Term Limits and Bounds on Policy Responsiveness in Dynamic Elections*

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Abstract

I analyze equilibria of a dynamic political agency model of elections with a two-period term limit in the presence of adverse selection and moral hazard. In equilibrium, office holders' policy choices are ordered by type; politicians exert positive effort in the first term of office; and as politicians become highly office motivated, the highest politician type mixes with positive probability between shirking and choosing arbitrarily high policies. Nevertheless, the commitment problem of voters imposes a bound on equilibrium expected effort exerted by politicians that holds uniformly across the level of office benefit and the rate of time discounting. In particular, when politicians are office motivated, voters are too demanding in equilibrium, and voter welfare would increase if it were possible to commit to a lower cutoff for re-election.

1 Introduction

An essential feature of representative democracy is the periodic reconsideration of political agents by their principal, the electorate. Elections allow voters to express approval or disapproval of their elected delegates, and at the same time they provide politicians with incentives to shun parochial interests in favor of the public good. The operation of these incentives is complicated by informational asymmetries—in the form of adverse selection and moral hazard—and by the extended time horizon over which interaction takes place. We would expect these electoral incentives to be mitigated by term limits but enhanced if the benefit of holding office per se is

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larger or the weight placed by politicians on short-term gains is smaller. These issues are properly addressed in a dynamic framework that explicitly accounts for informational asymmetries, but doing so requires precise attention to the ensuing technical complexities.

This paper considers elections in a dynamic political agency model similar to that analyzed by Banks and Sundaram (1998). In this framework, an office holder's policy choice (i.e., effort) is unobserved by voters and stochastically determines an outcome that is observable; moreover, politicians' preferences are indexed by their types, which are also unobserved by voters. Once an outcome is generated by the choice of a first-term office holder, voters must decide whether to replace the incumbent with a challenger, whereas politicians are automatically removed from office after their second term; and this process is repeated ad infinitum. The paper provides a partial characterization of stationary electoral equilibria for arbitrary parameterizations, along with sharper results for the case of highly office-motivated politicians. Of interest is the possibility that in response to electoral incentives, elected politicians decline the opportunity to shirk, foregoing policies close to their own ideal points and instead choosing policies that are good for voters; this phenomenon is referred to as *policy responsiveness.* Generally, office holders choose policies strictly higher than their ideal points, and as politicians become highly office motivated, the highest politician type mixes with positive probability between shirking (choosing policies close to its ideal point) and choosing arbitrarily high policies. Thus, a minimal level of policy responsiveness is achieved in equilibrium. However, the main substantive conclusion of the paper, collected in Theorems 4–6, is that the voters' equilibrium payoff is bounded above by the expected utility from the ideal point of the highest politician type. The upper bound holds generally across all parameter values. Moreover, it is robust to the introduction of very high types with very small prior probability, and for a given level of office benefit, the bound holds strictly regardless of the level of citizens' patience.

This bound on policy responsiveness described above is due to the *commitment problem of voters.* It is assumed that the electorate cannot write a binding contract to re-elect an incumbent following policy outcomes above a predetermined level. Rather, electoral outcomes are determined endogenously in equilibrium, where voters compare the expected stream of payoffs from re-electing the incumbent vs. the continuation value of a challenger; and as the voter's continuation value of a challenger increases, her incentive to re-elect the incumbent decreases, and elections lose their disciplining effect. More precisely, because second-term office holders simply choose their ideal points, the best case for voters is that a re-elected politician chooses the ideal point of the highest politician type. If the voters' continuation value of a challenger exceeded this payoff, then voters would always have an incentive to remove an incumbent after her first term to insert a more productive challenger. But then, of course, first-term office holders would have no incentive to depart from their ideal points in the first place. Therefore, the incentives of voters imply a general bound on the possibility of policy responsiveness: the continuation value of a challenger can never exceed the expected payoff from the ideal point of the highest politician type. Furthermore, if politicians are highly office motivated and voters are allowed to commit to a re-election standard at the beginning of the game, then the optimal cutoff for voters is below the equilibrium cutoff, and voter welfare would strictly increase if a lower cutoff were used. Thus, voters are too demanding in equilibrium relative to the optimum with commitment.

A contribution of this paper is the proof of existence of a perfect Bayesian equilibrium such that the policy choices of first-term office holders are history independent and the voters' decision to re-elect a first-term incumbent depends only on the observed policy outcome generated by the office holder's unobserved action in office. A byproduct of the existence result is the solution of an error in the proof of Banks and Sundaram's (1998) Proposition 3.1: those authors define a correspondence and attempt to apply Glicksberg's fixed point theorem to deduce existence of equilibrium, but the domain of their correspondence is not convex; thus, Glicksberg's theorem cannot be applied. The non-convexity arises because the authors incorporate a monotonicity property of policy strategies into the domain of their correspondence: they assume that the supports of policy strategies are weakly ordered by politician type. This is helpful to them because given policy strategies with this property, the voters' optimal responses comprise a convex set of cutoffs; in its absence, we are not guaranteed convexity of the set of optimal cutoffs. The approach of this paper is to omit monotonicity from the domain of the correspondence and to obtain the property ex post, after a fixed point is derived.

Section 2 contains a literature review. The model is described in Section 3, the concept of stationary electoral equilibrium is defined in Section 4, and preliminary results are set forth in Section 5. Existence and a partial characterization are provided in Section 6, along with a discussion of technical issues arising in Banks and Sundaram (1998). Section 7 sharpens the characterization for the case of highly office motivated politicians. Section 8 presents the bound on the possibility of policy responsiveness in the form of three theorems, Section 9 examines voter welfare under the commitment assumption, and Section 10 concludes. Proofs omitted from the text are provided in the appendix.

2 Related literature

Following the literature on electoral accountability, and consistent with the citizen-candidate approach of Osborne and Slivinski (1996) and Besley and Coate (1997), the dynamic political agency model analyzed in this paper assumes that neither candidates nor voters can make binding promises about future behavior. In particular, candidates cannot commit to policy platforms before an election, and thus they do not compete for votes in the manner of the Downsian electoral model; this is especially natural in the current framework, as actions of politicians are unobservable to voters, so that they would have no way of verifying that a given platform was implemented. Rather, electoral incentives arise from the desire of a first-term office holder to signal that she is a good type, in which case voters would prefer to re-elect the incumbent over the prospect of a relatively unknown challenger. This commitment problem of politicians is familiar from the accountability literature and featured in, e.g., Acemoglu et al. (2005), whereas the focus of the current paper is on the voters' lack of commitment power.

The electoral accountability literature traces to Barro (1973), in which there is a single politician type and policy choices are directly observable by voters, and Ferejohn (1986), who considers agency problems in which policy choices are subject to imperfect monitoring. Duggan (2000) analyzes elections under pure adverse selection, where policy choices are observable but politicians are privately informed about their preferences, and Bernhardt et al. (2004) consider the model with term limits and pork barrel spending.[1] The infinite-horizon model with a two-period term limit and combined adverse selection and moral hazard is the subject of Banks and Sundaram (1998), who show that in a stationary electoral equilibrium, each politician type chooses higher effort in the first term of office than the second (if re-elected), and that re-elected politicians are more productive on average than an untried challenger due to selection effects.

In the infinite-horizon model without term limits, Banks and Sundaram (1993) depart from the restriction to stationary electoral equilibria and show existence of equilibria in the class of trigger strategies, in which voters and politicians use history-dependent strategies that condition on past outcomes generated by an incumbent in addition to the voters' posterior beliefs. In particular, if the realized policy outcome falls below a given cutoff level during a politician's tenure, then the politician shirks (i.e., chooses zero effort) thereafter, and voters remove the incumbent from office. This approach has several shortcomings. First, even if the incumbent is a good type with arbitrarily high probability, there is always a positive probability that a bad outcome will be realized and that the trigger will be activated. Then voters will be compelled to replace an otherwise desirable incumbent; while optimal given the anticipated actions of the incumbent, this behavior may

[1] The adverse selection model is extended to allow for multiple dimensions by Banks and Duggan (2008), partisan challenger selection by Bernhardt et al. (2009), and valence by Bernhardt et al. (2011).

run counter to intuition. Second, the exact value of the trigger is not pinned down in the model, and in fact a continuum of levels can be supported in equilibrium. Third, the analysis relies on the assumption that all politician types are equivalent when they shirk; without that assumption, the trigger strategy construction breaks down, as voters may have an incentive to re-elect an incumbent who is a good type with high probability, even if it is known that she will shirk in the future.[2]

The informational structure of the dynamic political agency model is similar to the career concerns model of Holmstrom (1999) and the three-period version examined by Ashworth (2005).[3] In this environment, agents are differentiated by their production ability, and an agent's type is initially unknown to the principal and agent. Effort determines output via a noisy production technology, and thus learning about the agent's type is symmetric; in contrast, the dynamic political agency model of the current paper assumes that a politician's type is a preference parameter, and that this is private information of the politician. In the model of Holmstrom (1999), the agent is infinitely-lived and not subject to a term limit. He finds that when ability is persistent, the agent's type is learned over time, and effort exerted by the agent goes to zero as the value of signaling becomes small. A similar result is obtained by Ashworth (2005), as effort declines over time, and by Banks and Sundaram (1998), as first-term politicians exert positive effort to increase their chances of re-election.

The two-period version of the dynamic political agency model is analyzed in Duggan and Martinelli (2015). Although policy choices in the second period of this model are trivial due to endgame effects, the existence of equilibrium still requires a fixed point argument, because of the interaction between optimal policy choices in the first period and the updating of voter beliefs. The authors establish existence of equilibrium in which each type of politician mixes between at most two policies ("taking it easy" and "going for broke"), and they show that increasing office motivation leads to arbitrarily high expected policy outcomes in the first period. This finding contrasts with the results of the current paper, where the commitment problem of voters implies an upper bound on expected policy outcomes, and it points to a discrepancy between the two-period model and the infinite-horizon model with two-period term limit.[4] Thus, the analysis generates

[2]Related work on dynamic elections with an endogenous state variable includes Duggan and Forand (2014), who give conditions under which electoral equilibria solve the dynamic programming problem of a representative voter, and Battaglini (2014), who assumes parties can commit to fiscal platforms prior to each election and that these choices affect the level of public debt.

[3]Another branch of the electoral accountability literature examines the possibility of political inefficiencies due to pandering; cf. Canes-Wrone et al. (2001) and Maskin and Tirole (2004). In this approach, politicians have private information about the state of the world and may take action contrary to their information to secure re-election.

[4]That paper contains detailed discussion of related work in the two-period framework by Fearon

the perhaps surprising observation that the two-period model, which is more common in applications, is qualitatively different than the infinite-horizon model with a two-period term limit.[5]

Caselli et al. (2014) analyze a two-period model in which politician ability may take three values, voters are risk neutral, and voting is stochastic, i.e., voter payoffs for re-electing the incumbent are subject to a shock that is unobserved by the politician. The model further differs from the current paper in that policy outcomes are independent of effort exerted by the first-term politician; rather, effort expended by the politician increases the signal received by voters (indicating that the politician is a high ability type) but has no material impact on them. Because effort is wasted in their model, the optimal cutoff for re-election when voters can commit differs from the optimal cutoff studied in Section 9, below. Their problem is purely one of selecting good types, and not incentivizing effort in the first period, and Caselli et al. find that the equilibrium cutoff for voters is too low relative to the optimum.

Camara and Bernhardt (2015) examine a model of dynamic elections in which voters receive an exogenous signal of challenger quality. In the spirit of the current paper, where the cutoff for re-election is too demanding in equilibrium, the latter authors find that the equilibrium cutoff can be too strict. Their environment is spatial, and an incumbent is re-elected if and only if she chooses policy belonging to a "win set" centered at the median ideal point. They show that if voters receive a small increase in information about challenging candidates, then the win set becomes smaller as a consequence, making compromise more costly and decreasing the welfare of all voters.

3 Dynamic political agency model

This paper analyzes repeated elections to fill a political office that is subject to a two-period term limit. Elections are held over an infinite horizon, with periods indexed $t = 1, 2, \ldots$. In each period t, an incumbent office holder makes a policy choice $x_t \in X = \mathbb{R}_+$, a policy outcome $y_t \in Y \subseteq \mathbb{R}$ is drawn according to the distribution $F(\cdot|x_t)$, and a challenger is drawn without replacement.[6,7] If the incumbent is in her first term, then an election is held, and the winner takes office next period; and otherwise, if the incumbent is in her second term, then the challenger

(1999), Besley (2006), Ashworth and Bueno de Mesquita (2008), and Persson and Tabellini (2000).

[5] See Duggan and Martinelli (2015) for a survey of this literature.

[6] The set of challengers can be modeled as a separate pool, or if the electorate is a continuum, then challengers may be drawn from a continuous distribution over voters. To simplify the calculations of voters, it is assumed that the probability that any given voter is selected as challenger is zero.

[7] Banks and Sundaram (1998) assume that X is compact, which can appreciably change the structure of equilibria and lead to a continuum of pooling equilibria of an uninteresting form.

assumes office automatically. The preferences of politicians are represented by a type $j \in T = \{1, \ldots, n\}$, with $n \geqslant 2$, and are private information; voters do not observe the politicians' types. The policy choice x_t is also not directly observed by voters, but the outcome y_t is publicly observed. The types of politicians are identically and independently distributed, with $p_j > 0$ denoting the prior probability that a politician is type j. Consistent with the citizen-candidate approach, politicians and voters cannot make binding commitments regarding future actions, and thus political campaigns are suppressed in the analysis.

Voters, including politicians who are out of office, receive a payoff $u(y_t)$ from policy outcome y_t in period t, so that for simplicity, the electorate is homogeneous. A type j politician in office receives payoff $w_j(x_t) + \beta$ from policy choice x_t, where $\beta \geqslant 0$ is a non-negative office benefit.[8] Citizens have a common rate of time discounting, which is represented by the discount factor $\delta \in [0, 1)$. Given a sequence $x_1, x_2, \ldots$ of choices and a sequence $y_1, y_2, \ldots$ of outcomes, the total payoff of a citizen is the discounted sum of per period payoffs,

$$\sum_{t=1}^{\infty} \delta^{t-1} [I_t(w_j(x_t) + \beta) + (1 - I_t)u(y_t)],$$

which is written assuming the citizen in her role as politician is type j, and where $I_t \in \{0, 1\}$ is an indicator variable that takes a value of one when the citizen holds office in any period t and takes a value of zero otherwise.

Note that a politician who is removed from office does receive a flow of payoffs from future policy outcomes in their role as voter. This contrasts with the model of Banks and Sundaram (1998), where a politician removed from office receives a zero payoff thereafter. The latter assumption would seem appropriate for an agent who leaves the game after being terminated, but it is less compelling in the citizen-candidate context, where politicians are viewed as voters who serve in office for a time and, presumably, return to the role of voter after that time. In the current model, because they receive a flow of payoffs after they are removed from office, politicians' optimal policy choices will depend on the expected payoff generated by a randomly drawn challenger; this added realism complicates the equilibrium analysis, but all of the results of the paper carry over to the simpler setting in which a politician receives a zero payoff after leaving office.[9]

Assume voter preferences over policy outcomes are monotonic, so that $u: Y \rightarrow \mathbb{R}$ is strictly increasing, and write $\mathbb{E}[u(y)|x]$ for the voters' expected payoff from

[8] Banks and Sundaram (1998) do not explicitly parameterize politician payoffs by the office benefit term β, but their conditions (A5)–(A7) allow for the addition of a constant $\beta > 0$ to payoffs, so their analysis accommodates office benefit by incorporating it directly into the payoff functions w_j.

[9] Under the assumption that politicians receive a zero payoff after leaving office, we would modify (C7), below, so that for all types j, we have $w_j(\hat{x}_j) + \beta > 0$.

the distribution $F(\cdot|x)$ over policy outcomes determined by policy choice x. Since the electorate is homogeneous, the analysis assumes a representative voter in the sequel. Moreover, assume that while in office, the payoffs $w_j\colon X \to \mathbb{R}$ of each politician type are continuous. Thus, we have:

(C0) u is strictly increasing, and for all j, w_j is continuous.

In addition, we impose the following standard supermodularity assumption on politician payoffs:

(C1)
$$\text{for all } x, x' \in X \text{ with } x > x' \text{ and all } j < n, \quad w_{j+1}(x) - w_{j+1}(x') > w_j(x) - w_j(x').$$

Informally, the latter means that differences in payoffs are strictly monotone in politician type. In addition, assume that for each politician type j, the payoff function w_j has a unique maximizer $\hat{x}_j > 0$, that the ideal points of office holders are strictly ordered according to type, and that the maximized utilities are weakly increasing in type:

(C2)
$$\text{for all } j, w_j \text{ is uniquely maximized at } \hat{x}_j > 0, \text{ and } j < n \text{ implies } \hat{x}_j < \hat{x}_{j+1} \text{ and } w_j(\hat{x}_j) \leqslant w_{j+1}(\hat{x}_{j+1}).$$

Lastly, assume that the office holders' payoffs are unbounded below when policy choices are arbitrarily high:

(C3)
$$\text{for all } j, \lim_{x \to \infty} w_j(x) = -\infty.$$

By assumption of a unique maximizer, (C3) would be implied if, e.g., office holders' payoffs were concave.

A special case that highlights the potential applicability of the model is that in which politicians incur a cost for higher policy choices, with higher types weighting cost less. The model captures payoffs with the parameterized functional form

$$w_j(x) \quad = \quad (1 - \lambda + \lambda\theta_j)\left(v(x) - \frac{1}{\theta_j}c(x)\right) + \kappa_j,$$

where $v\colon X \to \mathbb{R}$ is strictly increasing and concave, $c\colon X \to \mathbb{R}_+$ is strictly increasing and strictly convex, $\lambda \in [0,1]$, and κ_j, θ_j are type-dependent parameters, with $\kappa_1 \leqslant \kappa_2 \leqslant \cdots \leqslant \kappa_n$ chosen to satisfy (C2) and with $0 < \theta_1 < \theta_2 < \cdots < \theta_n$.[10] The functional form for politicians' payoffs admits two sharper specifications that are

[10] In addition, to ensure existence of an interior maximizer $\hat{x}_j$, we can assume that v and c are differentiable, that $\theta_1 v'(0) > c'(0)$, and that $\lim_{x\to\infty} c'(x) = \infty$.

worthy of note. One common specification is *quadratic payoffs,* in which case $w_j(x) = -(x-\hat{x}_j)^2$. To obtain this, we set

$$v(x) = 2x,\ c(x) = x^2,\ \kappa_j = -\hat{x}_j^2,\ \lambda = 1,\ \theta_j = \hat{x}_j.$$

Another specification of interest is *exponential payoffs,* in which $w_j(x) = -e^{x-\hat{x}_j} + x + 1$. This is obtained by setting

$$v(x) = x,\ c(x) = e^x,\ \kappa_j = 1,\ \lambda = 0,\ \theta_j = e^{\hat{x}_j}.$$

Politician preferences are defined over policy choices, rather than outcomes, but this apparent difference from voters can be reconciled by setting the term $v(x)$ equal to the expected payoff $\mathbb{E}[u(y)|x]$ from policy outcomes generated by the choice x, so politicians share the voter's preferences over policy outcomes. Setting $\lambda = 0$, for example, an office holder differs from other citizens only by the cost term $(1/\theta_j)c(x)$, and higher policy choices are more costly for lower politician types.

Assume that the outcome distribution $F(\cdot|x)$ has a density $f(\cdot|x)$ with full support on the open, convex set $Y \subseteq \mathbb{R}$ for all x, and that this density is jointly continuous and single-peaked:

(C4) Y is open and convex, $f(y|x) > 0$ for all x and all y, and $f(y|x)$ is jointly continuous in (x,y) and strictly quasi-concave in y.

Assume also that f satisfies the standard monotone likelihood ratio property, or MLRP:

(C5) $$\text{for all } x > x' \text{ and all } y > y',\ \frac{f(y|x)}{f(y|x')} > \frac{f(y'|x)}{f(y'|x')}.$$

This implies that greater policy outcomes induce the voter to favorably update her beliefs about the policy adopted by the incumbent in the first period. As is well-known, the MLRP implies that when $x > x'$, the distribution $F(\cdot|x)$ first order stochastically dominates $F(\cdot|x')$, so that $F(y|x)$ is strictly decreasing in x. Next, let

$$y^- = \inf Y \quad \text{and} \quad y^+ = \sup Y$$

be the infimum and supremum of Y, where we allow $y^- = -\infty$ or $y^+ = \infty$ to capture unbounded policy outcomes. At times, it is useful to extend the domain of $F(\cdot|x)$ to include these bounds, in which case we specify that $F(y^+|x) = 1$ and $F(y^-|x) = 0$. Assume that arbitrarily extreme signals become arbitrarily informative, in a uniform way:

(C6) $$\text{for all } x > x',\ \lim_{(z,z',\bar{y})\to(x,x',y^+)} \frac{f(\bar{y}|z)}{f(\bar{y}|z')} \to \infty \text{ and } \lim_{(z,z',\bar{y})\to(x,x',y^-)} \frac{f(z|\bar{y})}{f(z'|\bar{y})} \to 0.$$

Note that (C6) is satisfied when x is a shift parameter for density h satisfying (C5) and a pointwise version of (C6).[11] A special case is that in which the policy choice x is a shift parameter on a fixed density over $Y = \mathbb{R}$, so the density can be written $f(y|x) = h(y-x)$ for a density $h(\cdot)$. Then (C4)–(C6) hold if, e.g., h is a normal density. When the policy choice is a shift parameter for h, the probability that the realized outcome is less than y given policy x is written $H(y-x) = F(y|x)$.

Given the structure imposed above on payoffs, it is natural to view the choice of x as an effort level, which stochastically determines an outcome y that can be viewed as a level of public good. Then a minimal criterion for policy responsiveness is that a type j office holder exert positive effort, i.e., she chooses $x > \hat{x}_j$, in her first term of office, and a more demanding criterion is that the expected effort of a newly elected challenger is large. Of special interest are the effect of varying model parameters such as office benefit, and a particular focus will be the possibility that policy responsiveness grows when office holders are highly office motivated.

4 Stationary electoral equilibrium

The analysis focuses on perfect Bayesian equilibria of the political agency model. Strategies in this dynamic game are potentially highly complex, as policy choices and votes could conceivably depend arbitrarily on observed histories of policy outcomes and electoral outcomes. To preclude implausible behavior by voters and politicians, I impose refinements that strengthen perfect Bayesian equilibrium by limiting the extent of history dependence exhibited by citizens' strategies. A *stationary strategy for a type j politician* is a pair (π_j^1, π_j^2), where $\pi_j^1 \in \Delta(X)$ specifies the politician's mixture over policy choices in her first term, and $\pi_j^2 : X \to \Delta(X)$ specifies the mixture of policy choices in her second term as a function of her earlier choice.[12] A *stationary strategy for the voter* is a measurable function

[11]To see this claim, assume:

$$\text{(C6}'\text{)} \qquad \text{for all } x > x',\ \lim_{y \to -\infty} \frac{f(y|x)}{f(y|x')} = \lim_{y \to y^+} \frac{f(y|x')}{f(y|x)} = 0.$$

Let $\hat{z}$ denote the mode of h. Note that given any $\varepsilon > 0$ with $\varepsilon < (x-x')/2$, for $\overline{y}$ sufficiently large that $\overline{y} - x$ exceeds $\hat{z}$, for $z \in (x-\varepsilon, \overline{y}-\hat{z})$, and for $z' < x' + \varepsilon$, we have

$$\frac{h(\overline{y}-z)}{h(\overline{y}-z')} \geqslant \frac{h(\overline{y}-x+\varepsilon)}{h(\overline{y}-x'-\varepsilon)} \to \infty$$

as $\overline{y} \to y^+$, where the limit follows from (C6′). The second limit in (C6) is obtained similarly.

[12]Given a closed subset S of finite-dimensional Euclidean space, the notation $\Delta(S)$ represents the set of Borel probability measures with support in S. Endow $\Delta(S)$ with the relative topology induced by the weak* topology on the space of signed Borel measures.

$\alpha: Y \to [0,1]$ specifying the probability that a first-term incumbent is re-elected as a function of policy outcomes. Note that π_j^1, π_j^2, and α could conceivably depend on histories of previous office holders, but stationarity isolates strategies in which such conditioning does not occur. A *belief system* is a function $\mu: Y \to \Delta(T \times X)$, where $\mu(\cdot|y)$ represents the voter's posterior beliefs about an incumbent's type and policy choice conditional on policy outcome y in her first term of office. Then the marginal $\mu_T(\cdot|y)$ gives the voter's posterior beliefs about the incumbent's type.

A strategy profile $\sigma = (\pi_1^1, \pi_1^2, \ldots, \pi_n^1, \pi_n^2, \alpha)$ is *sequentially rational* given belief system μ if no politician can gain by deviating to a different policy choice in any term of office, and for all policy outcomes y, the voter opts for the candidate that offers the highest expected discounted payoff conditional on her information. Beliefs μ are *consistent* with σ if for all $y \in Y$, $\mu(\cdot|y)$ is derived using Bayes' rule. A *stationary perfect Bayesian equilibrium* is an assessment $\psi = (\sigma, \mu)$ such that σ is sequentially rational given μ and such that μ is consistent with σ.

Further refinements reflecting two additional behavioral simplifications beyond stationarity will be imposed. First, write $V^I(y|\psi)$ for the voter's expected discounted payoff from re-electing a first-term incumbent conditional on policy outcome y, and write $V^C(\psi)$ for the voter's payoff from electing a challenger. A pair $\psi = (\sigma, \mu)$ is *deferential* if the voter favors the incumbent when indifferent, or more formally, given any policy outcome y, the voter votes for the incumbent if and only if $V^I(y|\psi) \geqslant V^C(\psi)$.[13] Second, ψ is *monotonic* if there is some utility cutoff $\underline{u} \in \mathbb{R} \cup \{-\infty, \infty\}$ such that for all policy outcomes y, the voter votes to re-elect the incumbent if and only if the payoff from y meets or exceeds that cutoff, i.e.,

$$\alpha(y) \quad = \quad \begin{cases} 1 & \text{if } u(y) \geqslant \underline{u}, \\ 0 & \text{else.} \end{cases}$$

Since u is strictly increasing, this entails that there is a cutoff outcome $\overline{y} \in Y \cup \{y^-, y^+\}$ such that a first term office holder is re-elected if and only if the policy outcome meets or exceeds $\overline{y}$. Here, of course, the cutoff $\overline{y} = y^+$ implies that no incumbent is ever re-elected, and $\overline{y} = y^-$ implies that all incumbents are always re-elected. Finally, $\psi = (\sigma, \mu)$ is a *stationary electoral equilibrium* if it is a stationary perfect Bayesian equilibrium that is deferential and monotonic.

Whereas stationarity implies that the continuation value of a challenger is history independent, the continuation value of re-electing a first-term incumbent depends, via the voter's posterior beliefs, on the policy outcome realized by the efforts of the office holder. Clearly, stationarity also implies that the equilibrium policy

[13] Banks and Sundaram (1998) do not impose the requirement that an equilibrium is deferential, and owing to their assumption that X is compact, equilibria may indeed require all types to pool on $\max X$, implying that the voter must sometimes reject an incumbent when indifferent.

choice of a second-term office holder, if elected, is simply her ideal point. Using the latter observation, the continuation values $V^I(y|\psi)$ and $V^C(\psi)$ are the unique solutions to the recursions

$$V^I(y|\psi) \quad = \quad \sum_k \mu_T(k|y)\left[\mathbb{E}[u(y)|\hat{x}_k] + \delta V^C(\psi)\right] \tag{1}$$

and

$$\begin{aligned} V^C(\psi) \quad = \quad & \sum_j p_j \int_x \Big[\mathbb{E}[u(y)|x] + \delta[(1-F(\overline{y}|x))(\mathbb{E}[u(y)|\hat{x}_j] \\ & + \delta V^C(\psi)) + F(\overline{y}|x)V^C(\psi)]\Big]\pi_j^1(dx). \end{aligned}$$

Solving for $V^C(\psi)$ explicitly, we have

$$V^C(\psi) \quad = \quad \frac{\sum_j p_j \int_x \left[\mathbb{E}[u(y)|x] + \delta(1-F(\overline{y}|x))\mathbb{E}[u(y)|\hat{x}_j]\right]\pi_j^1(dx)}{1-\delta\sum_j p_j \int_x [(1-F(\overline{y}|x))\delta + F(\overline{y}|x)]\pi_j^1(dx)}. \tag{2}$$

Stationary electoral equilibria then satisfy four necessary conditions. *First,* as noted above, a second-term office holder's mixture π_j^2 over policy choices puts probability one on her ideal point $\hat{x}_j$, i.e., for all j and all $x \in X$, $\pi_j^2(x)(\{\hat{x}_j\}) = 1$. *Second,* updating of voter beliefs follows Bayes' rule: conditional on policy outcome y, the posterior probability the politician is type j is

$$\mu_T(j|y) \quad = \quad \frac{p_j \int_x f(y|x)\pi_j^1(dx)}{\sum_k p_k \int_x f(y|x)\pi_k^1(dx)}.$$

Note that because the prior places positive probability on each type, we have $\mu_T(j|y) > 0$ for each type j. Moreover, it follows that the continuation value of re-electing the incumbent is continuous in the observed policy outcome. *Third,* since $V^I(y|\psi)$ is continuous in y, if the cutoff outcome $\overline{y}$ belongs to Y, then conditional on observing $\overline{y}$, the voter must be indifferent between re-electing a first-term incumbent and electing a challenger. Formally, using (1), this condition is

$$\sum_j \mu_T(j|\overline{y})\left[\mathbb{E}[u(y)|\hat{x}_j] + \delta V^C(\psi)\right] \quad = \quad V^C(\psi).$$

Simplifying, this means that the expected payoff from the incumbent's policy choice in the second term is equal to the (normalized) continuation value of a challenger:

$$\sum_j \mu_T(j|\overline{y})\mathbb{E}[u(y)|\hat{x}_j] \quad = \quad (1-\delta)V^C(\psi). \tag{3}$$

If $\overline{y} = y^+$, then an incumbent is never re-elected, and the assumption of deferential voting implies that the inequality

$$\sum_j \mu_T(j|\overline{y})\mathbb{E}[u(y)|\hat{x}_j] \quad < \quad (1-\delta)V^C(\psi),$$

must hold; and if $\overline{y} = y^-$, then a first-term office holder is always re-elected, and the inequality must hold weakly in the opposite direction. *Fourth,* each politician type, knowing that she is re-elected after the first term of office if and only if $y \geqslant \overline{y}$, mixes over optimal actions in her first term of office, i.e., π_j^1 puts probability one on solutions to

$$\max_{x \in X} w_j(x) + \delta\Bigg[(1-F(\overline{y}|x))[w_j(\hat{x}_j) + \beta + \delta V^C(\psi)] + F(\overline{y}|x)V^C(\psi)\Bigg]. \qquad (4)$$

When politician payoffs w_j are differentiable, if the cutoff $\overline{y}$ belongs to Y, then policy choices x in the support of π_j^1 solve the necessary first order condition, i.e.,

$$w_j'(x) \quad \leqslant \quad -f(\overline{y}|x)\delta[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)], \qquad (5)$$

with equality for $x > 0$.

Stationary electoral equilibrium is a refinement of perfect Bayesian equilibrium, so that after all histories, no citizen can increase her expected discounted payoff by deviating to another strategy (stationary or non-stationary). And although we allow in principle for behavior as a general function of updated priors, the two additional restrictions we impose—namely, deferential and monotonic voting—capture some intuitive ideas. The assumption of deferential strategies is a form of *prospective voting,* in which the representative voter acts as though pivotal in the election, and the assumption of monotonicity formalizes *retrospective voting,* in which a voter asks, "What have you done for me lately?" and votes to re-elect the incumbent if the policy outcome delivered by the politician satisfies a certain threshold. Thus, in a stationary electoral equilibrium, prospective and retrospective voting are compatible and both describe the behavior of voters, and the choices of office holders are optimal given these voting strategies.

5 Preliminary analysis

To facilitate the analysis, it is henceforth assumed that first-term office holders are in principle interested in re-election, a condition formalized as follows:

(C7) for all j, $w_j(\hat{x}_j) + \beta \quad > \quad \mathbb{E}[u(y)|\hat{x}_n]$, Thus, a first-term office holder prefers to remain in office, even if she can return to the electorate, and in all subsequent periods, outcomes are determined by the ideal point of the highest type. By Theorem 4, in Section 8, $\mathbb{E}[u(y)|\hat{x}_n]$ does indeed bound the (normalized) continuation value of a challenger in equilibrium, so (C7) means that all first-term office holders view re-election as inherently desirable. Note that by (C7), the objective function in (4) is never maximized at any policy choice $x < \hat{x}_j$, and thus the optimization problem of office holders can be restricted to policy choices $x \geqslant \hat{x}_j$ without loss of generality.

We can gain some insight into a first-term office holder's policy choice problem by reformulating it in terms of optimization subject to an inequality constraint. Define the new objective function

$$W_j(x,r;V) \quad = \quad w_j(x) + r\delta[w_j(\hat{x}_j) + \beta - (1-\delta)V],$$

which is the expected payoff if the politician makes policy choice x and is re-elected with probability r, minus a constant term. Here, V is a value that represents the voter's continuation value of a challenger, and denoting the voter's expected discounted payoff from the extreme ideal points as

$$\underline{V} \;=\; \frac{1}{1-\delta}\mathbb{E}[u(y)|\hat{x}_1] \quad \text{and} \quad \overline{V} \;=\; \frac{1}{1-\delta}\mathbb{E}[u(y)|\hat{x}_n],$$

we let V range over $[\underline{V},\overline{V}]$. Defining the constraint function

$$g(x,r) \quad = \quad r + F(\overline{y}|x) - 1,$$

the type j office holder's induced problem in her first term of office can then be formulated as

$$\begin{array}{c} \max_{(x,r)} W_j(x,r;V) \\ \text{s.t. } g(x,r) \leqslant 0. \end{array} \tag{6}$$

The special case in which w_j is concave and policy choices are a shift parameter for a fixed density h is depicted in Figure 1. Here, the objective function W_j is concave in (x,r) and quasi-linear in r, but the constraint set inherits the natural non-convexity of the distribution function H, leading to the possibility of multiple optimal policy choices, as in the figure.

Given $V \in [\underline{V},\overline{V}]$ and any cutoff $\overline{y} \in Y \cup \{y^+, y^-\}$, condition (C7) implies that an office holder's payoff from zero effort exceeds that from policy choices below her ideal point, and by (C3), zero effort also dominates arbitrarily high policy choices, so each type of politician has an optimal policy choice in the first term of office. Since the problem in (6) is continuous, the set of optimal policy choices is compact; henceforth, x_j^* denotes the greatest optimal policy choice of a first-term office

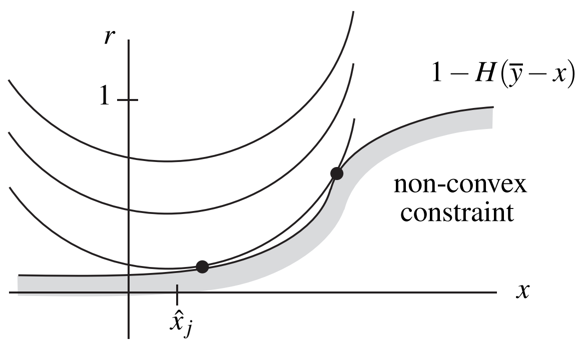

Figure 1: Politician's optimization problem

holder of type j, and $x_{*,j}$ denotes the least optimal policy choice. In much of what follows, dependence of these optimal policy choices on $\overline{y}$ and V is suppressed for notational simplicity, but note for now that $x_j^*(\overline{y},V)$ is upper semicontinuous on $Y \times [\underline{V},\overline{V}]$, and $x_{*,j}(\overline{y},V)$ is lower semicontinuous.

The next proposition establishes that the politicians' objective functions satisfy the important property that differences in payoffs are monotone in type, and it allows us to conclude that optimal policy choices are in fact ordered by type. We say that $W_j(x,1-F(\overline{y}|x);V)$ is *supermodular* in (j,x) if for all (j,x) and all (k,z) with $j>k$ and $x>z$, we have

$$\begin{aligned} & W_j(x,1-F(\overline{y}|x);V)-W_j(z,1-F(\overline{y}|z);V) \\ > \quad & W_k(x,1-F(\overline{y}|x);V)-W_k(z,1-F(\overline{y}|z);V). \end{aligned}$$

A well-known implication is that given an arbitrary value $\overline{y}$ of the cutoff, optimal policy choices are strictly ordered by type, i.e., for all $j<n$, if x_j solves

$$\max_{x\in X} W_j(x,1-F(\overline{y}|x);V)$$

and x_{j+1} solves the corresponding problem for type $j+1$, then $x_j < x_{j+1}$; in terms of the above conventions for greatest and least optimal policy choices, this is $x_j^* < x_{*,j+1}$. This ordering property will, in turn, be critical for establishing existence of equilibrium. The proof of supermodularity in the proposition follows the lines of Lemma A.3 of Banks and Sundaram (1998) and is omitted.

Proposition 1 *Assume (C0)–(C7). Let V be such that $\underline{V} \leqslant V \leqslant \overline{V}$, and let $\overline{y} \in Y \cup \{y^+,y^-\}$ be any cutoff. For each type j, the office holders' objective function, $W_j(x,1-F(\overline{y}|x);V)$, is super modular in (j,x); and the solutions to (6) are strictly* *ordered by type, i.e., for all $j < n$, we have $\hat{x}_j \leqslant x_{*,j} \leqslant x^*_j < x_{*,j+1}$. Given such V and $\overline{y} \in Y$, the maximum and minimum optimal policy choices, $x^*_j(\overline{y},V)$ and $x_{*,j}(\overline{y},V)$, are respectively upper and lower semi-continuous in their arguments.*

Now define the objective function of the voter, in ex ante terms, given mixed policy strategies $\pi^1_1,\ldots,\pi^1_n$ for first-term office holders and value V, as

$$\begin{aligned} U(\alpha,\pi^1_1,\ldots,\pi^1_n;V) &= \sum_j p_j \int_x \left[\int_y [(1-\alpha(y))(u(y)+\delta V) \right. \\ &\qquad \left. +\alpha(y)(u(y)+\delta\mathbb{E}[u(y)|\hat{x}_j]+\delta^2 V)] f(y|x)dy \right] \pi^1_j(dx), \end{aligned}$$

where $\alpha{:}Y \to [0,1]$ is any voting strategy. That is, each type j politician mixes according to π_j, and depending on the choice x, an outcome y is realized from the density $f(y|x)$, and the incumbent is re-elected with probability $\alpha(y)$. In case the challenger wins, the voter receives utility from y and subsequently receives the value of a challenger; and in case the incumbent wins, the voter additionally receives the expected payoff from the incumbent's ideal point before the value of a challenger. Then the induced problem of the voter prior to the realization of an outcome from the choice of a first-term office holder is

$$\max_\alpha U(\alpha,\pi^1_1,\ldots,\pi^1_n;V). \tag{7}$$

Consider $V \in (\underline{V},\overline{V})$, and let $\pi^1_1,\ldots,\pi^1_n$ be mixed policy strategies with supports that are strictly ordered by type. Arguments of Lemmas A.5 and A.6 of Banks and Sundaram (1998) establish that the optimal strategy is equivalent almost everywhere to a cutoff rule with cutoff $y^*(\pi^1_1,\ldots,\pi^1_n,V)$ belonging to Y. Restricted to cutoff rules, the voter's objective function can be written $U(\overline{y},\pi^1_1,\ldots,\pi^1_n;V)$, and this function is jointly continuous in its arguments. By the theorem of the maximum, the optimal cutoff $y^*(\pi^1_1,\ldots,\pi^1_n,V)$ is jointly continuous in its arguments, and examining the necessary first order condition with respect to $\overline{y}$, it solves

$$\frac{\partial U}{\partial \overline{y}}(\overline{y},\pi^1_1,\ldots,\pi^1_n;V) = \sum_j p_j \int_x ((1-\delta)V - \mathbb{E}[u(y)|\hat{x}_j]) f(\overline{y}|x)\pi^1_j(dx) = 0,$$

or equivalently, the optimal cutoff satisfies the indifference condition

$$\sum_j \mu_T(j|\overline{y})\mathbb{E}[u(y)|\hat{x}_j] = (1-\delta)V, \tag{8}$$

where $\mu_T(j|\overline{y})$ is derived via Bayes' rule. The solution to this indifference condition is unique when supports of mixed policy strategies are strictly ordered according to type, but uniqueness fails when all types pool on the same policy choice. In that case, updating does not occur, so $\mu_T(j|\overline{y})$ is constant in $\overline{y}$, and deferential voting would imply that the incumbent is always re-elected and thus $y^*(\pi^1_1,\ldots,\pi^1_n,V) = y^-$. This creates a discontinuity in the optimal cutoff that must be addressed in the analysis of existence. The next result states these findings.

Proposition 2 *Assume (C0)–(C7). Let V be such that $\underline{V} < V < \overline{V}$, and let $\pi^1_1,\ldots,\pi^1_n$ be mixed policy strategies with supports that are strictly ordered by type. The solution to (7) is unique up to a set of measure zero; it is equivalent almost everywhere to a rule with cutoff $y^*(\pi^1_1,\ldots,\pi^1_n,V)$ belonging to Y; and this cutoff is the unique solution to the first order condition (8). Given such a value and such mixed policy strategies, $y^*(\pi^1_1,\ldots,\pi^1_n,V)$ is jointly continuous in its arguments.*

Given a pair ψ with challenger continuation value satisfying the strict inequalities in Proposition 2, the above propositions imply that the four necessary conditions listed at the end of Section 4 are in fact sufficient. Indeed, given the cutoff $\overline{y}$, if each policy strategy π^1_j puts probability one on solutions to the maximization problem in (4), then Proposition 1 implies that the policy strategies are strictly ordered according to type; in turn, given these policy strategies $\pi^1_1,\ldots,\pi^1_n$, if the cutoff $\overline{y}$ satisfies the indifference condition (3), then Proposition 2 implies that the cutoff is optimal for the voter.

6 Existence of stationary electoral equilibria

Before proceeding to asymptotic characterizations and the analysis of policy responsiveness in the dynamic political agency model, existence of equilibria must be addressed. The next theorem establishes existence of stationary electoral equilibrium, and it provides a limited characterization of equilibria. Subsequent to the statement of the theorem, an informal description of the proof is given; a minimal responsiveness result under standard smoothness conditions is given; and the proof approach of Banks and Sundaram (1998) is discussed.

Theorem 1 *Assume (C0)–(C7). There is a stationary electoral equilibrium, and every stationary electoral equilibrium is given by policy strategies $\pi^*_1,\ldots,\pi^*_n$ for first-term office holders and a cutoff $y^* \in Y$ such that:*

(i) each type j politician mixes over policy choices using π^*_j in the first term of office; optimal effort is non-negative, i.e., for all j, $x_{*,j} \geqslant \hat{x}_j$; and optimal policy choices are strictly ordered by type, i.e., for all $j < n$, we have $x^*_j < x_{*,j+1}$,

(ii) the voter re-elects the incumbent if and only if $y \geqslant y^*$.

The proof of Theorem 1 is located in the appendix (along with other proofs omitted from the text) and consists of an application of Glicksberg's fixed point theorem to an appropriately defined correspondence. The first steps are to deduce an a priori bound, $\tilde{x}$, on equilibrium policy choices and to bound the equilibrium cutoff by an interval $[a,b]$; these steps are non-trivial but not the main focus of the discussion here. The domain of the correspondence is $\Delta([0,\tilde{x}])^n \times \Delta([a,b])$, which consists of $(n+1)$-tuples $(\pi_1,\ldots,\pi_n,\rho)$ of mixed policy strategies, π_j, for first-term politicians and a mixture over cutoffs, denoted ρ. The correspondence has the form

$$(\pi_1,\ldots,\pi_n,\rho) \mapsto \left(\prod_j \Delta(M_j(\pi_1,\ldots,\pi_n,\rho))\right) \times \Delta(C(\pi_1,\ldots,\pi_n,\rho)),$$

where $M_j(\pi_1,\ldots,\pi_n,\rho)$ consists of the optimal policy choices for a first-term politician of type j, given mixed policy strategies $(\pi_1,\ldots,\pi_n)$ and cutoff $\mathbb{E}[\rho]$, i.e.,

$$M_j(\pi_1,\ldots,\pi_n,\rho) \quad = \quad \arg\max_{x\in[0,\tilde{x}]} W_j(x, 1-F(\mathbb{E}[\rho]-x); V(\pi_1,\ldots,\pi_n,\mathbb{E}[\rho])),$$

and $C(\pi_1,\ldots,\pi_n,\rho)$ consists of the optimal cutoffs for the voter, i.e.,

$$C(\pi_1,\ldots,\pi_n,\rho) \quad = \quad \arg\max_{\overline{y}\in[a,b]} U(\overline{y},\pi_1,\ldots,\pi_n; V(\pi_1,\ldots,\pi_n,\mathbb{E}[\rho])).$$

In the above expressions, $V(\pi_1,\ldots,\pi_n,\mathbb{E}[\rho])$ is the voter's continuation value of a challenger induced by policy strategies $\pi_1,\ldots,\pi_n$ and cutoff $\mathbb{E}[\rho]$. Note that no ordering of mixed policy strategies is assumed in the domain, so there may be multiple optimal cutoffs, and the set $C(\pi_1,\ldots,\pi_n,\rho)$ need not be convex.

By taking mixtures over the set $C(\pi_1,\ldots,\pi_n,\rho)$, we ensure that the above correspondence has convex values; indeed, the conditions of Glicksberg's theorem are satisfied, and thus a fixed point $(\pi_1^*,\ldots,\pi_n^*,\rho^*)$ is assured. The use of mixtures over cutoffs in the definition of the correspondence is a technical device, and the mixture ρ does not have a behavioral analogue in the dynamic political agency model; in fact, an important feature in the construction is that randomness in the distribution ρ is extracted by reducing it to its mean in the calculation of the induced continuation value of a challenger. This implies that the mixtures π_j^* obtained from the fixed point put probability one on optimal policy choices in response to the cutoff $\mathbb{E}[\rho^*]$, and thus Proposition 1 implies that the strategies $(\pi_1^*,\ldots,\pi_n^*)$ are strictly ordered by type. By Proposition 2, this in turn implies that the voter has a unique optimal cutoff, so ρ^* is degenerate on some y^*, and thus $\mathbb{E}[\rho^*] = y^*$. That is, the strategies π_j^* are indeed optimal given cutoff y^*, and the cutoff y^* satisfies the voter's indifference condition. We conclude that $(\pi_1^*, \ldots, \pi_n^*)$ and y^* yield the desired stationary electoral equilibrium.

Next, we establish a minimal responsiveness result under standard differentiability conditions. Theorem 1 implies that equilibrium cutoffs belong to Y, so if politician payoffs are differentiable, then the first order condition (5) holds for each type j politician. Because optimal policy choices are at least $\hat{x}_j > 0$, the first order condition must in fact hold with equality. By (C7) and Theorem 4, in Section 8, the right-hand side of the first order condition is negative, and thus it follows that each type j politician exerts positive effort in the first term of office. This reasoning yields the following result, which extends Proposition 3.3 of Banks and Sundaram (1998) to the current setting.

Theorem 2 *Assume (C0)–(C7), and assume that for each type j, the payoff function w_j is differentiable. Every stationary electoral equilibrium is such that for each type j, we have $x_{*,j} > \hat{x}_j$.*

The proof of Theorem 1 addresses an error in the proof of Proposition 3.1 of Banks and Sundaram (1998). Those authors assume, in contrast to this paper, that the set X of policy choices and the set Y of possible cutoffs are compact.[14,15] Their approach to existence of equilibrium consists of a fixed point argument for a correspondence with domain equal to the set of profiles of mixed policy strategies that are weakly ordered by type, say $\Delta^n_{\geqslant}(X)$, crossed with Y. An element $(\pi_1, \ldots, \pi_n, \overline{y})$ of their domain determines best response mixed policy strategies and an interval of best response cutoffs, and their correspondence takes the form below,[16]

$$(\pi_1, \ldots, \pi_n, \overline{y}) \to \left(\prod_j \Delta(M_j(\overline{y})) \right) \times C(\pi_1, \ldots, \pi_n).$$

[14]Compactness of X can lead to trivial equilibria: when politicians are highly office motivated, there is a continuum of equilibria in which all politician types pool at the maximum effort level. This feature of the Banks-Sundaram model prevents a meaningful analysis of asymptotics of the sort in the next section.

[15]Compactness of Y leads to a technical difficulty, as Banks and Sundaram (1998) require Bayes' rule to be well-defined conditional on all outcome realizations, including the maximum and minimum. This implies the density $f(\cdot|x)$ is strictly positive at the endpoints of Y, precluding, e.g., the Beta density $f(y|x) = y^{\alpha}(1-y)^{1-\alpha}$, in which the policy choice x determines shape parameter $\alpha = x/(x+1)$.

[16]In the model of Banks and Sundaram (1998), optimal policies of the type j politician depend on $\overline{y}$ alone, and not the policy strategies $\pi_1, \ldots, \pi_n$. This owes to the fact that Banks and Sundaram (1998) assume that an out of office politician receives a fixed payoff of zero. Optimal cutoffs are derived from the dynamic programming problem of the voter in their framework and do not depend on $\overline{y}$; the latter difference is not important for the results.

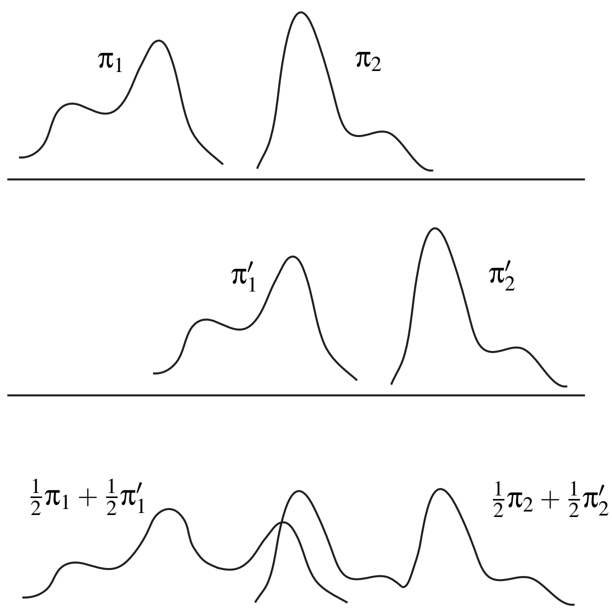

Figure 2: Non-convexity of Banks-Sundaram domain

Their argument attempts to apply Glicksberg's fixed point theorem to deduce an equilibrium, but while the set $\Delta^n_{\geqslant}(X)$ is compact, it is *not convex;* this is illustrated in Figure 2, where policy mixtures are represented for two types by density functions. In the top panel, the supports of π_1 and π_2 are separated by a positive amount, as are the supports of π'_1 and π'_2 in the middle panel. But taking convex combinations of π_1 and π'_1, and of π_2 and π'_2, the supports of the resulting mixtures overlap. This example demonstrates that the domain $\Delta^n_{\geqslant}(X) \times Y$ is not convex, violating the assumptions of Glicksberg's fixed point theorem and controverting the argument put forward by Banks and Sundaram (1998).

The approach of the current paper is to relinquish monotonicity of policy mixtures in the domain in order to satisfy convexity. Monotonicity is then recaptured after the attainment of a fixed point, through the ordering of best responses implied by supermodularity of the politicians' payoffs. Since the domain of policy mixtures no longer incorporates the ordering of policy choices by type, there will in general be multiple optimal cutoffs for the voter, and this set need not be convex. The solution is to convexify the optimal cutoffs by taking *mixtures* over them; this mixing does not have a behavioral interpretation in the model, and the mixing is actually extracted from the fixed point obtained by virtue of the fact that politicians respond only to the expectation of ρ. As such, it follows from supermodularity that the optimal choices of politicians are in fact ordered by type. Here, monotonicity of policy mixtures is like love: we let it go, and it comes back to us in the end.

7 Asymptotic policy choices

Theorem 1 provides a partial characterization of policy choices for arbitrary model parameters. More can be said when politicians are highly office motivated, i.e., when, fixing a discount factor δ, the office benefit β becomes large. The main result of this section shows that faced with highly office-motivated politicians, the voter becomes arbitrarily demanding, in the sense that the cutoff for re-election goes to the upper bound y^+. Furthermore, the type n politicians mix between arbitrarily high policy choices and choices close to her ideal point, while all other types shirk by making policy choices close to their ideal points.

A useful first step in the analysis of this section, and one that is used in the next section as well, is a modest lower bound on the voter's (normalized) continuation value of electing a challenger: as should be expected, this continuation value is at least equal to the voter's expected payoff from a randomly drawn politician's policy choice in the first term of office.

Lemma 1 *Assume (C0)–(C7). In every stationary electoral equilibrium, the (normalized) continuation value of a challenger is greater than or equal to the expected payoff to the voter from policy choices of first-term office holders, i.e.,*

$$(1-\delta)V^C(\psi) \quad \geqslant \quad \sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx). \tag{9}$$

The asymptotic characterization of stationary electoral equilibria is next. The result is actually somewhat more general than the above description, because it lets the discount factor vary with office benefit, as long as the product $\beta\delta$ diverges to infinity; thus, we capture the case of impatient citizens, as long as the increasing office benefit compensates for the decreasing discount factor. For the result, assume that voter payoffs are unbounded above:

(C8) $\lim_{y\to\infty} u(y) = \infty.$

The latter assumption is used to leverage the bound on policy responsiveness given in the next section, and it is obviously satisfied in typical cases such as log utility or risk neutral voters.

Theorem 3 *Assume (C0)–(C8). Let the office benefit* $\beta \geqslant 0$ *and discount factor* $\delta \in [0,1)$ *vary arbitrarily subject to* $\lim \beta\delta = \infty$. *For every selection of stationary* *electoral equilibria, the voter's cutoff diverges to y^+; the type n politicians in their first term mix between policy choices that are close to their ideal point and ones that are arbitrarily high, with small, positive probability on the latter; and all other type $j < n$ politicians make policy choices close to their ideal points in the first term, i.e.,*

(i) $\overline{y} \to y^+$,

(ii) $x_n^* \to \infty$ *and* $x_{*,n} \to \hat{x}_n$,

(iii) for all $j < n$, $x_j^* \to \hat{x}_j$,

(iv) for all $\eta > 0$ *and for* $\beta\delta$ *large enough, we have* $\pi_n^1([\eta,\infty)) > 0$,

(v) for all $\eta > 0$, *we have* $\pi_n^1((\hat{x}_n,\eta)) \to 1$ *and* $\pi_n^1([\eta,\infty)) \to 0$.

The proof of Theorem 3 first establishes that as office incentives become large, the voter's cutoff diverges to y^+. To see this, note that otherwise, there are two possibilities. First, there is a subsequence of cutoffs that converge to a limit $\overline{y} \in Y$, but then we can show that as office incentives become large, each politician type makes arbitrarily high policy choices in equilibrium, but this contradicts the bound on voter payoffs from policy choices of first-term office holders in Corollary 1 of Section 8. Second, there is a subsequence of cutoffs that diverges to y^-. Then conditional on the cutoff, the voter infers that lower types are relatively more likely, but then, using Lemma 1, the voter's expected payoff from re-electing the incumbent conditional on $\overline{y}$ is less than that of a challenger, contradicting the voter's indifference condition. We conclude that equilibrium cutoffs diverge to y^+.

The key step in the proof shows that there is no subsequence of optimal policy choices for any politician type that converges to a finite limit $\tilde{x}_j$ above the politician's ideal point. The proof proceeds by contradiction, supposing there is a subsequence of optimal policy choices with $x'_j \to \tilde{x}_j$ and $\hat{x}_j < \tilde{x}_j < \infty$. Then the gains from deviating to $\hat{x}_j$ from x'_j are non-positive along the subsequence:

$$\underbrace{\delta(F(\overline{y}|\hat{x}_j) - F(\overline{y}|x'_j))[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)]}_{\text{future losses}} \;\; \geqslant \;\; \underbrace{w_j(\hat{x}_j) - w_j(x'_j)}_{\text{current gains}}.$$

Now consider deviating in the opposite direction to $\tilde{x}_j + 1$. By L'Hôpital's rule,

$$\lim_{\beta\delta\to\infty} \frac{F(\overline{y}|x'_j) - F(\overline{y}|\tilde{x}_j + 1)}{F(\overline{y}|\hat{x}_j) - F(\overline{y}|x'_j)} \;\; = \;\; \infty,$$

and then it follows that for sufficiently high $\beta\delta$, the politician's payoff from $\tilde{x}+1$ is greater than from the optimal policy x'_j, i.e.,

$$\underbrace{\delta(F(\overline{y}|x'_j) - F(\overline{y}|\tilde{x}_j+1))[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)]}_{\text{future gains}}$$
$$> \underbrace{w_j(x'_j) - w_j(\tilde{x}_j+1)}_{\text{current losses}}.$$

a contradiction.

By the latter argument, if there is a subsequence of bounded optimal policy choices, then those choices must converge to the politician's ideal point, i.e., $x'_j \to \hat{x}_j$. If the greatest optimal policy choice x^*_n of the type n politician is bounded along any subsequence, then it follows that $x^*_n \to \hat{x}_n$; and since optimal policies are ordered by type, this implies that the optimal policy choices of each type $j < n$ converge to the ideal point of the politician. But then conditional on the cutoff, $\overline{y} \to y^+$, the voter's expected payoff from re-electing the incumbent is greater than that of a challenger, contradicting the voter's indifference condition. We conclude that $x^*_n \to \infty$. And by Corollary 1 of Section 8, the equilibrium policy strategies of the type n politician cannot place probability one on arbitrarily high policy choices, so we have $x_{*,n} \to \hat{x}_n$. Finally, because policy choices are ordered by type, we conclude all types $j < n$ shirk in the limit, i.e., $x^*_j \to \hat{x}_j$.

The asymptotic analysis reveals that as politicians become more office motivated, the policy choices of all politician types but the highest converge to their ideal points, i.e., their behavior approximates shirking in the limit. In contrast, the type n politicians mix over arbitrarily high policy choices, but with probability going to zero. Thus, to the extent that policy is responsive to voter preferences, this is driven by the choices of type n politicians in their first term of office. This leaves open the possibility that the probability of high policy choices decreases slowly relative to the increase in the type n politician's effort, with the net effect of these forces being that the voter's discounted payoff becomes high. We will see in the next section that this possibility is not realized.

8 Bounds on policy responsiveness

The main results of this section show that the voter's (normalized) continuation value of a challenger—and therefore expected payoff from the policy choices of first-term office holders—cannot exceed the expected payoff from the ideal point of the highest politician type. The reason is that as the voter's continuation value of a challenger increases, her incentive to re-elect the incumbent decreases, so that elections lose their disciplining effect. Second-term politicians exert zero effort in equilibrium, and so the voter's expected payoff from electing a politician to a second term is bounded above by the payoff generated by the highest ability type choosing her ideal point, plus the discounted continuation value of the challenger who would replace the incumbent after her second term. Therefore, if the continuation value of a challenger were too high, then the voter would opt for a challenger immediately, rather than delay one period, but then first-term office holders would in fact shirk in office. The next theorem establishes bounds on policy responsiveness, the lower bound following from Lemma 1 and the upper bound from the indifference condition of the voter; this result is then extended in the remainder of the section.

Theorem 4 *Assume (C0)–(C7). In every stationary electoral equilibrium, the (normalized) continuation value of a challenger is between the expected payoff from a randomly drawn politician's ideal point and the expected payoff from the ideal point of the type n politician, i.e.,*

$$\sum_j p_j \mathbb{E}[u(y)|\hat{x}_j] \;\; \leqslant \;\; (1-\delta)V^C(\psi) \;\; < \;\; \mathbb{E}[u(y)|\hat{x}_n]. \tag{10}$$

Proof: To prove the upper bound, consider any stationary electoral equilibrium, and note that the cutoff either belongs to Y or is equal to one of the extreme outcomes, i.e., $\overline{y} \in Y$ or $\overline{y} \in \{y^+, y^-\}$. In the first case, the indifference condition in (3), along with $\mu(n|\overline{y}) < 1$, yields the upper bound. In the second case, each politician type shirks, i.e., π_j places probability one on $\hat{x}_j$. Then the continuation value of a challenger is $V^C(\psi) = \frac{1}{1-\delta}\sum_j p_j \mathbb{E}[u(y)|\hat{x}_j]$, and the desired inequality follows from $p_n < 1$. For the lower bound, we apply the fact that equilibrium effort is non-negative, from Proposition 1, to inequality (9) of Lemma 1. ∎

A direct corollary of Theorem 4 is the sharper conclusion that the voter's expected payoff from policy choices of first-term office holders is also bounded above by the expected payoff from the ideal point of the type n politician. Indeed, otherwise there is a stationary electoral equilibrium with

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \;\; \geqslant \;\; \mathbb{E}[u(y)|\hat{x}_n].$$

Combining the latter inequality with (9), from Lemma 1, we have $(1-\delta)V^C(\psi) \geqslant \mathbb{E}[u(y)|\hat{x}_n]$, contradicting Theorem 4. Thus, for example, if voters are risk neutral and the density $f(\cdot|x)$ has mean x, then the bound on voter payoffs is simply $u(\hat{x}_n)$, and the expected policy outcome is bounded above by $\hat{x}_n$.

Corollary 1 *Assume (C0)–(C7). In every stationary electoral equilibrium, the expected payoff to the voter from policy choices of first-term office holders is less than the expected payoff from the ideal point of the type n politician, i.e.,*

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \quad < \quad \mathbb{E}[u(y)|\hat{x}_n].$$

A weakness of the bound on responsiveness derived above is that it is not robust to the introduction of very good types with very low probability. In particular, if we add a type $n+1$ with ideal point $\hat{x}_{n+1}$ greater than $\hat{x}_n$ and prior probability $p_{n+1} \downarrow 0$, then Corollary 1, applied to the modified model, dictates only that the voter's expected payoff from a first-term office holder is below $\mathbb{E}[u(y)|\hat{x}_{n+1}]$, which may be very high. This unsatisfying feature of the result is addressed in the following theorem, which provides an upper bound on the voter's expected payoff from a first-term office holder that is robust to the introduction of very good types with very low probability.

Theorem 5 *Assume (C0)–(C7). Consider any politician type k, and let prior beliefs vary subject to $\sum_{j>k} p_j \to 0$. Then the limiting expected payoff to the voter from the policy choices of first-term office holders is less than or equal to the expected payoff from the ideal point of the type k politician, i.e., for every selection of stationary electoral equilibria,*

$$\limsup \sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \quad \leqslant \quad \mathbb{E}[u(y)|\hat{x}_k].$$

Theorems 4 and 5 imply a general limit on policy responsiveness stemming from the commitment problem of voters, regardless of the office benefit or rate of discount. The next result gives a partial strengthening by showing that for a given level of office benefit, the voter's expected payoff from the policy choices of first-term office holders is bounded *strictly* below the expected payoff from the ideal point $\hat{x}_n$, even as we vary the discount factor. In fact, the statement of the result is stronger that this, in that we can allow the office benefit to become large, as long as the discount factor eventually offsets the increase in office motivation.

Theorem 6 *Assume (C0)–(C7). For every constant $c > 0$, there is a bound $\overline{u} < \mathbb{E}[u(y)|\hat{x}_n]$ such that for all levels of office benefit $\beta \geqslant 0$ and all discount factors $\delta \in [0,1)$ satisfying $\beta\delta \leqslant c$, in every stationary electoral equilibrium for parameters (β,δ), the expected payoff to the voter from policy choices of first-term office holders is below this bound, i.e.,*

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \quad \leqslant \quad \overline{u}.$$

To apply the previous result for a given level of office benefit, say β, we simply set $c = \beta$. The implication is that for a given benefit β, the voter's loss is not ameliorated by increased patience of citizens.

Corollary 2 *Assume (C0)–(C7), and fix the office benefit* $\beta \geqslant 0$. *There is a bound* $\overline{u} < \mathbb{E}[u(y)|\hat{x}_n]$, *independent of the discount factor, such that for every stationary electoral equilibrium, the expected payoff to the voter from policy choices of first-term office holders is below this bound, i.e.,*

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \quad \leqslant \quad \overline{u}.$$

9 Voter commitment

In a stationary electoral equilibrium, the voter's strategy emerges endogenously as a cutoff determined by the continuation value of a challenger: the voter re-elects the incumbent if and only if, conditional on the observed policy outcome, the expected discounted payoff of the incumbent weakly exceeds that of a challenger. It is assumed, consistent with the citizen-candidate approach, that voters cannot commit at the beginning of the game to a pre-determined cutoff. In turn, we have seen that this commitment problem of voters leads to a bound on the responsiveness of politicians. The alternative model with commitment raises questions of the precise mechanism by which voters could achieve such commitment, and it raises the question of how a heterogeneous electorate (which is represented by a single voter in the current analysis) would arrive at such a cutoff. Nevertheless, it is of normative interest to examine the optimization problem of the representative voter; in particular, we may wish to know whether the voter commitment problem leads to equilibrium cutoffs that are too high or too low. The results of this section provide a sharp answer to this question for the case in which office incentives are high.

The first simple observation in this vein is that the voter can achieve arbitrarily high payoffs from any fixed cutoff as office benefit becomes large. Given cutoff $\tilde{y}$, we say $\tilde{\pi} = (\tilde{\pi}_1, \ldots, \tilde{\pi}_n)$ is a *stationary electoral partial equilibrium* if each $\tilde{\pi}_j^1$ places probability one on solutions to

$$\max_{x \in X} w_j(x) + \delta\left[(1 - F(\tilde{y}|x))[w_j(\hat{x}_j) + \beta + \delta V^C(\tilde{\pi}|\tilde{y})] + F(\overline{y}|x)V^C(\tilde{\pi}|\tilde{y})\right]$$

and each $\tilde{\pi}_j^2$ places probability one on $\hat{x}_j^2$, where $V^C(\tilde{\pi}|\tilde{y})$ is the continuation value of a challenger given the politicians' strategies and fixed cutoff $\tilde{y}$.

Theorem 7 *Assume (C0)–(C7) and* $\delta > 0$. *Fix a cutoff* $\tilde{y} \in Y$, *and let the office benefit* $\beta \geqslant 0$ *become large. For every selection of stationary electoral partial equilibria, the voter's continuation value of a challenger becomes arbitrarily large, i.e.,* $V^C(\tilde{\pi}|\tilde{y}) \to \infty$.

Proof: Fix a cutoff $\tilde{y} \in Y$, and let $\beta \to \infty$. Suppose toward a contradiction that there is a subsequence such that the continuation values $V^C(\tilde{\pi}|\tilde{y})$ are bounded above. I claim that for each type j politician, $x_{*,j} \to \infty$. Otherwise, we may go to a further subsequence such that $x_{*,j} \to \tilde{x} < \infty$. Then the probability that the type j politician is re-elected after her first term given policy choice $x_{*,j}$ satisfies $1 - F(\tilde{y}|x_{*,j}) \to 1 - F(\tilde{y}|\tilde{x}) < 1 - \varepsilon$ for some $\varepsilon > 0$. Now let x' satisfy $1 - F(\tilde{y}|x') > 1 - \frac{\varepsilon}{2}$, and note that the value of the objective function in (4) at x' minus the value at $\tilde{x}$ is

$$\begin{aligned} & w_j(x') - w_j(\tilde{x}) + \delta(F(\tilde{y}|\tilde{x}) - F(\tilde{y}|x'))[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\tilde{\pi}|\tilde{y})] \\ > \quad & w_j(x') - w_j(\tilde{x}) + \frac{\delta\varepsilon}{2}[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\tilde{\pi}|\tilde{y})] \\ \to \quad & \infty \end{aligned}$$

as β becomes large, where we use the supposition that $V^C(\tilde{\pi}|\tilde{y})$ is bounded above, so that the term in brackets is positive. But when β is large and the policy choice $x_{*,j}$ approaches $\tilde{x}$, it follows that the choice is not optimal, a contradiction. ∎

An immediate consequence is that when politicians are highly office motivated, stationary electoral equilibrium voter cutoffs are suboptimally high. Specifically, Theorem 4 implies that the voter's equilibrium expected payoff from a challenger is bounded above by $\overline{V}$; Theorem 3 implies that the equilibrium cutoff $\overline{y}$ diverges to y^+ as the office benefit increases; and Theorem 7 implies that for a fixed $\tilde{y}$, the voters' payoff from a challenger exceeds $\overline{V}$ as $\beta \to \infty$. Therefore, when office benefit is large, the equilibrium cutoff exceeds $\tilde{y}$, and the equilibrium continuation value of a challenger is lower than it would be if voters could commit to $\tilde{y}$. This observation yields the following corollary.

Corollary 3 *Assume (C0)–(C8) and* $\delta > 0$. *Let the office benefit* $\beta \geqslant 0$ *become large. For any fixed* $\tilde{y}$, *if* β *is sufficiently high, then in each stationary electoral equilibrium* ψ, *the voters' equilibrium cutoff* $\overline{y}$ *exceeds* $\tilde{y}$, *and the continuation value of a challenger is less than the continuation value of a challenger in every stationary electoral partial equilibrium* $\tilde{\pi}$ *given cutoff* $\tilde{y}$, *i.e.,*

$$\tilde{y} < \overline{y} \quad \text{and} \quad V^C(\tilde{\pi}|\tilde{y}) > V^C(\psi).$$

The preceding result shows that when the office benefit is large, the voter's equilibrium payoff would be increased if she could commit to a lower cutoff; thus, the equilibrium cutoff is suboptimally high. However, the corollary leaves open the possibility that it is also suboptimally low. That is, in principle, it is also possible that if the voter could commit to some greater cutoff $\tilde{y} > \bar{y}$, then her payoff would increase relative to the equilibrium payoff. The next result imposes further structure on the model and resolves this question: when office benefit is sufficiently high, increasing the equilibrium cutoff cannot raise the payoff of the voter. For this result, we specialize the model by assuming that the policy choice x is a shift parameter on a fixed density over outcomes,

(C9) there is a fixed density h on $Y = \mathbb{R}$ such that for all x and all y, we have $f(y|x) = h(y-x)$,

and that politician payoffs are differentiable and concave,

(C10) for all j, w_j is differentiable and concave.

By our other assumptions, it follows that the density h is single-peaked as well. Next, we show that the voter's payoff is not increased by committing to a cutoff higher than a stationary electoral equilibrium cutoff.

Theorem 8 *Assume (C0)–(C10) and $\delta > 0$. For sufficiently large office benefit $\beta \geqslant 0$, if ψ is a stationary electoral equilibrium with cutoff $\bar{y}$, if $\tilde{y} > \bar{y}$, and if $\tilde{\pi}$ is a stationary electoral partial equilibrium given cutoff $\tilde{y}$, then the voter's continuation value of a challenger from committing to $\tilde{y}$ is lower than the equilibrium continuation value, i.e., $V^C(\tilde{\pi}|\tilde{y}) < V^C(\psi)$.*

We end by considering the optimization problem of the voter and stating the obvious corollary that equilibrium cutoffs exceed the voter optimum under commitment. To this end, we let $E(\tilde{y}|\beta)$ denote the set of stationary electoral partial equilibria π for fixed cutoff $\tilde{y}$, given office benefit β, and we formulate the voter's maximization problem as

$$\max_{\tilde{y}\in Y} \max_{\tilde{\pi}\in E(\tilde{y}|\beta)} V^C(\tilde{\pi}|\tilde{y}),$$

where we implicitly refine partial equilibria by selecting those with the highest continuation value of a challenger. Standard arguments imply that $V^C(\tilde{\pi}|\tilde{y})$ is jointly continuous in $(\tilde{\pi},\tilde{y})$, and that the equilibrium correspondence $(\tilde{y},\beta) \mapsto E(\tilde{y}|\beta)$ has closed graph. Given a level of office benefit β and a fixed cutoff $\tilde{y}$, office holders will not choose arbitrarily high policies, so policy choices are bounded, and thus $E(\tilde{y}|\beta)$ is compact, and the inner maximization problem is well-defined. Furthermore, by closed graph of the partial equilibrium correspondence, the maximum value of the inner optimization problem above is upper semicontinuous in $\tilde{y}$. Letting $\tilde{y} \to y^+$ or $\tilde{y} \to y^-$, policy strategies converge to the ideal points of the politician types; hence, arbitrarily low or high cutoffs lead to the lowest possible voter payoff, and so the outer maximization problem is also well-defined. Let

$$\tilde{Y}(\beta) \quad = \quad \left\{ \tilde{y} \in Y \mid \begin{array}{l} \text{there exists } \tilde{\pi} \in E(\tilde{y}|\beta) \text{ such that for all } y' \\ \text{and all } \pi' \in E(y'|\beta),\, V^C(\tilde{\pi}|\tilde{y}) \geqslant V^C(\pi'|y') \end{array} \right\}$$

denote the set of optimal cutoffs for the voter given office benefit β, a nonempty, compact set.

Corollary 4 *Assume (C0)–(C10) and* $\delta > 0$*. For sufficiently large office benefit* $\beta \geqslant 0$*, every stationary electoral equilibrium cutoff* $\overline{y}$ *exceeds the voter's optimal cutoffs, i.e.,* $\overline{y} > \max \tilde{Y}(\beta)$.

To see the result, let office benefit be high, and consider any stationary electoral equilibrium cutoff $\overline{y}$. By Corollary 3, it follows that $\overline{y}$ is not optimal for the voter; and by Theorem 8, there is no optimal cutoff above $\overline{y}$. We conclude that every optimal cutoff is below the equilibrium cutoff, i.e., $\overline{y} > \tilde{Y}(\beta)$, as required.

10 Concluding remarks

Policy responsiveness in dynamic elections is subject to a strict bound, owing to the commitment problem of voters: if first-term office holders generated utility greater than the ideal point of the type n politicians, then voters would have an incentive to replace office holders after their first term in order to reap the benefit from the effort of newly elected politicians; but then first-term office holders would shirk, instead. This contrasts with Duggan and Martinelli (2015), who analyze a two-period model of elections and show that increasing office motivation leads to arbitrarily high expected policy outcomes in the first period. There, an office holder in the first period has an incentive to choose high levels of policy to increase the likelihood of a high outcome, which signals to voters that she is a high type; at the same time, the bar for re-election increases, as voters become arbitrarily demanding. These forces are balanced in such a way that all "above average" politician types choose high policies in the first period, leading to arbitrarily high ex ante payoffs to the voter. At work is the fact that in the two-period model, voters do not suffer from the same commitment problem as in the infinite-horizon model: because the game ends and office holders simply choose their ideal points in the second period, there is no temptation to replace the incumbent with a fresh candidate. Thus, equilibrium behavior in the infinite-horizon model with two-period term limit is qualitatively different than in the two-period model, highlighting the importance of the time horizon in extracting the implications of dynamic equilibrium incentives.

A Proofs omitted from text

A.1 Proof of Theorem 1

The existence proof follows from an application of Glicksberg's fixed point theorem to an appropriately defined correspondence. It proceeds in a number of steps, the first of which bounds the possible equilibrium policy choices of office holders.

Step 1: There exists $\tilde{x}$ sufficiently large that for all types j, all cutoffs $\overline{y}$, and all values $V \in [\underline{V},\overline{V}]$, and all policy choices $x \geqslant \tilde{x}$, we have

$$W_j(\hat{x}_j, 1-F(\overline{y}|\hat{x}_j);V) \quad > \quad W_j(x, 1-F(\overline{y}|x);V).$$

Indeed, by (C3), we can choose $\tilde{x}$ such that for all $x \geqslant \tilde{x}$, we have

$$W_j(\hat{x}_j,0) \;=\; w_j(\hat{x}_j) \;>\; w_j(x)+\delta[w_j(\hat{x}_j)+\beta-(1-\delta)\underline{V}] \;=\; W_j(x,1),$$

so that the payoff from choosing the ideal point $\hat{x}_j$ and losing for sure exceeds the payoff from choosing x and winning with probability one, completing the step. ▫

Let $\Delta([\hat{x}_j,\tilde{x}])$ denote the set of Borel probability measures on $[\hat{x}_j,\tilde{x}]$. The next step shows that as electoral outcomes become certain, incentives weaken, and optimal policy choices in the first term converge to the office holder's ideal point.

Step 2: As $\overline{y} \to y^+$ or $\overline{y} \to y^-$, the optimal policies of each type j politician converge to the ideal point $\hat{x}_j$, irrespective of V, i.e., $x_j^*(\overline{y},V) \to \hat{x}_j$ uniformly in $V \in [\underline{V},\overline{V}]$. Indeed, Proposition 1 implies that for each type j, $x_{*,j}(\overline{y},V) \geqslant \hat{x}_j$. Now consider any $x > \hat{x}_j$, and note that the office holder's expected discounted payoff from $\hat{x}_j$ minus the payoff from choosing any $x > \hat{x}_j$ satisfies:

$$W_j(\hat{x}_j,1-F(\overline{y}|\hat{x}_j);V)-W_j(x,1-F(\overline{y}|x);V) \;\to\; w_j(\hat{x}_j)-w_j(x) \;>\; 0$$

as $\overline{y} \to y^+$ or $\overline{y} \to y^-$. If it is not the case that $x_j^*(\overline{y},V) \to \hat{x}_j$, then by compactness of $[0,\tilde{x}]$, we can assume that $x_j^*(\overline{y},V) \to x' > \hat{x}_j$ along some subsequence. Then

$$W_j(x_j^*(\overline{y},V),1-F(\overline{y}|x_j^*(\overline{y},V));V) \;\to\; w_j(x') \;<\; w_j(\hat{x}_j),$$

but then the office holder's expected discounted payoff from $\hat{x}_j$ exceeds that from $x_j^*(\overline{y},V)$ along the tail of the subsequence, a contradiction. We conclude that $x_j^*(\overline{y},V) \to \hat{x}_j$, completing the step. ▫

Step 3: The distance between optimal policy choices of different politician types as the voter's cutoff $\overline{y}$ and the continuation value V vary has a positive lower bound: there exists $\varepsilon > 0$ such that for all cutoffs $\overline{y}$, all values $V \in [\underline{V},\overline{V}]$, and all types $j < n$, we have $x_j^*(\overline{y},V)+\varepsilon \leqslant x_{*,j+1}(\overline{y},V)$. By Step 2, we can choose a sufficiently large interval $[y_L,y_H]$ and $\varepsilon' > 0$ such that for all $\overline{y}$ outside the interval, optimal policy choices differ by at least ε', i.e., for all $j < n$, we have

$|x_{*,j+1}(\overline{y},V) - x^*_j(\overline{y},V)| > \varepsilon'$. Moreover, by upper semi-continuity of $x^*_j(\cdot)$ and lower semi-continuity of $x_{*,j+1}(\cdot)$, the function $|x_{*,j+1}(\overline{y},V) - x^*_j(\overline{y},V)|$ attains a minimum on $[y_L, y_H]$, and this minimum is positive. Thus, there exists $\varepsilon'' > 0$ such that for all $\overline{y} \in [y_L, y_H]$, optimal policy choices differ by at least ε''. Finally, we set $\varepsilon = \min\{\varepsilon', \varepsilon''\}$ to complete the step. □

Let $\Delta^n(\varepsilon)$ denote the set of mixed policy profiles $(\pi_1,\ldots,\pi_n)$ such that for each type j, π_j has support in $[\hat{x}_j, \tilde{x}]$, and such that for all $j < n$ the support of π_j is separated from the support of π_{j+1} by a distance of at least ε, i.e.,

$$\max \operatorname{supp}(\pi_j) + \varepsilon \quad \leqslant \quad \min \operatorname{supp}(\pi_{j+1}).$$

Because it is defined by weak inequalities, the set $\Delta^n(\varepsilon)$ is compact in the relative topology inherited from the weak* topology on the space of signed Borel measures on $\Re^n$. The next step establishes joint continuity properties used later in the proof.

Step 4: For each type j, the expressions

$$\int_x f(\overline{y}|x)\pi_j(dx) \quad \text{and} \quad \int_x F(\overline{y}|x)\pi_j(dx)$$

are jointly continuous in $(\pi_j, \overline{y}) \in \Delta([\hat{x}_j, \tilde{x}]) \times Y$. To verify continuity of the first expression, let $(\pi_j, \overline{y})$ converge to $(\tilde{\pi}_j, \tilde{y})$. We have

$$\int_x f(\overline{y}|x)\pi_j(dx) \quad = \quad \int_x (f(\overline{y}|x) - f(\tilde{y}|x))\pi_j(dx) + \int_x f(\tilde{y}|x)\pi_j(dx). \tag{11}$$

Note that

$$\left| \int_x (f(\overline{y}|x) - f(\tilde{y}|x))\pi_j(dx) \right| \quad \leqslant \quad \max_{x \in [\hat{x}_j, \tilde{x}]} |f(\overline{y}|x) - f(\tilde{y}|x)|,$$

where the maximum is well-defined, since $[\hat{x}_j, \tilde{x}]$ is compact and the function $\tilde{f}(\overline{y}, x) \equiv f(\overline{y}|x) - f(\tilde{y}|x)$ is jointly continuous in $(\overline{y}, x)$. By the theorem of the maximum, joint continuity of $\tilde{f}$ implies that

$$\max_{x \in [\hat{x}_j, \tilde{x}]} |\tilde{f}(\overline{y}|x)| \;\rightarrow\; \max_{x \in [\hat{x}_j, \tilde{x}]} |\tilde{f}(\tilde{y}|x)| \;=\; 0.$$

Moreover, because $[\hat{x}_j, \tilde{x}]$ is compact and $f(\tilde{y}|x)$ is continuous in x, we have

$$\int_x f(\tilde{y}|x)\pi_j(dx) \;\rightarrow\; \int_x f(\tilde{y}|x)\tilde{\pi}_j(dx).$$

Combining these observations with (11), we see that $\int_x f(\overline{y}|x)\pi_j(dx)$ converges to $\int_x f(\overline{y}|x)\tilde{\pi}_j(dx)$. An analogous argument, with $F(\overline{y}|x)$ in place of $f(\overline{y}|x)$, establishes joint continuity of the second expression. This completes the step. □

Clearly, an implication of Step 4 is that the posterior probability that the office holder is type j, namely,

$$\mu_T(j|\overline{y}) \quad = \quad \frac{p_j \int_x f(\overline{y}|x)\pi_j(dx)}{\sum_k p_k \int_x f(\overline{y}|x)\pi_k(dx)},$$

is jointly continuous in $(\pi_1,\ldots,\pi_n,\overline{y}) \in \Delta^n(\varepsilon) \times Y$. Furthermore, using the expression for the voter's continuation value of a challenger in (2), we define the value induced by mixed policy strategies $\pi_1,\ldots,\pi_n$ and cutoff $\overline{y}$ by

$$\tilde{V}(\pi_1,\ldots,\pi_n,\overline{y}) \quad = \quad \frac{\sum_j p_j \int_x \left[\mathbb{E}[u(y)|x] + \delta(1-F(\overline{y}|x))\mathbb{E}[u(y)|\hat{x}_j]\right]\pi_j(dx)}{1-\delta\sum_j p_j \int_x [(1-F(\overline{y}|x))\delta + F(\overline{y}|x)]\pi_j(dx)}.$$

Next, we bound $\tilde{V}$ above by the expected discounted payoff from the ideal point of the highest type and below by the payoff from the lottery over ideal points:

$$V(\pi_1,\ldots,\pi_n,\overline{y}) \quad = \quad \max\left\{\sum_j p_j \frac{\mathbb{E}[u(y)|\hat{x}_j]}{1-\delta}, \min\left\{\frac{\mathbb{E}[u(y)|\hat{x}_n]}{1-\delta}, \tilde{V}(\pi_1,\ldots,\pi_n,\overline{y})\right\}\right\}.$$

Note that this expression is defined for an arbitrary profile in $\Delta([0,\tilde{x}])^n$, without any assumption that strategies are ordered by type, and Step 4 implies that $V(\pi_1,\ldots,\pi_n,\overline{y})$ is jointly continuous in $(\pi_1,\ldots,\pi_n,\overline{y}) \in \Delta^n(\varepsilon) \times Y$.

Now, let $\tilde{y}$ be the voter's optimal cutoff when office holders choose their ideal points, i.e., $\tilde{y}$ solves

$$\sum_j \left(\frac{p_j f(\tilde{y}|\hat{x}_j)}{\sum_k p_k f(\tilde{y}|\hat{x}_k)}\right)\mathbb{E}[u(y)|\hat{x}_j] \quad = \quad \sum_j p_j \mathbb{E}[u(y)|\hat{x}_j].$$

The next step shows that the optimal cutoff for the voter, if politician strategies converge to shirking in response to $\overline{y}$, converges to $\tilde{y}$ as $\overline{y} \to y^+$ or $\overline{y} \to y^-$. In general, the range of V is not contained in the interval $(\underline{V},\overline{V})$, but under the conditions of the next step, the induced value belongs to $(\underline{V},\overline{V})$, so that the optimal cutoff $y^*(\pi_1,\ldots,\pi_n,V(\pi_1,\ldots,\pi_n,\overline{y}))$ is well-defined.

Step 5: As $\overline{y} \to y^+$ or $\overline{y} \to y^-$, if policy strategies converge to shirking, then the induced value converges to the voter's discounted expected payoff when politicians choose their ideal points, and the optimal cutoff of the voter converges to $\tilde{y}$: for every sequence $(\pi_1,\ldots,\pi_n) \in \Delta^n(\varepsilon)$ such that each π_j converges weak to the unit mass on $\hat{x}_j$, we have*

$$(1-\delta)V(\pi_1,\ldots,\pi_n,\overline{y}) \;\to\; \sum_j p_j \mathbb{E}[u(y)|\hat{x}_j],$$

and

$$y^*(\pi_1,\ldots,\pi_n,V(\pi_1,\ldots,\pi_n,\overline{y})) \;\to\; \tilde{y}.$$

Whether $\overline{y}\to y^+$ or $\overline{y}\to y^-$, joint continuity of $V(\pi_1,\ldots,\pi_n,\overline{y})$, from Step 4, implies

$$(1-\delta)V(\pi_1,\ldots,\pi_n,\overline{y}) \;\to\; \sum_j p_j\mathbb{E}[u(y)|\hat{x}_j].$$

Write y^* for the optimal cutoff $y^*(\pi_1,\ldots,\pi_n,V(\pi_1,\ldots,\pi_n,\overline{y}))$, and note that it satisfies the indifference condition

$$\sum_j \mu_T(j|y^*)\mathbb{E}[u(y)|\hat{x}_j] \quad = \quad (1-\delta)V(\pi_1,\ldots,\pi_n,\overline{y}), \tag{12}$$

so the right-hand side of the indifference condition converges to the voter's expected payoff when politicians choose their ideal points. The left-hand side of (12), namely,

$$\sum_j\left(\frac{p_j\int_x f(y^*|x)\pi_j(dx)}{\sum_k p_k\int_x f(y^*|x)\pi_k(dx)}\right)\mathbb{E}[u(y)|\hat{x}_j],$$

is jointly continuous in $(\pi_1,\ldots,\pi_n,y^*)$, by Step 4. Thus, we need only argue that y^* does not diverge to y^+ or y^- along any subsequence.

To this end, first consider the case $y^*\to y^+$. For all $j<n$, let x_j maximize $f(y^*|x)$ over $x\in\mathrm{supp}(\pi_j)$, and let x_n minimize $f(y^*|x)$ over $x\in\mathrm{supp}(\pi_n)$. Note that for each j, we have $x_j\to\hat{x}_j$. Then as $\overline{y}\to y^+$, we have

$$\frac{p_j\int_x f(y^*|x)\pi_j(dx)}{\sum_k p_k\int_x f(y^*|x)\pi_k(dx)} \leqslant \frac{p_j\int_x f(y^*|x)\pi_j(dx)}{p_n\int_x f(y^*|x)\pi_n(x)} \leqslant \frac{p_jf(y^*|x_j)}{p_nf(y^*|x_n)} \to 0,$$

where the limit follows from (C6) and compactness of $\Delta^n(\varepsilon)$. This establishes that $\mu_T(n|\overline{y})$ goes to one as the cutoff increases. Thus, the left-hand side of (12) approaches $\mathbb{E}[u(y)|\hat{x}_n]$ as $y^*\to y^+$, a contradiction.

Next, consider the case $y^*\to y^-$ using a similar argument to that above. For all $j>1$, let x_j maximize $f(y^*|x)$ over $x\in\mathrm{supp}(\pi_j)$, and let x_1 minimize $f(y^*|x)$ over $x\in\mathrm{supp}(\pi_1)$. Note that for each j, we have $x_j\to\hat{x}_j$. Then as $\overline{y}\to y^-$, we have

$$\frac{p_j\int_x f(y^*|x)\pi_j(dx)}{\sum_k p_k\int_x f(y^*|x)\pi_k(dx)} \leqslant \frac{p_j\int_x f(y^*|x)\pi_j(dx)}{p_1\int_x f(y^*|x)\pi_1(x)} \leqslant \frac{p_jf(y^*|x_j)}{p_1f(y^*|x_1)} \to 0,$$

where the limit follows from (C6) and compactness of $\Delta^n(\varepsilon)$. This establishes that $\mu_T(1|\overline{y})$ goes to one as the cutoff decreases. Thus, the left-hand side of (12) approaches $\mathbb{E}[u(y)|\hat{x}_1]$ as $y^*\to y^-$, a contradiction. Therefore, y^* does not diverge to y^+ or y^- along any subsequence. By continuity of the left-hand side of the voter's indifference condition (12), the only possible limit of any subsequence is $\tilde{y}$, and we conclude that $y^* \to \tilde{y}$. This completes the step. ▫

The next step allows us to circumvent the discontinuity in $y^*(\cdot)$ anticipated prior to Proposition 2 and to bound the possible equilibrium cutoff of the voter. Note that under the conditions of Step 6, when a is low and b is high, and if policy strategies are optimal given $\overline{y} \notin (a,b)$, then the induced value $V(\pi_1,\ldots,\pi_n,\overline{y})$ belongs to $(\underline{V},\overline{V})$, so that the optimal cutoff $y^*(\pi_1,\ldots,\pi_n,V(\pi_1,\ldots,\pi_n,\overline{y}))$ is well-defined.

Step 6: There exist $a,b \in Y$ with a sufficiently low and b sufficiently high such that that for all $\overline{y} \in Y\backslash(a,b)$ and all $(\pi_1,\ldots,\pi_n) \in \Delta^n(\varepsilon)$, if there exists $V \in [\underline{V},\overline{V}]$ such that each π_j places probability one on solutions to

$$\max_{x\in[\hat{x}_j,\tilde{x}]} W_j(x,1-F(\overline{y}|x);V), \tag{13}$$

then

$$\underline{V} \ < \ V(\pi_1,\ldots,\pi_n,\overline{y}) \ < \ \overline{V} \tag{14}$$

and

$$a \ < \ y^*(\pi_1,\ldots,\pi_n,V(\pi_1,\ldots,\pi_n,\overline{y})) \ < \ b. \tag{15}$$

Indeed, let either $\overline{y} \to y^+$ or $\overline{y} \to y^-$. Let $(\pi_1,\ldots,\pi_n)$ be any selection of optimal mixed policy choices for the politicians given $\overline{y}$, i.e., each π_j places probability one on solutions to (13). By Step 3, we have $(\pi_1,\ldots,\pi_n) \in \Delta^n(\varepsilon)$, and Step 2 implies that for each j, the policy strategy π_j converges weak* to the unit mass on $\hat{x}_j$, uniformly in $V \in [\underline{V},\overline{V}]$. By Step 5, we then have

$$(1-\delta)V(\pi_1,\ldots,\pi_n,\overline{y}) \ \to \ \sum_j p_j \mathbb{E}[u(y)|\hat{x}_j],$$

and

$$y^*(\pi_1,\ldots,\pi_n,V(\pi_1,\ldots,\pi_n,\overline{y})) \ \to \ \tilde{y}.$$

Thus, we can choose $a,b \in Y$ such that for all $\overline{y}$ with $\overline{y} \leqslant a$ or $\overline{y} \geqslant b$, (14) and (15) hold. This completes the step. ▫

Let $\Delta([a,b])$ be the set of Borel probability measures with support in $[a,b]$, with elements denoted ρ. Let $\overline{y}(\rho) = \mathbb{E}[\rho]$ denote the mean of ρ, and note that $\overline{y}(\rho)$ varies continuously with the weak* topology on $\Delta([a,b])$.

We are now ready to define the fixed point correspondence $\Phi\colon \Delta([0,\tilde{x}])^n \times \Delta([a,b]) \rightrightarrows \Delta([0,\tilde{x}])^n \times \Delta([a,b])$. Given type j and $(\pi_1,\ldots,\pi_n,\rho)$ in the domain, let $M_j(\pi_1,\ldots,\pi_n,\rho)$ be the set of optimal policy choices for the type j politician:

$$M_j(\pi_1,\ldots,\pi_n,\rho) \quad = \quad \arg\max_{x\in[0,\tilde{x}]} W_j(x,1-F(\overline{y}(\rho)|x);V(\pi_1,\ldots,\pi_n,\overline{y}(\rho))).$$

Three features of this construction are noteworthy. First, the probability measure ρ is reduced to its mean, restricted to the interval $[a,b]$, so that no other information about this distribution is used. Second, the continuation value used by the politician is imputed by the policy strategies and cutoff $\overline{y}(\rho)$. Third, the objective function used in the definition of ϕ_j is jointly continuous in x and $(\pi_1,\ldots,\pi_n,\rho)$, and it follows from the theorem of the maximum. Then define

$$\Phi_j(\pi_1,\ldots,\pi_n,\rho) \quad = \quad \Delta(M_j(\pi_1,\ldots,\pi_n,\rho))$$

as the set of mixtures over optimal policy choices of type j politicians. The correspondence Φ_j then has convex values, and it inherits non-empty values and closed graph from M_j. Given $(\pi_1,\ldots,\pi_n,\rho)$ in the domain, we define $C(\pi_1,\ldots,\pi_n,\rho)$ to be the set of optimal cutoffs for the voter:

$$C(\pi_1,\ldots,\pi_n,\rho) \quad = \quad \arg\max_{\overline{y}\in[a,b]} U(\overline{y},\pi_1,\ldots,\pi_n;V(\pi_1,\ldots,\pi_n,\overline{y}(\rho))).$$

Then let

$$\Phi_{n+1}(\pi_1,\ldots,\pi_n,\rho) \quad = \quad \Delta(C(\pi_1,\ldots,\pi_n,\rho))$$

be the set of mixtures over optimal cutoffs. This correspondence has non-empty, convex values and closed graph. Finally, we define the fixed point correspondence Φ as the product of the correspondences defined above:

$$\Phi(\pi_1,\ldots,\pi_n,\rho) \quad = \quad \prod_{i=1}^{n+1} \Phi_i(\pi_1,\ldots,\pi_n,\rho).$$

The next step shows that Φ admits a fixed point, and that equilibrium policy strategies are strictly ordered by type.

Step 7: There exists $(\pi_1^*,\ldots,\pi_n^*,\rho^*)$ *in the domain such that* $(\pi_1^*,\ldots,\pi_n^*,\rho^*) \in \Phi(\pi_1^*,\ldots,\pi_n^*,\rho^*)$*, and the supports of* $\pi_1^*,\ldots,\pi_n^*$ *are strictly ordered according to type and are separated by a distance of at least* ε*, i.e.,* $(\pi_1^*,\ldots,\pi_n^*) \in \Delta^n(\varepsilon)$. Because Φ inherits non-empty, convex values and closed graph from its components, existence of a fixed point follows directly from Glicksberg's fixed point theorem, and Step 3 implies $(\pi_1^*,\ldots,\pi_n^*) \in \Delta^n(\varepsilon)$. □

Step 8: The cutoff $\overline{y}(\rho^*)$ *satisfies* $a < \overline{y}(\rho^*) < b$. Suppose toward a contradiction that either $\overline{y}(\rho^*) \leqslant a$ or $\overline{y}(\rho^*) \geqslant b$. Then since each π_j^* places probability one on policies that are optimal for the type j politician given cutoff $\overline{y}(\rho^*)$ and induced value $V(\pi_1^*,\ldots,\pi_n^*,\overline{y}(\rho^*))$, Step 6 implies that

$$\underline{V} \ < \ V(\pi_1^*,\ldots,\pi_n^*,\overline{y}(\rho^*)) \ < \ \overline{V}$$

and

$$a < y^*(\pi_1^*,\ldots,\pi_n^*,V(\pi_1^*,\ldots,\pi_n^*,\bar{y}(\rho^*))) < b.$$

By Proposition 2, the first set of inequalities imply that the unique solution to

$$\max_{\bar{y}\in Y} U(\bar{y},\pi_1^*,\ldots,\pi_n^*;V(\pi_1^*,\ldots,\pi_n^*,\bar{y}(\rho^*))$$

is $y^*(\pi_1^*,\ldots,\pi_n^*,V(\pi_1^*,\ldots,\pi_n^*,\bar{y}(\rho^*)))$, and the second set of inequalities implies that this cutoff belongs to the interval $[a,b]$. Thus, ρ^* places probability one on this cutoff, which implies

$$\bar{y}(\rho^*) = y^*(\pi_1^*,\ldots,\pi_n^*,V(\pi_1^*,\ldots,\pi_n^*,\bar{y}(\rho^*))).$$

But then we have $\bar{y}(\rho^*) \in (a,b)$, a contradiction. This completes the step. ▫

In the remainder of the proof, set $y^* = \bar{y}(\rho^*)$.

Step 9: The induced value $V(\pi_1^*,\ldots,\pi_n^*,y^*)$ *lies strictly between the voter's expected discounted payoff from the ideal points of the lowest and highest types:*

$$\underline{V} < (1-\delta)V(\pi_1^*,\ldots,\pi_n^*,y^*) < \overline{V},$$

and ρ^* *is the unit mass on* y^*. Indeed, the first inequality follows directly from construction of $V(\cdot)$, where we bound V below by the voters discounted expected payoff when politicians choose their ideal points. If the second inequality fails, then $V(\pi_1^*,\ldots,\pi_n^*,y^*) = \frac{\mathbb{E}[u(y)|\hat{x}_n]}{1-\delta}$. By Step 8, we have $a < y^* < b$, and thus $\rho^*([a,b)) > 0$, so there is a solution $\hat{y} \in [a,b)$ to

$$\arg\max_{\bar{y}\in[a,b]} U(\bar{y},\pi_1^*,\ldots,\pi_n^*;V(\pi_1^*,\ldots,\pi_n^*,y^*)).$$

The necessary first order condition at $\hat{y}$ yields

$$\sum_k \mu_T(k|\hat{y})\mathbb{E}[u(y)|\hat{x}_k] \geqslant (1-\delta)V(\pi_1^*,\ldots,\pi_n^*,y^*) = \mathbb{E}[u(y)|\hat{x}_n],$$

but this implies $\mu_T(n|\hat{y}) = 1$, which is impossible. We conclude that the second inequality holds as well. Since the induced value $V(\pi_1^*,\ldots,\pi_n^*,y^*)$ satisfies both inequalities, Proposition 2 implies that the optimal cutoff for the voter is unique, and thus ρ^* is degenerate and places probability one on y^*, completing the step. ▫

Define $\psi = (\sigma,\mu)$ so that for all j, $\pi_j^1 = \pi_j^*$ and π_j^2 places probability one on $\hat{x}_j$; that α is the cutoff rule with cutoff y^*; and that μ is derived via Bayes' rule.

Step 10: The assessment ψ *is a stationary electoral equilibrium.* In particular, $V^* = V^C(\psi)$, and ψ satisfies the necessary and sufficient conditions for equilibrium described in Section 4. This completes the proof of existence. Given an arbitrary stationary electoral equilibrium ψ, Theorem 4 in Section 8 implies that $\mathbb{E}[u(y)|\hat{x}_1] < (1-\delta)V^C(\psi) < \mathbb{E}[u(y)|\hat{x}_n]$, and thus properties (i) and (ii) follow from Propositions 1 and 2, respectively. ▫

A.2 Proof of Lemma 1

To prove the lemma, recall that the continuation value of a challenger satisfies

$$V^C(\psi) = \sum_j p_j \int_x \Big[\mathbb{E}[u(y)|x] + \delta[(1-F(\bar{y}|x))(\mathbb{E}[u(y)|\hat{x}_j] + \delta V^C(\psi)) + F(\bar{y}|x)V^C(\psi)]\Big]\pi_j^1(dx). \quad (16)$$

Note that

$$\begin{aligned}
&\sum_j p_j \int_x (1-F(\bar{y}|x))(\mathbb{E}[u(y)|\hat{x}_j] + \delta V^C(\psi))\pi_j^1(dx) \\
&= \int_{\bar{y}}^{y^+} \left[\sum_j \left(p_j \int_x f(\tilde{y}|x)\pi_j^1(dx)\right)(\mathbb{E}[u(y)|\hat{x}_j] + \delta V^C(\psi))\right] d\tilde{y} \\
&= \int_{\bar{y}}^{y^+} \left(\sum_k p_k \int_x f(\tilde{y}|x)\pi_k^1(dx)\right)\left[\sum_j \mu_T(j|p,\tilde{y})(\mathbb{E}[u(y)|\hat{x}_j] + \delta V^C(\psi))\right] d\tilde{y} \\
&= \int_{\bar{y}}^{y^+} \left(\sum_k p_k \int_x f(\tilde{y}|x)\pi_k^1(dx)\right) V^I(\tilde{y}|\psi)d\tilde{y} \\
&\geqslant V^C(\psi)\sum_k p_k \int_x (1-F(\bar{y}|x))\pi_k^1(dx),
\end{aligned}$$

where the second equality follows by multiplying and dividing by the denominator in the expression for Bayes' rule. Substituting into (16), we obtain

$$V^C(\psi) \geqslant \sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) + \delta V^C(\psi)\sum_j p_j \int_x (1-F(\bar{y}|x))\pi_j^1(dx) + \sum_j p_j \int_x F(\bar{y}|x)V^C(\psi)\pi_j^1(dx).$$

Re-arranging terms, this yields the desired inequality.

A.3 Proof of Theorem 3

Consider a sequence of stationary electoral equilibria as $\beta\delta \to \infty$, and note from Theorem 1 that equilibriums cutoff $\bar{y}$ belong to Y, and that equilibrium mixed policy strategies of first term politicians, π_j^1, are strictly ordered by type. We first prove (i). Indeed, suppose there is a subsequence of stationary electoral equilibria such that $\overline{y}$ is bounded away from y^+ and y^-, so going to a subsequence if needed, we can assume that $\overline{y} \to \tilde{y} \in Y$. I claim that for each type j politician, the least optimal policy choice diverges to infinity, i.e., $x_{*,j} \to \infty$. Otherwise, we can go to a subsequence if needed to suppose that $x_{*,j} \to \tilde{x} < \infty$. But the office holder's expected discounted payoff from choosing any $\overline{x} > \tilde{x}$ minus the payoff from $x_{*,j}$ is:

$$\begin{aligned}&w_j(\overline{x}) - w_j(x_{*,j}) + \delta\beta(F(\overline{y}|x_{*,j}) - F(\overline{y}|\overline{x}))\\ &\quad +\delta[(F(\overline{y}|x_{*,j}) - F(\overline{y}|\overline{x}))(w_j(\hat{x}_j) + \delta V^C(\psi)) - (F(\overline{y}|x_{*,j}) - F(\overline{y}|\overline{x}))V^C(\psi)].\end{aligned}$$

For $x_{*,j}$ close to $\tilde{x}$, (C5) implies $F(\overline{y}|x_{*,j}) > F(\overline{y}|\overline{x})$, and using the fact, from Theorem 4 in Section 8, that $\underline{V} \leqslant V^C(\psi) \leqslant \overline{V}$, this payoff difference is then greater than or equal to:

$$\begin{aligned}&w_j(\overline{x}) - w_j(x_{*,j}) + \delta\beta(F(\overline{y}|x_{*,j}) - F(\overline{y}|\overline{x}))\\ &\quad +\delta[F(\overline{y}|x_{*,j}) - F(\overline{y}|\overline{x}))(w_j(\hat{x}_j) + \delta\underline{V}) - (F(\overline{y}|x_{*,j}) - F(\overline{y}|\overline{x}))\overline{V}].\end{aligned}$$

Thus, because $\lim \beta\delta = \infty$, the liminf of the payoff difference goes to infinity, contradicting optimality of $x_{*,j}$. We conclude that $x_{*,j} \to \infty$ for each politician type j, as claimed. But then the voter's expected payoff from the policy choices of first-term office holders then diverges to infinity, contradicting Corollary 1 in Section 8. We conclude that $\overline{y}$ is not bounded away from y^+ and $\overline{y} \to y^-$.

Now suppose there is a subsequence such that $\overline{y} \to y^-$. Because policy choices are strictly ordered by type, it follows that for all $x_1, \ldots, x_n$ in the support of the politicians' policy strategies, we have $\hat{x}_1 \leqslant x_1 < x_2 < \cdots < x_n$. By (C6), for $\overline{y}$ sufficiently low, we have

$$f(\overline{y}|x_1) \;>\; f(\overline{y}|x_2) \;>\; \cdots \;>\; f(\overline{y}|x_n).$$

Thus, the coefficients on prior beliefs from Bayes' rule are ordered by type, i.e.,

$$\frac{\sum_x f(\overline{y}|x)\pi_1^1(x)}{\sum_k p_k \sum_x f(\overline{y}|x)\pi_k^1(x)} \;>\; \cdots \;>\; \frac{\sum_x f(\overline{y}|x)\pi_n^1(x)}{\sum_k p_k \sum_x f(\overline{y}|x)\pi_k^1(x)},$$

and we conclude that the voter's prior first order stochastically dominates the posterior distribution $\mu_T(\cdot|\overline{y})$, which implies

$$\sum_j p_j \mathbb{E}[u(y)|\hat{x}_j] \;>\; \sum_j \mu_T(j|\overline{y})\mathbb{E}[u(y)|\hat{x}_j].$$

From Lemma 1, inequality (9) holds, and since $\hat{x}_j \leqslant x_{*,j}$ holds for all j, we have $(1-\delta)V^C(\psi) \geqslant \sum_j p_j \mathbb{E}[u(y)|\hat{x}_j]$. Using (1), we then have

$$V^I(\overline{y}|\psi) \;<\; \sum_j p_j \left[\mathbb{E}[u(y)|\hat{x}_j] + \delta V^C(\psi)\right] \;\leqslant\; V^C(\psi),$$

contradicting the voter's indifference condition in (3). Thus, $\overline{y} \to y^+$, as desired.

To prove (ii) and (iii), we next show that for all types j, there is no subsequence of greatest optimal policy choices x_j^* that converge to a finite policy choice greater than the politician's ideal point; by the same argument, the least optimal policy choices $x_{*,j}$ also cannot converge to a finite policy choice greater than the ideal point. Indeed, suppose that there is some type j such that $x_j^* \to \tilde{x}_j$ with $\hat{x}_j < \tilde{x}_j < \infty$. Then for sufficiently large $\beta\delta$, we have $\hat{x}_j < x_j^* < \tilde{x} + \frac{1}{2}$. For these parameters, the current gain to the type j politician from choosing $\hat{x}_j$ instead of x_j^* is non-positive, and thus

$$\delta(F(\overline{y}|\hat{x}_j) - F(\overline{y}|x_j^*))[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)] \geqslant w_j(\hat{x}_j) - w_j(x_j^*). \qquad (17)$$

That is, the current gains from choosing the ideal point are offset by future losses. Note that, by (C5), we have

$$\frac{F(\overline{y}|x_j^*) - F(\overline{y}|\tilde{x}_j + 1)}{F(\overline{y}|\hat{x}_j) - F(\overline{y}|x_j^*)} \geqslant \frac{F(\overline{y}|\tilde{x} + \frac{1}{2}) - F(\overline{y}|\tilde{x}_j + 1)}{F(\overline{y}|\hat{x}_j) - F(\overline{y}|\tilde{x} + \frac{1}{2})}, \qquad (18)$$

and since $\overline{y} \to y^+$, the limit of the right-hand side above as $\beta\delta$ becomes large is indeterminate. By L'Hôpital's rule, the limit is equal to

$$\lim \frac{f(\overline{y}|\tilde{x} + \frac{1}{2}) - f(\overline{y}|\tilde{x}_j + 1)}{f(\overline{y}|\hat{x}_j) - f(\overline{y}|\tilde{x} + \frac{1}{2})} = \lim \frac{\frac{f(\overline{y}|\tilde{x}_j+1)}{f(\overline{y}|\tilde{x}+\frac{1}{2})}\left(\frac{f(\overline{y}|\tilde{x}+\frac{1}{2})}{f(\overline{y}|\tilde{x}_j+1)} - 1\right)}{\frac{f(\overline{y}|\hat{x}_j)}{f(\overline{y}|\tilde{x}+\frac{1}{2})} - 1} = \infty, \qquad (19)$$

where the limit follows from (C6). Then, however, the future gain from choosing $\tilde{x}_j + 1$ instead of x_j^* strictly exceeds current losses, i.e.,

$$\begin{aligned} &\delta(F(\overline{y}|x_j^*) - F(\overline{y}|\tilde{x}_j + 1))[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)] \\ &> \quad w_j(x_j^*) - w_j(\tilde{x}_j + 1), \end{aligned} \qquad (20)$$

for some parameters (β', δ'). To be specific, let

$$\begin{aligned} A &= \delta[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)] \\ B &= w_j(\hat{x}_j) - w_j(x_j^*) \\ C &= w_j(x_j^*) - w_j(\tilde{x}_j + 1), \end{aligned}$$

where A is evaluated at β and δ with $\beta\delta$ sufficiently large. Note that since $\hat{x}_j < \tilde{x}_j < \infty$, we have $\lim B > 0$ and $\lim C < \infty$. By (17), we have $(F(\overline{y}|\hat{x}_j) - F(\overline{y}|x_j^*))A \geqslant B$, and combining (18) and (19), we have

$$\frac{F(\overline{y}|x_j^*) - F(\overline{y}|\tilde{x}_j + 1)}{F(\overline{y}|\hat{x}_j) - F(\overline{y}|x_j^*)} > \frac{C}{B}$$

for sufficiently large $\beta\delta$. Combining these facts, we have

$$(F(\overline{y}|\hat{x}_j) - F(\overline{y}|x_j^*))A\left(\frac{F(\overline{y}|x_j^*) - F(\overline{y}|\tilde{x}+1)}{F(\overline{y}|\hat{x}_j) - F(\overline{y}|x_j^*)}\right) \quad > \quad B\left(\frac{C}{B}\right),$$

which yields (20) for some (β', δ'). This gives the type j politician a profitable deviation from x_j^*, a contradiction.

Now, suppose there is a subsequence such that the greatest optimal policy choice x_n^* of the type n politicians is bounded above by some policy choice, say $\overline{x}$. It follows that for all politician types j, we have $x_j^* \to \hat{x}_j$, so that the probability of re-electing an incumbent goes to zero, i.e., for all politician types j, we have $\int_x F(\overline{y}|x)\pi_j^1(dx) \to 1$. Re-writing (2), we have

$$\begin{aligned}(1-\delta)V^C(\psi) \quad &= \quad \left(\frac{1-\delta}{1-\delta\sum_j p_j \int_x [(1-F(\overline{y}|x))\delta + F(\overline{y}|x)]\pi_j^1(x)}\right) \\ &\qquad \cdot \sum_j p_j \int_x \Big[\mathbb{E}[u(y)|x] + \delta(1-F(\overline{y}|x))\mathbb{E}[u(y)|\hat{x}_j]\Big]\pi_j^1(x) \\ &\leqslant \quad \sum_j p_j \int_x \Big[\mathbb{E}[u(y)|x] + \delta(1-F(\overline{y}|x))\mathbb{E}[u(y)|\hat{x}_j]\Big]\pi_j^1(x).\end{aligned}$$

Taking limits as $\overline{y} \to y^+$, we see that

$$\limsup (1-\delta)V^C(\psi) \quad \leqslant \quad \sum_j p_j \mathbb{E}[u(y)|\hat{x}_j]. \tag{21}$$

But using $\overline{y} \to y^+$, we also have

$$\mu_T(n|\overline{y}) \quad = \quad \frac{p_n \int_x f(\overline{y}|x)\pi_n^1(dx)}{\sum_k p_k \int_x f(\overline{y}|x)\pi_k^1(x)} \quad = \quad \frac{1}{1+\sum_{k<n} \frac{p_k \int_x f(\overline{y}|x)\pi_k^1(dx)}{p_n \int_x f(\overline{y}|x)\pi_n^1(dx)}}.$$

For each type $k < n$, let x_k maximize $f(\overline{y}|x)$ over the support of π_k, as a function of $\overline{y}$; and let x_n minimize $f(\overline{y}|x)$ over the support of π_n, also as a function of $\overline{y}$. Then using (C6), we have

$$\frac{p_k \int_x f(\overline{y}|x)\pi_k^1(dx)}{p_n \int_x f(\overline{y}|x)\pi_n^1(dx)} \;\leqslant\; \frac{f(\overline{y}|x_k(\overline{y}))}{f(\overline{y}|x_n(\overline{y}))} \;\to\; 0$$

as $\overline{y} \to y^+$, and we conclude that $\mu_T(n|\overline{y}) \to 1$. By the indifference condition (3), we then also have $(1-\delta)V^C(\psi) \to \mathbb{E}[u(y)|\hat{x}_n]$, contradicting (21). We conclude that $x_n^* \to \infty$. By Corollary 1, it cannot be that the type n politicians place probability one on arbitrarily high policy choices as $\beta\delta$ becomes large, and it follows that $x_{*,n} \to \hat{x}_n$, which proves (ii). Moreover, since policy choices are ordered by type, this implies that for all $j < n$, we have $x_j^* \to \hat{x}_j$. This proves (iii).

Next, consider $\eta > 0$, and suppose there is a subsequence such that $\pi_n^1([\eta,\infty)) = 0$ for arbitrarily large $\beta\delta$. Then (2) again yields the implication that $(1-\delta)V^C(\psi) \to \sum_j p_j \mathbb{E}[u(y)|\hat{x}_j]$, and again $\mu_T(n|\bar{y}) \to 1$, yielding a contradiction as in the previous paragraph. This proves (iv).

Finally, consider $\eta > 0$, and suppose there exist $c > 0$ and a subsequence such that $\pi_n^1([\eta,\infty)) > c$. With (C8), Corollary 1 implies that $\lim_{\theta\to\infty} \pi_n^1([\theta,\infty)) \to 0$. In particular, there exists $\theta > \eta$ such that for $\beta\delta$ sufficiently large, we have $\pi_n^1([\theta,\infty)) < c/2$, which implies $\pi_n^1([\eta,\theta]) > \frac{c}{2}$. This implies that there is a subsequence of optimal policy choices for the type n politician converging to a finite policy choice greater than the ideal point $\hat{x}_n$, generating a contradiction as in the first part of the proof of (ii). Thus, we have $\pi_n^1([\eta,\infty)) \to 0$. Since $\pi_n^1((\hat{x}_n,\eta)) \cup \pi_n^1([\eta,\infty)) = 1$, we have $\pi_n^1((\hat{x}_n,\eta)) \to 1$. This proves (v), as required.

A.4 Proof of Theorem 5

To prove the result, consider any type k, and suppose toward a contradiction that there exist $\varepsilon > 0$ and a subsequence of stationary electoral equilibria with

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \quad \geqslant \quad \mathbb{E}[u(y)|\hat{x}_k] + \varepsilon \tag{22}$$

for $\sum_{j>k} p_j$ arbitrarily close to zero. Since the voter is indifferent between the incumbent and challenger conditional on the equilibrium cutoff $\bar{y}$, (3) implies

$$(1-\delta)V^C(\psi) \quad = \quad \sum_j \left(\frac{p_j \int_x f(\bar{y}|x)\pi_j^1(dx)}{\sum_{j'} p_{j'} \int_x f(\bar{y}|x)\pi_{j'}^1(dx)} \mathbb{E}[u(y)|\hat{x}_j] \right).$$

Combining these observations with inequality (9) of Lemma 1, we have

$$\sum_j \left(\frac{p_j \int_x f(\bar{y}|x)\pi_j^1(dx)}{\sum_{j'} p_{j'} \int_x f(\bar{y}|x)\pi_{j'}^1(dx)} \mathbb{E}[u(y)|\hat{x}_j] \right) \quad \geqslant \quad \mathbb{E}[u(y)|\hat{x}_k] + \varepsilon.$$

This in turn implies that there is a positive lower bound on the posterior probability, conditional on $\bar{y}$, that the incumbent's type belongs to $\{k+1,\ldots,n\}$, i.e., there exists $\lambda > 0$ such that for arbitrarily small $\sum_{j>k} p_j$, we have

$$\sum_{j>k} \frac{p_j \int_x f(\bar{y}|x)\pi_j^1(dx)}{\sum_{j'} p_{j'} \int_x f(\bar{y}|x)\pi_{j'}^1(dx)} \quad \geqslant \quad \lambda$$

Since $\sum_{j>k} p_j \to 0$, this implies that

$$\lim \frac{\sum_{j>k} \int_x f(\overline{y}|x)\pi_j^1(dx)}{\sum_{j\leqslant k} p_j \int_x f(\overline{y}|x)\pi_j^1(dx)} \quad = \quad \infty.$$

And since optimal policy choices are bounded along the sequence, this implies $\overline{y} \to y^+$. But then the policy choices for all type j politicians in the first term of office approach their ideal points, and since $\sum_{j>k} p_j \to 0$, this implies

$$\limsup \sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \quad \leqslant \quad \mathbb{E}[u(y)|\hat{x}_k],$$

contradicting (22), as required.

A.5 Proof of Theorem 6

To deduce a contradiction, suppose there is a constant $c > 0$ and a sequence of parameters (β, δ) such that $\beta\delta \leqslant c$ and for which the voter's expected payoff from the choices of first-term office holders approaches the expected payoff from the ideal point of the type n politician, i.e.,

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \quad \to \quad \mathbb{E}[u(y)|\hat{x}_n]. \tag{23}$$

Using inequality (10) of Theorem 4 and inequality (9) of Lemma 1, we have

$$\mathbb{E}[u(y)|\hat{x}_n] \;\geqslant\; (1-\delta)V^C(\psi) \;\geqslant\; \sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \;\to\; \mathbb{E}[u(y)|\hat{x}_n],$$

which implies the voter's (normalized) continuation value of a challenger has limit equal to the expected payoff from the ideal point of the type n politicians. For simplicity, going to a further subsequence if needed, we may assume that the discount factor converges to limit $\lim \delta \in [0,1]$.

I claim that the optimal policy choices of office holders are bounded along the sequence by some $\overline{x}$, for suppose otherwise. Going to a subsequence if necessary, we can assume that $x_j^* \to \infty$ for some type j. Fix an arbitrary policy choice $\tilde{x}$, and write the office holder's expected discounted payoff from choosing x_j^* minus the payoff from choosing $\tilde{x}$ as

$$\begin{aligned} & w_j(x_j^*) - w_j(\tilde{x}) + \delta\beta(F(\overline{y}|\tilde{x}) - F(\overline{y}|x_j^*)) \\ & \qquad + \delta\Big[(F(\overline{y}|\tilde{x}) - F(\overline{y}|x_j^*))(w_j(\hat{x}_j) - (1-\lim\delta)\mathbb{E}[u(y)|\hat{x}_n])\Big] \\ & < \quad 0. \end{aligned}$$

Taking limits, (C3) and $\lim \beta\delta \leqslant c$ imply that this difference becomes positive, contradicting optimality of x_j^*. Thus, as claimed, we can bound the optimal policy choices of the politicians along the sequence by some $\overline{x}$.

The indifference condition (3) then implies that the posterior probability that the incumbent is type n conditional on observing $\overline{y}$ goes to one, i.e.,

$$\frac{p_n \int_x f(\overline{y}|x)\pi_n^1(dx)}{\sum_k p_k \int_x f(\overline{y}|x)\pi_k^1(dx)} \;=\; \mu_T(n|\overline{y}) \;\to\; 1.$$

Because the equilibrium policy choices of the politicians belong to the compact interval $[0,\overline{x}]$, this implies that $\overline{y} \to y^+$. It follows that for each $x \in (\hat{x}_j, \overline{x}]$, the probability of re-election goes to zero, i.e., $F(\overline{y}|x) \to 0$, and the payoff from choosing the ideal point $\hat{x}_j$ exceeds the payoff from x in the limit: indeed, the limit of the difference in payoffs is just $w_j(\hat{x}_j) - w_j(x) > 0$. Thus, we have $x_j^* \to \hat{x}_j$ for each type j, but then

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \;\to\; \sum_j p_j \mathbb{E}[u(y)|\hat{x}_j],$$

contradicting (23). This establishes the result.

A.6 Proof of Theorem 8

Let $\beta \to \infty$, and suppose toward a contradiction that there is a subsequence of stationary electoral equilibria ψ with cutoffs $\overline{y}$ and a corresponding sequence of cutoffs $\tilde{y} > \overline{y}$ and partial equilibria $\tilde{\pi}$ such that $V^C(\tilde{\pi}|\tilde{y}) \geqslant V^C(\psi)$. For each politician type j, let π_j denote the equilibrium policy strategy for the type j politician, and let $x_{*,j}$ denote the lowest solution to (4). Let $\hat{z}$ denote the mode of the density h. Note from Theorem 3 that $x_{*,j} \to \hat{x}_j$ and since $\overline{y} \to \infty$, which implies $\overline{y} - x_{*,j} > \hat{z}$ when β is large enough, and thus $h(z)$ is decreasing in $z \geqslant \overline{y} - x_{*,j}$. Moreover, when β is large enough, we have

$$H(\overline{y} - x_{*,j}) - H(\hat{z}) \quad > \quad 1 - H(\overline{y} - x_{*,j}). \tag{24}$$

I argue that the strategy $\tilde{\pi}_j$ must place probability one on policies strictly less than $x_{*,j}$. The argument proceeds in two steps.

Step 1: We begin from the equilibrium ψ and examine the optimal policy choices of the type j politician when the voter's cutoff is increased from $\overline{y}$ to $\tilde{y}$, holding the continuation value of a challenger fixed at the equilibrium level, $V^C(\psi)$. Theorem 2 implies that in equilibrium, $x_{*,j}$ satisfies the first order condition in (5) with equality. Recall that we can view the politician's optimization problem as that of maximizing a quasi-linear objective function subject to a re-election constraint, as in

$$\max_{(x,r)} w_j(x) + r\delta[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)]$$
$$\text{s.t. } r + H(\overline{y} - x) - 1 \leqslant 0.$$

The increase in the cutoff has the effect of shifting the constraint in Figure 1 to the right, as depicted in the top panel of Figure 3.[17] Since the density h is single-peaked and $\overline{y} - x_{*,j}$ exceeds the mode of h, we have

$$w'_j(x_{*,j}) \quad < \quad -h(\tilde{y} - x_{*,j})\delta[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)], \tag{25}$$

and consequently the first order condition is violated at $x_{*,j}$. Now, define $\tilde{x} = x_{*,j} + \tilde{y} - \overline{y}$, so that the probability of re-election from choosing $\tilde{x}$ given cutoff $\tilde{y}$ is the same as the probability of re-election from choosing $x_{*,j}$ in equilibrium.

I claim that the value of the politician's objective function at policies $x \in (x_{*,j}, \tilde{x}]$, given cutoff $\tilde{y}$, is strictly below the value at $x_{*,j}$. Indeed, note that the reduction in probability of re-election when $\overline{y}$ is raised to $\tilde{y}$, given any policy x, is $H(\tilde{y} - x) - H(\overline{y} - x)$. For any policy $x \in (x_{*,j}, \tilde{x}]$, this reduction in probability is greater than the reduction at $x_{*,j}$ if and only if

$$H(\tilde{y} - x) - H(\overline{y} - x) \quad > \quad H(\tilde{y} - x_{*,j}) - H(\overline{y} - x_{*,j}).$$

This inequality is verified in two cases. In case $\overline{y} - x < \hat{z}$, then it is implied by

$$H(\tilde{y} - x) - H(\hat{z}) \quad > \quad 1 - H(\overline{y} - x_{*,j}),$$

and using $\tilde{y} - x \geqslant \tilde{y} - \tilde{x} = \overline{y} - x_{*,j}$, the latter inequality follows from (24). In case $\overline{y} - x \geqslant \hat{z}$, the reduction in re-election probability at x is greater than the reduction at $x_{*,j}$ if and only if

$$\int_{\overline{y}}^{\tilde{y}} h(y - x)dy \quad > \quad \int_{\overline{y}}^{\tilde{y}} h(y - x_{*,j})dy.$$

For every $y \in [\overline{y}, \tilde{y}]$, single-peakedness of h implies $h(y - x) > h(y - x_{*,j})$, so the latter inequality holds. By quasi-linearity of the politician's objective function, we conclude that the dashed indifference curve lies above the graph of $1 - H(\tilde{y} - x)$ between $x_{*,j}$ and $\tilde{x}$ in the bottom panel of Figure 3. Thus, the claim holds.

Moreover, I claim that the value of the politician's objective function at policies $x > \tilde{x}$, given cutoff $\tilde{y}$, is also strictly below the value at $x_{*,j}$. To see this, consult Figure 4, where $r = 1 - H(\tilde{y} - x_{*,j})$ and $r' > r$. Here, a is the horizontal distance

[17] Figure 3 illustrates the problem of a politician type who has multiple optimal proposals given cutoff $\overline{y}$; this is immaterial, as the analysis focuses only on the least optimal policy choice, $x_{*,j}$.

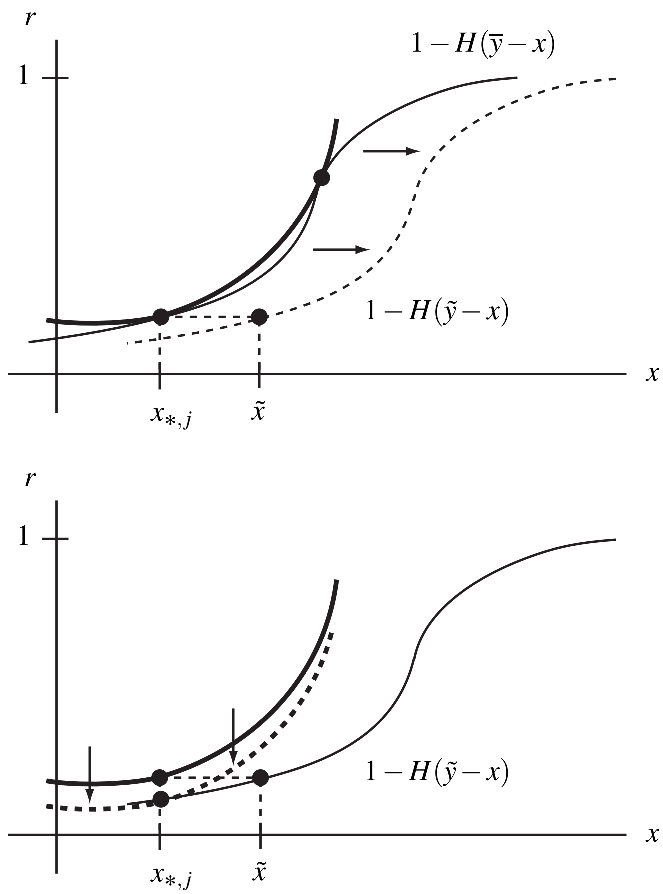

Figure 3: Politician's problem with commitment

between the level sets of the politician's objective function at height r, and $a+b$ is the horizontal shift in the constraint, i.e., $a+b = \tilde{y} - \bar{y}$. By the argument of the preceding paragraph, we have $b > 0$. Drawing a horizontal line at height $r' > r$, concavity in (C10) implies that the horizontal distance between the objective functions is no greater, so $c \leqslant a$, and because $x_{*,j}$ is optimal before the constraint shifts to the right, we also have $c + d \geqslant \tilde{y} - \bar{y}$. Combining these observations, we have

$$d \;\geqslant\; \tilde{y} - \bar{y} - c \;=\; a + b - c \;\geqslant\; b \;>\; 0.$$

Since this argument holds for all $r' \in (r, 1]$, we conclude that the dashed indifference curve lies above the graph of $1 - H(\tilde{y} - x)$ to the right of $\tilde{x}$, as in Figure 4. Thus, the claim holds. We conclude that the optimal policy choices for the politician given

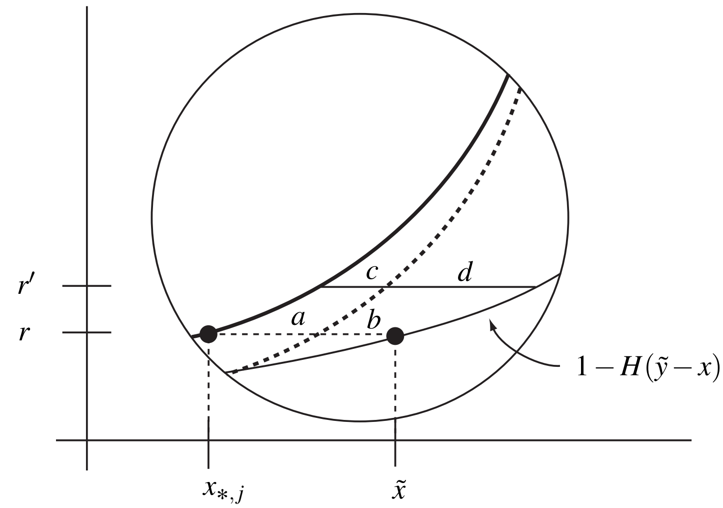

Figure 4: Politician's problem with commitment, again

cutoff $\tilde{y}$ all fall strictly below $x_{*,j}$. □

Step 2: We examine optimal policy choices when we replace the equilibrium continuation value $V^C(\psi)$ with the partial equilibrium value $V^C(\tilde{\pi}|\tilde{y}) \geqslant V^C(\psi)$. The effect is that the politicians' office incentives weakly decrease—geometrically, the level sets of the politicians' objective function become steeper, for the value of the objective function decreases more rapidly as the policy choice x moves away from the ideal point $\hat{x}_j$—with the consequence that optimal policy choices decrease. Let x' and x'' be policies such that $\hat{x}_j < x' < x''$, and let $r' = 1 - H(\tilde{y} - x')$ and $r'' = 1 - H(\tilde{y} - x'') > r'$ be the corresponding re-election probabilities. When the continuation value of a challenger increases from $V^C(\psi)$ to $V^C(\tilde{\pi}|\tilde{y})$, the decrease in the politician's objective function given policy choice x' is

$$r'\delta(1-\delta)[V^C(\tilde{\pi}|\tilde{y}) - V^C(\psi)],$$

and the decrease given policy choice x'' is

$$r''\delta(1-\delta)[V^C(\tilde{\pi}|\tilde{y}) - V^C(\psi)].$$

The latter quantity is greater than or equal to the former, as the politician's incentive to choose the greater policy is weaker. Thus, if x' is optimal for $V^C(\psi)$ and x'' is optimal for $V^C(\tilde{\pi}|\tilde{y})$, then x'' is optimal for $V^C(\psi)$ as well. Since optimal policies for $V^C(\psi)$ all fall strictly below $x_{*,j}$, it follows that all optimal policies for $V^C(\tilde{\pi}|\tilde{y})$ also fall strictly below $x_{*,j}$. □

We have seen that the optimal policy choices of all politician types decrease, and we conclude that the continuation value of a challenger from committing to $\tilde{y}$, with associated partial equilibrium $\tilde{\pi}$, is strictly lower than the equilibrium continuation value, i.e., $V^C(\tilde{\pi}|\tilde{y}) < V^C(\psi)$. This contradicts our initial supposition that $V^C(\tilde{\pi}|\tilde{y}) \geqslant V^C(\psi)$ as $\beta \to \infty$, as required.

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