Term Limits and Bounds on Policy Responsiveness in Dynamic Elections

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Outline

Motivation

Democracies rely on the possibility of future re-election as a way of disciplining political agents.

Is the retention tool effective? Do political agents respond to electoral incentives to exert effort while in office? If so, to what extent?

The analysis of these issues requires a model that is dynamic and accounts for reputational concerns of politicians.

Motivation (cont.)

I explore these questions in a dynamic model of political agency featuring:

- infinite horizon, discrete time
- two-period term limit
- adverse selection
- moral hazard
- citizen candidates.

Motivation (cont.)

I provide four main results:

- Existence of equilibrium, rectifying an error in the proof of Banks and Sundaram (1998).
- A characterization of equilibria when politicians are highly office motivated: the highest type of office holder mixes between shirking and arbitrarily high effort, while other types shirk.
- The voters' equilibrium expected payoff from policy choices of first-term office holders is bounded above by the expected payoff from the ideal point of the highest type.
- When office benefit is high, this negative welfare result arises because voters are too demanding.

Related literature

The literature on infinite-horizon models of political agency is small. Here is a selection.

- Early work: Barro (1973), Ferejohn (1986)
- Pure adverse selection: Duggan (2000), Bernhardt et al. (2004), Banks and Duggan (2008)
- Moral hazard and adverse selection: Banks and Sundaram (1993), Banks and Sundaram (1998)

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Timing

In each period,

- an office holder chooses effort $x \in \Re_+$,
- an outcome $y \in \Re$ is realized from density $f(\cdot|x)$,
- a challenger is randomly drawn,
- if incumbent is in her first term, the voter chooses between the candidates,
- otherwise the challenger is automatically elected,
- the process repeats in all subsequent periods.

Information

Preferences of office holders are indexed by a finite set of types $j = 1, \ldots, n$, which are private information.

Types are distributed identically and independently across candidates, with prior $p_j > 0$ on type j.

Effort x is not observed by the voter.

The outcome y is observed by the voter.

Payoffs

Voter's payoff from outcome y is $u(y)$, which is continuous and strictly increasing.

Type j office holder's payoff from effort x is $w_j(x) + \beta$, where $\beta \geq 0$ is office benefit.

Total discounted payoff of type j citizen is

$$\sum_{t=1}^{\infty} \delta^{t-1} [I_t(w_j(x_t) + \beta) + (1 - I_t)u(y_t)],$$

where $\delta \in [0, 1)$ is the common discount factor and $I_t \in \{0, 1\}$ indicates whether the citizen holds office in period t.

Assumptions on politician payoffs

Assume w_j satisfies

- differentiability
- supermodularity: for all x, x' with $x > x'$ and all $j < n$,

$$w_{j+1}(x) - w_{j+1}(x') \quad > \quad w_j(x) - w_j(x'),$$

- there is a unique ideal point $\hat{x}_j$, which is the unique critical point; moreover, $\hat{x}_1 < \hat{x}_2 < \cdots < \hat{x}_n$, and

$$w_1(\hat{x}_1) \;\leq\; w_2(\hat{x}_2) \;\leq \cdots \leq\; w_n(\hat{x}_n)$$

- $\lim_{x\to\infty} w_j(x) = -\infty$.

Special cases

These assumptions allow $0 < \theta_1 < \theta_2 < \cdots < \theta_n$ and

$$w_j(x) = \mathbb{E}[u(y)|x] - \frac{1}{\theta_j} c(x).$$

So, office holders are voters who incur a cost of effort.

Further special cases are $u(x) = x$ and

- quadratic payoffs: $w_j(x) = -(x - \hat{x}_j)^2$
- exponential payoffs: $w_j(x) = -e^{x - \hat{x}_j} + x$.

Assumptions on outcome distribution

Assume $f(y|x) = f(y-x)$ for a fixed density $f(\cdot)$ that is continuous with full support on $\Re$.

Assume f satisfies MLRP: for all $x > x'$ and all $y > y'$,

$$\frac{f(y-x)}{f(y-x')} > \frac{f(y'-x)}{f(y'-x')}.$$

Assume arbitrarily extreme signals become arbitrarily informative: for all $x > x'$,

$$\lim_{y \to -\infty} \frac{f(y-x)}{f(y-x')} = \lim_{y \to \infty} \frac{f(y-x')}{f(y-x)} = 0.$$

Stationary strategies

Focus on stationary strategies: choices of first-term office holders are history independent, the voter conditions only on the currently observed outcome using a cutoff rule, and second-term office holders simply shirk.

$\pi_j \in \Delta(\Re_+)$	mixed policy strategy of type j politicians
$\overline{y}$	re-election standard used by voter
σ	strategy profile
$\mu_T(\cdot\|y)$	belief assessment

Stationary electoral equilibria

A *stationary electoral equilibrium* is a pair $\psi = (\sigma, \mu)$ such that:

- for each j, π_j is optimal for the type j office holder given ψ,
- for each y, electing according to $\overline{y}$ is optimal for the voter given ψ,
- for each y, $\mu_T(\cdot|y)$ is derived from Bayes rule.

Let $V^I(y)$ be the voter's expected discounted payoff from re-electing a first-term incumbent, conditional on observing y.

Let V^C be the voter's expected discounted payoff from electing a challenger.

Outline

Elections with two periods

Several papers consider two-period models of electoral accountability, including: Fearon (1999), Besley (2006), Ashworth (2005), Acemoglu, Egorov, and Sonin (2013).

Duggan and Martinelli (2015) is closest to the current paper.

Here, the second period is trivial, and V^C is fixed at

$$V^C = \sum_j p_j \mathbb{E}[u(y)|\hat{x}_j].$$

But equilibria still must solve a complex fixed point problem.

Optimal policy choice

The type j politician's expected payoff from choosing x and being re-elected with probability r, minus a term independent of x, is:

$$W_j(x, r) \quad = \quad w_j(x) + r\delta[w_j(\hat{x}_j) + \beta - V^C],$$

Assume holding office is desirable: $w_j(\hat{x}_j) + \beta > V^C$.

Of course, given cutoff $\overline{y}$, probability of re-election from effort x is $r = 1 - F(\overline{y} - x)$

Optimal policy choice (cont.)

The maximization problem

$$\max_{x \in \Re_+} W_j(x, 1 - F(\overline{y} - x))$$

has a non-empty, compact set of solutions.

Let x_j^* and $x_{*,j}$ denote, resp., the maximum and minimum optimal effort levels.

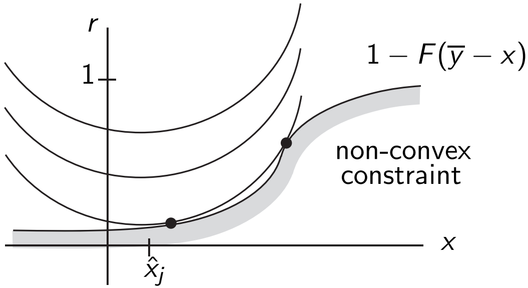
Differentiability implies positive effort: $x_{*,j} > \hat{x}_j$.

Auxiliary problem

We can decompose the problem of the type j office holder as:

$$\max_{(x,r)} W_j(x,r)$$
$$\text{s.t. } r + F(\overline{y} - x) - 1 \leq 0$$

This problem is non-convex and can have multiple solutions.

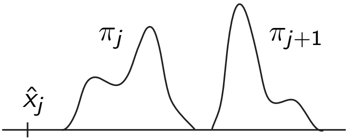


Implications

Arguments of Banks and Sundaram (1998) show that the objective $W_j(x, 1 - F(\overline{y} - x))$ is supermodular in (j, x).

Thus, given cutoff $\overline{y}$, optimal efforts are strictly ordered by type: for all $j < n$, we have $x_j^* < x_{*,j+1}$.

So in equilibrium, mixed policy strategies put probability one on positive effort and are strictly ordered by type.



Optimal re-election rule

Assuming mixed policy strategies are strictly ordered by type, arguments of Banks and Sundaram (1998) show that the optimal re-election rule for the voter is a cutoff, and this cutoff is unique.

Let $U(\overline{y}, \pi_1, \ldots, \pi_n)$ denote the voter's payoff from using cutoff $\overline{y}$ given mixed policy strategies $\pi_1, \ldots, \pi_n$.

Thus, the problem

$$\max_{\overline{y}} U(\overline{y}, \pi_1, \ldots, \pi_n)$$

has a unique solution, and the voter uses the unique optimal cutoff in equilibrium.

Results for two period model

Duggan and Martinelli (2015) show that an equilibrium exists.

In equilibrium, as office benefit $\beta \to \infty$, the voter's cutoff goes to infinity, and the effort levels of all "above average" types go to infinity as well.

In particular, a form of policy responsiveness holds:

$$\lim_{\beta \to \infty} \sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j(dx) \quad > \quad \mathbb{E}[u(y)|\hat{x}_n].$$

And if voter utility is unbounded (e.g., the risk neutral case), the expected payoff from a first-term politician goes to infinity.

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Similarity with two-period model

Now we parameterize these politician and voter problems by a value V that is appropriately bounded.

Then the type j politician solves

$$\max_{x \in \Re_+} W_j(x, 1 - F(\overline{y} - x); V),$$

and the voter solves

$$\max_{\overline{y}} U(\overline{y}, \pi_1, \ldots, \pi_n; V).$$

Similarity with two-period model (cont.)

Once again, given cutoff $\overline{y}$, optimal efforts are strictly ordered by type: for all $j < n$, we have $x_j^* < x_{*,j+1}$.

And if $\pi_1, \ldots, \pi_n$ are strictly ordered by type, then there is a unique optimal cutoff for the voter.

Assume $w_j(\hat{x}_j) + \beta > \mathbb{E}[u(y)|\hat{x}_n]$ for all j. By our bound on policy responsiveness, later, office is desirable.

Statement of existence

Theorem: There is a stationary electoral equilibrium, and every stationary electoral equilibrium is given by mixed policy strategies $\pi_1^*, \ldots, \pi_n^*$ for first-term politicians and a finite cutoff y^* such that:

(i) each type j politician mixes over policy choices using π_j^*, and the least optimal policy choice is greater than the ideal point of the politician, i.e., $x_{*,j} > \hat{x}_j$.

(ii) the supports of policy strategies are strictly ordered by type, and in fact for all $j < n$, we have $x_j^* < x_{*,j+1}$,

(iii) the voter re-elects the incumbent if and only if $y \geq y^*$.

Discussion of Banks-Sundaram

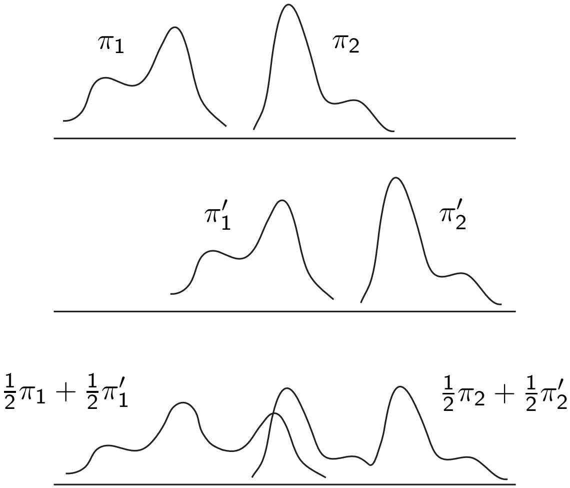
Banks and Sundaram (1998) construct a correspondence on the **domain of profiles of mixed policy strategies with supports weakly ordered by type,** crossed with an interval of possible cutoff outcomes.

A given $(\pi_1, \ldots, \pi_n)$ and $\overline{y}$ determine best response mixed policy strategies and an interval of best response cutoffs:

$$(\pi_1, \ldots, \pi_n, \overline{y}) \to \left(\prod_j \Delta(M_j(\overline{y})) \right) \times C(\pi_1, \ldots, \pi_n).$$

But this domain is not convex, and Glicksberg cannot be applied.

Discussion of Banks-Sundaram (cont.)



Existence proof: rough idea

An equilibrium is obtained as a fixed point of a correspondence:

$$\Phi(\pi_1, \ldots, \pi_n, \rho) = \left(\prod_j \Delta(\tilde{M}_j(\pi_1, \ldots, \pi_n, \rho)) \right) \times \Delta(\tilde{C}(\pi_1, \ldots, \pi_n, \rho)),$$

where ρ is a mixture over possible cutoffs (more on this later).

No ordering of mixed policy strategies assumed in the domain.

Let $V(\pi_1, \ldots, \pi_n, \overline{y})$ be the voter's continuation value of a challenger given mixed policy strategies $\pi_1, \ldots, \pi_n$ and cutoff $\overline{y}$.

Existence proof: rough idea (cont.)

Then I define for each type j

$$\tilde{M}_j(\pi_1, \ldots, \pi_n, \rho) = \arg\max_{x \in [0, \tilde{x}]} W_j(x, 1 - F(\mathbb{E}[\rho] - x); V(\pi_1, \ldots, \pi_n, \mathbb{E}[\rho])).$$

These are the optimal policy choices given mixed policy strategies $(\pi_1, \ldots, \pi_n)$ and cutoff $\mathbb{E}[\rho]$.

Note that randomness in the distribution ρ is extracted by reducing it to its mean.

Existence proof: rough idea (cont.)

And I define

$$\begin{aligned}\tilde{C}(\pi_1, \ldots, \pi_n, \rho) \\ &= \arg\max_{\overline{y} \in [a,b]} U(\overline{y}, \pi_1, \ldots, \pi_n; V(\pi_1, \ldots, \pi_n, \mathbb{E}[\rho])).\end{aligned}$$

These are the voter's optimal cutoffs given mixed policy strategies $(\pi_1, \ldots, \pi_n)$ and cutoff $\mathbb{E}[\rho]$.

The correspondence Φ satisfies the conditions of Glicksberg's theorem so there is a fixed point, $(\pi_1^*, \ldots, \pi_n^*, \rho^*)$.

Existence proof: rough idea (cont.)

Since politicians best respond to the cutoff $\mathbb{E}[\rho^*]$, the strategies $(\pi_1^*, \ldots, \pi_n^*)$ are strictly ordered by type.

So there is a unique optimal cutoff, so ρ^* is degenerate on some y^*, and $\mathbb{E}[\rho^*] = y^*$.

Then $(\pi_1^*, \ldots, \pi_n^*)$ and y^* yield the desired stationary electoral equilibrium.

Outline

Statement of result

In addition to the above, assume voter payoffs are unbounded:

$$\lim_{x \to \infty} u(y) \;=\; \infty.$$

Theorem: Let the office benefit $\beta \geq 0$ and $\delta \in [0, 1)$ vary subject to $\lim \beta\delta = \infty$. Then:

(i) $\overline{y} \to \infty$,

(ii) $x_n^* \to \infty$ and $x_{*,n} \to \hat{x}_n$,

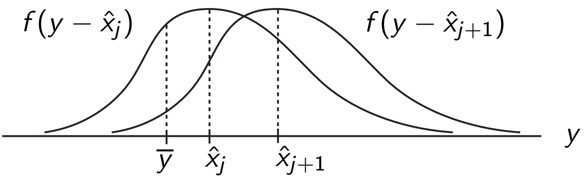
(iii) for all $j < n$, $x_j^* \to \hat{x}_j$,

(iv) for all $\eta > 0$ and for $\beta\delta$ large enough, we have $\pi_n([\eta, \infty)) > 0$,

(v) for all $\eta > 0$, we have $\pi_n((\hat{x}_n, \eta)) \to 1$ and $\pi_n([\eta, \infty)) \to 0$.

Proof: idea

Step 1: $\overline{y} \to \infty$. If $\overline{y} \to -\infty$, then all types shirk in the limit. Relative to the prior, lower types are more likely conditional on the cutoff.



This contradicts the voter's indifference condition.

Proof: idea (cont.)

Step 1 (cont.): If $\overline{y}$ has a subsequence with finite limit, then because $\beta\delta \to \infty$, the effort level of each office holder types goes to infinity. But this contradicts our bound on policy responsiveness, later. This proves (i).

Step 2: There is no subsequence of optimal effort levels x_j' for any type that converges to a finite limit above the office holder's ideal point. Suppose there is a sequence with $x_j' \to \tilde{x}_j$ and

$$\hat{x}_j < \tilde{x}_j < \infty.$$

Proof: idea (cont.)

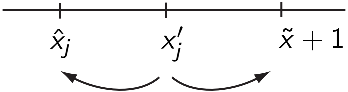
Step 2 (cont.): Then the gains from deviating from x_j' to $\hat{x}_j$ are non-positive:

$$\underbrace{\delta(F(\overline{y}-\hat{x}_j)-F(\overline{y}-x_j'))[w_j(\hat{x}_j)+\beta-(1-\delta)V^C(\psi)]}_{\text{future losses}}$$

$$\geq \quad \underbrace{w_j(\hat{x}_j)-w_j(x_j')}_{\text{current gains}}.$$

Proof: idea (cont.)

Step 2 (cont.): Now consider moving the other way to $\tilde{x}_j + 1$.



Using L'Hôpital's rule, we have

$$\lim_{\beta\delta\to\infty} \frac{F(\overline{y} - x'_j) - F(\overline{y} - \tilde{x}_j - 1)}{F(\overline{y} - \hat{x}_j) - F(\overline{y} - x^*_j)} \;=\; \infty.$$

Proof: idea (cont.)

Step 2 (cont.): And this implies that for sufficiently high $\beta\delta$,

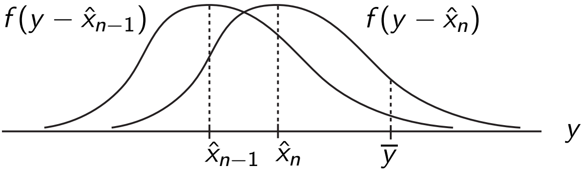
$$\underbrace{\delta(F(\overline{y} - x_j') - F(\overline{y} - \tilde{x}_j - 1))[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)]}_{\text{future gains}}$$

$$> \quad \underbrace{w_j(x_j') - w_j(\tilde{x}_j + 1)}_{\text{current losses}},$$

a contradiction.

Proof: idea (cont.)

Step 3: If any subsequence of highest optimal effort x_n^* is bounded, then by Step 2, we have $x_n^* \to \hat{x}_n$ along that subsequence. But then higher types are more likely conditional on the cutoff.



This contradicts the voter's indifference condition.

Proof: idea (cont.)

Step 3 (cont.): It follows that $x_n^* \to \infty$. Our bound on policy responsiveness implies that no subsequence of $x_{*,n}$ can diverge to infinity, so by Step 2, $x_{*,n} \to \hat{x}_n$. This proves (ii).

Step 4: Because mixed policy strategies are ordered by type, and using Step 2, it follows that for all j, $x_j^* \to \hat{x}_j$. This proves (iii).

Outline

Introduction

Modeling framework

Two-period benchmark

Infinite-horizon model

Asymptotic characterization

Bounds on policy responsiveness

Voter commitment

Conclusion

General bound

Theorem: For all levels of office benefit $\beta \geq 0$ and all discount factors $\delta \in [0,1)$, in every stationary electoral equilibrium, the expected payoff to the voter from a first-term office holder is strictly less than the expected payoff from the ideal point of the type *n* politician, i.e.,

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j(dx) \quad < \quad \mathbb{E}[u(y)|\hat{x}_n].$$

Proof: idea

Suppose that in some equilibrium, the inequality is violated, so:

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j(dx) \quad \geq \quad \mathbb{E}[u(y)|\hat{x}_n].$$

Because second-term office holders shirk and there is a possibility that $j < n$, the payoff from re-electing an incumbent is at most

$$V^I(\tilde{y}) \quad < \quad \mathbb{E}[u(y)|\hat{x}_n] + \delta V^C.$$

And because the voter can always choose $\overline{y} = \infty$, we have

$$V^C \quad \geq \quad \frac{1}{1-\delta} \sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j(dx).$$

Proof: idea (cont.)

Then we have

$$\begin{aligned} V^I(\tilde{y}) &< \mathbb{E}[u(y)|\hat{x}_n] + \delta V^C \\ &\leq \left(\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j(dx) \right) + \delta V^C \\ &\leq (1-\delta)V^C + \delta V^C \\ &= V^C. \end{aligned}$$

But then $\overline{y} = \infty$.

This implies that first-term office holders shirk, a contradiction.

Further bounds

We can show that for a given level β, the voters' expected payoff from policy choices of first-term office holders is bounded *strictly* below the payoff from the ideal point of the type n politician, with the bound uniform across δ.

That is, for all β, there exists $\overline{u} < \mathbb{E}[u(y)|\hat{x}_n]$ such that for all $\delta \in [0, 1)$, we have for every stationary electoral equilibrium,

$$\sum_j p_j \int_X \mathbb{E}[u(y)|x]\pi_j(dx) \quad \leq \quad \overline{u}.$$

Further bounds (cont.)

We can also show that if the type n politician has prior probability $p_n \to 0$, then the limiting expected payoff to voters from the policy choices of first-term office holders is less than or equal to the payoff from the ideal point of the type n politician.

That is, for all selections of stationary electoral equilibria, we have

$$\limsup \sum_j p_j \int_X \mathbb{E}[u(y)|x]\pi_j(dx) \quad \leq \quad \mathbb{E}[u(y)|\hat{x}_{n-1}]$$

as $p_n \to 0$.

Outline

Stationary electoral partial equilibria

Now assume that voters can commit to an arbitrary cutoff $\tilde{y}$.

Given $\tilde{y}$, we say $\tilde{\pi} = (\tilde{\pi}_1, \ldots, \tilde{\pi}_n)$ is a *stationary electoral partial equilibrium* if each $\tilde{\pi}_j$ places probability one on maximizers of

$$w_j(x) + \delta(1 - F(\tilde{y}|x))[w_j(\hat{x}_j) + \beta - (1 - \delta)\tilde{V}^C].$$

Here, $\tilde{V}^C$ is the continuation value of a challenger given the politicians' strategies and cutoff $\tilde{y}$.

Voter welfare from fixed cutoff

Theorem: Assume $\delta > 0$ and fix $\tilde{y}$. If office benefit $\beta \geq 0$ is sufficiently large, then for every selection of stationary electoral partial equilibria, we have $\tilde{V}^C \to \infty$.

Thus, if β is large, then in every stationary electoral equilibrium, we have

$$\overline{y} > \tilde{y} \quad \text{and} \quad V^C < \tilde{V}^C,$$

so equilibrium cutoffs for re-election are too high.

Voter welfare optima

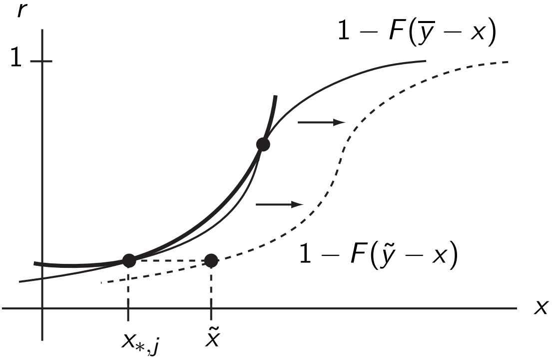
Assume each w_j is concave.

Theorem: Assume $\delta > 0$, and let office benefit $\beta \geq 0$ become large. For every stationary electoral equilibrium and every partial equilibrium with cutoffs $\tilde{y} > \overline{y}$, the continuation value of a challenger from committing to $\tilde{y}$ is lower than the equilibrium value: $\tilde{V}^C < V^C$.

Thus, the cutoffs that maximize voter welfare are below the equilibrium cutoffs.

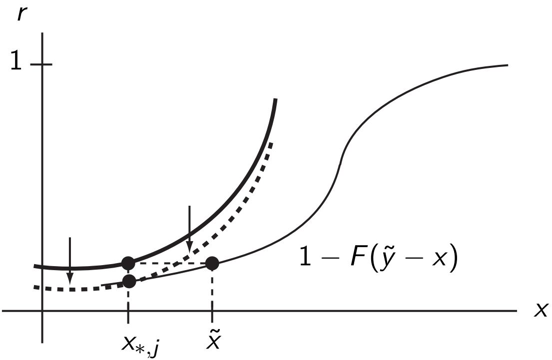
Proof: idea

Recalling the optimization problem of the type j politician over (x, r) pairs, the increase in the cutoff has the effect of shifting the constraint to the right.



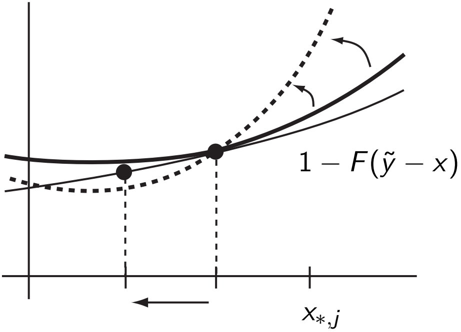
Proof: idea (cont.)

Fixing the continuation value at the equilibrium level, V^C, optimal efforts for the politician are all below the lowest equilibrium level $x_{*,j}$.



Proof: idea (cont.)

If we had $\tilde{V}^C > V^C$, then indifference curves would be steeper…



…causing optimal effort to decrease further.

Outline

Contributions

Existence of equilibrium, rectifying an error in the proof of Banks and Sundaram (1998).

Characterization of equilibria when politicians are highly office motivated: the highest type of office holder mixes between shirking and arbitrarily high effort, while other types shirk.

Bound on the voter's equilibrium expected payoff from effort of first-term office holders. The key difficulty is the commitment problem of the voter.

Suboptimally high cutoffs when office benefit is high.

$*$ Qualitative difference with the two-period model, where responsiveness is unbounded as office benefit increases.

Parting thought

Interestingly, the bound on responsiveness does not appear to depend on the presence of a term limit.

Consider the two-type model with no term limit, and suppose there is an equilibrium where the voter re-elects an incumbent if and only if her posterior belief that the incumbent type is high exceeds $\overline{b} \in (0, 1)$.

If the posterior belief b is very close to 1, then the incumbent will essentially shirk for an arbitrarily large duration. So the value of reelecting the incumbent satisfies

$$V^C \leq V^I(b) \to \mathbb{E}[u(y)|\hat{x}_2].$$