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The development of physical ideas concerning the interaction of plasma flows and electrostatic fields in dusty plasmas

V N Tsytovich

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V N Tsytovich Prokhorov General Physics Institute,
Russian Academy of Sciences,
ul. Vavilova 38, 119991 Moscow, Russian Federation
Tel. (7-495) 135 02 47
E-mail: tsytov@lpi.ru

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Abstract. **The concept of the interaction of an electrostatic field and plasma flows in a dusty plasma is reviewed. This approach helps to describe many aspects of dusty plasma physics. Of basic importance in this context are processes that plasma flows introduce into interactions between dust particles. Fluctuations in plasma flows, together with those in electrostatic fields, considerably modify these interactions, with the result that like-charged particles that are far apart start attracting one another, possibly leading to their pairing. Knowledge about the attraction between distant particles is traced from the early**

work of 1963 through modification and improvement to its present level, when it has become possible to qualitatively estimate the parameters of the dusty plasma – dust crystal transition, and to obtain the values for the coupling constant, dust particle separations, and the transition temperature consistent with observations. The self-energy of dust particles, exceeding both their kinetic energy and interaction energy, is discussed in terms of the role of its variations. Generation mechanisms and the role of regular plasma flows are examined. Self-excitation of regular and fluctuating plasma flows gives rise to structures such as dust voids, dust vortices, dust clumps, and helical dust structures. Self-organizing structures are frequently seen both in laboratory and natural conditions. Prospects for further research are addressed and problems yet to be solved reviewed.

1. Introduction

One of the goals of this review is to follow the development of ideas concerning the role of plasma flows in dusty plasmas, ideas that led to the modern concept of a dusty plasma as a special state of matter. The physical features of this state are (1) the presence of intense dissipation processes that make the system capable of forming dissipative self-organizing structures, (2) the capability of the charge of dust particles to change its sign, which makes the system 'open' and non-Hamiltonian, and (3) the nonlinearity of the screening of dust particles by polarization fields and plasma flows. By describing a dusty plasma by two fields, the plasma-flow field and the electrostatic field, it is possible to examine the unusual properties of the dusty plasma from a single standpoint and to substantially clarify earlier descriptions. Each of the two fields can have a regular component and a random component. While the fluctuations of electrostatic fields in ordinary plasma describe the collective interactions of plasma particles, the fluctuations in plasma flows together with the fluctuations in electrostatic fields substantially alter the interactions of the dust particles and lead to the attraction of like-charged dust particles at large distances. This can be considered the most characteristic property of dust systems.

Dusty plasma has become one of the most dynamically developing research areas, initiated by three large problems that confronted researchers in the early 1990s: commercial microwave etching, controlled-fusion research, and the discovery of the possibility of phase transitions [1, 2]. However, in the years that followed, attention shifted to the basics of dusty-plasma physics, whose studies focused more and more on the differences from other known states of matter [3 – 5]. The large body of data on this subject has been discussed in detail in Refs [1 – 5]. The main properties of dusty plasma as an unusual state of matter were first discussed in Ref. [3] (although the role of plasma flows was analyzed insufficiently there), as well as in the reviews that followed (see Refs [4, 5]). At the same time, observational data were gathered very rapidly; as a result, the interpretation of the previous observations changed dramatically in the course of about three years. An analysis of the role of plasma flows was largely absent in Refs [3 – 5], which, as is now clear, did not help to build approaches that would explain the observational data in a natural framework. Unfortunately, all attempts to interpret the data in Refs [3 – 5] lost their relevance very rapidly, either partially or fully. The goal of the present review is to describe the simple physics of the processes in light of the current research, which allows using a simple concept in interpreting the existing observational data in a language that employs the interaction of plasma-flow fields and electrostatic polarization fields.

2. Historical notes

2.1 General ideas

The idea that plasma crystals may form because of the attraction of dust particles was first proposed in Ref. [6], even before such crystals were discovered experimentally [7 – 10]. However, the analysis that followed showed that the attraction of like-charged dust particles may in general emerge only when the distances between the particles are large. Although this is sufficient information to attempt to explain the phenomenon of dust crystallization, the attraction over large distances was not examined in full in Ref. [6]; it was only proved that as the dust particles move closer, their total electrostatic energy decreases rather than increases (as it should if the particles repel each other). At the same time, Ref. [6] contained the 'seeds' of new ideas, whose importance became clear much later. Therefore, we here estimate to what extent modern ideas were reflected correctly in Ref. [6] and in what aspects they required development or served only as hints of ideas that are common today. The very logic of the first and subsequent studies is of interest in general and in and of itself. The trend has eventually led to the possibility of formulating some processes that are basic to dust-plasma physics, but it has also proved to be long and involved.

Today, the three main ideas of modern dusty-plasma physics (the three cornerstones, so to say) appear clear-cut only in light of the interaction of plasma flows and electrostatic polarization fields: (1) the dissipative nature of the system and self-organization, which is a natural consequence of the large absorption of flows on dust particles, depends on the polarization fields, which may inhibit the propagation of the flows to the surface of dust particles; (2) the openness and the non-Hamiltonian nature of the system are determined by the extent of variation in the flows that reach the dust particles; and (3) the nonlinearity of the screening may be altered by the interaction with the flows. Actually, these three cornerstones have one physical reason, and that is the large charges of dust particles, which lead to the onset of plasma flows whose action is comparable to or large than the action of the electrostatic field. This corresponds to the modern understanding of the phenomena underlying the physics of dusty plasma. This understanding was achieved gradually, although studies of the processes in question began relatively long ago. The details of the riddle of modern ideas fell into place and formed a more or less complete picture only gradually and, in some respects, followed a fairly logical pattern. We therefore attempt to follow the logic of studies and the gradual formation of the picture that we have today.

Already in the first studies, which took place more than half a century ago and focused mainly on astrophysical applications and only partially on laboratory studies [3], it was realized that the charges of dust particles are determined by electron and ion flows to the surface of dust particles. These studies produced ideas concerning plasma flows, but because only isolated dust particles were considered, it was assumed that such flows affect only the charges of separate particles — the particles were charged to large values of the order of those corresponding to a floating potential. What

was lacking was the understanding that the interaction of dust particles may be quite strong over distances much larger than the screening length.

The value of the charge can be estimated by simple arguments. If we introduce the dust-particle charge in units of the electron charge as Z_d, the dimensionless charge $z = Z_d e^2/aT_e$ must be of the order of unity (here, a is the size of a dust particle and T_e is the electron temperature). For a dust particle whose size is of the order of 3 – 10 μm and an electron temperature of the order of $1-2$ eV, the dust-particle charge is of the order $Z_d \approx (3-8)\times 10^3$. The electron and ion flows that make such charging possible do not disappear even when equilibrium values of dust charges are reached. Although the charges of dust particles reach their equilibrium values rather fast (and then remain constant), the dust particles continue to absorb plasma flows. This fact was largely ignored, and so was the role of plasma flows, with the charge of a dust particle assumed constant. Dust particles of the size adopted in this numerical example (or even smaller) are often encountered in outer-space or terrestrial objects and in laboratory experiments, and the number density of such particles in clouds may be relatively high. It was nevertheless assumed that for large number densities of such particles, only the saturation effect was important, when the flows did not suffice to charge the particles completely because the particles must share the plasma flows for attaining the corresponding charge. For some reason, it was also assumed that the interaction of the particles outside the screening length vanishes, without accounting for the fact that the interaction of plasma flows and polarization fields may dramatically change the nature of screening. There was the opinion that only the flow saturation effect is important and that it leads only to a decrease in the charge of individual particles when the charge density of the dust particles becomes comparable to the charge density of the electrons or ions, which may be characterized by the parameter $P = n_d Z_d/n_{i,0}$ (where n_d is the density of the dust and $n_{i,0}$ is the ion number density far from the dust particles). In particular, this situation occurs in the case where this parameter is close to unity. This effect was first discovered in [11], where the parameter was introduced that is presently known as the Havnes parameter $P_H = n_d a T_e/n_e e^2$ (it differs somewhat from the above parameter P).

Some remarks concerning the use of the Havnes parameter are in order. Compared to P, this parameter is not very convenient for making estimates, because P_H contains the value of the electron concentration, which varies significantly near dust particles and can lead to a number of misunderstandings in estimates. It is clear, however, that in conditions where the electron concentration far from the dust particles is close to the ion concentration $P = P_{H,0}/z$ (where $P_{H,0}$ is P_H at $n_e \approx n_{i,0}$), and because z is often of the order of unity, the parameters P and $P_{H,0}$ yield similar estimates. Below, we therefore use the parameter in its modified form, i.e., P. It is much more convenient to use in estimates of parameters of experiments and in numerical calculations, which has been demonstrated in many examples in Ref. [3].

One of the first studies of the effect of plasma flows on the asymptotic behavior of the potential of **individual isolated particles absorbing a plasma flow** can be found in Refs [12, 51]. The authors found that the electrostatic potential of a particle absorbing a plasma flow decreases at large distances as $1/r^2$ (rather than as the Coulomb potential $1/r$). This effect for separate particles was studied later in [13] (see also Ref. [3]), while the $1/r^2$ law was obtained in [14], based on the quasineutrality condition.

The importance of the role of plasma flows became more evident at the beginning of the 1990s in laboratory experiments in connection with the discovery of the role that dust clouds play in plasma etching processes [15], the discovery of dust crystals [7 – 10], and the discovery of the important role of dust particles in existing and future devices used in controlled fusion [2]. Experiments done at the International Space Station resolved many riddles in explaining the morphology of the observed dust structures [16]. It became possible to estimate the various processes and the role of plasma flows in laboratory experiments.

Only at the beginning of 2000 was a simple estimate of processes made where plasma flows become collective and are not determined by the sum of flows to individual particles (and where the estimate in [12] does not work) [17]. Because this estimate is important for what we discuss in this review, we make it here. For the sake of an example, we list the typical parameters for a device in which the formation of dust crystals was actually observed: the ion number density $n_i \approx 10^8$ cm^{-3}, the electron temperature $T_e \approx 1-3$ eV, $z \approx 3-4$, $Z_d \approx 3\times 10^3 - 10^4$, the ion temperature close to the room temperature, i.e., $\tau \equiv T_i/T_e \approx 10^{-2}$, $z/\tau \approx 300-400 \gg 1$, and the size of the dust particles $a \approx 5-10$ μm, with the result that the parameter P is somewhat less than but of the order of unity. With these parameters and $\tau \ll 1$, the Debye screening length is determined by the Debye ion radius $\lambda_{Di} = \sqrt{T_i/4\pi n_i e^2} \approx 35$ μm. We now assume that the length over which the flows to individual particles become meaningless and the flows become collective is the mean free path of ions absorbed by dust particles. We assume that the dust particles are uniformly distributed with a density n_d. Then the characteristic lengths over which the mutual effects of plasma flows to different dust particles manifest themselves are estimated as $\lambda_{Di}^2/aP \approx 200-300$ μm. It is important that these lengths are much smaller than the size of the system in laboratory conditions (often of the order of or greater than 10 cm = 100,000 μm), which is true even more for astrophysical objects. All this is an indication that plasma flows may have a strong effect on the properties of the entire system and, in particular, on the interaction between individual dust particles. The most interesting questions are: Can the interaction of dust particles over large distances become attractive and can the presence of plasma flows lead to 'pairing' of dust particles, formally similar to the pairing in superconductivity? The natural question was: Could this lead to the formation of clusters of dust particles and explain the observation of dust crystals? Hints that this is actually the case appeared immediately after the first experimental studies in which plasma dust crystals were observed, and there immediately emerged many indirect experimental indications that such attractive forces indeed exist in most laboratory experiments. But the main problem was from the very beginning to understand the physical processes that lead to such attraction and to determine the possible role of collective plasma flows in the appearance of such attraction.

In this review, we present the logical path of studies in this field and the chain of indirect arguments related to experimental observations that allow asserting that there are actually no other possibilities of explaining the phenomenon of plasma crystals than by the presence of attraction of dust particles. Such attraction emerges as a an inevitable conse-

quence of collectivization of plasma flows. This result was obtained only after a decade of intensive research that examined the occurrence of various structures (the occurrence could not be explained without bringing the idea of collective plasma flows into the picture), studied the pair interaction of particles in the presence of plasma flows, and compared the results with observational data. Plasma flows were first introduced in theoretical descriptions of structures and interactions of particles either according to the logic of the calculation or according to the necessity of obtaining a self-consistent picture of the phenomena, and special significance was attributed to them, contrary to the situation that was to become obvious later.

Here, it is convenient to use the concept of the field of plasma flows interacting with the electrostatic polarization fields, and to 'divide' the flows into regular plasma flows and random fluctuating plasma flows. We note that such a division of electrostatic fields into regular and random is a standard procedure in modern plasma physics. A dusty plasma may contain regular flows that are excited by the dust structure due to charging of dust particles. These flows act on the structures from the outside and are unable to penetrate the dust system deeper than the mean free path of the flow in the dusty plasma. A large system of dust particles always contains random dust flows, too. While the fluctuating fields in ordinary plasma are determined by particle interaction [21], the fluctuating flows change the interaction of dust particles in dusty plasma over large distances between them. It was found that such changes may even include a change in the sign of interaction, when repulsion of large charges is replaced by attraction. The story of how this conclusion was obtained is the topic of this review. Thus, the history of studies of attraction of dust particles is inseparable from the history of studies of the interaction of dust flows and electrostatic fields.

2.2 The first work [6] on the attraction of dust particles and its consequences

At present, when we have a fairly complete picture of the physical processes that produce the attraction of dust particles, it is only natural to judge, from a modern standpoint, the significance of the first work [6] in which the possibility of such attraction was discussed and where it was assumed that the presence of such attraction must lead to crystal formation. It turns out that many concepts discussed presently already existed in embryonic form or explicitly in Ref. [6], but at the beginning of the research some aspects that followed from the results in Ref. [6] were simply ignored. The fact used in Ref. [6] is that the charge of dust particles is not fixed and depends on the plasma flows to their surface and, as a consequence, the flows change as two isolated dust particles move closer and, hence, so does the self-energy of the particles, which is of the order of $Z_{\mathrm{d}}^2 e^2/2a$ for an individual isolated particle (where a is the dust particle radius). The charge of each particle must diminish as the particle separation becomes smaller. We repeat the arguments in Ref. [6]. As one particle approaches another particle, the potential at the surface of each particle is the sum of the potential of the particle proper and the potential of the other particle and is fixed by the floating potential corresponding to the electron temperature T_{e}. Then, if the distance r between the particles is much larger than the particle size a, the potential generated by the second particle is small compared to the potential of the particle proper, and, in terms of the parameter a/r, the charge of each particle approximately depends on the particle separation as

$$Z_{\mathrm{d}}(r) = \frac{Z_{\mathrm{d},\infty}}{1 + a/r} \approx Z_{\mathrm{d},\infty} - \frac{Z_{\mathrm{d},\infty} a}{r} \,,$$

where $Z_{\mathrm{d},\infty}$ is the charge of the dust particles when the distance between them is large ('infinite'). For simplicity, the interaction of the particles may be considered to obey the Coulomb law. Then the variation in the total self-energy of two particles, $2Z_{\mathrm{d}}^2 e^2/2a$, decreases by $-2Z_{\mathrm{d},\infty}^2/r$ as the particle separation decreases from infinity, i.e., decreases by a factor twice as large as the increase in the potential energy of the repulsion of two like-charged particles, $Z_{\mathrm{d},\infty}^2/r$, in which, in the first approximation, we can ignore the decreases in charge as the particles move closer to each other. This means that **as two like-charged dust particles move closer to each other, there is always a gain in electrostatic energy**. This result **remains valid as modern ideas are brought into the picture**. We also note that it is not altered when the particles are assumed screened by a field: the variation in self-energy is two times larger than the interaction energy and is of the opposite sign. This result is obtained only if we assume that the particle separation is much larger than the particle size and that the potential energy of their interaction is much lower than the self-energy of the dust particles.

But does this mean that the particles attract each other? This brings up the key issue that plays an important role in the entire physics of systems with varying charges and with a need to maintain these charges by introducing external flows, because electrostatic energy cannot be fully used in calculations of the acting forces and its spatial gradients do not determine the forces acting between the particles. In statics (when both particles are in a thermostat) [19], we must take the work done by external sources that maintain the particle charge into account [20, 21], because variation in the charge requires variation in the flows, which occurs because of the work done by external sources. This work may balance the variation in the self-energy, **but only if there is thermal equilibrium [19, 21]**. The plasma flows that do the charging not only affect the charges of the dust particles but also, by existing in the vicinity of the dust particles, may alter the polarization charges surrounding the particles and do work related to such variations in polarization. The answer to the basic question of whether the condition is satisfied that sources of flow do work related only to charging dust particles (as they do in thermal equilibrium) must generally be no. The work of external sources done on polarization charges of dust particles was not discussed in Ref. [6]. But if we temporarily ignore the role of polarization charges, we can, obviously, say that dust systems are in a nonequilibrium state and that dissipation of the flows in the absorption on dust particles must be balanced either by external flows or, in nonequilibrium conditions, the very statement that external sources balance the variations of self-energy is false. The following questions must be answered: What amount of work is done by external sources on the polarization charges for specific sources of plasma flows? And how does this work affect the interaction of particles, i.e., the forces that directly act on the particles (polarization forces and forces generated by the absorption of plasma flows)? Moreover, it would be interesting to know whether dust systems are thermodynamically open (similarly to dissipative structures) with constant energy and particle fluxes. These questions emerged immedi-

ately, but it also immediately became clear that no answer could be given to them if we remained within the framework of studies of the behavior of isolated dust particles, because complete absorption of the energy transferred from the source is determined by all dust particles of the system. These questions were posed and constantly resolved in further stages in the development of dusty plasma physics. In real experiments, the external sources are distributed in space almost uniformly (bulk ionization), and in such conditions the question arises of how the work of the sources affects the interaction of particles and how it can be transferred to the particles. This requires a specific study of various aspects of the generation of plasma flows in a system with external sources at all distances between the particles where such sources exist and an analysis of how the polarization charges and flow fields vary with the distance between particles, an analysis that does not rest on the energy principle or the free-energy principle. Nevertheless, in addition to the need to solve specific problems (which was quite clear according to the results in Ref. [6]), new generalizing aspects were present in Ref. [6], whose development led to fairly general statements only at the beginning of the current century. The new aspects mentioned in Ref. [6] or those that followed from the results in Ref. [6] (and whose importance became obvious only later) may be summarized as follows:

- Attention is drawn to the fact that the variability of the charge of dust particles is a fundamental property of dusty plasmas. Only at the beginning of the current century did this fact become common knowledge. Intensive research in this area began with what is known as programs of the non-Hamiltonian nature of dusty plasma systems.
- The interaction of like-charged dust particles may be attractive and lead to the formation of dust crystals. Only lately have the forces acting directly between the dust particles been studied, and it was found that there is attraction at large distances between such particles.
- The assertion that dust crystals may form because of attraction was stated before such crystals were discovered in experiments [7 – 10]. Only studies of the interaction of plasma flows and polarizations provided the means for a meaningful development of this assertion, predicted in Ref. [6].
- It became obvious that the forces acting between dust particles cannot be obtained as derivatives of the potential energy of the electrostatic interaction and must be determined directly in terms of the momentum transferred from the electrostatic fields and plasma flows to the dust particles.
- It was emphasized that the self-energy of dust particles is much higher than their mean kinetic energy or the mean energy of their interaction (today, it is obvious that the fundamental difference between dusty plasmas and ordinary media is that the self-energy of the particles in dusty plasmas is a variable quantity and can be interpreted as the internal variable energy of the system and can be altered by plasma flows).
- One of the consequences of the variability of charges is that the force $Z_\mathrm{d}\mathbf{E}$ acting on a dust particle in a potential electric field $\mathbf{E} = -\nabla\phi$ (where ϕ is the electrostatic field potential) is not a potential one; only now has it become clear (this fact was not stressed in Ref. [6]) that in the general case, the nonpotential nature of the electrostatic fields acting on dust particles sets dust systems apart from ordinary matter, with eddies being excited in the plasma, because $[\nabla \times Z_\mathrm{d}\mathbf{E}] = \left[(\nabla Z_\mathrm{d}) \times \mathbf{E}\right] \neq 0$ in the presence of a charge gradient of dust particles (such eddies are observed in most experiments involving dusty plasmas).

Thus, Ref. [6] already contained the rudiments of a number of entirely new implications, and the new area was constantly being developed, with each item mentioned above becoming increasingly involved. In our opinion, this has eventually led to a more exhaustive understanding of the physical picture of the phenomena characteristic of dusty plasmas. Naturally, none of the aspects were as clear when [6] went to press as they are now.

Below, we separately discuss the various ideas concerning the interaction of dust particles and the ideas about the internal self-energy of a dusty plasma.

2.3 The idea of Lé Sage shadow attraction of dust particles

Historically, the next stage in the studies initiated by Ref. [6] involved what is known as the Lé Sage [22] shadow attraction of two isolated dust particles [23, 24]. Ignatov [23], whose reasoning followed directly from the Lé Sage model [22], used what is known as the geometrical shadow principle, while in Ref. [24] the attraction of ions to dust particles increased the size of the shadow, a situation that could exist in modern experiments, where the real shadow is much larger than the geometrical one, with the shadow forces being approximately a hundred times stronger than those corresponding to the geometrical shadow.

We discuss these ideas in detail. We consider the interaction of two isolated dust particles separated by a distance much larger than their size, which means that we can think of them as being point-like, such that all variations in their charge can be ignored. It is important that each particle maintains its change because of plasma flows, which may be altered by any of the two dust particles. For a single dust particle, the plasma flow is spherically symmetric, but when there is another particle, which intercepts the flow to the first particle, the symmetry is broken. The flow directed to one particle is screened by the other particle, and the pressure of the flow from the outer side in relation to the region determined by the direction between the two particles is higher, which results in the attraction of the two particles, which is known as the Lé Sage shadow effect [22]. The name was taken from a long-forgotten work by Lé Sage (1782), where he attempted to explain universal gravity by the fact that each particle absorbs the ether, while the other particle screens this flux in proportion to the solid angle $\propto 1/r^2$, which leads to the $1/r^2$ law of gravity, corresponding to Newton's law of gravity. Real fluxes of electrons and ions act on dust particles and are absorbed by them (this is basically the ion transfer momentum). At first glance, shadow attraction is a simple consequence of the momentum conservation law, but the effect is not as simple as that unless we are dealing with the trivial geometrical shadow, as was the case with the first work on the Lé Sage attraction [23]. We must find the real size of the shadow. We also need to know the processes that determine it. Positive ions are attracted to negative-charged dust particles. A geometric shadow appears when $a \ll r$ within the solid angle a^2/r^2, and the force is determined by the momentum flux $nm_\mathrm{i}v^2\pi a^2$ [which must be averaged over the ion velocity (v) distribution]. Actually, we should speak of the effective surface area through which the flux passes and replace the radius a with $r_\mathrm{eff} \gg a$. This leads to the first ambiguity: Does r_eff determine the cross section πr_eff^2 of flux absorption by a dust particle? Determining absorption cross

sections requires using models. The second ambiguity resides in the value of the speed v reached by an ion on the particle surface when the ion comes from a region where attraction is weak and its speed is the thermal speed of ions with a temperature $T_{\rm i}$. This energy may be independent of the nature of the Coulomb field screening by the dust particle only if the distribution of the potential between the surface of the dust particle and regions far from its surface is such that no potential barriers appear in the path of the ions. If it is possible to use the energy conservation law when the ions travel from regions where the ion kinetic energy is much higher than the energy of their attraction to the dust particle (such attraction may take the effects of screening the Coulomb field by the dust particle into account), then in view of the attraction of ions to negative-charged dust particles, we have

$$r_{\rm eff}^2 = \left(1 + \frac{z}{\tau}\right) a^2 \approx \frac{z a^2}{\tau} \gg a^2$$

(the strong inequality is written for the condition $z/\tau \gg 1$, often satisfied in laboratory experiments). Here, the momentum flux is $m_{\rm i} v^2 n_{\rm i} \approx z T_{\rm e} n_{\rm i}$ because by the energy conservation law, the ions that reach the surface of a dust particle acquire the energy corresponding to the electron temperature $z T_{\rm e} \gg T_{\rm i}$ (the ions attracted to the dust particle acquire the energy corresponding to its floating potential). For the shadow force, we then obtain the estimate

$$F_{\rm at} \approx \frac{\eta_{\rm i} \pi a^2 z^2 T_{\rm e}^2 n_{\rm i}}{T_{\rm i}} \frac{a^2}{r^2} \qquad (1)$$

or $F_{\rm at} = (Z_{\rm d}^2 e^2/r^2)(a^2/4\lambda_{\rm Di}^2)$, where $\eta_{\rm i} < 1$ is the ion–dust particle attachment coefficient (often assumed to be equal to unity).

The above expression for the attraction force shows that this force is $\eta_{\rm i} a^2/4\lambda_{\rm Di}^2$ times weaker than the Coulomb repulsion, i.e., it would seem that we can ignore it in the same way as we ignored all variations in the force that were caused by charge variations. But this is not the same case, because in view of the conservation of flux (more exactly, of its shadow part), the shadow attraction force acts over distances greater than the screening length, while the Coulomb repulsion is screened at certain radii equal to the screening length (in general, nonlinear screening). The effect described by the above expression was first reported in Refs [14, 24], and the geometric shadow effect calculated in [23] was τ/z times weaker; the numerical factor in (1) was refined in [25].

The numerical calculations in Ref. [14] corroborated the law of variation of the charge of dust particles as the particles move closer (see above) and the fact that the influence of the shadow effect on the particles is much weaker than the one given by the above formula, found in Ref. [6]. For large distances between the particles ($r \gg a$), the effect of charge variation caused by the shadow effect is negligible. An estimate of the influence of the shadow effect on the charge of dust particles was first made in 1994 in [26], soon after [6]. But as we have shown, variations in charge (even those much larger than the one related to the shadow effect) have a very small effect on the interaction energy, while for the self-energy, it can be partially balanced by the work of external sources. Most important in the effect of the shadow force discussed here is momentum transfer (i.e., the force of particle interaction) and not the variation in the charge of dust particles, which in the first approximation is determined by an expression in Ref. [6] and is corroborated by the numerical calculations in Ref. [14].

The variation of screening of dust particles caused by the shadow effect and flow absorption by plasma particles was studied in [14, 24]. It was found that at large distances compared to the screening length, the quasineutrality condition for an individual particle leads to the Pitaevskii potential $\propto 1/r^2$ with the coefficient found by Pitaevskii. In this way, Pitaevskii's result received an additional interpretation.

In the case of many particles, computer simulation of the formation of dust clusters for linear screening in the presence of attraction (1) was performed in [27]. The researchers found a certain similarity of structures obtained as a result of numerical calculations involving linear screening and attraction described by Eqn (1) and the observed crystal structures; in particular, they observed the formation of a hexagonal crystal lattice. However, for complete compliance with and application to the existing experiments in which crystal structures with a large number of dust particles are observed, one small (it would seem) detail is lacking: verification of the condition for the applicability of shadow screening as a sum of independent screenings of flows of separate pairs of dust particles. This condition was satisfied in the numerical calculations in [27] with a moderate number of dust particles (in contrast to plasma crystals, such systems are often called dust clusters).

The question was whether shadow screening could be represented as a sum of independent screenings of flows of separate pairs of dust particles in crystal structures. Clearly, the independent addition of 'shadows' is possible only if the total number of particles in the system is small. It turns out that it is difficult to think of crystallization in a dusty plasma as a process for which shadow screening is responsible and to apply it to real crystals consisting of a large number of dust particles; it can only be used in dust systems with a moderate number of particles (dust clusters).

2.4 Limitations of approaches involving summation of particle pair interactions

From the very beginning, the limitations of using the interaction of pairs of isolated particles or processes of charging isolated particles to describe real systems with many dust particles were essentially clear from simple estimates. A criterion can be found from the condition that the plasma flows participating in the charging of dust particles and in shadow attraction do not interfere with each other or at least interfere insignificantly and that such interference can be ignored. Only in this case can we interpret the interaction of many particles as the sum of independent pair shadow attractions. The criterion consists in the smallness of the size of the system of dust particles compared to the dust-flow absorption length. This length can be estimated by the absorption cross section with a simple model, which was discussed in all the reviews on the subject and is based solely on conservation laws and the assumption that dust particles absorb plasma flows [3, 27]. The estimate yields $\lambda_{\rm Di}^2/aP$ as the mean free path of a flow with respect to absorption, which for most experiments amounts to about 200 μm and is 10–100 times smaller than the size of observed dust structures and, in particular, dust crystals [7–10]. Hence, under these conditions, shadow attraction may play a certain role in surface layers whose thickness is smaller than the mean free path and may determine the value of surface tension.

Recently, there has been an active discussion on the role of pair collisions of ions with neutral atoms, the collisions possibly increasing the flow of ions to dust particles [5]. We note that any increase in the absorption cross section on dust only reduces the mean free path of the flows and, in this way, limits the applicability of the pair interaction even more. Charge-exchange processes might play a special role here. Choi and Kushner [26] used the assumption (which has yet to be fully corroborated) that a thermal ion that emerges after a charge-exchange collision with a neutral atom is finally, in one way or another, absorbed by a dust particle. If we consider this assumption proved, then, according to Refs [5, 27, 28], within a narrow range of pressures of the neutral gas, the flows on the dust particle may increase by a factor of up to 100. This means that the absorption cross section for individual particles increases by a factor of approximately 100. Without discussing the validity of such estimates, we consider what this means from the standpoint of the applicability of these results to systems consisting of many particles. In this case, an essential decrease in the mean free path makes the effects for separate particles only applicable to systems consisting of a very small number of dust particles. For this, the number density of the dust particles in the dusty plasma must actually be very low. The problem becomes purely academic because it requires that the dust density decrease by six orders of magnitude [the above criterion for the absence of interference of dust flows changes by a factor of 100 (200 μm is replaced with 2 μm)]. Hence, we believe there is no real interest in a thorough study of the role of charge-exchange collisions in establishing entirely new phenomena reflecting the interaction of plasma flows and electrostatic fields, although, in theory at least, collisions with the neutral gas atoms are taken into account in the simplest model of ion friction on neutral-gas atoms.

2.5 Fluctuating and regular plasma flows

After the applicability limits of plasma-flow effects in a system of dust particles were established with the use of the results for isolated dust particles, the next problem (historically) that confronted researchers was how to describe systems containing a large number of dust particles. The first question was: How are the absorbed flows restored in real laboratory devices? Most devices of this kind have external ionization sources; in microwave discharges, there is continuous bulk ionization, which to a good approximation is homogeneous and ensures an influx of electrons and ions needed for absorption by dust particles [7–10]. In discharges in a constant external field [10], electrons are accelerated by the field to energies sufficiently high for ionization to occur, and a system of striations often emerges (maximum ionization and maximum plasma glow occur in regions where electrons acquire maximum energy). Through special experiments it was found that a dusty plasma ceases to exist as a system consisting of charged dust particles, electrons, and ions after the ionization source is switched off. It is therefore meaningful to consider a dusty plasma that is homogeneous on the whole only if the local absorption by dust balances the external ionization. But in this case, local inhomogeneities may appear in plasma flows, which, depending on conditions, may be directed at one dust particle or another, while averaged flows may be absent. Hence, it is reasonable to think of the flow fields $\mathbf{\Phi}$ as having two components, the regular $\langle\mathbf{\Phi}\rangle$ and the random $\delta\mathbf{\Phi}$, with $\mathbf{\Phi} = \langle\mathbf{\Phi}\rangle + \delta\mathbf{\Phi}$. Regular flows can be generated by flows from outside the dust systems and those excited within the system, but random flows constitute an indispensable component of any dust system with a large number of dust particles. We also note that in ordinary plasmas without dust and with particles moving with nonrelativistic speeds, the only electrostatic fields $\mathbf{E}$ are those that also have a regular component and a fluctuating component, $\mathbf{E} = \langle\mathbf{E}\rangle + \delta\mathbf{E}$, with the fluctuating component related to the discreteness of the system and describing the collision of plasma particles [18] (to be exact, they are described by the correlator $\langle\delta\mathbf{E}\,\delta\mathbf{E}\rangle$ determining the Landau–Balescu collision integral). Here, the particle interaction proves to be dynamically screened [18]. It would be hard to imagine that the fluctuation flows in a dusty plasma do not interact with electrostatic fluctuation fields and do not lead to correlators of fields and flows, $\langle\delta\mathbf{E}\,\delta\mathbf{\Phi}\rangle$, or to flow correlators $\langle\delta\mathbf{\Phi}\,\delta\mathbf{\Phi}\rangle$, which could significantly alter the interaction of dust particles. Thus, from the logic of research and simple physics, the natural assumption followed that the plasma flows, a specific feature of dusty plasmas, must alter the interactions of the dust particles to such an extent that they differ from interactions of ordinary particles with fixed charges. This meant that a new kinetics was needed in order to describe dusty plasmas. The first steps in building such a kinetics were taken in Refs [29, 30]. Theoretically, the kinetic approach is the most exhaustive, but it involves using complicated mathematics and constructing a system of new equations containing new collision integrals for the interactions between dust particles and the interactions of dust particles and the plasma electrons and ions. This constituted a certain stage in this area of research, and we do not wish to discuss it in full here, which would involve complicated calculations. Instead, we use simple qualitative ideas based on the moments of a distribution function, the equations for which naturally follow from the kinetic equations. However, instead of the 'true' distribution function, we use certain simple distributions, including thermal distributions, bearing in mind that using exact distributions may lead to variations in the numerical coefficients of the order of unity. Furthermore, there are effects, such as nonlinear screening, which so far have not been included in the kinetic approach (although this should not be too difficult), but have a simple explanation when the hydrodynamic description is used. Here, we must take all effects related to plasma flows into account, having in mind both random and regular components.

2.6 Probe particles and an important remark made in Ref. [31]

An extremely profound remark made in 1940 by Ginzburg [31] concerning the quantum theory of the Vavilov–Cherenkov effect [32] is closely related to the role of polarization charges that surround particles. It can be expanded to incorporate the additional polarization effects generated by the presence of plasma flows in a dusty plasma. First, however, we mention one remarkable property of particle interactions obtained from the Landau–Balescu collision integral [18] or, more precisely, the corrections made by Balescu. If we examine a plasma without dust, we bear in mind the need for further refinements when using analogies with known results in an ordinary plasma. The point is that in ordinary plasma, the collision integrals describe the collisions of 'dressed' particles, whose polarization 'cloud' is determined by the dielectric constant $\epsilon_{\omega,\mathbf{k}}$ at $\omega = \mathbf{kv}$, i.e., by the quantity $\epsilon_{\mathbf{kv},\mathbf{k}}$ (where ω is the frequency,

k is the wave vector, and **v** is the particle velocity prior to interaction). To be exact, the collisions are determined by the value of $1/|\epsilon|^2$, i.e., the interaction occurs via polarization surrounding the interacting particles, and each interacting (colliding) particle 'takes in' the field of another particle screened by polarization charges. This polarization is generated by all other particles in the system in their fluctuation motion [18]. It is important that if we introduce a probe particle, its potential (or field) is screened by exactly the same polarization charge with the same dielectric constant. For an arbitrary charge q, its electrostatic potential $\phi = q/2\pi^2 \int \mathrm{d}\mathbf{k}/k^2\epsilon_{\mathbf{kv},\mathbf{k}}$ is determined by the same value of the dielectric constant ϵ, and if the charge q is one of the charges of the plasma, i.e., is a probe charge, its screening is exactly the same as the screening of the charges in their interaction. Two issues are important here: the interaction is transmitted by polarization fields, and the polarization fields are generated by all the particles in fluctuations. The second issue is rarely stressed, as is the fact that the screening of particles in an ordinary plasma is a fairly complex physical process; for instance, static screening corresponds to the Debye screening $\phi = (q/r)\exp(-r/\lambda_{\mathrm{D}})$ (where λ_{D} is the Debye radius), while by the definition of a plasma, the Debye sphere contains a large number of particles and all the Debye spheres of individual particles overlap (although each has a field screened at the Debye radius). The concept of probe particles in determining their real interaction in a complex system consisting of many particles has proved very fruitful. It is based on the idea that the second particle in the interacting pair 'takes in' the polarization field as the 'inseparable' part of the field of the particle to which it reacts in the interaction.

We here recall an important statement made in Ref. [31] concerning the role of polarization fields in processes of emission of radiation, in particular Cherenkov radiation [32]. Ginzburg [31] was the first to clearly show that only the emitting particle loses its entire momentum and energy in such emission processes, although the polarization surrounding it is usually generated by lighter particles (the bound electrons of the medium in the case of Cherenkov radiation). This can be made even more clear if only electrons participate in polarization and the dielectric constant is determined solely by the electron mass. But the remark made in Ref. [31] amounted to stating that polarization electrons serve only as mediators, while the momentum and energy conservation laws in Cherenkov radiation, $\omega(\mathbf{k}) = \mathbf{kv}$, shows that the exchange of momentum and energy in the emission of radiation occurs only between the radiation field and the emitting particle. This was shown in the simplest and most transparent way in Ref. [31] in the quantum language: prior to emission, the energy of the particle was $E_{\mathbf{p}}$, but after emission it became (due to the conservation laws)

$$E_{\mathbf{p}+\hbar\mathbf{k}} \approx E_{\mathbf{p}} + \hbar\mathbf{k}\,\frac{\mathrm{d}E_{\mathbf{p}}}{\mathrm{d}\mathbf{p}} = E_{\mathbf{p}} + \hbar\omega_{\mathbf{k}}\,,$$

where $\omega_{\mathbf{k}} = \mathbf{kv}$. This idea was first generalized to scattering processes in Refs [33, 34] (transition scattering; see Ref. [35]) when the scattering particles may be extremely heavy but can scatter very strongly because of the light particles of polarization; for example, the scattering of ions may be determined by the electron mass [18]. Today, the idea is used in many areas of physics, with scattering determined by the square of the sum of the matrix elements of ordinary scattering and transition radiation. And yet one often meets with the recurrence of ideas from the past in the physics of scattering. At the onset of plasma physics, formulas were derived for the scattering of radiation on plasma fluctuations; these results are used everywhere in plasma diagnostics and are sometimes called Thomson scattering, ignoring the fact that such scattering on plasma fluctuations can be expressed in terms of the squares of the absolute values of the sums of the ordinary Thomson scattering and transition radiation amplitudes (this is a rigorous result). Here, the emission of radiation by the ions can be calculated with modifications of the incident polarization wave around the scattering ion taken into account. As regards the fact that the term 'Thomson scattering' is incorrectly applied to scattering in a plasma, we note that a straightforward physical picture whose basics were established so long ago in Ref. [31] clearly helps in understanding the physics of scattering processes. If we take transition scattering into account, the correspondence between scattering on individual particles and scattering on fluctuations is completely restored, at least in the first approximation, with the effects of absorption in the medium ignored. The same is true for other processes, e.g., for Bremsstrahlung [36, 37], where it suffices to add polarized Bremsstrahlung processes to obtain a complete picture of Bremsstrahlung. So far, in astrophysical works, it is also assumed that ions do not participate in scattering processes, and if the contrary is stated, the work is considered a curiosity. A curious fact, however, is that astrophysicists still have no full understanding of the depth of the ideas expressed in Ref. [31]. Actually, the scattering of ions plays an important role in many astrophysical problems, beginning with radiation transfer in solar interior [38] (where scattering on ions may dominate) and ending with the Syunyaev – Zel'dovich effect in the low-frequency range [39].

It is most interesting in all this story that polarization effects play such an important role in modern physics, including the physics of plasmas without dust. Hence, from the beginning of this century, the study of the screening of dust particles and their polarization 'clouds' in dusty plasmas became an important area of research. Of course, such research could have started much earlier if the scientific community had come to understand the deep physical level of the approaches founded in Ref. [31].

It can be said that polarization fields are virtual fields through which momentum is transferred from one interacting particle to another interacting particle. This is a fairly simple statement if we consider that fields may be polarization waves or the modes of the system (we note that superconductivity, related to the electron attraction, is transferred via phonons, which are modes of the system). The problem with dust systems is that polarization caused by fluctuations in electrostatic fields is mixed with polarization caused by fluctuations in the plasma flows, which generally cannot be separated from fluctuations in the electrostatic fields because of their interaction with the latter. Thus, only by the beginning of this century did it become obvious that attention should be focused on the properties of polarization and on the dielectric constant of dusty plasmas with plasma flows in it taken into account. Naturally, correctly accounting for plasma flows affects the modes of a dusty plasma and the nature of the interaction of dust particles through these modes (as in superconductors). It came as a big surprise at

the beginning of this century that in previous studies of the dielectric constant of dusty plasmas, the role of plasma flows was entirely ignored and that the data on the dusty plasma modes obtained before this period proved to be either basically erroneous or had very narrow applicability ranges and, naturally, corresponded to conditions where the role of plasma flows could be ignored. It is clear from the start that an elementary estimate of the applicability of the above results for the dielectric constant shows that the characteristic size $1/k$ is not only smaller than the mean free path of plasma flows for absorption by dust [$\lambda_{\mathrm{Di}} \times \lambda_{\mathrm{Di}}/aP$, where the second factor is always much greater than unity and, hence, the absorption length is much larger than the Debye radius, with the mean free path for characteristic values of the parameters in existing experiments with dusty plasmas being about 200 – 300 μm ($\lambda_{\mathrm{Di}} \approx 35$ μm)], but also much smaller than the characteristic size of the inhomogeneities and of the experimental devices. It is possible to ignore the effect of plasma flows either for wave numbers much larger than the reciprocal distances of this order or for a highly inhomogeneous system. For some reason, the second aspect was completely ignored by the first researchers in 1992 – 2000 (see, e.g., Refs [40 – 42] and many other works cited in reviews [3, 5]), who used erroneous formulas. In reviews [3], there are several examples of the role of plasma flows, while in reviews [4, 5] dusty plasma modes are examined without taking the balance of plasma flows into account, or the case of finite systems was considered, where absorption of flows is due to the presence of walls and is taken into account by a very crude phenomenological method. The research that followed the discovery of the need to take the balance of flows into account in any fairly large dust system involved studying the role of plasma flows in dusty plasma responses focused on the new qualitative manifestations of the interaction of flows and electrostatic polarizations and, in particular, on such situations where attraction of like-charged dust particles occurs.

2.7 Studies on the attraction of dust particles caused by modification of plasma flows

In 2000, a theoretical paper was published that for the first time correctly posed the problem of the ground state of a dusty plasma that emerged not only because of a balance in charges but also because of a balance in flows [17]. The paper also discussed a perturbation of this state caused by the dusty plasma modes. Before that work, the dielectric constant was always written in a form that followed from nowhere — the analogy with ordinary plasma was used, while the need to maintain the charges of dust particles by plasma flows was simply ignored [40 – 42]. The inapplicability of the approach in Refs [40 – 42] for modes with frequencies below the charging rate follows from simple considerations corroborated by experiments: it takes less than one wave period for a dusty plasma to disappear. This paradox resulted from an elementary error made in Refs [40 – 42], an error that emerged because the authors did not take into account that small deviations may be regarded only as deviations from a certain equilibrium, while the condition needed for the balance and equilibrium of flows was ignored completely, i.e., the ground state was a nonequilibrium one and, generally, even undefined. It must be noted, however, that prior to 2000 there were works in which a number of effects caused by plasma flows were taken into account. For instance, in 1996, a general correct statement was made [43] that all dusty plasma modes change significantly at frequencies below the charging rates of the dust particles and at wave numbers smaller than the reciprocal mean free path, but later, while examining the effect of charging processes on the propagation of ion-sound waves, the initial conditions needed for the balance of flows and their scattering by dust particles were ignored. Attempts were also made to take the Lé Sage shadow attraction of particles into account, and a dispersion equation similar to the equation for gravitational instability was derived [44]. The equation has a very narrow range of applicability: the condition that the shadow attraction is stronger than the additional Pitaevskii repulsion implies that r must be larger than $\lambda_{\mathrm{Di}}^2/a$ ($k < a/\lambda_{\mathrm{Di}}^2$), while the role of plasma flows can be ignored only if $r \ll \lambda_{\mathrm{Di}}^2/aP$ ($k \gg aP/\lambda_{\mathrm{Di}}^2$). In other words, the concentration of dust must be low, $P \ll 1$ (in most experiments, P is of the order of unity and may generally vary from 0 to 1).

The next step in describing the role of flows was taken in Refs [17, 45], where the balance of plasma flows was fully accounted for and where correct equations for ion-sound waves in plasmas were derived. The description of modes in a dusty plasma is directly related to the interaction of dust particles that is transmitted by these modes. But only in 2002, in writing review [3] the static dielectric constant was accidentally calculated [46] and it was found that its real part changes sign for small values of k, i.e., the repulsion of like-charged dust particles may be replaced by attraction at large distances between the particles. The authors examined the polarization charge around a point-like dust particle and demonstrated that attraction can exist at large distances, but they were unable to obtain a definite result because the approximation of point particles, where the calculation of the polarization field is especially simple, involves the problem of bypassing poles, inasmuch as circling a pole changes the sign of the dielectric constant. Only after this problem was solved in Ref. [47] did it become possible to widely use the attraction effect in interpreting the observational data. The progress achieved in Ref. [47] relies on the simple fact that all effects related to plasma flows are determined by the finite size of dust particles. This is obvious if only the shadow attraction is taken into account, but all variations in the dielectric constant are also determined by the finite size of dust particles. By solving the problem for finite-size particles, all results in Ref. [46] were reproduced in Ref. [47] in the limit where the particles are small compared with all other characteristic dimensions. Further investigation could follow this 'beaten path' with additional developments associated with plasma flows and the role of nonlinearity in screening. This made it possible to arrive at the current state of affairs, when an explanation is available for the large values of the coupling constants in the phase transitions into plasma-dust crystals, the relatively low phase-transition temperature, and the relatively large distances between dust particles in such phase transitions. All these aspects are examined below.

3. Linear interaction of plasma flows and electrostatic fields

3.1 Elementary estimates

We begin by making simple assumptions and then try to estimate the changes in the linear screening of a probe charge

in the presence of plasma flows. The term 'linear' means that the probe charge is small, and therefore the polarization charge surrounding it is proportional to the field strength of the probe charge. In an ordinary plasma without dust particles and plasma flows generated by such particles, the standard linear screening of probe particles carrying a charge Q corresponds to what is known as the Yukawa potential $\phi = Q\psi(r)/r$, where the screening factor

$$\psi = \exp\left(-\frac{r-a}{\lambda_{\mathrm{D}}}\right) \tag{2}$$

is determined by the Debye radius λ_{D}. Here, the screening factor is written for spherical particles of a finite radius a; the finiteness of the radiuses of dust particles is always important in examining plasma-flow effects, but the dust particles may be considered almost point-like in other manifestations, because typically $a \ll \lambda_{\mathrm{D}}$.

Some remarks concerning the terminology are in order. In plasma physics, the term 'Debye screening' is common and is used for point-like electrons and ions ($a = 0$ in the case where λ_{D} coincides with the Debye radius). In dusty-plasma physics, Eqn (2) with λ_{D} different from the Debye radius has come into use, and the term Yukawa potential, or Yukawa screening has become common. We believe that the use of this term is just as meaningful as that of the term Debye screening. For the Yukawa screening, the screening factor is exponential, but λ_{D} does not coincide with the Debye radius. Such terminology occurred because in comparing the results of observations and in attempting to fit these results to formula (2), it very often happens that λ_{D} does not coincide with the Debye radius. Moreover, a numerical preexponential factor occurs, indicating that the effective charge may differ from the particle charge. Below, we discuss in detail how to fit the actually observed or nonlinear screening to the Yukawa screening. But here we begin by discussing qualitatively new effects in screening in dusty plasmas.

It is only natural to assume from physical considerations that in a dusty plasma in the linear approximation, additional interaction of the polarization potential and plasma flows leads to additional exponentials in the screening factor (linear solutions can always be represented in such a form via Fourier transformations). But in the general case, the factors in the exponentials may be complex-valued. If there are two such exponentials, then

$$\psi(r) = \mathrm{Re}\left[\psi_a \exp\left(-\frac{r-a}{\lambda_1}\right) + \psi_b \exp\left(-\frac{r-a}{\lambda_2}\right)\right], \tag{3}$$

where, obviously, $\psi_a + \psi_b = 1$. The formula that links ψ_a and ψ_b must depend on the interrelation between the polarization fields and plasma-flow fields, i.e., on a certain coupling constant η. We consider the case where $\eta \ll 1$. Then, obviously, $\psi_b \propto \eta$. We select η such that the proportionality factor is unity, i.e., $\psi_b = \eta$. Then,

$$\psi(r) = \mathrm{Re}\left[(1-\eta)\exp\left(-\frac{r-a}{\lambda_1}\right) + \eta \exp\left(-\frac{r-a}{\lambda_2}\right)\right]. \tag{4}$$

The screening described by Eqn (4) differs dramatically from (3). When the coupling is weak, we can assume that λ_1 is close to λ_{D} and that λ_2 is of the order of or greater than the mean free path of the plasma flows, i.e., much larger than λ_{D}. This means that in the case of weak coupling of the polarization fields and the plasma-flow fields, the larger part of the charge of the dust particles is screened over distances of about the Debye radius, but the smaller part of the charge is screened over distances much larger than the distance for the larger part. Because the charges of dust particles are usually very large, this smaller part may indeed be very significant. Equation (4) also describes overscreening, when the polarization charge changes sign. There are two possibilities for this: either $\eta < 0$, when the polarization charge changes sign over distances very close to λ_1 [i.e., when the first exponential makes the contribution of the first term in (4) small], or λ_2 is complex-valued, when oscillations of the polarization charge emerge over distances of the order of λ_2. Qualitatively, these two possibilities correspond to real models describing the interaction of the electrostatic polarization fields and the plasma-flow fields.

3.2 Static equations for polarization fields and plasma-flow fields

We consider the simplest case, which is of interest in applications, where the electron temperature is much higher than the ion temperature and the flow velocities are much smaller than the electron thermal velocity. Then the electrons always match the distribution of ions, and the plasma flows can be determined from the ion continuity equations. The polarization fields are determined, as we know, from the Poisson equations. We use dimensionless variables, and therefore the distances r and the size of dust particles a are measured in units of λ_{Di}, the electrostatic field in units of $T_{\mathrm{i}}/\lambda_{\mathrm{Di}}$, the electrostatic potentials in units of T_{e}/e, the ion flows in units of $n_0\sqrt{2}\,v_{T\mathrm{i}}$, $v_{T\mathrm{i}} = \sqrt{T_{\mathrm{i}}/m_{\mathrm{i}}}$, and the ion and electron concentrations in units of n_0 ($n \to n_{\mathrm{i}}/n_0$ and $n_{\mathrm{e}} \to n_{\mathrm{e}}/n_0$). We define P as $n_{\mathrm{d}}Z_{\mathrm{d}}/n_0$, where n_0 is a certain constant concentration of ions or electrons far from the dust system or the ion concentration far from the probe charge. In these variables, the Poisson equation and the ion continuity equation do not contain undetermined constants and have the very simple form

$$\begin{aligned} &e\,\mathrm{div}\,\mathbf{E} \equiv e\,\frac{\partial}{\partial \mathbf{r}}\,\mathbf{E} = n - n_{\mathrm{e}} - P\,, \\ &\mathrm{div}\,\mathbf{\Phi} \equiv \frac{\partial}{\partial \mathbf{r}}\,\mathbf{\Phi} = a(I_{\mathrm{ion}} - \alpha_{\mathrm{ch}}Pn)\,. \end{aligned} \tag{5}$$

The second term $-a\alpha_{\mathrm{ch}}Pn$ was found from the ion absorption cross section $\sigma_{\mathrm{i}} = \pi a^2 z/\tau$, with α_{ch} being the coefficient in the ion flow absorbed by a dust particle. In the simplest model with $\tau \ll 1$, it is equal to the constant term $1/2\sqrt{\pi} = 0.282$. In the right-hand side of the second equation in (5), we introduced an additional source of flows caused by bulk ionization sources, I_{ion} (the factor a is written separately for convenience). An ionization source is needed in any system whose size is larger than the mean free path for flow absorption by dust particles. The term I_{ion} introduced in (5) describes such ionization [its dimensions follow directly from (5)]. The strength of radiation sources of ionization is usually almost independent of the electron concentration, while the strength of other sources (e.g., those used in dust experiments involving microwave fields) is proportional to the electron concentration. To illustrate, we consider the simplest case where

$$I_{\mathrm{ion}} = \alpha_{\mathrm{ion}} n_{\mathrm{e}}\,, \tag{6}$$

with $\alpha_{\rm ion}$ being the ionization constant and $n_{\rm e}$ the electron concentration.

These equations are sufficient for determining the main relation for the charge balance and flow balance, which are needed for the equilibrium state to exist. The conditions for the equilibrium of the charge and flow densities prior to the introduction of the probe charge (subscript '0') are given by

$$n_0 = 1\,, \quad n_{\rm e,0} = 1 - P_0\,, \quad \alpha_{\rm ion}(1-P_0) = \alpha_{\rm ch}(z_0)P_0\,, \qquad (7)$$

where the equilibrium dust charge z_0 is found from the equilibrium condition, which stipulates the equality of the ion and electron thermal fluxes to a dust particle,

$$\exp(-z_0) = \sqrt{\frac{m_{\rm e}}{m_{\rm i}\tau}}\,\frac{z_0}{1-P_0}\,.$$

This relation can be used to label the unperturbed state by a single parameter, P_0.

The linear equations for the perturbations of fields and flows caused by the introduction of a probe charge can be derived by assuming, to simplify matters, that the variation in the parameter $P = P_0 + P_q$ is related only to the variation in the charge of the dust particles in the dusty plasma, i.e., $z = z_0 + z_q$, $P_q = P_0 z_q/z_0$, $n = 1 + n_q$, and $n_{\rm e} = 1 - P_0 + n_{{\rm e},q}$:

$$\operatorname{div}\mathbf{\Phi}_q = a\alpha_{\rm ch}(z_0)P_0\left(\frac{n_{{\rm e},q}}{1-P_0} - n_q - \frac{z_q}{z_0}\left(1 + \frac{z_0}{\alpha_{\rm ch}(z_0)}\frac{\partial\alpha_{\rm ch}(z_0)}{\partial z_0}\right)\right). \qquad (8)$$

The final system of equations has the form

$$\operatorname{div}\mathbf{\Phi}_q = a\alpha_{\rm ch}(z_0)P_0\left(\frac{n_{{\rm e},q}}{1-P_0} - n_q - \frac{z_q}{z_0+\tau}\right), \qquad (9)$$

$$\operatorname{div}\mathbf{E}_q = n_q - n_{{\rm e},q} - P_0\,\frac{z_q}{z_0}\,, \qquad (10)$$

where z_q is the change in the charge of background dust particles caused by the introduction of the probe charge. We let Z_q denote the charge of the probe particle after it has been introduced in the dusty plasma (this is the equilibrium value of the charge, determined by the size a_q of the introduced probe particle, the temperature $T_{\rm e}$, and the plasma flows generated by the dusty plasma with the parameters P_0 and $\alpha_{\rm ion}$ determined before the probe charge was introduced). We assume that the field of the introduced charge is spherically symmetric, i.e.,

$$\begin{aligned}
\mathbf{E} &= \frac{\mathbf{r}}{r}\,\frac{Z_q e^2}{\lambda_{\rm Di}T_{\rm i}}\,\frac{\rm d}{{\rm d}r}\,\frac{\psi(r)}{r} \equiv \zeta_q\,\frac{\mathbf{r}}{r}\,\frac{\rm d}{{\rm d}r}\,\frac{\psi(r)}{r}\,,\\
\mathbf{\Phi} &= \frac{\mathbf{r}}{r}\,\frac{Z_q e^2}{\lambda_{\rm Di}T_{\rm i}}\,\frac{\rm d}{{\rm d}r}\,\frac{\chi(r)}{r} \equiv \zeta_q\,\frac{\mathbf{r}}{r}\,\frac{\rm d}{{\rm d}r}\,\frac{\chi(r)}{r}\,, \qquad (11)\\
\zeta_q &\equiv \frac{Z_q e^2}{\lambda_{\rm Di}T_{\rm i}}\,,
\end{aligned}$$

where we segregated the Coulomb factor $1/r$ in the potential of the electrostatic field, the flow is written in the same way as the field, ψ and χ are determined by Eqns (11) and correspond to the screening factors of the respective potentials, and the charge of the introduced dust particle is assumed to be negative and equal to $-Z_q e$. With

$$\begin{aligned}
&\frac{1}{r^2}\left(\frac{\rm d}{{\rm d}r}\,r^2\,\frac{\rm d}{{\rm d}r}\left(\frac{f}{r}\right)\right) = \frac{1}{r}\,\frac{{\rm d}^2 f}{{\rm d}r^2}\,,\\
&n_q = \zeta_q\,\frac{N_q}{r}\,, \qquad n_{{\rm e},q} = \zeta_q\,\frac{N_{{\rm e},q}}{r}\,, \qquad (12)\\
&z_q = \zeta_q\,\frac{\delta Z_q}{r}\,,
\end{aligned}$$

we can write system of equations (9) and (10) as

$$\begin{aligned}
&\frac{{\rm d}^2\chi}{{\rm d}r^2} = a\alpha_{\rm ch}P_0\left(\frac{N_{{\rm e},q}}{1-P_0} - N_q - \frac{\delta Z_q}{z_0}\right),\\
&\frac{{\rm d}^2\psi}{{\rm d}r^2} = N_q - N_{{\rm e},q} - P_0\,\frac{\delta Z_q}{z_0}\,.
\end{aligned} \qquad (13)$$

To complete the system of equations, we must use the linear relations that link N_q, $N_{{\rm e},q}$, and δZ_q to ψ and g. For this, we write the well-known relations for the balance of forces for electrons (equality of the electrostatic force and the pressure force) and ions (equality of the electrostatic force and the sum of the pressure force and the force of ion friction on the dust component caused by ions being scattered by dust particles), with the already used normalization condition in mind, as

$$\mathbf{E} = -\frac{1}{\tau n_{\rm e}}\,\frac{{\rm d}n_{\rm e}}{{\rm d}\mathbf{r}}\,, \qquad \mathbf{E}_q = -\frac{1}{\tau(1-P_0)}\,\frac{{\rm d}n_{{\rm e},q}}{{\rm d}\mathbf{r}}\,, \qquad (14)$$

$$\mathbf{E} = \frac{1}{n}\left(\frac{{\rm d}n}{{\rm d}\mathbf{r}} + \alpha_{\rm dr}(z)\,\beta(z)\mathbf{\Phi}\right), \qquad \mathbf{E}_q = \frac{{\rm d}n_q}{{\rm d}\mathbf{r}} + \alpha_{\rm dr}(z_0)\,\beta(z_0)\mathbf{\Phi}_q\,, \qquad (15)$$

$$\beta(z) \equiv \frac{za}{\tau}\,, \qquad \tau \equiv \frac{T_{\rm i}}{T_{\rm e}}\,, \qquad (16)$$

where $\alpha_{\rm dr}$ is the coefficient of momentum transfer (drag) from the ion flow to the dust particles when the latter are scattered by ions; in the linear approximation,

$$\alpha_{\rm dr}(z) = \frac{2}{3\sqrt{\pi}}\,(\ln\Lambda)\,, \qquad (17)$$

with $\ln\Lambda$ being the Coulomb logarithm. Generally, $\alpha_{\rm dr}$ depends on the charge z, but in the approximation linear in the flow, the second equation in (15) contains the value of the equilibrium charge z_0. Combining Eqns (14) and (15) with (11), we obtain

$$\begin{aligned}
&N_{{\rm e},q} = -\tau(1-P_0)\psi\,,\\
&N_q = \psi - \alpha_{\rm dr}(z_0)\,\beta P_0\chi = \psi - g\,, \qquad (18)\\
&g = \frac{\chi k_0^2}{\alpha_{\rm ch}(z_0)P_0 a} = \alpha_{\rm dr}(z_0)\,\beta P_0\chi\,,
\end{aligned}$$

$$k_0^2 = \alpha_{\rm dr}(z_0)\alpha_{\rm ch}(z_0)\,\beta(z_0)P_0^2 a\,. \qquad (19)$$

The system of equations is now fairly simple:

$$\frac{{\rm d}^2\psi}{{\rm d}r^2} = \frac{1}{\lambda_{\rm D}^2}\,\psi - g\,, \qquad \frac{{\rm d}^2 g}{{\rm d}r^2} = k_0^2\left(g - \frac{1}{\lambda_\Phi^2}\,\psi\right). \qquad (20)$$

Thus, we have found an expression for the coupling constant (introduced above) that relates the electrostatic polarization fields to the plasma-flow fields:

$$\eta = k_0^2 \,. \tag{21}$$

To be more specific, we give results for $\lambda_{\rm D}^2$ and λ_{Φ}^2 if variations in the charge of the background dust particles are ignored (i.e., $\delta Z_q = 0$):

$$\frac{1}{\lambda_{\rm D}^2} = 1 + \tau(1 - P_0)\,, \qquad \frac{1}{\lambda_{\Phi}^2} = 1 + \tau\,. \tag{22}$$

More general expressions for these parameters are discussed below.

3.3 Solutions of the linear equations

The system of coupled linear equations shows directly and very transparently how the polarization charge depends not only on the electrostatic potential ψ but also on the flow potential g, which in turn depends not only on the flow potential g but also on the electrostatic potential ψ. In the absence of flows, there is only the first term in the right-hand side of the equation for the electrostatic potential, whose solution is given by an increasing exponential and by a decreasing exponential, but it follows from physical considerations that only the decreasing exponential remains, and this exponential corresponds to the Yukawa potential [ion screening is described by the first term in the first relation in (22), while the electron screening is described by the second term in this relation]. Because system of equations (20) is a system of a higher order in derivatives with respect to r than the system of equations in the absence of plasma flows, in conditions where the relevant exponents are real, screening is described by at least two exponentials, which sets it apart from Yukawa screening, because with different exponentials involved, a fraction of the change must be screened, generally speaking, at much larger distances. In a certain sense, Eqns (16) resemble dispersion relations with different roots. The solution of system (20) is, of course, trivial. But it is important that the physical consequences following from them are

$$\psi = \mathrm{Re}\left\{\psi_+ \exp\left(-k_+(r-a)\right) + \psi_- \exp\left(-k_-(r-a)\right)\right\},$$
$$g = \mathrm{Re}\left\{g_+ \exp\left(-k_+(r-a)\right) + g_- \exp\left(-k_-(r-a)\right)\right\}, \tag{23}$$

where

$$k_{\pm}^2 = \frac{1}{2}\left(k_0^2 + \frac{1}{\lambda_{\rm D}^2}\right) \pm \sqrt{\frac{1}{4}\left(k_0^2 + \frac{1}{\lambda_{\rm D}^2}\right)^2 + k_0^2\left(\frac{1}{\lambda_{\Phi}^2} - \frac{1}{\lambda_{\rm D}^2}\right)}\,, \tag{24}$$

$$g_{\pm} = -\frac{k_0^2}{k_{\pm}^2 - k_0^2}\,\frac{1}{\lambda_{\Phi}^2}\,\psi_{\pm}\,. \tag{25}$$

Equations (22) confirm that estimate (3) used above is correct and allow finding explicit expressions for λ_1 and λ_2.

The linear approximation, as we see shortly, can be applied when $\beta = za/\tau \ll 1$; in addition, the ratio of the particle radius to the Debye radius is also often small, i.e., a is small, too, $P_0 < 1$ by definition, and, finally, $\alpha_{\rm ch}\alpha_{\rm dr} = \ln \Lambda/3\pi$. Hence, $k_0^2 \ll 1$, and in this limit,

$$k_+ \approx \frac{1}{\lambda_{\rm D}}\,, \qquad k_-^2 \approx k_0^2\left(1 - \frac{\lambda_{\rm D}^2}{\lambda_{\Phi}^2}\right). \tag{26}$$

This implies that k_-^2 is positive only if $\lambda_{\Phi}^2 > \lambda_{\rm D}^2$, which does not occur with relations (22), which were obtained with the variations in the charge of the dust particles ignored. Thus, in the case under consideration, $k_-^2 < 0$, and we can therefore set $k_-^2 = -1/\lambda_2^2$; instead of the second exponential, we then have a term that oscillates with distance, describing a system of the minima of the attraction potential of the probe particle. At these minima, the densities of the polarization charge have the same sign as the probe charge. The effect, often called overscreening, emerges at distances much larger than the Debye radius ($\approx 1/k_0\sqrt{\tau}$). The effect was first reported in Refs [46, 47] in the case where the ionization source strength is proportional to the electron number density, i.e., when relation (6) holds. The opposite case, where the ionization source strength is independent of the electron number density, was examined in Ref. [48], where it was shown that the second exponential is real (i.e., a fraction of the charge is screened only over distances much larger than the Debye radius). The relative fraction of the contributions of the two exponentials is determined by the values of the coupling constant $\eta = k_0^2$ and is found, for linear screening, from the boundary conditions at the surface of the probe charge.

We note that the attraction effects are enforced by the negative sign of the additional polarization charge generated by the plasma flows [i.e., $g > 0$; see the first equation in (20)]. Because, as shown above, flow-induced effects manifest themselves over distances greater than the Debye radius, to obtain the expressions in (26) that can describe dust-particle attraction, it suffices to use the quasineutrality condition, which can be obtained from the first equation in (20) with zero right-hand side, with the result that $g = \psi/\lambda_{\rm D}^2$. The second equation in (20) then leads to the expression in (26) for k_-^2.

3.4 Various generalizations of linear screening

In studying the possibility of attraction between dust particles, there is no need to know the amplitudes ψ_+ and ψ_- at the terms describing screening; it suffices to find the criteria that tell us when the exponent acquires an imaginary part. Then, irrespective of the dependence on the sign of the amplitudes, the sign of polarization changes at a certain distance and attraction wells emerge. This fact can be used in solving the obtained equations of type (20) as dispersion equations by substituting the exponential solutions and seeing whether the exponent is real or imaginary or has an imaginary component. Moreover, we can use the fact that attraction usually occurs at distances larger than those at which linear screening is present, and the solution may be considerably simplified by using the quasineutrality condition. In this section, we discuss the results of such an analysis (see Ref. [49]).

Variations in the charge density of background dust particles. It is fairly easy to take the variations in the charge of background dust particles into account and generalize the result to arbitrary values of the ion-to-electron temperature ratio $\tau = T_{\rm i}/T_{\rm e}$. We can use the local equations of charging of dust particles (the equality of the ion and electron flows) to express the perturbations of the charge of dust particles in terms of the perturbations of the electron and ion number

densities [49]. The result coincides with Eqns (20), but with certain modified ('renormalized') values of the characteristic lengths and coupling constants:

$$k_0^2 \to k_0^2 \, \frac{z_0+\tau}{1+z_0+\tau}\,, \qquad g \to g\left(1+\frac{P_0(z_0+1)}{z_0(z_0+\tau+1)}\right), \quad (27)$$

$$\frac{1}{\lambda_{\mathrm{D}}^2} \to \left(1+\frac{P_0(z_0+\tau)}{z_0(z_0+\tau+1)}+\tau\left(1-P_0+\frac{P_0(z_0+\tau)}{z_0(z_0+\tau+1)}\right)\right), \quad (28)$$

$$\frac{1}{\lambda_{\Phi}^2} \to \left(1+\frac{P_0(z_0+\tau)}{z_0(z_0+\tau+1)}\right)(1+\tau)\,. \quad (29)$$

Overscreening and attraction wells are always present here because

$$\frac{1}{\lambda_{\Phi}^2}-\frac{1}{\lambda_{\mathrm{D}}^2} = \tau P_0 > 0\,. \quad (30)$$

The occurrence of attraction for other types of ionization sources. The simplest example is the presence of an ionization source whose strength is independent of the electron number density. For many applications, ionization is usually done by electrons (as in all experiments in microwave discharges or dc discharges). Hence, the above example, where ionization is proportional to the electron concentration, is the most widespread. However, ionization is possible with other sources, such as resonance radiation and fast ions. For example, we analyze the case where

$$I_{\mathrm{ion}} = \alpha_{\mathrm{ion}}(n_{\mathrm{e}}+\gamma_{\mathrm{ion}})\,, \quad (31)$$

with α_{ion}, as previously, being the ionization constant proportional to the electron concentration and γ_{ion} being the ratio of the ionization constant independent of the electron concentration to the ionization constant proportional to the electron concentration. Depending on the value of γ_{ion}, the solutions of the equations describing the interaction of the electrostatic polarization fields and the plasma-flow-fields may correspond either to an oscillating screening factor or to an exponential screening factor [53, 55]. The criterion for the occurrence of attraction wells is then given by

$$\gamma_{\mathrm{ion}} < \frac{P_0 z_0(z_0+\tau)(1-P_0)}{z_0(z_0+\tau)(1-P_0)+z_0+\tau+P_0 z_0}\,, \quad (32)$$

with the right-hand side proportional to the parameter P_0. Similar criteria are possible for other models of ionization sources.

The role of collisions with neutral atoms. The friction force with which neutral atoms of a gas act on ions exceeds the friction force acting on the ions in the collisions with dust particles if

$$\frac{\lambda_{\mathrm{Di}}}{\lambda_{\mathrm{in}}} \geqslant \alpha_{\mathrm{dr}}\,\beta P_0\,, \quad (33)$$

where λ_{in} is the mean free path in ion–neutral collisions. With the values of parameters used in typical laboratory experiments, both the left-hand and right-hand sides of this inequality are of the order of or much smaller than unity. Thus, the case where friction on neutrals is predominant is quite realistic in a certain range of pressures of the neutral gas. Clearly, attraction is independent of the nature of the friction force and all results in the preceding sections can be applied to the case where friction on neutrals is predominant by introducing the substitution

$$\alpha_{\mathrm{dr}}\,\beta P_0 \to \frac{\sqrt{2}\,\lambda_{\mathrm{Di}}}{\lambda_{\mathrm{in}}}\,. \quad (34)$$

The role of regular plasma flows in collisions of ions with dust particles. In the presence of a regular plasma flow, far from a probe charge maintained by a certain external effective field E_{eff}, the distribution of polarization charges surrounding the probe charge becomes anisotropic and is different in the directions parallel to the flow and perpendicular to it. The external field generating the regular flow may determine the steady-state drift velocity of the ions, $u_0 \to u_{\mathrm{i}}/\sqrt{2}\,v_{\mathrm{Ti}}$, either because of the friction of the ions in collisions with dust particles or because of the friction in collisions with neutral atoms of the gas.

When collisions of ions with dust particles determine the ion drift velocity, we obtain the relation (which is Ohm's law in a dusty plasma):

$$E_0 = \alpha_{\mathrm{dr}}(z_0, u_0) P_0 u_0\,. \quad (35)$$

When perturbations introduced by a probe charge are considered for $u_0 \gg 1$, it is convenient to use the expressions for α_{dr} and α_{ch} in the limit where $u_0 \gg 1$,

$$\begin{aligned} \alpha_{\mathrm{dr}} &= \alpha_{\mathrm{dr}}(z_0,u_0) \approx \frac{1}{2u_0^3}\left(\ln\Lambda+\frac{\tau u_0^2}{z_0}+\frac{\tau^2 u_0^4}{z_0^2}\right), \\ \alpha_{\mathrm{ch}} &= \alpha_{\mathrm{ch}}(z_0,u_0) \approx \frac{1}{4u_0}\left(1+\frac{\tau u_0^2}{z_0}\right), \end{aligned} \quad (36)$$

to use the quasineutrality condition, and to introduce parameters that determine the occurrence of attraction between dust particles:

$$\begin{aligned} \tilde{k}_0^2 &= k_0^2\,\frac{1-P_0}{2u_0^2}\,, \\ \kappa &= \frac{2u_0^2\tau(P_0-\gamma_{\mathrm{ion}})}{\big(1+\tau(1-P_0)\big)(1-P_0+\gamma_{\mathrm{ion}})}\,, \\ U_0 &= \frac{4u_0^4}{\beta P_0 \ln\Lambda}\,. \end{aligned} \quad (37)$$

We let the distances along the flow be denoted by x and those perpendicular to it by $\mathbf{r}_{\perp}$. The dispersion equation obtained by substituting all variables in the form of expressions proportional to $\exp(-kx+\mathrm{i}\mathbf{k}_{\perp}\mathbf{r}_{\perp})$ (in the limit where $kU_0 \gg 1$) and then recalling that for $u_0^2 \ll z_0/\tau$, both z_0 and P_0 are also of the order of unity and $\kappa \ll 1$, has the solution

$$k = k_{\pm} = \frac{1}{2}\left(U_0\tilde{k}_0^2 \pm \sqrt{U_0^2\tilde{k}_0^4+4k_{\perp}^2}\,\right). \quad (38)$$

Here, the solutions are decaying in x, and we must select k_- for $x>0$ and k_+ for $x<0$. This means that there is neither overscreening nor particle attraction. For $\kappa \ll kU_0 \ll 1$,

$$k_{\pm} = \frac{1}{2}\left(U_0\tilde{k}_0^2 \pm \sqrt{U_0^2\tilde{k}_0^4-\frac{z_0}{1+z_0}\,4k_{\perp}^2}\,\right) \quad (39)$$

and screening is possible when $k_\perp^2 > \tilde{k}_0^4 U_0^2(1+z_0)/4z_0$. Finally, for $kU_0 \ll \kappa$, we have

$$k^2 = \frac{z_0}{1+z_0}(k_\perp^2 - \tilde{k}_0^2) \tag{40}$$

and attraction is possible under the weak condition $k_\perp < \tilde{k}_0$, because $k_0 \ll 1$. Thus, in the presence of external ion flows, often encountered in experiments, the collective interaction of electrostatic screening fields and plasma flows may, within a broad range of values of the parameters, lead to attraction and pairing of dust particles.

The role of regular plasma flows in collisions of ions with neutral gas atoms. When the ion drift velocity in a flow is high, $u_0 \gg 1$, the force of friction of the ions in collisions with neutral gas atom is [49]

$$-\alpha_{\text{in}}\mathbf{u}|\mathbf{u}|\,\frac{\lambda_{\text{Di}}}{\lambda_{\text{in}}}\,, \tag{41}$$

where α_{in} is a factor of the order of unity that depends on the type of gas. For the most interesting case where $\lambda_{\text{Di}}^2/\lambda_{\text{in}}^2 \ll 1/u_0^2\tau$ and for lengths along the flow much larger than the mean free path in the ion–neutral collisions, $k \ll \lambda_{\text{Di}}/\lambda_{\text{in}}$ (k is measured in units of $1/\lambda_{\text{Di}}$), we have

$$\frac{1}{2}\,k^2 = k_\perp^2 - \tilde{\tilde{k}}_0^{\,2}\,, \tag{42}$$

where

$$\tilde{\tilde{k}}_0^{\,2} = \frac{\tau\lambda_{\text{Di}}P_0^2 z_0 a}{4\lambda_{\text{in}}\big(1+z_0+P_0+\tau\big(1+z_0(1-P_0)\big)\big)}\,. \tag{43}$$

According to (42), attraction wells appear when $k_\perp^2 < \tilde{\tilde{k}}_0^{\,2}$.

3.5 Amplitudes of linear screening and the potential energy of dust-particle pairing

Of great importance is the fact that the effect of plasma flow on dust particles amounts to the appearance of a second exponential (or a cosine term oscillating with distance, the term in a certain sense also screening the potential of the probe particle, because it reduces the screening field to zero on the average). The first minimum in the potential is important; it describes attraction and possible pairing of dust particles. To determine the depth of the attraction potential well, we must find the amplitudes of the two screening factors. We begin with the case with no external ion flows, $u_0 = 0$. For small γ_{ion} and $k_0 \ll 1$, we then have

$$\psi \approx \psi_+ \exp\left(-\frac{r}{\lambda_{\text{D}}}\right) + \psi_- \cos\left(k_0 r\sqrt{1-\frac{\lambda_{\text{D}}^2}{\lambda_\Phi^2}}\right), \tag{44}$$

and for large γ_{ion} [51],

$$\psi \approx \psi_+ \exp\left(-\frac{r}{\lambda_{\text{D}}}\right) + \psi_- \exp\left(-k_0 r\sqrt{\frac{\lambda_{\text{D}}^2}{\lambda_\Phi^2}-1}\right). \tag{45}$$

The boundary conditions in the case of linear screening can be chosen on the surface of the probe charge. We assume that the size of the dust is small compared with the Debye radius and, moreover, compared with the characteristic range of the second term, caused by effects of interaction between plasma flows and electrostatic polarization fields. Normalization with the parameter ζ_q is selected in this case such that $\psi(a) \approx \psi_+ + \psi_- = 1$; furthermore, there are no additional flows to the surface of the dust particle except those related to charge variations at $u_0 = 0$, and $g(a) = g_+ + g_- = 0$. On the other hand, the first equation in (20) with $k^2 \ll 1$ yields $g_- = \psi_-/\lambda_{\text{D}}^2$ and the second equation in (2) with $k^2 \approx -1/\lambda_{\text{D}}^2$ yields $g_+ = -k_0^2\lambda_{\text{D}}^2/\lambda_\Phi^2\psi_+$, where

$$\begin{aligned}\psi_- &= k_0^2\,\frac{\lambda_{\text{D}}^4}{\lambda_\Phi^2}\,\psi_+ \approx k_0^2\,\frac{\lambda_{\text{D}}^4}{\lambda_\Phi^2}\,,\\ \psi_+ &= \left(1+k_0^2\,\frac{\lambda_{\text{D}}^4}{\lambda_\Phi^2}\right)^{-1} \approx 1-k_0^2\,\frac{\lambda_{\text{D}}^4}{\lambda_\Phi^2}\,.\end{aligned} \tag{46}$$

These relations are applicable for (44) and for (45). The sum of the coefficients of the two 'screening factors' is unity.

When $u_0 \gg 1$, the picture becomes more complicated because there is a difference between screening along the flow and perpendicular to the flow, but the result is roughly the same: the amplitude of the additional terms in screening have a relative order of $\tilde{k}_0^2$. This case requires an additional and detailed analysis; here, we limit ourselves to this estimate. The depth of the potential energy of attraction of two dust particles, U_{at}, is estimated as

$$U_{\text{at}} = -\frac{Z_{\text{d}}^2 e^2}{r_{\min}}k_0^2\,\frac{\lambda_{\text{D}}^4}{\lambda_\Phi^2} \approx Z_{\text{d}}^2 e^2 k_0^2 \pi\,\frac{\lambda_{\text{D}}^4}{\lambda_\Phi^2}\,\sqrt{\frac{1}{\lambda_\Phi^2}-\frac{1}{\lambda_{\text{D}}^2}}\,. \tag{47}$$

Although $k_0^2 \ll 1$, we usually have $Z_{\text{d}} \gg 1$, and the attraction potential may be significant.

3.6 The physics of dust-particle attraction effects

The physics of overscreening and attraction effects is clear from the above calculations: it is related to the accumulation of positive-charged ions in regions between the interacting negative-charged particles. Electrons that have been again produced by ionization, in view of their high mobility, easily leave these regions; there is usually enough time for them to become distributed according to Boltzmann's law and adjust themselves to the local value of the electrostatic potential. On the other hand, ions are unable to rapidly leave such regions due to friction and accumulate at fairly large distances between the interacting particles, and therefore the attraction effect manifests itself irrespective of the nature of the friction force. Plasma flows serve as the mechanism for accumulating positive charge in the regions between negative-charged dust particles. Direct generalization to the case of positive-charged dust particles is impossible because the plasma flows change and processes of charge buildup are altered, to say nothing about the fact that electrons (instead of ions) become the main particles of the flows. Interaction occurs via virtual waves, but these waves change dramatically in the presence of particle attraction, either bringing about instabilities of the gravitational type or forming bound states of dust particles, known as dust clusters or crystals. Everything depends on changes in the kinetic energy of the dust particles in the process of their self-compression due to attraction. A rapid decreases in the kinetic energy of dust particles in laboratory conditions is usually possible because of friction with atoms of the neutral gas. Such friction is significant because of the low ionization in gas discharges in which experiments with dusty plasma are typically conducted.

3.7 Demolished myths

One of the common myths often used in interpreting dusty plasma experiments is that in the case of small charges, the screening of particles in a dusty plasma is of the Debye type and that the potential of the particles can be approximated by the Yukawa potential. All that has been said above indicates that this is not so: at large distances, the Pitaevskii potential remains unscreened for individual particles, while for a large number of particles forming a system larger than the mean free path of plasma flows, a poorly screened component occurs in the potential that may correspond not only to repulsion but also to attraction. Here, collisions not only fail to restore the Yukawa potential but also enhance the effects associated with the poorly screened component of the potential. Thus, the myth of Yukawa screening is demolished even for small charges in dusty plasmas in conditions where the linear approximation for the polarization charge can be used (the polarization charge is proportional to the probe-particle potential).

Another myth used in most numerical calculations involving dusty plasmas was that by employing fixed potentials for the pair interaction of particles one can expect to describe the dynamics of a system consisting of a large number of dust particles. Here, too, all that has been said above indicates that the myth is untrue, because in all the above examples, the pair interaction and screening of individual particles depend on the parameter P_0, i.e., on the number density of the dust particles. Hence, when the distance between the dust particles changes, so does their pair interaction, and this should be taken into account in all calculations involving dust systems. This pair interaction may be called collective, and there is nothing unusual about it. We note that in ordinary plasma without dust particles, the pair interaction is given by the Yukawa potential, whose screening length depends on the number density of the plasma particles. It is difficult to assume that a dusty plasma is any different in this sense, and it is only natural to assume that the pair interaction of dust particles depends on the dust density. Of course, the above simple relations involve the mean local dust density. The nonlocality effect requires further detailed investigations in the future, but it is generally difficult to deny that the interaction is collective.

4. Nonlinear screening

4.1 Estimates of nonlinearities

Intensive experimental investigations of dusty plasmas began with the realization that even with small dust densities, the dust particles carrying large charges can interact strongly; this directly affects commercial applications such as plasma etching [3, 15], as well as problems associated with the formation of plasma crystals [7–10] and future controlled fusion devices, where the fact that the walls of the device are covered with dust may play a certain role in the possibility of generating thermonuclear energy. Roughly speaking, the parameter $P_0 = n_{\mathrm{d}} Z_{\mathrm{d}}/n_0$, which characterizes the ratio of the charge density of the dust particles to the number density of the plasma ions, must be of the order of unity (we recall that P_0 can vary between 0 and 1). If $n_0 \approx 10^8$ cm^{-3} and $n_{\mathrm{d}} \approx 10^4$ cm^{-3} in existing discharges, then Z_{d} must be roughly $(3-9)\times 10^3$, which is the case in modern experiments. But with such large charges, the screening is certainly nonlinear. We mean that near a dust particle, the ratio of the potential energy of the screening particles to their kinetic energy is much larger than unity.

To prove this, we make elementary estimates. Because electrons most often have the temperature about 2–5 eV and the ion temperature is about 0.02 eV, the value of τ amounts to roughly 0.01–0.005, and in view of the low ion temperature, ion nonlinearities are predominant. At the surface of a dust particle, $e\phi/T_{\mathrm{i}} \approx Z_{\mathrm{d}} e^2/aT_{\mathrm{i}} = z/\tau \approx 300-500$ for $z \approx 3$, i.e., the nonlinearities are very strong. In such conditions, the expansion of the ion polarization charge $\rho_{\mathrm{i}} \propto \exp(e\phi/T_{\mathrm{i}})$ in a power series in the parameter $e\phi/T_{\mathrm{i}}$, which leads to the linear Yukawa screening, is entirely unacceptable. It is amazing, however, that the Yukawa potential was used in many recent papers devoted to applications in experiments in which the main parameters corresponded to the above example. But maybe we should be less severe in judging such work and allow for the fact that the potential of a dust particle decreases as the distance increases. We assume that the distance is smaller than the linear Debye screening length λ_{Di} but larger than the size a of the dust particle (with the condition $\lambda_{\mathrm{Di}} \gg a$ usually satisfied). Here, the potential decreases as $\propto 1/r$ and such behavior continues with linear screening to approximately λ_{Di}, when $e\phi/T_{\mathrm{i}} \approx za/\tau\lambda_{\mathrm{Di}}$. Thus, although the nonlinearity is always strong at the surface of the dust particle, linear screening occurs only for $\beta \ll 1$, with

$$\beta = \frac{za}{\tau\lambda_{\mathrm{Di}}} = \frac{Z_{\mathrm{d}} e^2}{\lambda_{\mathrm{Di}} T_{\mathrm{i}}} \,. \tag{48}$$

It is important that in most experiments, $\lambda_{\mathrm{Di}}/a \approx 5-35$ and $\beta \approx 30-100$, i.e., there is no place for linear screening. Hence, the non-Yukawa form of screening is caused not only by the interaction of the polarization fields and plasma-flow fields over large distances but also by the strong nonlinearity of screening over small distances. Still, it should be noted that the nonlinearity may be weak for small particles, as shown by the first equality in (48). It would seem that this contradicts the second equality in (48), which appears to be independent of the particle size. But this is not the case, because the charge of the dust particles is proportional to their size, and the parameter z is practically independent of the particle size. While $a \approx 5-10$ μm in typical experiments, for the screening to become linear for the same values of the parameters, a must be no larger than 0.3 μm. Experiments with such small dust particles have been conducted, but to make the interaction between the dust particles strong, the particle size should be made as large as possible. In this case, the screening becomes nonlinear. It is amazing that this fact was almost completely ignored up to the year 2000 and Yukawa screening was used in the case of strongly interacting particles.

4.2 Gurevich nonlinearities [51] and the role of trapped particles [52]

The type of nonlinearity that has been best known since 1956 is the one described in detail in monograph [51]. It is commonly known as the Gurevich nonlinearity. We now consider the underlying physical assumptions and see whether these assumptions are always valid (see also Refs [52–60]). The monograph discussed the screening of the field of satellites, but in this respect, satellites are similar to dust particles, because the relevant size is much smaller than the screening length in both cases. Hence, while using the results in Ref. [51], we always have dust particles in mind, although there is no mention of them in Ref. [51]. The simplest way to obtain such a nonlinearity is described in

Ref. [53]. The main arguments in [53] consist in the assumption that the only particles (in our case, ions, because the nonlinearity is related mainly to the polarization charge of ions) that can participate in screening are those that can arrive to a dust particle from 'infinity,' i.e., from the region where the effect of the field of the dust particle is negligible. Because the potential of a negative-charge dust particle is negative, if the ions far from the particle are thermal with a temperature $T_{\rm i}$, then the criterion describing the situation where an ion can screen the dust particle is $\epsilon_{\rm i} > -e\phi$, where $\epsilon_{\rm i}$ is the local energy of ions at the distance r from the dust particle and ϕ is the local potential of the dust particle at the distance r. The charge density of ions at the distance r is then given by the formula

$$\rho_{\rm i}(r) = en_{\rm i,0}\,\frac{2}{\sqrt{\pi}}\exp\left(-\frac{e\phi}{T_{\rm i}}\right)\int_{-e\phi}^{\infty}\exp\left(-\frac{\epsilon_{\rm i}}{T_{\rm i}}\right)\sqrt{\frac{\epsilon_{\rm i}}{T_{\rm i}}}\,\frac{{\rm d}\epsilon_{\rm i}}{T_{\rm i}}$$
$$= en_{\rm i,0}\,\frac{2}{\sqrt{\pi}}\int_0^{\infty}\sqrt{\frac{\epsilon-e\phi}{T_{\rm i}}}\,\frac{1}{T_{\rm i}}\exp\left(-\frac{\epsilon}{T_{\rm i}}\right){\rm d}\epsilon\,, \qquad (49)$$

where the coefficient $2/\sqrt{\pi}$ follows from the standard normalization condition for the thermal distribution $(\int_0^{\infty}\exp(-y)\sqrt{y}\,{\rm d}y = \sqrt{\pi}/2)$, and $\epsilon = \epsilon_{\rm i} + e\phi$. If the nonlinearity is strong, i.e., $|e\phi/T_{\rm i}| \gg 1$, we can ignore ϵ in the radicand in the last relation in (49), with the result that

$$\frac{\rho}{en_{\rm i,0}} \approx \frac{2}{\sqrt{\pi}}\sqrt{\left|\frac{e\phi}{T_{\rm i}}\right|}\,. \qquad (50)$$

The exact expression for polarization charge (49) can be expressed in terms of the error functions and, naturally, is valid both for a strong nonlinearity and in the 'tail' of the distribution of the potential, where the nonlinearity is weak and $|e\phi/T_{\rm i}| \ll 1$. A detailed analysis of the cases where approximate relation (50) can be used in the nonlinearity region is given in Refs [54, 55, 61], where it is shown that this formula can be used up to $|e\phi/T_{\rm i}|$ values of the order of unity. We especially note that (49) and (50) are valid only for negative potentials or for positive values of the screening factor ψ [we recall that $\phi = -(Z_{\rm d}e/r)\psi(r)$, with $Z_{\rm d} > 0$]. The last condition becomes important in numerical solutions of the Poisson equation with potential (49); when negative values of ψ are obtained, they cannot be continued to large distances, which is sometimes erroneously done (see Ref. [56]). Clearly, as soon as the potential becomes positive and the screening factor negative, the limitations in energy disappear and we must simply use the Boltzmann distribution $[\rho_{\rm i} = en_{\rm i}\exp(-e\phi/T_{\rm i})]$. Most often, the emergence of negative values of the screening factor is caused by errors in the numerical calculations if other processes, such as the interactions of the electrostatic polarization fields and plasma flows discussed here, are ignored. So far, Eqn (49) has not taken these processes into account. But if they are taken into account, negative values of the screening factor certainly appear, which makes a change in (49) inevitable, with the Boltzmann or a different screening factor accounting for the interaction with plasma flows to come into play.

The condition opposite to $\epsilon_{\rm i} > -e\phi$, i.e., $\epsilon_{\rm i} < -e\phi$, means that particles with such energies cannot go to infinity, that is, they are trapped. We note that in deriving (49), the ion distribution must be assumed isotropic. Such isotropy is not typical for trapped ions, as shown in Refs [52–58]. We now discuss the role of trapped ions in detail. The problem was first examined in [59]. We note that **the Gurevich distribution does not take trapped ions into account** and that there are physical reasons for this. When a dust particle is introduced into a plasma, the dynamics of the growth of its charge, apparently, leads to a small number of trapped particles. If the distribution of the trapped particles is $\Phi^{\rm tr}(\epsilon_{\rm i}, \Omega)$, where Ω is the solid angle of the velocity of the trapped particles (normalized to ${\rm d}\Omega/4\pi$), the trapped particles prove to be an additional contribution to the charge density near the dust particle,

$$\rho_{\rm i}^{\rm tr} = en_{\rm i}\int_0^{-e\phi}\sqrt{\epsilon_{\rm i}}\,d\epsilon_{\rm i}\Phi^{\rm tr}(\epsilon_{\rm i}, \Omega)\,\frac{{\rm d}\Omega}{4\pi}\,. \qquad (51)$$

The only ions that impinge on the dust particle are those that come from regions where the potential of the dust particle is low compared with the kinetic energy of the ions, i.e., only nontrapped ions participate in screening. Their conversion into trapped particles may emerge because of collisions with neutrals. The most effective collisions are those with charge exchange, when a fast ion transforms into a fast neutral atom that leaves the system, and therefore the ions that remain are almost thermal with the temperature of the neutrals. This process is examined mainly in Refs [56–59]. It must be said that as an ion approaches a dust particle, its speed increases because it is attracted to the dust particle, with the energy of the ion far from the dust particle growing from the thermal ion temperature to the electron temperature at the surface of the dust particle, i.e., the ion energy increases by a factor of approximately 100 in the existing experiments. The charge-exchange cross section is approximately constant in this energy range (see Ref. [57]), and therefore the charge-exchange rate is proportional to the ion velocity and increases as the ion comes closer to the dust particle. As the distance to the dust particle decreases, collisions with charge exchange become progressively more important and their rate increases. Lampe et al. [52] and Goree [59] studied the paths of individual ions and determined the conditions that must be satisfied for ions with such paths to be transformed into trapped ions; in Ref. [58], the distribution density of the trapped particles was studied in greater detail by summing over the paths calculated for individual ions. Such an approach is justified when the number density of the trapped particles is very low, and in the earlier work [59] Goree came to the conclusion that this was indeed the case. Lampe et al. [58] arrived at the opposite conclusion and stated that trapped particles can completely screen the field of dust particles. But with such a number density of trapped particles, it is only natural to pose the question of the applicability of the initial assumptions. Trapped particles are distributed anisotropically, and we know from plasma physics that anisotropic distributions are highly unstable [60]. It is only natural then to assume that the trapped-particle distribution rapidly becomes isotropic and that this process is the fastest among all other process, including the absorption of ions by dust particles. The question of the instability growth rate of the distributions gathered in monograph [60] can be clearly formulated, but so far no answer has been given in the literature in view of the complexity of the problem of the instability of the highly anisotropic and strongly inhomogeneous distribution of trapped particles. Hence, here we can only make estimates based on the assumption that the process is the fastest, which the simplest estimates corroborate. Future studies will reveal for what critical number density of the trapped particles this

assumption is justified. Estimates give density values that are much smaller than those obtained in Ref. [60]. The reason is that both in Ref. [59] and in Ref. [58], the creation and disappearance rates of trapped particles are proportional to the charge-exchange rate, with the result that the number of trapped particles is independent of the charge-exchange rate. But if rapid isotropization is taken into account, the creation is proportional to the charge-exchange rate, while the disappearance is determined by the isotropization growth rate. We note that the angular distribution of trapped particles always contains a loss cone, where during the flight time particles touch a dust particle and are absorbed by it, while isotropization processes continuously supply new particles to this cone, which serves as a mechanism for the constant draining of trapped particles.

Here are some simple estimates. Let r denote a certain distance from a dust particle. A certain value $\phi(r)$ of the screened potential can be assigned to this distance. The distance can be marked either by a value of r or by a value of ϕ, because the two are in one-to-one correspondence. We assume that a charge-exchange collision occurs at this distance, with an ion with energy ϵ_{i} transformed into a thermal ion. Then the number of such events per unit time in one cubic centimeter of the plasma is given by

$$\nu_{\text{ch-ex}} \int_{-e\phi}^{\infty} \frac{v_{\mathrm{i}}}{v_{T\mathrm{i}}}\, \Phi^{\mathrm{ntr}}(\epsilon_{\mathrm{i}})\, \frac{2}{\sqrt{\pi}}\, \sqrt{\epsilon_{\mathrm{i}}}\, \mathrm{d}\epsilon_{\mathrm{i}}\,, \qquad v_{\mathrm{i}} = \sqrt{\frac{2\epsilon_{\mathrm{i}}}{m_{\mathrm{i}}}}\,, \tag{52}$$

where $\nu_{\text{ch-ex}} = v_{T\mathrm{i}}/\lambda_{\mathrm{in}}$ is the rate of charge-exchange collisions involving thermal ions, and the temperatures of the ions and neutrals are assumed to be equal. The most interesting distances are much larger than the particle size ($r \gg a$) and where the nonlinearity is strong ($-e\phi \gg T_{\mathrm{i}}$). The first condition is related to the fact that at small distances, both the linear and any nonlinear screening are weak (from physical considerations, a nonlinearity reduces the polarization charge and only increases the distances over which screening is significant). The second condition is natural if we are interested in the effect of collisions on the nonlinearity. In view of the first condition, the rate at which particles with energies lower than $-e\phi$ appear is, according to (52), $\nu_{\text{ch-ex}} \propto n_{\mathrm{ntr}}(2/\sqrt{\pi})(-e\phi/T_{\mathrm{i}})$ [here, we do the same operations as in deriving (50)]. The decrease in the number of such particles is determined by the quantity $n_{\mathrm{tr}}\pi p^2 v_{T\mathrm{i}}$, where p is the impact parameter and we took into account that the initial velocity of the ions that emerge as a result of charge-exchange collisions is equal to the thermal velocity of the neutrals, which is of the order of the thermal velocity of ions far away from the particle. Next, $p = r\sin\theta$, where θ is the angle between the ion velocity and the direction to the dust particle. In the absence of isotropization, p can be found from the energy and momentum conservation laws for the distances r and a, i.e., $pv = av_{\mathrm{d}}$ and $e\phi_{\mathrm{d}} + mv_{\mathrm{d}} \approx e\phi$, where v_{d} is the velocity of the ion tangential to the dust particle surface (here, we estimate the maximum impact parameter). Then,

$$p_{\max}^2 \approx a^2\left(\frac{e\phi - e\phi_{\mathrm{d}}}{T_{\mathrm{i}}}\right) \approx a^2\left(-\frac{e\phi_{\mathrm{d}}}{T_{\mathrm{i}}}\right),$$

and hence

$$\frac{n_{\mathrm{tr}}}{n_{\mathrm{ntr}}} \approx \sqrt{\frac{1}{n_{\mathrm{ntr}}\lambda_{\mathrm{in}}\pi a^2}\,\frac{\phi}{\phi_{\mathrm{d}}}}\,. \tag{53}$$

In most of the existing experiments, $\lambda_{\mathrm{in}} \approx 10^{-2}$ cm; $a \approx 10$ μm, and $n_{\mathrm{ntr}} \approx 10^8$ cm^{-3}, and we therefore have the estimate $n_{\mathrm{tr}}/n_{\mathrm{ntr}} \approx \sqrt{\phi/3\phi_{\mathrm{d}}}$, and although $\phi/\phi_{\mathrm{d}} \ll 1$, a certain increase in the neutral gas pressure (a decrease in λ_{in}) leads to an estimate that shows that the concentrations of trapped and nontrapped particles are comparable [in Ref. [59], the decrease in the number of trapped particles is also determined by charge-exchange collisions, with the result that the estimate of the number density of trapped particles is greater than the value in (53), where the decrease is determined by the loss cone]. Taking isotropization into account changes this estimate dramatically. Ions with values of p smaller than r then land in the loss cone and (53) must be replaced with

$$\frac{n_{\mathrm{tr}}}{n_{\mathrm{ntr}}} \approx \sqrt{\frac{1}{n_{\mathrm{ntr}}\lambda_{\mathrm{in}}\pi r^2}\left(\frac{-e\phi}{T_{\mathrm{i}}}\right)} \ll 1 \tag{53a}$$

and practically no trapped particles play an important role and the Gurevich nonlinearity can be used [51]. We believe that these estimates are even somewhat overstated and actually the role of trapped particles is even smaller. The result in (50) can easily be generalized to an arbitrary isotropic distribution of ions $f(\epsilon_{\mathrm{i}})$:

$$\rho_{\mathrm{i}} = n_{\mathrm{i},0}\sqrt{\left|\frac{-e\phi}{T_{\mathrm{eff}}}\right|}\,, \qquad T_{\mathrm{eff}} = \left(\frac{\int f(\epsilon_{\mathrm{i}})\sqrt{\epsilon_{\mathrm{i}}}\,\mathrm{d}\epsilon_{\mathrm{i}}}{\int f(\epsilon_{\mathrm{i}})\,\mathrm{d}\epsilon_{\mathrm{i}}}\right)^2. \tag{54}$$

In concluding this discussion, we note that the problem of trapped particles is not completely solved by this estimate, and a detailed study of the stability of the distribution of trapped particles is a pressing goal of a future analysis. However, it is only natural to say that **the problem of trapped particles is an important example of the large role of the interaction of polarization charges and plasma flows**.

4.3 A simple model of nonlinear screening

Nonlinear charge density (50) is not the only example of nonlinearities, because there can be nonlinearities caused by trapped particles with different types of collisions. To find the nonlinear potential, we must solve the nonlinear Poisson equations. In this respect, the simplest analytically solvable nonlinear equations for the screening factor ψ are instructive:

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}r^2} = \psi^{\nu}\,, \tag{55}$$

where ν is an arbitrary nonlinearity parameter. Equation (55) has explicit analytical solutions satisfying the conditions $\psi(1) = 1$ and $\psi(+\infty) = 0$, which differ dramatically for $\nu < 1$ and for $\nu > 1$:

$$\psi \propto \left(1 - \frac{r}{r_0}\right)^{2/(1-\nu)}, \qquad \nu < 1\,, \tag{56a}$$

$$\psi \propto \frac{1}{(1 + r/r_0)^{2/(\nu-1)}}\,, \qquad \nu > 1\,, \tag{56b}$$

where r_0 is the integration constant. The first distribution is cut off at a finite distance r_0, and the second becomes a power-law distribution when $r \gg r_0$.

4.4 Solution of the nonlinear Poisson equation

The above simple model serves as a hint for solving the nonlinear Poisson equation with the nonlinear charge density

$$\rho_{\mathrm{i}} = en_{\mathrm{i},0}\left(\left|\frac{e\phi}{T_{\mathrm{eff}}}\right|\right)^{\nu}, \qquad \operatorname{div}\mathbf{E} = 4\pi e\rho_{\mathrm{i}} \,. \tag{57}$$

With natural modification of the adopted notation and with the ion temperature replaced with the effective temperature in accordance with (54), Eqn (57) becomes

$$\lambda^2_{\mathrm{D,\,eff}} = \frac{T_{\mathrm{eff}}}{4\pi n_{\mathrm{i},0}e^2}\,, \qquad \beta_{\mathrm{eff}} = \frac{za}{\tau_{\mathrm{eff}}\lambda_{\mathrm{D,\,eff}}}\,, \qquad \tau_{\mathrm{eff}} = \frac{T_{\mathrm{eff}}}{T_{\mathrm{e}}}\,. \tag{58}$$

In the case of spherical symmetry, distribution (57) of the potential around a dust particle is reduced to a simple equation for the screening factor,

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}r^2} = r^{1-\nu}\psi^{\nu}\,, \qquad r \to \frac{r}{\lambda_{\mathrm{D,\,eff}}\,\beta_{\mathrm{eff}}^{-(3-\nu)/(1-\nu)}}\,. \tag{59}$$

This equation is reduced to the simplest possible form by eliminating the parameter β_{eff} via an appropriate normalization of the distance r in units of the effective Debye radius multiplied by a certain power of β_{eff}. When $T_{\mathrm{eff}} = T_{\mathrm{i}}$, accordingly, $\lambda_{\mathrm{D,\,eff}} = \lambda_{\mathrm{Di}}$ and $\beta_{\mathrm{eff}} = \beta$. The case of nonlinearity (54) corresponds to $\nu = 1/2$. In what follows, we keep in mind the possibility of such a simple generalization to the case of a nonthermal distribution of ions far from a dust particle and use the simple thermal distribution.

Equation (59) with $\nu < 1$ was solved numerically in Ref. [54]; as in the simple model in Section 4.3 with $\psi \to 0$ at finite distances, solutions of Eqn (59) were found with the boundary condition at the surface of the dust particle given by

$$\psi(a) = 1 + a\,\frac{\mathrm{d}\psi(a)}{\mathrm{d}a}\,, \tag{60}$$

where a is the size of the particle (for any normalization). With the normalization used here, $a \ll 1$, and the second term generally produces a small correction, which, however, must be taken into account because the numerical calculations must be done with great accuracy. The reason is that in fixing the second boundary condition, the value of $\mathrm{d}\psi(a)/\mathrm{d}a$ on the particle surface, there is always a mixture of increasing and decreasing solutions. In the above example of a simple model, this problem was solved by selecting one solution for the first integral of the equations (we had to use different solutions for $\nu < 1$ and for $\nu > 1$). Even in the linear approximation, we have two solutions, $\exp(-r/\lambda_{\mathrm{D}})$ and $\exp(+r\lambda_{\mathrm{D}})$, where the second solution is rejected on physical grounds. Charge density (57) has the same sign and corresponds to screening, i.e., to the solution that decreases as the distance from the dust particle increases. The method used to solve the Poisson equation can easily be verified in the case of linear screening: it shows that for meaningless values of $\mathrm{d}\psi(a)/\mathrm{d}a$, the solution always consists of a mixture of two exponentials. Nonlinear solutions are also a sum of increasing and decreasing solutions, whose ratio depends on the chosen value of $\mathrm{d}\psi(a)/\mathrm{d}a$. This value should be normalized such that only the decreasing solution remains. Usually, even if $\mathrm{d}\psi(a)/\mathrm{d}a$ is selected such that the amplitude of the increasing solution is small at $r = a$, the screening factor begins to decrease as we move away from the dust particle, but the increasing solution begins to come into play at a certain distance and then dominates. Moreover, calculations may yield negative values of ψ, which indicates the presence of a numerical error and the need to increase the accuracy of calculations and reduce the size of numerical steps in calculations. This fact was not taken into account in Ref. [61]. In Ref. [61], the steps were decreased down to $10^{-6}-10^{-8}$ over the length of 7 to 9 dimensionless lengths. The calculation procedure used in Ref. [54] amounted to the following: by varying $\psi'(a)$, the decreasing solutions were extended as far as possible, and the distances were adjusted by varying the initial value of $\psi'(a)$. This was accompanied by reducing the step size (and increasing the accuracy), which was continued until the result ceased to depend on the step size and accuracy and the decreasing solution reached zero with the accuracy to which the calculations were carried out. Clearly, these distances are greater than those at which the nonlinearity becomes weak. Further calculations with the initial expressions, which assume that the nonlinearity is strong, were meaningless, because the goal was to find the distances at which the nonlinearity becomes weak. But finding the solution of the nonlinear equation is a problem in its own right, because the goal is to find the distances at which the nonlinearity becomes weak. The resulting curve was approximated by approximate solution (56a), and the effective nonlinear screening radius $R(\nu, \beta)$ was found:

$$\psi(r) = \left\{\left(1 + a\,\frac{\mathrm{d}\psi(a)}{\mathrm{d}a}\right)^{(1-\nu)/2} - \frac{r-a}{R(\nu,\beta)}\right\}^{2/(1-\nu)} \tag{61}$$

or, in dimensional units [see Fig. 1a for $d(\nu)$],

$$R(\nu,\beta) = \lambda_{\mathrm{Di}}d(\nu)\,\beta^{(1-\nu)/(3-\nu)}\,. \tag{62}$$

The numerical calculations were carried out with the parameter ν in the range from 0.1 to 0.9 and the parameter β from 2 to 100 ($\nu = 0$ and $\nu = 1$ are singular values because the polarization charge density is constant at $\nu = 0$ and the distribution is of the Yukawa type at $\nu = 1$). Calculations were done mainly with $a = 0.15\lambda_{\mathrm{Di}}$, which is the value corresponding to the parameters of many experiments, in which the ion number density is of the order 3×10^7 cm^{-3} and $a \approx 9-10$ μm. Calculations were then extended to smaller values of a, which in the limit $a \ll 1$ were found to weakly depend on a. The value of $d(\nu)$ found as a result of these calculations is shown in Fig. 1a, and the values of $R(\nu,\beta)$ are shown in Fig. 1b.

The numerical calculations showed that the fit of the exact solution and (61) may be done with an accuracy less than 0.1%. Figure 1 suggests that $d(\nu)$ increases with ν from 1.4 at $\nu = 0.1$ to 20 at $\nu = 0.9$. This result agrees with the one obtained in Ref. [61]: $d = 3.4$ at $\nu = 1/2$. The expression for R reproduces the value $R = 3.6(\beta)^{1/5}\lambda_{\mathrm{Di}}$ obtained above for $\nu = 1/2$. The results of the calculations fully agree with the fact that within nonlinear screening, the length $R(\nu,\beta)$ corresponds to the distance at which the nonlinear potential vanishes. This distance is much larger than λ_{Di}. For strong nonlinearity, at $\beta \gg 1$, the radius $R(\nu,\beta)$ increases both with $\beta \gg 1$ and with ν. It is important that at $\nu = 0.9$, the screening is close to the Yukawa type (which corresponds to $\nu = 1$). This is indeed so if we take into account that

$$\left(1 - \frac{2r}{(1-\nu)\lambda_{\mathrm{Di}}R_{\mathrm{Y}}}\right)^{2/(1-\nu)} \to \exp\left(-\frac{r}{R_{\mathrm{Y}}\lambda_{\mathrm{Di}}}\right)$$

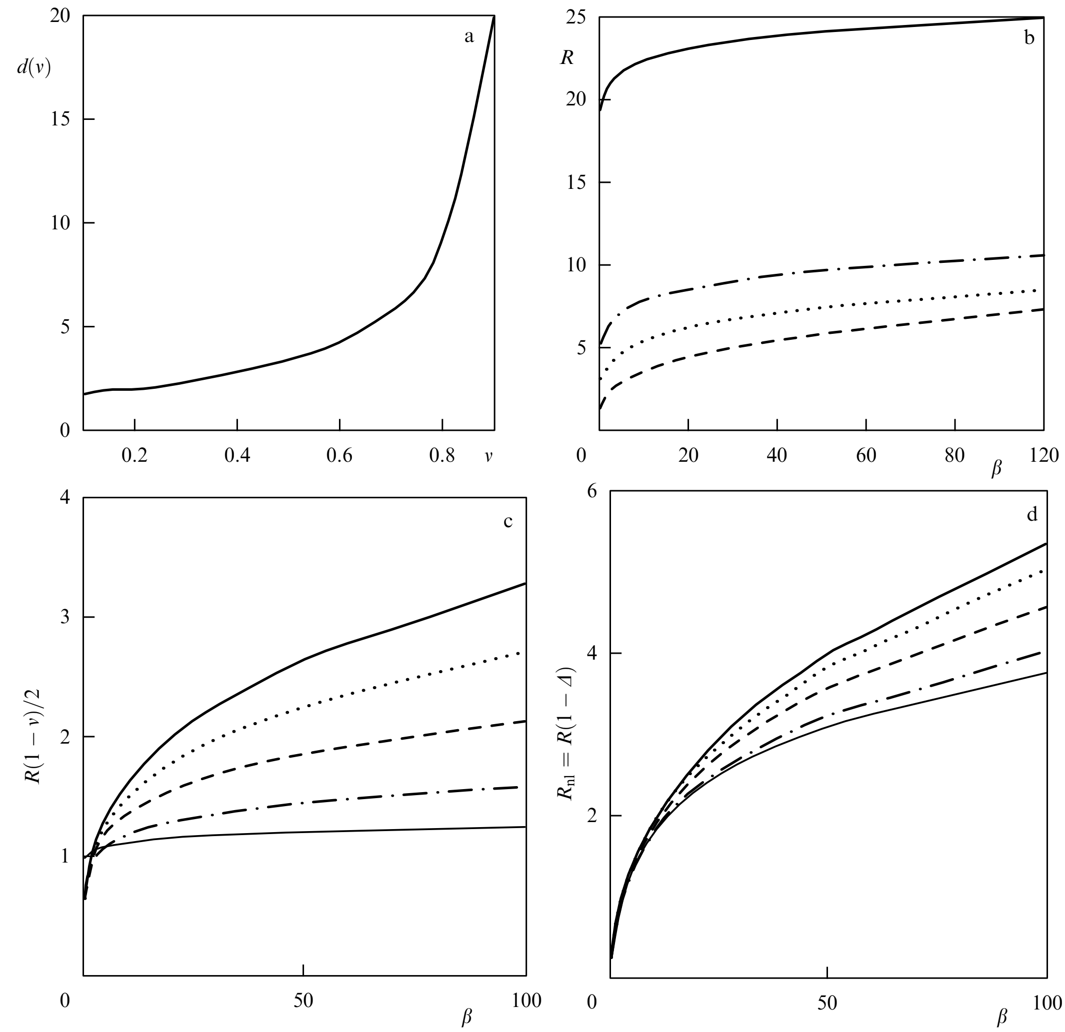

Figure 1. (a) The nonlinear screening coefficient $d(v)$ as a function of the nonlinearity parameter v; (b) the effective radius R in the nonlinear screening factor $\psi \approx (1 - r/R)^{2/(1-v)}$ (the approximate expression is valid for dust particles much smaller than the Debye radius, and the more exact expression is give in the main text), r and R are expressed in units of the ion Debye radius, the solid curve corresponds to $v = 0.9$, the dotted-dashed curve to $v = 0.5$, the dotted curve to $v = 0.3$, and the dashed curve to $v = 0.1$; (c) the same for $R(1-v)/2$ (this diagram shows that for v close to unity, the screening approaches the Debye type); and (d) the nonlinear screening radius $R_{\rm nl} = R\big(1 - \Delta(\beta, v)\big)$ corresponding to the situation where the ion potential and thermal energies are the same, and $\Delta = \Delta(\beta, v)$ satisfies the algebraic equation $\big(\Delta(\beta, v)\big)^{2/(1-v)}/\big[1 - \Delta(\beta, v)\big] = d(v)\beta^{-2/(3-v)}$.

as $v \to 1$. Hence, the more general type of screening of form (61) approaches the Yukawa screening only when v is close to unity. At $v = 0.9$, the Yukawa screening with $R_{\rm Y}(\beta)$ approximately describes the screening so long as r is not too close to R. To obtain the value $R_{\rm Y}$ for r not close to R, we must divide d by $2/(1-v) = 20$. The results of a comparison are shown in Fig. 1c, which suggests that at $v = 0.9$, the value $R_{\rm Y}$ is still a function of β and changes from $0.968\lambda_{\rm Di}$ at $\beta = 0.5$ to $1.245\lambda_{\rm Di}$ at $\beta = 100$. Still, the value of R differs somewhat from unity.

We note that Eqn (61), from the standpoint of using it to process the results of observations, is not more complicated than the Yukawa distribution and is determined by a single parameter (just as the Yukawa distribution is), the nonlinear screening radius. While fitting the observed results to the Yukawa screening requires two adjustable parameters, the effective exponential screening radius and the effective charge, using the more general factor (61) also requires only two constants, the effective nonlinear screening radius and the effective charge. The advantage of (61) is that this expression is more general and transforms into the Yukawa form as $v \to 1$.

Using Eqn (61), we can find the distances $r = R_{\rm nl}(\beta, v)$ at which the nonlinearity becomes weak and $e\phi/T_{\rm i} \leqslant 1$. Figure 1d depicts the results of numerical calculations of $R_{\rm nl} = R\big(1 - \Delta(v, \beta)\big)$, where $\Delta(v, \beta)$ satisfies a transcendental algebraic equation, which can be solved numerically for various values of v and β (naturally, $R_{\rm nl} < R$).

A simplified expression for the nonlinear screening factor $\psi(r)$ can be obtained in the limit where the screening radius is much larger than the particle size:

$$\psi(r) = \left(1 - \frac{r}{R(v, \beta)}\right)^{2/(1-v)}. \qquad (63)$$

In this expression, we can use the nonlinear radii found above via a more exact numerical calculation taking the finiteness of the dust particles into account.

4.5 Approximation of nonlinear screening by the effective Yukawa potential

In most existing experiments involving dusty plasmas, the length of the linear Debye screening of dust charges is much smaller than the length at which nonlinear process manifest

themselves (i.e., when $e\phi > T_{\rm i}$, with ϕ being the potential and $T_{\rm i}$ being the ion temperature). Hence, the screening of the charges of dust particles is mainly nonlinear. At the same time, even today, in many experimental works, the correlation effects are estimated on the assumption that screening is of the linear Yukawa type, that is, corresponds to the Yukawa potential $\phi \approx (q/r)\exp(-r/\lambda_{\rm D})$. Such use of the Yukawa screening is meaningful only if one can fit the real nonlinear potential to a certain effective adjustable Yukawa potential with an effective charge $q_{\rm eff}$ and an effective screening length $\lambda_{\rm eff}$ in the form $\phi \approx (q_{\rm eff}/r)\exp(-r/\lambda_{\rm eff})$. It is possible to decide whether such fitting is possible for an arbitrary nonlinear polarization density of the ion charge $\rho_{\rm ion} \propto e|\phi|^{\nu}$, $0 < \nu < 1$. It turns out that the approximation using the Yukawa potential works only up to distances from the dust charge that are no greater than a certain critical distance. This critical distance is shorter than the radius at which nonlinearities become weak. Under a further increase in distance, the discrepancy between the true nonlinear screening and its Yukawa approximation becomes large. At distances where nonlinear screening becomes weak, i.e., screening becomes linear, this discrepancy may amount to more than 240% for strong nonlinearity. In the range of distances for which nonlinear screening can be approximated by the Yukawa screening, the values of the adjustable parameters, the effective charge $q_{\rm eff}$ and the effective length $\lambda_{\rm eff}$, may differ substantially from the true charge q and the Debye screening length $\lambda_{\rm D}$. The discrepancies between nonlinear screening and its approximation by the Yukawa screening at large distances from the dust charge have a significant effect on the intensity of attraction of like-charged dust particles at large distances. Usually, in experiments, $\beta = za/\tau > 1$ and most often $\beta \gg 1$; to be exact, in most experiments, the parameter β varies from 5 to 100 and screening is strongly nonlinear. In this respect, it is amazing that the Yukawa screening is used for $\beta \gg 1$ [strictly speaking, such screening can be used only in the linear limit, where the potential of the screened particle is $(q/r)\exp(-r/\lambda_{\rm Di})$].

This could be justified only if the true nonlinear screening can be approximated by the Yukawa screening with the adjustable parameters $q_{\rm eff}$ and $\lambda_{\rm eff}$ (with the finiteness of the dust particles taken into account), i.e.,

$$\psi = q_{\rm eff}\exp\left(-\frac{r-a}{\lambda_{\rm eff}}\right). \tag{64}$$

But what are the conditions in which the true nonlinear screening potential can be approximated by the adjustable expression (64)? It turns out that depending on the type of nonlinearity, such an approximation works only up to certain small distances from the particle, and the smaller the distance, the stronger the nonlinearity. In the region where this fit is possible, the use of the Yukawa potential for processing observational data and finding the correlation functions or in comparing the data with theoretical approaches is justified. But our calculations show that the effective values of the charge $q_{\rm eff}$ and screening length $\lambda_{\rm eff}$ may differ significantly from the true charge and length of the linear Debye screening. Furthermore, beginning with certain distances, the errors in such fitting grow so fast that Eqn (64) ceases to be valid. The errors are especially large for strong nonlinearities and in the limit of distances where the nonlinear screening becomes linear. These distances play an important role in describing processes that are related to the influence of plasma flows in dusty plasmas on the interaction of dust particles, which leads to overscreening and to attraction between dust particles. In this way, the most important and characteristic effects in dusty plasmas cannot be described by the adjustable expression (64). Sometimes, however, Eqn (64) can be used. The specific values of the distances at which such fitting is still possible point to the possible range of applicability of the Yukawa approximation in processing observational data. Such fitting is done by numerical methods.

The best result of fitting have made it possible to do the following:

(1) find the two parameters $q_{\rm eff}$ and $\lambda_{\rm eff}$ as functions of two nonlinearity parameters in the range of a small distance from the charge, where such fitting is possible;

(2) find the distances at which this fitting yields large errors (larger than 20%);

(3) establish that the discrepancies between the exact nonlinear solution and the Yukawa approximation increases monotonically with the distance from the particle for all values of the nonlinearity parameters and that the stronger the nonlinearity, the greater the discrepancies; and

(4) determine the maximum difference between the exact nonlinear screening factor and the Yukawa approximation, $\Delta\psi/\psi$, for the maximum distances $r \approx R_{\rm nl}$ as a function of two nonlinearity parameters, β [see (48)] and ν.

The results of the calculations are shown in Fig. 2.

Figure 2a shows examples of fitting for a strong nonlinearity with $\nu = 0.1$, a Gurevich nonlinearity [51] with $\nu = 0.5$, and a weak nonlinearity with $\nu = 0.7$. Figure 2b shows the results for the maximum discrepancy between the screening factors for nonlinear screening and the Yukawa approximation at $r = R_{\rm nl}$. These discrepancies are most important in describing processes characteristic of dusty plasmas. Clearly, the discrepancies may be greater than 240% for a strong nonlinearity and greater than 130% for the Gurevich nonlinearity; even for a very weak nonlinearity with $\nu = 0.9$, they are substantial, about 12%. In addition, the discrepancies increase with β and are at their maximum for maximum values of β, which in the given numerical calculations amounted to 100. Figure 2c shows the effective screening length $\lambda_{\rm eff}$ as a function of two nonlinearity parameters. The results show that the discrepancies between the effective length and the Debye radius may be large, up to 2.5. Finally, Fig. 2d shows the dependence of the effective charge of the Yukawa fit on the nonlinearity parameter ν (the dependence on β is absent).

The above analysis suggests that the Yukawa approximation may be occasionally used with caution; the parameters of any specific experiment must always be checked for suitability. At high collision rates in low-temperature plasma, there is a condition where the parameter β moderates, ranging from 2 to 5, and where, according to Fig. 5, the Yukawa screening can be used as an approximation. It is also important that all the effects playing an important role in dusty plasmas and occurring because of the interaction of the electrostatic polarization fields and plasma flows at large values of β occur in the range of distances where the Yukawa approximation is inapplicable.

4.6 Ion scattering on a nonlinear potential of dust particles. The nonlinear drag coefficient and the friction force in nonlinear scattering [54]

When $\beta \gg 1$ [we recall that β is specified by Eqn (48)], the potential energy of the ion – dust particle interaction is higher than their mean kinetic energy up to $r = R_{\rm nl} > \lambda_{\rm Di}$.

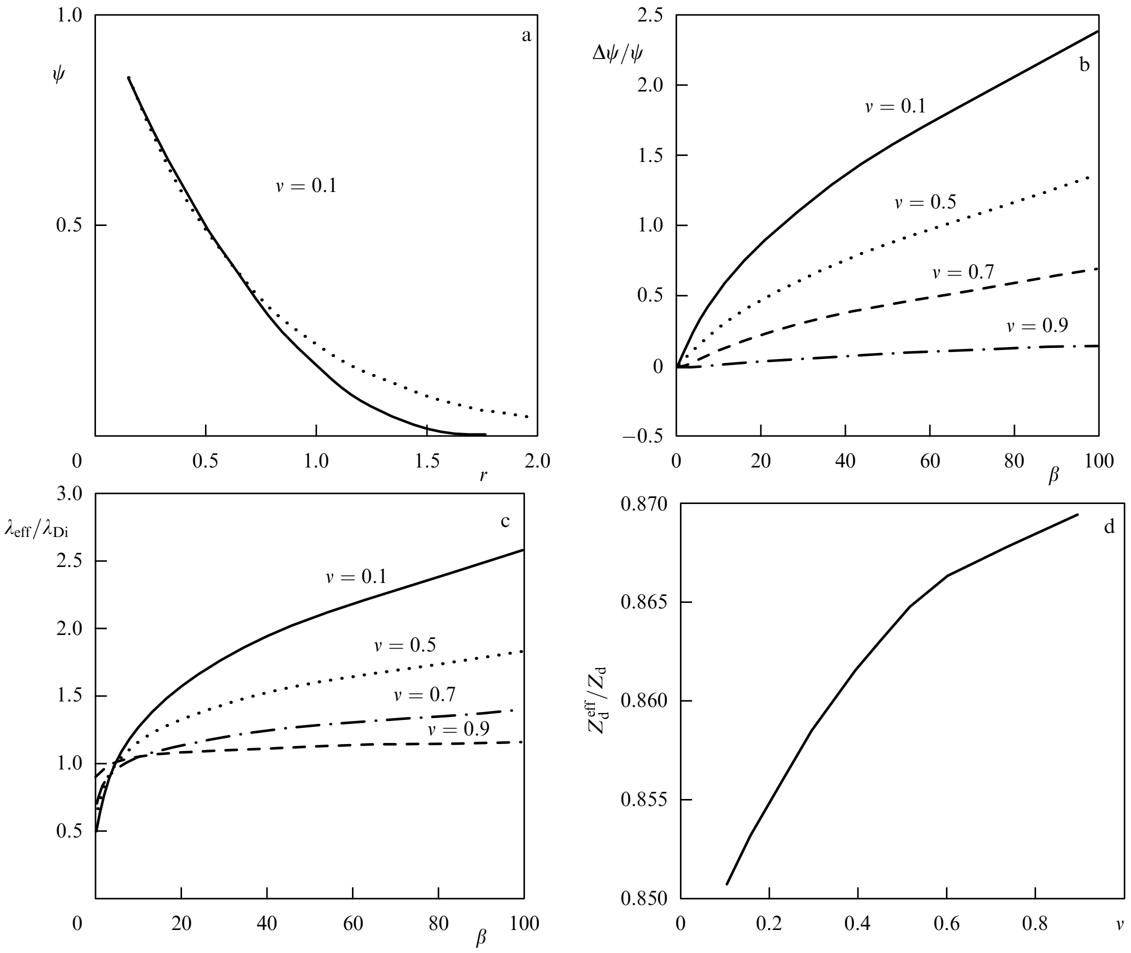

Figure 2. (a) Example of an approximation of the nonlinear screening factor ψ by the Yukawa factor at $v = 0.1$: the solid line represents nonlinear screening, the dotted line represents its approximation by the Yukawa screening (the radius is given in units of the Debye ion radius); (b) dependence of the relative deviation $\Delta\psi/\psi$ of nonlinear screening from its approximation by the Yukawa screening at $r = R_{\rm nl}$ as a function of the nonlinearity parameters β and v; (c) effective length of the Yukawa approximation, $\lambda_{\rm eff}/\lambda_{\rm Di}$, as a function of the two nonlinearity parameters β and v; and (d) effective charge of the Yukawa approximation as a function of the nonlinearity parameter v.

When the relation between the potential and kinetic energies is just the opposite, the scattering of the ion on the dust particle is small-angle, and is purely of the Coulomb type for $\beta \ll 1$ with impact parameters ranging from $\beta\lambda_{\rm Di}$ to $\lambda_{\rm Di}$. When $\beta \gg 1$, the main scattering is large-angle. Outside the range of impact parameters greater than $R_{\rm nl}$, the interval of distances where the linear effect operate remains small compared with $R_{\rm nl}$. For isolated particles, on which we now concentrate, this is obvious, because the interval of such distances must be of the order of $\lambda_{\rm Di}$, while $R_{\rm nl}$ is much larger than $\lambda_{\rm Di}$. But even if we account for many particles, when the interactions of polarization fields and plasma-flow fields become significant, this statement remains true, on the whole. The reason is that for $r > R_{\rm nl}$, the field of a dust particle is substantially weakened by nonlinear screening and is incapable of producing significant scattering. It must be taken into account that for ions of different energies, the radius at which the scattering ceases to be large-angle depends on the ion energy and corresponds to the situation where $|e\phi|$ is of the order of the ion kinetic energy $m_{\rm i}v_{\rm i}^2/2 = T_{\rm i}y^2$, where $y = v_{\rm i}/\sqrt{2}\,v_{T{\rm i}}$. This distance can be written as $R(v,\beta)\big(1 - \varDelta(v,\beta,y)\big)$, where $\varDelta(v,\beta,y)$ is the relative difference between the distance at which large-angle scattering occurs and the value of $R(v,\beta)$.

The equation for $\varDelta(v,\beta,y)$ that must be used in numerical calculations is given by

$$\frac{\big(\varDelta(v,\beta,y)\big)^{2/(1-v)}}{1 - \varDelta(v,\beta,y)} = y^2 d(v)\beta^{-2/(3-v)}\,. \tag{65}$$

It must be borne in mind that the right-hand side of Eqn (65) contains a constant of the order of unity. In finding the transport scattering cross section, we must account for the fact that the ion momentum changes, as a result of scattering by an angle of the order of unity, by $m_{\rm i}v_{\rm i}$ with a coefficient that varies from two (in strict backscattering) to a quantity of the order of unity (in scattering by large angles smaller than π). Collecting all the coefficients of the order of unity into a single coefficient η, we obtain the following expression for the transport scattering cross section:

$$\sigma_{\rm nl}(v,\beta,y) = \eta\pi\big(R(v,\beta)\big)^2\big(1 - \varDelta(v,\beta,y)\big)^2\,. \tag{66}$$

The total transport scattering cross section describes the friction force in the scattering of an ion flow on dust particles. When such a flow has a velocity much smaller than the thermal velocity of the ions, the distribution over the thermal

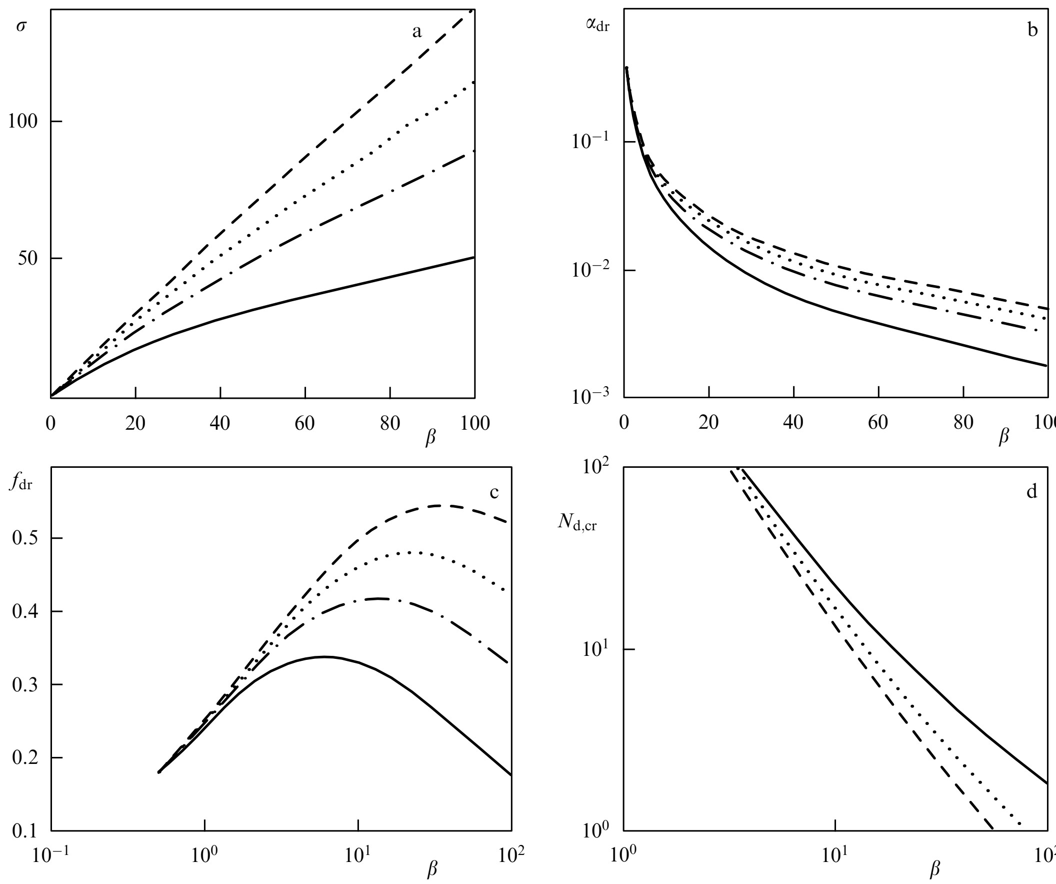

Figure 3. (a) Nonlinear transport scattering cross section σ of ions scattered by dust particles in units of $\pi\lambda_{Di}^2$ as a function of the parameter β for different values of v; (b) the same for the nonlinear drag coefficient α_{dr}; (c) the same for the nonlinear drag force $f_{dr} = \beta\alpha_{dr}$; and (d) the same for the total number $N_{d,cr}$ of dust particles inside the nonlinearity sphere of radius R_{nl}. In all figures, the solid line corresponds to $v = 0.9$, the dotted-dashed line to $v = 0.5$, the dotted line to $v = 0.3$, and the dashed line to $v = 0.1$.

velocities $y = v_i/\sqrt{2}\,v_{Ti}$ is given by

$$\mathbf{F}_{fr} \approx n_i v_{Ti} \sigma m_i \mathbf{u}_i = n_i T_i \sigma \frac{\mathbf{u}_i}{v_{Ti}}\,, \qquad f_{fr} \equiv \alpha_{dr}\beta\,,$$

$$\Phi_i(y,x) = \frac{2}{\sqrt{\pi}}(1+2yux)y^2 \exp(-y^2)\,, \tag{67}$$

$$\int_0^\infty \mathrm{d}y \int_{-1}^{1} \mathrm{d}x\, \Phi_i(y,x) = 1\,.$$

For the dimensionless friction force

$$\mathbf{F}_{fr} \to \frac{\mathbf{F}_{fr}\lambda_{Di}}{T_i}\,,$$

we have

$$\mathbf{F}_{fr} = -\beta\alpha_{dr} P\mathbf{u} = -f_{fr} P\mathbf{u}\,, \qquad \alpha_{dr} = \frac{1}{2\sqrt{2}\,\beta^2}\,\frac{\sigma(\beta)}{\pi\lambda_{Di}^2}\,,$$
$$\frac{\sigma(\beta)}{\pi\lambda_{Di}^2} = \frac{8\sqrt{2}}{3\sqrt{\pi}} \int_0^\infty \sigma(y) y^5 \exp(-y^2)\,\mathrm{d}y\,, \tag{68}$$

where α_{dr} is the drag coefficient for scattering on a nonlinear potential. In addition, it has proved convenient to introduce the effective cross section and the effective scattering radius using (66):

$$\alpha_{dr,nl}(v,\beta) = \frac{4}{3\beta^2\sqrt{\pi}} \int_0^\infty \sigma_{nl}(v,\beta,y) y^5 \exp(-y^2)\,\mathrm{d}y\,,$$
$$\sigma_{nl}(v,\beta) = 2\sqrt{2}\,\beta^2 \alpha_{dr,nl}(v,\beta) \equiv \pi R_{scat}^2(v,\beta)\,. \tag{69}$$

We note that for $\beta \ll 1$, the formulas yield the previous results for the drag force and the linear drag coefficient if the cross section of small-angle Coulomb scattering is used,

$$\sigma(y) = 4\pi\lambda_{Di}^2 \big(\beta(y)\big)^2 \ln\Lambda = \frac{\pi\lambda_{Di}^2\beta^2}{y^4} \ln\Lambda\,,$$
$$\alpha_{dr} = \frac{2}{3\sqrt{\pi}} \ln\Lambda\,, \tag{70}$$

where $\ln\Lambda$ is the Coulomb logarithm.

When $\beta \gg 1$, the results for the effective cross section, the drag coefficient, and the force of friction can be obtained only via numerical calculations that use the above formulas. The calculations encompassed strong nonlinearities with $v = 0.1$, Gurevich nonlinearities with $v = 0.5$, and weak nonlinearities with $v = 0.9$ (here, the Yukawa approximation holds). The value of β was varied from 2 to 100. Figure 3a shows the results of numerical calculations of the nonlinear scattering cross section.

For the sake of comparison, we note that in the linear approximation, where $\beta \ll 1$, the cross section increases in proportion to β^2. In the nonlinear case, the cross sections continue to increase with β, but not so fast. A more pictorial behavior of the drag coefficient in nonlinear scattering obtained as a result of numerical calculations with nonlinear screening, α_{dr} is illustrated by Fig. 3b, and that of the drag force $\alpha_{dr}\beta$ by Fig. 3c.

When $\beta \ll 1$, the drag coefficient was approximately constant or weakly (logarithmically) dependent on β, while

for $\beta > 1$ it rapidly decreases with increasing β. The friction force (the coefficient $f_{\rm dr}$) first increases with β (see Fig. 10) and then decreases, with its maximum at a certain value of β. The drag coefficient and the friction force increase with the nonlinearity. It is important to compare the drag coefficient and the friction force with the respective values at $v = 0.9$, when the potential of a dust particle is close to the Yukawa potential. To a certain extent, this case is of academic interest only, because the potential does not correspond to the linear Yukawa screening at large values of β. Chronologically, however, this case was examined earlier in [62], where large-angle scattering for the Yukawa potential was studied. On the other hand, as noted above, the criterion for large-angle scattering and the criterion that indicates when nonlinearities in the distribution of the polarization potential begin to play an important role simply coincide. Hence, they cannot be separated and the two effects must be considered simultaneously. Ignoring one effect underestimates the scattering cross section, the drag coefficient, and the friction force. Another important qualitative result is that the friction force does not increase linearly with β (as it does in the linear limit when $\beta \ll 1$), but for large values of β decreases with increasing β. This is important in a comparison with the experimental data, because not only this friction force but also the friction force in collisions with atoms of the neutral gas, which is independent of β, must be estimated. This means that as β increases, the force of friction on atoms of the neutral gas may dominate.

To summarize this section, here are the important aspects:

• An increase in the charge of dust particles needed for strengthening their interaction certainly leads to nonlinear screening. In all the experiments where the interaction of dust particles was very intense, the screening was strongly nonlinear.

• Only in a certain narrow interval of distance can nonlinear screening be approximated by the Yukawa screening, and the greatest discrepancies occur for distances where the screening becomes linear; the existing discrepancies by a factor of two to three occur when the effective length is modified by a factor of two, and if no such modification is done, the discrepancies are even greater. It is important that the derivatives of the nonlinear potential have much larger values in this range than in the case of the Yukawa screening.

• The criterion for the nonlinearity of screening and the criterion for large-angle scattering of ions are approximately the same, which dramatically changes the scattering pattern: for weak nonlinearities, the scattering is mainly small-angle, and for nonlinear screening, the main role is played by large-angle scattering.

• The dependence on β of the force of drag of the dust particles inflicted by the ion flow and the force of friction of ions on dust particles differs very strongly in the linear and nonlinear cases, and collisions with neutrals become more and more important as the nonlinearity of screening increases.

• All the above effects have been described only for isolated dust particles or in the case where there are many dust particles with a summation of the effects occurring independently on the dust particles, with plasma flows and their interactions with the screening electrostatic fields completely ignored.

4.7 A general discussion of the results. The role of collisions, external fields, and plasma flows

Up to this point, we have discussed the nonlinear screening of separate (isolated) particles. Various effects may alter such screening, and the main problem has been to find criteria when such alterations are possible. Among the effects important for understanding the experimental data are the presence of external fields and the resulting regular plasma flows, and the effect of collisions related to these flows, because regular flows occur as a result of the balance between external fields and collision-induced friction. We have discussed collisions in some detail when examining the possibility of particle trapping. Collisions still have to be theoretically studied in full, because, generally, examining their role requires the kinetic approach, which takes the specific conditions for dusty plasmas into account. In this section, we therefore limit ourselves to simple estimates and formulate the pressing problems.

Two important aspects come to mind here: (1) collisions cannot change thermal or Boltzmann distributions used to enforce nonlinear screening (the reason is that the collision integrals exactly vanish for thermal distributions), and (2) in most of the modern experiments in this area of research, the mean free paths for collisions are much longer than the nonlinear screening radius, and hence the nonlinearities are mainly collisionless, which justifies the use of the above results in the most interesting experiments. When the number density of neutrals is very high, i.e., at high gas pressures (for instance, in discharges at atmospheric pressure), the mean free path may be smaller than the described nonlinear length. In this case, hydrodynamic processes such as diffusion and viscosity may affect screening. So far, this problem has not been studied in full. Here, we discuss the question of how collisions can change the nature of nonlinear screening where the mean free path for collisions of ions with neutrals is much longer than the nonlinear screening length.

Collisions may affect screening if there is an external force maintaining the anisotropic distribution of ions or an ion flow. The simplest case is the presence of a uniform external electric field. If

$$E_0 \rightarrow \frac{eE_0\lambda_{\rm in}}{T_{\rm i}} \tag{71}$$

is the dimensionless force of an external field acting on ions far from the dusty particle and the collisions with a neutral are charge-exchange collisions and the charge-exchange rate is proportional to the ion speed (as is the case where the mean ion speed is much higher than the thermal speed) and, finally, all the ions have the same speed $u_0 \rightarrow v_{\rm i}/\sqrt{2}\,v_{\rm Ti}$, the balance between the electric field and the friction force yields [57]

$$E_0 = u_0^2\,, \tag{72}$$

which serves as Ohm's law in the system. In charge-exchange collisions, in contrast to ordinary collisions, in which the ion mainly changes the direction of its momentum, the ion becomes thermal after the collision, i.e., at mean velocities higher than thermal velocities, it loses almost all momentum. This leads to the conservation of the strong anisotropy of the ion distribution, which happens, however, only far from the dust particle. But how is this processes described in the kinetic approach? In equilibrium, the ion distribution is determined by an equation that involves only the electric field and

collisions with neutrals. We write this equation and its solution for the parallel component of the ion velocity, $v_{\parallel}/\sqrt{2}\,v_{T\mathrm{i}} = yx$, with $y = |v|\sqrt{2}\,v_{T\mathrm{i}}$ and $x = \cos\theta$:

$$E_0\,\frac{\mathrm{d}\Phi}{\mathrm{d}v_{\parallel}} = -|v_{\parallel}|\,,$$

$$\Phi(v_{\parallel}) \propto \exp\left(-\frac{v_{\parallel}^2}{u_0^2}\right), \qquad v_{\parallel} > 0\,, \tag{73}$$

$$\Phi(v_{\parallel}) = 0\,, \qquad v_{\parallel} < 0\,.$$

Here, we have taken into account that collisions depend only on the absolute value of the velocity and assumed that the longitudinal velocity is much higher than the transverse velocity. The solution of the kinetic equation [the first relation in (73)] can be found up to a constant that is finite for positive longitudinal velocities and must vanish for negative values of longitudinal velocities (the solution is increases unboundedly as $v_{\parallel} \to -\infty$). Bearing in mind that the ion distribution with respect to the ion velocity component perpendicular to the field is not affected by the external field and remains thermal, we can write the ion distribution as a function of the total dimensionless velocity $y = v/\sqrt{2}v_{T\mathrm{i}}$ and the cosine of the angle θ in relation to the field direction as follows:

$$\Phi(y,x) = n_{\mathrm{i}}\,\frac{2}{\pi^{3/2}u_0}\,\exp\left(-y^2(1-x^2) - x^2\,\frac{y^2}{u_0^2}\right), \quad x > 0\,, \tag{74}$$

$$\Phi(y,x) = 0\,, \quad x < 0\,.$$

Here, E_0 is expressed in terms of u_0 via Eqn (72). Distribution (74) is highly anisotropic, but it differs significantly from the Maxwellian distribution shifted by u_0. However, the mean value of the velocity along the field is $\sqrt{2/\pi}\,u_0$ and the mean value of the square of the velocity along the field, u_0^2, is close to the value for the shifted thermal distribution, although the spread in velocities is of the order of the velocity proper:

$$\sqrt{\langle v_{\parallel}^2\rangle - \langle v_{\parallel}\rangle^2} = u_0\,\sqrt{1-\frac{2}{\pi}}\,.$$

These properties are characteristic only of charge-exchange collisions. In [63], particular emphasis was placed on the strong 'smearing' of the ion distribution along the external field for charge-exchange collisions, but it was assumed that the collision rate is constant, which is true only for low drift velocities, much lower than thermal velocities. This means that the flow-related effects are weak and the more exact kinetic approach yields the same result as the thermal distribution shifted by the drift velocity. However, the opposite case described by distribution (74) is of interest here. So far, only the shifted Maxwellian distribution has been used at high drift velocities in dusty plasma physics. In particular, it was assumed that the drag coefficient α_{dr} for $\beta \ll 1$ decreases as $1/u_0^3$ when $u_0 \gg 1$. Here, we are discussing precisely the case of large drift velocities with $\beta \gg 1$ (this case has not been considered previously). We note that if we assume that charge-exchange collisions are predominant and (74) can be considered the correct distribution, then α_{dr} must also be corrected for $\beta \ll 1$. It is then easy to see that (for $\beta \ll 1$ and $u_0 \gg 1$) α_{dr} decreases as $1/u_0^2$ with increasing u_0 and $F_{\mathrm{fr}} \propto \alpha_{\mathrm{dr}}u_0 \propto 1/u_0$. This statement is significant if we use it to estimate the forces in interpreting the results of the existing experiments. Moreover, we must also estimate the friction force for $\beta \gg 1$ and $u_0 \gg 1$.

All this requires a careful study of the limits of distribution (74). This approximate distribution, valid for high drift velocities, may pose problems in view of the discontinuity of the solution at $v_{\parallel} = 0$. But the analysis in Ref. [64] shows that this is only the result of using the approximation $u_0 \gg 1$. This determines the need to find an exact solution, with no discontinuities. Even for $u_0 \gg 1$, the distribution changes its shape at $v_{\parallel}$ of the order of $v_{T\mathrm{i}}$, $u_0 \approx 1$, and begins to decrease with the parallel velocity, reaching values $1/u_0^2 \ll 1$ at $x = 0$, that is, smooth variation with a relatively small width $1/u_0^2 \ll 1$ occurs instead of a discontinuity in the distribution. Furthermore, there is still the problem of the stability of (74). When there are instabilities, elastic collisions may come into play, and this could lead to partial isotropization of the distribution.

We now examine a dust particle in the presence of an external field. With nonlinear screening, the field within the nonlinear screening radius is high, even higher than the external field, i.e., the ion distribution within the nonlinear screening radius is regulated by the field of the dust particle itself. Because the charge of a dust particle is spherically symmetric, the ion distribution within the nonlinear screening radius is isotropic, with the result that $\rho_{\mathrm{i}} \propto \sqrt{|\phi|}$ for the Gurevich potential with any isotropic distribution. Nonlinear screening is the same as the one described above for the Gurevich potential, but possibly with a certain effective ion temperature T_{eff}. The size of the potential of the dust particle at the edge of the nonlinearity sphere corresponds to the ion temperature or T_{eff}. The field of the particle at the edge of the nonlinearity sphere can be estimated at $eE \approx T_{\mathrm{eff}}/R_{\mathrm{nl}}$. This estimate corresponds to the smallest values of the almost perfectly screened field of the particle; naturally, as we move closer to the surface of the particle, the field increases substantially. According to (71), the external field is estimated as $eE_0 \approx (T_{\mathrm{i}}/\lambda_{\mathrm{in}})u_0^2$. If we use the minimum value of the field of the particle, we find that this field is predominant if $T_{\mathrm{eff}} > T_{\mathrm{i}}u_0^2R_{\mathrm{nl}}/\lambda_{\mathrm{in}}$. By the adopted condition, the right-hand side of this inequality contains the small parameter $R_{\mathrm{nl}}/\lambda_{\mathrm{in}} \ll 1$. Thus, to solve the problem, we must know T_{eff}.

At this point, it is important to note that outside the nonlinear screening radius, the responses are, by definition, linear. The relaxation of the ion distribution from highly anisotropic to isotropic occurs on the mean free path for collisions, when an ion distribution elongated along the external field is formed. In this region outside the nonlinear screening radius, the ion distribution is inhomogeneous. The kinetic approach acquires an additional term $v\,\mathrm{d}\Phi/\mathrm{d}r$ (here, we use dimensional variables), which can be compared with the term for which collisions are responsible, $\nu_{\text{ch-ex}}\Phi \approx v/\lambda_{\mathrm{in}}\Phi$, i.e., the characteristic relaxation length is of the order of $\lambda_{\mathrm{in}} \gg R_{\mathrm{nl}}$. On this fairly large length, the distribution changes from an isotropic distribution near the particle to a highly anisotropic one far from the particle. Here, we are confronted not only with the task of solving an inhomogeneous problem on the kinetic level but also with a problem related to the fact that strong variations in the ion flow distribution may occur in the linear region, where polarization fields are low. On the other hand, it is understood that the distribution of ions near a dust particle remains isotropic.

Logically, only two solutions are possible here: either the modification of the ion distribution extends to the nonlinear

region, where dust particle fields are high, or dust particle fields extend over distances much larger than the nonlinear screening radius that was calculated earlier for isolated particles. The second possibility has yet to be studied. However, we can immediately say that, actually, dust particle fields act over large distances from the particles if these dust particles are not considered isolated (this fact was stressed earlier); it is more realistic to consider them the constituents of a many-body system, where the interactions between plasma flows and electrostatic polarization fields become significant. The case of plasma flows moving at high velocities is the most complex one, with the result that the problem must be analyzed by first studying such interactions in the presence of nonlinear screening but in the absence of high-velocity external flows. Incidentally, we are presently unaware of explicit experiments involving flow velocities, and therefore we can only hypothesize how the polarization surrounding the particles changes when, on the grounds of independent ideas, we assume that the drift velocity of the ion flow changes significantly. We return to this problem in what follows.

We note that the problem of a high-velocity plasma flowing around large charges is extremely complex, both physically and mathematically, and has yet to be solved. The present ideas can be regarded only as preliminary ones, ideas that can help in solving it. We also note that the particle charge is usually determined from the condition that the potential on the particle surface is floating, i.e., the condition that the electron and ion flows to this surface balance each other. To complicate matters, the problem is not spherically symmetric. A simpler one would be the planar problem, but so far the problem of a plasma layer has not been solved, and therefore it comes as no surprise that the asymmetric problem has yet to find its solution. It must also be noted that in the simpler problem without external flows, the interactions between plasma flows and polarization fields at large distances with $r > R_{\rm nl}$ become linear. We described them in detail above with and without allowance for external fields leading to an ordered drift of ions.

The above material dealt with the hydrodynamic approach, which uses the various moments of the distribution function. Finding how these moments vary requires solving a kinetic (transport) problem, which is the goal of future investigations. The problem is simplified because the linear problem can be solved. If the interactions between plasma flows and polarization fields were not so significant, the field of a dust particle would extend outside the nonlinear screening radius into a thin layer of the order of $\lambda_{\rm Di} \ll R_{\rm nl} \ll \lambda_{\rm in}$. When such interactions are taken into account, the field of a dust particle extends much farther. Thus, the interactions between plasma flows and electrostatic fields must be taken into account in the entire transient region. This is also suggested by a comparison of the nonlinear screening radius and the dust particle separation done in Fig. 3d.

5. Interaction of electrostatic fields and plasma-flow fields in the presence of nonlinear screening

5.1 The nonlinear generalization

Generally, we must take nonlinearities in polarization fields and nonlinearities in the plasma flow field into account, i.e., we must use the exact relations between drift-velocity flows and the ion number density; for the latter, the Gurevich criterion for the contribution of only nontrapped particles to the ion number density must be used. The kinetic approach is more exact, but at the first stage, we can use equations for the moments of the distributions, the only variables that enter the Poisson equation and the continuity equation for the flows. An attempt at such a description was made in Ref. [65] in the case where ion–neutral collisions are negligible, the force of inertia of the ions is small (the flows are subsonic), and external electrostatic fields and the resulting regular flows far from the dust particles are also absent. As in the linear approach, the friction force is $F_{\rm fr} \propto -({\rm d}/{\rm d}r)(g/r)$, while the force of pressure in the equation for the ions is $({\rm d}/{\rm d}r)(\ln n/r)$. Moreover, we must not limit the analysis to strong nonlinearities, because the screening parameter passes through zero and the screening charge changes sign in the presence of attraction. Hence, when ψ is positive, we must use the general expression for the charge density and assume that there are no trapped particles and that all limitations on the energy range of the ions that may screen the dust particles are removed for negative ψ. The nonlinear density, which in the presence of strong screening $|e\phi/T_{\rm i}| \gg 1$ exhibits a square-root dependence on the potential $\rho_{\rm i} = en_{\rm i}I(x)$, $I(x) \approx 2\sqrt{x/\pi}$ with $x = -e\phi/T_{\rm i}$, is then replaced by

$$I(x) = \exp(x) + \left[2\sqrt{\frac{x}{\pi}} - \exp(x)\,\mathrm{erf}(\sqrt{x})\right]\frac{1}{2}\left(1 + \frac{x}{|x|}\right). \tag{75}$$

In addition, when ψ is small, we must take the contribution of electrons to the polarization near a dust particle into account. Then the balance equations for electrons and ions yield the following expressions (normalized to n_0) for the ion number density n and the electron number density $n_{\rm e}$:

$$\begin{aligned} n(r) &= I\left(\frac{\big(\psi(r) - G(r)\big)z_1 a}{r\tau}\right), \\ n_{\rm e}(r) &= (1 - P_0)\,I\left(-\frac{\psi(r)z_1 a}{r}\right). \end{aligned} \tag{76}$$

The Poisson equation and the continuity equation then become two coupled nonlinear equations:

$$\frac{{\rm d}^2\psi(r)}{{\rm d}r^2} = \frac{r\tau}{az_1}\left(n(r) - n_{\rm e}(r) - P_0\,\frac{z(r)}{z_0}\right), \tag{77}$$

$$\begin{aligned} \frac{{\rm d}^2 G(r)}{{\rm d}r^2} &= \left(\frac{{\rm d}G(r)}{{\rm d}r} - \frac{G(r)}{r}\right)\frac{\rm d}{{\rm d}r}\ln\left(\frac{z^2(r)}{n(r)}\right) \\ &\quad + k_0^2\,\frac{r\tau}{az_0}\,\frac{z^2}{z_0^2 n(r)}\left(\frac{n_{\rm e}(r)}{1 - P_0} - \frac{z(r)}{z_0}\,n(r)\right), \end{aligned} \tag{78}$$

where, as in the linear approach, $k_0^2 = (a/\lambda_{\rm Di})P_0\sqrt{\alpha_{\rm ch}\alpha_{\rm dr}z_0/\tau}$ is the coupling constant linking the electrostatic fields and the plasma-flow fields. The simplest form of the equation for the perturbed charge of dust particles is

$$\frac{{\rm d}z(r)}{{\rm d}r} = \frac{z(r)}{1 + z(r)}\left(\frac{1}{n_{\rm e}(r)}\frac{{\rm d}n_{\rm e}(r)}{{\rm d}r} - \frac{1}{n(r)}\frac{{\rm d}n(r)}{{\rm d}r}\right). \tag{79}$$

Clearly, these complicated nonlinear equations transform into the linear equations described above when all the plasma

parameters are weakly perturbed by a probe charge. In this sense, the nonlinear equations under discussion correspond to a simple generalization of the linear approach and are valid when the charges are large and, in particular, in the limit $\beta \gg 1$.

The first term in the right-hand side of Eqn (78) describes the additional nonlinearities caused by the dependence of the flows on the ion number density (this dependence can be ignored only in the linear approach). Here, we have written these equations only to show that it is generally possible to formulate nonlinear relations for describing the interaction of polarization fields and plasma flows. A direct solution of these equations primarily requires (as with any nonlinear equations) that we first analyze all possible features (especially singularities) and their physical interpretation. Such an analysis is a task for future researchers. So far, only some numerical solutions have been found in the range of not very small $\tau \approx 0.3$ (the most interesting values for applications are in the vicinity of $\tau \approx 0.01$); it has also been shown that nonlinearities do not destroy attraction, they only deepen the attraction potential wells [65]. A complete description requires scanning all the parameters of the problem and analyzing how the interaction of the particles changes. Furthermore, such complex nonlinear equations require using thoroughly worked-out numerical calculation procedures that do not take the interaction with flows into account. Allowing for nonlinearities only in polarization charges required a very finely tuned process in which both the increment in numerical calculations and their accuracy were refined. We can expect that equations that take the flows into account require just such a procedure. Only this procedure can guarantee that the answer with parameters that are of interest in modern experiments is meaningful and unique. This problem can be bypassed, however, by using certain physical assumptions that result in qualitative answers, which are the topic of the next sections. Here, we remark on the limitations of the hydrodynamic approach; a fuller answer could be obtained by using the kinetic description.

We also note that the problem of large charges being streamlined by external flows can be understood to a greater extent than the one achieved in the previous section. The total friction force must include the ion inertia force and the force of friction on neutrals $\approx (u^2 - u_0^2)$, where the term with u_0^2 describes the force exerted by the external field. The relations between the drift velocity and the flow are determined by more complex algebraic formulas, which nevertheless does not hinder generalization of the above system of nonlinear equations to the case with external flows. However, as mentioned above, the effective ion temperature depends on distance and the kinetic description is eventually more exact. But in view of the complexity of the kinetic description, there is at least hope that the hydrodynamic approach will produce a qualitative description.

5.2 Numerical solution of the problem under the assumption that the effects of interaction of polarization fields and plasma-flow fields at distances smaller than $R_{\rm nl}$ are weak in the presence of ionization proportional to the electron concentration

The problem of the nonlinear interaction of polarization fields and plasma-flow fields can be approached not from the mathematical standpoint but by using valid physical arguments supported by estimates. First, it must be noted that the nonlinear equations in the previous section allow a set of situations in which flows may be created by a dust particle itself. As is known, this is possible at high temperatures of the dust particles, which may be sources of plasma due to thermal emission and evaporation from the dust particle surface. We do not discuss such situations and limit ourselves to plasma flows that impinge on the dust particles and are absorbed by them. This is an ordinary situation at low dust-particle temperatures and with the existing plasma experiments, where the dust particles are at room (or close to room) temperature ≈ 0.02 eV and do not generate any flow. We begin with regions far away from the dust. Always, beginning at certain large distances, we can assume that the perturbations created by a dust particle are small (linear) and use nonlinear ideas concerning the interactions of polarization fields and plasma-flow fields. Such an approach was followed in the previous sections. It shows that up to certain distances, the value of g remains constant, but at larger distances, characteristic of absorption of flows by dust particles, the interaction becomes significant and changes not only the flows but also the polarization fields. The specified distances, as can easily be estimated, are larger than the nonlinear screening radius calculated above for individual dust particles. If we move from larger distances to the distance equal to the nonlinear screening radius, we find that at the nonlinear screening radius $g \ll 1$, for the strength of the ionization source independent of the electron concentration, Eqns (78) state that for $n \gg 1$, the value of g decreases as $r^3/R_{\rm nl}^3$ in approaching the dust particle from the distance equal to the nonlinear screening radius. Thus, while the value of g, which determines the plasma flows, is constant in the linear approximation, the flows play a less significant role in the nonlinear mode within the region of the nonlinear screening radius. This allows using the above solutions for the potential's distribution for nonlinear screening when all plasma flows are ignored. Naturally, at distances greater than the nonlinear screening radius, plasma flows must not be ignored. The most productive way to resolve this problem is to match the linear and nonlinear solutions at the nonlinear screening radius [66–69]. Matching necessarily involves a certain inaccuracy, but such an approach is justified by the fact that it leads to qualitative results. At distances equal to the nonlinear screening radius, we must write the continuity conditions for ψ and $\mathrm{d}\psi/\mathrm{d}r$. The values are determined by the properties of the nonlinearities, but not those on the surface of the charge. The reason is that the absolute value of $\mathrm{d}\psi/\mathrm{d}r$ is relatively large. As shown above, the linear screening factor for the ionization strength proportional to the electron concentration can be written as

$$\psi = A(\beta, v) \exp\left(-\lambda_1\left(r - R_{\rm nl}(\beta, v)\right)\right) + B(\beta, v) \cos\left(\lambda_2(r - R_{\rm nl}(\beta, v))\right) . \qquad (80)$$

We must bear in mind that we are examining the strong-nonlinearity case, and hence $\beta \gg 1$. This allows using the fact that for large β, the force of friction of ions in collisions with dust particles rapidly decreases as β increases, with the result that the force of friction of ions in collisions with neutral atoms is more important than the force of friction of ions in collisions with dust particles. Thus, the values of λ_1 and λ_2 are determined by linear relations, while the

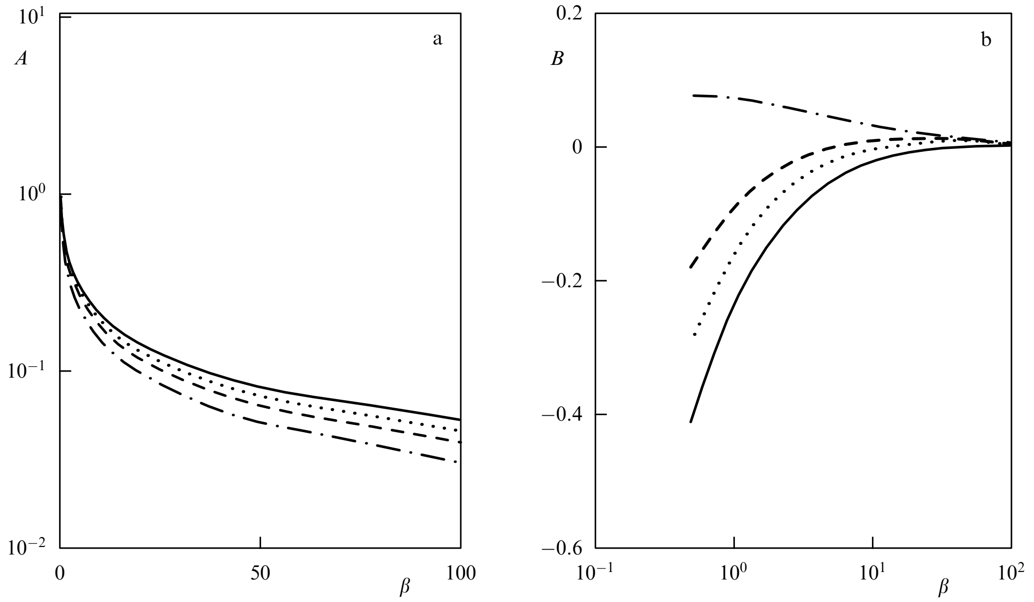

Figure 4. The coefficients A (a) and B (b) of collective nonlinear screening as functions of the parameter β; the solid lines correspond to $v = 0.9$, the dotted-dashed lines to $v = 0.5$, the dotted lines to $v = 0.3$, and the dashed lines to $v = 0.1$.

coefficients A and B are determined by the matching with the nonlinear screening potential (to be exact, A and B are determined by the continuity condition for the potential and the field at the nonlinear screening radius R_{nl}). These conditions yield

$$A(\beta, v) = \big(\Delta(\beta, v)\big)^{(1+v)/(1-v)}$$
$$\times \left[\frac{2}{1-v}\cos\big(\lambda_2 R(\beta, v)\big(1-\Delta(\beta, v)\big)\big)\right.$$
$$\left. - \lambda_2 R(\beta, v)\Delta(\beta, v)\sin\big(\lambda_2 R(\beta, v)\big(1-\Delta(\beta, v)\big)\big)\right]$$
$$\times \Big[\lambda_1 R(\beta, v)\cos\big(\lambda_2 R(\beta, v)\big(1-\Delta(\beta, v)\big)\big)$$
$$- \lambda_2 R(\beta, v)\sin\big(\lambda_2 R(\beta, v)\big(1-\Delta(\beta, v)\big)\big)\Big]^{-1}, \qquad (81)$$

$$B(\beta, v) = \big(\Delta(\beta, v)\big)^{(1+v)/(1-v)}$$
$$\times \left[\lambda_1 R(\beta, v)\Delta(\beta, v) - \frac{2}{1-v}\right]$$
$$\times \Big[\lambda_1 R(\beta, v)\cos\big(\lambda_2 R(\beta, v)\big(1-\Delta(\beta, v)\big)\big)$$
$$- \lambda_2 R(\beta, v)\sin\big(\lambda_2 R(\beta, v)\big(1-\Delta(\beta, v)\big)\big)\Big]^{-1}. \qquad (82)$$

Further calculations must be done numerically. The results of numerical calculations of these coefficients are shown in Fig. 4 at $\tau = 0.1$ and $a/\lambda_{\mathrm{in}} = 0.5$ and at relatively high ionization strengths, when $P_0 \approx 0.9$.

Figure 4 clearly shows that the effective charge remaining for collective interaction rapidly decreases as β increases. It is most important that **nonlinear screening leads to negative values of the coefficient B for $\beta \gg 1$**, which indicates that attraction may occur immediately outside the nonlinear screening radius, i.e., **at distances much smaller than in the linear case** ($\beta \ll 1$). The difference between nonlinear screening ($\beta \gg 1$) and linear screening ($\beta \ll 1$) amounts not only to the size of the potential at distances where effects of the interactions of polarization fields and plasma-flow fields begin to operate but also to the fact that the continuity of the potential and the field uniquely determines the parameters A and B and, in this way, fully determines g, the flow to the surface of a dust particle, and hence fully determines the plasma flow velocity $u \to u/\sqrt{2}\, v_{T_{\mathrm{i}}}$ directed to this surface. Using the calculated values of A and B, we can verify that this flow is small compared with the thermal flow and that collective interactions do not significantly change the dust particle charges.

To find the position of the first potential minimum of attraction and the depth of the potential well, numerical calculations that use the obtained values for the amplitudes of the collective interactions were carried out. Because B is negative, the first potential minimum of attraction can be located close to the nonlinear screening radius R, which is greater than the nonlinear screening radius $R_{\mathrm{nl}} = R(1 - \Delta)$. As β increases, the absolute value of B decreases and the distance to the first minimum may rapidly increase, because it is determined not by the negative value of B but by the change of the sign of $\cos(\lambda_2 r)$, which becomes negative only at large distances. Figure 5 illustrates this.

The binding energy, i.e., the value of the minimum in the potential well of attraction, determines the transition temperature or the energy needed for trapping a dust particle by the attraction well. This energy rapidly reduces as soon as the position of the minimum is determined by the sign of $\cos(\lambda_2 r)$. Hence, we present the results in Fig. 6 on two energy scales (Figs 6a and b).

The results shown in Fig. 6 imply that the trapping energy may be high if the potential well is located near R_{nl} (Fig. 6a) and is caused by attraction, which is responsible for a substantial change in plasma flows impinging on a dust particle (the coefficient B is negative). For very large values of β, the trapping energy is shown in Fig. 6b and is much lower than the trapping energy for smaller values of β. This means

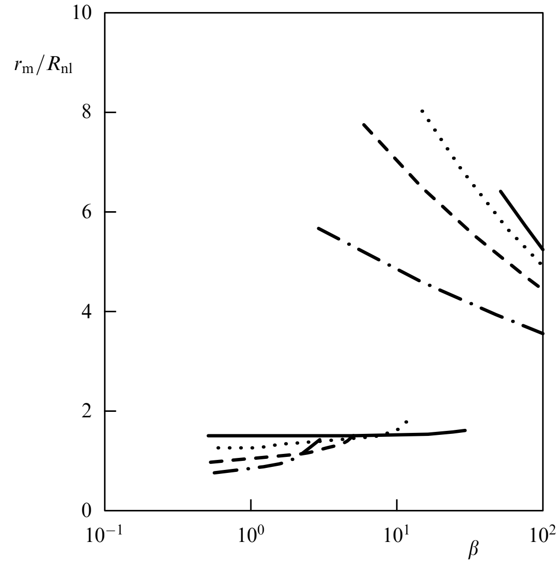

Figure 5. Dependence of the potential minimum of attraction of dust particles on the parameter β for a source of ionization proportional to the electron concentration.

that in experiments where β is smaller, there must be more durable dust crystals with a high transition temperature, and this fact is verifiable through experiments. It can also be assumed that the different types of crystal lattice observed in experiments in different parts of the discharge are related to the rapid changes in the position of the potential well of attraction caused by variations in β.

5.3 Numerical solution of the problem under the assumption that the effects of interaction of polarization fields and plasma-flow fields at distances smaller than $R_{\rm nl}$ are weak in the presence of ionization that is independent of the electron concentration

For ionization that is independent of the electron concentration, both the power balance [69] and the ground state of the dusty plasma change, with $\alpha_{\rm ion} = P_0/2\sqrt{\pi}$; $n_{{\rm e},0} = 1 - P_0$; here, the Havnes parameter of the ground state is completely expressed in terms of the ionization coefficient, defined in the same way as in the previous case. In the limit $\tau \ll 1$ and in the case where the ion friction force is determined by collisions with neutral gas atoms, both factors in the exponents of the collective interaction, λ_1 and λ_2, are positive:

$$\lambda_1 \approx \sqrt{1 + \frac{P_0}{1+z}}\,, \tag{83}$$

$$\lambda_2 \approx \frac{k_0}{\lambda_1}\sqrt{\tau\left(1 - P_0 + \frac{1}{z}(1+P_0)\right)}\,, \tag{84}$$

where

$$k_0^2 \approx P_0 \tilde{a}\,\frac{z}{1+z}$$

is the coupling constant in the case where the ionization source is independent of the electron concentration, $\tilde{a} \to a/(\sqrt{\pi}\lambda_{\rm in})$. In the linear approximation $\beta \ll 1$,

$$\psi = \psi_a \exp(-\lambda_1 r) + \psi_b \exp(-\lambda_2 r)$$

and both amplitudes are positive, $\psi_a > 0$ and $\psi_b > 0$, i.e., there is no attraction (but one of the exponentials decreases at distances much larger than the Debye radius). In the nonlinear limit, we write the collective interaction in a form in which the exponentials become equal to unity at the nonlinear screening radius (just as we did in the previous case):

$$\psi = A_c \exp\big(-\lambda_1(r - R_{\rm nl})\big) + B_c \exp\big(-\lambda_2(r - R_{\rm nl})\big)\,. \tag{85}$$

Continuity of the potential and the field at the nonlinear screening radius yields

$$A_c(\beta,\nu) = \Delta(\beta,\nu)^{(1+\nu)/(1-\nu)}\,\frac{2/(1-\nu) - \lambda_2 R \Delta(\beta,\nu)}{\lambda_1 R_{\rm nl}(\beta,\nu) - \lambda_2 R_{\rm nl}(\beta,\nu)}\,, \tag{86}$$

$$B_c(\beta,\nu) = -\Delta(\beta,\nu)^{(1+\nu)/(1-\nu)}\,\frac{2/(1-\nu) - \lambda_1 R \Delta(\beta,\nu)}{\lambda_1 R_{\rm nl}(\beta,\nu) - \lambda_2 R_{\rm nl}(\beta,\nu)}\,. \tag{87}$$

The results of numerical calculations of these coefficients (at the same value of the Havnes parameter as in Fig. 4 for an

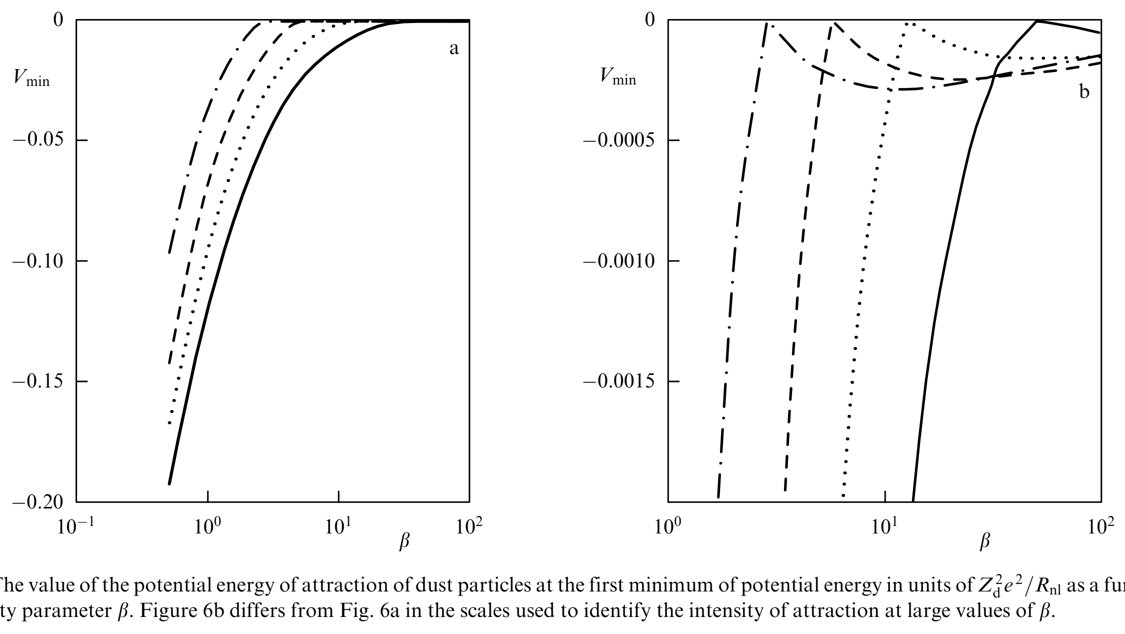

Figure 6. The value of the potential energy of attraction of dust particles at the first minimum of potential energy in units of $Z_{\rm d}^2 e^2/R_{\rm nl}$ as a function of the nonlinearity parameter β. Figure 6b differs from Fig. 6a in the scales used to identify the intensity of attraction at large values of β.

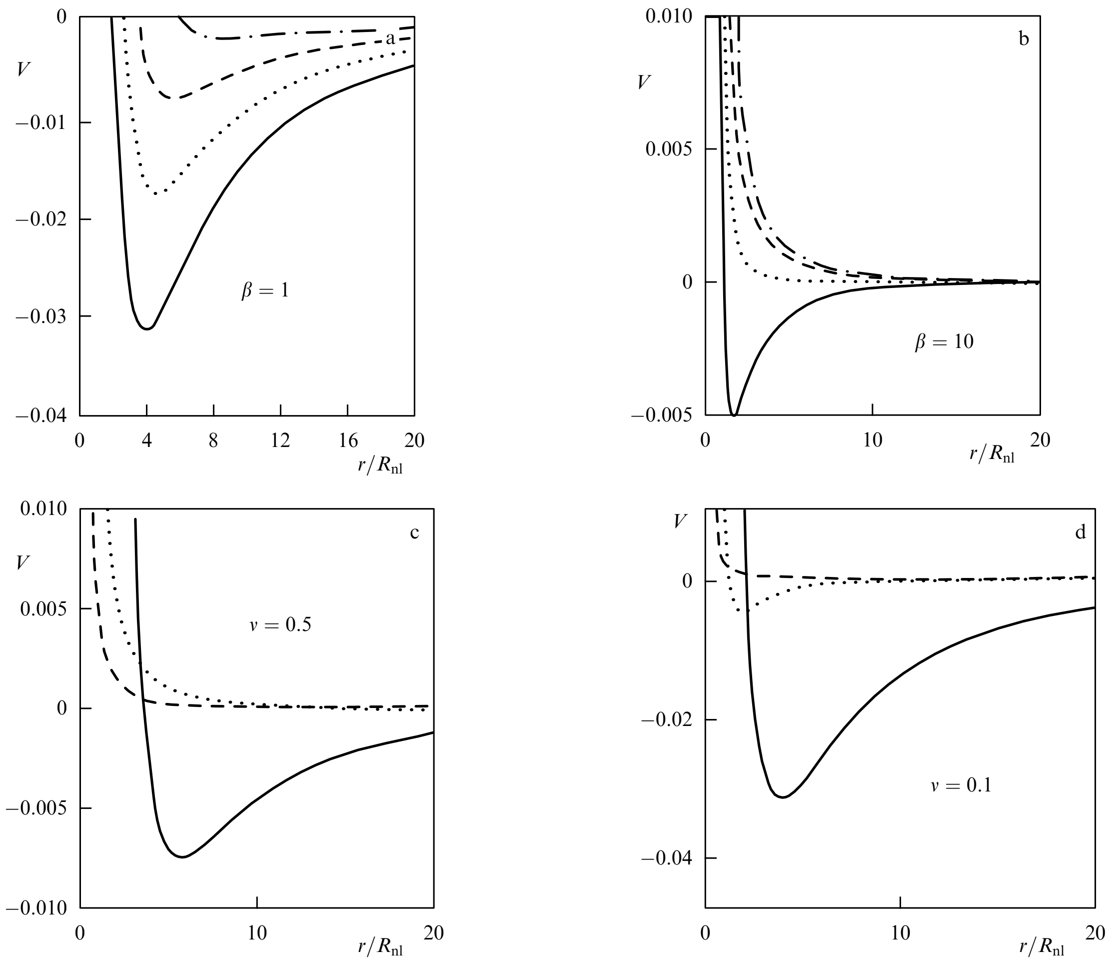

Figure 7. Dependence of the attraction potential energy in units of $Z_{\rm d}e^2/R$ on the distance in units of $R_{\rm nl}$ for a constant source of ionization. In Figs 7a and b, solid lines correspond to $v = 0.1$, dotted lines to $v = 0.3$, dashed lines to $v = 0.5$, and dotted-dashed lines to $v = 0.7$. In Figs 7c and d, solid lines correspond to $\beta = 1$, dotted lines to $\beta = 10$, and dashed lines to $\beta = 100$.

ionization source proportional to the electron concentration) are very similar to those in Fig. 4. The reason is that the values of λ_1 and λ_2 are close and the value of the term with the cosine in the interaction with an ionization source proportional to the electron concentration is close to unity (for the adopted numerical values of the parameters at the boundary), in the same way as the exponential in the second term is close to unity (the argument of the cosine in the previous results is $\lambda_2 R_{\rm nl} \ll 1$).

The fact that B is negative remains valid for a source independent of the electron concentration, but no jump in the position of the potential well should occur because there is no term with a cosine; instead, there are two exponentials and the potential well must vanish when β is large. At the same time, the fact that B is negative is important because it shows that in contrast to linear screening, where only two exponentials appear for a constant ionization source, there is always the effect of attraction of dust particles in nonlinear screening (we note that the presence of a second slowly decaying exponential term in the nonlinear collective interaction is also of considerable interest because a fraction of the charge of the dust particle is screened only at very large distances). Here, we give an example of the dependence of the potential on the distance when the ionization source is independent of the electron concentration; the example clearly shows the onset of attraction between dust particles (Figs 7a and b).

Figurers 7c and d show the dependence of the potential energy on the distance for a fixed value of v but different values of β.

The attraction potential well disappears as β increases and is practically absent at $\beta = 100$. Here, collective attraction for an ionization source that is independent of the electron concentration differs from that for an ionization source proportional to the electron concentration, where collective attraction emerges at practically any value of β.

As β increases, the attraction potential well becomes shallower, disappears at a certain value of β, and reappears only at $\beta = 100$.

Such behavior is also predicted by the numerical calculations of the position of the first minimum of attraction shown in Fig. 8a. The well disappears at the endpoints of the depicted curves.

Figures 8b and c demonstrate the results of calculating the energy at the potential minimum as a function of β. The potential well disappears when the energy at the potential minimum vanishes.

The above results clearly demonstrate the universality of the dust-particle attraction effects when $\beta \gg 1$. The case where the ionization source is proportional to the electron

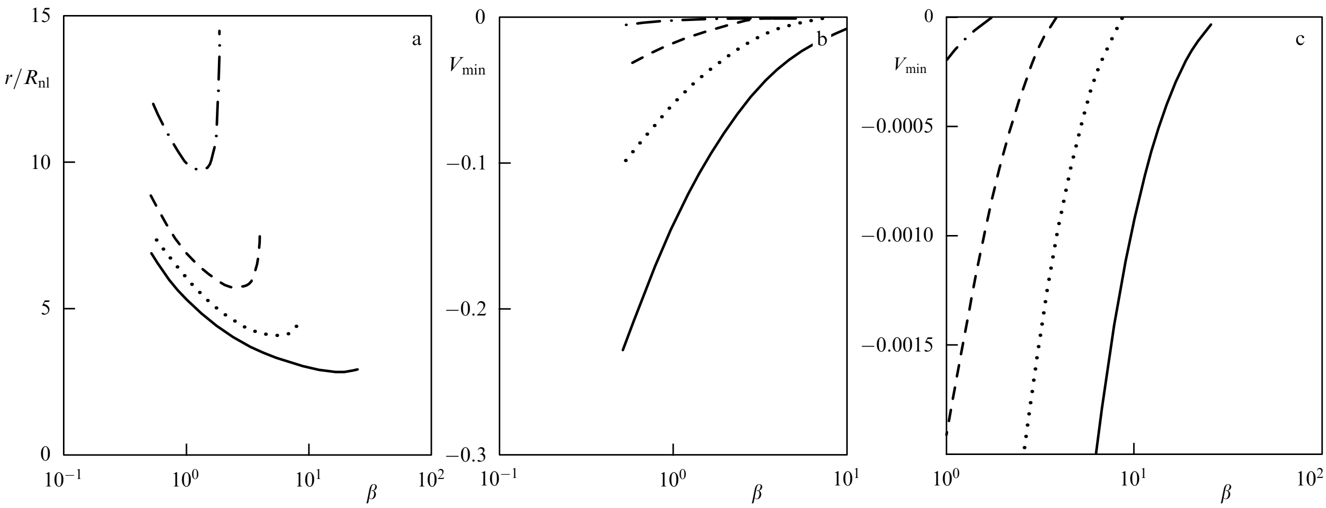

Figure 8. (a) Dependence of the position of the first minimum of attraction of dust particles on the parameter β; (b, c) dependence of the attraction energy $V_{\min}$ at the first potential minimum of attraction in units of $Z_{\mathrm{d}}^2 e^2/R$ on the nonlinearity parameter β. Figure 8c differs from Fig. 8b in the scales used to identify the intensity of attraction at large values of β.

concentration has more advantages over the case where the ionization source is independent of the electron concentration because in the former case the attraction potential well occurs at any value of β.

6. Pairing of dust particles and an explanation of the main parameters of plasma crystals

6.1 A general qualitative picture of the interaction of dust particles

All the previous material can be regarded as a set of examples that point to the presence of attraction of dust particles at large distances exceeding the nonlinear screening radius. Each example may correspond to certain limited applicability conditions, but the examples are numerous and, on the whole, give a general picture that leaves no doubt that there must be attraction between like-charged dust particles. This attraction occurs in the linear region, where mutual perturbations of two particles satisfy the superposition condition. This may serve as the main argument in the problem of mutual variation of the polarization of two interacting particles. Finally, on the basis of everything that has been said above, we can conclude that it is precisely the polarization field that describes the interaction of dust particles. Summarizing the above results and their possible development, we can schematically represent the potential of the interaction of two probe dust particles in a dusty plasma by the diagram in Fig. 9.

We temporarily concentrate on the general pattern of the interaction of like-charged dust particles, which repel each other at short distances and attract each other at a distance r_{m}. The previous analysis shows that $\psi_{\mathrm{m}} \ll 1$ and that r_{m} is greater than $R_{\mathrm{nl}} \approx (4-8)\lambda_{\mathrm{Di}}$ by a factor of two to three, with $\lambda_{\mathrm{Di}} \gg a$. Dust particles usually have moderate temperatures T_{d}, of the order of the neutral-gas temperature, due to friction with the neutral gas. It is natural to assume that the particles become localized in the attraction potential well, first forming bound pairs of dust particles and, later, long-range order and dust crystals. For the crystallization criterion, we can use the Lindemann criterion that the kinetic energy of dust particles is approximately equal to the depth of the attraction potential well [65], $Z_{\mathrm{d}}^2 e^2 |\psi_{\mathrm{m}}|/r_{\mathrm{m}} \approx T_{\mathrm{d}}$. We can introduce the coupling constant Γ as the ratio of the Coulomb energy of the interacting particles at their average separation r_{m} to the energy of thermal motion in the phase transition, $T_{\mathrm{d,cr}}$. The simplest way to estimate this quantity is by using the experimental data. The transition temperature is also related to ψ_{m}. The result is

$$\Gamma = \frac{1}{\psi_{\mathrm{m}}}\,, \tag{88}$$

$$\frac{T_{\mathrm{d,cr}}}{T_{\mathrm{e}}} = \frac{Z_{\mathrm{d}} z a}{r_{\mathrm{m}}}\,\psi_{\mathrm{m}} = \frac{Z_{\mathrm{d}} z a}{r_{\mathrm{m}} \Gamma}\,. \tag{89}$$

Thus, these relations contain parameters that can be measured in existing experiments. The maximum potential energy of the interaction, $V_{\max}$, is reached when the distance between the particles is of the order of their size a (the reduction of the charge of the particles as they move closer to each other is of the order of their charge at large distances if

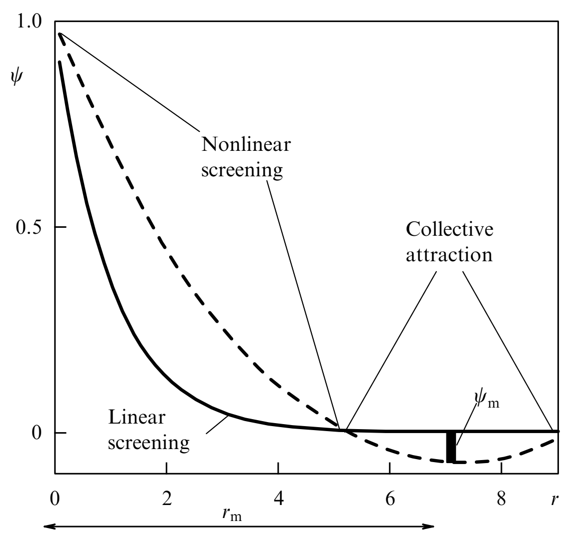

Figure 9. Schematic diagram representing the dependence of the screening factor on distance in the presence of dust-particle attraction.

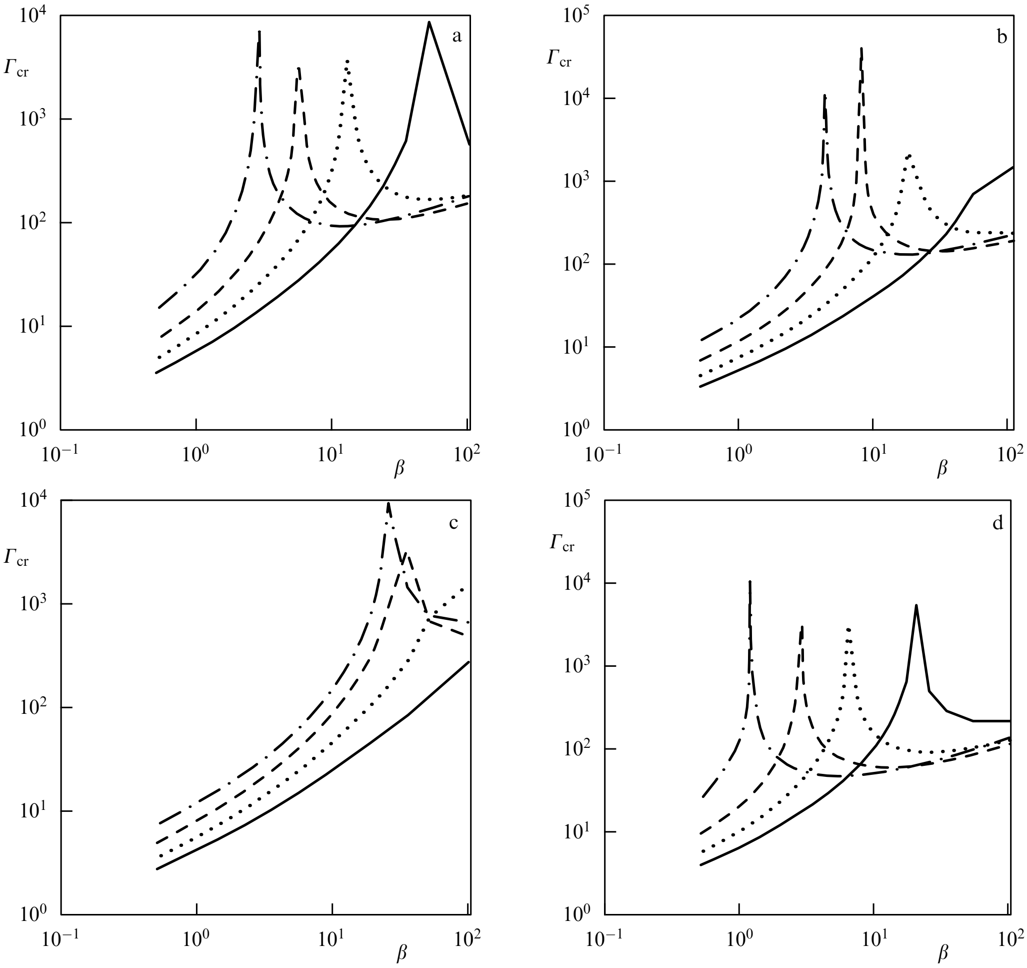

Figure 10. Dependence of the coupling constant Γ_{cr} on the parameter β for different values of the parameter of collisions with neutral gas atoms, the ionization coefficient, and the ion-to-electron temperature ratio values given in the text.

the distance is approximately a, which does not change the estimate of V_{max}), and $V_{max}/T_e \approx Z_d z$, i.e., $T_{d,cr}/V_{max} \approx (a/r_m)\psi_m \ll 1$ because either $a/r_m \ll 1$ or $\psi_m \ll 1$; it is usually assumed that $T_{d,cr}$ is much lower than the dust temperature before the phase transition, as well as the temperature after the phase transition (melting the crystal requires heating the dust component). The latter is related to the dissipation of the dust-component energy due to friction with the neutral gas. These ideas are presented here only in order to make another general statement: in contrast to the common belief that a strong interaction between the dust particles is required for crystal dust structures to form in a dusty plasma, the mean particle separations in the attraction model are so great that, **the interaction of these particles is weak on the average and manifests itself over distances at which the interaction of polarization fields and plasma-flow fields becomes a dominating factor**. In this sense, the idea that strong interactions are needed for the formation of dust crystals is also **a myth**. Equations (88) and (89) contain several parameters that make it possible to verify them by using experimental data.

6.2 Results of numerical calculations of Γ

The above results concerning the position and size of the attraction minimum in the model that uses the weakness of the interaction between flows and polarization fields over small distances from the dust particles allow using Eqn (89) to obtain the numerical dependences of the parameter Γ on the nonlinearity parameters β and ν. The results shown in Fig. 10 point to the fact that the values of Γ are high in this model, up to $3 \times 10^3 - 10^4$, a situation impossible with any model of strong interaction of dust particles [66].

Figure 10a depicts the dependence of Γ_{cr} on β for $\tau = 0.1$, $a/\lambda_{in} = 0.5$, and $P_0 = 0.9$; Fig. 10b, for $\tau = 0.001$, $a/\lambda_{in} = 0.1$, and $P_0 = 0.9$; Fig. 10c, for $\tau = 0.03$, $a/\lambda_{in} = 0.3$, and $P_0 = 0.25$; and Fig. 10d, for $\tau = 0.01$, $a/\lambda_{in} = 1$, and $P_0 = 0.25$. The results show how the coupling constant varies with the temperature ratio $\tau = T_i/T_e$ and with the pressure of the gas in terms of a/λ_{in} and the ionization degree α_{ion}. We note that peaks in the distributions appear because the potential wells, for which the negativity of the collective-interaction amplitude B is responsible, disappear, and the first potential minimum emerges only because of variations in the cosine in the collective-interaction amplitude. All the examples suggest that relatively large values of Γ_{cr} occur only at large values of the parameter β: the values of Γ_{cr} may become as high as $10^3 - 10^4$. Earlier theoretical approaches never predicted such large values of Γ_{cr} for $\beta \gg 1$, but such values of Γ_{cr} have been observed in experiments.

6.3 Comparison with experiments

The first experiments in which plasma crystals were discovered go back to the period from 1994 to 1996 [7–10]. Since the

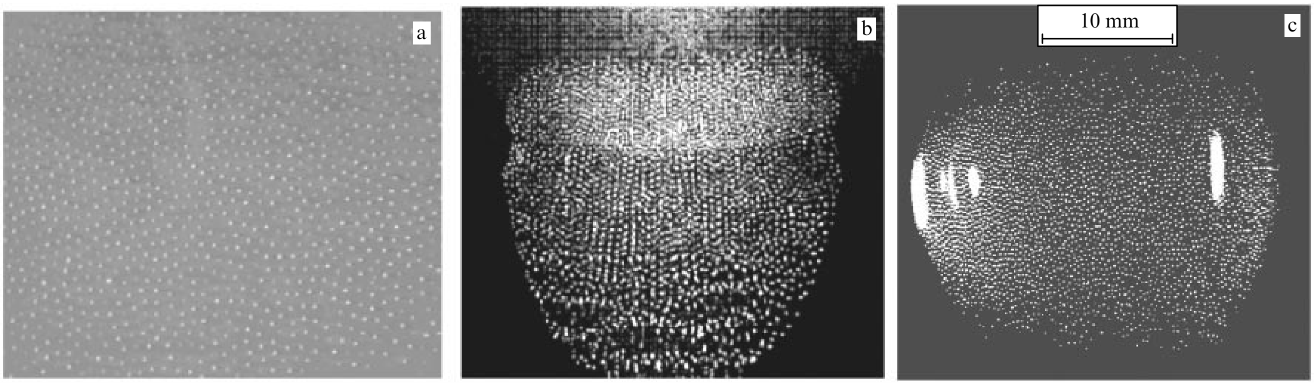

Figure 11. (a) Crystal structure of a dusty plasma crystal in the plane perpendicular to the vertical axis (the direction of gravity) in the laboratory experiment in [7]; (b) crystal structure observed in [10] in the laboratory experiment; and (c) crystal structure observed in [105] in the laboratory experiment. The vertical axes in Fig. 10b and c correspond to the direction of gravity in the laboratory experiments.

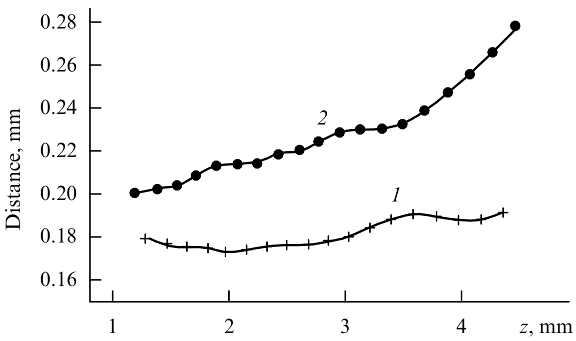

Figure 12. The distance between the neighboring planes in a crystal as a function of the vertical coordinate z. Curve *1* represents the dependence on z of the distance between the horizontal planes in Ref. [10], where the number of planes is 18 and the distance between them varies very slowly. Curve *2* represents the mean distance between dust particles in the crystal plane as a function of the vertical coordinate z.

late 1990s, plasma crystals have been studied in many experiments. In particular, the particle separation in the transitions, the dust temperature immediately after the melting of the dust crystals, and the value of the coupling constant Γ were determined. Figure 11 shows an example of a well-formed crystal structure in which the particle separation was a constant quantity in three laboratory experiments [7–10], and Fig. 12 presents the experimental data on the dependence of the particle separation on height in the Earth's gravitational field [7]. In conditions of microgravity, this distance depends only weakly on the positions of the particles in the crystal.

The first feature that attracts attention is the relatively low value of the transition temperature, about 0.1 – 0.8 eV, compared to the maximum energy of interaction when the particles touch each other. This occurs at the ion temperatures about 0.02 eV, which is much lower than the transition temperature; in this sense, the potential well is not shallow but the transition temperature is much lower than the maximum interaction energy [by the factor $Z_d T_e z r_m / a$)], which is of the order of 100 – 300 keV at $Z_d \approx 3 \times 10^3 - 10^4$, $z \approx 3$, and $T_e \approx 2$ eV. All this irrevocably suggests that particle interaction is weak at a distance of the order of r_m. Measuring the charge of dust particles has been the subject of direct experiments in conditions close to those that correspond to crystal formation and lead to values close to those used in the estimates. Although such measurements are not very accurate, it is hard to imagine the variations in Z_d amounting to several orders of magnitude. The estimates include the particle size, which is known to be ≈ 10 μm, and the distance $r_m \approx 200-250$ μm. Hence, the ratio $Z_d T_e z r_m / a$ can be determined accurately from experiments. The value of λ_{Di} is known with less accuracy, but the approximate estimate is 35 – 50 μm. This quantity, however, is present in estimates in a number of experiments and is used to compare the experimental and the theoretically predicted values of Γ. Such estimates account for the fact that the Coulomb field is screened, and for unknown reasons the screening is of the Yukawa type. Then

$$\Gamma \to \Gamma \exp\left(-\frac{r_m}{\lambda_{eff}}\right), \tag{90}$$

where λ_{eff} is a certain effective Yukawa screening length. The value of λ_{eff} obtained in this manner turns out to be 10 to 30 times larger than λ_{Di} and in no way corresponds to the value of the approximation of the true nonlinear screening by the Yukawa screening (we discussed this approximation above). For nonlinear screening, the experimental particle separation values can be matched to numerical results with satisfactory accuracy. As regards Γ, its theoretical values correspond to the unscreened Coulomb interaction, which can be determined fairly well from the value of the charge of dust particles and the observed value of r_m. Such a calculation yields $\Gamma \approx 3 \times 10^3 - 10^4$, which is in satisfactory agreement with the value expected from the discussed model of particle interaction. A discussion is needed here of the effect of constant ion flows over large distances between dust particles. Such a dependence is possible in experiments conducted in conditions of the Earth's gravity. However, flows exist near the discharge walls in the plasma sheath or at distances of the order of the mean free path in the presheath. The speeds of the ion flow in the presheath increases from zero to speeds of order of the speed of sound. But the mean free path is estimated at 200 – 300 μm, while the number of crystal layers varies from 10 to 20 with their separation being about 200 – 300 μm, and therefore only the first layers of the crystal may be in the field of the ion flows, while the theoretical estimates belong to regions far from the crystal

surface. Moreover, the data shown in Fig. 12 demonstrate only a weak dependence of the distances between crystal layers on height, while such dependence certainly exists if ion flows strongly influence the particle interactions, because the drift velocity should vary significantly over such distances. In addition, the interaction of flows and electrostatic fields becomes an important factor at distances much shorter than the mean free path in collisions of ions with neutral atoms.

There is repeated mention in experimental works of the fact that a plasma crystal in terrestrial conditions 'floats,' as it were, in the plasma sheath and essentially modifies it. It is stated that in the first layer, there is a sort of 'equilibrium' between the force of the electric field of the sheath and the force of gravity, while the next layers 'rest' on this layer. This behavior is not corroborated by experiments, but the presumed strong modification of the plasma sheath certainly exists. The main question, however, is how strong this modification may be. There are reasons to believe that in the presence of a crystal, the sheath is modified more strongly, such that the effect of the electric field of the sheath and the related external ion flow on the wall weakens considerably.

The arguments supporting these ideas are simple and are corroborated by numerical calculations [70–72]. First, we recall that a source of ionization generating the electrons and ions of the plasma exists in the discharge gap between the discharge walls. The walls are under a floating potential and a steady state emerges when the number of expected ion–electron pairs is balanced by their absorption by the walls, which become negative-charged (up to the floating potential) because of the high mobility of electrons and the wall field generates a plasma sheath with an ion flow toward the wall. This is the standard model of a plasma sheath. But if a dust cloud whose size is greater than the plasma flow absorption length forms in front of the wall, a redistribution of the sheath potential occurs. Indeed, if the dust layer (or a dust plasma crystal) is much larger than the ion mean free path in the flow, the dust layer serves as a similar 'wall,' only distributed in space. Thus, the equilibrium and complete balance of forces and plasma generation and absorption occur in the sheath in such a way that the sheath has a 'zero' plane where the flow changes sign, being directed toward the wall before this plane and toward the dust layer after the plane. Numerical calculations show that an equilibrium configuration is possible only if such a 'zero' plane emerges. This result was obtained for the planar geometry [72] and the spherical geometry [71] of electrodes within a broad range of possible parameters. According to the numerical calculations, the flow velocities of ions toward the dust layer are moderate, of the order of the thermal velocity of the ions. Of course, it would be ideal if there were experimental corroboration of this effect, but its physical meaning is obvious. There are very few experimental works in which the ion velocity distribution in plasma sheaths was measured. The best known is the one in [73], were the distribution of ions in the plasma sheath was measured in the absence of dust (by the resonance fluorescence method). It must be noted that measurements near walls are very difficult. So far, there have been no such measurements in the presence of a dust layer, and we can rely only on numerical calculations and the fact that the mean free path of the ions moving with a speed of the order of the ion thermal speed is of the order of a single crystal layer. Hence, ignoring the surface phenomena in dust crystals, we can say that the existing experiments irrevocably indicate that regular external plasma flows have little effect on the interaction of dust particles, on the distances between particles, and on transitions into the crystal state.

6.4 A discussion of other crystallization models and some experiments

It is extremely difficult to compare the above interpretation of how dust crystals form with other possible interpretations because none of them predicts all three main parameters of phase transitions. A numerical study of the criterion for crystallization with strong interactions in the one-component plasma model in [74] produced the value $\Gamma = 170$; to make it agree with the experimental data, the Yukawa screening model was employed, with the value of the factor in the exponent fitted with $\lambda_{\rm eff}$ such that the observed value of Γ would coincide with the predicted strong interaction. The researchers did not explain the other parameters. The obtained value of $\lambda_{\rm eff}$ was not explained on the basis of any reasonable model and turns out to be too large for any local screening when the Yukawa model is used. And if nonlinear screening is approximated by the Yukawa screening, the discrepancy between $\lambda_{\rm eff}$ and the observed values is several-fold.

We can use a purely phenomenological approach by taking the value of the screening length $\lambda_{\rm eff}$ from paper [75], where experiments with pair collisions of dust particles were performed. But as we have noted, the conditions in which the formation of crystals is observed are such that the system is much larger than the mean free path for the absorption of plasma flows by dust particles. In these conditions, we cannot speak of pair interaction of isolated dust particles. Moreover, the results in [75] have not yet received a clear theoretical interpretation, because the experiments were carried out in conditions close to the discharge wall in strong electric fields and, apparently, in strong ion flows to the discharge walls. In these experiments [75], there were strong ionization sources in the region where dust particles interacted, such that not only wall inhomogeneities but also charge-exchange collisions, which may change the nonlinearities [76], play an important role; we can therefore say that a complete theory interpreting the observations in [75] requires an analysis of many effects that, as a rule, are not very important for the problem of plasma crystals but could be of interest in interpreting these experiments (see Ref. [75]). Hence, it is impossible to use the results in [75] in the problem of plasma crystals.

We note that **not a single alternative approach, with the exception of that discussed in the previous section, explains the fact that the ratio of the maximum interaction energy at the melting point coincides with the value of Γ.**

Intricate experiments that directly involve a system of many particles are needed if the particle interaction potential that follows from the model of interaction of plasma flows and polarization fields is to be verified. There is, of course, the possibility of introducing some kind of 'tracer' particles into the system of dust particles, such that in all their main properties determining their interaction with flows, the tracer particles are identical to the other particles comprising the crystal but, naturally, their collisions cannot be used to determine the interaction potential. We could assume that the tracer particles differ in size (but not in weight, because the difference in weight is essential in terrestrial gravity, with particles of different weights gathering in spatially distinct regions), but the interactions of particles of different sizes

with plasma flows must be different. However, beginning with the experimental discovery of plasma crystals, there were indirect indications of the attraction of dust particles. This was first discussed in the experiments in [76, 77], where it was found that under the local knockout of dust particles of a plasma crystal by a laser beam, the particles moving along the beam always tend to become localized in the next possible equilibrium positions in the crystal lattice. The researchers also observed prolonged motion of an isolated dust particle around a cluster of dust particles (the total charge of the cluster was sufficiently large to generate a plasma flow toward the cluster strong enough for the necessary attraction force to be generated) [78]. Indications that attraction exists were later discussed in Refs [79, 80]. Suggestions for using helical dust structures that consist of a moderate number of large dust particles in a system of many small particles satisfying the condition of collectivity of plasma flows were made in Ref. [61]. However, such a system is capable of detecting effects related to flows in a dense system of small particles in which the large particles serve as probe particles and require conducting a large number of modes in microgravity experiments. It appears, however, that a new idea of an intricate experiment is needed to study the pair interaction of dust particles in a system of many particles. Of course, a detailed investigation of the vibrational modes of the crystals could also produce important information. Such experiments have already been conducted for 3D crystals [81], as well as 2D [82, 83] and 1D [84] crystals, but a theory of the vibrational modes with the interaction of plasma flows taken into account has yet to be developed, and therefore there is nothing we can compare with the theory (so far only the Yukawa screening has been used [82 – 84]).

6.5 The possibility of measuring attraction by the dispersion of dust sound

It is not essential to study the modes in a crystal state. The presence of attraction due to the interaction of plasma flows and electrostatic fields changes the dispersion equations for dust acoustic waves [85, 86], which acquire a form similar to that of the equations for gravitational instability,

$$\omega^2 = k^2 V_{\mathrm{daw}}^2 - 4\pi G_{\mathrm{eff}} n_{\mathrm{d}} m_{\mathrm{d}} \,, \tag{91}$$

where

$$V_{\mathrm{daw}}^2 = \frac{Z_{\mathrm{d}} P_0 T_{\mathrm{i}}}{m_{\mathrm{d}}\big(1 + P_0/(1+z_0)\big)} \tag{92}$$

is the squared velocity of dust acoustic waves, and the effective gravitational constant G_{eff} is given by

$$G_{\mathrm{eff}} = \frac{Z_{\mathrm{d}}^2 e^2 k_0^2 \lambda_{\mathrm{Di}}^2}{m_{\mathrm{d}}^2\big(1 + P_0/(1+z_0)\big)} \,. \tag{93}$$

This equation can be compared to the equation for ordinary gravitational instability,

$$\omega^2 = k^2 v_{\mathrm{s}}^2 - 4\pi G n m \,, \tag{94}$$

where G is the ordinary gravitational constant and v_{s} is the ordinary speed of sound in matter. Its estimate can be given for the typical experimental parameters of a dusty plasma [7 – 10]: $a \approx 10$ μm, $z_0 \approx 3$, $T_{\mathrm{e}} \approx 3$ eV, $m_{\mathrm{d}} \approx 2 \times 10^{-9}$ g, and $P_0 \approx 0.5$. Then, $G_{\mathrm{eff}} \approx 72.6$ dyn cm^2 g^{-2}, i.e., G_{eff} is approximately ten orders of magnitude larger than the ordinary gravitational constant $G = 6.67 \times 10^{-8}$ dyn cm^2 g^{-2}. For the above estimate, we used the value of the maximum drag coefficient for nonlinear screening, $\alpha_{\mathrm{dr}}\beta \approx 0.4$, with $\beta \approx 30$ – 50 (see Fig. 3b). Generally, Eqn (91) suggests that there is strong attraction when the distance between the dust particles is small. This attraction is different from true gravitational attraction because it acts only between dust particles, while ordinary gravity acts between any particles of matter. Just as with ordinary gravity, setting the frequency equal to zero in dispersion equation (91), we find the critical wave number at which the attraction becomes predominant and gravitational instability sets in for ordinary gravity, $k_{\mathrm{eff}} = 2\pi/L_{\mathrm{J,\,eff}}$, with

$$L_{\mathrm{J,\,eff}} = 2\pi\,\frac{\lambda_{\mathrm{Di}}^2}{a z_0 P_0}\sqrt{\frac{P_0\tau(1+z_0)}{\alpha_{\mathrm{dr}}\alpha_{\mathrm{ch}}(1-P_0)}} \tag{95}$$

being the effective Jeans length. This length is close, in the order of magnitude, or even somewhat greater than the particle separation in a plasma crystal.

A number of important questions arise in connection with these relations. Some can be answered immediately, while for others we can state the prospects for further investigations.

• Why so far has the relation $\omega^2 = k^2 V_{\mathrm{daw}}^2$ always been used (with the mass and temperature for ordinary sound replaced by the respective quantities) [40 – 42] instead of the correct expression for the dispersion relations for dust acoustic waves (91)? The answer is that from the general theoretical standpoint, a simple error was committed in Refs [40 – 42]: the initial state, small deviations from which determine the dispersion relation for dust acoustic waves, was defined incorrectly or, to be exact, was not defined at all. The researchers used only the quasineutrality condition, which, as discussed in detail at the beginning of this review, is not enough. The balance condition for the flows in the continuity equation must also be used, which was not done, and this made some sense only for a finite-size system smaller than the mean free path for flow absorption. However, in a bounded system, the system inhomogeneity must be taken into account, which was not done in Refs [40 – 42]. Generally, expansions in plane waves and wave vectors **k** have meaning in an infinite system but only approximate meaning in a spatially bounded system. Taking the inhomogeneities into account in a bounded system comprises an important chapter in plasma physics and has been discussed in detail in monographs (e.g., see Ref. [60]). The results discussed in Refs [40 – 42] make sense only to the extent to which they coincide with (91), i.e., for values of the wave numbers at which the effective gravity caused by particle attraction can be ignored. The inaccuracy in defining the ground state reveals itself in the absence of effective attraction. This inaccuracy was removed in Ref. [45], but Eqn (91) was not derived in the process because the force of drag of the dust particle by plasma flows generated by the perturbations caused by dust acoustic waves was ignored.

• The nature of the effective gravity caused by the force of drag of the dust particles in dust acoustic waves is very simple. Disturbances in the balance of flows manifest themselves in the flow continuity equation through the perturbations of dust and ion concentrations and the charge of dust particles. In this sense, the propagation of perturbations generated by dust acoustic waves is accompanied by plasma-flow perturbations. This implies that not only does the electric field act on the dust particles (as was assumed in Refs [40 – 42, 45]), but so

does the drag force, whose magnitude is determined by perturbations in the dust and ion density. In this way, we obtain a closed system of linear relations in which both density variations generate flows and the flows alter the density variations, which is described by Eqn (91).

• We note that relations similar to (91) were derived earlier in Ref. [44] for effects for which shadow attraction (the Lé Sage effect) with a much smaller effective gravitational constant is responsible. For many particles, shadow attraction is clearly applicable only when the flows of the different particles do not interfere with each other, i.e., the system is smaller than the mean free path for flow absorption, while Eqn (91) holds in just the opposite case. In this sense, the two cases are complementary to each other. On the other hand, the dispersion relation derived in Ref. [44] (and in Refs [40–42]) is applicable only for large wavelengths and spatially bounded systems.

• A question arises: Does the obtained gravitational instability serve as the initial stage in crystallization? This could be the case, but also possible is the formation of condensations and rarefactions of plasma with no crystallization in each condensation. Plasma flows then increase in the direction of formed condensations of dust particles: rapid redistribution of electrons occurs, while ions are attracted to condensations of negative-charged dust particles and generate flows that sustain or even amplify the inhomogeneities of dust particles. Such instability is universal and was first studied in Ref. [17] with all types of friction taken into account. It was found that there is always an instability, irrespective of the friction forces, at $k = 0$ [the same result follows from Eqn (91)] and it is universal in just this sense. Incidentally, Ref. [17] was the first paper that took the power balance in the equations for plasma flows into account and correctly defined the dusty-plasma ground state. The universality of this instability suggests that structurization of dusty plasma similar to the well-known structurization caused by gravitational instability is possible [17].

• Another question is: How is the Jeans length of the effective gravitational instability related to the attraction of dust particles connected with this instability? Of course, there can be no ideal correspondence, because the potential well is nonlinear and dust acoustic waves correspond to linear perturbations (calculations of the dust-particle polarization involve the spherical symmetry condition, while dust acoustic waves are plane waves). At the same time, the dependences on parameters such as λ_{Di}, τ, and P_0 are practically the same. It is not quite clear how to derive the pair interaction from the effective gravitational constant, but a relation that yields an estimate for the attraction potential energy can be used:

$$V = -\frac{m_{\mathrm{d}}^2 G_{\mathrm{eff}}}{r} = -k_0^2 \lambda_{\mathrm{Di}}^2 \frac{Z_{\mathrm{d}}^2 e^2}{r} \,. \tag{96}$$

For simplicity, we present this result in the limit where P_0 is of the order of unity. However, it coincides perfectly with the earlier formula for particle attraction if the interaction of plasma flows and polarization fields are taken into account, in the same way as Jeans length (95) coincides with the approximate position of the first minimum of the attraction potential well. This fact can be used (theoretically at least) to measure the attraction of dust particles if Eqn (91) is used for the dispersion of dust acoustic waves.

• But why in earlier experiments in detecting and measuring the dispersion of dust acoustic waves [87–89] was the attraction effect not measured? The answer to this question consists of two parts. First, the experiments were carried out with parameters that differed significantly from the parameters of experiments in which dust plasma crystals are observed today: they are 'suited' to measuring dust acoustic waves. Second, moderate variations in these parameters would allow reaching those values of wave numbers k. The size L of the device and the size a of the particles were about 3 cm and 0.6 μm, respectively, and the required criterion $L > \lambda_{\mathrm{Di}}\ (\lambda_{\mathrm{Di}}/aP_0)$ for $\lambda_{\mathrm{Di}} \approx 34-60$ μm was satisfied with a large margin at $P_0 = 0.5$ (values of P_0 of the order of unity can easily be achieved in the experiments) and seemed to be given by $3 \gg 0.6$. A distinctive change in the dispersion curve is shown by the dashed line in Fig. 13, which also shows the experimental points [87, 88]; a solid line represents the inaccurate theoretical curves.

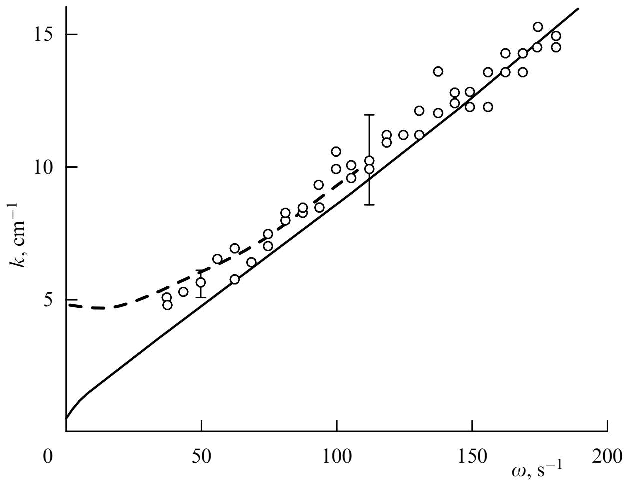

Figure 13. Frequency dependence of the wave number for the measurements described in Ref. [89]; the circles correspond to measured values, the solid line corresponds to the theoretical model adopted in Ref. [89], and the dashed line represents the dispersion curve that follows from the correct theory that takes the interactions of polarization fields and plasma-flow fields into account (this theory predicts that the frequency vanishes at certain finite values of the wave number, and the point at which the dispersion curve intersects the vertical axis allows experimentally measuring the intensity of attraction of dust particles).

Clearly, the experimental points exhibit a tendency to 'bend.' The dashed (correct) dispersion line ends at the extrapolation of ω to zero and yields a constant value related to the attraction of particles. It is advisable to use large particles if we want to fix the attraction more distinctly and to measure the constant k_0. For particles with sizes in the 5–10 μm range, the main criterion is satisfied only with high accuracy; moreover, the length $2\pi/k_0$ should be measurable. In the experiments described in Refs [87–89], the smallest measured value of k was 5 cm^{-1}. The value of k_0 can be estimated by various theoretical models. The linear screening model is the simplest one, but Eqn (91) is also valid for nonlinear screening. The estimate of k_0 for the experiments with $\beta \approx 0.4$ described in Refs [92, 93] is $k_0 \approx 1/\lambda_{\mathrm{Di}}\sqrt{(1/3\pi)\beta a/\lambda_{\mathrm{Di}}} \approx 3-4$ cm^{-1}, which yields wavelength values somewhat smaller than those corresponding to k in these experiments (about 5 cm^{-1}). If we assume that the small upward 'bend' in the observed curve in Refs [87, 88] at small values of ω is caused by the particle attraction, then in the limit as $\omega \to 0$ we obtain $k_0 \approx 3-4$ cm^{-1}. For larger dust particles (about 5–10 μm in diameter), the value of k_0 must

be close to 6 – 8 cm^{-1} and the attraction of dust particles can be accurately measured even if the variations in the parameters of the experiments are not very large.

The main conclusion that can be drawn is that in the experiments in [87 – 89], the attraction could not be accurately measured. We believe, however, that it is not very difficult to alter the conditions such that attraction can be measured with sufficient accuracy. For this, (a) large dust particles with $a > 2$ μm must be used, (b) the ion temperature must be increased as much as possible, and (c) plasma whose density is at least 10^9 cm^{-3} must be used to reduce the Debye screening length and increase the ratio a/λ_D.

With all this in mind, we nevertheless note that the estimates were done for a certain model of attraction and drag forces acting on a dust particle. It is possible that other models will yield weaker attraction and smaller values of k_0. One should anticipate the possibility of measuring the smallest values of k_0.

Crystal states can also be used to measure modes, but it should be recalled here that wave numbers of the order of k_0 correspond to the reciprocal-lattice length, and the role of the crystal lattice may be significant. Hence, we must first study the role of attraction in the modes of the crystal lattice of plasma crystals.

7. Surface effects and the role of regular plasma flows

7.1 Regular plasma flows and their generation by dust structures

The relation between regular and random plasma flows may be extremely complicated. The sources of external flows may be localized outside the plasma structure, but may also be created by the structure itself. If the flow sources and plasma sources are distributed fairly uniformly in the space occupied by dusty plasma and the flow mean free path is much shorter than the existing inhomogeneities or the size of the system, such flows rapidly become random within the system. If the flow sources are localized outside the system, the system is 'bombarded' by directed, regular external flows. Irrespective of whether there are regular external plasma flows, the plasma structures (aggregates of dust particles) must also generate external flows directed toward the structures (aggregates of dust particles). Indeed, in steady-state conditions, each dust particle absorbs a plasma flow of a certain magnitude to sustain its charge. As the number of particles increases in the aggregate, so does the plasma flow up to the point where the structure size exceeds the mean free path of the flow in the structure. It is natural to call a layer whose thickness is of the order of the mean free path the surface layer. Thus, the surface of any aggregate of dust particles of a finite size is subjected to the action of the plasma flows generated by this layer. Earlier, we thought of ionization processes as the mechanism for compensation of plasma loss on dust particles, and ionization was considered homogeneous. But if ionization is inhomogeneous and the plasma flows are generated outside the structure, steady-state dust structures emerge as a result of the generation of flows outside the structures being balanced by the absorption of these flows by the structure.

Research into the existence and balance of forces at each point in such structures was first done by numerically solving the exact system of nonlinear equations describing the balance of forces for ions, electrons, and dust, together with the equation for dust-structure charging. These balance conditions and the conditions for the existence of equilibrium structures were studied for planar structures [71, 72, 90, 91] and cylindrical and spherical structures [70 – 72] in the absence of ionization within their range of existence and with absorption of flows within the structures taken into account. This research immediately **showed that steady-state structures exist for certain magnitudes of the external flows impinging on the structure and that the size of such structures is determined by the mean free path of the flows actually generated by the structure itself**. These results appear to be quite natural. We mention them here only to stress two aspects. First, the fact that larger structures require ionization to occur within the structure and, second, that the surface layers of such structures are characterized by the fact that random flow fields existing with the structures become comparable to the regular flow fields generated by the structure near its boundary. In this surface layer, the attraction of dust particles, discussed earlier, does not fully operate and a shadow (Lé Sage) attraction becomes significant. Hence, the structure of the surface layers of both crystals and noncrystalline structures is complex and so far has not been studied sufficiently well either experimentally or theoretically. This is true primarily of the region where random and regular flows are of the same order of magnitude. It is obvious, however, that the presence in the surface layers of an additional external flow impinging on the structure and caused by the charge of the dust particles manifests itself as an effective surface tension. This is corroborated by observations in microwave discharge plasma [92] (Fig. 11a), in dc discharges [5, 93, 105, 107] (Fig. 11b), and in discharges at cryogenic temperatures [94] (here, the electrons have a temperature of several electronvolts, while the ions and the gas have low cryogenic temperatures) and in the observations described in Ref. [92], which is illustrated by Fig. 11c.

All this immediately points to a tendency of spherical structure formation and, hence, to the presence of the effective surface tension. To estimate the surface tension σ, we do not have to solve a complex surface problem; we should simply divide the size of the flow impinging on the structure by the flow mean free path, $\sigma = n_i T_i u^2 a P/\lambda_{Di}^2$, with $u = v_i/\sqrt{2}\, v_{Ti}$. The flow is determined by the flow speed, for which we can use the data of numerical calculations [95, 86], which suggest that (a) the flow speed in conditions of a small effect of ion – neutral collisions does not exceed the mean thermal speed of ions, and (b) in conditions of a large effect of ion – neutral collisions, the flow speed does not exceed the speed of sound. Here, we limit ourselves to this rough estimate of the flow speed in order to estimate the surface tension σ. There is also another estimate of the flow speed worth mentioning. As noted above, dust layers change the structure of plasma sheaths by generating flows directly toward the sheath. Hence, in terrestrial conditions, this corresponds to a vertical-stable equilibrium, where not only the flows and their direction change in the sheath but also the electrostatic fields of the sheath are altered. In the case of a plasma crystal, equilibrium is achieved by the combined action of (vertical) gravity and the flow, via the drag force, on the dust particles. There is also the question of horizontal balance (in addition to vertical balance under gravity). Naturally, we are speaking here only of surface layers, because in the inner region, crystals are formed by attraction forces, which become weaker as we move toward the surface, but also horizontal

flows are generated that can lead to self-confinement effects. There is certainly nothing remarkable in this, because examples of numerical calculations of plasma structures with distant sources suggest that such self-confinement is possible [90, 91]. When self-confinement is present, the boundary is free and the crystals can be called the free-boundary crystals. But are the crystals observed in laboratory experiments crystals with a free boundary related to the effective surface tension and is it possible to actually grow a plasma crystal with a free boundary?

It is worth mentioning that all the laboratory experiments conducted so far used weak confining fields, and yet estimates show that the forces related to these fields are of the same order of magnitude as forces of surface tension, i.e., strictly speaking, the observed plasma crystals cannot be interpreted as free-boundary crystals. There is one exception, however: the crystals observed by astronauts of the International Space Station (ISS) in microgravity [95] (Fig. 14), where there were no special fields directed at right angles to the wall of the discharge chamber.

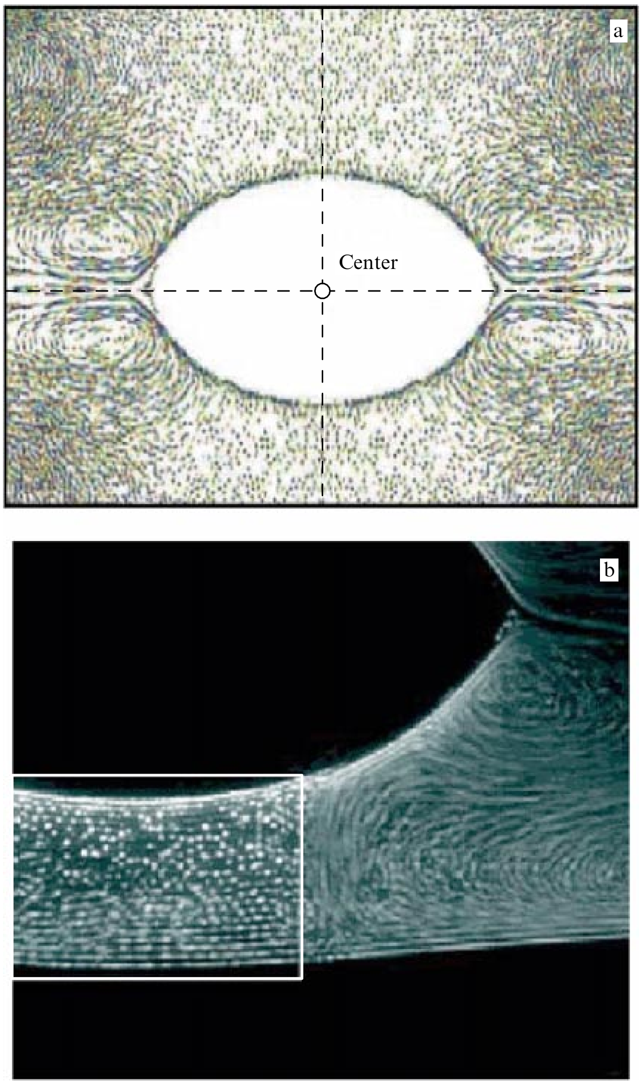

Figure 14. (a) The results of an ISS experiment in which a large dust void was detected, and (b) the magnified part of Fig. 14a where the formation of a dust plasma crystal was observed.

The distributions of flows and fields in the system were not measured in these experiments, and it is therefore difficult to decide whether confining fields in the transverse direction were absent. The direction toward the discharge wall was identified correctly, but no variations in the particle separation in relation to the distance to the wall were detected. This suggests that flows did not significantly alter the attraction of dust particles in this direction and that, most likely, the dust particles significantly changed the structure of the plasma sheath. In the transverse direction, such changes are fixed and, possibly, the observed crystal was a crystal with free boundaries. Figure 14a also depicts a large central void; the mechanism of formation of such a void is discussed below.

7.2 The physics of dust-void formation and the mechanism of formation of sharp boundaries of structures

A dust void is a structure that contains not a single dust particle. The most probable physics of dust-void formation is that the dust particles are swept away by plasma flows. Dust voids for very small ($< 0.1-0.3$ μm) dust particles were first observed in [96]. Importantly, the voids had **very sharp boundaries**. Even in the preliminary stages of their experiment, the researchers found that the numerical solution of the problem of balance of dust structures in Ref. [91] (which was nearing completion) exhibited the absence of solutions without sharp boundaries for dust-void structures. This fact led to collaboration and the first paper on the physics of dust-void formation and on structures with sharp boundaries [97]. Such discontinuities differ dramatically from all previously known discontinuities, because only the dust density or its derivatives have a discontinuity at the surface, on one side of which there is no dust. It must be noted, however, that the first to formulate the problem was Goree [98], who in 1999 (before the three main papers [90, 97, 99] were completed) presented a report to the American Physical Society (without any mention of the sharpness of boundaries, however). Today, the physics of dust-void formation seems very simple. With ionization in the central region of a dust formation, there emerges a plasma flow from this region that sweeps the dust out, due to drag; this amplifies the ion flow from the central region up to the point where the full balance of forces is restored. This full balance determines the size of the dust void. Numerical calculations [97, 71] have corroborated the sharpness of the void boundaries, but void size obtained via numerical calculations has proved much smaller than the one observed. Various attempts have been made to achieve agreement. In the process, observations of dust voids have become numerous, and especially interesting was the explanation of the central voids in the ISS experiment [95] (see Figs 14 and 15) (Fig. 14 shows the dust void observed in experiments with dust particles of equal size, and Fig. 15 shows the distribution of dust particles of two sizes).

The first possible explanation was that voids are caused not by flows but by a thermophoretic force, but this did not agree with the sharpness of the boundaries and estimates of thermophoretic forces in the experiment [96]. The second possibility was to assume that ion – neutral collisions play a significant role. These collisions do not enter the balance of forces for dust particles, but could indirectly affect the void size through nonlinear interactions. This assumption proved partially true, but it did not fully explain the void size in the ISS experiment [95]. Finally, the third assumption made in [101] was that for large particles used in the ISS experiment, taking large-angle scattering into account may substantially

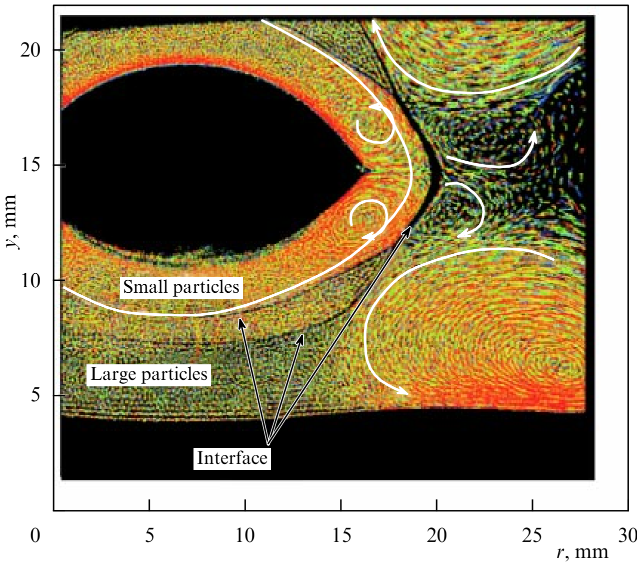

Figure 15. Results of the ISS experiment with dust particles of two sizes. The sharp boundaries of the void were predicted in Refs [90, 91], and the sharp and narrow boundary between particles of different sizes was explained on physical grounds and as a result of numerical calculations in Ref. [70]. The results illustrate the generation of multiple dust vortices.

increase the drag force. We note that paper [101] was the first in which processes of large-angle scattering by large charges of dust particles were discussed in quantitative terms. Before we describe this approach, we must explain how flows may create sharp boundaries [102]. The flows meet the first layers of a plasma formation, where they are partially absorbed, with the result that a smaller drag force acts on the deeper layers, and this continues until the flows are completely absorbed. Hence, the outer surface layers of dust move faster than the inner layers and compress the surface layer, making it sharp. Hydrodynamic estimates show that the thickness of this layer with respect to the void size is of the order of $1/Z_{\rm d} \ll 1$, because $Z_{\rm d} \gg 1$ in most experiments [90, 91]. Thus, the existing approaches use this obviously 'crude' parameter and the balance between pressure forces and drag forces, without going into the details of the kinetic description, which may lead to weak 'halos' of fast ions and electrons near the boundary, with their nonthermal distribution taken into account, because of acceleration on dust particles (the problems of the kinetic description are discussed in the last section).

7.3 A discussion of the various approaches to void formation

The most important statement made in Ref. [101] was that for charges $Z_{\rm d} \approx 10^3 - 10^4$, the so-called Coulomb radius, i.e., the distance over which the kinetic energy of ions is of the order of the Coulomb energy of the interaction with dust particles, is much larger than the Debye screening length $\lambda_{\rm Di}$, and therefore large-angle scattering for the impact parameters much larger than the Debye radius becomes important. This statement is true and was first formulated in Ref. [101]. Later, however, the Yukawa screening was used, although it is inapplicable in the case where the interaction energy is much higher than the kinetic energy and large-angle scattering occurs. A complete theory with nonlinear screening taken into account has been discussed earlier in this review and yields a value for the drag force for $\beta \gg 1$ that is approximately two to four times higher than for $\beta \approx 1$ [54, 55]. This is sufficiently high if we want to explain the results of observations of voids at the ISS and is larger than the result provided by Yukawa screening, which cannot be used anyway in the case under consideration. To avoid any misunderstanding, we mention two inaccuracies in the statements made in Ref. [101] that refer to the comparison with the fictitious drag coefficient obtained by extrapolating the results for $\beta \ll 1$ to the case where $\beta \gg 1$ (incidentally, this method was used by most researchers before Ref. [101] appeared). For $\beta \ll 1$, the result depends on the Coulomb logarithm, which can be calculated in different ways: (1) by examining the collisions of individual particles, when extrapolation to $\beta \gg 1$ yields a drag force that tends to a constant value, while the true drag force is much larger than the fictitious one obtained through extrapolation; (2) by using the Landau – Balescu Coulomb logarithm, where extrapolation yields a result for the drag force that is proportional to $1/\beta$, which is much smaller than the true value; or (3) by using that for dust particles, even when $\beta \ll 1$, there is a small range of impact parameters smaller than $\lambda_{\rm Di}$ for which large-angle scattering occurs and extrapolation yields a drag force proportional to β and the correct value of the drag force smaller than the one obtained by extrapolating its value for $\beta \ll 1$. The last case is the correct one and the nonlinearity always reduces the drag coefficient. Actually, various incorrect extrapolations have been used by different researchers, including the extrapolations in Ref. [101]. All this is not very important when compared to the fictitious, incorrect values for the drag force. According to Ref. [54], using the correct drag force with nonlinear screening leads to satisfactory agreement with the observed void sizes.

In such a comparison, not only the effect in [101] with the nonlinearity of screening taken into account [54] but also the dependence of the drag force on the ion drift velocity is essential. For the dust-void region, the numerical calculations in Refs [93, 99] always show that both the electric field strength and the drift velocity, which determines the ion flow, increase rapidly with the distance from the center of the void. The two forces act in opposite directions and the electric field increases faster from the center of the void, such that equilibrium always exists in all numerical calculations even without refining the drag force; it was the equilibrium that determined the drag force [54], which among other things strongly depends on the drift velocity at the void surface. These calculations show that the drag force at the void may be considerably enhanced by the large value of the drift velocity and there is often no need to increase the drag coefficient in order to obtain a self-consistent picture of void formation. It must also be noted that according to Refs [97, 99], the drag force is strongly dependent on the charge of the dust particles, whose value depends on the electron and ion number densities at the void surface, which is also controlled by the self-consistent nature of the distribution of all the parameters inside and outside the void. A self-consistent theory with the dependence of all the parameters on the distance from the center inside and outside the void taken into account also controls the value of the charge of the dust particles at the void surface, which also has a marked effect on the predictions for the void size. Such numerical investigations have allowed determining the dependence of dust-void sizes on the ionization power input, and the results happened to be in good agreement with the observational data [103]. The next step amounted to introducing the concept of virtual voids

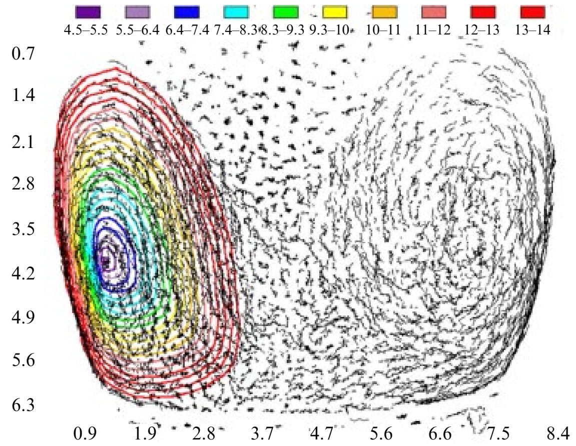

Figure 16. One of the convective cells discovered in cylindrical discharges [107]; different degrees of shading correspond to different dust-particle speeds, which are given in millimeters per second in the upper part of the figure. Depending on the discharge parameters, the central part of the cell may contain either a cylindrical dust crystal or a random distribution of dust particles. Convective cells are located one atop the other with sizes varying because of gravity in the vertical direction.

[104], which are imaginary surfaces for which a balance of forces for dust particles occurs, although no particles exist on such surfaces. In this connection, a number of experiments were carried out, which showed that some particles relax and remain on the surface of virtual voids [105]. The researchers were also able to see how the layer near the surface of a virtual void thickens as the number of injected particles increases, which is due to their interaction caused by the appearance of the interaction between flows and electrostatic fields.

7.4 Convective dust cells near structure surfaces

The surfaces of structures may be surrounded by convective cells of dust particles. The emergence of convection was first discussed in Ref. [106], and cells were clearly visible in the ISS experiment [95]. Convection stems from the nonpotential nature of the electrostatic fields due to the spatial dependence of the charges of dust particles: $e\nabla \times Z_{\rm d}\mathbf{E} = e[\nabla Z_{\rm d} \times \mathbf{E}]$. Calculations show that the charge distribution is spatially inhomogeneous at the boundaries of structures. For cylindrical crystals, good convective dust cells are clearly visible in Fig. 16, taken from Ref. [107], and their explanation within the framework of the theory of self-consistent cylindrical structures was obtained through numerical calculations in Refs [108–110], while the authors of [111–113] did the same for other inhomogeneities of the charges of dust particles.

Cylindrical structures generate plasma flows toward the axis of the cylinder, which may lead to self-confinement. In conditions where the size of the cylinder in the discharge with dust particles is smaller than the mean free path, helical crystal-like structures form [114–116].

7.5 Problems associated with two-dimensional and helical clusters

The simplest and best-studied subjects in this area of research are two-dimensional clusters, which, as they were, represent a surface layer [117–119]. However, clusters are also studied in conditions of transverse confinement (usually, in a parabolic electrostatic potential) and, so far, in terrestrial gravity, which requires using particle levitation in the sheath's electrostatic field, which generates a strong flux of ions directed toward the wall. Clusters that contain only a few particles (starting with two) and those containing many particles (more than 150) have been studied. Clusters of the second type are sometimes called flat dust clusters; with a small number of particles, they have a shell structure, just like atoms of ordinary matter (but in the presence of external transverse confinement). The pair interactions can apparently be summed when there are few particles, but the role of collective flows is unclear with a large number of particles, because such collectivity is certainly not present in the direction perpendicular to the cluster plane (along the external flow), while mutual shading of flows occurs in the cluster plane when the transverse size of the cluster is much larger than the mean free path of the flow across the cluster. This anisotropy does not allow using the above results for isotropic flows, however. This problem has yet to be solved theoretically. Apparently, the shadow (Lé Sage) attraction operates in the cluster plane, but in modified form. For the known pair interaction, there exists a general theory of equilibrium in the collective vibrational modes of the cluster, one that is widely used in theoretical studies of two-dimensional crystals [116–119]. The main problem here amounts to determining the type of pair interaction in the presence of a strong flow across the cluster plane. Apparently, charge-exchange collisions may also play an important role [120, 121]. However, as discussed above, there must be two regions in the vicinity of a dust particle: one in which the field of the particle itself is predominant, where the distribution of the potential is approximately isotropic and spherically symmetric, and the other at distances ranging from about the mean free path to the nonlinear screening radius, where the distribution of the ion flow must undergo a significant transformation. Examples of such an abrupt change are well known and, usually, the ion distribution parameters experience discontinuities. Such discontinuities have yet to be studied, and we can only assume that they emerge in roughly the same way as in other examples. But it is almost inevitable that the ion distributions are elongated along the flows at large distances. This oblongness became known as wake and was first studied in flows whose speed exceeds the speed of sound in [122–125]. In this case, the wake is simply the Cherenkov radiation consisting of ion-sound waves. However, it turned out later that the presence of supersonic flows is not necessary for wake elongation to appear in a distribution with a positive-charge concentration in the wake. In flat clusters, there may be vibrational modes when dust particles are displaced in the cluster plane and modes in which dust particles are displaced at right angles to the cluster plane. The latter are subjected to interactions related to the presence of a wake, while according to Ref. [126], the interaction of transverse and longitudinal modes leads to a transition of the cluster from an ordered distribution of particles to a disordered one (the crystal melts). This phenomenon has been observed in experiments, and an explanation is given in Ref. [126]. The results of the specially designed experiments in [127, 128] corroborated the presence of the interaction of particles located below the two-dimensional layer with a wake field of the particles in the layer. All this suggests that physical processes of a special kind occur in flat clusters and that the processes differ from those occurring in large systems. Wake attraction of particles was suggested as a way to explain the elongation of particle distributions in crystals formed in plasma sheaths [129] (this

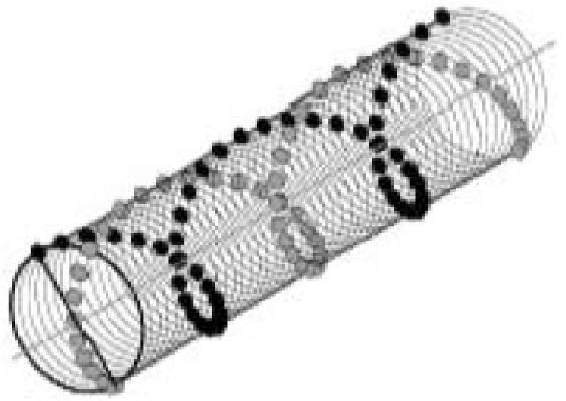

Figure 17. Schematic image of a twofold dust helical structure similar to DNA. The bifurcations of the helix that were discovered in Ref. [61] may serve as memory elements and used for modeling the evolution of biological systems.

phenomenon is sometimes observed in crystals) and as a model of plasma crystals in Ref. [130]. But, as noted above, this effect does not actually work with large systems, because the sheath usually exhibits ion-flow inversion and near the crystal the flow is weak in such systems. This is not true for flat crystals, which in laboratory experiments in the presence of terrestrial gravity in the sheath are in a field of very large plasma flows. To interpret such experiments, we must first establish the nature of particle interaction for nonlinear flows behind a large charge. As noted above, this problem most likely cannot be solved without employing the kinetic approach combined with a study of the emergence of instabilities and the related random fields (in other words, an approach dealing with a turbulent flow behind large charges has to be developed). In this respect, computer simulation by the particle-in-cell method [131] is hardly acceptable because it requires that the development of the turbulent fields be closely monitored, and the result for real systems differs dramatically from that obtained through computer simulation by the particle-in-cell method.

It would be ideal for experiments to be conducted in microgravity conditions. Then, however, using plane geometry is meaningless and, instead, cylindrical geometry is preferable. Cylindrical clusters have been thoroughly studied analytically and in computer simulations [114–116]. When there is an additional degree of freedom along the axis, helical clusters appear in planes perpendicular to the axis (an example of such a cluster is shown in Fig. 17). As the number of particles in the plane of such strictures increases, the structures transform into cylindrical dust crystals. When the structures are studied in microgravity conditions, systems with large flows can be discarded and, instead, the known interactions of particles can be used. Such experiments have been planned for the ISS crew [132].

7.6 Self-organization processes in dusty plasmas and the maximum size of dust crystals

It is only natural to assume that the 'splitting' of dusty plasma into dust structures and voids is the final nonlinear stage in the development of the universal instability of structurization [17]. Individual structures are best suited for studying self-organization phenomena. They are highly dissipative in view of the large rate of flow absorption by the structure. These flows feed the structures. The systems are open and may help in modeling self-organization processes. For this, the interaction of such structures and their evolution must be studied. Helical structures with bifurcations of the rotation angle could be of special interest here [112–115]. The study of such structures may help to model biological systems. In view of the large amount of material in this area of physics, Sections 6 and 7 contained only a brief exposition of the subject, and we selected only the aspects that are directly relevant to the manifestation of plasma flows in the formation of dust structures and in the structures themselves.

8. A discussion of the prospects for further research

8.1 Nonlinear interactions of plasma flows and polarization fields: discontinuities and singular lines

From what we have discussed in this review, it is clear that dusty plasmas present a new and very interesting (for applications) concept of the interaction of plasma flows and electrostatic fields, many aspects of which are in the initial stages of investigation. However, the manifestations of the already discovered phenomena are so rich that we can expect new intensive development in this area of research. Many theoretical problems are waiting for a solution. Nonlinear equations for such an interaction have been studied only in the simplest form and many assumptions have to be checked. We can be sure of surprises that nonlinear interactions have in store, primarily in manifestations of special features, special surfaces, etc. Of course, there are physical reasons for them, and yet they have not been studied. It is known, for instance, that even the simplest equations of hydrodynamics exhibit discontinuities in the form of shock waves, tangential discontinuities, etc. We should expect other surprises in the description of nonlinear interactions of flows and electrostatic fields. The one thing that is clearly understood today is that such an interaction is indeed nonlinear.

8.2 Development of the kinetics of dusty plasmas

In the past, considerable effort went into building the kinetics of dusty plasmas [29]. The main idea was to take the effects of plasma flows and the charging of dust particles into account. New collision integrals were derived and many new qualitative results were obtained. But their direct use in describing the most important effects of dust-particle attraction was not achieved because ion scattering by dust particles was considered in the first approaches as small-angle scattering, and therefore the results of the theory pertained only to the limit where $\beta \ll 1$. Among the initial ideas discussed in Refs [29, 30] was the idea of dividing fluctuations into high-frequency ones, for which ion and electron motion is responsible, and low-frequency ones, for which dust particles are responsible. This idea was not used in full, because, as is easy to see, the condition imposed on β affects high-frequency fluctuations. It is quite possible to generalize this theory to the case where $\beta \gg 1$ if large-angle ion scattering is taken into account in the initial relations. In accordance with the nonlinear scattering discussed above, such scattering is determined by the frequency $n\pi v R_{\mathrm{nl}}^2(\beta, v, v)$ (above, we used the variable $y = v/\sqrt{2}\,v_{T\mathrm{i}}$ instead of v, but it is easy to determine the dependence of the cross section on v even without assuming that the ion distribution is thermal). There

are several simple models that help to describe large-angle scattering, e.g., by taking into account that at distances longer than $R_{\rm nl}$, small-angle scattering occurs in a thin layer and can be ignored. For example, such a model can be based on the assumption that large-angle scattering can be approximated by backscattering. This assumption yields a small error in the cross section in the form of a factor of the order of unity. We must then introduce an additional term describing back-scattering and proportional to $f_{\rm i}(\mathbf{v}) - f_{\rm i}(-\mathbf{v})$ into the initial equation. This shows that the limitation $\beta \ll 1$ can be lifted completely and the kinetic theory can be developed for arbitrary β. This generalization requires much work, as shown by the experience in building a theory for $\beta \ll 1$, but such a development is urgently needed if we want to correctly interpret the interaction of flows and electrostatic fields.

The kinetics of bounded dusty plasmas and a kinetic description of the observed sharp boundaries should also be developed. Here, much depends on the nonthermal distribution of the ions and electrons near the boundaries, accelerated by dust particles in the bulk of the plasma. The kinetic boundary conditions imposed on the distribution functions must be determined first. One of the first proofs that sharp boundaries exist in the kinetic approach was given in [133].

8.3 Interaction of dust structures and polycrystals

Another important problem is the interaction of self-organizing dust structures of various types, including structures of various geometric configurations and helical clusters. This could be used for modeling biological structures and, possibly, in designing helical structures for data transmission. The problem of the dynamics of self-organizing structures also requires further detailed investigation, because the linear theory of the universal instability [17] of dusty plasmas can only be considered a starting point in such an investigation. In view of this, finite dust crystals should also be regarded as self-organizing dust formations. Regarding the universality of the instability of structurization [17], there is the question of the maximum possible size of a dust single crystal and the question of whether large dust crystals are always polycrystals.

8.4 Experimental problems in the diagnostics of dusty plasma parameters

Notwithstanding their diversity, the existing experimental data are actually not very helpful, because they provide fairly complete information about dust particles but only scant information about the other components, the electron and ion distributions. At the same time, this information could be very important for a detailed comparison of theory and experiment. Of course, we can assume that, thanks to self-organization, the ion and electron distributions are to a certain extent determined by the distribution of dust particles. But the presence and extent of self-organization must by itself constitute a topic for investigation.

8.5 New techniques and new manifestations in astrophysics

A complete understanding of the physics of the interaction of flows and electrostatic fields may lead to new achievements in technology and in fabricating new materials. In this regard, we are unable to dwell on this topic any further and recommend reviews [3, 134–136], which may give the reader an idea of the numerous technological applications of dusty plasma and of how understanding the physics of the interaction of flows and electrostatic fields may help change these applications or in the discovery new ones. It is difficult to discuss these aspects any further in view of the limited size of this review.

As regards astrophysics, the important role of the dust component is well known [137–141]: dust determines, to a great extent, the processes in interstellar space, the formation of stars and planets, the structure of planetary rings, and much more. Dust clouds have been thoroughly studied on the basis of the data in the IR range from the recently launched Spitzer Space Telescope. As examples to illustrate the important role of the so-far ignored interactions of flows and electrostatic fields, we present only proofs of the fallibility of some current myths in this area of research:

(1) the interaction of dust particles is inessential if the distances between the particles exceed the Debye screening radius; obviously, this is not so because the interaction of flows and electrostatic fields extends over much larger distances;

(2) the observed structurization of dust clouds is unrelated to the interaction of dust particles; clearly, the new effective gravitational instability related to particle attraction can be responsible for such structurization;

(3) there can be no formation of large aggregates of dust particles; clearly, self-organizing spherical dust structures may exist as perfectly balanced (in all their components) equilibrium structures (dust stars, so to say), and their contribution to the balance of dark matter must be estimated; and

(4) there are many other examples, such as the observed agglomeration of dust behind a shock front and the role of dust-particle attraction in such agglomeration (see Refs [137, 142, 143]). And again, there is no space left here for a detailed discussion of these examples.

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