Why Physics Uses Second Derivatives

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Abstract

I defend a causal, reductionist account of the nature of rates of change like velocity and acceleration. This account identifies velocity with the past derivative of position, and acceleration with the future derivative of velocity. Unlike most reductionist accounts, it can preserve the role of velocity as a cause of future positions and acceleration as the effect of current forces.

I show that this is possible only if all the fundamental laws are expressed by differential equations of the same order. Consideration of the continuity of time explains why the differential equations are all second order. This explanation is not available on non-causal or non-reductionist accounts of rates of change.

Finally, I argue that alleged counterexamples to the reductionist account involving physically impossible worlds are irrelevant to an analysis of the properties that play a causal role in the actual world.

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1 Background

There is a traditional debate in the metaphysics of physics about the status of velocity, acceleration, and other rates of change of quantities.[1] One traditional

[1] I intend my analysis to apply to whichever rates of change are causally relevant in the actual world. I talk throughout as though the actual world is roughly Newtonian, but I

view, called reductionist, follows Bertrand Russell in saying that the velocity and acceleration of an object at a time are themselves entirely grounded in the fundamental facts about the position of that object at all times. The non-reductionist, on the other hand, argues that these rates of change are metaphysically separate from the position facts.

A primary argument given by many non-reductionists claims that these rates of change must somehow play a role in causally determining the evolution of the world. This requirement poses challenges for standard reductionist accounts. I will neither attempt to argue against such a causal requirement that non-reductionists apparently have strong intuitions of, nor attempt to give an account of causation in terms that make sense to a reductionist. Instead, I will just show that a certain form of reductionism is compatible with the proposed causal requirement.

However, these considerations do put some constraints on the nature of rates of change that could be causally relevant. I will argue that the reductionist account I give is consistent with a causal requirement, but that it wouldn't be if there were laws appealing to the third derivative of position, as well as the force laws that actually exist. Thus, I claim this causal reductionist account can help explain why the fundamental physical laws don't appeal to third derivatives of quantities. Since this fact about the fundamental physical laws is otherwise apparently inexplicable, this may give some further motivation for the reductionist to accept the causal constraints, and for the causal theorist to accept a reductionist view of rates of change.

2 Grounding

The traditional reductionist view, due to Bertrand Russell, says that the velocity of an object at a time t just is the time derivative of the position of that object, at t. The time derivative is taken to be the limit of the ratio of distance traveled in a period of time, as the length of that period goes to 0. That is, the velocity of an object at an instant t is that value v_t, if any, that satisfies the formula:

$$\forall(\epsilon > 0)\exists(\delta > 0)\forall t' \left(|t' - t| < \delta \rightarrow \left| \frac{x_{t'} - x_t}{t' - t} - v_t \right| < \epsilon \right),$$

where t' quantifies over times, and $x_{t'}$ and x_t are the positions of the object at times t' and t respectively. v_t is that value (if any exists) such that for any degree ϵ of approximation one would like, by choosing t' close enough to t (that is, within δ), one can guarantee that the average velocity of the object between t and t' is that close to v_t. That is, for any t' in $\langle t - \delta, t + \delta \rangle$, it must be that $\frac{x_{t'} - x_t}{t' - t}$ is within ϵ of v_t.[2]

believe that what I say should apply to any rates of change that appear in the fundamental presentations of any laws that have been proposed as potentially actual.

[2] I use the notation $[a, b]$ to denote the interval from a to b that includes both endpoints, and angle brackets $\langle a, b\rangle$ on one or both ends to indicate that the relevant endpoint is excluded. It

There are some difficult questions here about the nature of velocity on such a reductionist view. Some might say it means there is no such thing—talk of velocity and acceleration should be eliminated in favour of talk of positions at times. Others might say that velocity exists, but that it is identical to some collection of positions at times, or some fact about such a collection. Others will say that velocity and acceleration exist as separate properties of an object, but that they are not fundamental—the fundamental features of an object are its position, mass, charge, etc. at various times, while velocity, acceleration, and various other non-fundamental features are merely *determined by* or *grounded in* the distributions of these fundamental features over time. I will talk in the latter way, though my main claims may be available to proponents of the eliminativist or identity views.

I won't assume much about the nature of this grounding relation, apart from the following: if some property P of an object at t is grounded in some properties of the object at some times, then it is impossible for the object to have those other properties at those times without having that value of P at t. I will also appeal at various points to some fundamental properties being 'sufficient to ground' some non-fundamental property. This will mean that the non-fundamental property is grounded in some subset of these fundamental properties, but the formulation is neutral as to whether or not all of these fundamental properties actually play a role in grounding the non-fundamental one.

At any rate, on the account on which v_t is the derivative of the position of an object, v_t is not grounded in the fundamental facts at that time—it is possible for all the fundamental properties of all objects at t to be the same as they actually are, while the positions at other times are such as to make the derivative of the position be something other than what it actually is. However, for any positive time interval Δ, the positions of the object in $\langle t-\Delta, t+\Delta\rangle$ are sufficient to ground the velocity at t. I will make the following definition, adapting an idea from Arntzenius:[3]

> A (two-sided) neighbourhood property at t is a property of an object that is not grounded in the fundamental properties of the object at t, but, for every interval $\langle t-\Delta, t+\Delta\rangle$, the fundamental properties of the object across that interval are sufficient to ground it.

The traditional reductionist idea is that the velocity and acceleration of an object are to be understood as neighbourhood properties, which are grounded in the locations of that object over time.

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is also quite common to use round parentheses (a, b) to indicate that the relevant endpoints are excluded, but this presents an ambiguity, as a pair of numbers enclosed by parentheses is also used as the notation for an ordered pair of numbers rather than an interval.

[3]His version of this definition is in [Arntzenius, 2000]. In that paper he also considers a view on which there are no instants in time, but merely intervals of arbitrarily short length. If that view of time is correct, it may cause problems for the positions I take, but I will not consider such a view here.

3 Causation

As Arntzenius and Lange note, however, such neighbourhood properties don't satisfy the intuitive demands for the causal role of a property like velocity. For instance, for an object that experiences no forces, the Newtonian laws guarantee that the position $x_{t+\delta}$ of an object at any time after t is given by

$$x_{t+\delta} = x_t + v_t \cdot \delta.$$

It is natural to think that this equation furthermore indicates a kind of *causal* determination that v_t has on the future positions $x_{t+\delta}$. But if v_t is a neighbourhood property, then this may not make sense. If v_t might be at least partly grounded in $x_{t'}$ for t' later than t, then v_t doesn't seem to be the kind of property that could partly causally determine $x_{t'}$.

To make the argument more explicit, consider the following two principles:

> Cause-Ground Transitivity: If A is part of the ground of B, and B partly causally determines C, then A partly causally determines C.[4]
>
> Causal Irreflexivity: A does not partly causally determine A.[5]

Using these two principles, we can see that if the velocity of an object partly causally determines each of the future positions of an object, then none of those future positions can be part of the ground of the velocity, or else they would partly causally determine themselves.

I won't say much more about what this sort of causal determination is, or how it is that a non-fundamental feature of the world can partly causally determine a fundamental feature of the world. Traditional reductionists will reject any argument based on causal determination because of the strangeness of these issues, especially in the setting of fundamental physics. I will instead concede the cogency of this notion to the non-reductionist, and show that there is a reductionist account compatible with the principles I will mention.

Lange describes one reductionist view of velocity that is not ruled out by the considerations above: one can say that velocity is the *past* derivative of position. That is, we define the 'past velocity' of an object at a time t to be that value v_t^p (if any), that satisfies the formula:

$$\forall(\epsilon > 0)\exists(\delta > 0)\forall t', t'' \left((t - \delta < t', t'' < t) \rightarrow \left| \frac{x_{t'} - x_{t''}}{t' - t''} - v_t^p \right| < \epsilon \right).$$

On this definition, v_t^p is entirely determined by the values $x_{t'}$ for t' in $\langle t - \delta, t \rangle$. Because of the way I have defined it, v_t^p doesn't depend on x_t itself, but only on the positions of the object at earlier times. It might seem more natural

[4] A principle like this for scientific explanation, rather than causation, is defended in [Lange, 2012].

[5] This principle, and some others to occur later, may well need to be modified to account for cases where time travel occurs. However, it seems plausible that none of the cases to be discussed in this paper involve time travel, so I will ignore the possibility throughout.

to include x_t as part of the definition of v_t^p, but this definition that only uses positions at times earlier than t will be useful when I start to consider second and higher derivatives later on.[6]

Echoing the earlier definition, I will define:

> A past neighbourhood property at t is a property of an object that is not grounded in the fundamental properties of the object at t, but, for every interval $\langle t - \Delta, t\rangle$, the fundamental properties of the object across that interval are sufficient to ground it.

By the above definitions, past velocity is a past neighbourhood property. It is grounded entirely in past positions of the object. It can partly causally determine the future positions of the object, with no problems for transitivity or irreflexivity. The first part of my reductive analysis will be to say that the notion of 'velocity' that plays a causal role in classical mechanics is this past velocity.

Lange considers and rejects one argument against attributing causal significance to these past velocities, before giving his own argument later on. The argument he rejects is one he attributes to Bigelow and Pargetter, which says that causation involving past velocity requires a certain kind of action at a temporal distance. They suggest that the force with which a meteor impacts Mars is related to the velocity of the meteor, but if the velocity of the meteor is grounded in facts about the past location of the meteor, then those past location facts seem to have causal consequences now. 'This requires the meteor to have a kind of "memory"—what it does to Mars depends not only on its current properties but also on where it has been.'[Bigelow and Pargetter, 1990, p. 72] As Lange points out though, this may not be so bad—since the velocity at a time is grounded in location facts at any arbitrarily small time interval, the meteor doesn't have to have any positive length of 'memory'. There is no positive ϵ such that the velocity at t depends essentially on positions before $t - \epsilon$. (This will be discussed further in section 6.) And as Lange points out, this appears to be the best we can do in a world where time is continuous. If present fundamental quantities at t are caused by properties at some earlier time t', then there would be action at a temporal distance. So if they are to be caused by properties that are earlier than t, then they must be caused by past neighbourhood properties at t.[7]

[6]The definition I give here for this one-sided open-ended derivative is exactly the one-sided limit of the standard two-sided derivative, and thus the only way this could differ from the one-sided closed-ended derivative is if the standard two-sided derivative changes discontinuously, in which case the open-ended derivative may exist while the closed-ended derivative doesn't. If the laws guarantee that derivatives always change continuously, then this means that the two values will always agree. Thus, the explanatory payoff I show later for using the open-ended derivative will suggest that we should use this open-ended derivative rather than the slightly simpler closed-ended one.

[7]I would also like to point out that the meteor example in any case depends on a mistake. The relevant forces that are exerted by the meteor on the surface of Mars are primarily forces of electrostatic repulsion between the electrons in the molecules that make up the meteor and the electrons in the molecules that make up the surface of Mars. These forces are in fact

The problem Lange raises instead involves considering these rates of change as *effects*. His argument [Lange, 2005, p. 448] assumes something like the following principle:

> Existential Forwards Causation: If A partly causally determines B, then for any set S_A of sufficient grounds for A and any set S_B of sufficient grounds for B, some member of S_A is not temporally later than some member of S_B.

In a causal interpretation of classical Newtonian physics, the present acceleration of an object is causally determined by the present masses and locations of all objects. If acceleration were a past neighbourhood property, then there would be a sufficient set of grounds for it that are all temporally earlier than the present, which would thus violate Existential Forwards Causation. (Again, a non-causal reductionist can deny the problem by taking a Humean view of laws, on which they express regularities of the universe that are not causal, but my account will preserve the causal interpretation of the laws.)

So Lange considers one other alternative, which involves the notion of a future derivative. The future derivative of a quantity y at a time t is that value z_t^f (if any), that satisfies the formula:

$$\forall(\epsilon > 0)\exists(\delta > 0)\forall t', t'' \left((t < t', t'' < t + \delta) \rightarrow \left| \frac{y_{t'} - y_{t''}}{t' - t''} - z_t^f \right| < \epsilon \right).$$

I similarly define:

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entirely determined by the *positions* of the molecules in the meteor and the molecules in Mars, and don't directly depend on the velocity at all. (And by Newton's third law, the forces that Mars exerts on the meteor are exactly equal and opposite to the forces that the meteor exerts on Mars.)

So why is it that a low velocity meteor might have very little effect on the surface of Mars, while a sufficiently high velocity meteor might leave an enormous crater? This is simply because a very high velocity meteor will travel much farther than a very low velocity meteor will, if the same forces are exerted on them. The high velocity meteor will get close enough to a very large number of molecules on Mars that the forces it exerts will push them aside, while the low velocity meteor will be stopped before it gets near any molecules that are far from the surface. (As to the question of why molecules themselves are large enough to exert significant electromagnetic forces on each other, that involves some tricky issues in quantum mechanics that are beyond the scope of this article. [Lieb, 1990])

I suspect that in general, fundamental forces are always determined by the locations (and masses and charges and other fundamental properties) of objects at a time, and never directly depend on velocities or other neighbourhood properties. The only direct fundamental causal relevance of the velocity of an object at a time is to the future positions and velocities of that same object. All other effects of velocity are intermediated through these future positions of the object. This is certainly true for Newtonian gravitation. Whether it is also true for classical electromagnetism depends on some subtle issues—are the electric and magnetic fields both fundamental quantities, or is one just some sort of derivative of the other?

There may also be non-fundamental forces in which velocity plays a causal role, but I suspect these can be explained in terms of fundamental causation that doesn't involve velocity. However, this claim about velocity is not relevant to my main argument, and it can be given up without any effect on my reductionist views or the explanation of why fundamental physics doesn't use third derivatives.

> A future neighbourhood property at t is a property of an object that is not grounded in the fundamental properties of the object at t, but, for every interval $\langle t, t+\Delta\rangle$, the fundamental properties of the object across that interval are sufficient to ground it.

Thus, the future derivative of a fundamental quantity is a future neighbourhood property. Present properties of an object can causally determine future neighbourhood properties of an object even if we assume a stronger principle:

> Universal Forwards Causation: If A partly causally determines B, then there is a set S_A of sufficient grounds for A and a set S_B of sufficient grounds for B, such that no member of S_A is temporally later than any member of S_B.

However, Lange's worry is that although past neighbourhood properties can serve as causes, and future neighbourhood properties can serve as effects, neither one can serve as both. And this is his central argument:

> Any difference between a body's location at one moment and its location at some later moment must have been caused by the body's having non-zero velocity at various intervening moments, any difference between a body's velocity at one moment and its velocity at some later moment must have been caused by the body's undergoing non-zero acceleration sometime in the intervening period, and every acceleration is caused by a force. [Lange, 2005, pp. 434-5]

Lange says forces cause accelerations, which cause velocities, which cause positions. If velocities are past derivatives, then they can be causes, and if they are future derivatives, then they can be effects. But neither interpretation allows them to be both causes and effects. Thus, Lange's proposed causal chain is impossible, even allowing these one-sided derivatives in the reductionist analysis of velocity and acceleration.

4 The proposal

I claim that once we understand things properly, this problem will go away. The fundamental causal chains are shorter than Lange thinks, so that the problem doesn't arise. However, Lange's worry about the possibility of one (past or future) neighbourhood property being both an effect and a cause of other fundamental and neighbourhood properties at a time will help explain why it is that all fundamental physical laws involve only second-order differential equations.

I have proposed above that part of my reductionist solution involves identifying the notion of velocity that plays a causal role in mechanics with past velocity, the past derivative of position. The other half of my solution is to identify the notion of acceleration that plays a causal role in mechanics with the *future* derivative of *past* velocity. Lange is never explicit about whether the acceleration he considers as a future derivative is the future derivative of future velocity or the future derivative of past velocity. For my view, it is important that it be the future derivative of past velocity. I will symbolise this quantity as a^{pf}, with the first superscript indicating that the first derivative of position is calculated from the past, to get past velocity, and the second superscript indicating that the second derivative is calculated from the future, to get the future derivative of past velocity.

First, notice that the future derivative of past velocity is indeed a future neighbourhood property, on my characterisation. To see this, note that a_t^{pf}, as the future derivative of v^p, is determined by the set of values $v_{t'}^p$ at all t' in the interval $\langle t, t+\Delta\rangle$. Thus, I claim, any properties that are sufficient to ground all of these $v_{t'}^p$ are sufficient to ground a_t^{pf}. Now consider some t' in this interval. Let $\Delta' = t' - t$. Because v^p is a past neighbourhood property, the fundamental properties in the interval $\langle t' - \Delta', t'\rangle$ are sufficient to ground $v_{t'}^p$. But by definition of t' and Δ', this interval is contained entirely in $\langle t, t+\Delta\rangle$. Thus, for any Δ, the fundamental properties of the object in the interval $\langle t, t+\Delta\rangle$ are sufficient to ground a_t^{pf}.

This is where the open-end feature of my definition of these derivatives is important. If the future derivative z^f of a quantity y at t involved the value y_t of that quantity at t, then if y itself were a past derivative, then y_t would depend on values of the fundamental quantity at times before t. Thus, if these past or future derivatives and neighbourhoods were closed at one end, then the mixed derivatives would have to be two-sided neighbourhood properties, rather than one-sided ones. Because of the way I have defined things, the future derivative of any fundamental or neighbourhood property (past or future) is itself a future neighbourhood property, and the past derivative of any such property is itself a past neighbourhood property. Under the alternate closed-end definition, any mixed derivatives would be two-sided neighbourhood properties.

Therefore, if acceleration is the future derivative of velocity, and velocity is the past derivative of position, then Universal Forwards Causation allows acceleration to be set by the present masses, charges, and positions of objects, and allows velocity to play a role in determining future positions. This is a reductionist view that is compatible with a causal understanding of the laws.

Hartry Field argues against a causal understanding of the laws, claiming that laws involving differential equations have a 'very different character' from causal laws:

> for instance, instead of directly connecting things at two different times[...], a differential equation involves a single time only: it determines the rate at which a quantity changes at a given time t from the value of it and other quantities at that very time. [Field, 2003, p. 438]

Field suggests that even a non-reductionist understanding of rates of change would require a non-causal understanding. But by thinking of derivatives as one-sided neighbourhood properties, we can read a differential equation as stating how past neighbourhood properties and present properties together causally determine future neighbourhood properties. Although there is not a strict temporal separation between causes and effects, Universal Forwards Causation seems to be the heart of the traditional notion, which Field thinks is incompatible with laws phrased as differential equations. (I thank Brad Skow for pointing me to this discussion.)

Lange says that forces cause accelerations, which cause changes in velocity, which cause changes in position. But on the view I defend here, forces cause accelerations, while accelerations are *grounded in* future velocities and positions. Future velocities and positions are (perhaps indirectly) caused by whatever causes present acceleration, but are not caused by the acceleration itself. Acceleration is an effect, and only a cause in the indirect sense that the future velocities that constitute it can themselves cause things even farther in the future. Velocity causes things, and is only an effect in the indirect sense that it can be partly causally determined by events even farther in the past, and that velocities over time constitute accelerations, which are effects. Lange's chain would be problematic if accelerations at a time had to causally determine velocities at the same time. But the fundamental causal determination is just forces and velocities at a time determining positions and accelerations on a future neighbourhood of that time.

There is still something odd here. Velocity plays a role in causally determining future positions of objects, even though velocity is not itself a fundamental feature of the world. How one understands this will depend on how one understands the notions of grounding and causation that are involved. There are parallels to cases of mental and social causation, in which non-fundamental entities (like minds, nations, and electorates) are able to cause physical effects. Perhaps such causation is best understood in terms of the causal powers of the fundamental entities that ground the non-fundamental entity. The non-fundamental entity (whether a mind, a nation, an electorate, or a velocity) might be thought of as a notational device for abbreviating a causal power that is fundamentally a collective power of the fundamental entities that ground the non-fundamental one. Or perhaps there is some special way to understand higher level entities as fundamentally playing a causal role despite not being fundamental entities themselves. At any rate this is an important question to answer for theories that allow causation to cross levels of fundamentality, but it appears to be no easier or harder than answering similar questions in philosophy of mind and society. But the challenge to the reductionist view has generally been a challenge involving causal chains and loops, and not this level-crossing problem, which applies just as well to many views that are traditionally unrelated to the analysis of rates of change in physics.

5 Why no third derivatives?

To see the full significance of these causal principles, we must consider why it is that the velocity of an object at a time is said to play a role in the causal determination of future positions of that object. And to see this, we must consider how the determination of the derivative of a quantity relates to determination of the value of that quantity itself.

If z is the derivative of some quantity y then fixing z over some time interval $\langle t_1, t_2 \rangle$ does not suffice to fix all the values of y over that interval. However, fixing z over an interval while also fixing the value of y at any one time in that interval (or even on an endpoint) will suffice to fix the value of y at all other times in that interval. In particular, for any t' in the interval $\langle t_1, t_2 \rangle$, we will have $y_{t'} = y_{t_1} + \int_{t_1}^{t'} z_t dt$. Thus, if a fundamental law causally determines the future by setting a derivative of some quantity, then the present value of that quantity must be part of that causal determination of the future as well. More generally, if a fundamental law causally determines the future by setting the nth derivative of some quantity, then the present value of that quantity, and its first $n-1$ derivatives, must be part of that causal determination of the future as well.

Universal Forwards Causation means that whatever quantities play a role in causally determining the future positions of an object must be grounded entirely in the past and present. Thus, if a fundamental law causally determines the future by setting the nth derivative of some quantity, then the present value of that quantity, and its first $n-1$ derivatives, must be grounded entirely in the past and present. So these first $n-1$ derivatives must all be past derivatives. However, the nth derivative itself must be a future derivative, in order to be causally determined by the present.

Thus, if there is a fundamental force law, which operates by setting the second derivative of position, then the first derivative of position that is causally relevant (velocity) must be a past derivative, while the second derivative (acceleration) must be a future derivative. However, if there were a fundamental 'yank'[8] law, which operated by setting the third derivative of position, then the first *two* causally relevant derivatives of position (velocity and acceleration) would have to be past derivatives. So if there were both a fundamental force law and a fundamental yank law, then acceleration would both have to be a past derivative and a future derivative, which is a contradiction.

Similar reasoning applies if there were any two fundamental causal laws that operated by setting different order derivatives of the same quantity. Universal Forwards Causation means that whatever fundamental causal laws operate by setting derivatives of a quantity must all operate by setting derivatives of the same order. Since there is at least one force law, any fundamental law determining positions must be a force law. There could be other fundamental laws determining other fundamental quantities by means of a first-derivative law or a third-derivative law, but each fundamental quantity that has laws governing its evolution must only have laws that set a single order of derivative. Strictly speaking, I have not explained why there are no third derivatives, but just why all derivatives that appear in the laws are of the same order. But an explanation of why there is a second derivative law would complete this explanation.

[8]'Yank' is the term used by engineers for the time derivative of force, and 'jerk' is the time derivative of acceleration.

Note that I have focused here on *fundamental* causal laws. It is conceivable that there are non-fundamental laws involving third derivatives. For instance, on roller coasters and airplanes, there are engineering limits on how high the jerk (the third derivative of position with respect to time) is allowed be. From the reference frame inside the vehicle, the acceleration feels like an additional component of gravity, so a large jerk means that the apparent direction of gravity inside the vehicle changes quickly. Thus, a motion with high jerk can apparently cause people to fall over. However, the 'falling over' is caused by the apparent gravity being in one direction, while the person is braced against gravity in a different direction. So this causation can be reduced to causation by apparent forces, rather than by jerk. And a 'fall' due to apparent gravity is not really a fall at all—rather, it is a situation where the vehicle rushes up to meet the person, and the motion of the vehicle is caused in standard ways by the shape of the track or the lift on the wings. It is all *really* forces and velocities causing motions, so there is no need for a direct causal role of any derivative other than the past derivative, and no direct effects other than the future derivative of the past derivative. Similar analyses will presumably apply to other non-fundamental equations of motion derived from the fundamental ones.

6 Why any derivatives?

Why do the fundamental laws set derivatives at all? Why don't the fundamental laws directly determine fundamental quantities?

To answer this question, we should think further about what it takes for a law to be properly causal. Arntzenius objects to the causal significance of past neighbourhood properties for reasons related to those of Bigelow and Pargetter. They object to causation at a positive temporal distance, but I side with Lange in saying that these neighbourhood properties don't actually involve a positive temporal distance, but can be thought of as somehow 'infinitesimal'.[9] Arntzenius thinks that even the use of an infinitesimal neighbourhood poses problems. 'One will have to give up on the idea that the truly instantaneous features of a system, or the world, can determine the future developments [...] if there is a velocity dependence of developments.' [Arntzenius, 2000, p. 192] 'If one believes that any influence that past states have upon future states must be mediated through states at all intermediate times, then one must disbelieve this theory.'[Arntzenius, 2000, p. 205] But while Arntzenius thinks this is a problem, I think that neighbourhood causation, despite its infinitesimal temporal distances, is actually *essential* to the notion of causality when time is continuous.

This view is defended by [Harrington, ms]. He considers a 'Humean' picture on which states of the world at various times are metaphysically and logically independent of one another. On this picture, any fundamental metaphysical

[9] This notion of an 'infinitesimal' is an informal one, and not the formal notion from non-standard analysis.

relations between quantities must relate quantities at a single time. But then there could be no relations across times, and nothing about these relations could tell the difference between times that are later than the present and times that are earlier than the present, or between times that are close to the present and times that are far away. The relation between times and real numbers would merely be one of cardinality, not topology. The only way to get the familiar linear structure of time to have any metaphysical reality is if there is some sort of metaphysical dependence between properties across times.

But if we reject causation at a positive temporal distance, then the only way to have this is to have laws involving neighbourhood properties. To get proper continuity with the past, we need laws that involve past neighbourhood properties, and to get proper continuity with the future, we need laws that involve future neighbourhood properties. If time were discrete, we could get a sort of continuity by allowing quantities at one time to cause quantities at the *next* moment in time. But if time is dense, so that there is no 'next' moment, then we need the neighbourhood properties. Thus, we get:

> Neighbourhood Causation: The fundamental causal laws must use present properties and past neighbourhood properties to determine future neighbourhood properties.[10]

With this constraint there is no fundamental direct causation at any positive temporal distance, but there are causal connections across neighbourhoods both from the past and into the future. This is causation by means of properties that are extended over infinitesimal neighbourhoods, which Arntzenius objects to, but which according to Harrington and Lange seems to be essential to the topological connectedness of time.

Thus, the fundamental laws can't just relate fundamental quantities. They must involve some neighbourhood properties as well. And the most natural sort of neighbourhood properties appear to be derivatives. Perhaps there is some alternative, but any other neighbourhood property would appear to be just as complicated. Thus, the causal topology of time seems to explain why the laws involve derivatives.

We might even be able to say slightly more. In order to get a causal connection that is properly continuous in both directions of time, the laws must involve both past neighbourhood properties and future neighbourhood properties. A natural way to involve both is to have a law that sets a second derivative (which would be a future neighbourhood property) and thus uses a first derivative (which would be a past neighbourhood property) as an initial condition. And this is in fact how classical Newtonian physics works. In quantum mechanics, the Schrödinger equation directly sets the first derivative of the quantum wave-function, which must thus be a future neighbourhood property. But as long as some other feature of the laws involves a past neighbourhood property,

[10] A view like this is defended in Chapter 1 of [Lange, 2002]. Also, unlike the Forwards Causation constraints, this one only depends on a 'local' direction of time, and thus might be able to be adapted to cases of time travel or circular time.

there will still be proper continuity in both directions. These appear to be the two simplest ways to get the appropriate causal connections in both directions, and it is striking that the best candidate laws are of these forms. The causal reductionist view described above can give an explanation of this feature of the laws.

7 Counterexamples?

In addition to the causal challenges to reductionism, many non-reductionists have proposed apparent counterexamples to the view. Lange considers a series of these counterexamples, discussed in [Bigelow and Pargetter, 1990], [Carroll, 2002], and elsewhere. But I will argue that these are only apparent.

These counterexamples all take a particular form. They describe a conceivable world, in which some object takes on a particular series of positions, and interacts with other objects in particular ways. They then invoke some intuition we have about the velocity of that object (perhaps including the intuition that there is no such thing as the velocity of that object), and show that it disagrees with the value of the relevant derivative of position. My response to each will be the same—I will accept that the intuition captures something that is causally relevant to the situation, but deny that this is relevant to the actual properties of velocity and acceleration that we want to analyse. The words 'velocity' and 'acceleration' refer to whatever properties are actually causally relevant, but the worlds of the proposed counterexamples are ones in which different properties are causally relevant. If we had lived in those worlds, we would have used the words 'velocity' and 'acceleration' in the way suggested by the proponents of the counterexample, but since we don't, we shouldn't expect our words to apply to the property that happens to be causally relevant. The fact that velocity and acceleration are actually causally relevant leads us to confuse them with the properties that are causally relevant in the worlds of these examples.

Some examples involve objects that come into or go out of existence at specific moments in time, and thus either have no past neighbourhood properties (if they just came into existence) or no future neighbourhood properties (if they are just about to go out of existence). Others involve worlds where there are no nomological connections between times, or where the nomological connections are extremely chancy, in which some object merely happens to have a trajectory over some interval of time in which its derivatives are defined, so that these reductionist velocities and accelerations exist, even though they don't play the causal role that is supposed to be essential to them. I will focus on the example discussed on pp. 59-62 of [Carroll, 2002], though my response to the others will be the same.

In this example, Carroll considers an instantaneous perfectly elastic collision of the sort commonly described by naive Newtonian physics. One object is at rest at all instants before t, while another object of the same mass is moving towards it at a constant speed. At all moments after t, the incoming object is at rest, immediately next to where the outgoing object had been, while the outgoing object is moving away from the incoming one at the same constant speed with which the incoming one had been approaching. The question is about which object (if either) is in motion exactly at time t, which is the instant at which the collision occurs.

On the view of motion as past derivative of position, the incoming object is in motion at t, while the outgoing object is at rest. On the view of motion as future derivative of position, the outgoing object is in motion at t, while the incoming object is at rest. On the view of motion as two-sided derivative of position, neither object is in motion, since the two-sided derivative doesn't exist at this time. However, Carroll argues that any of these answers gets things wrong. In fact, he claims, the positions of these two objects at all times do not suffice to determine which is in motion exactly at t. This may depend on further facts about the world.

In particular, Carroll considers two worlds that this interaction might take place in. In World 1, the effects of any interaction are present immediately, at the same instant that the interaction takes place. In World 2, on the other hand, the effects of an interaction are only present at instants after the one in which the interaction takes place. Carroll argues that the differences between World 1 and World 2 can be grounded in interactions other than this sort of velocity transfer—for instance, transfers of heat, electric charge, and other potentially fundamental quantities might take place in one way or the other, and this could determine whether the laws require us to view the velocity as having transferred immediately, or only being transferred after the moment of collision. (One might contest Carroll's claim that two worlds really could differ in the relevant way, but I will concede this to him, and instead illustrate my general reply to all the proposed counterexamples.) Carroll says,

> To have given a successful definition of instantaneous motion in terms of the derivative of the position function, it must be true that *it is necessarily true that an object is moving at a time if and only if the value of the derivative of the object's position function at that time is nonzero.* Science fiction examples, so long as they are genuinely possible, can show this claim to be false without establishing anything about how motion takes place in the actual world. [Carroll, 2002, p. 62]

Thus, if we agree with Carroll that which object is in motion depends on which world the interaction takes place in, then we must deny any identification of motion with any past, future, or two-sided neighbourhood property of an object.

However, I don't agree with Carroll. I claim that in both worlds, the incoming object is in motion at t, while the outgoing object is at rest. It may seem to us that the facts about motion should be different in the two worlds, but I claim this is only because of a mistake in our intuitions. In this case, our intuitions track the causal role. We observe that *whatever is causally relevant* in World 1 can't be a past neighbourhood property, and conclude that *motion* is not a past neighbourhood property.

However, this argument only works if *it is necessarily true that motion is causally relevant to future positions of objects*. And this just isn't the case. There are conceivable worlds in which *no* property is causally relevant to the future positions of objects, and I claim that Carroll's worlds are ones in which a *different* property is causally relevant. To say that the *same* properties must causally determine the future positions in these worlds is to say that it is necessary that a moving object that experiences no force continues to change its position. If we don't want to be committed to the claim that such laws of motion are necessary, then we must allow that motion can be present *without* having these effects, and this is exactly how I understand Carroll's example.

Most of the rest of the cases clearly involve worlds with different causal laws, so that our intuitions track some property other than the one that is actually causally relevant. The one difficult case is due to Lange himself. This case involves an infinitely thin spherically symmetric shell of mass.[11] On the Newtonian gravitational force law, the net force on any massive object inside the shell is zero, but the net force on any massive object outside the shell is non-zero. Thus, an object that is leaving the shell has a non-zero future derivative of velocity at the moment when it is on the border, while an object that is entering the shell has a zero future derivative of velocity at the moment when it is on the border. Thus, it looks like the presence or absence of the acceleration at that moment in time, considered as the future derivative of velocity, depends not only on the positions and masses of all objects at that moment in time, but also which direction the object is moving, which seems to contradict the dependence expressed by Newtonian gravitation.

However, this case only arises if there can be an infinitely thin shell with non-zero mass, and if an object can pass through this shell without disturbing it in any way. It appears to be the case that all non-zero masses occupy a non-zero volume, and that matter can't pass through other matter without disturbing it. Even if these features of the world are contingent, Newton's third law guarantees that if the shell acts gravitationally on the object then the object must also act gravitationally on the shell, in a way that would tend to deform the shell out of its perfectly spherical shape, which would make the force non-zero at all points. The example only arises by ignoring many physically real features of the world. This is natural for a homework assignment in an introductory physics class, but this is just for mathematical simplicity. The actual world doesn't contain infinitely thin non-interacting shells with positive mass, just as it doesn't contain perfectly rigid bodies of infinite elasticity, or completely smooth surfaces with zero friction.

My response to these cases is similar to that of [Meyer, 2003]. He accepts that there may be causally relevant properties in other worlds in addition to the reductionist derivative properties. The difference between my view and Meyer's is that on my view, velocity and acceleration do in fact feature in a

[11] In Lange's discussion, this is a shell of charge, but since the mathematics works out the same for gravity as for electrostatic attraction, I will stick with the gravitational example to avoid worries about the connection between electricity and magnetism on a reductionist view of rates of change.

causal understanding of the actual world. On his view, the way to understand laws that involve velocity and acceleration is merely as constraints that the full history of the universe must obey. But on my view, we can instead understand these laws as telling us something about the causal structure of trajectories over time. Thus, my view is able to reconcile the simplicity of Meyer's reductionism with the causal explanatory power of non-reductionist views.

I accept the metaphysical significance of two relations of determination—causation and grounding. Meyer (like traditional Russellians) accepts that velocity and acceleration are grounded in position, but denies that they causally determine any positions. Lange (like other non-reductionists) accepts that velocity and acceleration cause positions, but denies that they are grounded in them. On my view, velocity and acceleration can be grounded in some positions and bear causal relations to others. By accepting the significance of both of these relations, and constraints on their interaction (namely, that they can't form loops, and that causation must go from past neighbourhoods through the present to future neighbourhoods), I can partially explain why the physical laws fundamentally take the form of force laws.

The modal strength of the explanation will depend on the modal strength of these constraints—I don't know if it is necessary that the physical world has the sort of causal structure that I propose, or merely contingent. But the strength of this explanation, together with the strength of the intuitions behind causation and grounding, determines the extent to which this causal reductionist view is better than the non-causal reductionist or the causal non-reductionist.

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