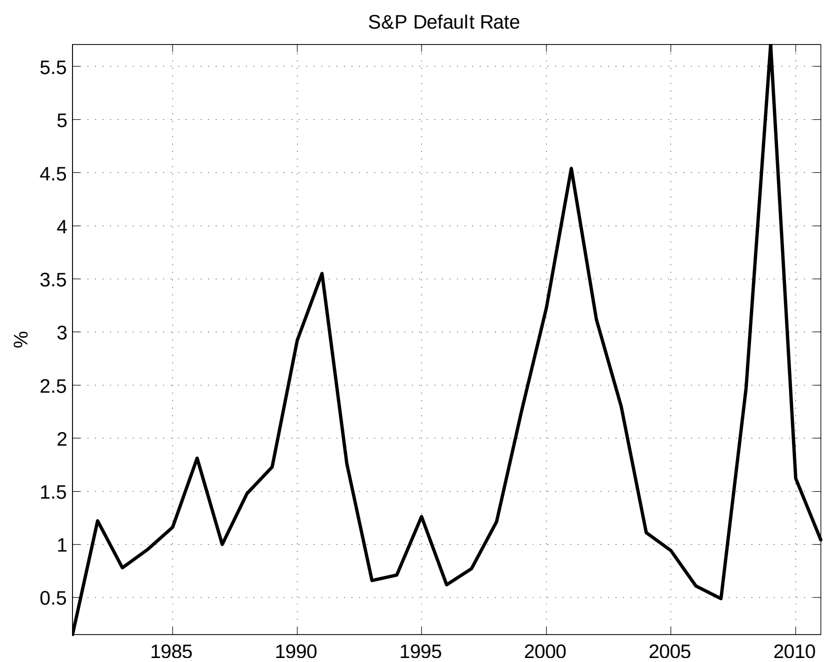

Figure 2: S&P default rate. Note: this is the percentage of S&P rated issuers that defaulted in a given year.

macroeconomic literature that studies in general equilibrium the effects of these frictions. Moreover, the social and private costs of these frictions could be quite different. One limitation of some studies (including, to some extent, the Brown and Masa study discussed above) is that it is difficult to disentangle the effects of economic from financial distress.

There has been some recent work focusing on the effect of financial frictions on the demand for labor (e.g. Chugh (2013); Petrovsky-Nadeau (2014); Quadrini et al. (2013)).

Also to discuss = domino effects and bankruptcy waves (e.g., Caballero and Simsek)

Large working capital literature, but rather ad-hoc constraints, exogenous.

2 Model

This section lays out the model; it describes the technology and finance assumptions, the nature of default costs, the household problem, and derives the equations characterizing the equilibrium.

2.1 Technology

In order to study the effect on default risk between customers and suppliers, one needs to incorporate intermediate inputs; I follow the simple "roundabout" production structure of Basu (1995). I assume that there is a continuum of industries (of arbitrary measure M), that each produce a different good. These goods are then combined using a constant return to scale, perfectly competitive technology to produce the final good, which is used either for consumption or as intermediate input in each industry. (There is no investment or capital for simplicity.) Each industry, indexed by j, is competitive, and there is a measure one of firms in each industry, indexed by i. Hence, the total output of industry j is

$$Y_j = \int_0^1 y_{ij} di,$$

and the total final goods production is

$$Y = M^{1-\sigma} \left(\int_0^M Y_j^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}.$$

In equilibrium, all industries make identical choices, so that $Y_j = Y$. The relative price of all industry goods is one, i.e. the industry output price equals the final goods price, which is the numeraire.

Going now to study the problem faced by each firm in industry j, we assume a Cobb-Douglas production function with decreasing return to scale,

$$y_{it} = z_t x_{it} n_{it}^{\alpha} m_{it}^{\beta},$$

where m is the input, n is labor, z is aggregate productivity, which follows an exogenous stochastic process, and x is idiosyncratic productivity, which follows an exogenous stationary stochastic process with invariant distribution $\mu(x)$.

2.2 Entry and Exit

The entry and exit process is kept simple to simplify aggregation. Each period, firms' owners have to decide whether to pay some exogenous costs and remain in control of the firm, or default. These costs can be interpreted either as a debt repayment, or as fixed costs of production. These costs take two parts: first, there is a fixed number b. (A future version of this paper will endogenize the choice of b, for instance by using the trade-off model of debt.) Second, there is a random cost shock η, with mean 0 and cdf $H(.)$. This shock is *iid* over time and across firms. The randomness captures pure profit windfall, that could be either positive or negative (e.g., a lawsuit that must be settled, or a one-time capital gain or profit from special circumstances).[6] Each period, some firms will decide to default rather than continue operating and pay both costs. These exiters are replaced with new firms, which we assume have exactly the same idiosyncratic productivity x as the old firms. The most natural interpretation is that these firms are immediately restructured, with creditors taking over the firm. This assumption keeps the analysis very simple since the cross-sectional distribution of firms according to x is constant as a result, equal to $\mu(x)$, i.e. there is no truncation of the firm distribution. Firms will turn out to be heterogeneous only in this productivity dimension.

2.3 Timing and Default Costs

The timing of the model is as follows. At the beginning of each period, the productivity shocks z and x are realized. Firm x then decides to hire $n(x)$ workers and $m(x)$ units of the final good as input. The cash flow shock η is then realized. Two possibilities arise: either the owners decide to continue running the firm; in this case they obtain as dividend the profits from today's production, less $\eta+b$. This dividend may be negative, which means that there is no financial friction preventing recapitalization (i.e. equity issuance). The second possibility is that the firm owners decide not to repay $\eta + b$; in this case, they lose control of the firm and obtain 0, i.e. they exercise their limited liability option to walk away from the firm. For clarity, we assume that this default has no direct effect on the firm's operations - workers provide labor, inputs are delivered, and output is produced and sold. (The net proceeds from these operations go to the new owners of the firm, which could be the creditors, or even the government, if it is the ultimate insurer; the identity of the new owner is irrelevant since this only affects the distribution of wealth among agents, which has no effect as we will assume a representative agent economy.)

The central assumption of the paper is that workers and suppliers bear a cost if the firm goes into default. Specifically, I assume that the cost borne by each worker is $\theta_w w(x)$ where $w(x)$ is their wage, and θ_w is a parameter; and similarly for suppliers the cost is $\theta_p p(x)$ where $p(x)$ is the input price. There are different interpretations for this reduced form. A literal interpretation is that workers or suppliers do not receive the payments (e.g. wages or account payables) to which they are entitled. For instance, the new firm owners might renegotiate salaries and benefits; or there might be no new owners, and the government might only partially insure for unpaid wages and account payables; or it might take time and effort, or be costly to recover unpaid wages and invoices. A broader interpretation (that goes outside the model) is that there are other costs of default that are borne by workers or suppliers, such as time spent unemployed or looking for new customers, or long-term wage losses because of displacement, or loss of relationship capital.

[6] Empirically, EBITDA contains a large very transitory component.

2.4 The firm owner's problem

The firm owner has to choose how many workers to hire, how many materials to buy, and in which states to default. His problem is summarized by the following Bellman equation:

$$V(x,\omega) = \max_{n,m\geq 0} E_\eta \max\left(0, \pi - b - \eta + E_{\omega',x'} M(\omega,\omega')V(x',\omega')\right).$$

Here V is the equity value, ω is the aggregate state of this economy, to be discussed below, $M(\omega,\omega')$ is the stochastic discount factor, which in equilibrium equals the marginal rate of substitution of the representative household, and π is the operating profit:

$$\pi(n,m;x,\omega) = zxn^\alpha m^\beta - w(x,\omega)n - p(x,\omega)m.$$

Note that the input prices $w(x,\omega)$ and $p(x,\omega)$ depend on the firm's characteristics, i.e. its productivity x: workers and suppliers care about the likelihood of default of their employer or customer, hence they will charge different prices depending on the default risk.[7] This Bellman equation reflects that, as discussed above, the equityholder has a default option that allows him to walk away from the firm following the observation of η. Dividends $\pi - b - \eta$ are allowed to be negative, so that equityholders can freely inject funds in the firm if it is profitable. Hence, we do not model "liquidity driven defaults" i.e. situations where firms end up in default for lack of cash, despite positive net present value.[8]

In this problem, the firm takes as given the wage $w(x,\omega)$ and prices $p(x,\omega)$, which will be determined in equilibrium (in the next section).

Cleary, the firm defaults if $\eta \geq \eta^*(x,\omega)$, where

$$\eta^*(x,\omega) = zxn(x,\omega)^\alpha m(x,\omega)^\beta - w(x,\omega)n(x,\omega) - p(x,\omega)m(x,\omega) - b + E_{\omega',x'}\left(M(\omega,\omega')V(x',\omega')\right),$$

and the probability of default is hence

$$PD(x,\omega) = 1 - H\left(\eta^*(x,\omega)\right).$$

To find input demands, rewrite the problem as:

$$\begin{aligned} V(x,\omega) &= \max_{n,m\geq 0} \int_{-\infty}^{\eta^*(x,\omega)} zxn^\alpha m^\beta - w(x,\omega)n - p(x,\omega)m - b - \eta + E_{\omega',x'}\left(M(\omega,\omega')V(x',\omega')\right) dH(\eta) \\ &= \max_{n,m\geq 0} H\left(\eta^*(x,\omega)\right)\left(zxn^\alpha m^\beta - w(x,\omega)n - p(x,\omega)m - b + E_{\omega',x'}\left(M(\omega,\omega')V(x',\omega')\right)\right) \\ &\quad - \int_{-\infty}^{\eta^*(x,\omega)} \eta dH(\eta) \end{aligned}$$

This implies that the standard first-order conditions hold:

$$\begin{aligned} \alpha zxn\left(x,\omega\right)^{\alpha-1} m(x,\omega)^\beta &= w(x,\omega), \\ \beta zxn\left(x,\omega\right)^{\alpha} m(x,\omega)^{\beta-1} &= p(x,\omega). \end{aligned}$$

[7] By analogy with the sovereign or corporate debt literature, the reader might prefer to define the input prices as equilibrium schedules, e.g. $w(n;x,\omega)$ with $\partial w/\partial n > 0$. However, given that equityholders maximize value, it is easy to show that in the model $\partial w/\partial n = 0$ in equilibrium, so that the two equilibrium concepts lead to the same first-order conditions (see appendix for complete proof).

[8] Liquidity driven defaults can lead to an alternative interesting mechanism, with "negative spirals": as the firm becomes closer to default, its profitability deteriorates because of the default wedge, leading to lower cash flows and further increases in the likelihood of default.

The economic intuition is that equityholders make promises for wages and prices, but honor these promises only in the non-default states. They also receive the marginal product only in the non-default states. Hence, they equate marginal costs and benefits in non-default states.

We can use these equations to obtain the input demand as a function of the prices:

$$n(x,\omega) = \left(\frac{\alpha}{w(x,\omega)}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{p(x,\omega)}\right)^{\frac{\beta}{1-\alpha-\beta}} (zx)^{\frac{1}{1-\alpha-\beta}},$$
$$m(x,\omega) = (zx)^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{w(x,\omega)}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{p(x,\omega)}\right)^{\frac{1-\alpha}{1-\alpha-\beta}},$$

To find the equilibrium prices $w(x,\omega)$ and $p(x,\omega)$, we turn to the household problem.

2.5 Households

We assume a representative household with expected utility preferences over total consumption and hours worked, $E\sum_{t=0}^{\infty}\beta^t U(C_t, N_t)$. Implicitely, there is perfect risk-sharing among workers, and the head of this "large family" decides to allocate family members to work or leisure, and to work at different firms x with wages $w(x,\omega)$. The household problem is to send $n^s(x,w)$ people to work at firms with productivity x, taking into account the wages offered by these firms and that there will be a loss in the event that the firm goes under. Mathematically,

$$\max_{\{n^s(x)\}} U(C(\omega), N(\omega))$$

$$s.t. \quad : \quad C(\omega) = \Pi(\omega) + \int_0^\infty n^s(x,w)w(x,w)(1-\theta_w PD(x,\omega))d\mu(x),$$
$$\text{and } N(\omega) \quad = \quad \int_0^\infty n^s(x,w)d\mu(x),$$

where $\Pi(\omega)$ are firm profits, rebated lump-sum to the household. Agents can also trade shares of the different firms that exist in this economy, however this trade "washes out" since there is no real investment, and it is thus omitted for simplicity.

This problem leads directly to the first-order conditions:

$$\forall x \geq 0, -\frac{U_2(C(\omega), N(\omega))}{U_1(C(\omega), N(\omega))} = w(x,\omega)(1-\theta_w PD(x,\omega))). \tag{1}$$

This equation simply equates the marginal cost of working (the left hand-side, which is the standard expression for the marginal rate of substitution) and the benefit to working for a firm x (the right-hand side), which reflects the expected costs of default. This equation also reflects a compensating differential: workers require higher wages to work in more risky firms. In any competitive equilibrium, they must be indifferent between working at two firms with different productivities x and x' :

$$w(x,\omega)(1-\theta_w PD(x,\omega))) = w(x',\omega)(1-\theta_w PD(x',\omega))). \tag{2}$$

The other first-order condition with respect to asset holdings shows that the household prices all assets in this economy, leading to the usual expression for the stochastic discount factor,

$$M(\omega,\omega') = \frac{\beta U_1(C(\omega'), N(\omega'))}{U_1(C(\omega), N(\omega))}.$$

Finally, we note that the same reasoning determines the input price $p(x,\omega)$. The supplier (the producer of the final good) expects to receive $p(x,\omega)$ with probability $1-PD(x,\omega)$, but with probability $PD(x,\omega)$ he will receive only $(1-\theta_p)p(x,\omega)$. The total expected reward from selling is thus $p(x,\omega)(1-\theta_p PD(x,\omega))$. This must equal the reward from selling the good to consumers (who are assumed not to default), at price one. Hence,

$$p(x,\omega) = \frac{1}{1-\theta_p PD(x,\omega)}.$$

2.6 Resource constraints and aggregation

There is no capital in this economy, hence the aggregate resource constraint for goods reads:

$$C(\omega) + M(\omega) = GO(\omega) - \zeta DC(\omega),$$

where $DC(\omega)$ are default costs, and $\zeta \in [0,1]$ is a parameter that measures the fraction of default losses that are real resource costs (rather than transfers). Default costs include both the (standard) losses born by bondholders, and the (novel to this paper) losses born by workers and suppliers. Mathematically,

$$\begin{aligned} DC(\omega) &= \theta_w \int_0^\infty w(x,\omega)n(x,\omega)PD(x,\omega)d\mu(x) \\ &\quad + \theta_p \int_0^\infty p(x,\omega)m(x,\omega)PD(x,\omega)d\mu(x) \\ &\quad + \theta_b \int_0^\infty \int_{\eta^*(x,\omega)}^\infty \left(A(x,\omega)-\eta\right) dH(\eta)d\mu(x), \end{aligned}$$

and $A(x,\omega)$ is the enterprise value of the firm.[9] Since bonds are not priced ex-ante in this economy, the only effect of bondholder losses ($\theta_b < 1$) is to generate a wealth effect. In the interest of clarity, I will focus on the case where $\zeta = 0$. This implies that there is no wealth effect of default losses, and

$$C(\omega) + M(\omega) = GO(\omega),$$

so that GDP $Y(\omega) = GO(\omega) - M(\omega)$ equals consumption.[10] Last, the aggregate resource constraint for labor reads

$$N(\omega) = \int_0^\infty n(x,\omega)d\mu(x).$$

2.7 State space and computation

The key remark is that the only aggregate state is $z : \omega = z$. This results from (i) net worth is not a state variable for firm since there is no constraint on equity issuance; (ii) the distribution of x is constant. This simplifies substantially the numerical computation. It is important in particular given the focus of the paper on nonlinearities which require a precise solution.

[9] It is given by the recursion

$$A(x,\omega) = \pi(x,\omega) + E_{\omega',x'} M(\omega,\omega')A(x',\omega').$$

[10] A slight accounting issue arises, since it is not clear how national accounts would deal with the price heterogeneity of inputs. One may hence redefine the measured input value as $PM(\omega) = \int_0^\infty p(x,\omega)m(x,\omega)d\mu(x)$ and GDP as $GDP(\omega) = GO(\omega) - PM(\omega)$.

Intuitively, to solve the model we need factor demand from firms, which depend on prices, which depend on default risk; and default risk depends on the firm values, which depend on profits which depend on factor demand. Hence, we need to solve jointly for all policy functions. Specifically, we need to find firm-specific individual factor demand and prices, $n(x,\omega)$, $m(x,\omega)$, $p(x,\omega)$, and $w(x,\omega)$, firm-specific values, default cutoff and probabilities $V(x,\omega)$, $\eta^*(x,\omega), PD(x,\omega)$ as well as aggregates $GO(\omega), M(\omega), C(\omega), N(\omega), M(\omega,\omega')$. The algorithm is as follows: we make an initial guess for default cutoffs $\eta^*(x,\omega)$ (e.g. a very high number i.e. no default) and for consumption $C(\omega)$; we deduce the default probability $PD(x,\omega) = 1 - H\left(\eta^*(x,\omega)\right)$; given this, we find firm-specific prices:

$$p(x,\omega) = \frac{1}{1-\theta_p PD(x,\omega)},$$

$$w(x,\omega) = \frac{\overline{w}(\omega)}{1-\theta_w PD(x,\omega)},$$

where $\overline{w}(\omega) = \frac{U_2(C(\omega),N(\omega))}{U_1(C(\omega),N(\omega))}$ is an hypothetical risk-free wage. We then obtainthe input demands and output supply of each firm:

$$n(x,\omega) = (zx)^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{w(x,\omega)}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{p(x,\omega)}\right)^{\frac{\beta}{1-\alpha-\beta}},$$

$$m(x,\omega) = (zx)^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{w(x,\omega)}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{p(x,\omega)}\right)^{\frac{1-\alpha}{1-\alpha-\beta}},$$

$$y(x,\omega) = (zx)^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{w(x,\omega)}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{p(x,\omega)}\right)^{\frac{\beta}{1-\alpha-\beta}}.$$

We can then calculate the value function

$$V(x,\omega) = H(\eta^*(x,\omega))\eta^*(x,\omega) - R(\eta^*(x,\omega))$$

where $R(x) = \int_{-\infty}^{x} \eta dH(\eta)$, as well as aggregate gross output, inputs and labor:

$$GO(\omega) = \kappa_y z^{\frac{1}{1-\alpha-\beta}} \overline{w}(z)^{-\frac{\alpha}{1-\alpha-\beta}} \int_0^\infty x^{\frac{1}{1-\alpha-\beta}} \left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}} d\mu(x),$$

$$M(\omega) = \kappa_m z^{\frac{1}{1-\alpha-\beta}} \overline{w}(z)^{-\frac{\alpha}{1-\alpha-\beta}} \int_0^\infty x^{\frac{1}{1-\alpha-\beta}} \left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(1-\theta_p PD(x,\omega)\right)^{\frac{1-\alpha}{1-\alpha-\beta}} d\mu(x),$$

$$N(\omega) = \kappa_n z^{\frac{1}{1-\alpha-\beta}} \overline{w}(z)^{-\frac{1-\beta}{1-\alpha-\beta}} \int_0^\infty x^{\frac{1}{1-\alpha-\beta}} \left(1-\theta_w PD(x,\omega)\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}} d\mu(x),$$

where $\kappa_y = (\alpha)^{\frac{\alpha}{1-\alpha-\beta}} (\beta)^{\frac{\beta}{1-\alpha-\beta}}$, $\kappa_m = (\alpha)^{\frac{\alpha}{1-\alpha-\beta}} (\beta)^{\frac{1-\alpha}{1-\alpha-\beta}}$ and $\kappa_n = (\alpha)^{\frac{1-\beta}{1-\alpha-\beta}} (\beta)^{\frac{\beta}{1-\alpha-\beta}}$, and $\overline{w}(\omega) = U_2\left(C(\omega), N(\omega)\right) / U_1(C(\omega), N(\omega))$ is the hypothetical risk-free wage.

We can finally verify that the guesses are consistent, i.e. that $C(\omega) = Y(\omega) - M(\omega)$, and that $\eta^*(x,\omega) = \pi(x,\omega) - b + E_{\omega'}\left(M(\omega,\omega')V(x',\omega')\right)$. If not, we update our guesses using some relaxation. This procedure was found to converge fairly rapidly.[11]

3 Theoretical Results

I first discuss how the model generates a labor and a TFP wedge. I then discuss under which conditions the model admits multiple equilibria.

[11] We also discretized x and z for simplicity using the Rouwenhorst algorithm.

3.1 Labor and TFP Wedges

Macroeconomic shocks - such as a shock to aggregate productivity z - will affect the economy in different ways. One lens through which the mechanisms can be seen is the decomposition of labor and TFP wedges implied by the model. Equation (1) clearly suggests that a labor wedge will arise in this economy, and will depend on the likelihood of default. However this equation relates the MRS to the wage offered by an individual firm rather than a measure of aggregate wage or aggregate MPL. To conform to the macroeconomic literature, I hence define the labor wedge as

$$1-\tau_t = \frac{-\frac{U_2(C_t,N_t)}{U_1(C_t,N_t)}}{\frac{\alpha}{1-\beta}\frac{GDP_t}{N_t}},$$

where $\frac{\alpha}{1-\beta}$ is the share of labor in value added.[12] Using the expressions for GDP and labor demand obtained in section 2.7, it is easy to find the expression for the labor wedge in the model:[13]

$$1-\tau(\omega) = \frac{\int_0^\infty x^{\frac{1}{1-\alpha-\beta}} \left(1-\theta_w PD(x,\omega)\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}} d\mu(x)}{\int_0^\infty x^{\frac{1}{1-\alpha-\beta}} \left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}} d\mu(x)}.$$

While apparently complicated, this formula is quite useful, as it shows that the aggregate labor wedge is the ratio of two productivity weighted averages of firm-specific wedges, which are directly related to default risk. In some special cases, we can simplify this further. First, if $\theta_w = 0$ (and regardless of the value of θ_p), or if $PD(x,\omega) = 0$ for all x, we have $\tau(\omega) = 0$: there is no labor wedge, since there is no distortion to the exchange of labor. This is obviously at odds with the data. Second, if all firms have the same default risk, $PD(x,\omega) = PD(\omega)$,[14] then $\tau(\omega) = \theta_w PD(\omega)$. We now have a positive labor wedge, and one that is higher in times when the default likelihood is higher, i.e. in recessions. The magnitude of the labor wedge further depends directly on the employee losses θ_w as well as the likelihood of default. Given the sharp movements in default risk, and the significant losses to workers of losing their job, this variation could be important. Third, and more generally, when the default likelihood depends both on x and ω, the aggregate labor wedge will be approximately a weighted average of these firm-level distortions.

Turning now to the distortions in aggregate TFP, we can summarize these by considering the Solow residual implied by the model as $TFP = \frac{GDP}{N^{\frac{\alpha}{1-\beta}}}$. Straightforward manipulations (available in appendix) show that

$$TFP = z^{\frac{1}{1-\beta}} \frac{\int_0^\infty x^{\frac{1}{1-\alpha-\beta}} \left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}} d\mu(x)}{\left(\int_0^\infty x^{\frac{1}{1-\alpha-\beta}} \left(1-\theta_w PD(x,\omega)\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}} d\mu(x)\right)^{\frac{\alpha}{1-\beta}}}.$$

Again this is a ratio of productivity-weighted default wedges. It is easy to see that if there is no default risk, $PD(x,\omega) = 0$, or if $\theta_w = \theta_p = 0$, then $TFP = z^{\frac{1}{1-\beta}} E(x^{\frac{1}{1-\alpha-\beta}})^{\frac{1-\beta-\alpha}{1-\beta}} = TFP^*$. This formula

[12] Some studies (e.g. Shimer (2010)) calculate the labor wedge assuming that the econometrician does not know the preference parameters of the utility function or the technology parameter $\frac{\alpha}{1-\beta}$. These are then picked to generate an "average" labor wedge of 40%, similar to the data. Here instead we assume that the econometrician knows these parameters. This has little effect on the results; the alternative approach generates a proportional, but more volatile labor wedge.

[13] See calculation details in appendix.

[14] This is the case, for instance, if the standard deviation of x is small relative to the standard deviation of η.

reflects the usual aggregate productivity z, adjusted for (i) reallocation of inputs across firms, (ii) the "multiplier" effect of inputs on productivity (Jones (2012)). Second, if default risk is the same for all firms, $PD(x,\omega) = PD(\omega)$, then

$$TFP = TFP^* \left(1 - \theta_p PD(\omega)\right)^{\frac{\beta}{1-\beta}},$$

so that measured TFP falls as default risk increases. The logic is that the economy uses too little inputs because of the default risk to suppliers. Finally, if $PD(x,\omega)$ depends on x, there will be a further misallocation as high productivity firms become "too big" because their low default risks allows them to hire more inputs and workers, to the point where the marginal product is lower in the more productive (and safest) firms. Small, unproductive, risky firms have too few inputs on the other hand. The logic here is similar to Restuccia and Rogerson (2008).

3.2 Multiple equilibria

For some parameter values, this model admits multiple equilibria. Intuitively, if people expect a firm to default, they will require high wages and prices to supply labor or inputs, which will reduce the firms' profits and hence makes the firm more likely to default, validating the initial beliefs. More precisely, the equation determining the default cutoff is

$$\eta^* = \pi - b + J,$$

where $J = E_{\omega',x'}\left(M(\omega,\omega')V(x',\omega')\right)$ is the future value, and π is the equilibrium profit, which itself depends on η^* since it affects factor prices:

$$\begin{aligned} \pi &= (1-\alpha-\beta)y, \\ &= (1-\alpha-\beta)\,(zx)^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha(1-\theta_w PD(x,\omega))}{\overline{w}(\omega)}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\beta\left(1-\theta_p PD(x,\omega)\right)\right)^{\frac{\beta}{1-\alpha-\beta}}, \end{aligned}$$

hence the equation determining η^* given the future value J can be written:

$$\eta^* = \kappa_\pi\,(zx)^{\frac{1}{1-\alpha-\beta}}\,\overline{w}(\omega)^{-\frac{\alpha}{1-\alpha-\beta}}\left(1-\theta_w(1-H(\eta^*))\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(1-\theta_p\left(1-H(\eta^*)\right)\right)^{\frac{\beta}{1-\alpha-\beta}} - b + J(\omega,x),$$

where $\kappa_\pi = (1-\alpha-\beta)\alpha^{\frac{\alpha}{1-\alpha-\beta}}\beta^{\frac{\beta}{1-\alpha-\beta}}$.

Start by considering multiple equilibria for a single firm, that is taking $\overline{w}(\omega)$ as given. We also will assume that the future value J is also taken as given. The question is then simply whether this equation has more than one solution in η^*. The left hand side obviously rises monotonically from minus infinity to plus infinity; the right-hand side is also increasing monotonically in η^* from a finite value $\left(\kappa_\pi\,(zx)^{\frac{1}{1-\alpha-\beta}}\,\overline{w}(\omega)^{-\frac{\alpha}{1-\alpha-\beta}}\,(1-\theta_w)^{\frac{\alpha}{1-\alpha-\beta}}\,(1-\theta_p)^{\frac{\beta}{1-\alpha-\beta}} - b + J\right)$ to a higher finite value $\left(\kappa_\pi\,(zx)^{\frac{1}{1-\alpha-\beta}}\,\overline{w}(\omega)^{-\frac{\alpha}{1-\alpha-\beta}} - b + J\right)$. The intermediate value theorem implies that there must be one solution at least; however there may be several depending on the shape of the distribution H of cost shocks.

Which parameters determine whether there are multiple equilibria? Clearly, if $\theta_w = \theta_p = 0$, then the right-hand side function is flat so there is a single solution. Moreover, intuitively if J is large relative to π, i.e. the future value is significant, it is less likely that there are multiple equilibria, since the right-hand side is nearly flat. However, I do not have a formal argument. [The case with aggregate multiple equilibria is to be added.]

The numerical solution method starts from a guess of zero default and hence is likely to converge on the lowest default equilibrium; it hence abstracts from "runs".

4 Numerical Results

This section first discusses the model parameters, then illustrates the implications of the model by solving it numerically.

4.1 Parametrization

The parametrization of the model is still in progress; the results here are preliminary and should be understood as "illustrative" more than "realistic". The time period is one quarter. The discount factor is $\beta = .995$. The labor share in gross output is $\alpha = .3$ while the input share is $\beta = .3$. Preferences are assumed to have a unit IES to consumption and a Frisch elasticity equal to 4. The process for aggregate productivity z is an AR(1) with $\sigma_\varepsilon = 0.7\%$ and $\rho_z = .98$. The process for idiosyncratic productivity is an AR(1) with $\sigma_x = 13\%$, and $\rho_x = .85$.The parameters b and σ_η are set to target a mean default rate of 1% per quarter with a standard deviation of 0.5%. The default costs are assumed to be $\theta_w = \theta_p = 1$.

4.2 Amplification

Figure 3 calculates the response of the economy as aggregate total factor productivity z varies. The blue line depicts the response of the model without default losses (i.e. $\theta_p = \theta_w = 0$); in this case, the model collapses to the standard RBC model (without capital). Given log utility, employment is independent of z. Output responds hence one-for-one, aggregate TFP is measured without error, and there is no variation in the labor wedge. The red line depicts the response of the model with $\theta_w > 0$ and $\theta_p > 0$. First, the top two panels show that employment and output decline more when z falls. The bottom graphs establish the proximate causes of this larger recession: first, aggregate TFP falls further than z itself implies, and second there is a significant countercyclical labor wedge. The mechanism is that as z falls, more firms become close to default and hence affected by the default wedges. This reduces labor and input demand for all firms, and leads to lower TFP through lower input usage and through misallocation of labor and inputs across firms. Overall, the figure demonstrates that the model generates amplification of TFP shocks. The table summarizes this simply by calculating the volatility of GDP, employment and measured TFP for both the frictionless model and the model with frictions.

4.3 Time-varying sensitivity to macroeconomic shocks

To now illustrate that the response to the same shock is larger when many firms are close to default, I depict in figures 4 and 5 the elasticity of output and employment to a TFP shock, as a function of

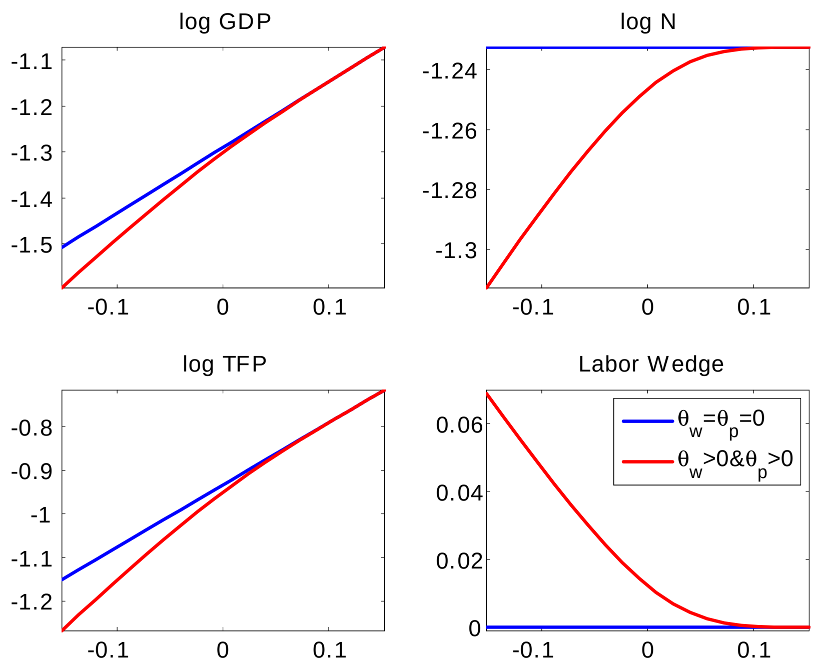

Figure 3: Effect of aggregate productivity z on output Y, employment N, measured TFP, and the measured labor wedge τ.

	$\theta_p = \theta_w = 0$	$\theta_w > 0, \theta_p > 0$
$\sigma(\Delta \log N)$	0.0	0.20
$\sigma(\Delta \log GDP)$	1.00	1.18
$\sigma(\Delta \log TFP)$	1.00	1.12
$\sigma(\Delta \log Z)$	0.70	

Table 1: Standard deviation of growth rates of employment, GDP, measured TFP, and aggregate productivity.

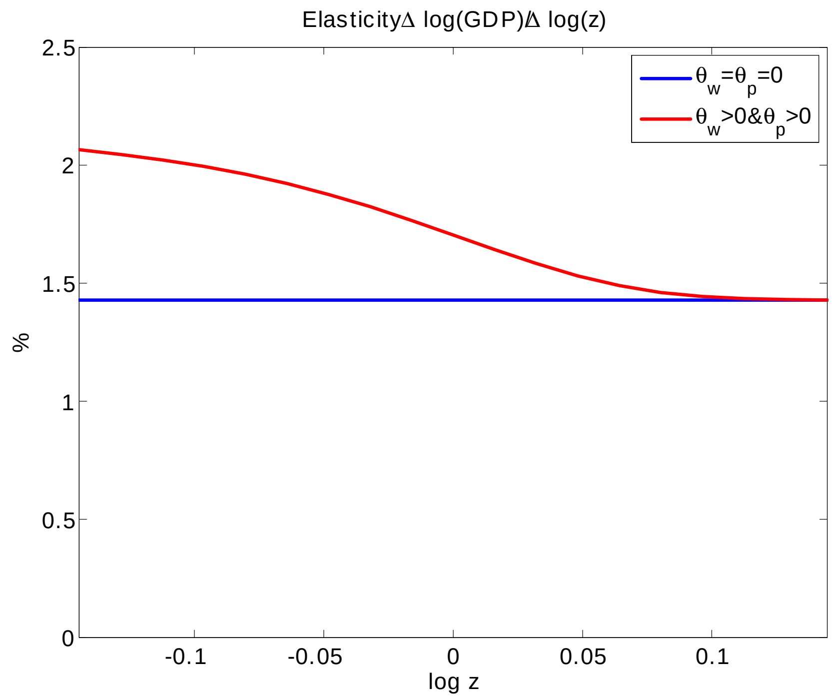

Figure 4: Elasticity of GDP to a productivity shock, as a function of the intial level of TFP. Blue line = model without default wedges; red line = model with default wedges.

the underlying level of TFP. The key results from these pictures are (a) without default wedges (blue line), this elasticity is constant, as the model is indeed log-linear; (b) with positive default wedges, the response of the economy to a TFP shock (positive or negative) is indeed bigger when TFP is low to start with, i.e. when more firms are close to default.

4.4 Countercyclical volatility of output and employment

The time-varying sensitivity to macroeconomic shocks demonstrated in the previous subsection implies a time-varying volatility of employment and output. To illustrate this in an empirically realistic way, figures 6 and 7 present the result from a long model simulation. In these figures, each dot represents a time period, with the associated current level of TFP on the x-axis, and the associated macro volatility, proxied as the standard deviation of the growth rate of employment or output over the next 20 quarters, on the y-axis. Note that the z shock is homoscedastic, so the standard deviation of the growth of z is constant. However, when z is low, more firms are close to default, the sensitivity to a macro shock is larger, and consequently employment and output become more volatile.

4.5 Countercyclical cross-sectional dispersion

For the same reason that the model generates time-varying aggregate volatility, it also generates time-varying cross-sectional dispersion of firm variables such as employment or sales, even though the cross-sectional dispersion of productivity x is constant. “Bad” firms with low idiosyncratic productivity x are more affected by low realizations of z, implying a “fanning out” of the cross-sectional distribution of

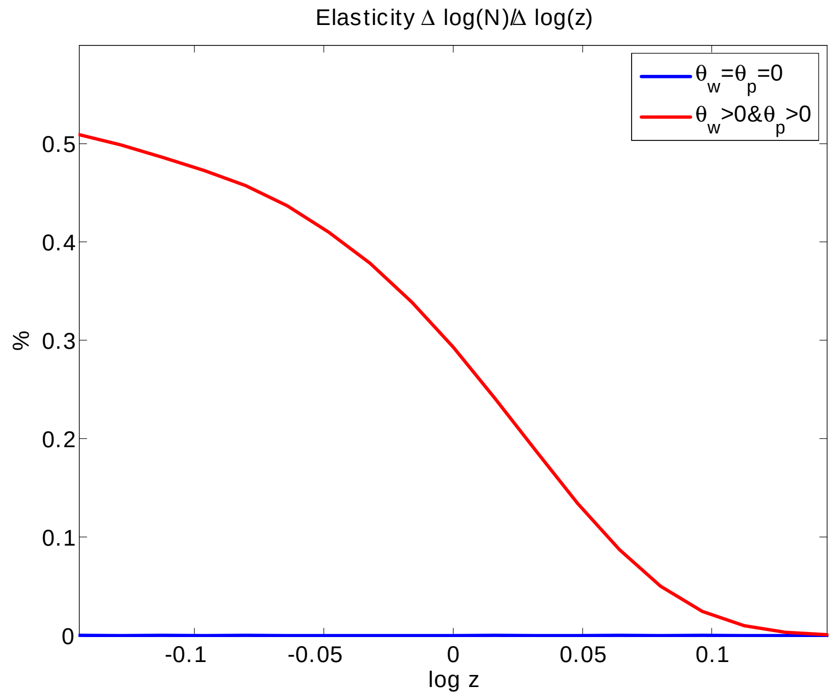

Figure 5: Elasticity of employment to a productivity shock, as a function of the intial level of TFP. Blue line = model without default wedges; red line = model with default wedges.

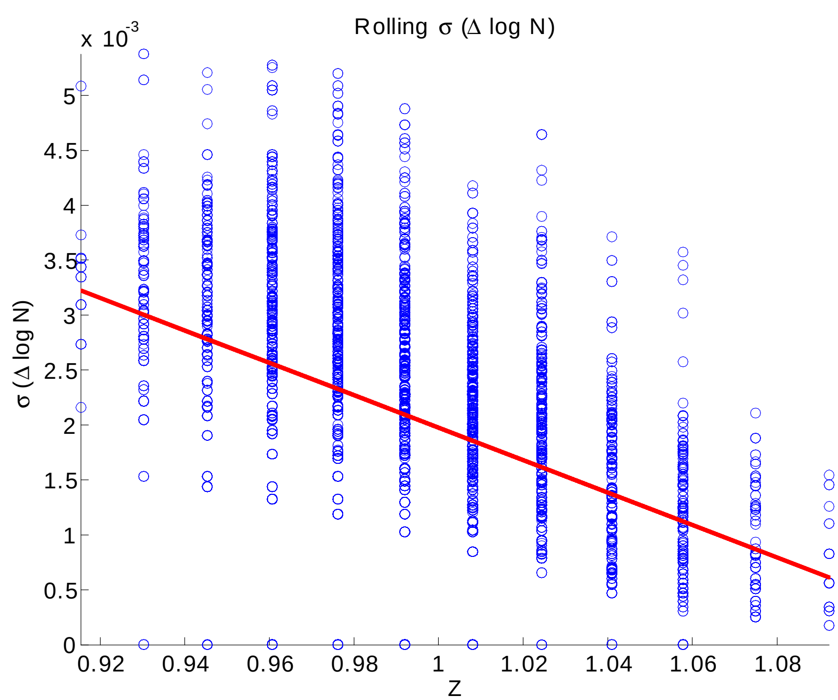

Figure 6: Rolling standard deviation of employment growth in the next 20 quarters, as a function of the current level of productivity, with a fitted regression line.

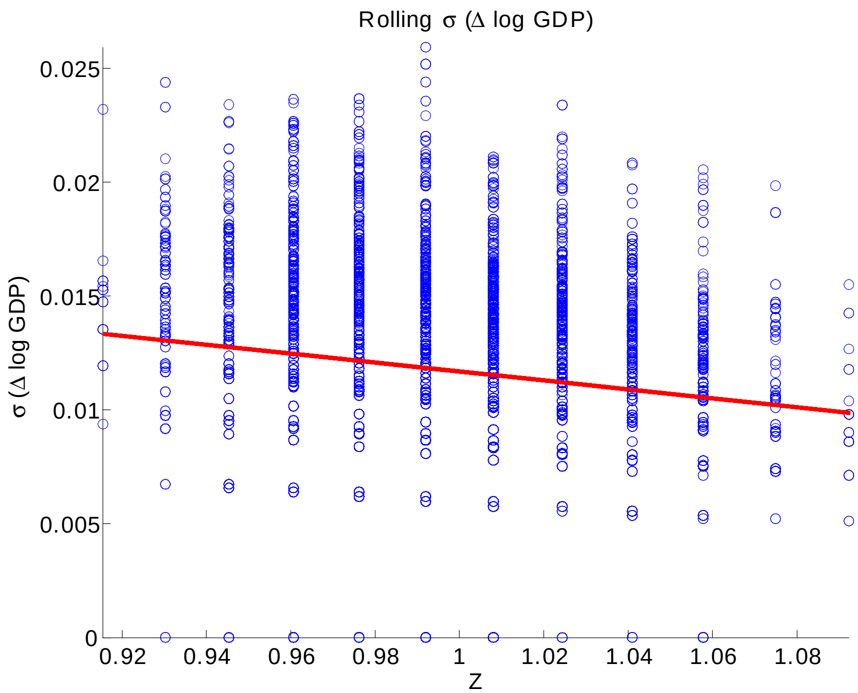

Figure 7: Rolling standard deviation of GDP growth in the next 20 quarters, as a function of the current level of productivity, with a fitted regression line.

sales or employment in bad times. This is illustrated in figures 8 and 9 that depict the cross-sectional standard deviation of log sales (resp. employment) as a function of aggregate productivity.

4.6 Negative skewness of macroeconomic aggregates

The policy functions depicted above may suggest that the model generates important asymmetries. However, these asymmetries are somewhat subtle to quantify. In particular, growth rates of GDP or employment exhibit almost no asymmetry or skewness: the large drops when z is low if there is a negative shock, are offset by equally large increases when z is low if there is a positive shock. However, the level of GDP and employment do exhibit some moderate negative skewness, not inconsistent with the data. The following tables summarize the skewness and kurtosis of the underlying (discretized) shock z, as well as GDP, employment and measured TFP, both in growth rates and in levels. There is essentially no skewness generated by the model for growth rates, but there is a tad of negative skewness for the level of employment. We also observe some kurtosis for employment both in growth rates and leves. (Note: data to be added.)

4.7 Alternative shocks

While the analysis so far has been confined to aggregate productivity shocks, the mechanisms outlined operate for any reasonable aggregate disturbance. In particular, shocks that affect asset prices can now directly affect the economy. To illustrate this, I introduce time-varying risk aversion in the model. To

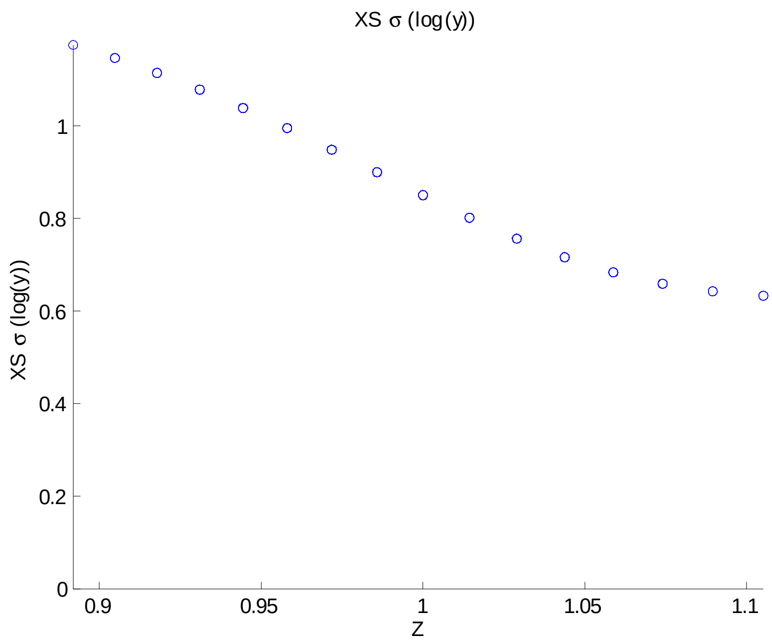

Figure 8: Cross-sectional standard deviation of log sales, as a function of the current level of aggregate productivity z.

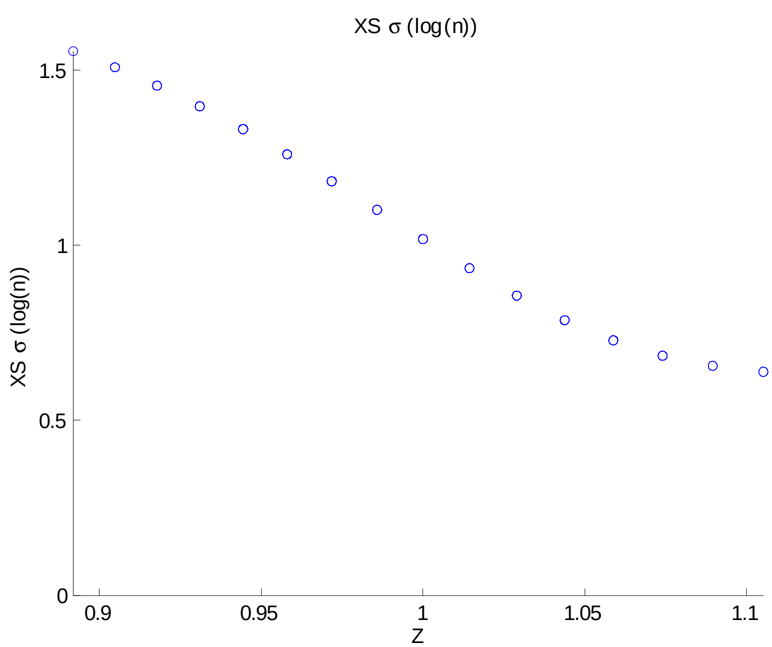

Figure 9: Cross-sectional standard deviation of log employment, as a function of aggregate productivity z.

		$\Delta \log z$	$\Delta \log Y$	$\Delta \log N$	$\Delta \log TFP$
Skewness	Frictionless	-0.02	-0.02	NA	-0.02
	Benchmark	-0.02	-0.02	0.00	-0.02
Kurtosis	Frictionless	0.30	0.30	NA	0.30
	Benchmark	0.30	0.30	0.60	0.30

Table 2: Skewness and kurtosis of growth rates of macroeconomic variables, in the frictionless and benchmark versions of the model. NA means skewness/kurtosis undefined as the standard deviation is zero.

		z	Y	N	TFP
Skewness	Frictionless	0.01	0.01	NA	0.01
	Benchmark	0.01	-0.02	-0.36	-0.00
Kurtosis	Frictionless	-0.12	-0.12	NA	-0.12
	Benchmark	-0.12	-0.12	0.17	-0.12

Table 3: Skewness and kurtosis of the (HP-filtered) level of macroeconomic variables, in the frictionless and benchmark versions of the model.

do so in a clean way, I use Epstein-Zin preferences:

$$V_t = \left((1-\beta) u(C_t, N_t)^{1-\sigma} + \beta E_t \left(V_{t+1}^{1-\gamma_t} \right)^{\frac{1-\sigma}{1-\gamma_t}} \right)^{\frac{1}{1-\sigma}},$$

and assume that γ_t follows a Markov chain that approximates an AR(1) process. A shock to risk aversion is a shortcut, frequently used in the finance literature or even financial press, to proxy for a "panic" whereby agents suddenly dislike risky assets. In a standard business cycle model, such as the current model with $\theta_w = \theta_p = 0$, a shock to risk aversion γ_t has no effect on output or employment.

In contrast, in the model with default wedges, an increase in risk aversion reduces output and employment. Higher risk aversion reduces equity values, which increases the risk of default, and hence the associated wedges. As a result, output and employment contract. This channel connects asset prices and the macroeconomy in a different way than the standard Q-theory, which emphasizes that lower asset prices lead to lower incentives to invest.

This result is illustrated in figure 10, which depicts the level of output, employment, measured TFP, and the measured labor wedge, as a function of the level of risk aversion. As risk aversion (stochastically) rises, the economy contracts, partly because of lower TFP, and partly because of a higher labor wedge. The magnitudes, however, remain fairly small in the current calibration.

4.8 Summary of model implications

To summarize, this section has demonstrated five model implications: (1) the effect of a fundamental shock (such as a z shock) is larger if there are financial distress costs (amplification); (2) the amplification effect of financial distress costs depends on the state of the economy and is larger when more firms are close to default, leading to time-varying volatility of macroeconomic time series; (3) the cross-sectional

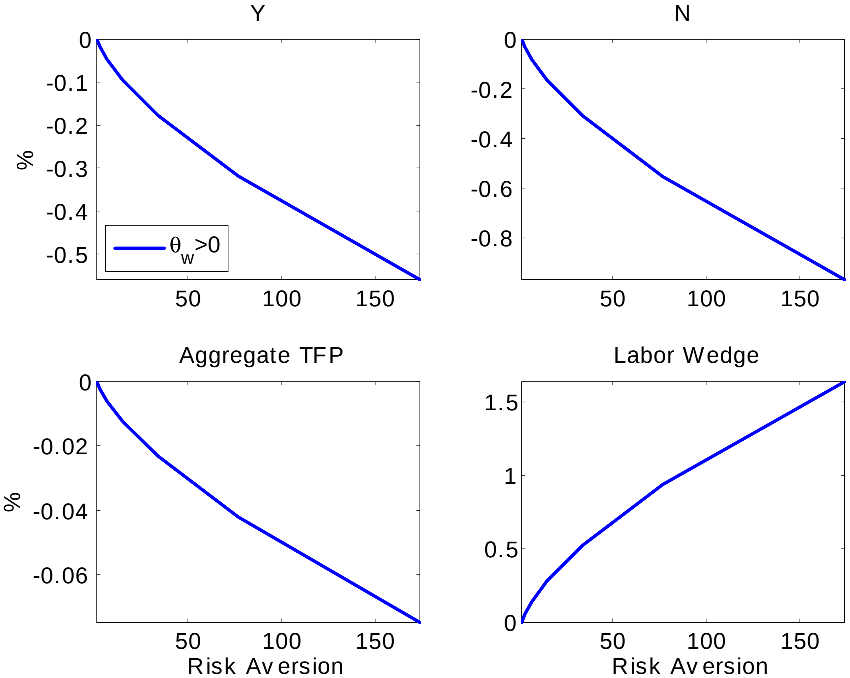

Figure 10: Output, Employment, Measured aggregate TFP, and Measured Labor Wedge, as a function of risk aversion γ.

standard deviation of firm outcomes is also countercyclical for the same reason; (4) the model generates some negative skewness of macroeconomic aggregates in level; (5) any shock that affects equity value or likelihood of default will generate a contraction in economic activity. Overall, we expect the labor wedge, economic activity, the share of firms with high leverage, and macroeconomic volatility to be correlated.

5 Empirical Evidence

This section is a first attempt at assessing the mechanism discussed in the previous section. I proceed in three steps. First, panel data regressions provide some support for the basic idea that firms that are close to default (as measured by their leverage or distance to default) have sales and employment more sensitive to aggregate fluctuations. Second, I construct the cross-sectional distribution of (market) leverage and find that it varies substantially over time. In particular, the number of firms "close to default" (e.g. with a leverage above a threshold value) is strongly procyclical. This result holds for various definitions of leverage and various thresholds. In contrast, the median leverage is not clearly cyclical. Third, I investigate the time series relations between (i) aggregate uncertainty, (ii) the labor wedge, (iii) measures of economic activity, and (iv) the share of firms that are close to default. While some of the correlations are consistent with the model, others are not. However, an interesting finding is that the share of firms that are close to default appears to be a useful statistical predictor of economic activity - more so than the average or median leverage that is typically used in empirical studies such as Kollman and Zeugner (2013).

5.1 Cross-sectional evidence

A key empirical implication of the theory is that firms with high leverage or high default risk are more sensitive to aggregate fluctuations. This section provides some simple reduced-form evidence consistent with this implication (see also Sharpe (1994) for related evidence). I discuss below some potential endogeneity concerns.

Using firm-level annual panel data from Compustat, I estimate the regression,

$$\Delta \log S_{it} = \alpha + \delta \Delta \log GDP_t + \gamma Z_{i,t-1} + \beta Z_{i,t-1} \Delta \log GDP_t + X_{it}\rho + \varepsilon_{it},$$

where S_{it} is real sales of firm i in year t, $Z_{i,t-1}$ is a dummy variable equal to one if a firm is "close to default" (as defined below), $\Delta \log GDP_t$ is the growth rate of real GDP, and X_{it} are some potential controls. The coefficient of interests are γ, which measures by how much "being close to default" reduces sales growth, and especially β, which measures the extra sensitivity to GDP growth of high leverage firms. I also estimate a similar regression using employment growth as the dependent variable. The motivation for the dummy variable (rather than a continuous measure of being close to default) is that the effects are likely to be highly nonlinear.

I use two measures of "close to default". The first is simply leverage, i.e. the ratio of debt to equity market value; in this case I define "close to default" as a leverage above 1. The second measure, the Merton distance-to-default, defined as

$$DD = \frac{\log \frac{\text{firm value}}{\text{debt}} + \mu_V - \frac{\sigma_V^2}{2}}{\sigma_V},$$

where μ_V and σ_V are the drift and volatility of the firms' asset value, as estimated using the equity value and equity volatility.[15] I define "close to default" as a distance to default less than 4. For comparison, this corresponds to the typical value for firms just below investment grade rating.[16] The results continue to hold, however, if one uses other thresholds than 1 for leverage and 4 for distance-to-default.

The Compustat sample used is fairly standard; it includes all non-financial, non-utilities, domestic firms, with a December fiscal-year, from 1970 to 2011. Leverage is defined as the ratio of total debt (long-term debt plus short-term) over equity market value (i.e. (dltt+dlc)/(csho*prcc_f)), but we also consider alternative definitions including net-of-cash leverage (substracting cash and short-term investments from debt on the denominator ((dltt+dlc-che)) and net-of-cash and trade credit leverage (substracting receivables and adding receivables, (dltt+dlc+ap–rectp-che)).

The first table reports the results for sales growth, using Z =leverage. First, estimates of γ are consistently negative and highly significant. Being high leverage is associated with lower sales growth

[15] The data on distance to default were provided by Egon Zakrajsek.

[16] The following table summarizes the value in my data:

S&P long-term rating	Median DD
AAA or AA	11.3
A	8.6
BBB	6.3
BB	4.4
B	2.7
CCC or below	0.7

Dependent variable:$\Delta \log S_{it}$	(1)	(2)	(3)	(4)	(5)	(6)
Leverage definition	(debt)		(debt-cash)		(debt-cash-trade)	
γ	-0.075	-0.068	-0.063	-0.051	-0.057	-0.055
t-stat	14.3	10.5	12.4	8.1	10.8	8.2
β	0.52	0.50	0.49	0.49	0.38	0.37
t-stat	3.6	3.4	3.5	3.4	2.6	2.5
Firm fixed-effects	n	y	n	y	n	y
Observations	69377					

Table 4: Sales growth sensitivity as a function of leverage. Robust standard errors.

Dependent variable:$\Delta \log N_{it}$	(1)	(2)	(3)	(4)	(5)	(6)
	(debt)		(debt-cash)		(debt-cash-trade)	
γ	-0.062	-0.060	-0.055	-0.051	-0.049	-0.051
t-stat	15.1	11.3	13.2	9.4	11.4	8.8
β	0.44	0.51	0.44	0.51	0.38	0.45
t-stat	3.7	4.2	3.7	4.2	3.1	3.6
Firm fixed-effects	n	y	n	y	n	y
Observations	69377					

Table 5: Employment growth sensitivity as a function of leverage.

of 5-8% going forward. Obviously, this coefficient does not measure the “effect” of high leverage, since leverage is endogenous: negative shocks to expected sales drive equity value lower and leverage higher, so the causation runs “both ways”, and we expect a negative coefficient even in the absence of any financial distress cost. On the other hand, one might have expected some mean-reversion of sales for firms that had negative shocks the previous year.

Second, the coefficient β is positive and significant. The typical firm in Compustat has a sensitivity of sales growth to GDP around 2; a coefficient β of 0.5 reflects that high leverage firms have a sensitivity around 2 instead. This is economically important.[17] Finally, note that both coefficients β and γ are fairly stable across different definitions of leverage. Table 2 turns to the employment results, which are very similar overall. In Compustat, the typical firm has a sensitivity of employment to GDP around 1.5. Hence the estimated β is, if anything, even more economically important for employment than for sales.

[17] In this case too, a potential source of bias is that firms with different cyclical sensitivities in their real activities (in finance language, firms that have high asset beta) may choose a different leverage. For instance, a firm with high sensitivity may choose a lower leverage. This would tend to bias our estimated β towards zero. On the other hand, in a business cycle downturn, the market values of more cyclical firms may fall more, leading them to have higher leverage ex-post, which would bias our estimated β up. Hence, the overall effect is unclear.

Dependent variable:$\Delta \log S_{it}$	(1)	(2)	(3)	(4)
γ	-.10	-.12	-.12	-.11
s.e.	(.01)	(.01)	(.01)	(.01)
β	0.89	0.69	0.57	0.62
s.e.	(0.23)	(0.23)	(0.24)	(0.24)
Firm fixed effects	n	y	y	y
Industry controls	n	n	y	y
Firm controls	n	n	n	y
Observations		60,631		

Table 6: Sales growth sensitivity as a function of distance to default. Robust standard errors.

Dependent variable:$\Delta \log N_{it}$	(1)	(2)	(3)	(4)
γ	-.08	-.12	-.11	-.10
s.e.	(.01)	(.01)	(.01)	(.01)
β	0.70	0.71	0.59	0.60
s.e.	(0.16)	(0.16)	(0.16)	(0.16)
Firm fixed effects	n	y	y	y
Industry controls	n	n	y	y
Firm controls	n	n	n	y
Observations		60,631		

Table 7: Employment growth sensitivity as a function of distance to default. Robust standard errors.

Tables 3 and 4 provide the results using Z = distance to default. Very similar to the leverage variable, we find γ to be very negative and highly significant, and β to be positive and fairly large. This holds even when industry dummies, industry dummies interacted with GDP, and standard firm controls (such as Tobin's Q or Profitability) are included.

5.2 Business cycle variation in the cross-sectional distribution of leverage

A second key piece of the mechanism is that the number of firms close to default varies substantially over time. To provide some light on this topic, Figures 11 and figure 12 provide the distribution of net-of-cash leverage in 2006Q1 and 2009Q1. First, note that there is a wide distribution, with the typical firm having essentially zero net leverage. But the fanning out of the distribution during the recession is impressive: there were many more firms with high net leverage in early 2009 than in early 2006. There were also significantly more firms with negative net leverage. This result also continues to hold for the alternative definitions of leverage.

One possible interpretation of these figures is that for the typical firm, financial distress is not a

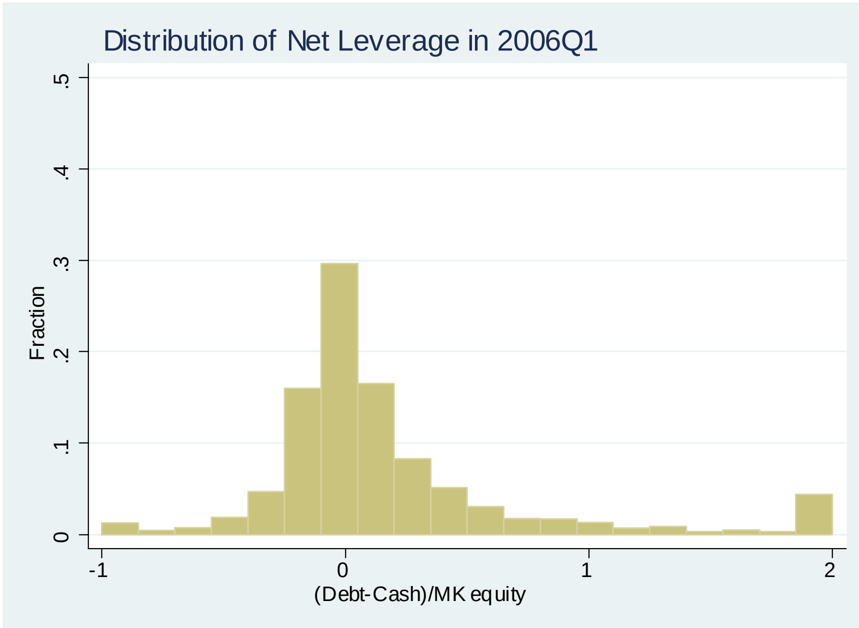

Figure 11: Distribution of net-of-cash leverage in 2006q1 in Compustat (nonfinancial firms). Net-of-cash leverage is (debt-cash)/(market value of equity).

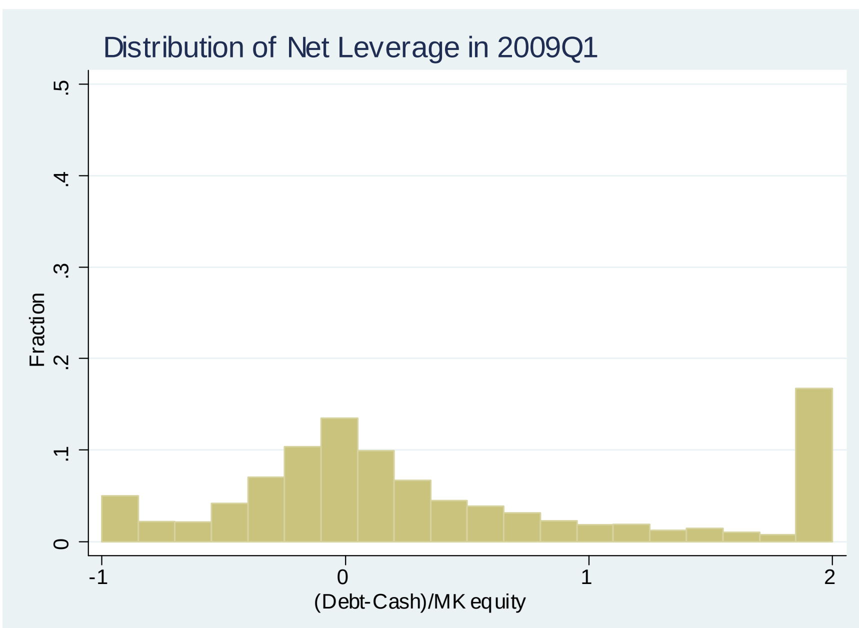

Figure 12: Distribution of net-of-cash leverage in 2009q1 in Compustat (nonfinancial firms). Net-of-cash leverage is (debt-cash)/(market value of equity).

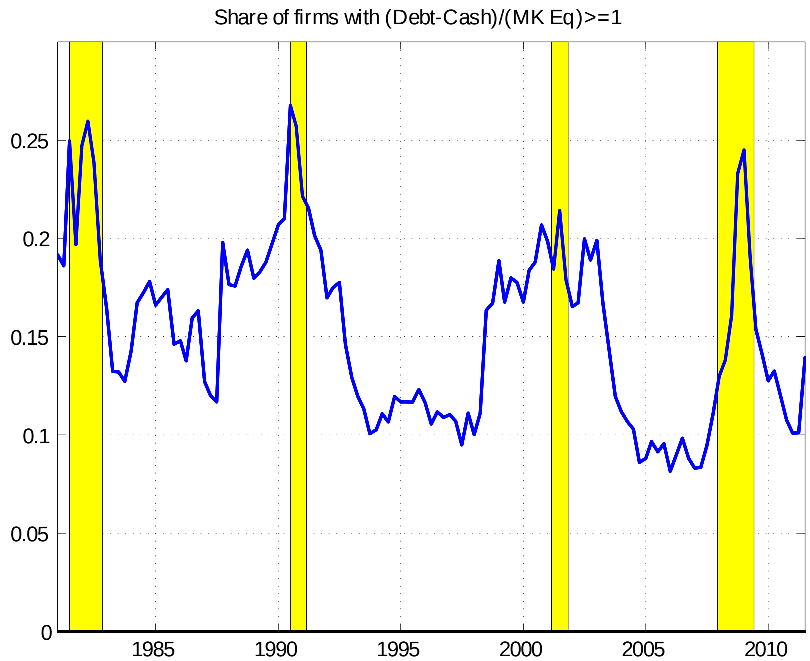

Figure 13: Share of firms with (debt-cash)/)(market value of equity) $\geq$ 1. Compustat, nonfinancial firms.

concern. But it is a concern for a small numbers of firms in "good times" like 2006, and for a significant fraction of firms in "bad times" like 2009. Hence, one really wants to track the time series of the number of firms with high leverage. Figure 13 plots the time series, which clearly exhibit a strong countercyclicality. This series remains very similar if we weight firms by sales rather than just counting firm units, as shown in figure 14; the main difference is some additional noise, as some large firms may go above or below the threshold.

An interesting fact is that this share variable behaves quite differently from the median or average leverage, depicted in figure 15. This suggests that there is some interesting information in the "tail" of the leverage distribution.

5.3 Time series evidence

The model makes strong prediction regarding the association of the following variables: (i) the share of firms close to default, (ii) the labor wedge, (iii) macroeconomic uncertainty, and (iv) economic activity. This section discusses the empirical correlation between these time series.

5.3.1 Data construction

The share of firms close to default is constructed as in the previous section. The labor wedge is constructed as in Shimer (2010). Specifically, assuming a representative agent with utility

$$E\sum_{t=0}^{\infty}\beta^t\left(\log(c_t)-\gamma\frac{n_t^{1+\phi}}{1+\phi}\right),$$

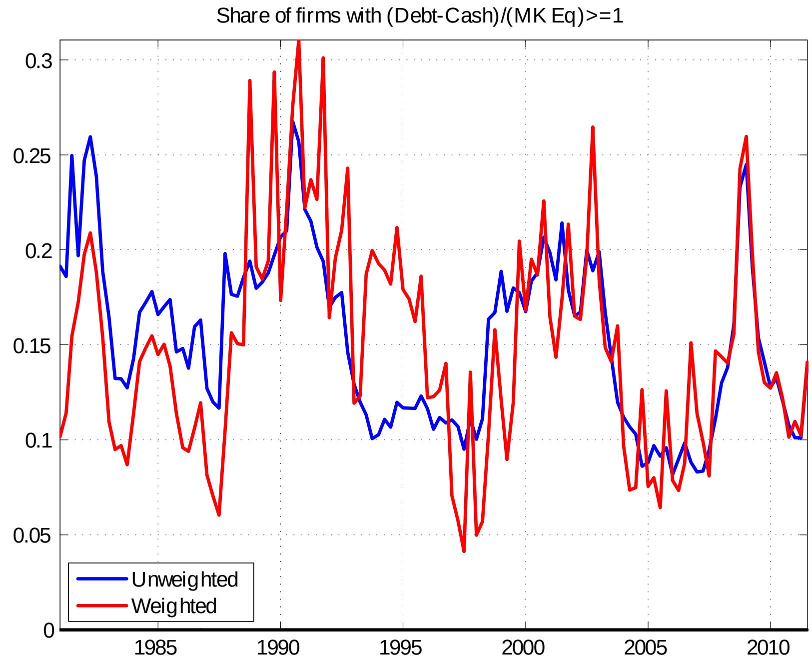

Figure 14: Share of firms with (debt-cash)/)(market value of equity) ≥ 1, both weighted by sales and unweighted. Compustat, nonfinancial firms.

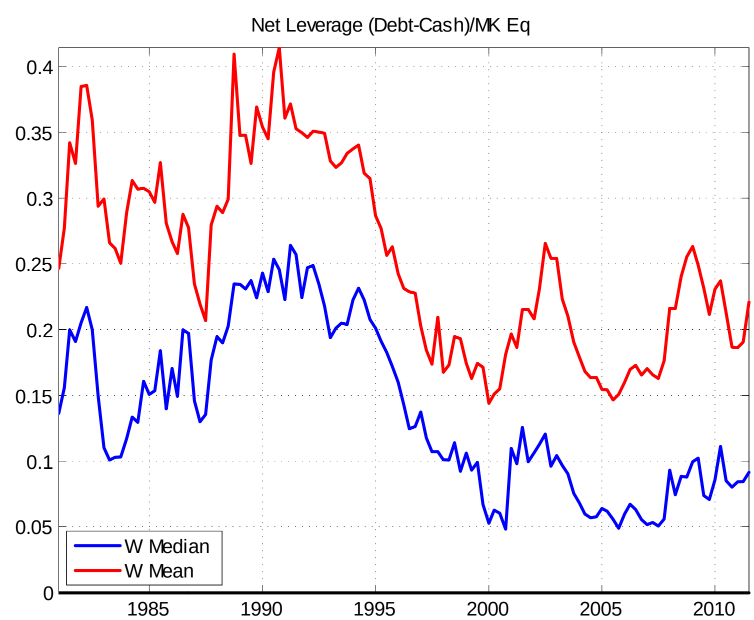

Figure 15: Weighted mean and median of net-of-cash leverage (debt-cash)/(mkt value of equity). Compustat, nonfinancial firms.

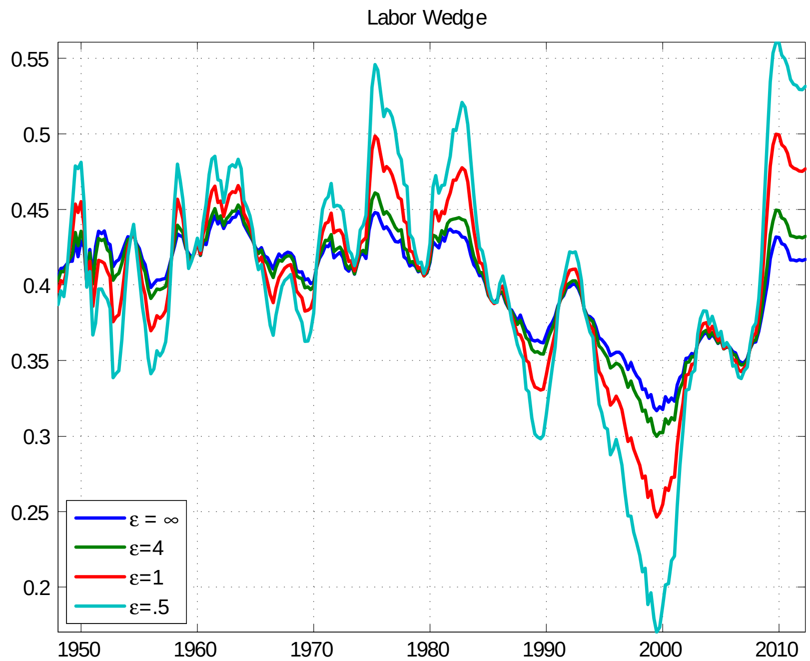

Figure 16: Labor wedge, constructed as in Shimer (2010) and as explained in the text.

and a production function $y_t = k_t^{\alpha} (z_t n_t)^{1-\alpha}$, the first-order condition for labor implies

$$w_t(1-\tau_t) = (1-\alpha)\frac{y_t}{n_t}(1-\tau_t) = \gamma c_t n_t^{\phi},$$

so that

$$\tau_t = 1 - \frac{\gamma}{1-\alpha}\frac{c_t}{y_t} n_t^{1+\phi}.$$

Using nondurable per-capita consumption, hours worked per-capita, and real GDP, we construct the right-hand-side for given value of ϕ. The coefficient $\frac{\gamma}{1-\alpha}$ is picked so that τ_t is on average equal to 0.4. Obviously, the elasticity of labor supply (the inverse of the parameter ϕ) matters for this construction, but the labor wedge is a puzzle regardless of the value of ϕ. Figure 16 depicts the labor wedge implied by different values of ϕ. This series is highly countercyclical with respect to employment, and quite countercyclical with respect to output. Figure 17 illustrates this by plotting together HP filtered log labor wedge and HP filtered log hours.

We consider two macroeconomic uncertainty measures. First, we use the stock market volatility, constructed as the standard deviation of realized daily returns within a quarter. Second, we use the uncertainty measure constructed by Jurado, Ludvigson and Ng (2013). They use a large dataset of macro and financial indicators and estimate the average standard deviation of the unforecastable component of these time series. This uncertainty measure is depicted in figure 18. It has spikes in 2008, 1981, and 1975, but not much action in the 1991 or 2001 recessions.

5.3.2 Relations between time series (highly incomplete)

This section currently presents some reduced-form relations.

Correlations

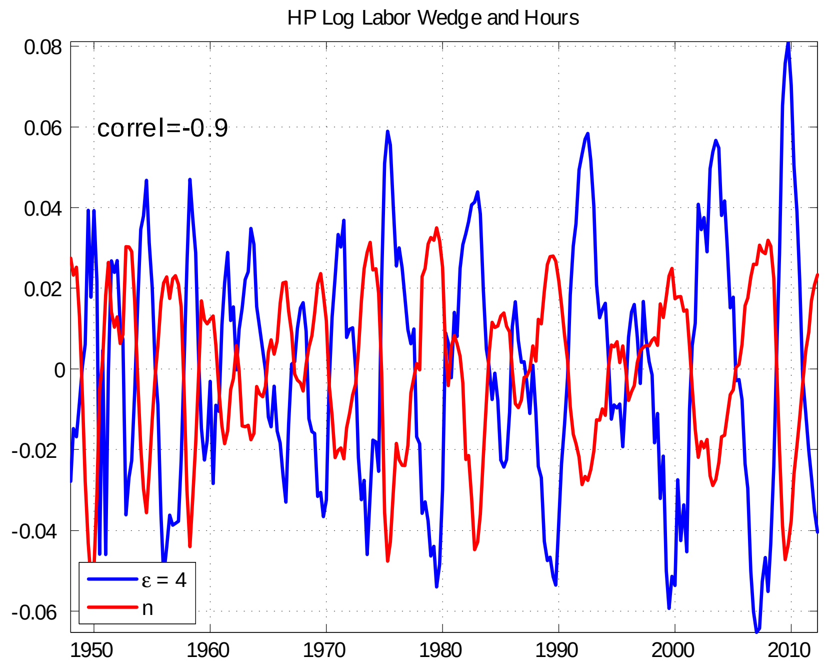

Figure 17: HP filtered log labor wedge and HP filtered log hours. US data.

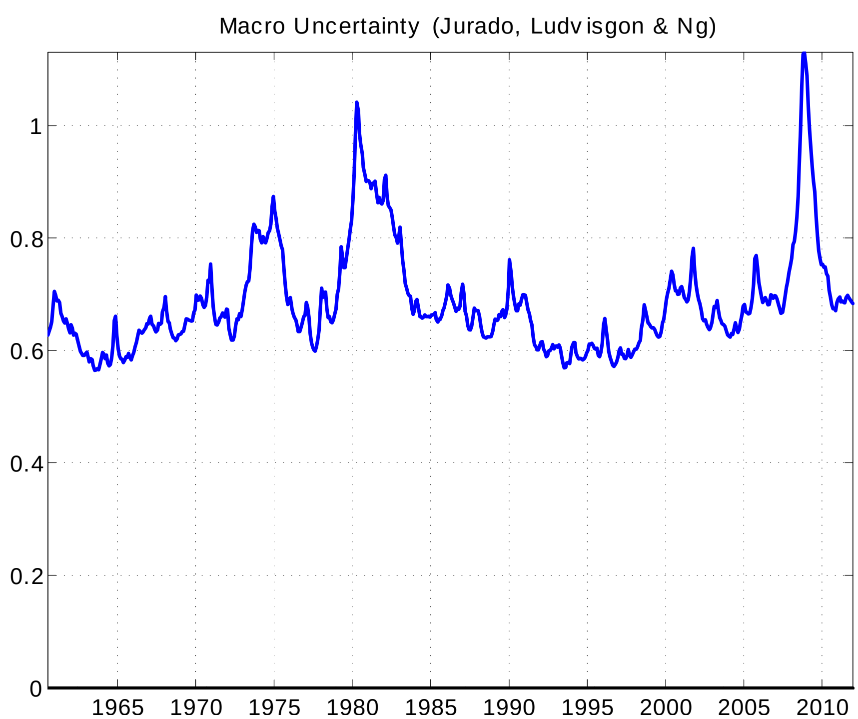

Figure 18: Macroeconomic uncertainty measure. Source: Jurido, Ludvisgon, and Ng (2013).

	StockVol	Unc	Share	Median	Average	GDP	Hours	Labor Wedge
Stock market volatility	1	0.52	0.46	-0.03	0.15	-0.16	0.02	0.00
Uncertainty JLN		1	0.48	0.24	0.33	-0.20	-0.05	0.42
Share high leverage			1	0.62	0.84	-0.23	-0.10	0.11
Median leverage				1	0.93	-0.07	0.04	0.25
Average leverage					1	-0.13	-0.01	0.17
GDP (HP)						1	0.88	-0.53
Hours (HP)							1	-0.61
Labor Wedge								1

Table 8: Correlations. 1980q1-2011q4.

Table 3 presents the correlation between the key macroeconomic time series studied here. Interesting, and consistent with the model, uncertainty is strongly correlated with the share of firms with high leverage (0.48 or 0.46 depending on the measure of uncertainty). In contrast, the median or average leverage have much weaker correlation (from -0.03 to 0.33). Moreover, the labor wedge is significantly correlated with macro volatility (0.42). The correlation between leverage and the labor wedge is weaker however, and the share of firms with high leverage does not outperform here the median or average leverage.

Forecasting GDP growth

Kollman and Zeugner (2012) show that average leverage forecasts negatively GDP growth. I show that this relation is significantly stronger if one uses as measure of leverage not the average leverage, but the share of firms with high leverage. To show this, run the regression

$$\Delta \log GDP_{t+1} = a + b\Delta \log GDP_t + cZ_t + \varepsilon_{t+1},$$

where Z_t is either average or median leverage, or the share of firms with high leverage. Table 5 summarizes the results. Clearly, the share of high-leverage firms has more explanatory power than just the average or median.

6 Conclusion

TBA

7 References (incomplete)

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	100×c	t-stat	R^2
Average (unweighted)	-0.0046	-0.66	.235
Average (weighted)	-0.0024	-0.27	.233
Median (unweighted)	-0.0052	-0.92	.238
Median (weighted)	-0.0024	-0.27	.233
Share (unweighted)	-0.0330	-2.20	.262
Share (weighted)	-0.0276	-2.47	.269

Table 9: Forecasting GDP growth using its own lag and measures of leverage.

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8 Appendix

8.1 Sample construction in Compustat

TBA

8.2 Model solution

The equilibrium is characterized by the following equations in $C(\omega), Y(\omega), M(\omega), N(\omega)$; and $y(x,\omega), m(x,\omega), n(x,\omega), \eta^*(x,\omega)$

$$C(\omega) = Y(\omega) - M(\omega)$$

$$\alpha z x n(x,\omega)^{\alpha-1} m(x,\omega)^{\beta} = w(x,\omega)$$

$$\beta z x n(x,\omega)^{\alpha} m(x,\omega)^{\beta-1} = p(x,\omega)$$

$$y(x,\omega) = z x n(x,\omega)^{\alpha} m(x,\omega)^{\beta}$$

$$\begin{aligned} GO(\omega) &= \int_0^\infty y(x,\omega) d\mu(x) \\ M(\omega) &= \int_0^\infty m(x,\omega) d\mu(x) \\ N(\omega) &= \int_0^\infty n(x,\omega) d\mu(x) \end{aligned}$$

$$\frac{U_2(C(\omega), N(\omega))}{U_1(C(\omega), N(\omega))} = \overline{w}(\omega)$$

$$\eta^*(x,\omega) = \pi(x,\omega) - b + E_{\omega'}\left(M(\omega,\omega')V(x',\omega')\right)$$

$$V(x,\omega) = H(\eta^*(x,\omega))\eta^*(x,\omega) - R(\eta^*(x,\omega))$$

$$R(x) = \int_{-\infty}^{x} \eta dH(\eta)$$

We can simplify this and obtain the input factor demand and output supply for each firm:

$$n(x,\omega) = \left(\frac{\alpha}{w(x,\omega)}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{p(x,\omega)}\right)^{\frac{\beta}{1-\alpha-\beta}} (zx)^{\frac{1}{1-\alpha-\beta}},$$

$$m(x,\omega) = (zx)^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{w(x,\omega)}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{p(x,\omega)}\right)^{\frac{1-\alpha}{1-\alpha-\beta}},$$

$$y(x,\omega) = (zx)^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{w(x,\omega)}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{p(x,\omega)}\right)^{\frac{\beta}{1-\alpha-\beta}}.$$

We next define $\kappa_y = (\alpha)^{\frac{\alpha}{1-\alpha-\beta}} (\beta)^{\frac{\beta}{1-\alpha-\beta}}$, $\kappa_m = (\alpha)^{\frac{\alpha}{1-\alpha-\beta}} (\beta)^{\frac{1-\alpha}{1-\alpha-\beta}}$ and $\kappa_n = (\alpha)^{\frac{1-\beta}{1-\alpha-\beta}} (\beta)^{\frac{\beta}{1-\alpha-\beta}}$. Note that

$$p(x,\omega) = \frac{1}{1-\theta_p PD(x,\omega)}$$

$$w(x,\omega) = \frac{\overline{w}(z)}{1-\theta_w PD(x,\omega)}$$

We obtain formulas for aggregate gross output, inputs and labor:

$$GO(\omega) = \kappa_y z^{\frac{1}{1-\alpha-\beta}} \overline{w}(\omega)^{-\frac{\alpha}{1-\alpha-\beta}} \int_0^\infty x^{\frac{1}{1-\alpha-\beta}} \left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}} d\mu(x),$$

$$M(\omega) = \kappa_m z^{\frac{1}{1-\alpha-\beta}} \overline{w}(\omega)^{-\frac{\alpha}{1-\alpha-\beta}} \int_0^\infty x^{\frac{1}{1-\alpha-\beta}} \left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(1-\theta_p PD(x,\omega)\right)^{\frac{1-\alpha}{1-\alpha-\beta}} d\mu(x),$$

$$N(\omega) = \kappa_n z^{\frac{1}{1-\alpha-\beta}} \overline{w}(\omega)^{-\frac{1-\beta}{1-\alpha-\beta}} \int_0^\infty x^{\frac{1}{1-\alpha-\beta}} \left(1-\theta_w PD(x,\omega)\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}} d\mu(x).$$

We can define the measured intermediate inputs (using the prices):

$$PM(\omega) = \int_0^\infty p(x,\omega) m(x,\omega) d\mu(x),$$

and hence GDP is defined as

$$\begin{aligned} GDP(\omega) &= GO(\omega) - PM(\omega) \\ &= z^{\frac{1}{1-\alpha-\beta}} \overline{w}(\omega)^{-\frac{\alpha}{1-\alpha-\beta}} (\kappa_y - \kappa_m) \int_0^\infty x^{\frac{1}{1-\alpha-\beta}} \left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}} d\mu(x). \end{aligned}$$

8.3 Algebra with Epstein-Zin preferences

Suppose preferences are now given by

$$W = \left(u(C,N)^{1-\sigma} + \beta E \left(W\prime^{1-\gamma} \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{1-\gamma},$$

Here N is total employment, and u is the felicity function of the big family. For instance, $u(C,N) = C^{\upsilon}(1-N)^{1-\upsilon}$. The parameter σ is related to the IES while the parameter γ reflects risk aversion over static gambles.

The real risk-free wage is given by the usual formula,

$$\overline{w}(\omega) = \frac{u_2(C(\omega),N(\omega))}{u_1(C(\omega),N(\omega))}.$$

For instance, if $u(C,N) = C^{\upsilon}(1-N)^{1-\upsilon}$, then

$$\overline{w}(\omega) = \frac{1-\upsilon}{\upsilon} \frac{C(\omega)}{1-N(\omega)}.$$

The SDF is

$$M(\omega,\omega') = \beta \frac{U_1(C(\omega'),N(\omega'))}{U_1(C(\omega),N(\omega))} \frac{U(C(\omega'),N(\omega'))^{-\sigma}}{U(C(\omega),N(\omega))^{-\sigma}} \frac{W(\omega)^{\sigma-\gamma}}{E\left(W(\omega')^{1-\gamma}\right)^{\frac{\sigma-\gamma}{1-\gamma}}}.$$

The other equations of the model are unchanged. The only added complexity is that the value $W(\omega)$ needs to be found as part of the solution.

8.4 Calculations of Wedges

The labor wedge is defined as:

$$1-\tau(\omega) = \frac{U_2(C,N)/U_1(C,N)}{\frac{\alpha}{1-\beta}\frac{GDP}{N}}.$$

Plugging in the expressions for N and GDP, we obtain:

$$1-\tau(\omega)=\frac{\overline{w}(\omega)}{\frac{\alpha}{1-\beta}}\frac{\kappa_n z^{\frac{1}{1-\alpha-\beta}}\overline{w}(\omega)^{-\frac{1-\beta}{1-\alpha-\beta}}\int_0^\infty x^{\frac{1}{1-\alpha-\beta}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{1-\beta}{1-\alpha-\beta}}\left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}}d\mu(x)}{z^{\frac{1}{1-\alpha-\beta}}\overline{w}(\omega)^{-\frac{\alpha}{1-\alpha-\beta}}\left(\kappa_y-\kappa_m\right)\int x^{\frac{1}{1-\alpha-\beta}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}}d\mu(x)},$$

which simplifies to:

$$\begin{aligned}1-\tau(\omega) &= \frac{1-\beta}{\alpha}\frac{\kappa_n}{\kappa_y-\kappa_m}\frac{\int_0^\infty x^{\frac{1}{1-\alpha-\beta}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{1-\beta}{1-\alpha-\beta}}\left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}}d\mu(x)}{\int x^{\frac{1}{1-\alpha-\beta}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}}d\mu(x)}, \\ &= \frac{\int_0^\infty x^{\frac{1}{1-\alpha-\beta}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{1-\beta}{1-\alpha-\beta}}\left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}}d\mu(x)}{\int x^{\frac{1}{1-\alpha-\beta}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}}d\mu(x)}.\end{aligned}$$

Note that we used

$$\frac{\kappa_n}{\kappa_y}=\frac{(\alpha)^{\frac{1-\beta}{1-\alpha-\beta}}(\beta)^{\frac{\beta}{1-\alpha-\beta}}}{(\alpha)^{\frac{\alpha}{1-\alpha-\beta}}(\beta)^{\frac{\beta}{1-\alpha-\beta}}}=\frac{(\alpha)^{\frac{1-\beta}{1-\alpha-\beta}}}{(\alpha)^{\frac{\alpha}{1-\alpha-\beta}}}=\alpha,$$

and

$$\begin{aligned}\frac{\kappa_y-\kappa_m}{\kappa_n^{\frac{\alpha}{1-\beta}}} &= \frac{(\alpha)^{\frac{\alpha}{1-\alpha-\beta}}(\beta)^{\frac{\beta}{1-\alpha-\beta}}-(\alpha)^{\frac{\alpha}{1-\alpha-\beta}}(\beta)^{\frac{1-\alpha}{1-\alpha-\beta}}}{(\alpha)^{\frac{\alpha}{1-\beta}\frac{1-\beta}{1-\alpha-\beta}}(\beta)^{\frac{\alpha}{1-\beta}\frac{\beta}{1-\alpha-\beta}}} \\ &= \frac{(\alpha)^{\frac{\alpha}{1-\alpha-\beta}}\left(\beta^{\frac{\beta}{1-\alpha-\beta}}-\beta^{\frac{1-\alpha}{1-\alpha-\beta}}\right)}{(\alpha)^{\frac{\alpha}{1-\alpha-\beta}}(\beta)^{\frac{\beta}{1-\alpha-\beta}}} \\ &= 1-\beta\end{aligned}$$

and

$$\begin{aligned}\frac{\kappa_y-\kappa_m}{\kappa_n} &= \frac{(\alpha)^{\frac{\alpha}{1-\alpha-\beta}}(\beta)^{\frac{\beta}{1-\alpha-\beta}}-(\alpha)^{\frac{\alpha}{1-\alpha-\beta}}(\beta)^{\frac{1-\alpha}{1-\alpha-\beta}}}{(\alpha)^{\frac{1-\beta}{1-\alpha-\beta}}(\beta)^{\frac{\beta}{1-\alpha-\beta}}} \\ &= \frac{1-\beta}{\alpha}\end{aligned}$$

Turning to measured TFP, we have the definition of

$$\begin{aligned}TFP &= \frac{GDP}{N^{\frac{\alpha}{1-\beta}}} \\ &= \frac{z^{\frac{1}{1-\alpha-\beta}}\overline{w}(\omega)^{-\frac{\alpha}{1-\alpha-\beta}}\left(\kappa_y-\kappa_m\right)\int_0^\infty x^{\frac{1}{1-\alpha-\beta}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}}d\mu(x)}{\left(\kappa_n z^{\frac{1}{1-\alpha-\beta}}\overline{w}(\omega)^{-\frac{1-\beta}{1-\alpha-\beta}}\int_0^\infty x^{\frac{1}{1-\alpha-\beta}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{1-\beta}{1-\alpha-\beta}}\left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}}d\mu(x)\right)^{\frac{\alpha}{1-\beta}}} \\ &= z^{\frac{1}{1-\beta}}(1-\beta)\frac{\int_0^\infty x^{\frac{1}{1-\alpha-\beta}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}}d\mu(x)}{\left(\int_0^\infty x^{\frac{1}{1-\alpha-\beta}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{1-\beta}{1-\alpha-\beta}}\left(1-\theta_p PD(x,\omega)\right)^{\frac{\beta}{1-\alpha-\beta}}d\mu(x)\right)^{\frac{\alpha}{1-\beta}}}\end{aligned}$$

8.5 Quantity-dependent schedules

This section demonstrates that the equilibrium where firms take as a given a wage and input price schedule has the same first-order conditions as the equilibrium studied in the paper. To simplify the notation we let $s=(\omega,x)$. The equilibrium is now characterized by:

$$p(n,m;s)(1-\theta_p PD(\eta^*(n,m;s))=1 \tag{3}$$

$$w(n,m;s)(1-\theta_w PD(\eta^*(n,m;s)))=\overline{w}(s) \tag{4}$$

Denote by J the future value if the firm continues. The equity value is:

$$V(s) = \max_{n,m} \int_{-\infty}^{\eta^*(n,m;s)} \big(zxn^\alpha m^\beta - w(n,m;s)n - p(n,m;s)m - \eta - b + J\big)\, dH(\eta),$$

and the default threshold is:

$$\eta^*(n,m;s) = zxn^\alpha m^\beta - w(n,m;s)n - p(n,m;s)m - b + J. \tag{5}$$

From now on forget that s is an argument. The first-order conditions are given by:

$$\int_{-\infty}^{\eta^*} \left(\alpha zxn^{\alpha-1}m^\beta - w - n\frac{\partial w}{\partial n} - m\frac{\partial p}{\partial n}\right) dH(\eta) = 0,$$

$$\int_{0}^{\eta^*} \left(\beta zxn^{\alpha}m^{\beta-1} - p - m\frac{\partial p}{\partial m} - n\frac{\partial w}{\partial m}\right) dH(\eta) = 0.$$

Since the integrand is independent of η, we can immediately simplify these to

$$H(\eta^*)\left(\alpha zxn^{\alpha-1}m^\beta - w - n\frac{\partial w}{\partial n} - m\frac{\partial p}{\partial n}\right) = 0, \tag{6}$$

$$H(\eta^*)\left(\beta zxn^{\alpha}m^{\beta-1} - p - m\frac{\partial p}{\partial m} - n\frac{\partial w}{\partial m}\right) = 0. \tag{7}$$

The goal is now to obtain expressions for $\frac{\partial w}{\partial n}, \frac{\partial w}{\partial m}, \frac{\partial p}{\partial n}, \frac{\partial p}{\partial m}$ so as to simplify these equations. First, differentiate the pricing equations 3-4 with respect to n and m, and obtain:

$$\frac{\partial p}{\partial m}(1 - \theta_m P(\eta^*)) - \theta_m P'(\eta^*)p\frac{\partial \eta^*}{\partial m} = 0,$$

$$\frac{\partial p}{\partial n}(1 - \theta_m P(\eta^*)) - \theta_m P'(\eta^*)p\frac{\partial \eta^*}{\partial n} = 0,$$

$$\frac{\partial w}{\partial m}(1 - \theta_w P(\eta^*)) - \theta_w P'(\eta^*)w\frac{\partial \eta^*}{\partial m} = 0,$$

$$\frac{\partial w}{\partial n}(1 - \theta_w P(\eta^*)) - \theta_w P'(\eta^*)w\frac{\partial \eta^*}{\partial n} = 0,$$

where $P(\eta)$ is the default probability. This implies that

$$\begin{aligned}
\frac{\partial p}{\partial m} &= \frac{\theta_m P'(\eta^*)p}{1 - \theta_m P(\eta^*)}\frac{\partial \eta^*}{\partial m}, \\
\frac{\partial w}{\partial m} &= \frac{\theta_w P'(\eta^*)w}{1 - \theta_w P(\eta^*)}\frac{\partial \eta^*}{\partial m}, \\
\frac{\partial p}{\partial n} &= \frac{\theta_m P'(\eta^*)p}{1 - \theta_m P(\eta^*)}\frac{\partial \eta^*}{\partial n}, \\
\frac{\partial w}{\partial n} &= \frac{\theta_w P'(\eta^*)w}{1 - \theta_w P(\eta^*)}\frac{\partial \eta^*}{\partial n}.
\end{aligned}$$

Second, differentiate the default cutoff equation 5 with respect to n and m and obtain:

$$\frac{\partial \eta^*}{\partial n} = \alpha zxn^{\alpha-1}m^\beta - n - n\frac{\partial w}{\partial n} - m\frac{\partial p}{\partial n},$$

$$\frac{\partial \eta^*}{\partial m} = \beta zxn^{\alpha}m^{\beta-1} - n\frac{\partial w}{\partial m} - m\frac{\partial p}{\partial m} - p,$$

so that:

$$\frac{\partial \eta^*}{\partial m} = \big(\beta zxn^{\alpha}m^{\beta-1} - p\big) - \left(n\frac{\theta_w P'(\eta^*)w}{1 - \theta_w P(\eta^*)} + m\frac{\theta_m P'(\eta^*)p}{1 - \theta_m P(\eta^*)}\right)\frac{\partial \eta^*}{\partial m}$$

and hence:

$$\frac{\partial \eta^*}{\partial m} = \frac{\beta z x n^{\alpha} m^{\beta-1} - p}{1 + n\frac{\theta_w P'(\eta^*) w}{1-\theta_w P(\eta^*)} + m\frac{\theta_m P'(\eta^*) p}{1-\theta_m P(\eta^*)}}.$$

Given this, we can rewrite the FOC 6 and obtain that $\frac{\partial \eta^*}{\partial m} = 0$. As a result, $\frac{\partial p}{\partial m} = \frac{\partial w}{\partial m} = 0$. Similarly, we obtain $\frac{\partial \eta^*}{\partial n} = \frac{\partial p}{\partial n} = \frac{\partial w}{\partial n} = 0$ at the optimum. It follows that the equilibrium equations simplify to the case studied in the main text.