Beliefs, Sentiments and the Cross Section of Equity Returns

Manuel Macera

Colorado State University

Motivation

Behavioral finance rationalizes a variety of stock markets events, such as bubbles and crashes, based on two assumptions: sentiment and limits to arbitrage.

Financial decisions within the firm can also be driven by sentiment:

- Many corporate decisions are made under incomplete information.
- Firms act based on their beliefs regarding the world they live in.
- Firms' capital structure and valuation depend on those decisions.

What are the consequences of assuming that decisions at the firm level are made by sentiment-driven agents?

Model of Bayesian decision making:

- Sentiment is a subjective belief about the state of the world.
- It does not need to reflect the true state of the world.
- It is rather justified by individual histories.

Sentiment offers a new perspective to understand stylized facts regarding the behavior of equity returns .

Motivation

Two main findings:

1. Leverage and Equity Returns:
   - The presence of growth options associated with bayesian uncertainty alters the link between equity returns and leverage.
   - Controlling for these growth options strengthens the positive relationship between these two variables.
2. Sentiment shifts must affect mostly small firms (sentiment seesaw)
   - Hard-to-arbitrage stocks are overvalued in high sentiment states and undervalued in low sentiment states.
   - The effect of sentiment shifts is larger for firms with larger growth options.

Related Literature

Recent literature shows that modeling firm behavior regarding financial decisions can take us a long way in rationalizing stylized facts in finance.

- Gomes and Schmid (JF 2010): the presence of growth options alters the link between leverage and equity returns. Other papers in the same spirit include Chen (JF 2010), Garlappi and Yan (JF 2011) and Obreja (RFS 2013).

Bayesian uncertainty can also generate real option effects.

- Grenadier and Malenko (JF 2010): introduce Bayesian uncertainty regarding the permanence of shocks into a real option model of valuation. They focus on the option value to learn about the nature of past shocks.

Plan

1. Model Setup.
2. Analytical Results.
3. Simulation.
4. Numerical Results.

Model

A risk neutral firm faces the following investment problem:

- By paying the investment cost I the firm implements stage i and obtains the perpetual cash flow $R_i X(t)$.
- The firm is free to invest in at any time, but investment must be sequential, e.g. stage i must be implemented before stage $i+1$.
- The evolution of $X(t)$ consists of two components: a geometric Brownian motion with drift and a jump process that affects cash flows negatively.

Formally, the evolution of $X(t)$ is given by:

$$dX(t) = \mu X(t) + \sigma X(t) dW(t) - \varphi X(t) dN(t) \tag{1}$$

where:

1. $dW(t)$ is the increment of the Wiener process.
2. $dN(t)$ is a counting process that corresponds to the negative shocks.

The intensity of the negative shocks is denoted by $\lambda \in \{\lambda_0, \lambda_1\}$ with $\lambda_0 < \lambda_1$.

Firms observe X_t and N_t, but cannot distinguish if $\lambda = \lambda_0$ or $\lambda = \lambda_1$. They attach probability $p(t)$ that the intensity of the jump process is actually λ_0.

As time evolves, firms update this probability in a Bayesian fashion:

- While there is no shock the belief that the intensity is low will appreciate continuously according to:

$$dp(t) = \Delta\lambda(1 - p(t))p(t) \tag{2}$$

- When a shock occurs, the belief depreciates discretely to:

$$\hat{p}(t) = \frac{\lambda_0}{\lambda(p(t))} p(t) \tag{3}$$

where $\Delta\lambda = \lambda_1 - \lambda_0$ and $\lambda(p) = p\lambda_0 + (1 - p)\lambda_1$

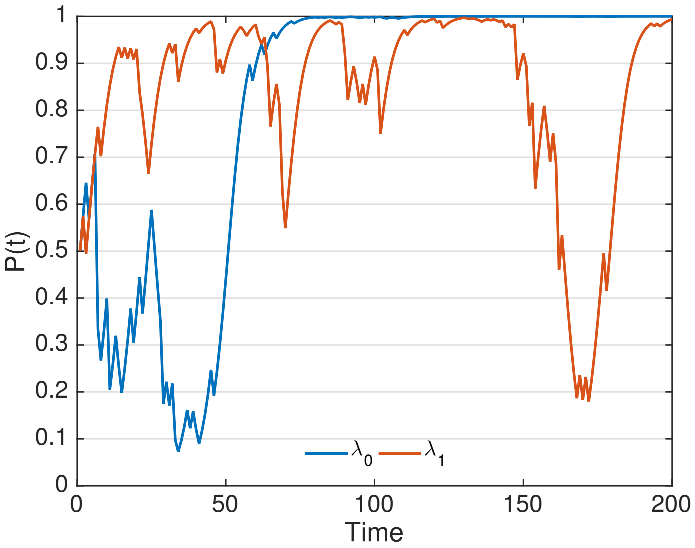

Figure 1: Sample Paths for the Evolution of Beliefs

I assume there is an upper bound N on the number of stages that can be implemented.

Consider a firm that has built $N-1$ stages already. Under full information, stage N would add to the value of the firm:

$$S_N(X) = \mathbb{E}\left[\int_0^\infty (R_N - R_{N-1})X(t)dt \mid X(0) = X\right] \quad (4)$$

which corresponds to the net present value of future cashflows. This expression can be written as follows:

$$S_N(X) = \frac{R_N - R_{N-1}}{r + \varphi\lambda - \mu} X \quad (5)$$

Notice that if $\lambda = 0$ or $\varphi = 0$ we obtain the usual NPV expression.

But recall that firm can't distinguish λ. Hence, the present value of adding stage N to the firm at the belief level p is given by:

$$S_N(p, X) = p\frac{R_N - R_{N-1}}{r + \varphi\lambda_0 - \mu}X + (1-p)\frac{R_N - R_{N-1}}{r + \varphi\lambda_1 - \mu}X \tag{6}$$

In making the investment decision, the firm holds an option to learn about the true nature of the shock. I denote this option value by $G_N(p)$ and proceed to characterize it.

The value of a firm that has already implemented $N-1$ stages and has the option to invest in the N^{th} stage is given by:

$$V_{N-1}(p) = \underbrace{\sum_{i=1}^{N-1} S_i(p)}_{\text{PV Term}} + \overbrace{G_N(p)}^{\text{Option Value Term}}$$

I can write:

$$\sum_{i=1}^{N-1} S_i(p) = (p\Lambda_0 + (1-p)\Lambda_1)R_{N-1}X$$

where $\Lambda_i = \dfrac{1}{r + \varphi\lambda_i - \mu}$.

Ito's Lemma implies that while the option to invest is not exercised, $G_N(p)$ must satisfy:

$$rG_N(p) = G_{N,p}\frac{dp}{dt} + \lambda(p)\left[G_N(\hat{p}) - G_N(p)\right] \tag{7}$$

The solution to this differential equation is given by:

$$G_N(p) = B_N(1-p)\left(\frac{1-p}{p}\right)^{\mu_N} \tag{8}$$

where B_N and μ_N are constants that I need to solve for.

Investment will occur when the agent is sufficiently sure that $\lambda = \lambda_0$. In other words, there is a cutoff belief $\overline{p}_N$ that triggers investment. At $\overline{p}_N$ I must have:

$$G_N(\overline{p}_N) = S_N(\overline{p}_N, X) - I \qquad \text{and} \qquad G_{N,p}(\overline{p}_N) = S_{N,p}(\overline{p}_N)$$

The first condition imposes indifference at $\overline{p}_N$. The second condition requires optimality in the choice of $\overline{p}_N$. I can use the first condition to pin down B_N:

$$B_N = \frac{S_N(\overline{p}_N, X) - I}{\Omega_N(\overline{p}_N)}$$

where $\Omega_N(p) \equiv (1 - p)\left(\frac{1-p}{p}\right)^{\mu_N}$

Proposition (Optimal Investment Decision)

Stage N will be implemented when the subjective belief p reaches the threshold $\overline{p}_N$ given by:

$$\overline{p}_N(X) = \frac{\mu_N(I - \Lambda_1(R_N - R_{N-1})X)}{(\mu_N + 1)(\Lambda_0(R_N - R_{N-1})X - I) + \mu_N(I - \Lambda_1(R_N - R_{N-1})X)}$$

where μ_N solves $r + \lambda_1 + \mu_N \Delta\lambda = \lambda_1 \left(\frac{\lambda_1}{\lambda_0}\right)^{\mu_N}$.

More generally, the value of a firm of *size* i is given by the present value of future cash flows plus the option to invest.

$$V_i(p) = (p\Lambda_0 + (1-p)\Lambda_1)R_iX + G_i(p)$$

I can proceed backwards to characterize the option value of implementing stage $i+1$ to the firm. The appropriate boundary conditions are

$$G_i(\overline{p}_i) = S_i(\overline{p}_i) - I \qquad \text{and} \qquad G_{i,p}(\overline{p}_i) = S_{i,p}(\overline{p}_i)$$

for $i = 1, ..., N-1$.

How does the value of the option to invest depends on p?

Proposition (Monotonicity of Option Values and Probability cutoffs)

The option to invest is increasing and convex in the value of the subjective belief p. Moreover, the optimal probability cutoffs are increasing in size, e.g. $\overline{p}_i < \overline{p}_{i+1}$*, and the constant terms satisfy* $\mu_i = \mu$ *and* $B_i > B_{i+1}$ *for all* i*.*

Introducing Debt:

I assume the following regarding debt:

- It takes the form of a consol bond that pays coupon c_i.
- It can only be issued to finance investment.

The debt schedule $\{c_i\}$ is exogenously given and I choose it such that there is a positive relationship between size and financial leverage.

Shareholders now receive $R_i X_t - c_i$ per period. Value Matching condition will now look like:

$$G_i(\overline{p}) = S_i(\overline{p}) - \frac{c_i}{r} - \left(I - \frac{\Delta c_i}{r} \right)$$

where $\Delta c_i \equiv c_i - c_{i-1}$ denotes the marginal increase in the coupon payment if stage i is implemented.

Effect of beliefs on equity returns

To characterize how the equity value of the firm responds to changes in beliefs, I define the equity β as the elasticity of the equity value of the firm with respect to the state variable:

$$\beta_i^p = 1 + \frac{c_i}{rV_i} - \frac{\overbrace{(\mu+1)}^{<0} B_i \Omega(p)}{(1-p)V_i(p)}$$

$$\beta_i^X = 1 + \frac{c_i}{rV_i} + \frac{I\Omega(p)}{V_i(p)\Omega(\overline{p})}$$

Implications of the Theory

The two main implications of this theory:

1. Growth options are more important for small firms and they affect equity returns positively .

2. Small firms are more sensitive to changes in sentiment: the sensitivity of β_i to p is decreasing in i.

Empirical Strategy

Think of a bunch of firms each with the same problem just described.

- Individual history of shocks will deliver a firm size distribution: a lucky firm turns out to be larger than an unlucky firm.
- A small firm holds larger option values ($B_i > B_{i+1}$).

Simulation

I create an artificial dataset:

- I set values for all the exogenous elements of the model.
- I simulate the model with $N = 2,500$ firms over $T = 500$ periods.
- For each firm I calculate statistics such as size, leverage, BTM, etc.
- I generate portfolios and track their performance over time.

How does the model conform with the data?

Table 1: Average monthly realized returns of portfolios sorted by either book leverage or market leverage.

	Low	**2**	**3**	**4**	**High**
Book Leverage	0.34	0.34	0.35	0.37	0.42
Market Leverage	0.32	0.34	0.36	0.38	0.43

Results: Portfolio Sorts II

Table 2: Average monthly realized returns for portfolios sorted first by size and then by market leverage.

	Market Leverage				
Size	**Low**	**2**	**3**	**4**	**High**
Small	0.35	0.37	0.39	0.41	0.45
2	0.33	0.35	0.37	0.39	0.42
3	0.33	0.34	0.35	0.37	0.42
4	0.32	0.32	0.35	0.37	0.45
Large	0.29	0.30	0.31	0.35	0.46

Results: Portfolio Sorts II

Table 3: Average monthly realized returns for portfolios sorted first by book to market and then by market leverage.

	Market Leverage				
BTM	**Low**	**2**	**3**	**4**	**High**
Low	0.34	0.33	0.34	0.34	0.33
2	0.34	0.34	0.34	0.34	0.34
3	0.36	0.36	0.35	0.36	0.35
4	0.38	0.38	0.38	0.38	0.37
High	0.43	0.43	0.43	0.43	0.42

Results: Portfolio Sorts II

Table 4: Average monthly realized returns for portfolios sorted first by size and then by book leverage.

	Book Leverage				
Size	**Low**	**2**	**3**	**4**	**High**
Small	0.32	0.33	0.34	0.37	0.40
2	0.32	0.34	0.35	0.37	0.42
3	0.33	0.33	0.36	0.37	0.43
4	0.33	0.34	0.36	0.38	0.45
Large	0.33	0.34	0.36	0.39	0.50

Table 5: Average monthly realized returns for portfolios sorted first by book to market and then by book leverage.

	Book Leverage				
BTM	**Low**	**2**	**3**	**4**	**High**
Low	0.33	0.32	0.33	0.33	0.33
2	0.34	0.34	0.34	0.34	0.33
3	0.35	0.36	0.35	0.35	0.35
4	0.38	0.38	0.38	0.38	0.37
High	0.44	0.44	0.44	0.44	0.43

Summary:

1. Equity returns are positively related with leverage. The relationship is stronger when I use market leverage rather than book leverage.
2. After controlling for firm size, the link between leverage and equity returns strengthens across all size portfolios.
3. Controlling for book-to-market completely washes out the relationship between leverage and return.

The model conforms well with the broad patterns of the data.

Baker and Wurgler's Sentiment seesaw:

- Hard to arbitrage stocks are more sensitive to sentiments shifts: they tend to be overvalued in high sentiment states and undervalued in low sentiment states.
- When sentiment is measured to be high, hard to arbitrage stocks display lower future returns on average.

Results: Portfolio Sorts III

Table 6: Average monthly realized returns of portfolios sorted by subjective belief.

	Low	**2**	**3**	**4**	**High**
Subjective Belief	0.37	0.37	0.39	0.41	0.42

Results: Portfolio Sorts III

Table 7: Average monthly realized returns of portfolios sorted first by size and then by subjective belief.

	Subjective Belief				
Size	**Low**	**2**	**3**	**4**	**High**
Small	0.39	0.42	0.43	0.44	0.47
2	0.39	0.37	0.40	0.41	0.42
3	0.37	0.37	0.39	0.41	0.42
4	0.37	0.36	0.38	0.39	0.41
Large	0.35	0.35	0.37	0.38	0.38

Summary

- I presented a model of real options that formalizes the idea that decision at the firm level can be sentiment driven.
- Sentiment is modeled as a subjective belief regarding the true state of the world, which is updated in a Bayesian fashion.
- I explored the predictions of the model through simulation and verified that it conforms well with broad patterns of the data.