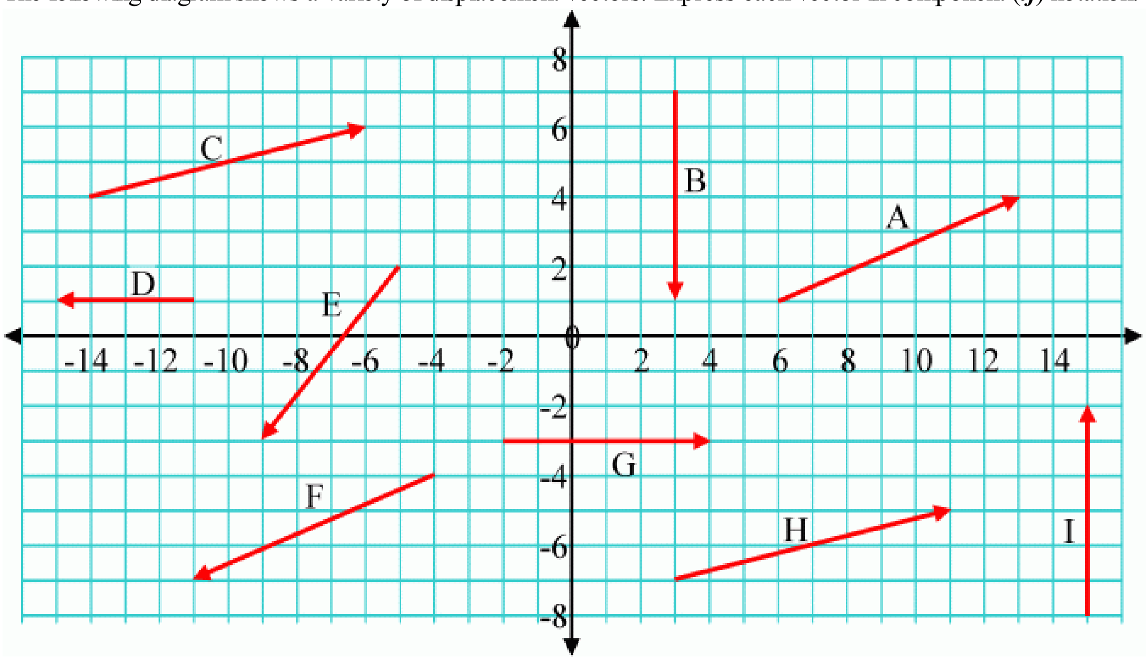
Physics 1100: Vector Solutions

1. The following diagram shows a variety of displacement vectors. Express each vector in component (***ij***) notation.



(i) $\vec{A} = \hat{i}7 + \hat{j}3$ (ii) $\vec{B} = -\hat{j}6$

(iii) $\vec{C} = \hat{i}8 + \hat{j}3$ (iv) $\vec{D} = -\hat{i}4$

(v) $\vec{E} = -\hat{i}4 - \hat{j}5$ (vi) $\vec{F} = -\hat{i}7 - \hat{j}3$

(vii) $\vec{G} = \hat{i}6$ (viii) $\vec{H} = \hat{i}8 + \hat{j}2$

(ix) $\vec{I} = \hat{j}6$

Note that a vector such as (i) may be written as **A** = ***i***7 + ***j***3 when typed, as it is easier to produce since arrow and hat symbols are not common, or as $\vec{A} = \langle 7,3 \rangle$ in math class.

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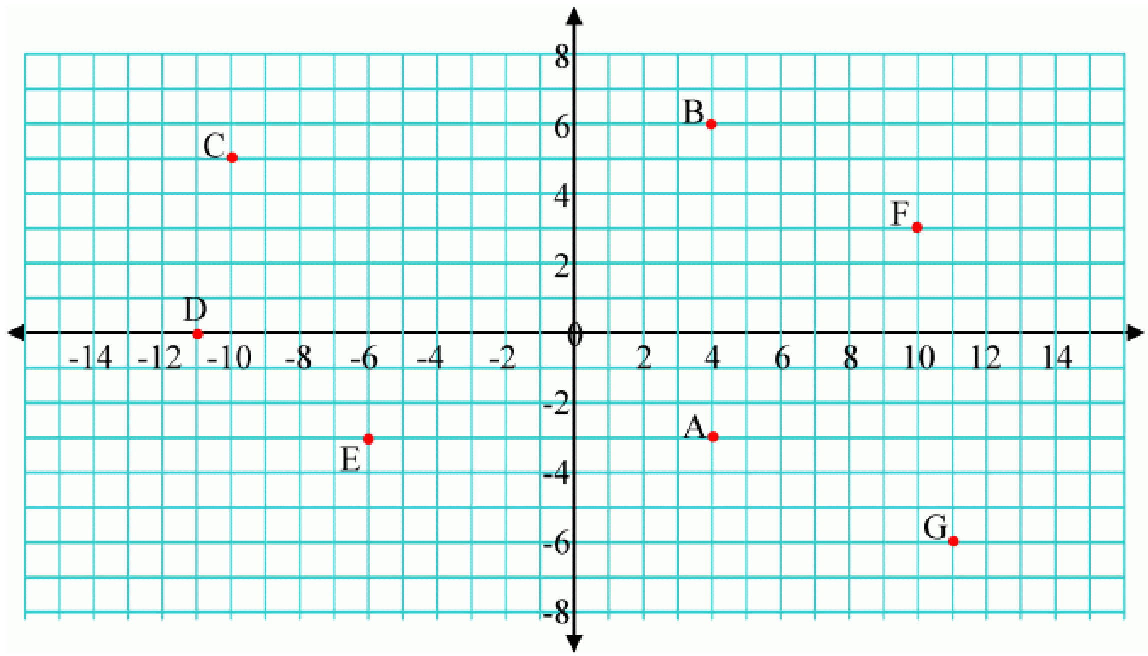
2. Find the vectors that point from A to the other points B to G. Express each vector in component (***ij***) notation.



(i) $\vec{A}_B = \hat{j}9$ (ii) $\vec{A}_C = -\hat{i}14 + \hat{j}8$

(iii) $\vec{A}_D = -\hat{i}15 + \hat{j}3$ (iv) $\vec{A}_E = -\hat{i}10$

(v) $\vec{A}_F = \hat{i}6 + \hat{j}6$ (vi) $\vec{A}_G = \hat{i}7 - \hat{j}3$

Note that a vector such as (i) may be written as $\mathbf{A}_B = \mathbf{j}9$ when typed as it is easier to produce since arrow and hat symbols are not common or as $\vec{A}_B = \langle 0,9 \rangle$ in a math class.

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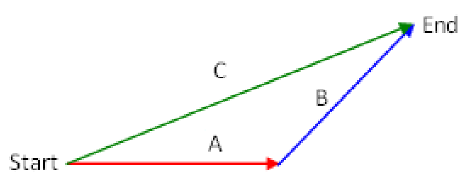
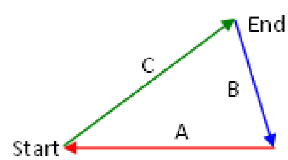
3. Write vectors equations for each diagram below.

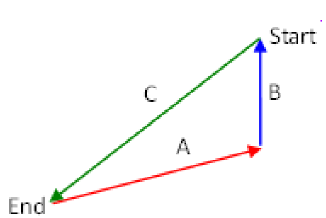
(a)



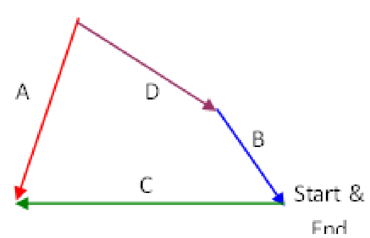
(b)



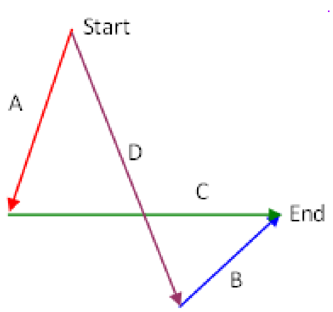
(c)



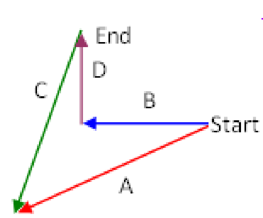
(d)



(e)



(f)



(a) $\vec{C} = -\vec{A} + -\vec{B}$ (b) $\vec{C} = \vec{A} + \vec{B}$

(c) $\vec{C} = -\vec{B} + -\vec{A}$ (d) $\vec{C} - \vec{A} + \vec{D} + \vec{B} = 0$

(e) $\vec{A} + \vec{C} = \vec{D} + \vec{B}$ (f) $\vec{A} - \vec{C} = \vec{B} + \vec{D}$

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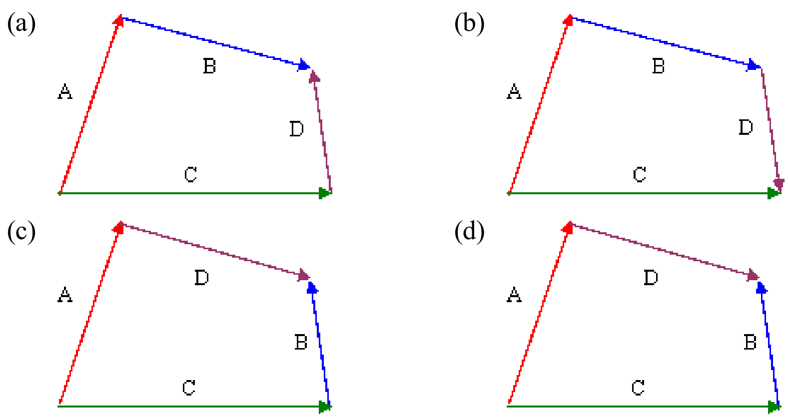
4. Sketch vector diagrams for the following vector equations.

(a) $\vec{A} = \vec{C} + \vec{B} + \vec{D}$ (b) $\vec{A} + \vec{B} = \vec{C} + \vec{D}$

(c) $\vec{A} = \vec{C} + \vec{B} - \vec{D}$ (d) $\vec{A} + \vec{B} = \vec{C} - \vec{D}$



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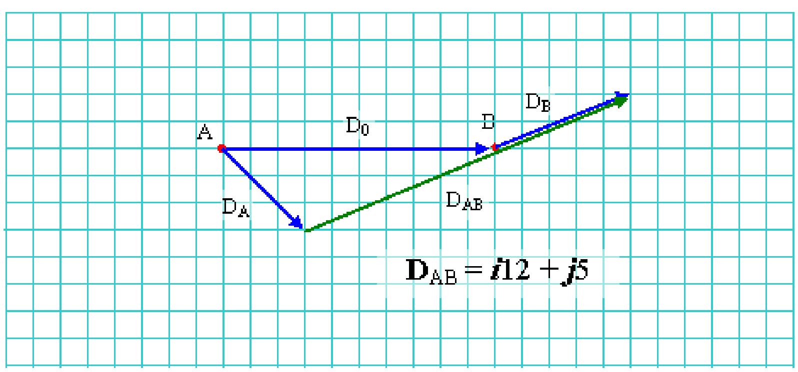
5. Person B is 10 m to the right of person A. Person B walks a distance $\vec{D}_B = \hat{i}5 + \hat{j}2m$ and person A walks a distance $\vec{D}_A = \hat{i}3 - \hat{j}3m$. Sketch neatly the situation on graphpaper and from the drawing determine the vector that points from A to B, $\vec{D}_{AB}$. Write out the vector equation for the situation. Confirm that the numerical solution and the graphical solution agree.



Numerically the solution is:

$$\begin{aligned} \mathbf{D}_{AB} &= \mathbf{D}_0 + \mathbf{D}_B - \mathbf{D}_A \\ &= \boldsymbol{i}10 + [\boldsymbol{i}5 + \boldsymbol{j}2] - [\boldsymbol{i}3 - \boldsymbol{j}3] \\ &= \boldsymbol{i}12 + \boldsymbol{j}5 \end{aligned}$$

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6. Find the unit vectors that point from A to the other points B to G in Question #2. Express each vector in component (***ij***) notation.

Unit vectors are vectors of length 1 that point in the desired direction. The best known unit vectors are ***i*** and ***j*** which point in the positive x and y directions respectively. You generate unit vectors by first find a vector that points the right way and then dividing by the magnitude of that vector, $\hat{r} = \frac{\vec{r}}{r}$.

(i)	$\vec{A}_B = \hat{j}9$	$A_B = 9$	$\hat{A}_B = \frac{\vec{A}_B}{A_B} = \frac{\hat{j}9}{9} = \hat{j}$
(ii)	$\vec{A}_C = -\hat{i}14 + \hat{j}8$	$A_C = \sqrt{(-14)^2 + (8)^2} = 16.125$	$\hat{A}_C = \frac{\vec{A}_C}{A_C} = -\hat{i}0.868 + \hat{j}0.496$
(iii)	$\vec{A}_D = -\hat{i}15 + \hat{j}3$	$A_D = \sqrt{(-15)^2 + (3)^2} = 15.297$	$\hat{A}_D = \frac{\vec{A}_D}{A_D} = -\hat{i}0.981 + \hat{j}0.196$
(iv)	$\vec{A}_E = -\hat{i}10$	$A_E = 10$	$\hat{A}_E = \frac{\vec{A}_E}{A_E} = -\hat{i}$
(v)	$\vec{A}_F = \hat{i}6 + \hat{j}6$	$A_F = \sqrt{(6)^2 + (6)^2} = 8.485$	$\hat{A}_F = \frac{\vec{A}_F}{A_F} = \hat{i}0.707 + \hat{j}0.707$
(vi)	$\vec{A}_G = \hat{i}7 - \hat{j}3$	$A_G = \sqrt{(7)^2 + (-3)^2} = 7.616$	$\hat{A}_G = \frac{\vec{A}_G}{A_g} = \hat{i}0.919 - \hat{j}0.394$

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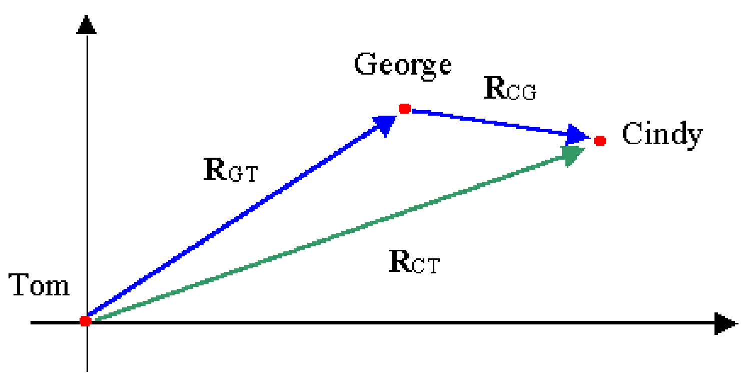
7. The relative position of George to Tom, in metres, is ***i***10 + ***j***15. The relative position of Cindy to George is ***i***6 – ***j***4. What is the relative position of Cindy to Tom?

A good graph let's us see the relationship quite simply.



Clearly, $\mathbf{R}_{CT} = \mathbf{R}_{CG} + \mathbf{R}_{GT} = [\boldsymbol{i}6 - \boldsymbol{j}4] + [\boldsymbol{i}10 + \boldsymbol{j}15] = \boldsymbol{i}16 + \boldsymbol{j}11$.

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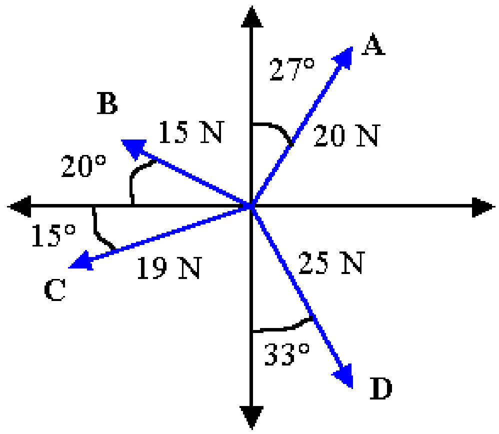
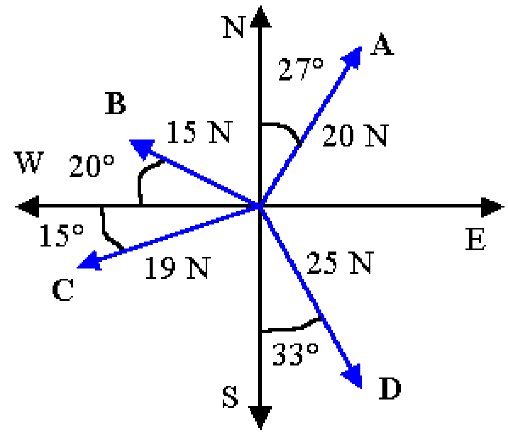
8. State the vectors in the diagrams below in standard form.

(a)

(b)





(a) **A** = 20 N at 53° a.h.

B = 15 N at 160° a.h.

C = 19 N at 195° a.h. or **C** = 19 N at 165° b.h.

D = 25 N at 303° a.h. or **D** = 25 N at 57° b.h.

(b) **A** = 20 N at 27° E of N or **A** = 20 N at 53° N of E

B = 15 N at 20° N of W or **B** = 15 N at 70° W of N

C = 19 N at 15° S of W or **C** = 19 N at 75° W of S

D = 25 N at 33° E of S or **D** = 25 N at 57° S of E

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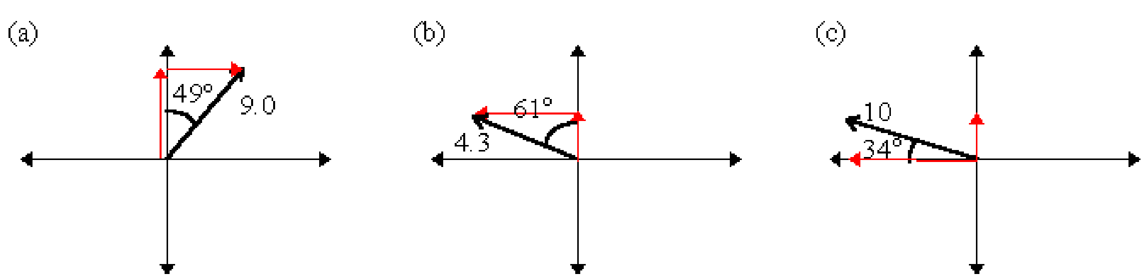
9. Neatly sketch the x and y components on the graphs of the vectors shown below. Indicate the sign of the x and y components of the vectors shown below. Express the vectors in component (***i***, ***j***) form.



(a) **A** = ***i***[+9.0sin(49°)] + ***j***[+9.0cos(49°)]

= ***i***[6.7924] + ***j***[5.9045]

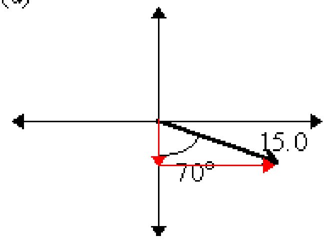
(b) **B** = ***i***[–4.3sin(61°)] + ***j***[+4.3cos(61°)]

= ***i***[–3.7609] + ***j***[2.0847]

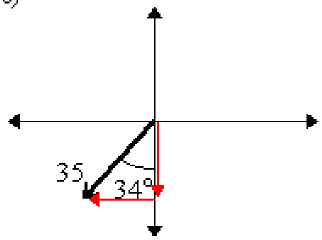
(c) **C** = ***i***[–10cos(34°)] + ***j***[+10sin(34°)]

$= \boldsymbol{i}[-8.2904] + \boldsymbol{j}[5.5919]$

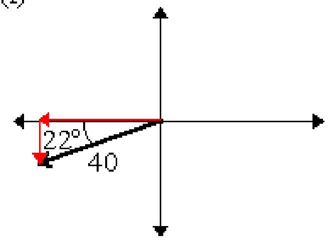
(d)



(e)



(f)



(d) $\mathbf{D} = \boldsymbol{i}[+15 sin(70°)] + \boldsymbol{j}[-15 cos(70°)]$

$= \boldsymbol{i}[14.0954] + \boldsymbol{j}[-5.1303]$

(e) $\mathbf{E} = \boldsymbol{i}[-35 sin(34°)] + \boldsymbol{j}[-35 cos(34°)]$

$= \boldsymbol{i}[-19.5718] + \boldsymbol{j}[-29.0163]$

(f) $\mathbf{F} = \boldsymbol{i}[-40 cos(22°)] + \boldsymbol{j}[-40 sin(22°)]$

$= \boldsymbol{i}[-37.0874] + \boldsymbol{j}[-14.9843]$

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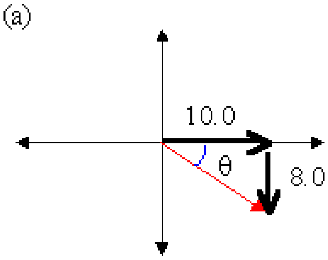
10. Use the parallelogram method to sketch in the resultant vector which has the components shown in the diagrams below. In each case, write the vector in component (***i***, ***j***) form. In each case write the vector in standard form.

(a)



A =10***i*** – 8***j***

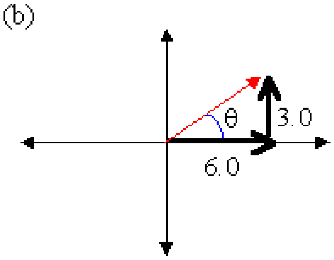
$A = [(10.0)^2+(8.0)^2]^{½}$
$= 12.8062$

$\theta = arctan(8/10)$

$= 38.66°$

A = (12.81, 38.7° b.h.)

(b)



B = 6***i*** + 3***j***

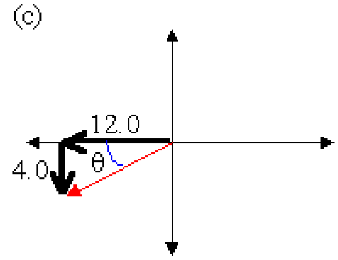
$B = [(6.0)^2+(3.0)^2]^{½}$
$= 6.7082$

$\theta = arctan(3/6)$

$= 26.57°$

B = (6.71, 26.6° a.h.)

(c)



C = -12***i*** – 4***j***

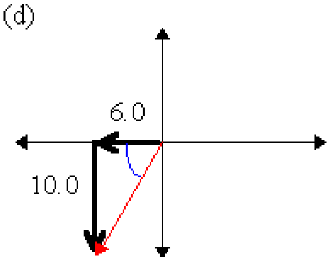
$C = [(12.0)^2+(4.0)^2]^{½}$
$= 12.6491$

$\theta = arctan(4/12)$

$= 18.43°$

C = (12.65, 198.43° a.h.)

(d)



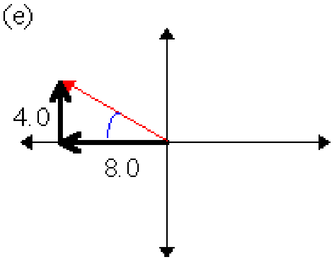
$\mathbf{D} = -6\boldsymbol{i} - 10\boldsymbol{j}$

$D = [(6.0)^2+(10.0)^2]^{1/2}$
$= 11.6619$

$\theta = \arctan(10/6)$
$= 59.04°$

$\mathbf{D} = (11.66, 239.0° \text{ a.h.})$

(e)



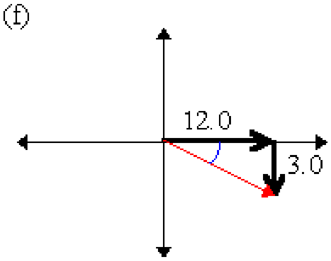
$\mathbf{E} = -8\boldsymbol{i} + 4\boldsymbol{j}$

$E = [(8.0)^2+(4.0)^2]^{1/2}$
$= 8.9443$

$\theta = \arctan(4/8)$
$= 26.57°$

$\mathbf{E} = (8.94, 153.4° \text{ a.h.})$

(f)



$\mathbf{F} = 12\boldsymbol{i} - 3\boldsymbol{j}$

$F = [(12.0)^2+(3.0)^2]^{1/2}$
$= 12.3693$

$\theta = \arctan(3/12)$
$= 14.043°$

$\mathbf{F} = (12.37, 14.0° \text{ b.h.})$

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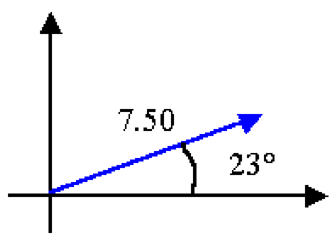
11. Convert the following vectors to component form. Include sketches.

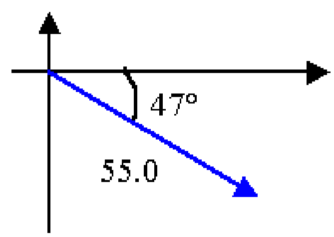
(a) **A** = 7.50 m/s^2 at 23° a.h.
(b) **B** = 55 m at 47° b.h.
(c) **C** = 15.5 m/s at 155° a.h.
(d) **D** = 42 m at 35° S of E
(e) **E** = 120 m/s at 41° W of N
(f) **F** = 75 N at 15° S of W

(a)



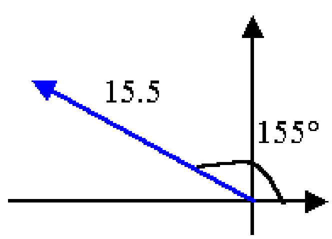
$\mathbf{A} = \boldsymbol{i}[+7.5\cos(23°)] + \boldsymbol{j}[+7.5\sin(23°)]$
$= \boldsymbol{i}[6.9038] + \boldsymbol{j}[2.9305]$

(b)



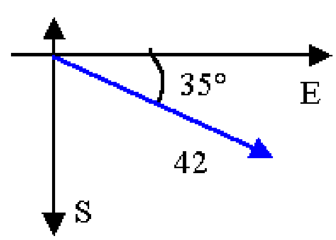
$\mathbf{B} = \boldsymbol{i}[+55\cos(47°)] + \boldsymbol{j}[-55\sin(47°)]$
$= \boldsymbol{i}[35.5099] - \boldsymbol{j}[40.2245]$

(c)



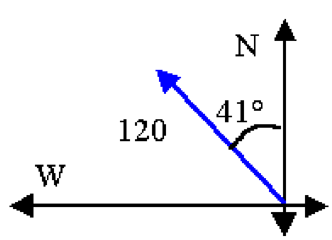
$\mathbf{C} = \boldsymbol{i}[+15.5\cos(155°)] + \boldsymbol{j}[+15.5\sin(155°)]$
$= \boldsymbol{i}[-14.0478] + \boldsymbol{j}[6.5506]$

(d)

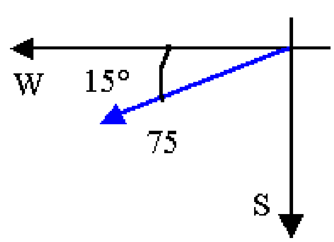


$\mathbf{B} = \boldsymbol{i}[+42\cos(35°)] + \boldsymbol{j}[-42\sin(35°)]$
$= \boldsymbol{i}[34.4044] - \boldsymbol{j}[24.0902]$

(e)



(f)



$\mathbf{E} = \boldsymbol{i}[-120\,sin(41°)] + \boldsymbol{j}[+120\,cos(41°)]$
$= \boldsymbol{i}[-78.7271] + \boldsymbol{j}[90.5651]$

$\mathbf{F} = \boldsymbol{i}[-75\,cos(15°)] + \boldsymbol{j}[-75\,sin(15°)]$
$= -\boldsymbol{i}[72.4444] - \boldsymbol{j}[19.4114]$

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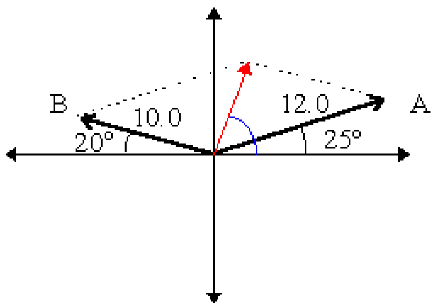
12. For the following, sketch the addition of the given vectors. Use the component method to find the resultant vector. State the result in standard form.

(a)



$\mathbf{A} = \boldsymbol{i}[+12\,cos(25°)] + \boldsymbol{j}[+12\,sin(25°)]$
$= \boldsymbol{i}[10.8757] + \boldsymbol{j}[5.0714]$

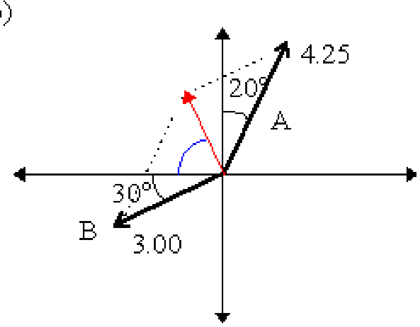
$\mathbf{B} = \boldsymbol{i}[-10\,cos(20°)] + \boldsymbol{j}[+10\,sin(20°)]$
$= \boldsymbol{i}[-9.3969] + \boldsymbol{j}[8.4916]$

$\mathbf{C} = \mathbf{A} + \mathbf{B}$
$= \boldsymbol{i}[10.8757-9.3969] + \boldsymbol{j}[5.0714+8.4916]$
$= \boldsymbol{i}[1.4788] + \boldsymbol{j}[8.4916]$

$C = [(1.4788)^2+(8.4916)^2]^{½}$
$= 8.6194$
$\theta = arctan(8.4916/1.4788)$
$= 80.12°$

$\mathbf{C} = 8.62$ at $80.1°$ a.h.

(b)



$\mathbf{A} = \boldsymbol{i}[+4.25\,sin(20°)] + \boldsymbol{j}[+4.25\,cos(20°)]$
$= \boldsymbol{i}[1.4536] + \boldsymbol{j}[3.9937]$

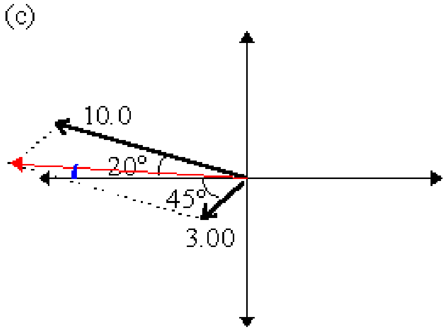
$\mathbf{B} = \boldsymbol{i}[-3\,cos(30°)] + \boldsymbol{j}[-3\,sin(30°)]$
$= \boldsymbol{i}[-2.5981] + \boldsymbol{j}[-1.5]$

$\mathbf{C} = \mathbf{A} + \mathbf{B}$
$= \boldsymbol{i}[1.4536-2.5981] + \boldsymbol{j}[3.9937-1.5]$
$= \boldsymbol{i}[-1.1445] + \boldsymbol{j}[2.4937]$

$C = [(1.1445)^2+(2.4937)^2]^{½}$
$= 2.7438$
$\theta = arctan(2.4937/1.1445)$
$= 65.35°$

$\mathbf{C} = (2.74, 114.7°$ a.h.)

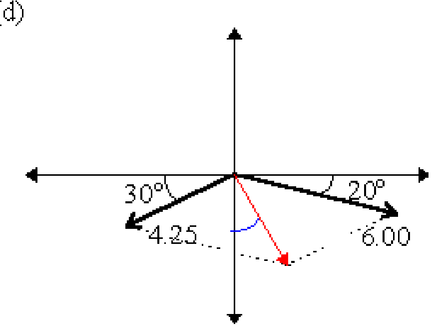
(c)



$\mathbf{A} = \boldsymbol{i}[-10\,cos(20°)] + \boldsymbol{j}[+10\,sin(20°)]$
$= \boldsymbol{i}[-9.3969] + \boldsymbol{j}[3.4202]$

$\mathbf{B} = \boldsymbol{i}[-3\,cos(45°)] + \boldsymbol{j}[-3\,sin(45°)]$
$= \boldsymbol{i}[-2.1213] + \boldsymbol{j}[-2.1213]$

(d)



$\mathbf{A} = \boldsymbol{i}[+6\,cos(20°)] + \boldsymbol{j}[-6\,sin(20°)]$
$= \boldsymbol{i}[5.6382] + \boldsymbol{j}[-2.0521]$

$\mathbf{B} = \boldsymbol{i}[-4.25\,cos(30°)] + \boldsymbol{j}[-4.25\,sin(30°)]$
$= \boldsymbol{i}[-3.6806] + \boldsymbol{j}[-2.125]$

$\mathbf{C} = \mathbf{A} + \mathbf{B}$

$= \boldsymbol{i}[-9.3969-2.1213] + \boldsymbol{j}[3.4202-2.1213]$

$= \boldsymbol{i}[-11.5182] + \boldsymbol{j}[1.2989]$

$C = [(-11.5182)^2+(1.2989)^2]^{½}$

$= 11.5912$

$\theta = arctan(1.2989/11.5182)$

$= 6.43°$

C = 11.6 at 173.6° a.h.

$\mathbf{C} = \mathbf{A} + \mathbf{B}$

$= \boldsymbol{i}[5.6382-3.6806] + \boldsymbol{j}[-2.0521-2.125]$

$= \boldsymbol{i}[-1.1445] + \boldsymbol{j}[-4.1771]$

$C = [(1.9576)^2+(-4.1771)^2]^{½}$

$= 4.6131$

$\theta = arctan(1.9576/4.1771)$

$= 25.11°$

C = 4.61 at 64.9° b.h.

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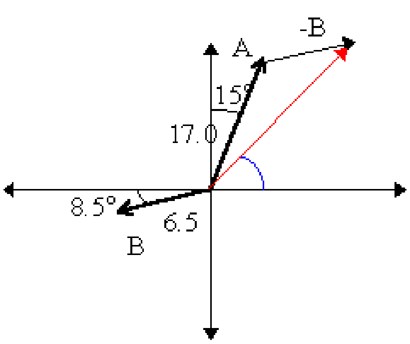
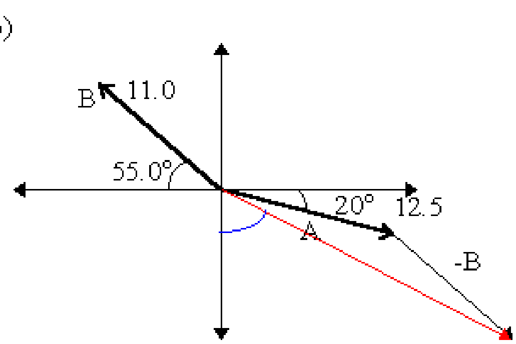
13. For the following, subtract the vector **B** from vector **A** and sketch the result.

(a)



(b)



(a)

$\mathbf{A} = \boldsymbol{i}[+17sin(15°)] + \boldsymbol{j}[+17cos(15°)]$

$= \boldsymbol{i}[4.3999] + \boldsymbol{j}[16.4207]$

$\mathbf{B} = \boldsymbol{i}[-6.5cos(8.5°)] + \boldsymbol{j}[-6.5sin(8.5°)]$

$= \boldsymbol{i}[-6.4286] + \boldsymbol{j}[-0.9608]$

$\mathbf{D} = \mathbf{A} - \mathbf{B}$

$= \boldsymbol{i}[4.3999+6.4286] + \boldsymbol{j}[16.4207+0.9608]$

$= \boldsymbol{i}[10.8285] + \boldsymbol{j}[17.3815]$

$D = [(10.8285)^2+(17.3815)^2]^{½}$

$= 20.4786$

$\theta = arctan(17.3815/10.8285)$

$= 58.08°$

D = 20.5 at 58.1° a.h.

(b)

$\mathbf{A} = \boldsymbol{i}[+12.5cos(20°)] + \boldsymbol{j}[-12.5sin(20°)]$

$= \boldsymbol{i}[11.7462] + \boldsymbol{j}[-4.2753]$

$\mathbf{B} = \boldsymbol{i}[-11cos(55°)] + \boldsymbol{j}[11sin(55°)]$

$= \boldsymbol{i}[-6.3093] + \boldsymbol{j}[9.0107]$

$\mathbf{D} = \mathbf{A} - \mathbf{B}$

$= \boldsymbol{i}[11.7462+6.3093] + \boldsymbol{j}[-4.2753-9.0107]$

$= \boldsymbol{i}[18.0555] + \boldsymbol{j}[-13.2860]$

$D = [(18.0555)^2+(-13.2860)^2]^{½}$

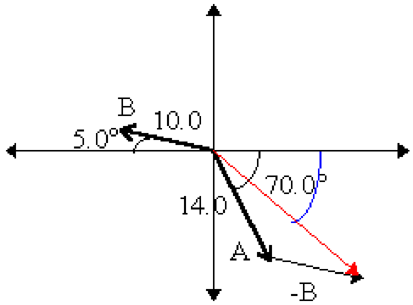
$= 22.4169$

$\theta = arctan(13.2860/18.0555)$

$= 36.35°$

D = 22.4 at 36.4° b.h.

(c)



$\mathbf{A} = \boldsymbol{i}[+14\,cos(70^\circ)] + \boldsymbol{j}[-14\,sin(70^\circ)]$
$= \boldsymbol{i}[4.7883] + \boldsymbol{j}[-13.1557]$

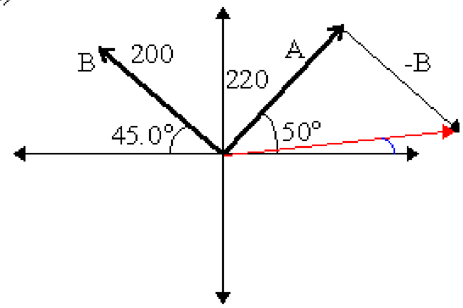
$\mathbf{B} = \boldsymbol{i}[-10\,cos(5^\circ)] + \boldsymbol{j}[+10\,sin(5^\circ)]$
$= \boldsymbol{i}[-9.9619] + \boldsymbol{j}[+0.8716]$

$\mathbf{D} = \mathbf{A} - \mathbf{B}$
$= \boldsymbol{i}[4.7883+9.9619] + \boldsymbol{j}[-13.1557-0.8716]$
$= \boldsymbol{i}[14.7502] + \boldsymbol{j}[-14.0273]$

$D = [(14.7502)^2+(-14.0273)^2]^{½}$
$= 20.3551$
$\theta = arctan(14.0273/14.7502)$
$= 43.56^\circ$

D = 20.4 at 43.6° b.h.

(d)



$\mathbf{A} = \boldsymbol{i}[+220\,cos(50^\circ)] + \boldsymbol{j}[+220\,sin(50^\circ)]$
$= \boldsymbol{i}[141.4133] + \boldsymbol{j}[168.5298]$

$\mathbf{B} = \boldsymbol{i}[-200\,cos(45^\circ)] + \boldsymbol{j}[+200\,sin(45^\circ)]$
$= \boldsymbol{i}[-141.4214] + \boldsymbol{j}[141.4214]$

$\mathbf{D} = \mathbf{A} - \mathbf{B}$
$= \boldsymbol{i}[141.4133+141.4214] + \boldsymbol{j}[168.5298-141.4214]$
$= \boldsymbol{i}[282.8346] + \boldsymbol{j}[27.1084]$

$D = [(282.8346)^2+(27.1084)^2]^{½}$
$= 284.1307$
$\theta = arctan(27.1084/282.8346)$
$= 5.47^\circ$

D = 284.1 at 5.5° a.h.

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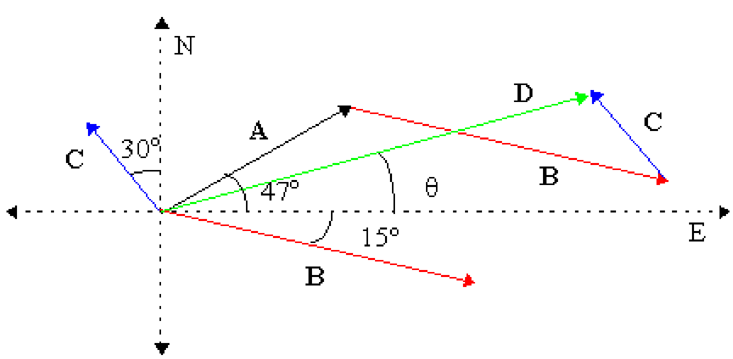
14. A person walks 57.0 m at 47.0° north of east, turns and walks 72.0 m at 15.0° south of east, and then turns and walks 24.0 m 30.0° west of north.
(a) How far and at what angle is the person's final position from his/her initial position?
(b) In what direction would the person have to head to return to his/her initial position?

(a) For vector problems, we first draw a neat sketch of the vectors and the vector operation of interest. Here we are adding three vectors.



Then to solve the problem numerically, we break the vectors into their components.

$\mathbf{A} = \boldsymbol{i}[57\,cos(47^\circ)] + \boldsymbol{j}[57\,sin(47^\circ)] = \boldsymbol{i}[38.8739] + \boldsymbol{j}[41.6872]$

B = ***i***[72*cos*(15°)] + ***j***[–72*sin*(15°)] = ***i***[69.5467] + ***j***[–18.6350]

C = ***i***[–24*sin*(30°)] + ***j***[24*cos*(30°)] = ***i***[–12] + ***j***[20.7846]

Next we add them to get the components of vector **D**.

D = ***i***[38.8739 + 69.5467 + –12] + ***j***[41.6872 + –18.6350 + 20.7846] = ***i***[96.4206] + ***j***[43.8368]

Then we convert to polar coordinate form. Using Pythagoras' Theorem, D = $[(96.4206)^2 + (43.8368)^2]^{1/2}$ = 105.92 m. The angle θ = *arctan*($|D_y/D_x|$) = *arctan*(43.8368/96.4206) = 24.45° . Thus the person's displacement is 106 m at 24.4° north of east.

(b) To return, the person would have to travel in the direction opposite to his/her displacement, i.e. 106 m at 24.4° south of west.

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15. Momentum is an important vector quantity which we will encounter later in this course. Momentum is a product of mass and velocity (**p** = m**v**). A car of mass 1200 kg and velocity 22.0 m/s at 30.0° south of west is hit by a second car of mass 1450 kg travelling at 18.0 m/s at 55.0° west of north.
(a) Find each car's momentum in standard form.
(b) Find the magnitude and direction of the total momentum of the two cars.

(a) The magnitude of the momenta of the cars are:

$$p_1 = m_1 v_1 = (1200\ \text{kg})(22\ \text{m/s}) = 2.64 \times 10^4\ \text{kg-m/s};$$

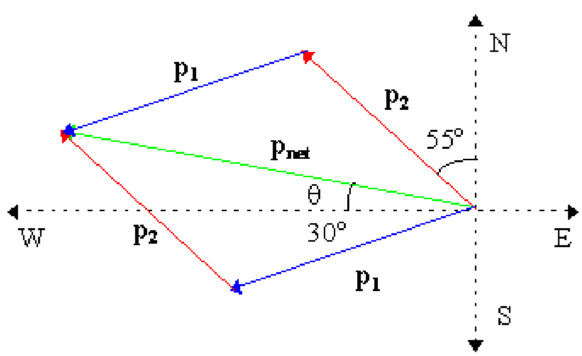
$$p_2 = m_2 v_2 = (1450\ \text{kg})(18\ \text{m/s}) = 2.61 \times 10^4\ \text{kg-m/s}.$$

Therefore

$$\mathbf{p}_1 = 2.64 \times 10^4\ \text{kg-m/s at } 30.0° \text{ S of W}$$

$$\mathbf{p}_2 = 2.61 \times 10^4\ \text{kg-m/s at } 55.0° \text{ W of N}$$

(b) For vector problems, we first draw a neat sketch of the vectors and the vector operation of interest. Here we are adding two vectors.



Then to solve the problem numerically, we break the vectors into their components:

$\mathbf{p}_1$ = ***i***[–(2.64× 10^4)*cos*(30°)] + ***j***[–(2.64× 10^4)*sin*(30°)] = ***i***[–2.2863 × 10^4] + ***j***[–1.3200 × 10^4]

$\mathbf{p}_2$ = ***i***[–(2.61× 10^4)*sin*(55°)] + ***j***[(2.61× 10^4)*cos*(55°)] = ***i***[–2.1380 × 10^4] + ***j***[1.4970 × 10^4]

Next we add them to get the components of vector $\mathbf{P}_{net}$.

$\mathbf{P}_{net}$ = ***i***[–2.2863 × 10^4 + –2.1380 × 10^4] + ***j***[–1.3200 × 10^4 + 1.4970 × 10^4] = ***i***[–4.4243 × 10^4] + ***j***[0.1770 × 10^4]

Then we convert to polar coordinate form. <>Using Pythagoras' Theorem, $p_{net} = [(4.4243\times 10^4)^2 + (0.1770\times 10^4)^2]^{½} = 4.428 \times 10^4$ kg-m/s. The angle $\theta = arctan(|p_{net\ y}/p_{net\ x}|) = arctan(0.1770/4.4243) = 2.3°$. Thus the net momentum of the two cars 4.428×10^4 kg-m/s at 2.3° north of west.

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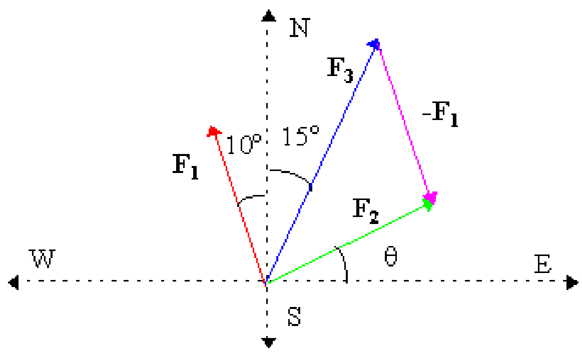
16. Forces are vector quantities. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a body such the total force $\mathbf{F}_3$ has a magnitude of 150 N at 15.0° east of north. If $\mathbf{F}_1$ has magnitude 100 N at 10.0° west of north, what is the magnitude and direction of $\mathbf{F}_2$?

    We are looking for $\mathbf{F}_2 = \mathbf{F}_3 - \mathbf{F}_1$, a vector subtraction. For vector problems, we first draw a neat sketch of the vectors and the vector operation of interest.



    Then to solve the problem numerically, we break the vectors into their components:

    $\mathbf{F}_1 = \boldsymbol{i}[150 sin(15°)] + \boldsymbol{j}[150 cos(15°)] = \boldsymbol{i}[38.823] + \boldsymbol{j}[144.889]$

    $\mathbf{F}_3 = \boldsymbol{i}[-100 sin(10°)] + \boldsymbol{j}[100 cos(10°)] = \boldsymbol{i}[-17.365] + \boldsymbol{j}[98.481]$

    We subtract the components to get the components of vector $\mathbf{F}_2$.

    $\mathbf{F}_2 = \boldsymbol{i}[38.823 - (-17.365)] + \boldsymbol{j}[144.889 - 98.481] = \boldsymbol{i}[56.188] + \boldsymbol{j}[46.408]$

    Then we convert to polar coordinate form. Using Pythagoras' Theorem, $F_2 = [(56.188)^2 + (46.408)^2]^{½} = 72.875$ N. The angle $\theta = arctan(|F_{2y}/F_{2x}|) = arctan(46.408/56.188) = 39.55°$. Thus the other force is 72.9 N at 39.6° north of east.

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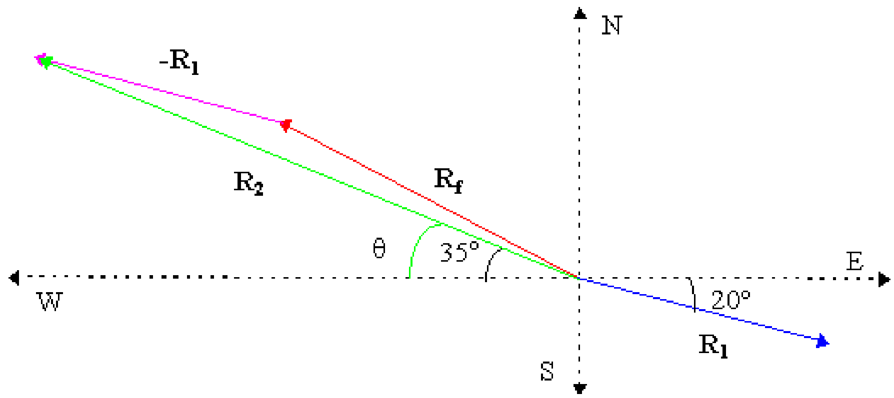
17. A person is located 225 m and 35.0° north of west from her initial position. She walked from her initial position to her final position in two stages. In the first stage of her walk, she walked 150 m at 20.0° south of east. How far and in what direction did she walk in the second part?

    We are looking for $\mathbf{R}_2 = \mathbf{R}_f - \mathbf{R}_1$, a vector subtraction. For vector problems, we first draw a neat sketch of the vectors and the vector operation of interest.



Then to solve the problem numerically, we break the vectors into their components:

$\mathbf{R}_f = \boldsymbol{i}[-225cos(35°)] + \boldsymbol{j}[225sin(35°)] = \boldsymbol{i}[-184.309] + \boldsymbol{j}[129.055]$

$\mathbf{R}_1 = \boldsymbol{i}[150cos(20°)] + \boldsymbol{j}[150sin(20°)] = \boldsymbol{i}[140.954] + \boldsymbol{j}[-51.303]$

Next we subtract them to get the components of vector $\mathbf{R}_2$.

$\mathbf{R}_2 = \boldsymbol{i}[-184.309 - 140.954] + \boldsymbol{j}[129.055 - -51.303] = \boldsymbol{i}[-325.263] + \boldsymbol{j}[180.358]$

Finally we convert to polar coordinate form. Using Pythagoras' Theorem, $R_2 = [(325.263)^2 + (180.358)^2]^{½} = 371.921$ m. The angle θ = *arctan*($|R_{2y}/R_{2x}|$) = *arctan*(180.358/325.263) = 29.01° . Thus the other displacement is 372 m at 29.0° north of west.

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