Quiz 12

Let $X$ and $Y$ be random variables with the following joint pdf.

$$f_{X,Y}(x, y) = \begin{cases} 
  1/2, & x \geq 0, y \geq 0, x + y \leq 1 \\
  3/2, & x \geq 0, y \geq 0, 1 \leq x + y \leq 2 
\end{cases}$$

Find $P(Y \geq X/2)$.

Solution

The region corresponding to the intersection of $Y \geq X/2$ and the support is the triangle $\{(0, 0), (1, 0), (1, 1/2)\}$. This triangle can be divided into two triangles $A$ and $B$ where the pdf over $A$ equals 1/2 and the pdf over $B$ equals 3/2. So

$$P(Y \geq X/2) = |A|(1/2) + |B|(3/2)$$

where $|A|$ is the area of $A$ and $|B|$ is the area of $B$. The point shown by an arrow is obtained by solving the equations

$$\begin{cases} 
  y = x/2, \\
  x + y = 1,
\end{cases}$$

hence the point is $(2/3, 1/3)$. Thus

$$P(Y \geq X/2) = |A|(1/2) + |B|(3/2) = \frac{1}{12} + \frac{1}{8} = \frac{2 + 3}{24} = \frac{5}{24}.$$