Maths Made Magic

A handbook of magical mathematical tricks for you to learn

Jason Davison and Peter McOwan

Meet the Authors

Jason Davison is a Master of Mathematics (MMath) student at Warwick University. This is where his interests in magic and Maths began to combine. He is a semi-professional magician, putting his skills to use at weddings and business events. He will be teaching with TeachFirst and is enthusiastic about teaching Maths using magic tricks to make it fun for kids.

Peter McOwan is a Professor of Computer Science at Queen Mary, University of London. He has a particular interest in artificial intelligence. He uses his life long interest in magic to help teach mathematics and science concepts and was awarded the IET Mountbatten medal in 2011 for promoting computer science to diverse audiences.

Notes to Teachers

We hope you will be able to use this book in the classroom to help you teach many of the basic concepts in mathematics in an engaging and entertaining way. All the tricks are self-working and easy to do, and have been tested in UK classrooms. For UK teachers the tricks have been mapped to many of the topics for mathematics at Key Stage 4 however teachers in other countries will also find this material useful. The book may be photocopied for non-commercial use, and there are a number of easy to copy worksheets also included.

A sound understanding of basic mathematics underpins science and engineering. We hope this book helps you teach these subjects and gives you and your pupils the opportunity to develop presentation and communication skills. It may also open the door for others creating new mathematical magic and start some people on the road to a fascinating new hobby too.

Acknowledgments

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Further reading either to learn more maths or to take up magic as a hobby

Martin Gardner, Mathematics, Magic and Mystery, Dover Publications (or any of his numerous maths, magic and puzzle books)

Karl Fulves, any of his ‘Self Working’ series from Dover Publications

Paul Zenon, Street Magic, Carlton Books Ltd.

To learn about some amazing advanced maths, tricks and the history of mathematical magic see Magical Mathematics By Persi Diaconis and Ron Graham

Free to download pdf books

Manual of Mathematical Magic www.mathematicalmagic.com

The Magic of Computer Science Vol 1 and Vol2 www.cs4fn.org/magic

Illusioneering, the Magic of STEM www.illusioneering.org

This book is dedicated to Kinga Garriott de Cayeux. A new world of magic awaits.

Introduction

Dear Magic Makers and Practitioners of the Noble Art of Mathematics,

Within the pages of this book you will find inventive illusions and mind-melting magic to amaze you, your friends and family. You will also explore some fundamental mathematics powering the tricks. Learning and entertaining go hand in hand in magic. Understanding the secret techniques, combined with fantastic presentation and practice can make any trick unbelievable.

The greatest magic tricks in the world always have simple but cleverly hidden solutions. Many are mathematical, but the presentation is so brilliant that the audience's minds don't want to look for the solution. An audience may even stumble upon the correct solution but will immediately ignore it if your presentation makes them want to believe in magic. Magicians world-wide also know that the quality of your presentation can also affect the memories of your magic. Creating your great presentation means that when your magic is remembered in the minds of your spectators, it will be embellished, and seem even greater. This is what magicians call the art of creating astonishment.

In this book each nugget of magic and illusion is divided into 3 parts: "The Tale", where you will be given the background and information needed to understand each magical effect, "The Magic", where the effect will actually happen, and "The Secret", where the illusion will be explained further, and the hidden power of mathematics will be made clear to you.

If you are performing the tricks and illusions found here to your audience then take care to not break the magician's code. Practice hard and make sure the secrets stay secret. Finding out your own style for performing the tricks and illusions will help to give you a truly magical aura. There is enough material in this book for you to put on a show, take up magic as a hobby or even start a magical career!

The book will also let you explore some of the great theories of mathematics behind the tricks, mathematics which you will be able to use in whatever career you follow. Perhaps it will inspire you to discover more and deeper astonishment in the realm of mathematics as well.

We hope this book will help you entertain, amaze and learn.

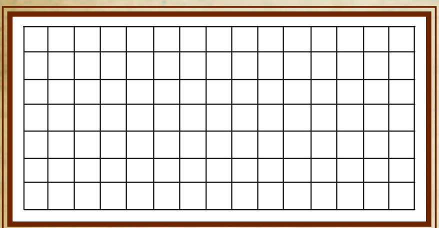
Contents

The Vanishing Potion and the Mathematics of Area

You master a spell to make a square vanish

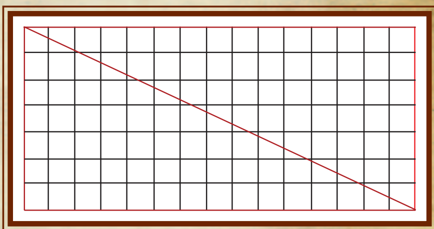
The Tale

If we have a rectangle that is 5cm tall and 13cm wide what is the area of the rectangle? In this grid we can see that if the space between each parallel line is 1cm (parallel lines are two lines that point in exactly the same direction) then each square is a “square centimetre” or $1cm^2$. So when we ask what the area of the rectangle is, we are asking how many square centimetres the rectangle contains.

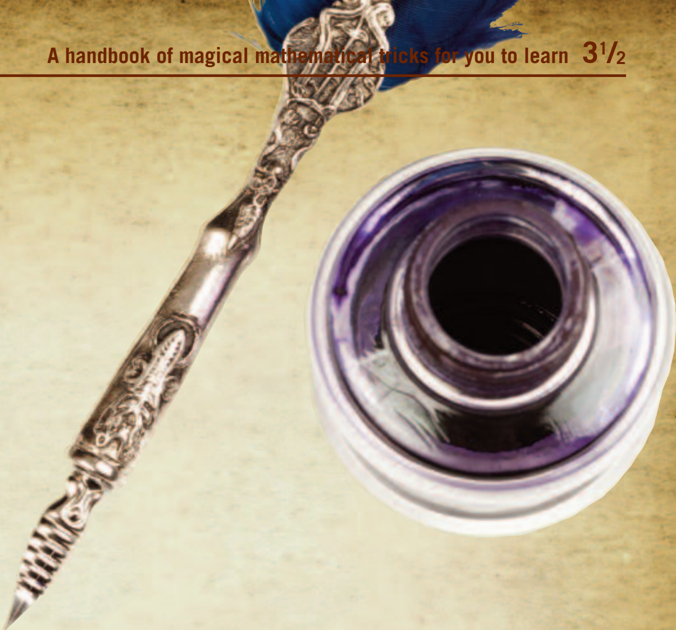


Imagine you have a group of apples and you arrange them so there are 5 rows and each row has 13 apples. In total you then have 5x13=65 apples. The same is true if we now think of the squares in the rectangle. We can think of the rectangle as having 5 rows of squares. Each row has 13 squares in it. So the total number of square centimetres is 5x13=65. So the area of the rectangle is $65cm^2$.

Now let’s look at the area of a triangle. Imagine we have a triangle that is 5cm tall and 13cm wide. What is the area of the triangle? Well, imagine we had two of these triangles. We could then put them together to make a rectangle that is 5cm tall and 13cm wide.

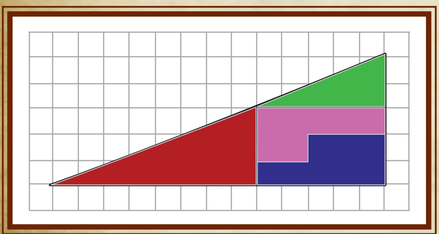


We know from before that a rectangle like this has an area of $65cm^2$. This is then also the area of 2 triangles. So if the area of 2 identical triangles is $65cm^2$ then the area of 1 triangle must be half of the area of the rectangle. So it is $65cm^2$ divided by 2. This is $32.5cm^2$.

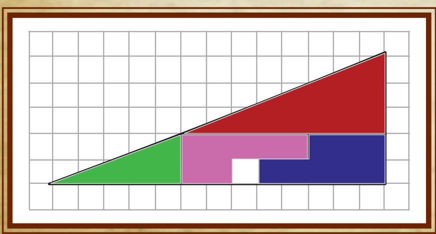


The Magic

Now for the magic! Look at this triangle. We can see it is 5cm tall and 13cm wide. I have cut the triangle up into different shapes ▼.



Look at what happens now when I move the shapes around. We can make another triangle the same size that is still 5cm tall and 13cm wide.



So the area of the triangle should be the same but now there is a piece missing. This surely doesn't make sense. This picture seems to show that the area of the shapes from the first triangle ($32.5cm^2$) plus an extra $1cm^2$ gap still makes a triangle that is also $32.5cm^2$. Impossible. Where is this extra square centimetre coming from?

The Secret

This is a trick. The shapes to begin with don't actually make a triangle. The longest edges (the "hypotenuses") of the smaller triangles are not perfectly parallel. So the longest edge of the main big triangle is not perfectly straight. There is a small bend which means that the area of all the shapes together in the first picture is a little bit less than the area of the triangle we think we are looking at. When we put the shapes in the second position the longest side of the triangle is still not straight; it bends outwards very slightly. The difference in area of the large triangles made by changing the smaller shapes

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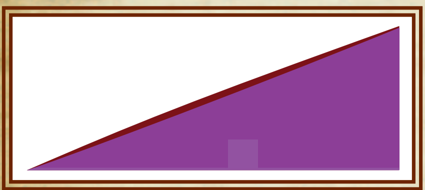
The Vanishing Potion and the Mathematics of Area

around is equal to $1cm^2$. You can see this extra $1cm^2$ along the long edge if we overlap the two triangles made:



One reason why the small shapes don't perfectly make a large triangle is that the two smaller triangles aren't "similar". If two shapes are similar then they have matching angles (but the lengths of the sides might be different – one of the shapes must look like a zoomed in/out version of the other shape). The angles in the small triangles don't perfectly match the relevant angles in the big triangle we would like them to fit into. At the start of the trick I pretended that the triangles fitted perfectly, but they don't.

Clearly the total (sum) area of all the smaller shapes stays the same no matter what pattern they are placed in. If we work out this area, the area of the pink shape is $7cm^2$, the area of the blue shape is $8cm^2$, and the area of the green triangle is the height multiplied by the length divided by 2 = 2x5/2 = 5. In the same way the area of the red triangle is 3x8/2 = 12. So the total area is $7+8+5+12=32cm^2$ which is half a square centimetre less than the area of the triangle we thought the shapes fitted into.

Thought Witchery using the power of the Mathematics of Algebra

Mysteriously, a number thought of by you is drawn from your mind

The Tale

Algebra is where we study and use numbers that are a little mysterious. When we use algebra we might actually not know exactly what the numbers are. They could be 3, 100 or -2.455, but we do know something about how the numbers combine together and using algebra we can work them out. This may sound a little confusing, so let's do some magic.

The Magic

Now for the magic! Let's walk through and explore….

TRICK VERSION 1 Get your calculators out or switch your phone to calculator mode, and think of any number.

Write this number down on a piece of paper.
Now add 5,
Multiply by 2,
Square (i.e. times the number you now have by itself),
Minus 100,
Divide by your original number,
Minus 40,
Divide by your original number,
Add 4.

If my powers of mind reading are strong enough you should now have the number 8.

How did this work? Well we can actually explain it with algebra. First let's try a simpler one:

Continued...

Thought Witchery using the power of the Mathematics of Algebra

TRICK VERSION 2
Write down a number,
Add 5,
Multiply by 3,
Minus 15,
Divide by your original number,
Add 7.

Again, if my powers of mind reading are strong enough you should now have the number 10.

Or an even simpler one:

TRICK VERSION 3
Write down a number,
Add 5,
Minus your original number.

Again, if my powers of mind reading are strong enough you should now have the number 5. Are you starting to see how I'm tricking you?

The Secret

Now to explain this let me first tell you that unfortunately I am not a true mind reader. Although you may think your original choice of number would change the number at the end, in fact it doesn't make any difference what your start number is. For example in version 3, I have no idea what number you picked at the start. So for me you picked the number "?". It's just a question mark for me, I have no idea what the number is.

Now let's look at the third version of the trick.

First I ask you to write a number,
(for me this gives "?")

Then I ask you to add 5,
(for me this gives "? + 5")

Then I ask you to minus your original number
(for me this gives "? + 5 - ?")

One useful technique in mastering algebra is called rearranging an equation. To start off we can see what happens when we just use numbers, it's obvious you get the same answer, 2, if you write 3 + 7-8 = 7 + 3 - 8 = 3 - 8 + 7 = 2. What's important to remember here is that 3 and 7 are positive numbers, so there is always a "+" sign in front of them (but when we write the 3 or the 7 at the front of the equation that "+" sign is invisible, if we wanted to do it longhand we could write + 3 + 7 - 8 = 2, but that could be confusing, it could look like something was maybe missing at the front, so we lose that + at the front, and so long as we remember it's really there everyone is happy). In our example the number 8 is negative, there is always a "-" sign in front. As long as we keep the signs in front of the right numbers we can change the order as much as we like and it will give the same result. So we can change the order of "? + 5 - ?" to "? - ? + 5" = "+ 5" = 5.

In algebra, we don't usually use the "?" sign. We use "x" or another letter to represent the number we don't know.

We can also "simplify" brackets. This means that if we had "$3 \times (x + 5)$" we could write it as "$3 \times x + 3 \times 5$"= "$3x + 15$" without brackets. Check this is true for yourself:

Look at:
"$x \times (y + z) = x \times y + x \times z$"

Choose any three numbers. Swap "x" for the first number, "y" for the second number, "z" for the third number. Then check that the equation above is true.

Let's look at example 2 and use "x":

You write down a number:
(for me this gives "x")

Then I ask you to add 5:
(for me this gives "$x + 5$")

Then I ask you to multiply by 3:
(for me this gives "$3 \times (x + 5)$" = "$3x + 15$" from the rule above)

Continued...

Thought Witchery using the power of the Mathematics of Algebra

Then minus 15:
(for me this gives "3 × *x*" or "3*x*")

Divide by your original number:
(for me this gives 3)

Add 7:
(for me this gives 10)

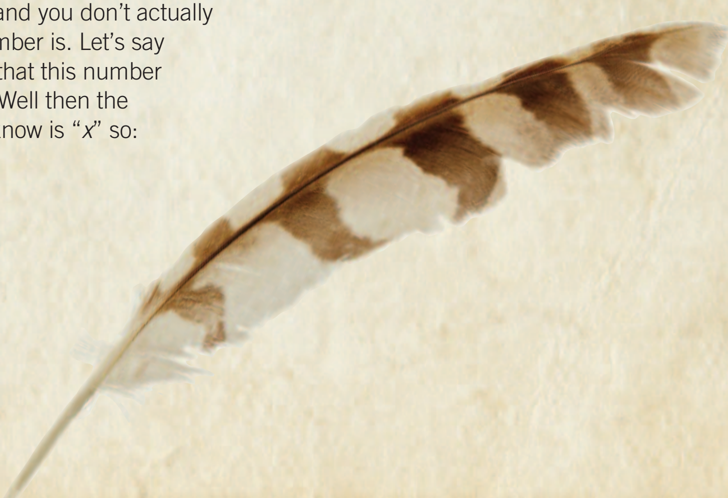
So again, the final number doesn't have an "*x*" in it which means the answer will always be 10 and it doesn't matter what the start number that you choose is.

This trick gives you an introduction into algebra and how it works. In algebra there is a number ("*x*") and you don't actually know what the number is. Let's say someone told you that this number minus 5 gives 12. Well then the number we don't know is "*x*" so:
$x - 5 = 12$

The equals sign means both sides of the equation are the same number. Take "x - 5"and add 5. This gives "x - 5 + 5" which equals "x".
$x - 5 + 5 = x$

But from the first part we know that $x - 5 = 12$, so replace the $x - 5$ with 12:
$12 + 5 = x$
$17 = x$

We have basically added 5 to both sides of the equations. Now we know the mystery number *x* is 17.



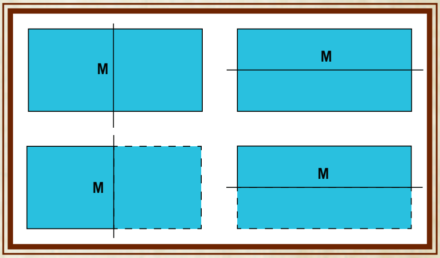
Enhanced Senses Incantation using the power of the Mathematics of Symmetry

An enchantment allowing you to increase the power of your senses to find a card just by looking at the faint fingerprint mark left by your spectator.

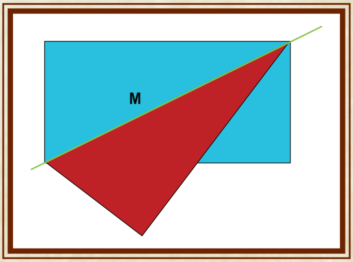
The Tale

There are three types of symmetry you'll need to know about: Line symmetry, Plane symmetry and Rotational symmetry. Things are "line" symmetric or "plane" symmetric if you can place a mirror on a line or plane and the shape looks the same with or without the mirror. It is perfectly reflected in the mirror. A "Plane" is a completely flat surface like a piece of card.

For an example of line symmetry look at this diagram of a rectangle. Imagine we place a mirror at the line "m". We can see how the rectangle has two lines of symmetry. The dotted lines in the lower diagrams are what the shape on the "m" side of the mirror would look like if it was reflected in the mirror. Notice that the reflected image looks the same as the original rectangle.



In this diagram we try to find symmetry by placing a mirror on the diagonal line between two corners. The red shape shows what the part of the rectangle above the mirror looks like when it is reflected. You can see it does not match the rectangle. If the rectangle was square there would be diagonal symmetry as well.



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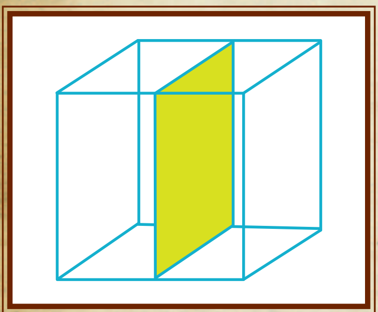
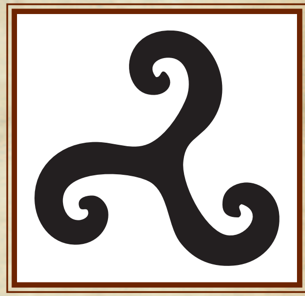
Enhanced Senses Incantation using the power of the Mathematics of Symmetry

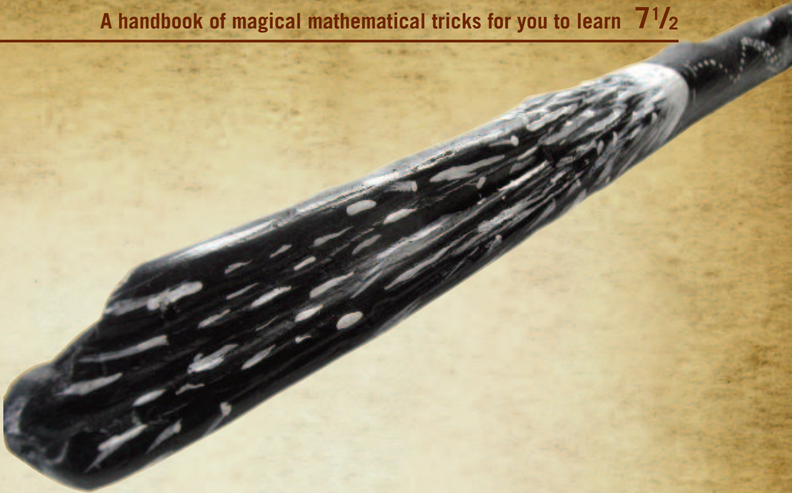
Plane symmetry is similar to line symmetry but we look at 3D shapes. For example, a plane of symmetry of the cube is shown by the yellow plane. If there was a mirror where the plane is the cube would look the same if we looked through the mirror as it would without the mirror.



Rotational symmetry is when we can fix a point on a shape, rotate the shape around that point an angle more than 0 and less than 360 such that the shape looks the same after it's been rotated.

Look at this shape. If we rotate the shape around the centre 120 degrees and 240 degrees the shape will look exactly the same, so it has rotational symmetry.





The Magic

Now for the magic! Look through your pack of cards. You will notice that for example the Jacks, Queens and Kings have rotational symmetry around the middle of the card. You could turn the card 180 degrees (upside-down) and it would look the same. There are some cards this is not true for. Look at the 7s, they look different if you turn them upside-down, they have no rotational symmetry. Now remove all the cards that have no rotational symmetry.

(Try and do this yourself, but in case you need a hand, these will be: the Ace, 3,5,6,7,8,9 of Hearts, Clubs, Spades and 7 Diamonds). Now arrange these cards so most of the suit symbols (e.g. the heart symbols) are facing the right way. For example, if the 7 Clubs has 2 clubs facing up and 5 pointing down turn it so 5 are pointing up and two down instead. Do this for all the cards, the 7 of Diamonds is different because the diamonds look the same either way but put it so the pattern of the diamonds is the same as on the Hearts and Clubs, with 5 of the symbols in the top half of the card. Now put all these cards that are not "rotationally symmetric" on the top of the other cards. You are now ready for the trick.

Spread the cards on a table face up to show your spectator that it is a normal pack. Then spread the cards face down on the table and point at the bottom card on the deck. As you move your finger from the bottom to the top card tell your spectator to say stop whenever they want, but make sure by the time you've said this, your finger is already in the top TWENTY TWO cards (the ones not rotationally symmetric). When they say "stop" bring your finger down on a card. Get them to take that card. Collect the cards up again and turn the deck around so that when they put their card back in it will be the other way round (in terms of the symmetry). You can then give the pack a normal shuffle. Your spectator can even normally shuffle them too.

Now tell them you will look for their invisible fingerprint on their card with your amazing heightened vision skills. Look at

Continued...

Enhanced Senses Incantation using the power of the Mathematics of Symmetry

the spectator's fingers and "study" their fingerprints (this is just acting). Now deal the cards out one by one on the table, "looking for fingerprints". Secretly you will know when you see their card as it is the only non-rotationally symmetric card facing the wrong way! Magic.

The Secret

That's most of what you should know about symmetry. But as well as this make sure you understand the "order" of rotational symmetry. The order is the number of different positions you can put a shape into by rotating it to give a shape that is the same. For example, a triangle has order 3 because you could turn it so that any of the three corners are at the top and the triangle would still look the same.

Unique Conjuring using the Mathematics of Inequalities

Somehow, I know the potion you will create

The Tale

Here is a spell using inequalities. The symbols you'll need to know for this are $<$ $>$ (and $\leq$ $\geq$). You place these symbols where you might find an equals sign "=" (so between two mathematical phrases). But instead of the phrases on both sides of the symbol being equal, one is larger than the other (but with $\leq$ $\geq$ both sides can also be equal). The arrow points to the smallest side of the equation. You can think of it as a greedy crocodile symbol that will always try to eat the bigger phrase. So for example we can say $7>4$. We should also look at how these symbols work in Algebra. If we were asked to show a region where $x>y$ then we should draw the graph of $x=y$ and then shade the region above the line as for all these (x,y) co-ordinates $x>y$.

The Magic

Now for the magic! First you must create your potion. To do this, shuffle your pack of cards. Each order the cards can be in represents a different potion. Once you're happy with your potion you'll need to find the substance that potion creates with this process:

Put a Joker on the top of the pack and then turn the pack and hold it face up so you can see the faces.

Work out "14 - the value of the first card" (Ace = 1, Jack = 11, Queen = 12, King = 13). If this new number is BIGGER than the value of the second card then put these two cards to the back behind the Joker. If the new number is smaller or equal to the second card then get rid of the two cards on the table somewhere.

Do this repeatedly until you get to the Joker then put the Joker to the bottom again.

Continued...

Unique Conjuring using the power of the Mathematics of Factorials

Now work out "11 - the value of the first card" (Ace = 1, Jack = 11, Queen = 12, King = 13). If this new number is SMALLER than the value of the second card then put these two cards to the back behind the Joker. If the new number is bigger or equal to the second card then get rid of the two cards on the table somewhere.

Do this repeatedly until you get to the Joker again. Hopefully you'll now be holding a few cards (if you don't have any left shuffle all the cards and try again). Now get rid of either the first two cards or the last two cards. Do this again until you only have two cards left in your hand and add the value of these two cards together. Now to find what your potion was worth find this new number in the list. The sum of these two cards must be between 2 (if they are two Aces) and 26 (if they are two Kings):

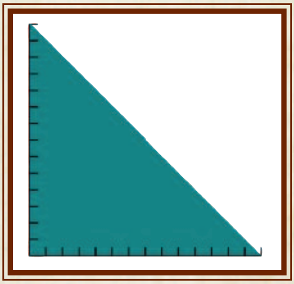
Gold	2	Iron	18	Moonlight	7	Stone	13
Silver	19	Silk	10	Grass	15	Starlight	5
Gold	2	Iron	18	Moonlight	7	Stone	13
Silver	19	Silk	10	Grass	15	Starlight	5
Bronze	14	Copper	4	Cotton	16	Ice	21
Diamond	6	Rubies	9	Ivory	26	Oak	8
Steel	23	Emeralds	17	Polish	3	Wool	20
Stone	12	Leather	22	Smoke	25	Pine	11
Tin	24	Rust	26				

If the predictive powers of this spell book are strong then you will create Stone. Magic?

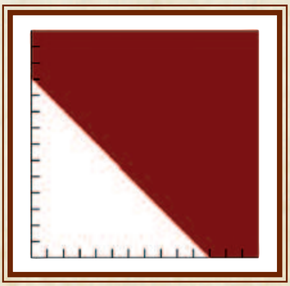
The Secret

To understand this trick we must work with inequalities. There are 52 cards in a pack. As this is an even number we can group the pack into pairs and move cards in each pair. Each pair in your shuffled pack represents a co-ordinate on a graph. The first card in the pair represents the “x” co-ordinate, the second card is the “y” co-ordinate. So the pair of cards (Ace, Two) represent the point that is 1 unit across and 2 above the 0 point of the graph (the 0 point is called the origin).

First, I asked you to keep all pairs such that: “14 – value of first card is bigger than the value of the second card” and remove the others. Representing this with the co-ordinates means we only keep the co-ordinates such that y<14-x. Let’s represent this on the blue graph; we draw the line of y=14-x and shade in the co-ordinates on the side of the line such that the “y co-ordinate” is less than “14 – the x co-ordinate” to get this:



So all the pairs you now had represented co-ordinates in this blue section of the first graph. Second, I asked you to only keep the pairs such that “11-value of first card is smaller than the value of the second card”. Again, using co-ordinates, this means we only keep the co-ordinates such that y>11-x. So again this would give the red graph that looks like this:

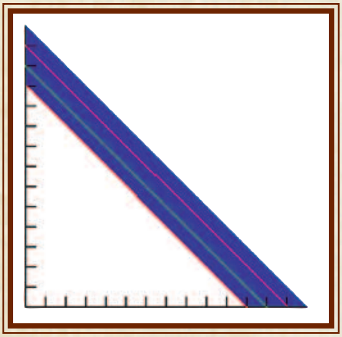


So all the pairs you now had also represented co-ordinates in the red section of the second graph. At this

Continued...

Unique Conjuring using the power of the Mathematics of Factorials

point the cards you held were co-ordinates in both the red and the blue sections. As they were in both sections, they must have been in the band where the section meets, and that looks like this:



A lot of the co-ordinates in this band are not whole numbers. The only co-ordinates that are whole numbers and that are inside this band will be points that are on either the green or red lines. These are the lines $y = 12 - x$ and $y = 13 - x$. For any x, y co-ordinate on these lines, $x + y$ will then be 12 (for the first line) or 13 (for the second). You can check for yourself, pick any value of x. Use the line equation "$y = 12 - x$" to find the corresponding value of y. This gives you the co-ordinate at x that is on the green line. Now add x and y and you should get 13. So when you eliminate pairs and end up with just one pair, that pair will add to either 12 or 13.

Now look at the list of items again, you'll notice that both 12 and 13 are worth stone. So once again, the maths helped us to create an impossible feat.

Mind Control Spell and the Mathematics of Fractions and Percentages

Control your spectators' minds so they all have the same thoughts

The Tale

A fraction is written as two numbers separated by a line like this: ½. A fraction can represent a quantity of things you might have (like magic apples). You probably recognise "½" as a half and you'd find it easy to know how much ½ an apple was. Well what if I gave you 64/128 apples or 7/3 apples?

Try thinking of a fraction like this: the bottom number (the denominator) is the number of equal pieces we divide one apple into. The top number (the numerator) is the amount of these pieces we have. So if you had ½ apples: take one apple and divide it into 2, then take 1 of these pieces, this will be a half. If we have 64/128 apples: we take one apple and divide it into 128 pieces, then you take 64 of these pieces. But how many pieces would be left? If we take 64 pieces from 128 then there will be 64 pieces left. This must mean that we have taken half the amount of pieces and "64/128" = "½".

This helps to shows us something important about fractions: if you divide the numerator and denominator by the same number then the fraction has the same meaning. For example, if we divide the numerator and denominator of 64/128 by 64 then on the top we get "1" and on the bottom we get "2". So 64/128 is the same as ½.

Second, if the numerator is larger than the denominator then we can simplify the fraction (if this is the case then we say the fraction is "top heavy"). If we have 7/3 apples then how many do we have? Well we divide apples into 3 equal parts each and then take 7 of these parts just like before. But if we take 7 parts then we could group 3 of these parts back together to make a whole apple and leave 4 parts. Then we could group another 3 together to make another whole apple and leave 1 part. This gives us 2 whole apples and 1 part of an apple that was divided into 3. We say this is the same as 2 ⅓. So to simplify top heavy fractions we must divide the

Continued...

Mind Control Spell and the Mathematics of Fractions and Percentages

numerator by the denominator (there may be a remainder). The result of this becomes the number of whole apples and the remainder become the new numerator on the new fraction.

The Magic

Now for the magic! Get a few spectators together and make sure they have calculators. Tell them to think of any one of the single digits on the calculator (except 0) then get them to press the digit they are thinking of 3 times (so they will have 444 or 222 on their screen, for example). Now in their head get them to add the three digits together. Now ask them to press divide on the calculator, type in the number in their head and press = for the result. They should now all concentrate strongly on the new number on the screen. Get them to shout out that new number together. They may be surprised that they just said exactly the same thing!

The Secret

This trick is not a coincidence, it uses the rules of fractions we just discussed. Let's see what happens if the original digit chosen was 1. Then the three digit number would be 111. Adding the three digits would give us $1 + 1 + 1 = 3$. Then $111/3 = 37$. However they might not have chosen 1, it could have been any digit. As we don't know what this digit is let's call it "y" and do the trick with this using algebra. (First notice that 555 is just 5×111 and 888 is just 8×111 etc. so $999/9 = 111$).

So if their original digit was "y"

Then their 3 digit number would be "yyy"

Adding the three digits would give
"$y + y + y = 3y$"

Then dividing "yyy" by $3y$ gives "yyy"$/3y$.

Then by the rules of fractions that we discussed we can divide the numerator and denominator by "y" to get $111/3 = 37$. So the original digit doesn't matter, we will still always get 37.

It would also be useful to introduce percentages here. The symbol for a percent is “%”. A percent is simply a fraction. “Percent” basically means “/100”. So if I gave you 50% of my glazed frogs then you would get 50/100 of them, dividing the numerator and denominator by 50 means this fraction is the same as ½. So I would be giving you ½ of my frogs.

Did you know– Magicians don't just do magic?

Did you know that many magicians have changed the world. Harry Houdini was one of the first men to fly a plane in Australia. Another magician. John Neville Maskelyne, invented a typewriter, a mechanical cashier and a coin operated lock that went on to be used in commercial coin operated vending machines.

Your Animal of Destiny and the Mathematics of Circles and Rounding

The Wizard of Dreams predicts your animal of destiny

The Tale

Get your protractors ready, in the next trick you'll need to measure some angles. The trick will tell you about some of the crazy properties of circles but they'll be explained after the trick. You'll also need to know about "rounding". Rounding gives us an approximate number. For example, if I give you a number with many decimal places (for example 3.14159265) we could say that this number is approximately 3 as the closest whole number is 3 (this is "rounding to the nearest whole number). If I asked you to give me the number "to one decimal place" I will want you to write the number with 1 digit after the decimal point (so 3.1).

But we must be careful. Imagine if I asked you to round 1.9 to the nearest whole number. It's not as simple as just writing "1". We have to see which whole number is closest to 1.9 and actually "2" is "nearer" to 1.9 than 1 is. So if we round 1.9 to the nearest whole number we get "2".

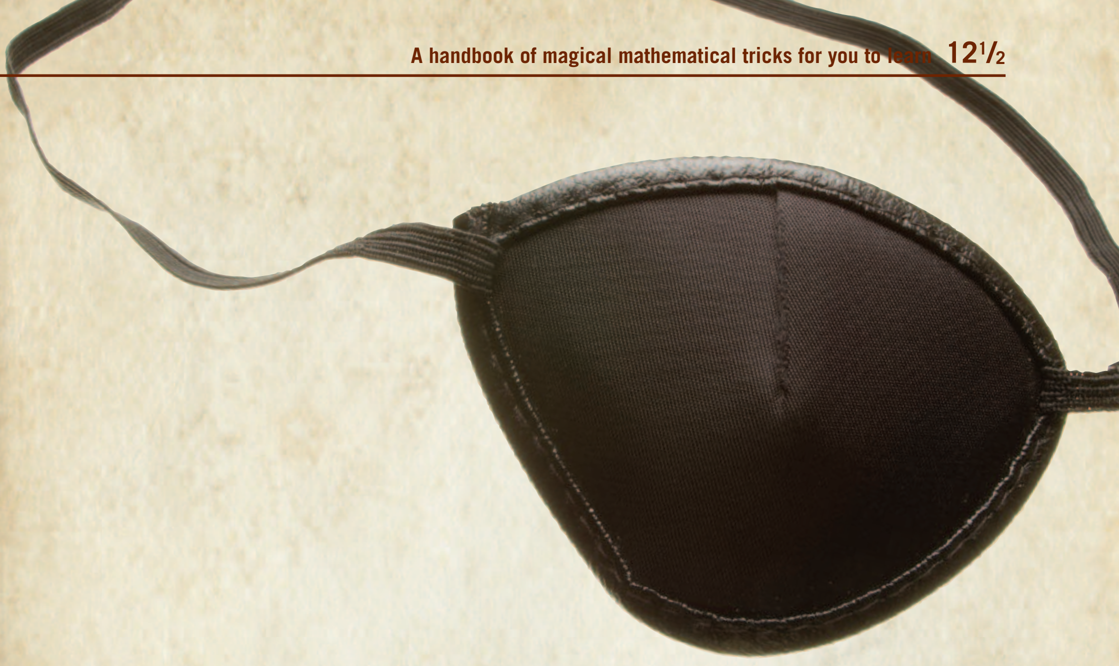
Here is the rule we use: We must look at the digit after the one we want to write down.

- If this digit is 0,1,2,3,4 then we write our answer down normally.
- If this digit is 5,6,7,8,9 then we add 1 to the last digit of our answer.

For example, if we are rounding 3.14152965 to three decimal places we would want to write down 3.141, but because the number after 3.141 is 5 (which is in the second list) we have to add 1 to the last digit of our answer to get 3.142.

Let's try it again: let's round 3.14152965 to four decimal places. We would want to write 3.1415 and the next number is 2 (which is in the first list) so we can leave our answer as 3.1415.

One last time: let's round 3.14152965 to five decimal places. We would want to write 3.14152 and the next number is 9 (which is in the first list) so we have to add 1 to the last digit of our answer to get 3.14153.



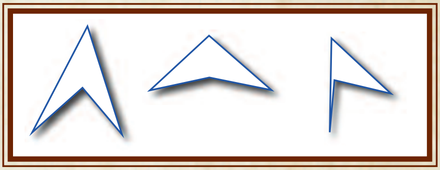
The Magic

Now for the magic! Here are the Wizard of Dreams' three Crystal balls:

There are larger copies of the crystal balls at the back of the book for you to photocopy and draw on.

To reveal your "animal of destiny" you'll have to use the crystal balls in a certain way. I will ask you to draw on them, make sure you use a ruler so you definitely get straight lines! You could use a compass to make copies of the circles. Put dots at the centres.

On the first ball draw an arrow like one of these



The only rules are that one corner must be on the red dot in the middle of the crystal ball and the other 3 points must touch the edge of the crystal.

On the second ball draw a straight line that goes from one edge of the crystal to the other and goes through the red dot. Now put a dot on any point on the edge of the crystal and join this point to the two ends of the line to make a triangle.

On the third crystal draw a four sided shape (it can be a square or rectangle or any crazy shape that has four sides ,as long as the edges don't cross each other), just make sure each corner touches the edge of the crystal.

Now we will use these shape to work out your "animal of destiny".

Continued...

Your Animal of Destiny and the Mathematics of Circles and Rounding

In the first ball measure the angle at the tip of the arrow inside the arrow, and measure the angle at the red dot outside of the arrow. Add the two angles together and divide by the angle you measured at the tip. Write down your answer and label it "*a*".

In the second ball look at the corner of the triangle that is not touching the long edge through the middle. Measure the angle at this corner. Write it down and label it "*b*".

In the third ball pick any two opposite corners of your shape. Measure the angle at these two corners. Add the two angles together. Write down the total and label it "*c*".

Now divide the number c by the number b and add it to the number in a. [So do (c/b)+a]. Round it to the nearest whole number (find the whole number that is closest to it).

Now look for your number here and find your animal of destiny:

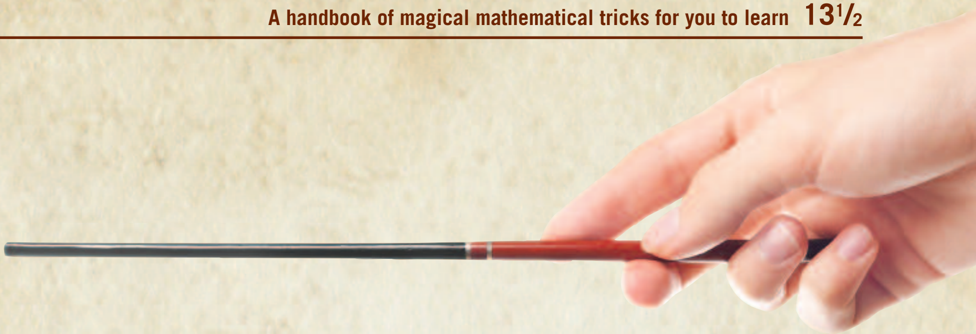
1) Cat
2) Dog
3) Moose
4) Fish
5) Monkey

Now would it surprise you if I told you that before this book was written the Wizard of Dreams said they would predict what your animal of destiny was? They told the animal to me and asked me to write it in the book. They said your animal of destiny would be a monkey. Are they right?

The Secret

The Wizard of Dreams is actually not as mystical as you might think. They always knew your animal would be a monkey. This is because of the strange properties of circles. Here are the facts that they used:

- In the first crystal: If you draw a dart, the angle at the centre is always twice the angle at the tip. So if I asked you to measure the angle at the tip, it would be "x", the angle at the centre would be "$2x$" then adding these gives "$x + 2x = 3x$" then dividing by the angle at the tip ("x") gives "$3x/x$" = 3. It doesn't matter what the angles are, we always get 3.



- In the second crystal: The straight line through the centre is called the "diameter". If we draw a triangle in a circle where one edge is the diameter, the final point on the edge of the crystal will always have an angle of 90 degrees.

- In the third crystal: If you draw a 4 sided shape in a circle, opposite angles always add up to 180 degrees.

- So the number from the third crystal (180) divided by the number from the second crystal (90) gives 2. Then we add the number from the first crystal (3), which always gives 5, which always leads to the monkey! Sometimes when measuring angles it is hard to measure the angles exactly, which means that the angles you get might be a little bit different to the ones I said you should get. This is why at the end I asked you to round the number to the nearest whole number (to get rid of any possible small problems there were in measuring the angles).

The Challenge of the Magic User and the Mathematics of Ratios and Algebra

You play a game of cards against a magic user, the Wizard of Snakes, who aims to imitate your movements

The Tale

We have just looked at fractions. Let's now look at something similar called "ratios". Ratios help us compare one quantity to another. For example, let's say we make a potion that is made with one cup of moon water and 2 cups of lizard juice. We then say that "the ratio of moon water to lizard juice is 1:2". If we have a potion made from 2 cups of crushed beetles and 4 cups of blended newts then the ratio of beetles to newts is 2:4. This means that for every 2 portions of beetles there are 4 portions of newts. Or for every one portion of beetles there are 2 of newts. Having 4 sweets for each 2 children is the same as having 2 sweets for each child. In the same way that we can cancel down fractions by dividing the top (numerator) and bottom (denominator) by the same number, we can also do the same to ratios by dividing both sides by the same number.

The Magic

Now for the magic! Get your cards (make sure it is a full pack of 52). You are about to play a game of chance against the mysterious magic user, the Wizard of Snakes. He is sitting opposite you right now under his cloak of invisibility. Shuffle your cards so that there is no way the Wizard of Snakes can predict the outcome of the game. Although in this game he will secretly try to mimic you as closely as possible. At the end of the game if he manages to mimic you perfectly then he wins. You are going to deal 4 piles of cards, one face up pile and one face down pile for you, and the same for the dark magic user. Start by turning over the first card. If it is black then place it in the Wizard of Snakes' face up pile then deal him another face down card in his face down pile; if it is red then place it in your face up pile and deal yourself another face down card in your face down pile. Continue doing this until you have dealt out all the cards. Each player now has 2 piles, one face up, one

face down. Now count the number of face up reds you have, and count the number of face up blacks the magic user has. It might be that one player has more than the other and has some extra cards. This is clearly not fair so take the extra face up cards away and add them to the OTHER player's face DOWN pile so that now both players have the same number of face up cards.

Now to see if the Wizard of Snakes has won the game by mimicking you correctly; Turn over your face down cards and work out (and write down) the ratio of red cards to black cards (red:black). Then turn over the Wizard of Snakes' face down cards and work out (and write down) the ratio of black cards to red cards (black:red). The Wizard of Snakes has beaten you if they correctly controlled the cards so they would have the same ratio as you. Knowing the Wizard of Snakes, I am pretty confident that this powerful magician has won and that the ratios are the same, How can I be so confident? You are about to find out.

The Secret

So did the invisible Wizard of Snakes actually predict your final ratio or was this another example of maths?

When you finished dealing out the cards (before you moved the extra face up cards around) you had four piles, I don't know how many cards were in these piles so I'll label them with letters (with 'u' and 'd' to represent face-ups and face-downs):

Your Piles:	**Wizard of Snakes piles.**
U	u
D	d

Now either your "u" pile or the magic user's "U" pile might have been bigger than the other. This is where I asked you to move the extra cards to the other person's face down cards. Let's say u>U (if U>u we will be doing exactly the same calculation but with writing U and u the other way around). So we moved the extra cards ("u-U") to give this:

Continued...

The Challenge of the Magic User and the Mathematics of Ratios and Algebra

Your Piles:	**Wizard of Snakes' piles.**
U	U <<< These piles now have the same number of cards
D+(u-U)	d

Also remember that in the trick whenever you dealt a face up card, you immediately dealt the same person a face down card. This means for each player (before you moved the extra cards) there were the same number of face up cards as there were face down cards (so U=u and D=d). If we put these into the bottom left section we get:

Your Piles:	**Wizard of Snakes' piles.**
U	U
D+(d-D)	d

Which simplifies to make:

Your Piles:	**Wizard of Snakes' piles.**
U	U
d	d

So we know at the end that the two face down piles have the same number of cards in them.

- As the pack of cards originally had the same number of reds as blacks, and the number of reds and blacks in the face up piles are the same, it can only be that the total number of red and total number of blacks in the face down piles added together are the same (we can call this amount of red cards [or black cards] in both the face down piles together "x")

- We also know that the number of face down cards you have is the number of face down cards the Wizard of Snakes has. This is because the number of extra face up cards in u must also equal the number of extra face down cards in d (from $u = d$ and $U = D$). Adding the extra cards to the D pile then gives the face down piles the same amount of cards (we can call this amount of cards "Y").

- We should also point out that $x = Y$. In other words, the number of red cards in both the face down piles together = the number of cards in one pile. This is from $2x$ (the amount of face down reds + the amount of face down blacks) = $2Y$ (the amount of cards in both face down piles. If $2x = 2Y$ then $x = Y$.

- So call the amount of red cards in your face down pile "*r*" then the amount of red cards in the Wizard of Snakes' face down pile is $x - r$. The amount of face down blacks he then has is $Y - (x - r)$ which is the same as $Y - x + r$ which is the same as $Y - Y + r$ which is the same as r. So the number of red cards in your face down pile is the same as the number of blacks in the Wizard of Snakes' face down pile. Similarly the number of black cards in your face down pile is the same as the number of reds in the Wizard of Snakes' face down pile by the same logic. This then produces identical ratios.

So in the end the Wizard of Snakes has actually secured the win with a mixture of Algebra and logical thinking.

The Movement Control Charm and the Mathematics of Vectors

This charm allows me to predict your movements

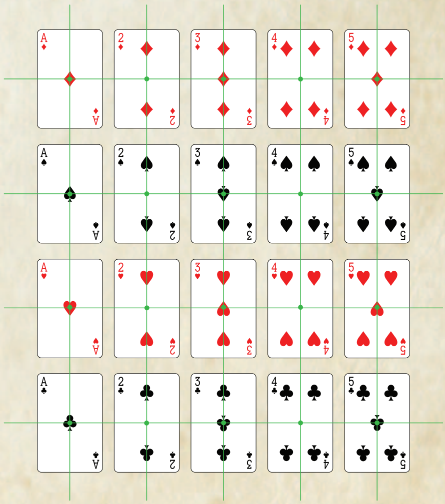
The Tale

Magicians shorthand: H=Hearts, S=Spades, D=Diamonds, C=Clubs

A "vector" is a line that has direction and size. For example, imagine we are positioned at "start" (the very bottom left hand corner of the grid) and we can only move along the vertical and horizontal lines (not diagonally). I'll call the places where the vertical and horizontal lines meet "points".

Now to get to the number columns we move right 2 points to get to the 2 column, or move right 3 points to get to the 3 column etc. So to get to the 5H we'd move across 5 points to the 5 column and then up 2 points to get to the hearts row leaving us at 5H. We have gone across 5 and up 2 (notice we always go across and find the number column first rather than going up to the suit first, we are finding 5H not H5).

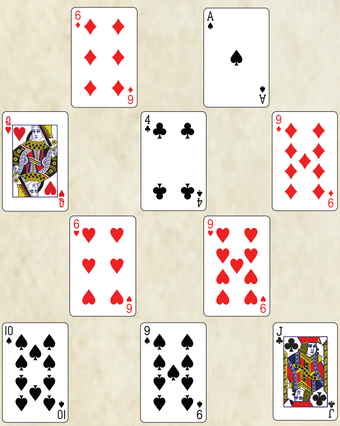
As we have gone across 5 and then up 2 we write this as (5:2). This is called a vector. If we move to the right, or up then we are moving a positive number of points. But if we move to the left or down (so we're moving in the opposite direction) this is a negative number of points (there'll be a minus sign).



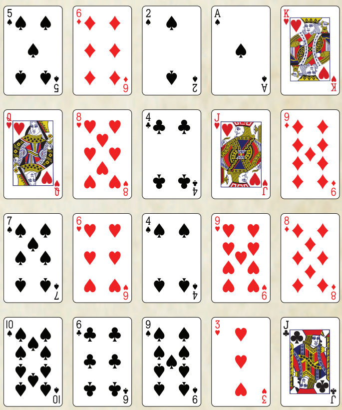
For example, imagine we are at the 2D and want to get to the 5H: Remembering that we find the correct number first we would have to travel right 3 to get to the 5D then down 2 to get to the 5H. We have gone right 3 and down 2 so the vector is (3 : -2).

The Magic

Now for the Magic! Think of one of these cards,



Now I will add more to confuse you but keep your card in mind:



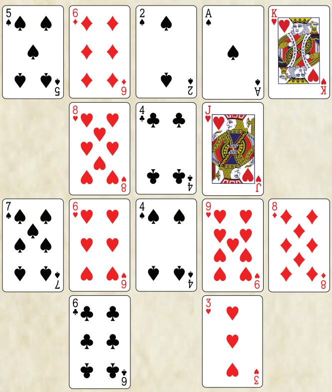
Now think of a vector with small numbers so that the two numbers in the vector add to an odd number. Then move along this vector from your original card to get to a new card. Make sure your vector doesn't take you off the edge of the square of cards and takes you to another card. To make it easier, notice that moving along a vector

Continued...

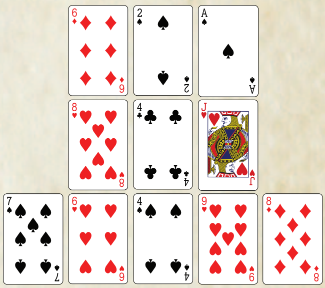
The Movement Control Charm and the Mathematics of Vectors

where the numbers add up to an odd number is the same as taking an odd number of moves in total (no diagonals). It is similar for evens.

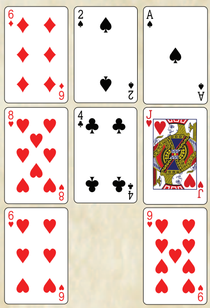
In this trick each time I ask you to move along a vector, make sure that vector takes you to a new card. Now with the power of magic I know the card you are now on so I'll remove some of the others you aren't standing on to prove it….



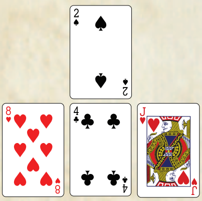
Now do the same thing again; make sure the numbers in your vector add up to an odd number. Again I think I know which card you are on so I'll remove some of the rest…



Now move along another vector where the numbers of the vector add up to an even number. Now I'll remove some more that I know you're not thinking of…



Wow, I'm getting closer to track down your movements. Move along another vector where the numbers add to an odd number. Now I'll remove more cards I know you're not thinking of.



Finally, move along a vector where the numbers add up to an odd number.

After this, using my powers of magic, I think I can say that you are now thinking of the **four of clubs**. Was I right? Maybe it was magic, or maybe it was maths (or maybe a little of both).

The Secret

The reason why this works is a little bit cheeky. To begin with I asked you to think of a card (let's call the cards in the first picture "set 1"). Notice that to get from your card to any other card in the first picture you always have to move along a vector where the numbers of the vector add up to an even number. If you try to move to the empty spaces (where I put the new cards – call these cards "set 2") you have to move along a vector where the numbers of the vector add up to an odd number. So when I ask you to move an "even" vector or an "odd" vector, I always know when you'll be in set 1 or set 2. As I know which set of cards you are on, I can get rid of some of the cards from the other set that you are not in. If I keep doing this there will be less and less cards you can be on until I know your final card, as it is the only card left you can choose. This bit about "odd" and "even" vectors isn't something you will need for the GCSE exam.

Mind Expansion Jinx and the Mathematics of Mental Methods

Your mental power is expanded to unbelievable levels as you conjure up an ever popular magic square

The Tale

It will be important to know (or to be able to work out) the square numbers from 1^2 to 15^2 as well as the cubes of 2,3,4,5 and 10. As well as this you'll need your multiplication and division skills. In terms of multiplication, if you have two fairly long numbers it is often easier to look at the shorter number in sections of hundreds, tens and units and then do the multiplication in parts like this:

532 x 52

This is the same as:

532 x (50 + 2)

Using algebra we can then simplify by multiplying 532 with each number in the bracket to get:

(532 x 50) + (532 x 2)

If it's easier to see, you can write it like this:

((532 x 5) x 10) + (532 x 2)

Remember, multiplying by 50 is as easy as multiplying by 5 and then multiplying by 10 by adding a "0" at the end.

Do each bracket individually to get:

26,600 + 1,064

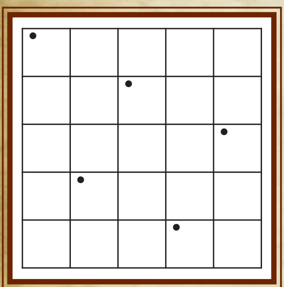
Then do the sum:

27,664

There are little tricks in multiplication and division you will discover. For example in this next spell it is useful to know that dividing by 5 is the same as multiplying by 2 and then dividing by 10.

The Magic

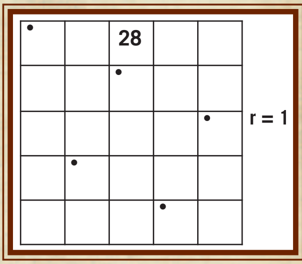
Now for the magic! This time you will perform an incredible trick for someone else. Draw out a grid like this one with 5 squares across and 5 down. Also draw in the dots.



Tell your spectator to think of any number and that for this extreme mental challenge it should be between 60 and 500. Tell them that using their number you will create an incredible gift for them that shows how much of a genius you are.

First you must take the number they give you and subtract 60. Now work out the next smallest number from this that ends with a 5 or a 0. How many do you need to subtract to get to this number? Call the amount you subtract the “remainder” and make a note of it. Now take the number you found that ends in 5 or 0 and divide it by 5 (use the tip above) and write the answer to this in the middle square of the top row.

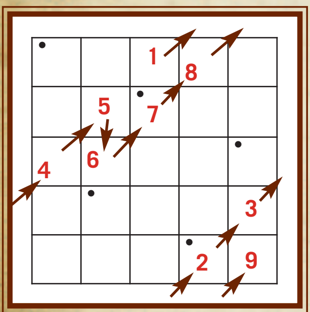
For example, if your spectator picked 201, subtract 60 to get 141. The next lowest number that ends in a 5 or 0 is 140. We have to subtract 1 to get there so the remainder is 1. And 140/5 = 28 so this goes in the middle square of the top row.



You should now fill in every square one by one by completing each square that is diagonally up and to the right of the one you just wrote in with the number that comes after the one you just wrote (so 28, 29, 30, 31…). Each time you can’t move diagonally up and to the right move one square down instead. This diagram shows the order you fill squares in:

Continued...

Mind Expansion Jinx and the Mathematics of Mental Methods



Each time you get to the edge of the grid imagine the grid connects top to bottom and left to right so that if you go off the top you come back at the bottom, and if you go off the right you come back at the left.

So the complete order you fill in the squares should be like this:

•17	24	1	8	15
23	5	•7	14	16
4	6	13	20	•22
10	•12	19	21	3
11	18	25	•2	9

Practice by filling an empty grid with the numbers 1-25 in the right order.

When you do this with your spectator's number, you start with the number you wrote in the middle square of the top row and count up in the same way moving from square to square. BUT when you land on a square with a dot in it, you must also add the remainder to the number you planned to put there. See if you can follow our example where the start number is 28 and the remainder is 1:

•45	51	28	35	42	
50	32	•35	41	43	
31	33	40	47	•50	r = 1
37	•40	46	48	30	
38	45	52	•30	36	

Do the same thing with the spectator's number and once you have made the grid hand it back to them. Ask them if they can see where you've hidden their original number (in the example it was 201). They will say that they can't see it anywhere. Then tell them that in fact their number is everywhere in the grid, get them to add the numbers in any row, or in any column or even the diagonals and they will add up to their number! Incredible! Clearly only someone with a truly magical brain could have the mental power to produce this.

The Secret

One more tip to help your mental methods; if your numbers have decimals or lots of "0s" at the end you can divide or multiply by 10s and take the 10s out of a bracket to make it easier.

e.g.

6000 x 5.32
= (6 x 1000) x (532 / 100)
= (6 x 532) x 1000 / 100
= (6 x 532) x 10

The Prediction Hex and the Mathematics of Sequences

The outcome of a random number generator is predicted

The Tale

A sequence is a set of numbers that follow a certain rule. Each number in the sequence is called a "term". We can often create a formula that tells us what the nth term is (in other words the formula will be an equation with the letter "n" in, when we replace the "n" with "1" the equation will tell us the 1st number in the sequence, when we replace "n" with "2" the equation will tell us the 3rd number in the sequence etc.)

Here are some examples of sequences:

Sequence:	Rule:	Formula:
2,4,6,8,10,12…	Even numbers	(formula is: 2n)
1,3,5,7,9,11,…	Odd numbers	(formula is: 2n-1)
1,4,9,16,25,36,…	Square numbers	(formula is: n^2)
1,10,100,1000,10000	Powers of 10	(formula is: $10^{(n-1)}$)

If we know the terms of the sequence, how would we find the formula for that sequence? Well a useful tool is to write out the sequence and underneath each pair of numbers write the difference of those two numbers like this:

4 7 10 13 16
3 3 3 3

A number in the second row is the difference of the two numbers above it. Notice that in the bottom row there is only one number repeated. In this case where the repeated number is 3 we know the formula has a "3n" in it (if the repeated number was 4 it would have a "4n" in it etc.). We then try "3n" as the formula, but notice how for each term "3n" gives us the

wrong answer. But it is wrong by the same amount (1) each time. So we can correct the formula we tried and now write the formula as "3n+1". Try it and it works!

The Magic

Now for the magic! Take the picture cards out of a pack and arrange the number cards into two piles like this: First lay down two 10s separately face up, then on top of each 10 put a 3 face up, then on top of each 3 put a 6, then a 9, a 2, 5, 8, Ace, 4, 7, 10, 3, 6, 9, 2, 5, 8, Ace, 4, 7.

So the two piles are: 10, 3, 6, 9, 2, 5, 8, Ace, 4, 7, 10, 3, 6, 9, 2, 5, 8, Ace, 4, 7

The 10s are on the bottom, 7s on the top and all the cards are facing up.

Now put your piles side by side. This is your random number generator. You can cut both piles as many times as you like to give you a random 2 digit number (pretend that the 10s are 0s) (to "cut" the cards take as many cards as you like from the top of a pile and place them on the bottom of the same pile so a new number is face up and all the cards are still face-up).

Now find a random number by cutting both piles like this as many times as you like. Have you done that? Good.

Now look at the number on top of the first pile. Either this number, or "10 + this number" or "20 + this number" will divide by 3 to give a whole number. Work out which one of these divides by 3 and divide that number by 3 to get a new number. Add 1 to that new number and remove that many cards from the top of pile 1 and put them on the bottom of the pile.

Now look at the number on top of the second pile, either this number, or "10 + this number" or "20 + this number" will divide by 3 to give a whole number. Work out which one of these divides by 3 and divide that number by 3 to get a new

Continued...

The Prediction Hex and the Mathematics of Sequences

number. Add 2 to that new number and remove that many cards from the top of pile 2 and put them on the bottom of the pile.

You now have your two piles which together show you a new random number with 2 digits. You could have had any random number between 00 and 99 to begin with which would then give you a random calculation to work out how to find the random number you are looking at now. But with my powers of predicting the future, I can tell you that the number you're looking at now is…74

The Secret

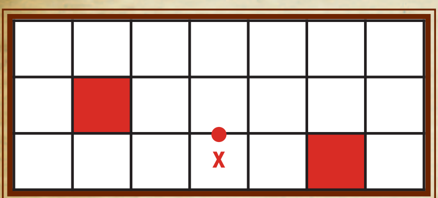
So how does this work? Well if you cut one of the piles so there is a 10 on the bottom (which for us is a "0"), then spread it out, you will notice there is a sequence. Each card is 3 more than the one before it. Any second digit has been ignored so "12" becomes "2" and "14" becomes "4" for example. Touch any card and try to work out how many cards that is away from the "10" before it. Remember when wanting to find a particular term in a sequence we have a formula with "n"s. If each term in the formula is 3 more than the one before it then the formula has "3n" in it. Well here we are doing the opposite, instead of multiplying by 3 we are trying to divide by 3, trying to do the reverse of the formula by taking a card in the sequence and finding out where that number is (2nd, 3rd, 4th number etc.). Moving this number of cards to the back is just like going back this amount in the sequence (if we are on the 3rd number in the sequence and we put 3 cards to the back it takes us back to the 10 [which is the same as "0"] – the same is true for every number). So the first part of what I asked you to do takes you back to 10 no matter what number you start on, taking away 1 more card or 2 more cards (the second part of what I asked) just makes sure your final number is interesting and not 10,10.

Transformation Hocus-Pocus and the Mathematics of Transformations

The power to transform objects

The Tale

There are four different types of transformation you should know about. For the moment I'll give you the power to carry out two of them. The first is "Translation" in which you translate an object or shape by a vector. For example, look at this grid: The red square on the right is a copy of the red square on the left that has been translated along a vector of (4:-1) (So to the right by 4 units and down one unit).



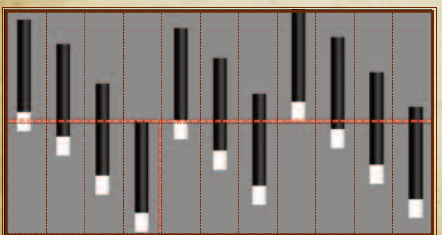
This isn't the only type of transformation we could have done to move the square. We could have also carried out a "rotation" around the point "*x*" by 180 degrees. To do this, imagine we have a copy of this page and we lay it on a board. We then put a pin through the page where the x dot is and spin the page until it has rotated 180 degrees (a half turn). At this time, the original red square on the left will now have moved to where the right hand one is.

Continued...

Transformation Hocus-Pocus and the Mathematics of Transformations

The Magic

Now for the magic! Let's put the powers of transformations to the test and see what wonderful things they can do. Make a copy of this page or use the diagram pages at the end of the book to copy.



Now transform the cut outs like this: First rotate the "MAGIC!" 180 degrees, that is turn it upside down. You'll notice that now instead of saying "MAGIC!" it has transformed and now says "MATH". Now let's try something with the picture of the set of magic wands. Count how many magic wands there are in the picture when the pieces fit together as shown on the page, and write down this number. Carefully cut along the dotted lines in the diagram then put the pieces back as they were.

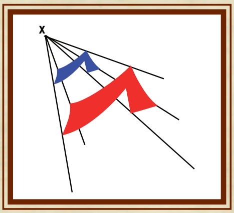
Now the magic! Swap the bottom left piece and exchange it with the bottom right piece (translate the bottom left piece by [7,0] and translate the bottom right by [-4,0]). Now count the wands again with the pieces put together in this order. One of the magic wands has mysteriously disappeared! The transformation has occurred; can you work out where the wand went?

The Secret

There are two more types of transformations that you should know about:

In a reflection, a shape is moved around a line to give a new shape (the "image"). Each point in the image is the same distance from the line as the corresponding point in the original shape. It's just like putting a mirror where the line is:

The final type of transformation is called an "Enlargement". To perform an enlargement we must have a shape, a "centre of enlargement" and a "Scale factor" by which we enlarge the shape by to create the new shape (the "image"). We then draw lines between points on the original shape and the centre of enlargement. Measure these lines, and then multiply the lengths by the scale factor. This then gives the distance between the corresponding point on the image and the centre of enlargement along the same line. Looking at a diagram of this will make it much clearer:



In this diagram the blue shape has been enlarged by a scale factor of 2 around the centre of enlargement "*x*" to give the red shape. The distance between a point on the shape and the centre of enlargement is doubled. We could also say the red shape has been enlarged by a scale factor of 1/2 around the centre of enlargement "*x*" to give the blue shape.

The Wizard of Mischief's payment and the Mathematics of Powers and Indices

The Wizard of Mischief once saved the kingdom from impending doom. The kingdom's master asked what this wizard would like in return for his good deed. The mischievous Wizard replied, "I'd like a chessboard; on the first square of the chessboard please place 2 grains of rice, and on each square after place double the amount of grains that are on the square before. So on the first there are 2, then 4 then 8 etc". The kingdom's master thought it odd to request such an odd and simple reward but he agreed. A week later the master came to the Wizard and told him it wouldn't be possible as the kingdom was not rich enough for this reward. The cunning magic user laughed and told the master it was all a playful joke.

Why did the master struggle to give the magic user his reward? Well we must learn about powers and indices. Indices are small numbers we place to the top right of a normal number. For example 6^5. The index tells you how many times we must multiply the larger number to itself. So 6^5 is $6 \times 6 \times 6 \times 6 \times 6$.

The magic user asked, simply, for 2 grains of rice on the first square. (This is 2^1 – we are multiplying only one 2).

But then asked for double on the next square (This is $2 \times 2 = 2^2 = 4$).

Then double again on the next square (This is $2 \times 2 \times 2 = 2^3 = 4$).

This is continued until we get to the 64th square which would have 2^{64} grains of rice. You might be surprised at how large this number is. It is

18,446,744,073,709,551,616

grains of rice! This is why the master had trouble; there wasn't enough rice in the kingdom.

Haunted Woods Magic and the Mathematics of Locus

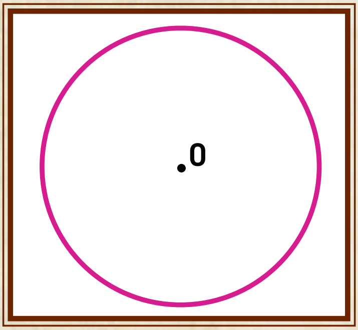
A card chosen by you is hidden in the haunted woods and found

The Tale

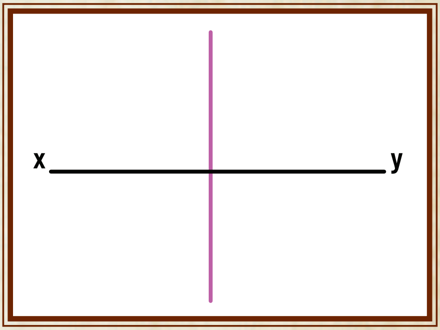
"Locus" is the path given by certain points that satisfy a given rule. You may be asked to draw several of the following examples on one diagram (In the examples the locus is given by the purple line). Each line will tell you information about where the specific point could be. The points where locus lines cross is where several rules are satisfied by that point.

e.g.

1) The locus of a point that is a fixed distance (e.g. 2cm) from the point O :



2) The locus of a point that is equidistant from two other points *x* and *y*. This is the perpendicular bisector of the line that joins *x* and *y*:



3) The locus of a point that is a fixed distance from the line joining *x* and *y*

Continued...

Haunted Woods Magic and the Mathematics of Locus

The Magic

Now for the magic! In a moment I will show you 5 cards. I will ask you to think of one. I will then have a look through the cards and try to pick out the one I think you chose. Then I will hide it in the haunted woods for you to dig up and retrieve.

So here are the 5 cards; memorise one of them:

Now concentrate on it. With my mental telepathy skills I will read your mind. I think I've worked out which card is yours, I'll remove it from the pack :

I'll remove it from the pack as you turn the book upside down. Did I manage to remove it successfully? Quick let's find it again in the haunted forest. Here is a map of the forest (the height and width of each square is 1 mile):

First turn your card you are thinking of into two numbers:

For the first number:

- A Jack becomes a 1, a Queen becomes a 2, a King becomes a 3.

For the second number:

- A Club becomes a 1, a Heart becomes a 2, a Spade becomes a 3, a Diamond becomes a 4.

So, for example, the Queen of Spades becomes (2,3).

I will now tell you where I buried your card with a riddle using locus. Use the examples from earlier to find your card again.

The Mountains

5	14	3	18	7
10	17	12	1	20
19	6	8	15	16
2	13	11	9	4

The Wall (bottom) — **The River** (right side)

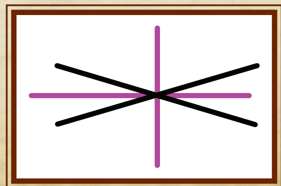
The first number of your card is how many miles away from the wall your card is. Your card is also more than 1 mile away from the centre of the mountains. Add 1 to the second number of your card and this is how many miles away from the river your card is buried. Now find the number on the square that your card is buried at. This is a page number. If I guessed your card correctly then you should turn to this page number of this book and see your card next to the number.

The Secret

How did I know which card you would think of and how did I know to bury it in the right place so you would find it again? The answer is that in fact every single card I showed you at the beginning was exchanged for another. So the truth was that I removed every card, so I knew I must have removed yours.

I also buried all five of the initial cards in the forest and, changing what your card was, changed the burial place. So in fact, your card was never actually known.

Another locus you may need to know is the locus of a point that is the same distance from two crossing lines. The locus will be on one of the two lines that bisect the angles where the crossing lines meet.



Again, these are the purple lines in the diagram.

There is a larger copy of this map for you to use at the back of the book.

Thought Reading Sorcery and the Mathematics of Factors

Gives you the power to read your spectator's hidden thoughts

The Tale

The factors of a particular number (for example 15) are the numbers that divide exactly into that particular number (the factors of 15 are 1,3,5,15 as each of these numbers divides 15 exactly, leaving no fractions or decimals).

A prime number (for example, 11) is a number that has only two factors. These factors are 1, and itself (11). There are no other numbers that can divide the prime number to give a whole number.

A prime factor is a factor that is also a prime number. Every number can be written as a set of prime numbers multiplied together. For example, $18 = 2 \times 9$ but 9 is 3×3. So $18 = 2 \times 3 \times 3$, it is now written as the multiplication of prime numbers only.

The Magic

Now for the magic! This time you will perform the magic on someone else. Make sure they have a calculator and get them to multiply together any single digit numbers (single digit numbers are: 1,2,3,4,5,6,7,8,9) until they have a long number that takes up the whole calculator screen or a number that is pretty long. A "digit" is the name for the symbols used when writing numbers, for example the digits of 238 are 2 and 3 and 8. When they have done this tell them they can now choose one digit on their calculator screen. The rest of the numbers they should read out to you in any order they like, not reading the number they are hiding. You tell them you will then read their mind and tell them the number they are hiding. Secretly what you are doing is adding up the individual numbers they tell you (for example they might tell you 3,6,7,9,3,5 and in your head you work out $3 + 6 + 7 + 9 + 3 + 5$). When you have done this you will have a number in your head (for example, 15) then find the next highest number in this list:

9,18,27,36,45,54,63,72,81.

For example, if the number in your head now is 15 then the next highest number is 18. Then minus the number in your head from the number you just looked at in the list. The answer to this is the number they are hiding from you. (So if the number in your head is 15, the next highest number in the list is 18 and 18 - 15 = 3, so they are hiding the number 3). Tell them this number and pretend you have read their minds.

If the number you have in your head (after the adding) is already in the list, then the number they are hiding is either a 0 or a 9. You can say to them that you aren't sure because they have been quite sneaky but you know it's either a 0 or a 9. You can then make a guess, if they seem sneaky they will usually hide the 0 from you, if you guess and it is wrong then you know it's the other one.

The Secret

The reason why this works is to do with factors. Your participant has multiplied lots of small numbers together to make a big number. For example they might have done 4 × 6 × 3 × 6 × 9 × 1 × 4 × 2 × 5. There are 3 digits on a calculator that have factors of 3 (these digits are 3 which is obviously divisible by 3, 6 which is 3× 2 and 9 which is 3 × 3) If the final number has at least two of these as factors then it must have at least two 3s in its prime factors. For example let's look at 4 × 6 × 3 × 6 × 9 × 1 × 4 × 2 × 5 again and look at the prime factors. Well the prime factors of 4 are 2 × 2, the prime factors of 6 are 2 × 3, the prime factors of 9 are 3 × 3, so if we replace these numbers in 4 × 6 × 3 × 6 × 9 × 1 × 4 × 2 × 5 by their prime factors we get 2 × 2 × 2 × 3 × 3 × 2 × 3 × 3 × 3 × 1 × 2 × 2 × 2 × 5. If we are multiplying together numbers, we can do it in any order we like (2 × 3 is the same as 3x2, and 2 × 3 × 2 is the same as 2 × 2 × 3). So we can group all the numbers that are the same together to get 1 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 3 × 3 × 5. We can then swap a 3 × 3 for a 9 as 3 × 3 = 9.

Continued...

Thought Reading Sorcery and the Mathematics of Factors

This means that the final number has a factor of 9. It is very likely that this will happen as you are asking for a big number and it is likely they will press one of 3,6 or 9 more than once.

Now we use a secret of maths. Any number that is divisible by 9 (for example, the number your spectator makes on their calculator) has digits that add up to another number that is divisible by 9. The list I gave you above was just a list of the first few numbers that were divisible by 9. So you know the sum of all the numbers on the screen must add up to one of the numbers on this list. So when you add up your numbers all you will need to do is work out how much more you need to add to get a number in the list (a divisor of 9) and this will be their number. If they left out a 0 or a 9 then the numbers they give you will already add up to a number on the list and so you have to make the guess of which number they left out.

A Magic User's Investigation of the Mathematics of Sine and Cosine

Investigate when the actions of the sinful magic users will be resolved

The Tale

Later we will see that using Pythagoras' theorem we can find out the length of one side of a right angled triangle (a triangle where one of the angles is 90 degrees) if we know the lengths of the other two. How about the angles of a right-angled triangle? If we know 2 angles can we know the third? Or if we know 2 lengths can we work out an angle? The important answer lies in this piece of magic.

The Magic

With so many magic users in the world right now have you ever wondered how strong the negative effects of the evil magic users are? Well luckily your calculator comes with buttons that give you a measure of the sin caused by the world's evil magic users and the cost of their actions at any particular time. The measures come between 1 and -1, the higher the number, the more sin/cost is being created.

Try it for yourself, have a look at how much sin is being caused by the magic users right now by using the "sin" button on your calculator. For the sin you'll want to replace "t" in the following equation with the current time: Sin(30 × t). You can put in any time you like (you can even put in decimals e.g. two thirty is half past two so 2.5). If you try out different times you will see how the sin varies with each time. A similar thing happens with cos (measuring the cost).

Now pick any time of day and write down this time (like 6: but don't pick 6, this is just an example!). We will now look at the sin caused at this time, the cost it takes to repair the damage, and the time by which this damage will be repaired.

To work out this damage repair time use this formula:

Damage repair time: Cos(90 - 30t)/ Sin(30t).

Continued...

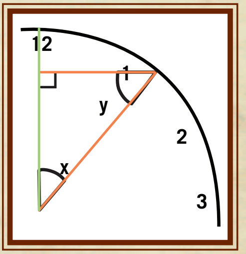
A Magic User's Investigation of the Mathematics of Sine and Cosine

If you're getting worried about the damage caused by the magic users at this time you have chosen to investigate then keep calm; there is an announcement in the back of the book.

The Secret

So perhaps you have read the message from the chief magic user. Perhaps you have noticed that even though you chose the time to investigate and that the calculation seemed random, he still knew your calculations would show the damage would be fixed by 1. Well as usual the damage repair time should always be 1. To explain this let's look at part of the face of a clock.

Notice that when you choose your time, if it is in this section of the clock, the line the hour hand would make can be joined to the 12 o'clock line horizontally to make a right-angled triangle.



Now I should tell you the truth about the sin and cos buttons. They do not actually measure the sin and cost of a magic user's actions. They actually tell us the angles within a right-angled triangle. We are looking now at the angle shown at the centre of the clock (x).

We'll call:

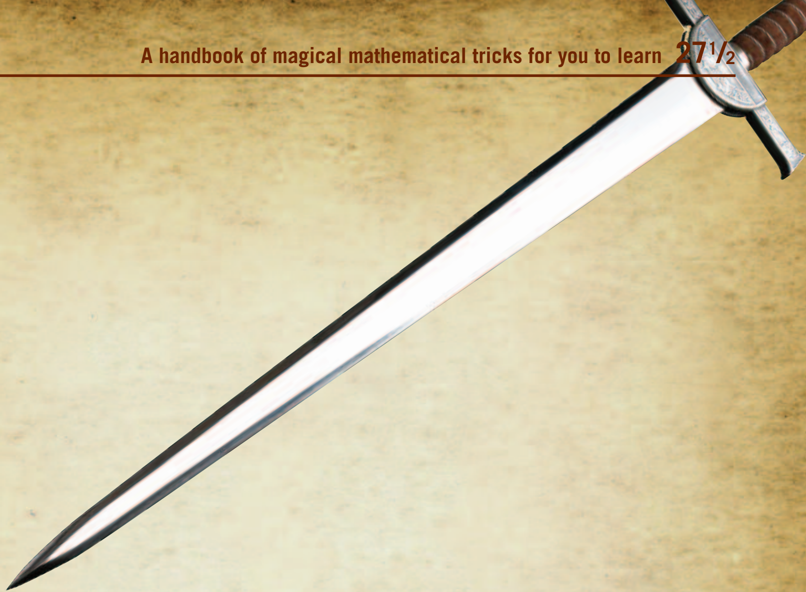
The longest side (clock hand pointing to the 1) the Hypotenuse.

The side opposite the angle "x" the Opposite.

The remaining side next to the angle "x" the Adjacent.

"Sin" actually represents the "Sine" function (a "function" is something we would do to a number like squaring or doubling). If we use the sine function on an angle we will be given a number equal to Opposite / Hypotenuse.

Sin(x) = O/H



"Cos" actually represents the "Cosine" function. If we use the cosine function on an angle we will be given a number equal to Adjacent / Hypotenuse.
Cos(x) = A/H

Also

"Tan" represents the "tangent" function. If we use the tangent function on an angle we will be given a number equal to Opposite / Adjacent.
Tan (x) = O/A

Practicing with these functions will show you that if you know some information about the sides and other angles of a right angled triangle you can often find out any other information you need.

Now notice that if I take the time you chose on the clock and multiply it by 30, this actually gives me the angle "x" made between the hour hand and the 12 o'clock direction. Now that I have the angle in the triangle I can use the sine and cosine functions.

First remember that the angles in a triangle add up to 180. You know the square angle is 90 so the other angle we haven't looked at must be 180 - 90 - x = 90 - x. We can call this angle y. But when we are working with angle y the labels we give to the sides change. In fact the Opposite for angle x is the Adjacent for angle y and the Adjacent for angle x is the Opposite for angle y. This means that cos(y) = sin(x) (because y = 90 - x) or:
Cos(90 - x) = sin(x)

This gives
Cos(90 - x) / sin(x) = 1

But this is when the xs are in angle form. Originally you were asked to find the angles as a time, and to convert the time to angle we saw you have to multiply by 30. This gives:
Cos(90-30t) / sin(30t) = 1

So the answer will always be 1!

Impossibility Enchantment and the Mathematics of Probability

An impossible event occurs not once but three times

The Tale

Let's look at probability. The "probability" that something will happen tells you how likely it is to happen. The "probability" is a number between 0 and 1. If the number is 0 then it means there is no probability (no chance) that it will happen. If the probability is 1 then it means it will definitely happen. The closer the "probability" is to 1, the more likely it is to happen.

Remember in fractions $\frac{0}{6} = 0$, $\frac{6}{6} = 1$. If the top and bottom are the same number then it is equal to 1. Remember, in fractions we can always divide the top and bottom of the fraction by the same number. So in 6/6 we can divide the top and bottom by 6 to get $\frac{1}{1} = 1$. Or in $\frac{2}{6}$ we can divide the top and bottom by 2 to get $\frac{2}{6} = \frac{1}{3}$.

For example, the probability of flipping a coin and it landing on heads is ½. The top number says how many different outcomes there are to the action we are looking at (we are looking at 1 outcome – that the coin lands on heads). The bottom number says how many total outcomes there are (there are 2 different outcomes, heads or tails). This gives ½.

As another example, what is the probability that we will roll a dice and get a number 5 or 6? The top number in the fraction is the number of outcomes we are looking at (2 outcomes as we are looking at the probability the dice gives a 5 or it gives a 6). The bottom number is the total number of outcomes there are (there can be 6 outcomes, it can be 1 or 2 or 3 or 4 or 5 or 6). So the "probability" we get a 5 or 6 is 2/6 and by the rules of fractions we can divide the top and bottom numbers by 2. This gives ⅓.

The Magic

Now for the magic! I will perform the crazy magic on you! Take a pack of cards (it must be a complete pack of 52 with no cards missing). Pick out any three cards and put them to the side. Also write the names of these three cards down on a piece of paper.

Now take the rest of the cards and deal them face down into 4 piles next to each other. Deal them so that in the first pile there are 12 cards, in the second pile there are 15, in the third pile there are 15 and in the last pile there are 7.

Now take the three cards you picked out at the beginning. Place the first card on the first pile and then take some cards from the second pile and put them on top of the first card on the first pile (it doesn't matter how many you take, it's your choice). Then put the second card on the second pile and again take some cards from the third pile and put them on top of the second card on the second pile. Then put the third card on the third pile and again take some cards from the fourth pile and put them on top of the third card on the third pile.

Then put the fourth pile on top of the third pile, then that pile on top of the second, then that pile on top of the first so you have one pile of cards with the cards you chose somewhere randomly in the pack.

Hold the pack face down. What is the chance the top card is one of your 3 cards that you chose at the beginning?

Well we are looking at the 3 possible outcomes that the top card is the first card you picked, the second card or the third card.

The total number of outcomes the top card could be is 52 as there are 52 cards in the pack so there are 52 different cards the top card could be.

This means that the "probability" that the top card is one of your chosen cards is $\frac{3}{52}$.

Continued...

Impossibility Enchantment and the Mathematics of Probability

It is not very likely, this number is closer to 0 than to 1, so get rid of that top card and put it on the table. Let's say that the card now on top could be one of your cards, so let's save it just in case and put it on the bottom of the pack.

Now what is the chance the new top card is one of your three cards? There is still 3 ways the top card could be your card but now there is one less card in the pack (there are now 51). So assuming you didn't just get rid of one of your cards the chance the new top card is your card is 3/51. Again, this is not very likely, so get rid of this card as well, put it on the table, and again let's say the next card could be your card, so save it and put it on the bottom of the pack just in case.

Now do this again and again. Put the top card on the table, then put the next card on the bottom of the pack, then the next card on the table, then the next on the bottom… . Do this until you only have 3 cards left. Remember what happened in the trick, you selected 3 cards, then they were placed in random parts of the pack, then slowly you got rid of all the cards until only 3 cards were left. It seems it would be very unlikely that these 3 cards were the cards you picked at the beginning but turn them over. If everything has gone perfectly, you should now magically be holding your first 3 cards.

The Secret

This is a magic trick and should always work, but I'm keeping this one a secret! See if you can work out how you ended up with your 3 cards. Now for something a little more advanced: What would be the probability of taking three cards out of a pack, putting them back, shuffling, and taking another three cards out and these new three cards are the same as your first ones? Well what is the probability the first card you pick is the same? It is $\frac{3}{52}$ again.

Now one of the three cards have been removed, so now there are only 2 outcomes we're looking at left and only 51 cards. So what is the probability the second card you pick is one of the first 3? It is now $\frac{2}{51}$.

And then the probability the third card you pick is also one of the three is $\frac{1}{50}$.

If we want to look at the probability of several different events happening, we multiply the probabilities together. For example, the probability of getting two heads is the probability of getting a head first and the probability of getting a head second so = ½ x ½. = ¼. In the same way we would guess the probability of picking out the three cards is $\frac{3}{52} \times \frac{2}{51} \times \frac{1}{50}$. But in reality, there are 6 different ways to pick out the three cards. If we label the three cards A, B, C, we could pick them out in the orders: ABC, ACB, BAC, BCA, CAB, CBA. As there are 6 ways to pick the cards, the probability of picking the three cards is actually 6 times more likely than what we just guessed so it is $6 \times \frac{3}{52} \times \frac{2}{51} \times \frac{1}{50}$. = $\frac{36}{132600}$ = 0.00027149… Which is a lot closer to 0 than 1. This means that the probability of the trick working seems to be so low, almost impossible! Or maybe it's magic.

Your Element of Destiny and the Mathematics of Pythagoras' theorem and Simultaneous equations

Mystic Mandy predicts your element of destiny

The Tale

You are about to learn a bizarre fact to do with triangles. Get your rulers out for this trick. Before we go into it we'll revise important words to do with triangles:
A triangle is "right-angled" if one of it's angles is 90 degrees. It's also important to know that the longest side of a right-angled triangle is called the "hypotenuse".

We should also mention "simultaneous equations". This is a set of equations that use the same letters or symbols (for example, x and y). Usually we try to find points where all of the equations are correct at the same time. This can be done in two ways. To look at these two ways let's look at an example of simultaneous equations:
$Y = x$ and $y = -x + 2$

The first way we can solve the equations is graphically. This means we draw the lines that represent the two equations on the same graph and find where they cross.

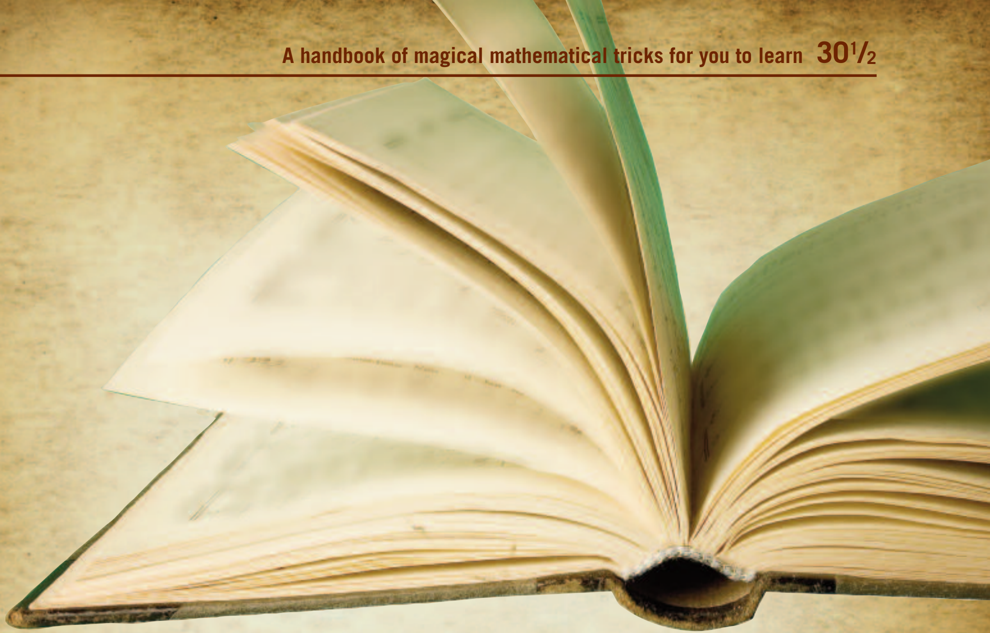
Each line from each equation shows you when that particular equation is correct. So at the points where the lines cross we know both equations are correct (as the point is on both of these lines). The x and y values of this point then must solve both equations and are then the solution to the simultaneous equations.

The second way to solve the equations is algebraically. One way to solve them algebraically is "substitution". This is where we substitute information from one equation into the other equation. The first equation above says that x=y, so in the second equation we can replace x with y to get **$y = -y + 2$.**

Then if we add y to both sides we get
$y + y = 2$

which is the same as
$2y = 2$

then we divide both sides by 2 to get
$y = 1$



Now we can put y = 1 into the first equation to get 1 = x. So the solution to the equations are x = 1, y = 1. Sometimes we may have to manipulate the equation first before we can substitute.

The other way to solve the equations algebraically is with "elimination". The equals sign means that the items on both sides of the sign are exactly the same. So y=x means that adding y is the same as adding x. We can use this on the second equation and add y to the left side and x to the right side, (we are adding the same thing to both sides so the second equation must still be true – if this is hard to imagine then try it with numbers, if you had an equation like 2 × 4 = 8 then after adding 1 to both sides the equation still must be true, [2 × 4] + 1 = 8 + 1).

After doing this we get

$y + y = x - x + 2$

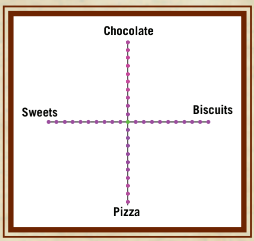
which is the same as

$2y = 2$

In the same way as before this means **$Y = 1$ and then $x = 1$.**

The Magic

Now for the magic! You'll need your rulers for this one. The Wizard of Dreams claims that from knowing how much you like certain foods they can tell you your element of destiny (fire, earth, water, air). Take a look at this diagram there is an enlarged version at the back of the book:



On the diagram there are four lines coming away from the centre green dot. You should put a mark on each line on one of the purple/pink dots. If you like the food that is at the end of the line a lot, then put a mark on a purple/pink dot that is far away from the green dot. If you don't like the food then put the mark closer to the green dot.

Continued...

Your Element of Destiny and the Mathematics of Pythagoras' theorem and Simultaneous equations

Now draw a line from your "chocolate" mark to the "biscuit" mark and label this line "*a*".

Now draw a line from your "biscuit" mark to the "pizza" mark and label this line "*b*".

Now draw a line from your "pizza" mark to the "sweets" mark and label this line "*c*".

Now draw a line from your "sweets" mark to the "chocolate" mark and label this line "*d*".

Now get your ruler and measure the length of line a, write "*a* =" and then write the length. Do the same thing for line *b*, *c* and *d*.

Now get your calculator and work out what a^2, b^2, c^2 and d^2 are. Write the answers down as well.

Now write down the answer to "$a^2 + c^2$" and write the answer to "$b^2 + d^2$" and divide the answer of "$a^2 + c^2$" by the answer of "$b^2 + d^2$".

So you are doing: "$(a^2 + c^2) / (b^2 + d^2)$". Round the answer to this to the nearest whole number. Now find this number here and find your element of destiny:

1) Fire
2) Earth
3) Water
4) Air

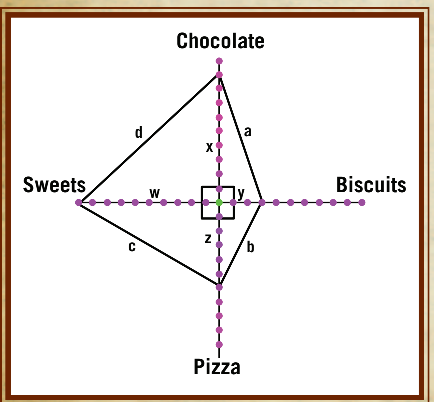
Now would it surprise you again if the Wizard of Dreams predicted what your element of destiny was and commanded I write it here in this book? Your element of destiny is FIRE!

The Secret

The Wizard of Dreams is not as mystical as you might think, he actually always knew you would end up choosing fire. How did he's know this? Well it was with the help of "Pythagoras' theorem". In this theorem if "h" is the length of the longest side of a right-angled triangle and "a" and "b" are the lengths of the two other sides. Then it is always true that $a^2 + b^2 = h^2$.

So how did this help? Well first notice that the shape you have drawn is divided up by the vertical and horizontal lines. This makes 4 right-angled triangles that are touching each other. Their "hypotenuses"

are on the outside. Now, let's look at the shape you drew (remember, I didn't know how much you would like each food, it could have been anything from the 1st dot to the 10th dot). As I don't know the lengths of the small sides we will label them "*x*", "*y*", "*z*" and "w" just like in algebra. And remember I asked you to label the lines you drew as "*a*", "*b*", "*c*", "*d*". So let's put these symbols on a diagram:



Remember, I don't know what any of the actual lengths are, which is why I am using letters and algebra. Now we will use Pythagoras' theorem on the four triangles. The theorem tells us that:

$x^2 + y^2 = a^2$
$y^2 + z^2 = b^2$
$z^2 + w^2 = c^2$
$w^2 + x^2 = d^2$

Now we use the ideas from solving simultaneous equations (elimination method). Take the first equation and minus the second equation so:

$x^2 + y^2 - y^2 - z^2 = a^2 - b^2$
if we "cancel" the $y^2 - y^2$

$x^2 - z^2 = a^2 - b^2$
If we do the same thing with the 3rd and 4th equation (try it) we get:
$z^2 - x^2 = c^2 - d^2$

So we have:

$x^2 - z^2 = a^2 - b^2$ and $z^2 - x^2 = c^2 - d^2$

Continued...

Your Element of Destiny and the Mathematics of Pythagoras' theorem and Simultaneous equations

If we add these two equations together we get:
$x^2 - z^2 + z^2 - x^2 = a^2 - b^2 + c^2 - d^2$

Cancelling again gives:
$0 = a^2 - b^2 + c^2 - d^2$

Rearranging gives:
$a^2 + c^2 = b^2 + d^2$

Dividing by "$b^2 + d^2$":
$(a^2 + c^2)/(b^2 + d^2) = 1$

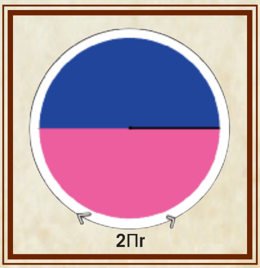
Notice that this equation is what I asked you to do with the lengths of the sides. So just like Wizard of Dreams' last experiment, it doesn't matter what shape you draw, you will always end up with 1!

The Impossible Drinking Glass and the Mathematics of Circumference and Circle Area

Take a glass, a pint glass if you can find one, and place it on the table. Which do you think is larger: the length around the top of the glass, or the distance from the top of the glass to the table? Make a guess. Now put a pack of cards underneath the glass. Now which do you think is larger, the length around the top or the distance from the top of the glass to the table? Now take the cards out the box, put the cards on the box and the glass on top of that. Now which is larger? Try putting a book underneath too and ask the same question. If you thought the distance to the table was larger, it is quite likely you were wrong! Get a piece of string and wrap it round the top of the glass. Put a mark where the end of the string meets the string again so you have a measure of the length. Now hold the string up against the side of the glass and use the mark to see how many things you would need to put under the glass for the two lengths to be equal. The optical illusion caused by the shape of the glass may surprise you! Your brain will take a guess at the length around the outside of the glass by looking at the distance of the "diameter" (the distance from one point on the rim to the opposite point – we'll call it "d"). But the distance around the outside ("the circumference") is in fact ∏ × d. ∏ is a number on your calculator that roughly equals 3.141. So for the two lengths to be equal the length to the table must be nearly 3 times the diameter of the glass.

∏ is defined to be the length of the circumference divided by the diameter – this is the same for all circles.

Whilst we are talking about circumference we should also mention the area of a circle. The area of a circle is $\prod r^2$. Here is a short explanation why. First let's label the distance from the centre to the edge of a circle "r", this is the "radius" – it is half of the diameter. If the circumference is "∏d" then this must also be equal to "2∏r" as d = 2r. The circle below is labelled with these lengths.

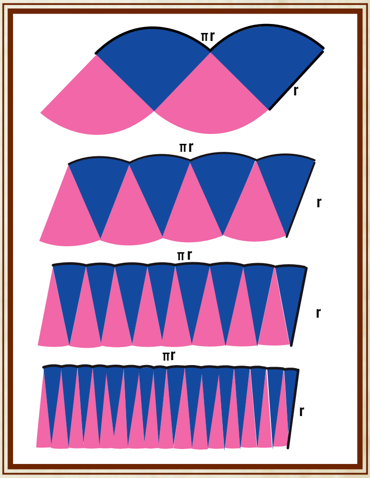


Continued...

The Impossible Drinking Glass and the Mathematics of Circumference and Circle Area

Notice that if the entire circumference is $2\prod r$ then the circumference around the blue half only must be half of this, so it is $\prod r$.

If we then cut the circle into quarters and place them together differently we can label the edges with these lengths. Then cutting the circle into smaller and smaller pieces and rearranging them you will see that we can make something more and more similar to a rectangle. The area of the rectangle is the "height *x* width" which is $\prod r$ x r which is $\prod r^2$ which must then equal the area of the circle.

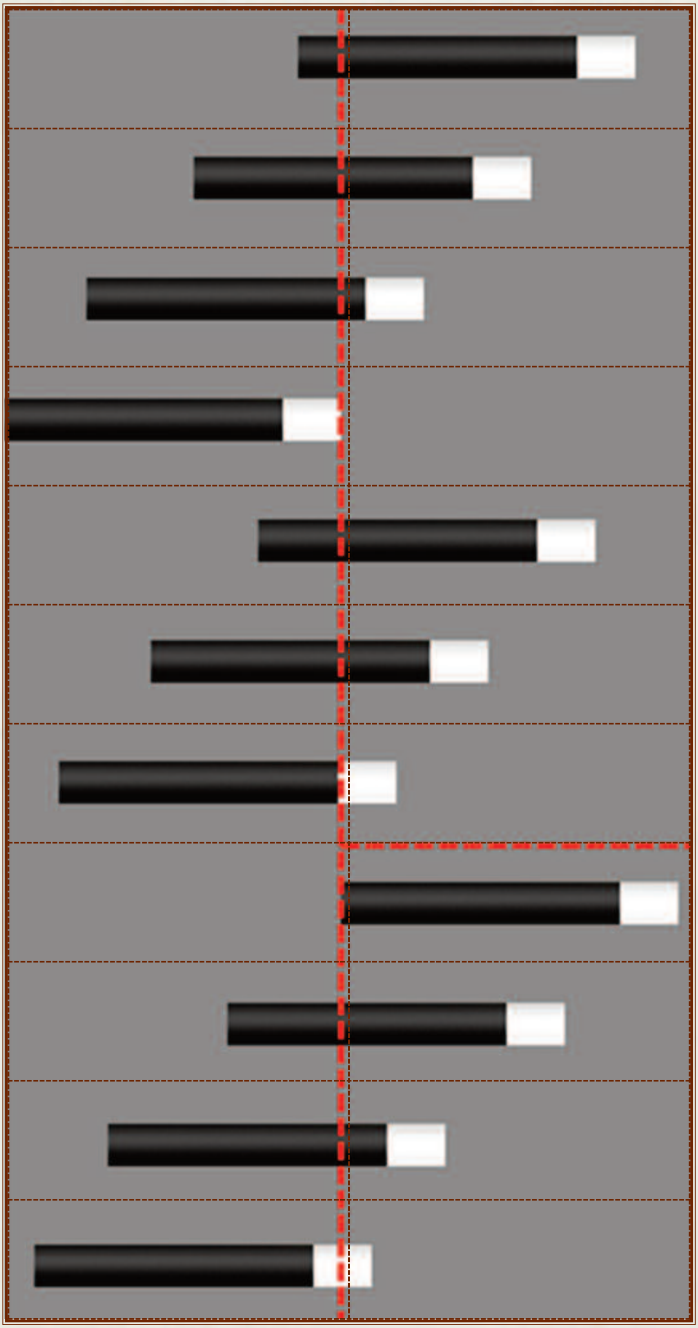


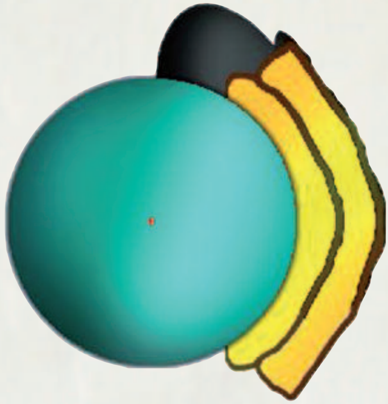
Dear All,

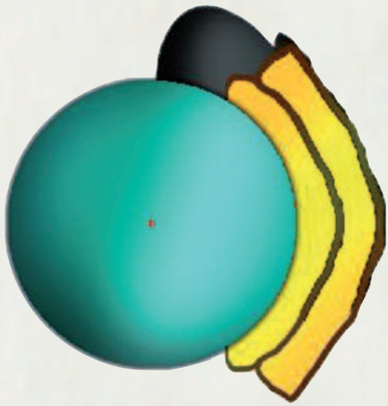
This is a message from the national magic users council, we've heard you might be worried about the damage caused by the magic users that sin. I can assure you that everything will be fine. For the time that you are enquiring about in particular the damage will be cleared up completely by one o'clock as I'm sure you have calculated already. How did we know the clear-up time for your particular enquiry? Well, we are magic users.

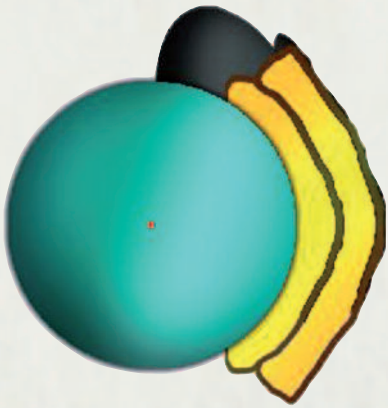
Yours,

Chief Magician.





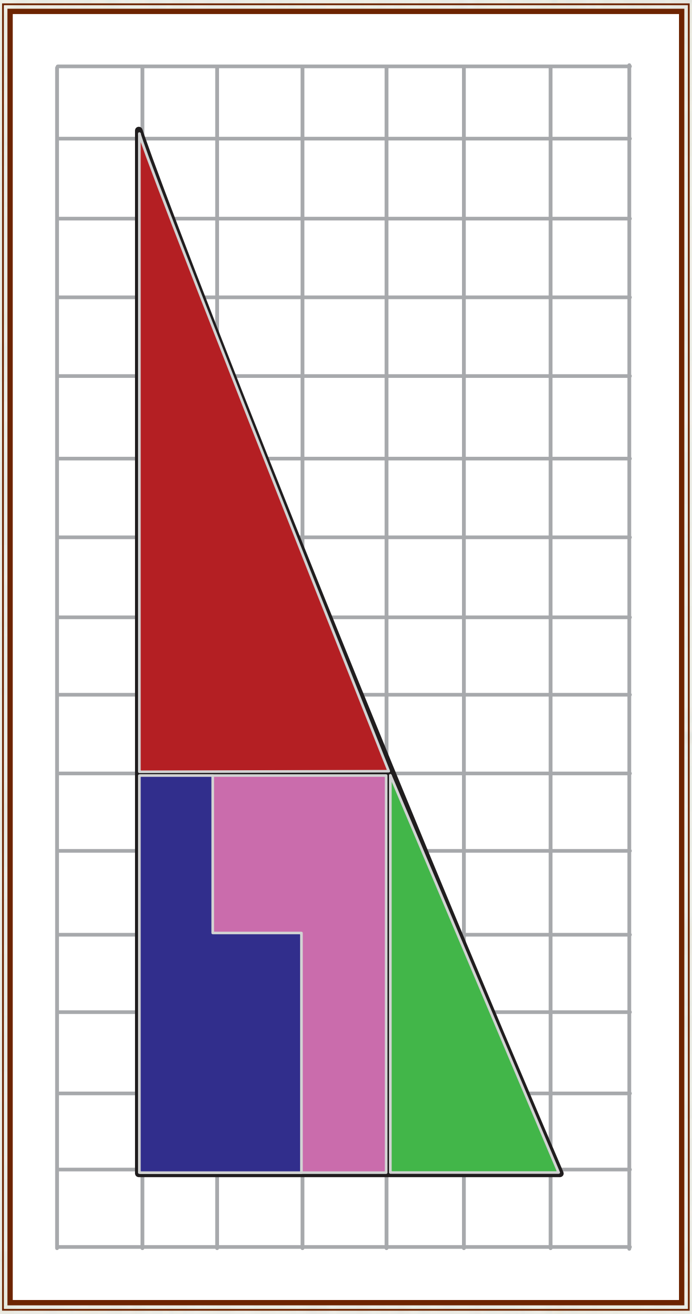


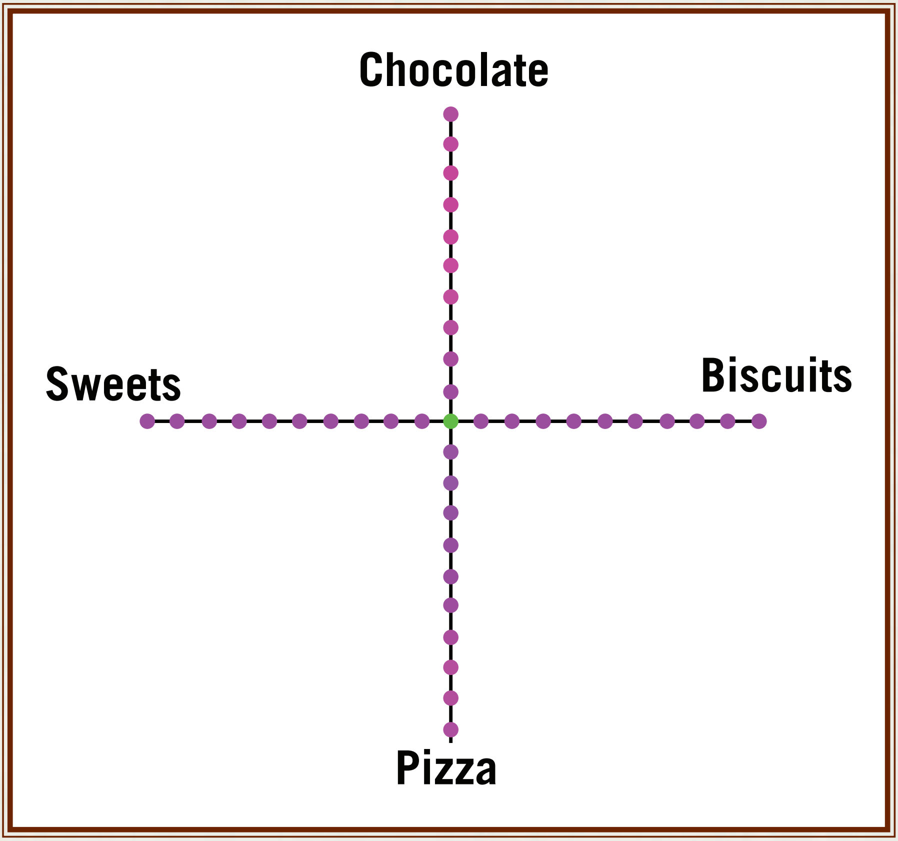


The Mountains

5	14	3	18	7
10	17	12	1	20
19	6	8	15	16
2	13	11	9	4

The Wall




Chocolate
Sweets
Biscuits
Pizza

You can cut out these images and secretly swap one for the other behind your back, see page 24 for details of the trick.

Use this parchment to create your own mathematical spells and ideas.

P.S.

In case you're wondering where the wand that vanished in Transformation Hocus-Pocus went, all is revealed at http://www.cs4fn.org/magic/teleportingrobot.php

This book was developed as part of the Strategic Partnership between Queen Mary, University of London and the University of Warwick.