Problem 1 Let X_t be a geometric Brownian motion with *drift* $\mu \in \mathbb{R}$ and volatility $\sigma \in \mathbb{R}^+$, given by

$$\frac{dX_t}{X_t} = \mu dt + \sigma dB_t \tag{1}$$

Let $\alpha \in \mathbb{R}^+$ be a constant discount rate and $\mu < \alpha$. Let V_t be the present value of X_t given by

$$V_t = E\left[\int_t^\infty e^{-\alpha(u-t)} X_u du | \mathcal{F}_t\right] \tag{2}$$

where $u \geq t$ and $\mathcal{F}_t$ is the σ-algebra generated by B_t. Show that V_t is a geometric Brownian motion with the same drift and volatility of the X_t process.