Comments on the Logic of Justification

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Abstract

This note presents some comments and questions related to two papers of Sergei Artemov: "The Logic of Justification" [1] and "The Ontology of Justifications in the Logical Setting" [2]. This note was prepared by invitation for the event *Logic across the University: Foundations and Applications* held at Tsinghua University, 14–16 October 2013, in Beijing, China.

1 Justification and Proof

I have come to think of the Justification Logic (JL) approach to reasoning about justification (or "evidence," broadly construed) as more paradigmatic than dogmatic. By this I mean that the justification-combining operations found in a specific JL system should be viewed as just one way to reason about justification, as opposed to an assertion of "the one true way" for reasoning about justification. In this sense, the main message I get from the JL approach is that we can use in-language syntactical bookkeeping to describe and characterize reasoning in a stepwise fashion, and this can be leveraged to provide a more nuanced formal account of knowledge and belief. In particular, by introducing justifications for the first time as separate logical entities, Justification Logic aims to persuade the logic community to consider the previously missing justification component as a key ingredient in logics of knowledge and belief.

While most JLs are based on the *sum* (i.e., justification aggregation) and *application* (i.e., justification Modus Ponens) operations, we should not be overly serious about this. When it comes to reasoning about justification, many other operations are also of interest, even some that are not logically sound. Indeed, much everyday, "real life" reasoning is not logically sound and, even worse, sometimes logically flawed. But there are ways to recover from flawed reasoning via "backtracking" or "revision," and all of this is worthy of consideration in a general study of formal justification. Examples of operations to consider: various other sound operations (e.g., a justification version of Modus Tollens), nonmonotonic or default operations (e.g., a justification version of "φ normally follows from ψ"), commonsense induction (e.g., a justification version of "if every φ-situation seen so far is also ψ, conclude $\varphi \rightarrow \psi$"), and even fallacies or other logically flawed operations (e.g., a justification version of "from $\varphi \rightarrow \psi$ and ψ, conclude φ"). Of course the unsound operations must

come with some means of recovery from error, perhaps in a justification-friendly adaptation of Belief Revision theory, which, in the interest of full disclosure, is part of an ongoing project of mine.

In essence, the general picture I see is the following: justification is in some sense about "telling a story" as to why something is the case, and the quality of the justification has to do with the quality of the story (within a particular context and for a particular purpose). So far most work in JL has focused on extremely well-behaved "stories": Hilbert-style proofs constructed from basic assumptions using proof concatenation (i.e., sum), Modus Ponens (i.e., application), and possibly other logically sound operations on proofs. But there is room for a broader perspective, wherein we focus less on proofs and more on potentially unsound justificatory "stories" that are intended to support certain assertions, though this support may be highly subjective or contextual and is subject to revision upon receipt of additional information. From this perspective, the proof-based JLs studied to date are only a first approximation to a generalized study of subjective justification, the key feature of which is an adaptation of the in-language syntactical bookkeeping mechanisms already developed in the JL literature. Of course this extra syntactical structure comes with a cost: typically easy theorems (e.g., Replacement [6]) become difficult because formulas now include detailed, highly syntax-dependent "stories" asserting subjective justifications, and making sure everything is in order with these often requires nontrivial trickery. Nevertheless, what we gain in exchange is the opportunity for fine-grained analysis of a purported justification—something it seems we cannot do absent extra-modal syntactical baggage—and this sometimes makes it worth the additional effort.

Question 1. What would a Justification Logic-style syntactic bookkeeping mechanism for general justificatory "stories" look like?

Question 2. How might justificatory "stories" be compared in terms of their quality or persuasiveness (within a particular context and for a particular purpose)?

2 Justifications and Omniscience

One of the persistent philosophical complaints about modal logics for knowledge and belief is that these logics attribute "too much" knowledge or belief to the agent.[1] The syntactic bookkeeping mechanisms of Justification Logic suggest one way of addressing this: roughly speaking, larger justifications are needed for more distant conclusions, so explicit knowledge or belief (i.e., knowledge or belief witnessed by a specific justification) comes with a specific "cost" [3, 4, 5, 7, 8]. Nevertheless, there is still another kind of omniscience present: in most JLs, justifications are always "out there," even if they are too large for anyone to possibly know them. The question then becomes one of determining which justifications an agent has in hand and how it is that the agent comes to gain (or lose) justifications as a result of further consideration, receipt of additional information, or simple passage of time.

Question 3. What is the cleanest way to study the justifications of non-omniscient agents, taking into account the fact that the available justifications change over time?

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[1]I borrow this formulation from Melvin Fitting [5].

3   Conclusion

Inasmuch as there is structure to the story one gives in favor of an assertion, there is in some sense a "logic" afoot, and the syntactical bookkeeping mechanisms used in Justification Logic can be adapted to provide a fine-grained representation of these "logics." So far the focus has been on the usual things one thinks of when speaking of formal logic: sound axioms and rules of inference, proofs, and the like. But when it comes to everyday justifications, additional flexibility is required. In this short note I have tried to provide the briefest sketch of what this might mean for the study of Justification Logic. My hope is that those who wish to take this approach seriously as a general study of justification will agree with me that the JL literature has only scratched the surface, and so there is a great deal of interesting work still to be done.

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