Adaptive Linear Programming Decoding of Polar Codes

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  - However, this LP decoder performs poorly over a BAWGN channel.
- **Contributions**
  - Modified adaptive LP decoder for short blocklength polar codes over a BAWGN channel
  - Polar code sparse factor graph reduction algorithm

Introduction to LP Decoding

J. Feldman, M. J. Wainwright, and D. R. Karger, "Using linear programming to decode binary linear codes," *IEEE Trans. Inform. Theory*, vol. 51, no. 3, pp. 954–972, March 2005.

Linear Programming Decoding

- ML decoding of a binary linear block code $\mathcal{C}$ can be approximated using a LP decoder which solves the relaxed linear programming problem

$$\text{minimize} \quad \boldsymbol{\gamma}^T \mathbf{x} \quad \text{s.t.} \quad \mathbf{A}\mathbf{x} \leq \mathbf{b}, \quad \mathbf{x} \in [0,1]^N$$

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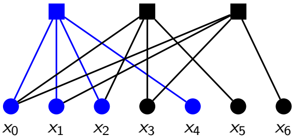
- The fundamental polytope, $\mathcal{Q} = \{\mathbf{x} \in \mathbb{R}^N \mid \mathbf{A}\mathbf{x} \leq \mathbf{b}, \;\; 0 \leq x_i \leq 1\}$

Generation of LP Constraints

- $\mathbf{Ax} \leq \mathbf{b}$ is a set of constraints (linear inequalities) generated by every check j in the parity-check matrix $\mathbf{H}$ of $\mathcal{C}$ as follows

$$\sum_{i \in V} x_i - \sum_{i \in \mathcal{N}(j) \setminus V} x_i \leq |V| - 1; \text{ for all } V \subseteq \mathcal{N}(j) \text{ s.t. } |V| \text{ is odd}$$

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$



$$|V| = 1 : \begin{aligned} x_0 - x_1 - x_2 - x_4 &\leq 0 \\ -x_0 + x_1 - x_2 - x_4 &\leq 0 \\ -x_0 - x_1 + x_2 - x_4 &\leq 0 \\ -x_0 - x_1 - x_2 + x_4 &\leq 0 \end{aligned}$$

$$|V| = 3 : \begin{aligned} x_0 + x_1 + x_2 - x_4 &\leq 2 \\ x_0 + x_1 - x_2 + x_4 &\leq 2 \\ x_0 - x_1 + x_2 + x_4 &\leq 2 \\ -x_0 + x_1 + x_2 + x_4 &\leq 2 \end{aligned}$$

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- Every parity check j provides $2^{|N(j)|-1}$ constraints where $|N(j)|$ is the degree of the parity check.
- Hence to represent $\mathcal{Q}$, the number of constraints needed grows exponentially with the check node degrees.

LP Decoding of Polar Codes

N. Goela, S. Korada, and M. Gastpar, "On LP decoding of polar codes," in *Proc. IEEE Inf. Theory Workshop (ITW)*, August 2010.

Polar Codes

E. Arikan, "Channel Polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," *IEEE Trans. Inform. Theory*, vol. 55, no. 7, pp. 3051–3073, July 2009.

Polar Codes

- Achieve capacity of symmetric binary input discrete memoryless channels under successive cancellation (SC) decoding

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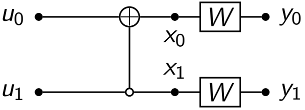

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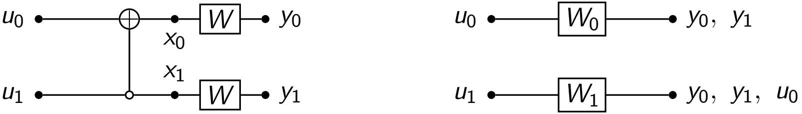

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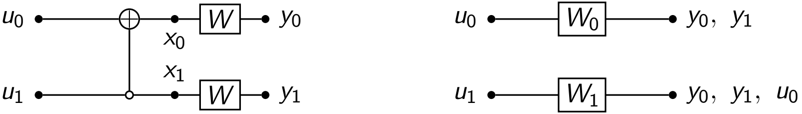

- As number of polarization steps $m \to \infty$, $NC(W)$ of the bit-channels become noiseless and $N(1 - C(W))$ of them become completely noisy. $[N = 2^m]$
- Transmit information only on the noiseless bit-channels and set the inputs to noisy bit-channels to some known value (frozen bits).

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Polar Code Generator and Parity Check Matrix

I. Tal and A. Vardy, "How to construct polar codes", *IEEE Trans. Inform. Theory*, vol. 59, no. 10, pp. 6562–6582, October 2013

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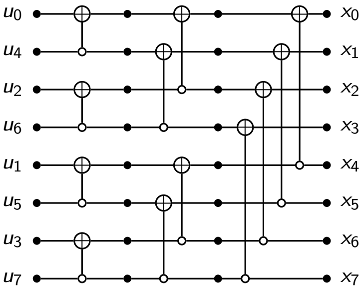

$$G_N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

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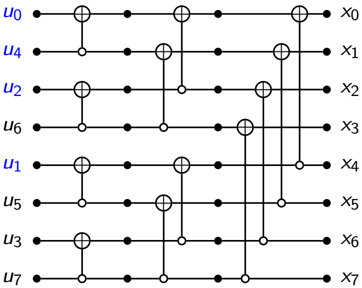

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- We use the Tal-Vardy algorithm to choose the frozen bit indices and set them to 0.

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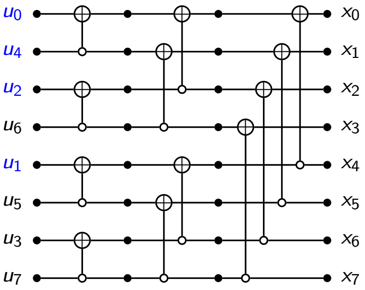

$$G_N = \begin{bmatrix} \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{1} & \cancel{0} & \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ \cancel{1} & \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

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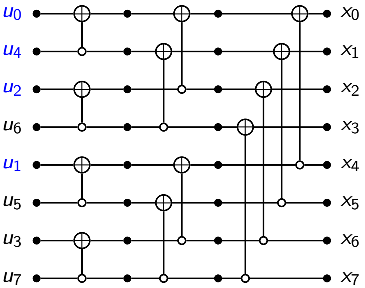

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$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

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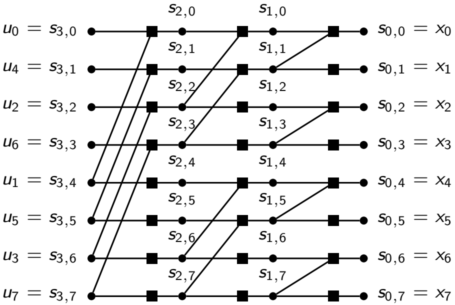
- However, the parity check matrix is high-density and the maximum possible check node degree is N.
- The degree N parity check needs to be represented using 2^{N-1} constraints, which is impractical, even for short blocklengths (e.g., 32, 64).
- It was shown that LP Decoding for a polar code on the polytope $\mathcal{Q}$ fails for all the three channels: $BEC(\epsilon)$, $BSC(p)$, $BAWGNC(\sigma)$.

A sparse representation based LP polytope



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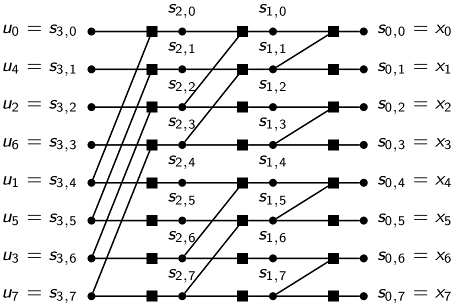

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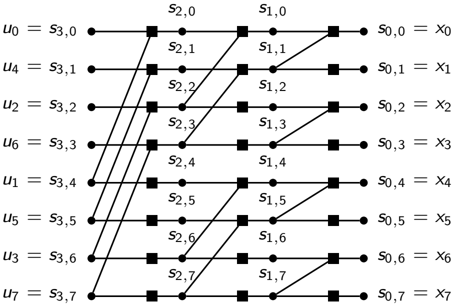

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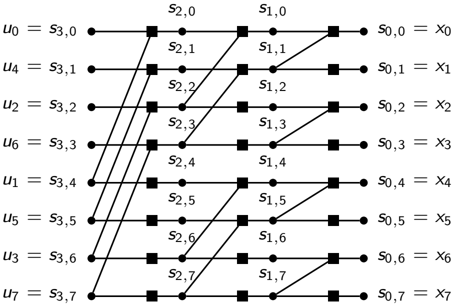

- Polytope $\mathcal{P}$ is defined using local polytopes ($\mathcal{P}_j$) of check nodes

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$$T = \{u_i = 0 \mid u_i \text{ is a frozen bit}\}$$

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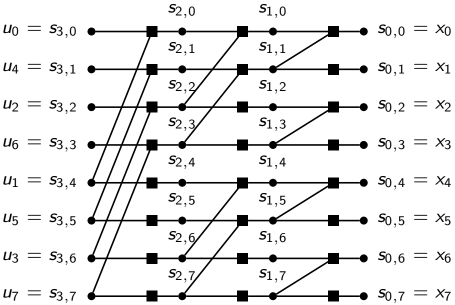

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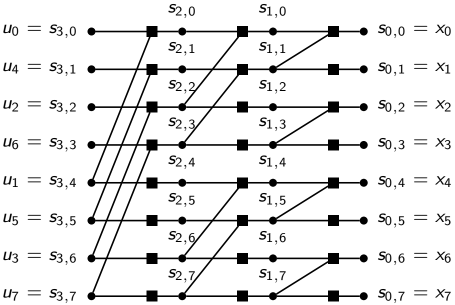

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- For a BEC, LP decoding over $\mathcal{P}$ is equivalent to BP decoding over the sparse factor graph and hence achieves capacity.
- For a BAWGNC, polytope $\mathcal{P}$ is insufficient and hence results in very poor Frame Error Rate (FER) performance.

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Introduction to ALP Decoding

- An adaptive LP decoder solves a sequence of LP decoding problems with the addition of intelligently chosen constraints (cuts) at every iteration.

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- ALP decoder
  - Solve initial LP with constraints

$$x_i \geq 0 \;\text{ if }\; \gamma_i \geq 0; \quad x_i \leq 1 \;\text{ if }\; \gamma_i < 0$$

  - Search all parity checks to find cuts, add cuts to the LP and solve the LP again.
  - Repeat until no cuts found.

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- Adaptive cut generation ALP (ACG-ALP) decoder based on efficient cut-search and cut-inducing RPC generation algorithms.

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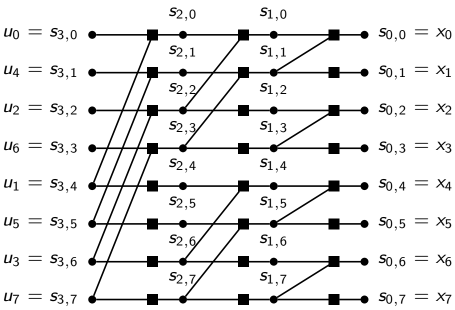
Representations of a Polar Code

Sparse Factor Graph, $\mathbf{H}_{\mathcal{P}}$



Channel Polarization Transform

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Parity-check Matrix, H

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Modified ACG-ALP Decoder for Polar Codes

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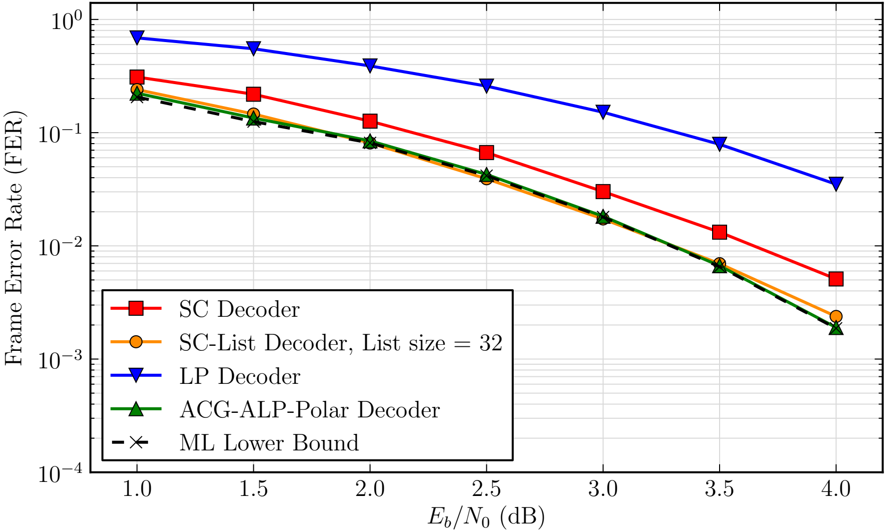
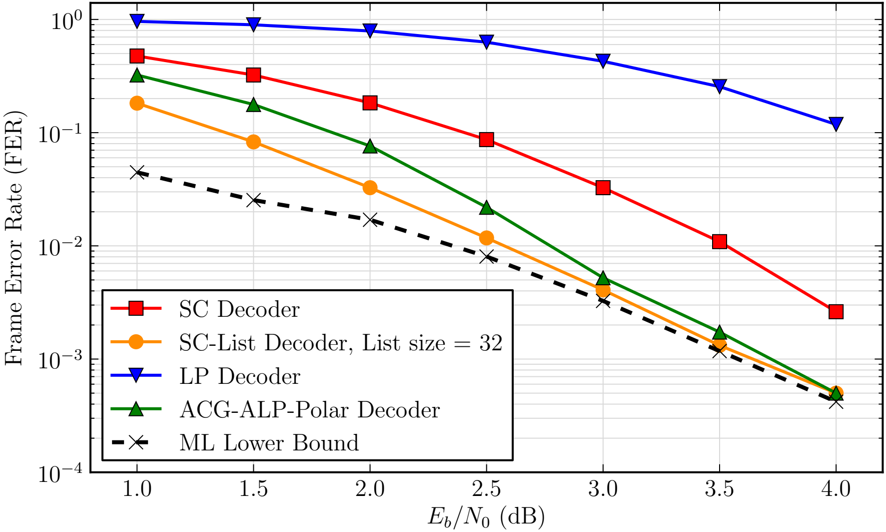
  - Transforming the parity check matrix $\mathbf{H}$ to provide cut-inducing RPCs
  - Apply the cut search algorithm to these RPCs.
- This combination of constraints from $\mathbf{H}_{\mathcal{P}}$ and $\mathbf{H}$ provides the smallest LP decoding time complexity.

Simulation Results

FER Performance of a (64, 32) Polar Code over BAWGNC

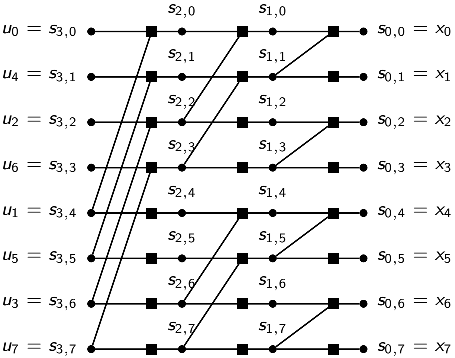


FER Performance of a (128, 64) Polar Code over BAWGNC



Polar Code Sparse Factor Graph Reduction

Polar Code Sparse Factor Graph



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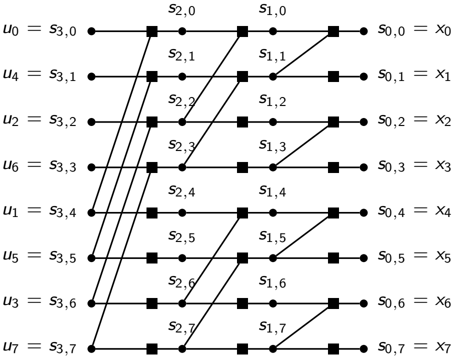

- Exactly half of the check nodes are degree-3 and half are degree-2.

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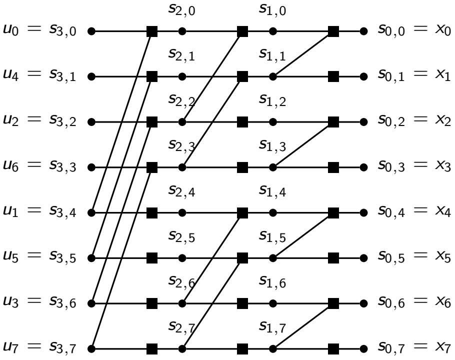

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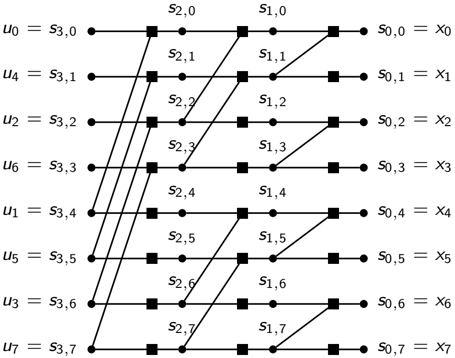
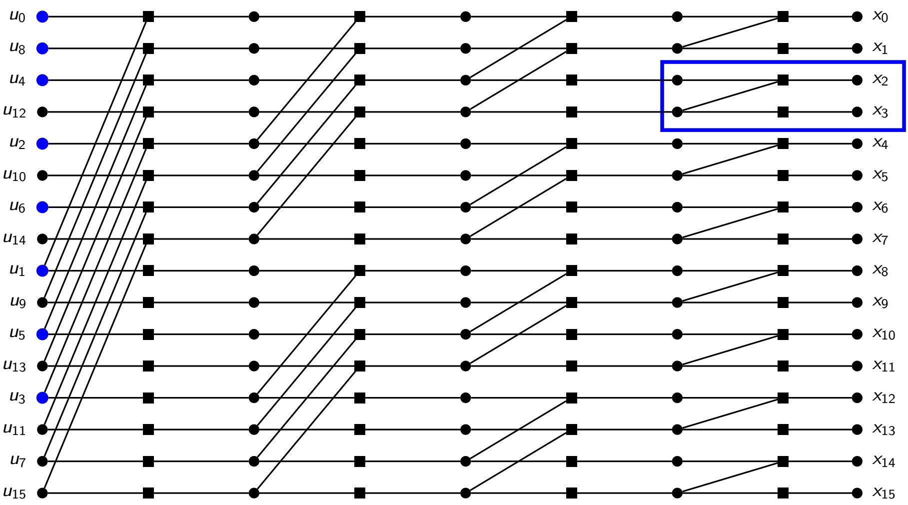


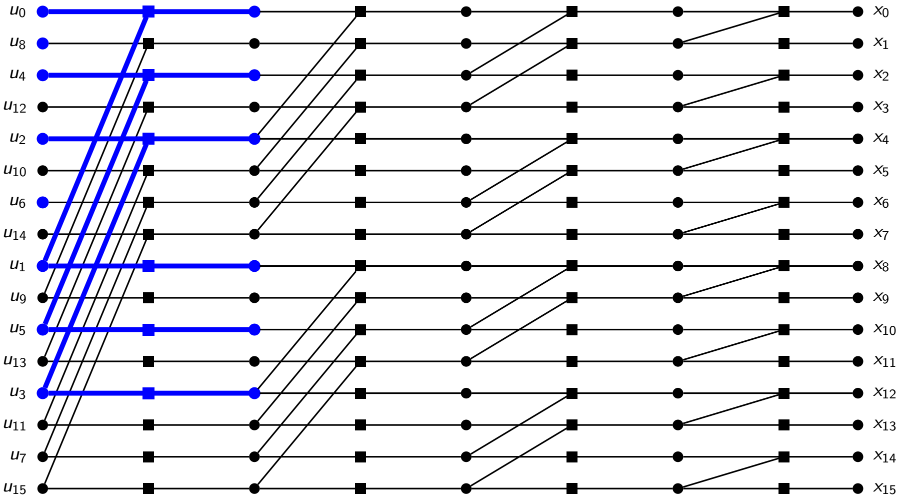
- Exactly half of the check nodes are degree-3 and half are degree-2.
- Degree-2 check nodes lead to redundant variable nodes which may not be required.
- Can we reduce the number of nodes needed to represent a polar code factor graph?

An Example

(16, 8) Polar Code Sparse Factor Graph



Step 1: Propagate Frozen Bit Pairs and Eliminate



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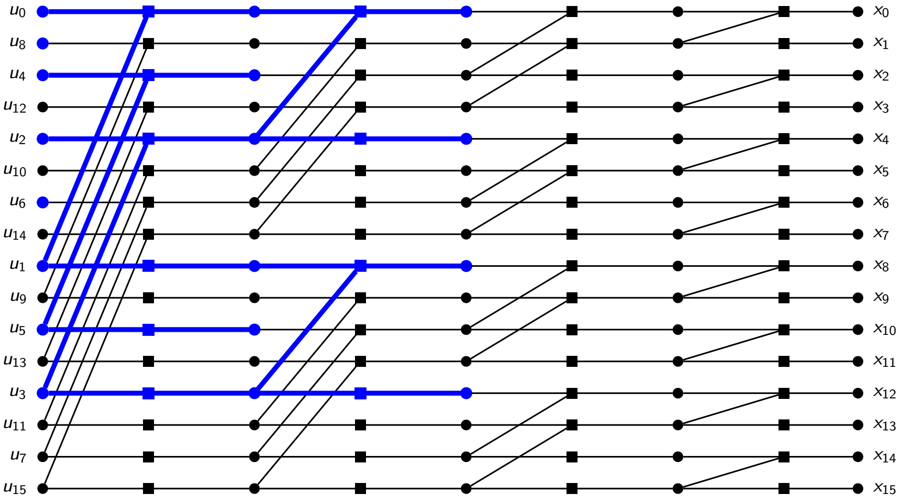

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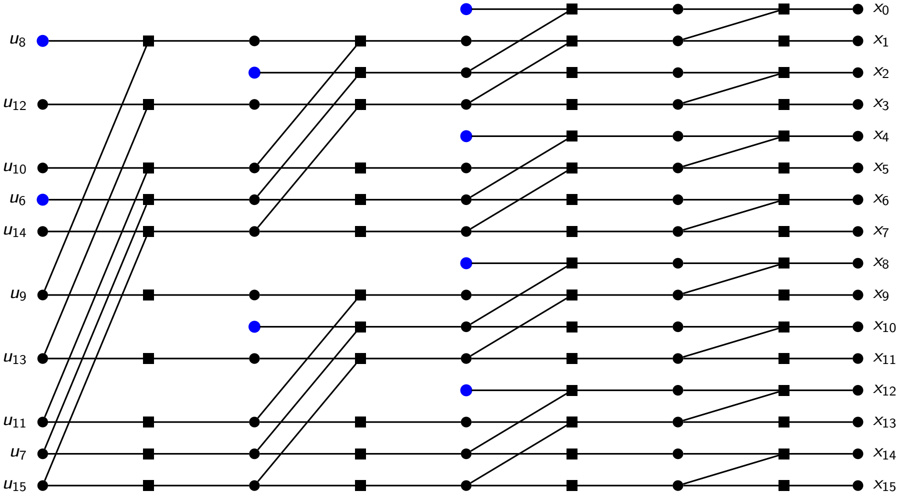
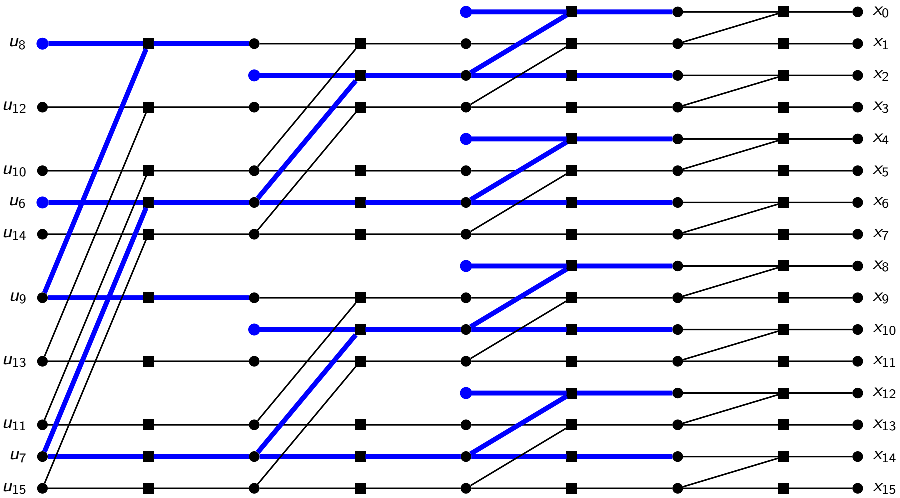


Step 2: Reduce using a Single Frozen Bit



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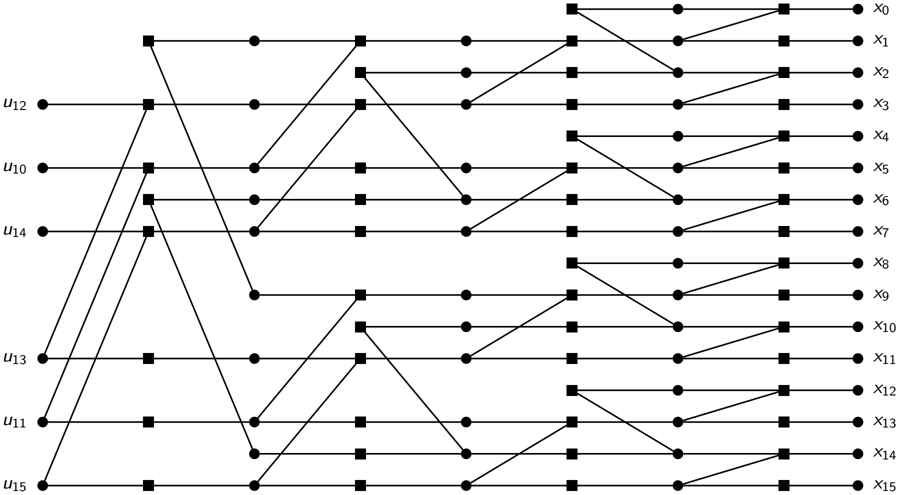
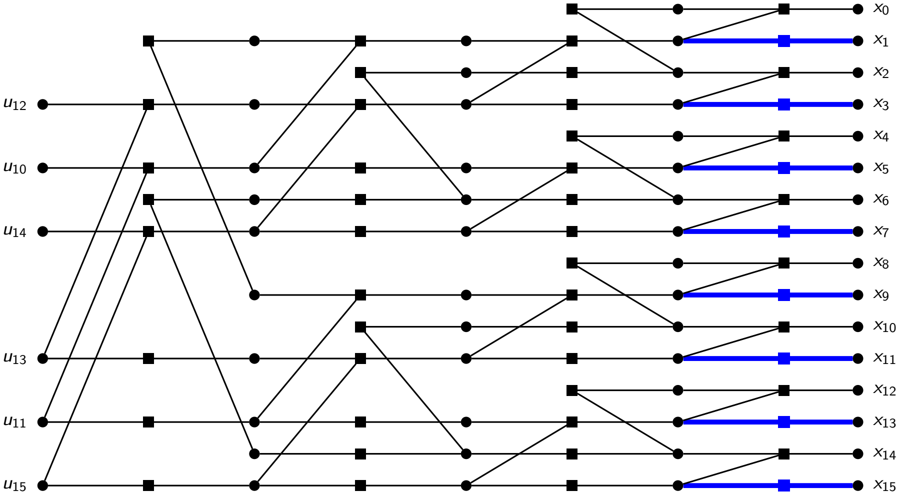


Step 3: Iteratively reduce Degree-2 check nodes



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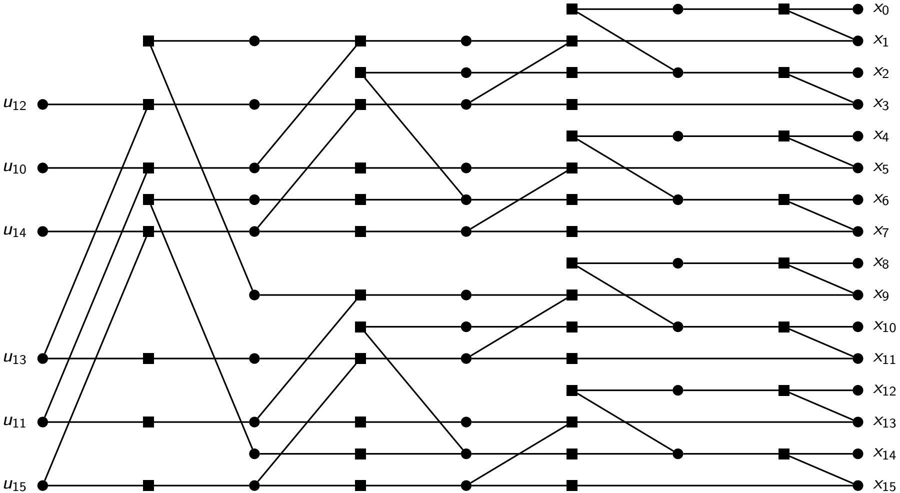

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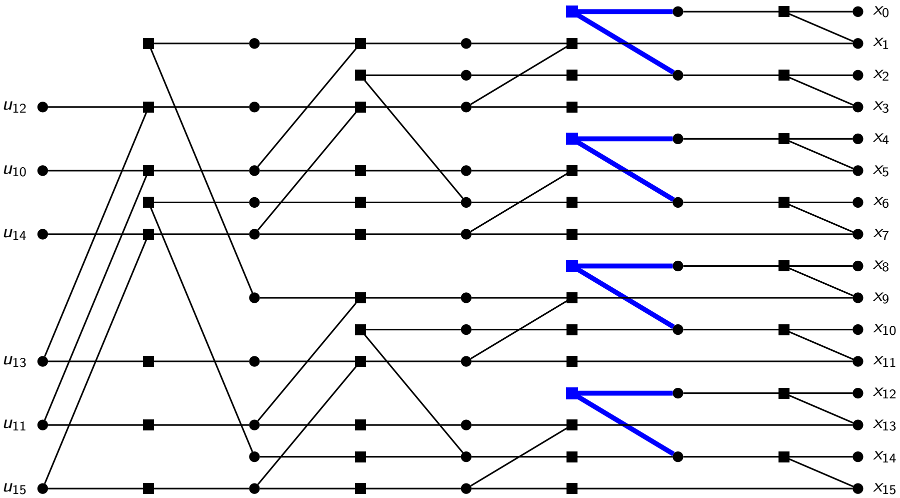

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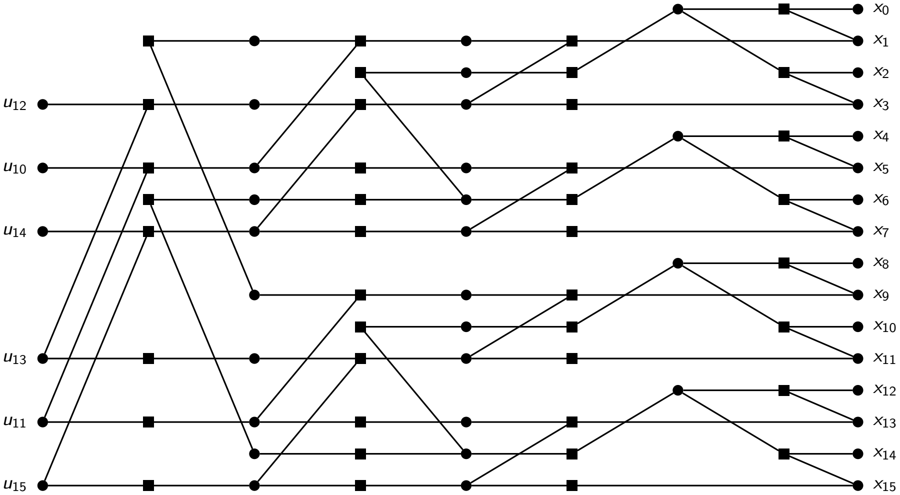

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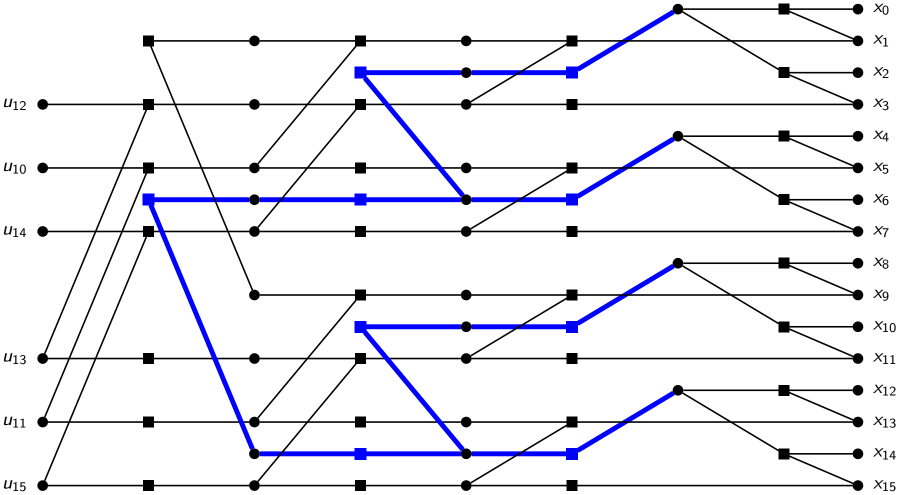

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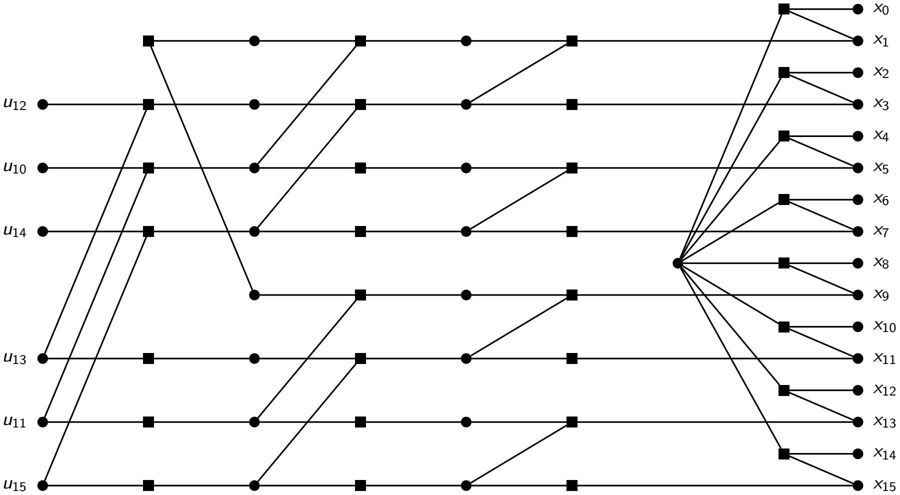

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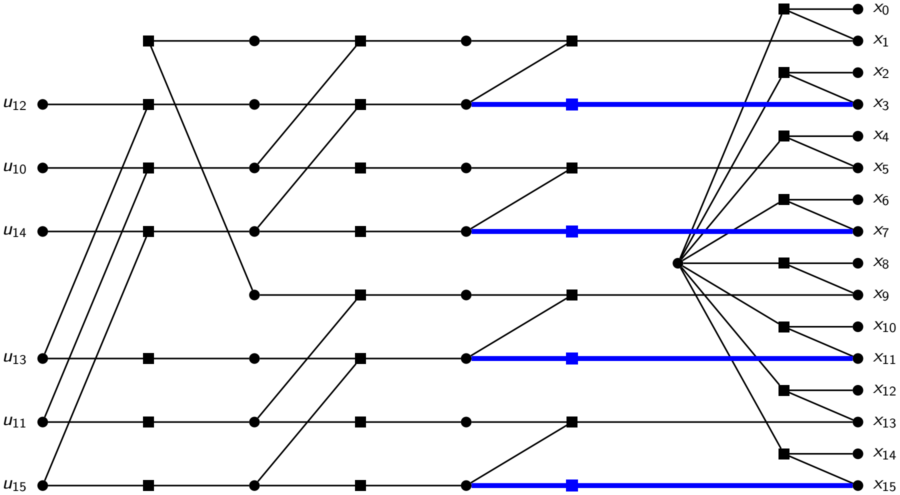

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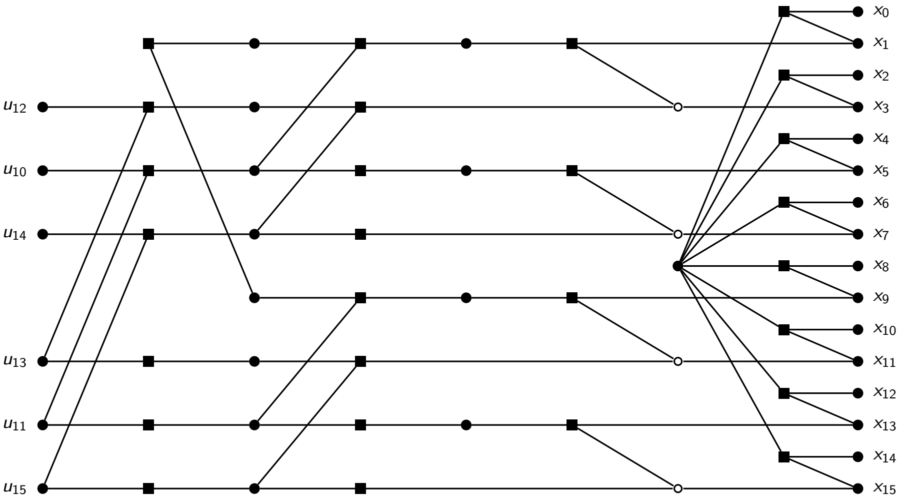

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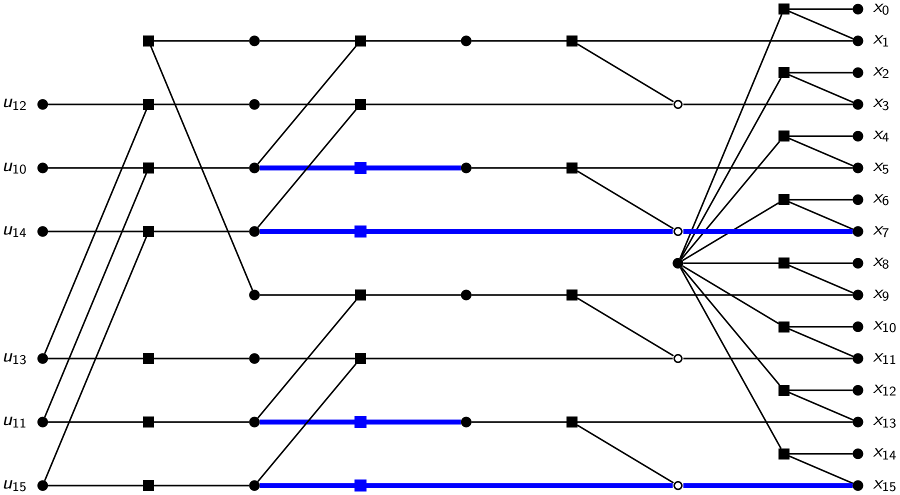

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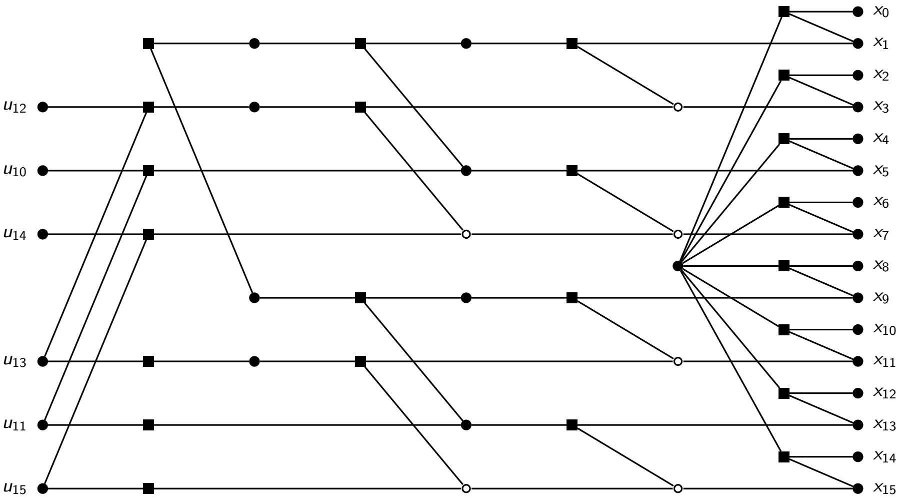

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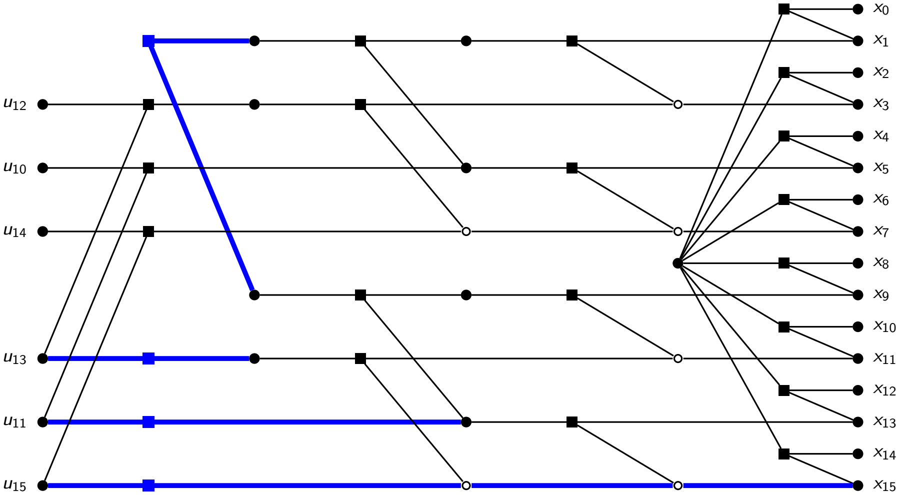

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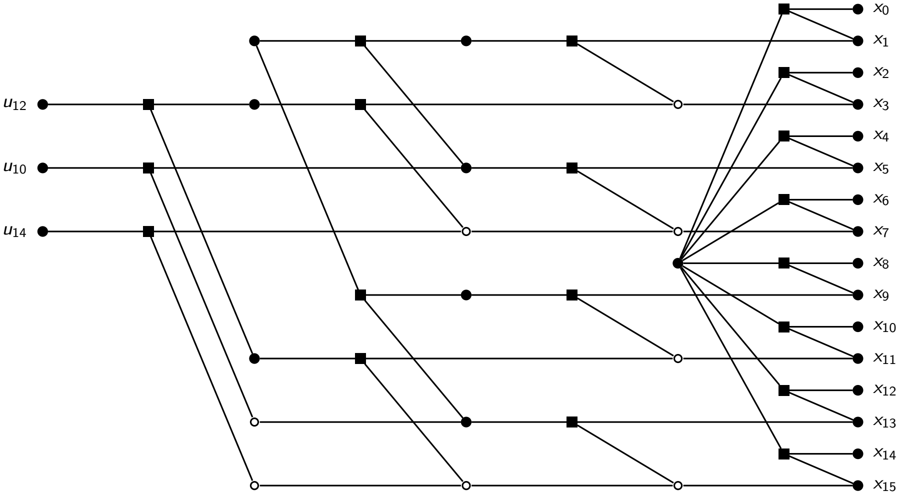
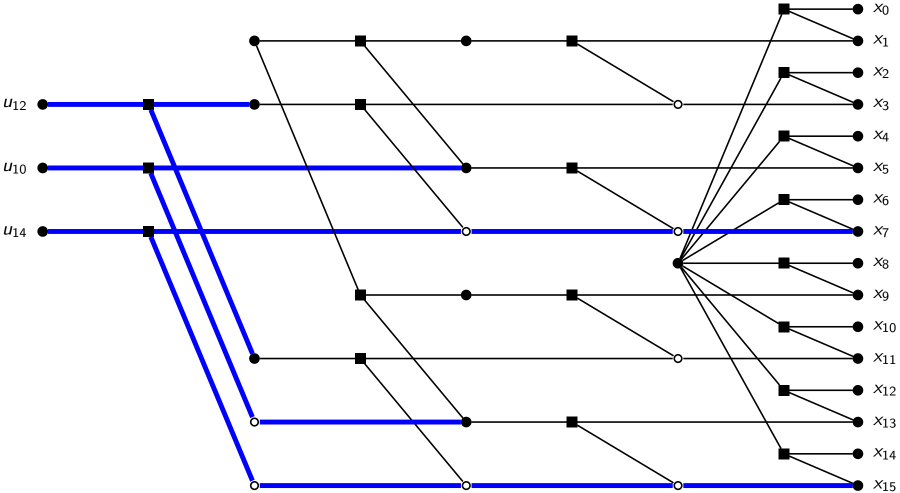


Step 4: Iteratively reduce Degree-1 variable nodes



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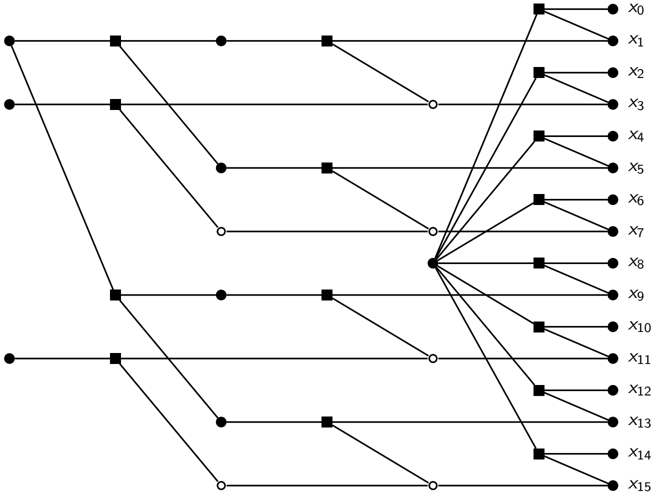

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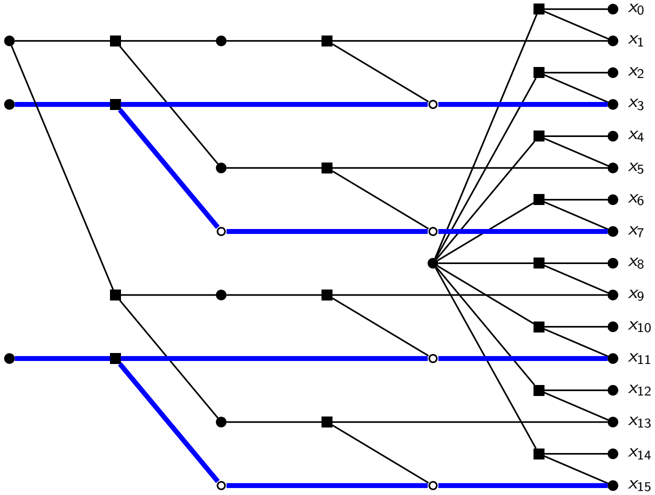
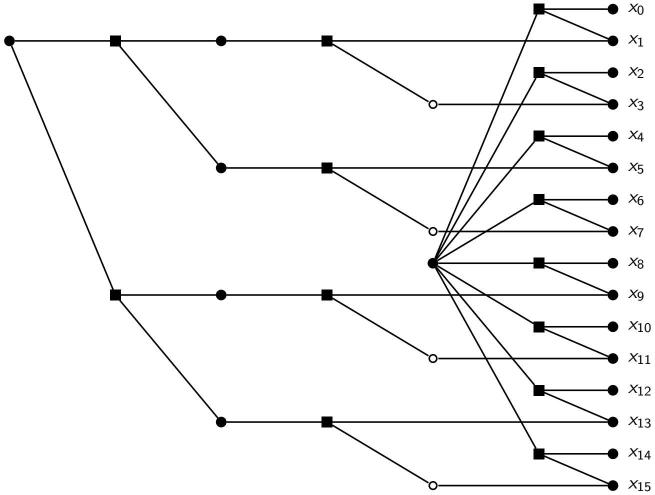


Reduced Factor Graph for a (16, 8) Polar Code



Results for Polar Code Reduced Factor Graph (RFG)

Lemma

A polar code reduced factor graph (RFG) $\mathbf{H}_{\mathcal{R}}$ consists of only degree-3 check nodes.

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A polar code reduced factor graph (RFG) $\mathbf{H}_{\mathcal{R}}$ consists of only degree-3 check nodes.

Theorem

Let $\mathcal{R}$ be the fundamental polytope of the reduced factor graph $\mathbf{H}_{\mathcal{R}}$

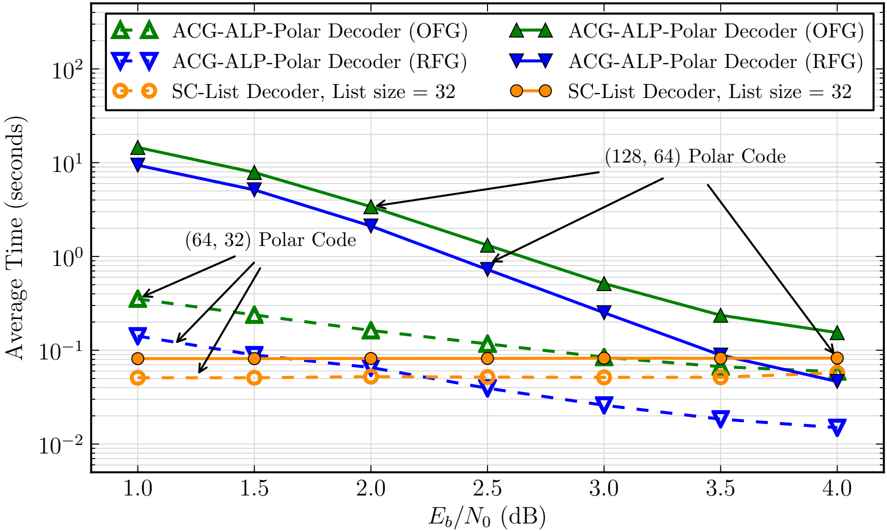
$$\mathcal{R} \subset [0,1]^d, \quad \text{where dimension } d = f(N, r)$$

Let $\mathcal{P}$ be the polytope obtained from the original sparse factor graph and frozen bit information of the polar code.

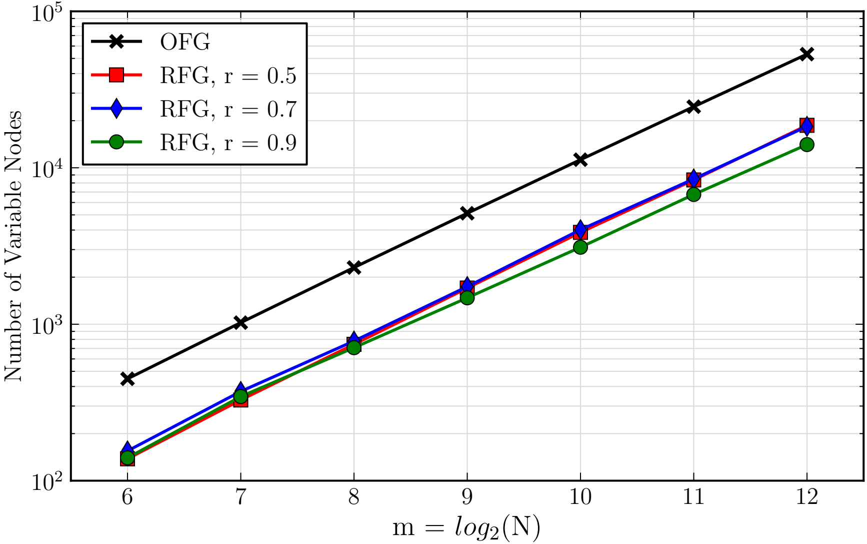
Let $\tilde{\mathcal{P}}$ be the projection of $\mathcal{P}$ on to the d variables in $\mathbf{H}_{\mathcal{R}}$. Then

$$\mathcal{R} = \tilde{\mathcal{P}}$$

Decoding time complexity improvement using RFG



Reduced representation complexity using RFG



Conclusion

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Conclusion

- A modified ACG-ALP decoder for polar codes which provides close to ML performance and is suitable for short blocklength polar codes.
- A polar code sparse factor graph reduction algorithm which provides a reduced complexity representation for the initial LP polytope used in the modified ACG-ALP decoder.
- The reduced factor graph also decreases the decoding time complexity of the ACG-ALP decoder without degrading its error rate performance.

Acknowledgement

- Prof. Paul H. Siegel
- Aman Bhatia
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Thank You!