Quiz 5

\[ X \sim Bin(n, p), \text{ } p \text{ is given and } n \text{ is unknown. Find the ML estimate of } n \text{ if } X \text{ is observed.} \]

\[ p_x(X) = \binom{n}{X} p^X (1-p)^{n-X} \]

\[ \frac{\binom{n}{X} p^X (1-p)^{n-X}}{\binom{n-1}{X} p^X (1-p)^{n-1-X}} = \frac{n}{n-X} (1-p) \geq 1 \Rightarrow n(1-p) \geq n-X \Rightarrow np \leq X \Rightarrow n \leq \frac{X}{p} \]

\[ \frac{X}{p} \text{ is not an integer: Max } n \text{ occurs at } n = \left\lfloor \frac{X}{p} \right\rfloor \]

\[ \frac{X}{p} \text{ is an integer: Max } n \text{ occurs at } n = \frac{X}{p} \text{ or } n = \frac{X}{p} - 1 \]