Exercise 34. Use the Poincaré lemma to show that there exists a smooth function $g: \mathbb{R} \to \mathbb{R}$ such that
\[
\sin x + x^2 \exp(g(x)) - 1)g'(x) + g(x) \cos x + 2x \exp(g(x)) = 0.
\]

Definition. For any real numbers $0 \leq r \leq s$, let $T^2(r, s) \subset \mathbb{R}^3$ denote the torus
\[
T^2(r, s) := \left\{ (x, y, z) \in \mathbb{R}^3 \mid (x^2 + y^2 + z^2 + s^2 - r^2)^2 = 4s^2(x^2 + y^2) \right\}.
\]
Let us write $ST(r, s)$ for the union $\bigcup_{\rho \in [0, r]} T^2(\rho, s)$.

Exercise 35. What is the volume of $ST(r, s)$?

Exercise 36. What is the surface area of $T^2(r, s)$?

Exercise* 37. Consider the following vector field on $\mathbb{R}^3 - \{(0, 0, 0)\}$:
\[
\varphi(x, y, z) = \frac{1}{x^2 + y^2 + z^2}(-y, x, (z - 1/4)(z + 1/4) - x^2 - y^2).
\]
(a) Make a good faith attempt to draw this vector field (or some flow lines).
(b) Is this vector field a gradient vector field?