Quiz 9

Let $X$ be a random variable with mean $\mu = 12$ and variance $\sigma^2 = 12$. Find the probability of the event $\{X \leq 12 - \sqrt{12}\}$ if

1. $X$ is Normal.
2. $X$ is Uniform.

You may assume $\sqrt{12} = 3.5$.

Solution

1. 

$$P \left( X \leq 12 - \sqrt{12} \right) = P \left( \frac{X - 12}{\sqrt{12}} \leq -1 \right) = \Phi (-1) = Q (1) = 0.1587.$$  

2. This part can be solved with the same method as example 3.6.5. Alternatively, one can define the standard uniform random variable: Let $U \sim \text{Uni} [a, b]$ be the standard uniform RV. We need to find $a$ and $b$. Since $E \hat{U} = \frac{a + b}{2} = 0$, we have $a = -b$. Furthermore, $\text{Var} [\hat{U}] = \frac{(b-a)^2}{12} = 1$. So

$$\frac{(b-a)^2}{12} = \frac{4b^2}{12} = 1 \Rightarrow b = \sqrt{3}.$$ 

Hence, $\hat{U} \sim \text{Uni} [-\sqrt{3}, \sqrt{3}]$. Now

$$P \left( X \leq 12 - \sqrt{12} \right) = P \left( \frac{X - 12}{\sqrt{12}} \leq -1 \right) = P \left( \hat{U} \leq -1 \right) = \frac{-1 - (-\sqrt{3})}{2\sqrt{3}}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{3}} = \frac{\sqrt{12} - 2}{2\sqrt{12}} = \frac{1.5}{7} = 0.21.$$