

**Problem 2**

Consider the Cox-Ingersoll-Ross model for interest rates written in the differential form

$$dR_t = (\alpha - \beta R_t) dt + \sigma \sqrt{R_t} dB_t$$

where  $\alpha, \beta, \sigma$  and  $R_0 \in \mathbb{R}_+^*$ . In the integral form it is given by

$$R_t = R_0 + \int_0^t (\alpha - \beta R_u) du + \int_0^t \sigma \sqrt{R_u} dB_u$$

Find the semi-analytical solution for  $R_t$  and then compute the  $E(R_t)$  and  $Var(R_t)$ . (Hint: Start working with the differential form and use the integrating factor  $\exp(\beta t)$ .)