A Model of Loan Commitments

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Abstract

Bank lending to firms in the U.S. is mostly implemented through lines of credit. A line of credit grants firms access to funds for a predetermined period and up to a limit. In this paper, we develop a model of bank lending through lines of credit and we study the use of lines of credit during recessions. We find that the use of lines of credits increases during recessions. Moreover, when the recession is caused by a shock to the real economy, as opposed to a financial crisis, firms are more likely to request an increase in the credit limits.

JEL: G21, E44,O16. **Keywords:** Bank Lending, Credit Card, Credit Crunch, Real Activity

1. Introduction

Bank lending to firms in the U.S. is mainly implemented through loan commitments or revolving credit lines. According to the Survey of Terms of Business Lending released by the Federal Reserve Board, about 75% of Commercial and Industrial Loans (C&I hereafter) made by all commercial banks is made under commitment. This paper studies firms' use of credit lines during recessions. Recessions are frequently associated with a slowdown in lending activity. The distinctive feature of loans made through lines of credit is that such slowdown can potentially be reflected not in a decrease in outstanding debt but rather in a decrease in credit limits. For example, during the Great Recession, the economic downturn that occurred in the late 2000's and that is considered the largest decline since the Great Depression, the stock of loans did not fall at all when the crisis was at its peak. A substantial decline in bank lending was indeed observed later, when the economy was already going through a deep recession. However, since the real activity slowed down first, it is not clear whether the decline in lending was due to banks being reluctant to lend or due to firms not willing to borrow in the absence of good projects to be financed. That is, it was not clear if it was supply or demand driven.

These observations suggest that the evolution of outstanding loans during recessions is not necessarily informative about whether the economy is experiencing a credit crunch. Since a great portion of lending to firms is already committed through lines of credit, it is crucial to follow also the evolution of the unused balances of loan commitments[1]. In this sense, a credit crunch could imply a decline of newly extended loans but not necessarily of committed ones. Moreover, in the presence of committed loans, a lag in the fall of loans is consistent with a credit crunch as long as the unused balances decline simultaneously[2].

To better understand the evolution of bank lending during recessions, this paper proposes

[1]Throughout the paper, the terms "Loan commitments" and "Credit lines" will be used indistinctly. The main features of this type of contracts are detailed in Section 2.

[2]For a discussion on this debate during the Great Recession, see Chari et al. (2008) and Ivashina and Scharfstein (2010)

a model of loan commitment contracts. In the model, these contracts play an essential role in production since they allow firms to overcome cash shortages. We allow firms to choose optimally the contracts that better fit their financial needs. This means that firms are allowed to choose their credit limit as well as their debt balances. We use the model to analyze the evolution of these variables during recessions. We consider recessions triggered by two types of negative shocks: 1) an exogenous deterioration of terms of lending (which we call a "financial shock") and 2) an exogenous deterioration of business prospects (a "real shock"). We find that the use of lines of credit increases during recessions. Moreover, when the recession is caused by a shock to the real economy, as opposed to a financial crisis, firms are more likely to request an increase in the credit limits.

In terms of the modeling choice, our model borrows from the analytical framework used in most quantitative macroeconomic models, e.g. agents facing idiosyncratic shocks within an incomplete market structure. We add two important ingredients into this environment. First, we do no restrict borrowing to one period debt contracts, which gives firms more flexibility to manage their debt balances. More importantly, we allow borrowing limits to be endogenously determined in equilibrium. In this way, we stand against the idea that these limits are set exogenously (or even unilaterally) and examine how these arrangements are shaped by the optimal choices of economic agents. We do not take a stand on what constraints indebtedness. We allow firms to optimally choose their limits recognizing that this will imply to keep them fixed for a while. In the real world, many contracts have fixed duration which could push the firm to try more expensive sources of financing when they are pushed against their debt limits and they cannot change it.

It is important to mention that firms have also access to external sources of funds, not only bank loans. The importance of bank lending relative to these other sources is a debatable issue. Conventional wisdom suggests that bank loans are important at least for small and medium sized firms. This type of firms have a strong dependence for trade credit which in practice impose the necessity of transforming maturities given that in general revenues do not arrive when expenditures are due. As for large firms, the broader access to external funds seems to imply that bank lending can be easily substituted. We believe that some of the properties of bank lending made to firms are essential for their operation as well. For instance, just as an individual makes use of their credit card for day to day purchases instead of liquidating mutual fund shares, it is plausible to imagine that firms prefer to draw funds from loan commitments rather than issue equity or bonds.

Our model provides a useful framework to interpret aggregate trends. The model is suitable to perform quantitative analysis since it delivers predictions for the aggregate level of loans, credit limits and unused balances that can be meaningfully compared to U.S. data. Of course, the virtues of disaggregated data are unquestionable, but the correct interpretation of the aggregates grants policymakers the ability to make opportune interventions when needed. This paper contributes to this goal.

Related Literature The interest on loan commitments starts as early as the 80's. The theoretical literature focused on explaining why these commitments exist. One view is that they provide adequate credit to overcome revenue volatility (see Campbell (1978), and Thakor (1982)). Others see them as a solution to a moral hazard problem (see Boot et al. (1987), Berkovitch and Greenbaum (1991)) or adverse selection (see Thakor and Udell (1987) and Kanatas (1987)). Other stream of the theoretical literature has focused on the credit risk exposure through balance sheet effects that loan commitments might have in banks (see Avery and Berger (1991)).

In the economic literature, Holmstrom and Tirole (1998) model of financial intermediation sees the existence of lines of credit as a private solution to the problem of avoiding inefficient discontinuation of projects in the presence of lack of commitment. Through lines of credit, the intermediary can redistribute excess liquidity from lucky firms to unlucky ones, something that the market would not do due to ex post differences in liquidity needs. Intermediaries play the essential role of acting as liquidity pools that avoid the waste of liquid funds.

Finally, Shockley (1995) studies the link between loan commitments and capital structure and argues that rather than being a substitute for, loan commitments can actually complement with other sources of funds and lower the cost of borrowing and therefore affects the debt structure of firms.

We start our analysis by documenting the evolution of bank lending to firms during recessions. In Section 3 we setup the model. In Section 4 we discuss the properties of the optimal decision rules and perform steady state comparisons. Section 5 is devoted to the analysis of transitional dynamics and the effects of aggregate uncertainty. Section 6 provides some concluding remarks.

2. Data description

The purpose of this section is twofold. First, we want to stress the importance of loan commitment contracts in bank lending to firms in the United States. Second, we want to document the evolution of both the used and unused balances of these loans during recessions. The particular behavior of these two variables is the main empirical observation that motivates this paper.

2.1. Features of Bank Lending to firms

A distinctive feature of bank lending to firms is that it is generally implemented through loan commitment contracts (also known as revolving credit facilities or lines of credit). These contracts grant firms access to funds for a predetermined period and up to a credit limit (the committed amount). In exchange for the option to access funds at will, firms are charged an interest rate on the funds borrowed and a commitment fee. In most cases, the interest rate is expressed as a spread over the bank's prime rate or the LIBOR rate and the fee is proportional to the committed amount.

Several papers have documented the use of credit lines in the cross section of firms. For instance, Sufi (2009) reports that about 75% of public firms have a line of credit and that funds borrowed from these lines represent more than a quarter of their outstanding debt[3]. In addition, Campello et al. (2010) report that about the same fraction of private firms possesses a line of credit[4]. Furthermore, they find that 92% of large firms (those with total sales above $1 billion) have a credit line and 75% of small firms do. Conditional on having ever borrowed from those lines, there is a higher fraction of small firms that have positive debt balance in their credit line and small firms draw down more funds, as a fraction of the committed amount they have available. Overall, the evidence suggests that lines of credit are an important source of funds for firms in the U.S. and, despite the fact that small firms may rely on them more than large firms, the use of these instruments is widely spread among all firms.

At the aggregate level, the Survey of Terms of Business Lending (hereafter STBL) released by the Federal Reserve Board, reveals that a large share of C&I loans originated by all commercial banks is made under commitment. The Survey collects data on loans made during the first full business week in the middle month of each quarter (February, May, August, and November). The sample includes 348 domestically chartered commercial banks and 50 U.S. branches and agencies of foreign banks. We use the statistical releases since 1997 to characterize the more salient features of this type of loans. In Figure 1, we plot the share of total loans that are made under commitment, classified by loan size. On average, this share is around 75% of all C&I Loans and it reaches almost 90% for loans of medium size. These aggregate figures are broadly consistent with the evidence cited in the previous paragraph.

Figure 2 shows that the interest rate on these loans closely tracks the evolution of the policy interest rate. Using movements in interest rates to infer the causes behind the evolution of aggregate quantities is known to be misleading due to the possibility of a flight to quality

[3]While standard firm level databases do not include information on credit lines, Sufi (2009) combines data on Compustat from 1996 to 2003 and information from the annual 10-K SEC filings to derive summary statistics on lines of credit usage in the U.S. For a detailed description of the data, see Section 1 of Sufi (2009).

[4]Campello et al. (2010) use data from a survey realized to 794 CFOs from 31 countries in North America, Europe and Asia. The statistics reported above correspond only to the U.S. sample.

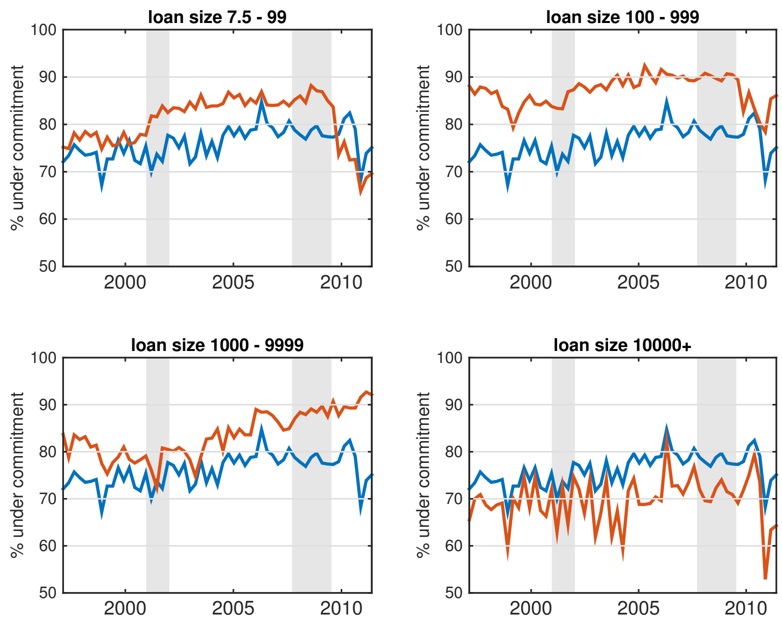

Figure 1
Percentage of C&I Loans made under Commitment by Loan Size
Loan size is expressed in thousands of U.S. dollars. The blue line in each figure corresponds to the average across different loan sizes. Shaded columns correspond to NBER recessions. Source: Survey of Terms of Business Lending.

effect during bad times. Interest rates could reflect the creditworthiness of those firms that can actually borrow funds rather, than the interest rate offered to those that want to borrow funds. This observation underscores the importance of exploiting additional data sources as we do in this paper. Finally, Figure 3 shows that the duration of loan commitments is quite large with an average of about 10 months in bad times and 15 months in good times.

2.2. Bank Lending During Recessions

Lending activity is known to be highly procyclical. As we have argued above, interest rates are not useful to identify what drives that cyclicality. The Senior Loan Officer Opinion Survey on Bank Lending Practices (SLOOS hereafter) released by the Federal Reserve Board collects useful information in this respect. In this survey, senior officers are asked to evaluate intensity of lending practices in (a) tightening standards, (b) increasing spreads and (c)

Figure 2
Weighted Average Loan Rate by Loan Size
The blue line in each figure corresponds to the average across different loan sizes. Shaded columns correspond to NBER recessions. Source: Survey of Terms of Business Lending.

demand strength. We use these responses to prepare Figure 4. The data reveals that the last two recessions were periods of considerable tightening of lending terms and standards. However, the data also shows a simultaneous weakening in the demand for bank loans. As we mention in the introduction, two stories fit this pattern of responses. The tightening of standards and terms of lending could be a consequence of a weaker demand for loans. Alternatively, the weaker demand for loans could be a consequence of tighter lending conditions. The survey does not help to elucidate which of these two stories is more adequate.

We complement this finding by turning to data from the Report of Condition and Income (Call Report hereafter) released by the Federal Deposit Insurance Corporation. On the top of Figure 5 we plot the evolution of used and unused balances of C&I loans as a percentage of GDP. On the bottom, we plot the implicit return and the realized charge-off rates[5]. A

[5]Before the first quarter of 2010 banks were not required to report separately the unused part of commercial and industrial loans. The data we use corresponds to the item “Other unused commitments” which

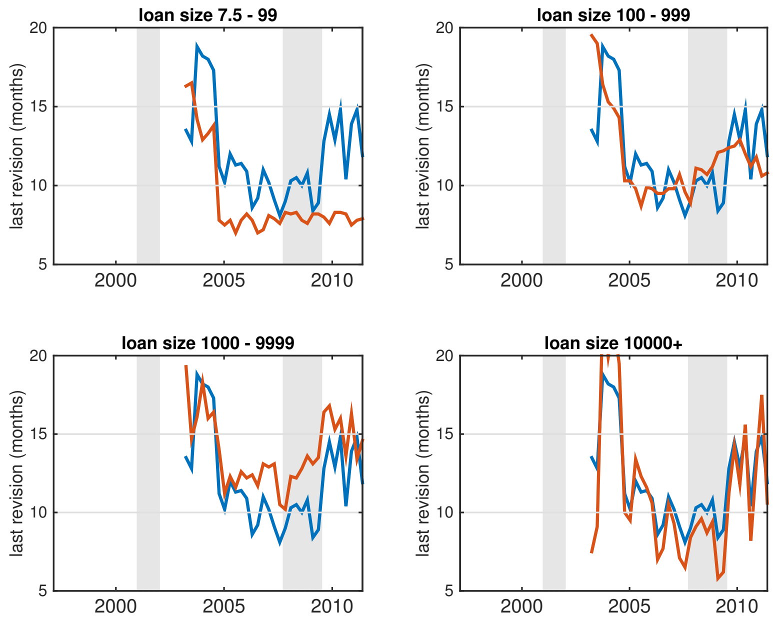

Figure 3
Months since Last Revision by Loan Size
The blue line in each figure corresponds to the average across different loan sizes. Shaded columns correspond to NBER recessions. Source: Survey of Terms of Business Lending.

useful way to describe the data on the top of the figure is by focusing on the size of the gap between both lines. In the last recession, one observes an immediate widening of this gap which lasts for several quarters. Towards the end of the recession, this gap starts to narrow. This evolution contrasts with that corresponding to the previous recession, in which the gap starts to narrow from the outset. Furthermore, notice that such narrowing seems to be present in the 1991 recession period as well. It is also interesting to note that the increment in charge-off rates for these loans increases several quarters after the recession is called. However, one should be careful in interpreting this as evidence of a lagged response in delinquency rates since the evolution of the implicit return seems to indicate loans performed

includes the securities offered to firms (e.g. overdraft facilities, commercial lines of credit, retail check credit and related plans) and excludes those offered to households (e.g. home equity lines of credit). According to the new information released, about 45% of these "Other unused commitments" corresponds to C&I loans so we plot 45% of the original series as a proxy to measure the unused balances of C&I loans before 2010. In addition, in line with the information from the STBL mentioned above, we consider only 65% of the total outstanding balance of C&I loans as a proxy of the used balances of credit lines.

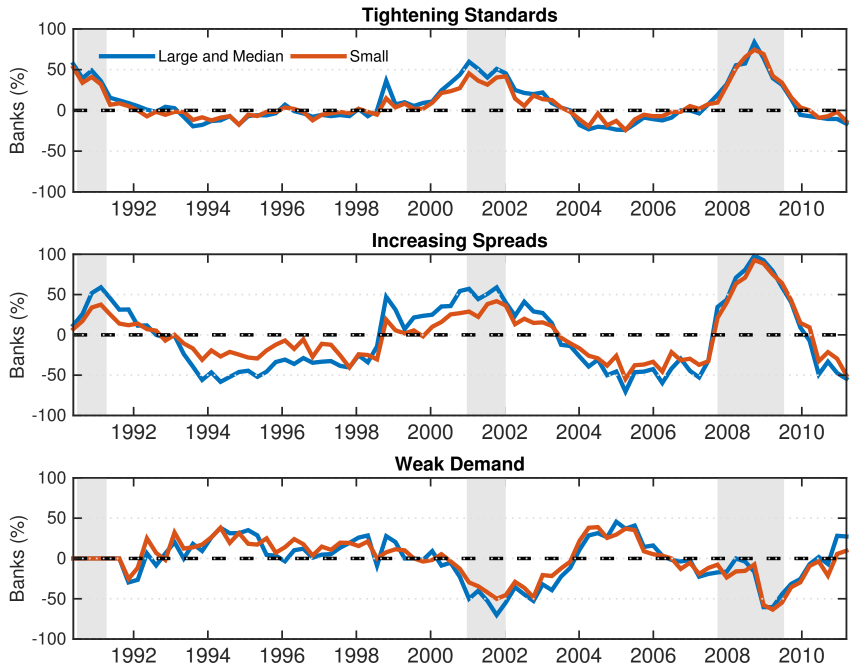

Figure 4
Evolution of Lending Practices
Banks report answers using a numerical scale that goes from -2 to +2. The results are released as the average over all answers. The shaded columns indicate NBER recessions. Source: Senior Loan Officer Opinion Survey on Bank Lending Practices.

badly from the outset, and only months later they became non-accrual.

To sum up, except for an initial period in which firms withdrew from their committed lines of credit, loans decreased during the Great Recession perhaps because banks made them more expensive. At the same time, unused credit balances decreased because credit limits were drastically reduced. At first sight, the data seems to suggest that the economy was going through a credit crunch. However, an alternative explanation as likely to fit the pattern observed in the data is a deterioration of firms' business prospects. The lack of good projects not only induces a reduction in indebtedness but also makes larger credit limits less attractive. As long as these limits are determined based on the value that firms assign to them, the data could display a decrease in both the used and the unused credit balances as a result of a decrease in this value. In our opinion, the idea of borrowing limits exogenously

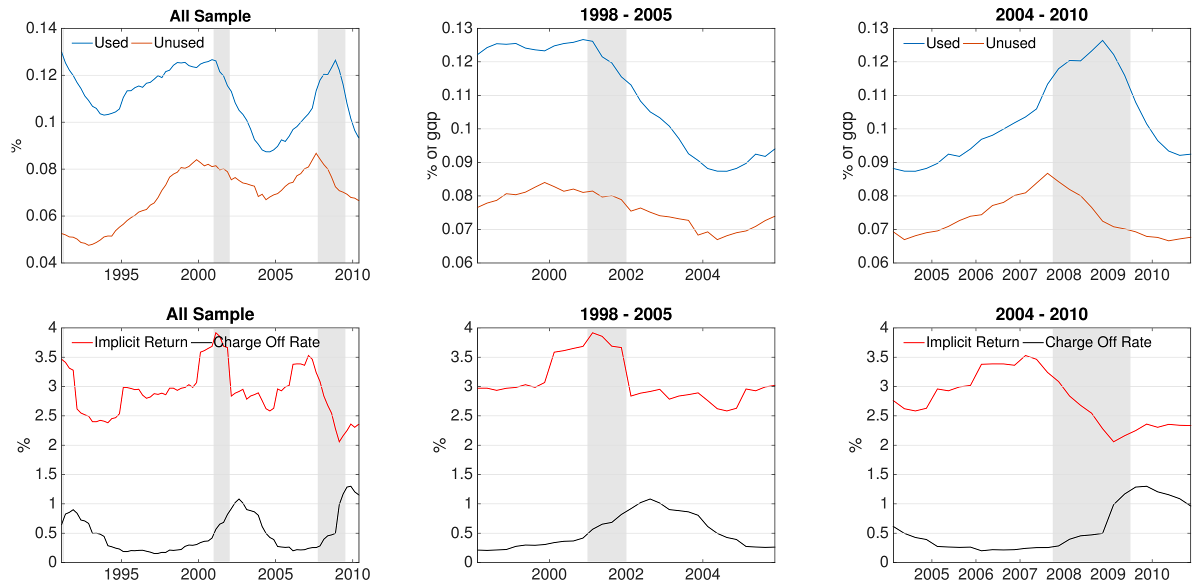

Figure 5

Commercial and Industrial Loans

The implicit return is calculated as the interest income on C&I loans over the stock of C&I loans. The shaded columns indicate NBER recessions. Source: Call Reports

given or unilaterally set by banks favors the credit crunch hypothesis and precludes other candidate explanations. We argue instead that the evolution of used and unused credit balances are equilibrium outcomes which should be understood as arising from the optimal choices of economic agents. In order to address this issue, we develop in the next section a model of loan commitments which we use later to study the behavior of the economy when it is hit by aggregate shocks.

3. A model of loan commitments

We consider the problem of a firm that faces uncertain revenues and finances the purchases of its inputs through a line of credit. We start with an example of how the possibility of changing limits affects available funds to the firm. We then describe the model recursively. When we populate the environment with a continuum of firms, the model admits a stationary distribution that can be used to calculate aggregate statistics. Steady state comparisons are reserved for the next section.

3.1. An Example

Consider a firm that operates a decreasing returns to scale technology using a single input factor h, which we call labor. Their revenues per period are given by the following function

$$R\left(z_t, h_t\right) = A z_t h_t^{\alpha} - w h_t \tag{1}$$

where $w > 0$ is the wage per unit of labor. The random variable z_t realizes every period after the labor decision has been made. Firms rely on their ability to borrow funds up to a limit L_t in order to pay for the wage bill. Once revenues are realized, cash is used to pay debt and dividends.

Suppose the borrowing limits were not fixed but vary over time. Let $\{L_t\}$ be an arbitrary sequence of credit limits defining the constraint on the outstanding debt balances, b_t, that the firm can carry over across periods. In this environment, a firm maximizing the expected discounted value of dividends, d_t would solve

$$\begin{aligned} \Omega\left(\{L_t\}\right) = \max_{\{h_t, d_t, b_{t+1}\}} \quad & \mathbb{E}_0\left[\left\{\sum \delta^t g\left(d_t\right)\right\}\right] \\ s.t. \quad & d_t + \left(1+r\right) b_t \leq R\left(z_t, h_t\right) + b_{t+1} \\ & 0 \leq w h_t \leq L_t - (1+r) b_t \\ & 0 \leq (1+r) b_{t+1} \leq L_{t+1} \end{aligned} \tag{2}$$

where δ is the firm's discount factor, and r is the interest rate charged over debt balances. Notice we allow dividends to be valued according to the function g. Let $a_t = L_t - (1+r)b_t$ denote the unused balances in period t. We can rewrite the constraint set of Problem 1 as follows

$$
\begin{aligned}
& d_t + \frac{a_{t+1}}{1+r} \leq R\left(z_t, h_t\right) + a_t - \frac{r}{1+r} L_t + \frac{1}{1+r}\left(L_{t+1} - L_t\right) \\
& 0 \leq w h_t \leq a_t \\
& 0 \leq a_{t+1} \leq L_{t+1}
\end{aligned}
$$

In this modified constraint set, a firm manages debt by choosing directly the unused balances in its line of credit. Observe that when dividends are paid, cash in hand includes the expression $\left(L_{t+1} - L_t\right)$. Therefore, varying credit limits across time affect the resources available to the firm and, as a consequence, its dividends payout decision. The possibility of running into losses because of the presence of uncertainty suggests the value of larger credit limits will depend both on the individual and the aggregate state. If firms were allowed to chose their credit limits, their will condition their choices on their state and this will have implications for the aggregate used and unused balances of loan commitments. To further explore this line of reasoning, in the next section we lay out a model in which firms choose these limits and they have to pay a proportional fee every time they do it.

3.2. The Model

Time is discrete and the time horizon is infinite. There is a unit measure of firms that hire labor h as an input to obtain revenues $R(z, h)$, where $z \in Z \subset \mathbb{R}$ is an idiosyncratic revenue shock. For simplicity we assume these shocks are i.i.d. over time. The analysis in sections 4 and 5 considers Markov shocks so we add z as an individual state variable. Since labor must be paid in advance, firms rely on loan commitment contracts to finance their production. A contract θ is a triple consisting of a credit limit $L(\theta)$, an interest rate $r(\theta)$, and a commitment fee $q(\theta)$. These contracts are not perpetual, which implies that the borrowing capacity of firms varies over time. We model this feature by introducing a random variable $\lambda \in \{0, 1\}$ with $\Pr[\lambda = 1] = \gamma$, which indicates whether or not the firm is allowed to switch to a new contract θ' within a period. This modeling choice allows to consider different scenarios for

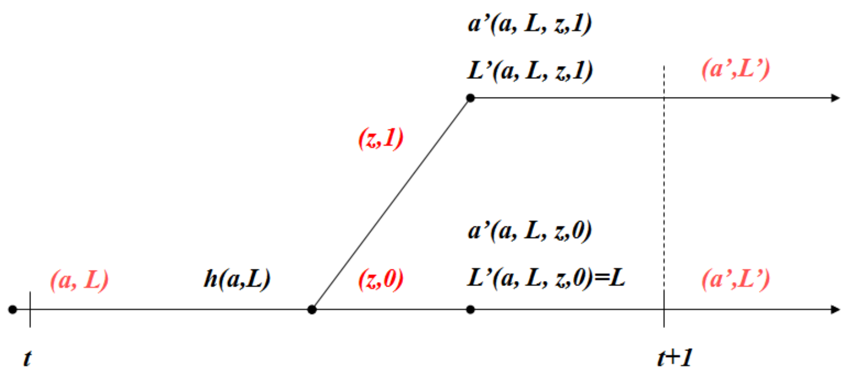

Figure 6
Timing within a period

the average duration of a contract. More explicitly, in the model there is a one to one mapping between the probability of switching contracts ($\lambda = 1$) and the average duration of a contract. We consider the case in which r and q are taken as given, so that choosing a contract is equivalent to choosing a credit limit. The timing of events is as follows. A firm enters the period with an unused balance a and a credit limit L. Labor services are decided and paid for at the beginning of the period. Subsequently, all uncertainty is revealed and firms learn their idiosyncratic shocks (z, λ). Since bad shocks translate into low revenues, contracts with larger limits are preferred insofar they allow to overcome more contingencies. This timing is summarized in Figure 6.

Recursive formulation A *production plan* is a collection of functions h, $d(z, \lambda)$, $a'(z, \lambda)$, and $L'(z, \lambda)$ defined over $\mathbb{R}_+ \times \mathcal{L}$. The first three functions map into the real line, whereas the last maps into $\mathcal{L}$. The set $\mathcal{L}$ over which the choice of the credit limit takes place is bounded above by $\overline{L}$. Firms' choices must satisfy four conditions. First, labor services can only be paid by drawing funds from the unused part of their credit line:

$$0 \leq wh \leq a \tag{3}$$

When firms are not allowed to switch contracts, we have that $\lambda = 0$ and $L'(z, 0) = L$. The only remaining choices are how much dividends to pay out, $d(z, 0)$, and how much unused credit balance to carry forward to the next period, $a'(z, 0)$. These decisions must satisfy the following constraints

$$
\begin{aligned}
0 &\leq a'(z,0) \leq L && (4) \\
d(z,0) + \frac{a'(z,0)}{1+r} &\leq R(z,h) + a - \left(\frac{r}{1+r} + q\right) L && (5)
\end{aligned}
$$

The first constraint states that the unused balances cannot exceed the outstanding credit limit. Observe that the flow of funds constraint now incorporates the payment of the commitment fee denoted by q. When firms are allowed to switch contracts, $d(z, 1)$, $a'(z, 1)$ and $L'(z, 1)$ are chosen so that they satisfy

$$
\begin{aligned}
0 &\leq a'(z,1) \leq L && (6) \\
d(z,1) + \frac{a'(z,1)}{1+r} &\leq R(z,h) + a - \left(\frac{r}{1+r} + q\right) L + \left(\frac{1}{1+r} - q\right)\left(L'(z,1) - L\right) && (7)
\end{aligned}
$$

Let $\Omega(a, L)$ be the value of a firm with unused balances a and a credit limit L. This value must solve the following functional equation

$$
\Omega(a,L) = \max \mathbb{E}\left[g\left(d(z,\lambda)\right) + \delta\Omega\left(a'(z,\lambda), L'(z,\lambda)\right)\right] \quad (8)
$$

where the maximization on the right hand side is made over *production plans* and subject to (3) - (7). Observe that we do not assume limited liability, which implies that unused balances can be replenished by running negative dividends. We consider the following functional form for g

$$
g(d) = \begin{cases} d & if \quad d \geq \overline{d} \\ d - \chi\left(\overline{d} - d\right)^2 & if \quad d < \overline{d} \end{cases}
$$

where we allow $\chi > 0$ to consider the case in which paying out low dividends is costly for the firm. We can interpret this modeling choice in several ways. It could be that firms replenish unused credit balances by using other sources of funds, which are costly when used in a relatively small scale. Alternatively, one might think that firms anticipating liquidity shortages respond by hoarding cash, which is costly since those funds could have been used in other profitable investment projects.

4. Steady state analysis

Our environment admits a stationary distribution over individual states. We consider a Markov structure for the productivity shock z. The stationary distribution is a fixed point of the operator T that maps the space of probability measures defined over the space $\mathcal{S} \equiv (\mathbb{R}_+ \times \mathcal{L} \times Z)$ into itself, which is defined using the optimal decisions of the firm. We denote such a distribution by Ψ^*. Our steady state analysis proceeds as follows. We first discuss the properties of the optimal policies, which have been solved for numerically. We then use the distribution Ψ^* to define the aggregate statistics we are interested in and perform comparative statics. We delay the study of transitional dynamics to section Section 5.

4.1. Baseline economy

We classify the parameters in two groups, depending on whether they affect the real side or the financial side of the firm. We have chosen plausible parameter values so that we can make meaningful comparisons with actual data. Table 2 reports all the parameters and the values we have chosen.

We normalize the wage to one. The discount factor was chosen so that firms are impatient enough to be willing to borrow. In steady state, this choice implies a rate of utilization of the credit line (e.g., B/L) of about 60%. The grid for idiosyncratic productivity shocks was chosen so that operating at the unconstrained level of labor delivers negative revenues following bad shocks. The choice of the productivity shock implies a variance of 8% and a

Table 1
Baseline calibration

Parameter	Symbol	Value
1. Real Parameters		
Discount Factor	δ	.92
Dividend Threshold	$\overline{d}$	2
Aggregate Productivity	A	3
Idiosyncratic Shocks	Z	$\{.57; 1.18\}$
Transition Matrix	z	$[.7, .3; .6, .4]$
Labor share	α	.67
2. Financial Parameters		
Upper Credit Limit	$\overline{L}$	25
Probability of Switching Contract	γ	.2
Commitment Fee	q	.06
Interest Rate	r	.045 - .055

mean of 1 while the aggregate level of productivity was set to three. The value for the labor share is standard.

Regarding the financial side of the model, we specified the grid of credit limits available to the firm so that with the maximum credit limit, an unconstrained firm can finance three times its wage bill. The probability of having the option to change the credit line contract is set so that the average duration of a credit line is five quarters. This number is consistent with the information reported in the STBL regarding the average number of months passed since the terms of new C&I loans were set. The fee was set to 6% of the credit limit and the interest rates vary from 4.5% in good times to 5.5% during recessions.

4.2. Optimal Policies

In this section we report and discuss the optimal policies that solve the right hand side of the functional equation (8). The optimal choices of the credit limits and unused balances are of particular interest. Since firms are heterogeneous, they will optimally choose different limits when they are allowed to do so. The policy functions are depicted in the four panels of Figure 7.

The optimal labor choice is shown in Panel (A). We can observe that there are three regions of interest. When the unused balances are low, firms will hire as much labor as possible, the constraint (3) is effectively binding and firms hit their credit limit. On the other hand, when the line of credit remains highly unused, the labor choice is optimal in the sense that it maximizes expected revenues $\mathbb{E}\left[R\left(z,h\right) \mid z_{-}\right]$. There is a middle range of values for the unused balances for which the labor choice is interior. This is a consequence of the presence of borrowing limits and the choice of the functional form of g. As we motivated above, low revenues might require the firm to adjust its scale of production in order to avoid decreasing dividends. Finally, for a given credit limit, the optimal choice of h is increasing in the unused balances a.

Unused balances are perceived as an asset to the firm insofar they are essential to undertake production. As a consequence, the optimal decision rules are standard and resemble those that arise in a standard income fluctuation problem with incomplete markets. In the context of our model, firms build up their unused balances in good times and deplete them in bad times, irrespectively of whether they are allowed to switch contracts or not.

Finally, Panel (D) displays the optimal choice of the credit limit when firms are allowed to switch contracts. As we mentioned earlier, larger limits are valuable as they allow firms to overcome more contingencies. At the same time, firms understand that bad luck cannot last forever and hence, the larger the limit, the lower the marginal value of increasing it. Since firms are charged a proportional fee every time they choose a new contract, they will not choose arbitrarily large limits.

4.3. Comparative Statics

It is straightforward to calculate aggregate statistics in the steady state version of our model. We are ultimately interested in how these aggregates respond to aggregate shocks. The aggregate level of credit limits, used and unused credit balances are calculated using the

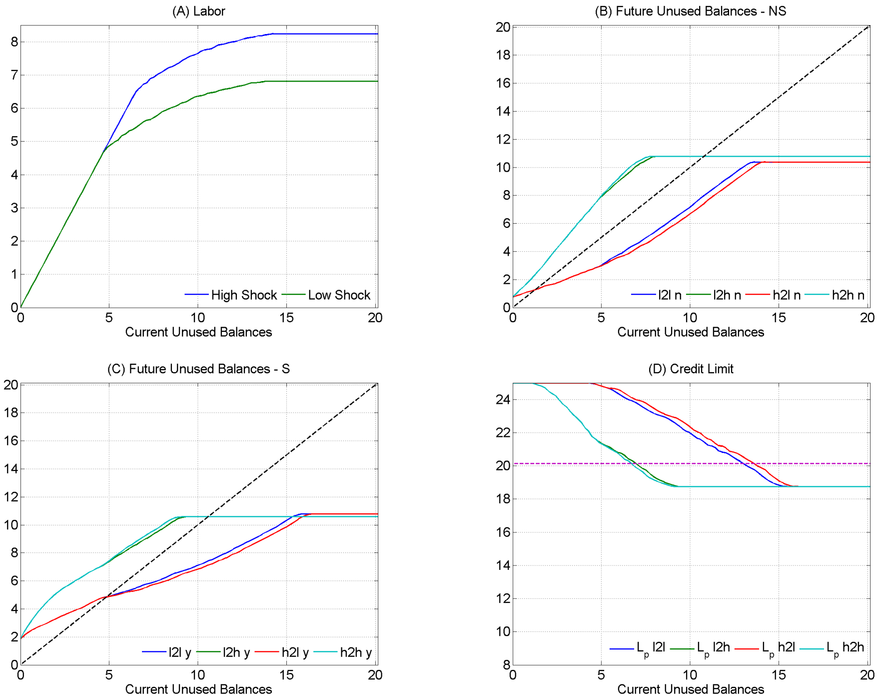

Figure 7
Optimal Policies
Panel (B) and (C) display the optimal choices of future unused balances when the firm is not allowed to change contracts (NS) and when it is allowed to do so (S).

stationary distribution Ψ^* as follows:

$$\begin{aligned} \mathbb{L} &= \int L\Psi^*\left(da, dL, dz\right) \\ \mathbb{B} &= \int \left(L-a\right)\Psi^*\left(da, dL, dz\right) \\ \mathbb{UB} &= \int a\Psi^*\left(da, dL, dz\right) \end{aligned} \tag{9}$$

In order to get some insight about what the model can deliver we perform the following exercise. We calculate the steady state in four different scenarios. The first two differ only on the interest rate charged over the used balances. By studying how the aggregates vary when we move from a low to a high interest rate, we gain some insight about the consequences of an exogenous increase in the cost of funds. We associate this exercise with the effects of a recession originated from a *financial shock* since the perturbation is external to the firm. The

Table 2
Steady state effects of recessions

	Interest Rate r		Productivity A	
	Low	High	High	Low
1. Aggregates				
Agg Credit Limit (L)	22.27	19.43	20.91	19.93
Used Balances (B)	12.90	9.59	11.41	11.24
Unused Balances (UB)	9.37	9.84	9.50	8.69
2. Ratios				
B/L	.58	.49	.55	.56
L/Y	1.87	1.60	1.83	2.16

other two scenarios differ only on the level of aggregate productivity as given by the value of A in Equation 1. In this case, by comparing the aggregates, we associate the results with the effects of a recession originated from a *real shock*, since the perturbation is technological and internal to the firm.

We summarize the results of this exercise in Table 1. As it can be seen, steady state responses differ both in magnitude and in the direction depending on the source of the recession. When the recession comes from a financial shock, the aggregate credit limit and the outstanding debt decrease, whereas the aggregate unused balance goes up. In contrast, as the last two columns of Table 1 show, when the recession comes from a real shock, both the used and the unused balances fall. This suggests that following a perturbation coming from the real side, the aggregate credit limit will fall by more than the used balances.

To understand these results, notice that a higher interest rate makes debt more expensive so firms reduce the used balances of their credit lines. This reduction makes less likely that the firm will hit the credit limit and as a result larger credit limits become less valuable. On the other hand, when aggregate productivity goes down firms find optimal to operate at a smaller scale. As a consequence, the shadow value of larger limits and unused balances go down.

The results so far indicate that the origin of perturbations have qualitatively different implications regarding the evolution of the aggregates. To complete the analysis, we shall evaluate how these aggregates evolve over time when these shocks hit the economy.

5. Transitional dynamics

In this section, we further study the effects of aggregate shocks by computing the transitional dynamics of the economy when entering a recession. To ease the exposition, we refer to the scenarios 1 and 3 computed in the previous section as the good states and to scenarios 2 and 4 as the recessions or bad states. Let $S = \{G, B\}$ denote the aggregate state. We focus on the transition from the good state to the bad state. We perform two sets of experiments which differ on how firms form expectations about future aggregate shocks. The details are explained below.

5.1. Unexpected Recession

In this exercise, we assume that the recession is unexpected. In particular, firms take as given the current aggregate productivity level and the interest rate they pay on their loans and make their decisions as if these values will never change. The steady states of these deterministic economies are the ones computed in Table 2. When firms are surprised by a bad shock, they instantaneously adjust their optimal decisions to the new scenario. The economy goes through a transition where firms' used and unused balances and credit limits gradually adjust until they are consistent with the bad steady state balances. More precisely, we assume the economy reaches the new steady state in T periods. Using the transition matrix implied by the optimal behavior of firms during recession, we compute the sequence of distributions $\left\{\Psi^{G*}, \Psi_1^B, \Psi_2^B, ..., \Psi_{T-1}^B, \Psi^{B*}\right\}$, where Ψ_S^* represents the stationary distribution of firms for $S = \{G, B\}$. We then use these distributions to compute the corresponding aggregates as defined in (10). For a sufficiently large T, this procedure delivers the transitional dynamics.

In Figure 8 we show the evolution of credit limits, used and unused credit balances during the transition. With respect to average levels, the real shock corresponds to a decrease of -3.3% in the aggregate productivity while the interest rate remains at the average level of

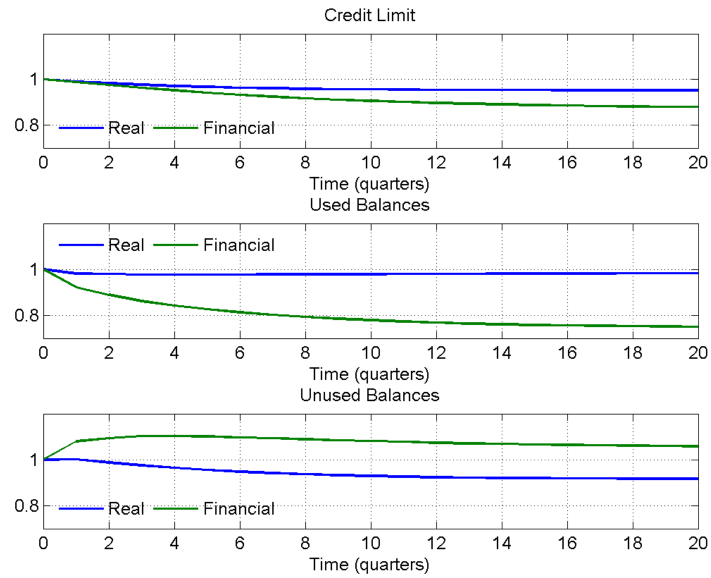

Figure 8
Transitional Dynamics after an Unexpected Bad Shock

5%. Analogously, the financial shock corresponds to an increase of 1% in the interest rate while the aggregate productivity remains at the average value of 3. The values are normalized to one at the moment of the state switch. The picture shows that a higher interest rate makes debt more expensive so firms reduce the used balances of their credit lines. This reduction implies that it is less likely that the firm will hit the credit limit. Hence, larger limits become less valuable and firms gradually adjust their limits to a lower level as well. On the other hand, a real shock reduces the holdings of unused balances and credit limits. Although the effects of the financial shock are larger, the model in general delivers little action in terms of its responses to shocks.

5.2. Aggregate Uncertainty

The second experiment introduces aggregate uncertainty. In this economy, firms make decisions knowing that a recession is probable. Firms understand that they live in a stochastic world where the aggregate state of the economy changes with positive probability. Since we assumed that prices are exogenous in our model, the introduction of aggregate uncertainty is relatively simple and only costly in terms of the computational burden. In particular, we need to extend the model to allow firms to condition their optimal decisions on the aggregate state of the economy. We assume all uncertainty occurs simultaneously. Consequently, the decision rules are calculated in the same way as before, but a cross sectional stationary distribution no longer exists. The computation of the aggregates is analogous to the previous experiment with the caveat that, in this case, the optimal decisions and hence, the transition matrix we use to update the distribution, take into account the stochastic nature of the economy.

We analyze the transitional dynamics of the economy when it switches to the bad state after a long period of stability in the good state. The exercise consists on simulating a 50 year realization of the good state after which a permanent switch to the bad state takes place. The approach of the exercise is close to that used by Diaz-Gimenez and Prescott (1997) to study the real effects of different monetary policies. We chose the Markov transition matrix for the aggregate state so that the average duration of the good state is 2.5 years (10 model periods) and the average duration recessions is 6 months (2 model periods). For the financial shock, we chose the interest rates values to be $r(g) = 4.5\%$ and $r(b) = 5.5\%$. It is important to highlight the fact that the aggregate financial shock does not affect the transition matrix of the idiosyncratic productivity process. We modeled the real shock as an increase of 3.3% in aggregate productivity during good times and a decrease of 3.3% in bad times with respect to the average level. In both cases, the value of the other parameter (A in the first case, r in the second) is set to its expected value with respect to the invariant distribution of the proposed transition matrix. Figure 9 shows the transitional dynamics after the recession hits. When recessions are expected, the effects of real and financial shocks change substantially. First of all, the aggregates move in the same direction regardless of the nature of the shock. Second, the magnitude of these movements are larger after the real shock. To understand this result, observe that our choice of the transition matrix for the aggregate shock implies that the recession is perceived as transitory by the firms. In the case of the financial shock,

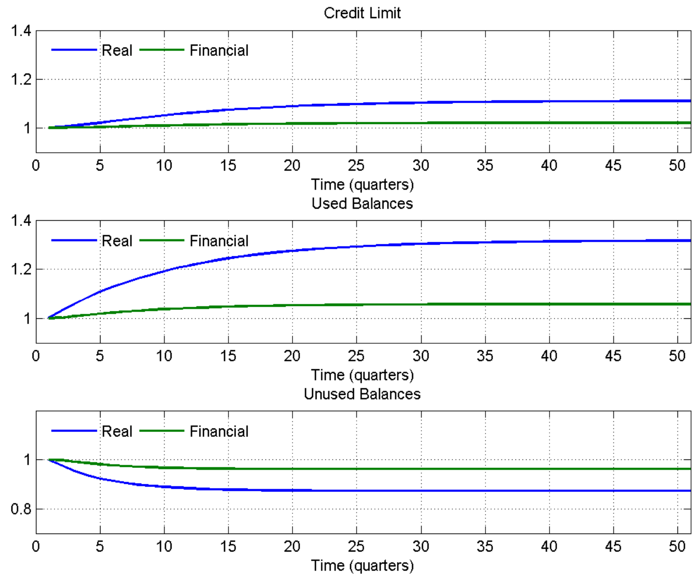

Figure 9
Transitional Dynamics after a Bad Shock with Aggregate Uncertainty

this explains the increase of the used balances. Firms do not adjust their debt levels since they expect the interest rate to go down in the short run. As for the real shock, since the fall in revenues is only transitory firms maintain their scale of production and let the unused balances decrease. By doing so, they avoid reducing dividends. Furthermore, as unused credit balances decrease, firms face the risk of hitting the credit limit. As a consequence, firms are likely to request increases in their credit limits during recessions.

The size and the direction of the effects of both real and financial shocks depend on the parametrization of the model we have chosen. The numerical exercises presented here are meant to provide a flavor of how the model of loan commitments we present in this paper serves as a useful framework to do meaningful comparisons with real banking data.

6. Concluding Remarks

The extensive use of lines of credit by firms in the U.S. poses a challenge to policymakers who follow the evolution of bank lending in an attempt to identify any sign of a credit crunch that could push the economy into a recession. The reason is that such credit crunch could take the form of a reduction on credit limits, and no change observed in the outstanding debt. For this reason, the evolution of loans during recessions is not sufficient information and policymakers need to also follow the evolution of credit limits ad the unused credit balances. To assist this goal, this paper proposes a model of loan commitments. In the model, credit lines allow firms to overcome cash shortages and we let firms choose optimally the contracts that better fit their financial needs. Although larger credit limits are valuable to the extent that they allow to overcome more contingencies, firms must pay interest and fees in order to access these funds.

Our model provides a novel and useful framework to interpret aggregate trends during recessions. The model has great potential to perform quantitative analysis since it delivers predictions for the aggregate level of loans, credit limits and unused balances that can be meaningfully compared to the U.S. banking data. We use the model to analyze bank lending practices during recessions. We propose two sources of recessions: 1) an exogenous deterioration of the terms of lending (what we denote a "financial shock") and 2) an exogenous deterioration of the business prospects (that we call a "real shock"). We find that the use of lines of credit increases during recessions. Moreover, when the recession is caused by a shock to the real economy, as opposed to a financial crisis, firms are more likely to request an increase in the credit limits.

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