Applied Engineering Economy

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Table of Content

1 Introduction to Engineering Economy

1.1 Introduction

Engineering economy deals with the evaluation of systems, products, and services in relationship to their costs. Engineering economy is a field that addresses the dynamic environment of economic calculations and principles through the prism of engineering. It is a fundamental skill that all successful engineering firms employ in order to retain competitive advantage and market share.

Engineering economy studies various financial and economic problems pervasive to engineers in a variety of industries. Engineering economy is a topic that all industry-bound students should learn because of its real-world applications. Engineering economy poses numerous benefits because it allows those in industry to make strategic decisions for their companies. While macroeconomic and financial competencies are key for business operations, engineering economics further provides a mechanism for decision-making. It forces engineers to think twice before making many choices in everyday operations, such as process configurations, materials, production size, and other economic factors. Daily decisions by the engineering firms (based on an economic framework) will decide how successful and profitable that company is.

1.2 Definitions

Time value of money is the idea that money has a different value now than it will in the future. This is due to a number of dynamic variables, such as inflation and interest rates. These values are standardized through present and future value calculations, thereby equalizing the time dependent variables. This is very important for engineers because these calculations provide an intuition as to how money should be spent and saved, how cash flow should be negotiated in contracts, and how interest rates can affect the present and future values.

Cost analysis is a key tenant for balancing a business's budget, as well as for calculating the viability of a project. Engineers can compare the costs and benefits of a project and determine whether the benefits outweigh the costs enough to entertain the project. Each are then further broken down into subcategories. There are fixed costs (initial infrastructure), variable costs (each additional input), total variable costs (aggregate of all inputs), and total costs (fixed costs plus total variable costs). Meanwhile, the benefits can comprise of total revenues (final sales), marginal revenues (each additional sale), and profits (final sales minus total costs).

Interest is another concept that is important to economical engineers. Many times, engineering firms take out significant loans to finance construction of major projects. Having a clear understanding of the cost of borrowing money is crucial to making appropriate business decisions. For instance, if the costs of a five-year long project (after accounting for the annually compounded interest rate) exceed the revenues collected, then it would be unwise to pursue the project. Many construction companies are highly cognizant of interest rates (mortgage rates in particular), because if mortgage rates are too high, many people can't afford to finance buying new houses. Thus, once the demand for homes dries up, construction companies must pursue other avenues to make money.

Economic fluctuations characterize the changes in the market economy as peaks, recessions, troughs, or expansions. Each of these four stages has a direct impact on the choices made by businesses, particularly construction companies. During a recession, decisions made by the Federal Reserve and U.S. Government provide a signal for the direction of the economy. For example, if the Federal Reserve decides to engage in expansionary monetary policy, they will lower interest rates in order to make it cheaper to borrow money for business operations. In response, many firms can take advantage of the temporary stimulus and invest heavily.

Depreciation is the loss of value in an asset over time. During the 2008 housing market collapse, many homeowners saw the value of their homes depreciate tremendously, leading them to go underwater on their mortgages – many to a point where their loans exceeded the values of their homes. Depreciation plays a major influence on engineering firms; it is important for engineers to calculate the "wear and tear" that activities have on their expensive equipment. This allows them to calculate how much it costs their firm to operate a piece of equipment for a period of time, and how much they should recoup annually to compensate for these costs. Furthermore, since capital depreciation is tax-deductible, savvy engineers can save their firms tremendous amounts of money.

The marriage between economics and engineering is one that is crucial to the success of engineers in the 21st century; the interdisciplinary nature of the topic offers key insight into the underlying mechanisms that drive daily business operations. Engineering economics is an integral component to many engineering curricula across the country, covering a wide variety of topics including the time value of money, cost analysis, interest rates, economic fluctuations, depreciation, and everything in-between. Furthermore, it has been noted by renowned engineer John Hayford that engineering and economics "help to develop the very valuable habit of thinking in terms of groups rather than of individuals." By understanding and implementing the outcomes, framework, and tools for actively teaching engineering economics, future engineers can continue evolving as problem solvers and innovators.

2 Foundations of Engineering Economy

2.1 Introduction

The need for engineering economy is primarily motivated by the work that engineers do in performing analyses, synthesizing, and coming to a conclusion as they work on projects of all sizes. In other words, engineering economy is at the heart of making decisions. These decisions involve the fundamental elements of cash flows of money, time, and interest rates. This section introduces the basic concepts and terminology necessary for an engineer to combine these three essential elements in organized, mathematically correct ways to solve problems that will lead to better decisions.

2.2 Description and Role in Decision Making

Decisions are made routinely to choose one alternative over another by individuals in everyday life; by engineers on the job; by managers who supervise the activities of others; by corporate presidents who operate a business; and by government officials who work for the public good. Most decisions involve money, called **capital** or **capital funds**, which is usually limited in amount. The decision of where and how to invest this limited capital is motivated by a primary goal of **adding value** as future, anticipated results of the selected alternative are realized. Engineers play a vital role in capital investment decisions based upon their ability and experience to design, analyze, and synthesize. The factors upon which a decision is based are commonly a combination of economic and noneconomic elements.

Engineering economy deals with the economic factors. By definition, engineering economy involves formulating, estimating, and evaluating the expected economic outcomes of alternatives designed to accomplish a defined purpose. Mathematical techniques simplify the economic evaluation of alternatives.

People make decisions; computers, mathematics, concepts, and guidelines assist people in their decision-making process. Since most decisions affect what will be done, the time frame of engineering economy is primarily **the future**; therefore, the numbers used in engineering economy are **best estimates of what is expected to occur**. The estimates and the decision usually involve four essential elements: **cash flows**, **times of occurrence of cash flows**, **interest rates for time value of money**, and **measure of economic worth for selecting an alternative**.

Since the estimates of cash flow amounts and timing are about the future, they will be somewhat different than what is actually observed, due to changing circumstances and unplanned events. In short, the variation between an amount or time estimated now and that observed in the future is caused by the stochastic (random) nature of all economic events. **Sensitivity analysis** is utilized to determine how a decision might change according to varying estimates, especially those expected to vary widely.

The criterion used to select an alternative in engineering economy for a specific set of estimates is called a measure of worth. The measures developed and used in this document are: Present worth (PW), Future worth (FW), Annual worth (AW), Rate of return (ROR), Benefit/cost (B/C), Capitalized Cost (CC), Payback Period, Economic Value Added (EVA), and Cost Effectiveness.
All these measures of worth account for the fact that money makes money over time. This is the concept of the **time value of money**.

It is a well-known fact that money **makes** money. The time value of money explains the change in the amount of money **over time** for funds that are owned (invested) or owed (borrowed).
This is the most important concept in engineering economy.

The time value of money is very obvious in the world of economics. If it is decided to invest capital (money) in a project today, it is inherently expected to have more money in the future than what was invested. If money is borrowed today, in one form or another, it is expected to return the original amount plus some additional amount of money.

Engineering economy is equally well suited for the present, future, and for the **analysis of past cash flows** in order to determine if a specific criterion (measure of worth) was attained.

2.3 Performing an Engineering Economy Study

An engineering economy study involves many elements: problem identification, definition of the objective, cash flow estimation, financial analysis, and decision making. Implementing a structured procedure is the best approach to select the best solution to the problem.

The steps in an engineering economy study are as follows:

- Identify and understand the problem; identify the objective of the project.
- Collect relevant, available data and define viable solution alternatives.
- Make realistic cash flow estimates.
- Identify an economic measure of worth criterion for decision making.
- Evaluate each alternative; consider noneconomic factors; use sensitivity analysis as needed.
- Select the best alternative.
- Implement the solution and monitor the results.

Technically, the last step is not part of the economy study, but it is, of course, a step needed to meet the project objective. There may be occasions when the best economic alternative requires more capital funds than are available, or significant non-economic factors preclude the most economical alternative from being chosen. Accordingly, the fifth and sixth steps above may result in the selection of an alternative different from the economically best one. Also, sometimes more than one project may be selected and implemented. This occurs when projects are independent of one another. In this case, the fifth, sixth, and seventh steps vary from those above.

2.3.1 Problem Description and Objective Statement

A succinct statement of the problem and primary objective(s) is very important to the formation of an alternative solution. As an illustration, assume the problem is that a coal-fueled power plant must be shut down in the near future due to the production of excessive sulfur dioxide. The objectives may be to generate the forecasted electricity needed for the near future and beyond, plus to not exceed all the projected emission allowances in these future years.

2.3.2 Alternatives

These are stand-alone descriptions of viable solutions to problems that can meet the objectives. Words, pictures, graphs, equipment and service descriptions, simulations, etc. define each alternative. The best estimates for parameters are also part of the alternative. Several parameters include equipment first cost, expected life, salvage value (estimated trade-in, resale, or market value), and annual operating cost (AOC), which can also be termed maintenance and operating (O&M) cost, and subcontract cost for specific services. If changes in income (revenue) may occur, this parameter must be estimated.

Detailing all viable alternatives at this stage is crucial. For example, if two alternatives are described and analyzed, one will likely be selected and implementation initiated. If a third, more attractive method that was available is later recognized, a wrong decision was made.

2.3.3 Cash Flows

All cash flows are estimated for each alternative. Since these are future expenditures and revenues, the results of the third step usually prove to be inaccurate when an alternative is actually in place and operating. When cash flow estimates for specific parameters are expected to vary significantly from a point estimate made now, risk and sensitivity analyses (fifth step) are needed to improve the chances of selecting the best alternative. Sizable variation is usually expected in estimates of revenues, AOC, salvage values, and subcontractor costs. Estimation of costs, the elements of variation (risk), and sensitivity analysis is discussed later on in this document.

2.3.4 Engineering Economy Analysis

The techniques and computations that will be learned and used throughout this text utilize the cash flow estimates, time value of money, and a selected measure of worth. The result of the analysis will be one or more numerical values; this can be in one of several terms, such as money, an interest rate, number of years, or a probability. In the end, a selected measure of worth mentioned in the previous section will be used to select the best alternative.

Before an economic analysis technique is applied to the cash flows, some decisions about what to include in the analysis must be made. Two important possibilities are taxes and inflation. Federal, state or provincial, county, and city taxes will impact the costs of every alternative. An after-tax analysis includes some additional estimates and methods compared to a before-tax analysis. If taxes and inflation are expected to impact all alternatives equally, they may be disregarded in the analysis; however, if the size of these projected costs is important, taxes and inflation should be considered. Also, if the impact of inflation over time is important to the decision, an additional set of computations must be added to the analysis and details will be discussed later on in this document.

2.3.5 Selection of the Best Alternative

The measure of worth is a primary basis for selecting the best economic alternative. For example, if alternative A has a rate of return (ROR) lower than that of alternative B, then B is better economically; however, there can always be **non-economic** or **intangible factors** that must be considered and that may alter the decision. There are many possible noneconomic factors and several typical ones are:

- Market pressures, such as need for an increased international presence.
- Availability of certain resources, e.g., skilled labor force, water, power, tax incentives.
- Government laws that dictate safety, environmental, legal, or other aspects.
- Corporate management's or the board of director's interest in a particular alternative.
- Goodwill offered by an alternative toward a group: employees, union, county, etc.

Once all the economic, non-economic, and risk factors have been evaluated, a final decision of the "best" alternative is made.

At times, only one viable alternative is identified; in this case, the **do-nothing (DN) alternative** may be chosen provided the measure of worth and other factors result in the alternative being a poor choice. The do-nothing alternative maintains the status quo.

In economic analysis, financial units (dollars or other currency) are generally used as the tangible basis for evaluation. Thus, when there are several ways of accomplishing a stated objective, the alternative with the lowest overall cost or highest overall net income is selected.

2.4 Professional Ethics and Economic Decisions

Many of the fundamentals of engineering ethics are intertwined with the roles of money and economics-based decisions in the making of professionally ethical judgments. The terms **morals** and **ethics** are commonly used interchangeably, yet they have slightly different interpretations. Morals usually relate to the underlying tenets that form the character and conduct of a person in judging right and wrong. Ethical practices can be evaluated by using a code of morals or **code of ethics** that forms the standards to guide decisions and actions of individuals and organizations in a profession, for example, electrical, chemical, mechanical, industrial, or civil engineering. There are several different levels and types of morals and ethics:

- **Universal or common morals:** These are fundamental moral beliefs held by virtually all people. Most people agree that to steal, murder, lie, or physically harm someone is wrong. It is possible for actions and intentions to come into conflict concerning a common moral.

- **Individual or personal morals:** These are the moral beliefs that a person has and maintains over time. These usually parallel the common morals in that stealing, lying, murdering, etc. are immoral acts. It is quite possible that an individual strongly supports the common morals and has excellent personal morals, but these may conflict from time to time when decisions must be made.

- **Professional or engineering ethics:** Professionals in a specific discipline are guided in their decision making and performance of work activities by a formal standard or code. The code states the commonly accepted standards of honesty and integrity that each individual is expected to demonstrate in her or his practice. There are codes of ethics for medical doctors, attorneys, and, of course, engineers. Although each engineering profession has its own code of ethics, the **Code of Ethics for Engineers** published by the National Society of Professional Engineers (NSPE) is very commonly used and quoted. As with common and personal morals, conflicts can easily rise in the mind of an engineer between his or her own ethics and that of the employing corporation. Like many people during a declining national economy, retention of this job is of paramount importance to the family and the engineer. Conflicts such as this can place individuals in real dilemmas with no or mostly unsatisfactory alternatives. When an engineering economy study is performed, it is important for the engineer performing the study to consider all ethically related matters to ensure that the cost and revenue estimates reflect what is likely to happen once the project or system is operating.

2.5 Interest Rate and Rate of Return

Interest is the manifestation of the time value of money. Computationally, interest is the difference between an ending amount of money and the beginning amount. If the difference is zero or negative, there is no interest. There are always two perspectives to an amount of interest: **interest paid** and **interest earned**. These are illustrated in Figures 2.a and 2.b. Interest is paid when a person or organization borrowed money (obtained a loan) and repays a larger amount over time. Interest is earned when a person or organization saved, invested, or lent money and obtains a return of a larger amount over time. The numerical values and formulas used are the same for both perspectives, but the interpretations are different.

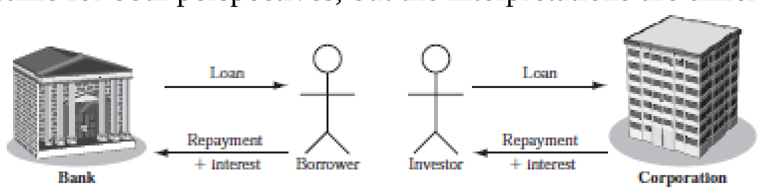

Figure 2.a **Figure 2.b**

Interest paid on borrowed funds (a loan) is determined using the original amount, also called the principal:

Interest Paid = Amount Owed now – Principal [1]

From the perspective of a saver, a lender, or an investor, **interest earned** (Figure 2.b) is the final amount minus the initial amount, or principal.

Interest Earned = Total Amount Now – Principal [2]

When interest paid over a specific time unit is expressed as a percentage of the principal, the result is called the interest rate:

Interest Rate (%) = (Interest Accrued per Time Unit) *(100%) / Principal [3]

Keeping in mind that:

Total Accrued = Deposit + (Deposit)*(Interest Rate) [4]

The time unit of the rate is called the **interest period**. By far the most common interest period used to state an interest rate is 1 year. Shorter time periods can be used, such as 1% per month. Thus, the interest period of the interest rate should always be included. If only the rate is stated, for example, 8.5%, a 1-year interest period is assumed.

Interest earned over a specific period of time is expressed as a percentage of the original amount and is called rate of return (ROR):

Rate of Return (%) = (**Interest Accrued per Time Unit) *100% / Principal [5]**

The time unit for rate of return is called the interest period, just as for the borrower's perspective. Again, the most common period is 1 year.

The term **return on investment** (ROI) is used equivalently with ROR in different industries and settings, especially where large capital funds are committed to engineering-oriented programs. The numerical values in Equations [3] and [5] are the same, but the term interest rate paid is more appropriate for the borrower's perspective, while the rate of return earned is better for the investor's perspective. Remember, end of the period means end of interest period, not end of calendar year.

Example 1:

An employee borrows $10,000 on May 1 and must repay a total of $10,700 exactly 1 year later. Determine the interest amount and the interest rate paid.

Solution:

The perspective here is that of the borrower since $10,700 repays a loan. Apply Equation [1] to determine the interest paid:

Interest Paid = $10,700 - 10,000 = $700
Equation [2] determines the interest rate paid for 1 year:
Percent Interest Rate = ($700) *(100%)/$10,000 = 7% per year

Example 2:

(a) Calculate the amount deposited 1 year ago to have $1000 now at an interest rate of 5% per year.

(b) Calculate the amount of interest earned during this time period.

Solution:

(a) The total amount accrued ($1000) is the sum of the original deposit and the earned interest. If X is the original deposit:

Total Accrued = Deposit + (Deposit)*(Interest Rate)
$1000 = X (0.05) + X*(1 + 0.05) = 1.05 X

The original deposit is:

X = 1000/1.05 = $952.38

(b) Apply Equation [2] to determine the interest earned:

Interest = $1000 - 952.38 = $47.62

In Examples 1 and 2, the interest period was 1 year, and the interest amount was calculated at the end of one period. When more than one interest period is involved, e.g., the amount of interest after 3 years, it is necessary to state whether the interest is accrued on a simple or compound basis from one period to the next. This topic is covered later in this document.

Since **inflation** can significantly increase an interest rate, some comments about the fundamentals of inflation are warranted at this early stage. By definition, inflation represents a decrease in the value of a given currency. That is, $10 now will not purchase the same amount of gasoline for your car (or most other things) as $10 did 10 years ago. The changing value of the currency affects market interest rates.
In simple terms, interest rates reflect two things: a so-called real rate of return plus the expected inflation rate. The real rate of return allows the investor to purchase more than he or she could have purchased before the investment, while inflation raises the real rate to the market rate that we use on a daily basis.

The safest investments (such as government bonds) typically have a 3% to 4% real rate of return built into their overall interest rates. Thus, a market interest rate of, say, 8% per year on a bond means that investors expect the inflation rate to be in the range of 4% to 5% per year. Clearly, inflation causes interest rates to rise.

From the borrower's perspective, the rate of inflation is another interest rate tacked on to the real interest rate, and from the vantage point of the saver or investor in a fixed-interest account, inflation reduces the real rate of return on the investment. Inflation means that cost and revenue cash flow estimates increase over time. This increase is due to the changing value of money that is forced upon a country's currency by inflation, thus making a unit of currency (such as the dollar) worth less relative to its value at a previous time. It is seen that the effect of inflation in that money purchases less now than it did at a previous time. Inflation contributes to:

- a reduction in purchasing power of the currency.
- an increase in the CPI (consumer price index).
- an increase in the cost of equipment and its maintenance.
- an increase in the cost of salaried professionals and hourly employees.
- a reduction in the real rate of return on personal savings and certain corporate investments.

In other words, inflation can materially contribute to changes in corporate and personal economic analysis.

2.6 Terminology and Symbols

The equations and procedures of engineering economy utilize the following terms and symbols. Sample units are indicated.

P = value or amount of money at a time designated as the present or time 0. Also P is referred to as present worth (PW), present value (PV), net present value (NPV), discounted cash flow (DCF), and capitalized cost (CC); monetary units, such as dollars.

F = value or amount of money at some future time. Also, F is called future worth (FW) and future value (FV); monetary units, such as dollars.

A = series of consecutive, equal, end-of-period amounts of money. Also, A is called the annual worth (AW) and equivalent uniform annual worth (EUAW); monetary units, such as dollars per year, euros per month.

n = number of interest periods; years, months, days.
i = interest rate per time period; percent per year, percent per month.
t = time, stated in periods; years, months, days.

The symbols P and F represent one-time occurrences: A occurs with the same value in each interest period for a specified number of periods. It should be clear that a present value P represents a single sum of money at some time prior to a future value F or prior to the first occurrence of an equivalent series amount A.

It is important to note that the symbol A always represents a uniform amount (i.e., the same amount each period) that extends through consecutive interest periods. Both conditions must exist before the series can be represented by A.

The interest rate i is expressed in percent per interest period, for example, 12% per year. Unless stated otherwise, assume that the rate applies throughout the entire n years or interest periods. The decimal equivalent for i is always used in formulas and equations in engineering economy computations.

All engineering economy problems involve the element of time expressed as n and interest rate i. In general, every problem will involve at least four of the symbols P, F, A, n, and i, with at least three of them estimated or known.

Example 3:

Today, a person borrowed $5000 to purchase furniture for his new house. He can repay the loan in either of the two ways described below. Determine the engineering economy symbols and their value for each option.

(a) Five equal annual installments with interest based on 5% per year.
(b) One payment 3 years from now with interest based on 7% per year.

Solution:

(a) The repayment schedule requires an equivalent annual amount A, which is unknown.
P = \$5000, i = 5% per year, n = 5 years, A = ?

(b) Repayment requires a single future amount F, which is unknown.
P = \$5000, i = 7% per year, n = 3 years, F = ?

Example 4:

You plan to make a lump-sum deposit of \$5000 now into an investment account that pays 6% per year, and you plan to withdraw an equal end-of-year amount of \$1000 for 5 years, starting next year. At the end of the sixth year, you plan to close your account by withdrawing the remaining money. Define the engineering economy symbols involved.

Solution:

All five symbols are present, but the future value in year 6 is the unknown.
P = \$5000, A = \$1000 per year for 5 years, F = ? at end of year 6, i = 6% per year, n = 5 years for the A series and 6 for the F value.

Example 5:

Last year Lara's grandmother offered to put enough money into a savings account to generate \$5000 in interest this year to help pay Lara's expenses at college. *(a)* Identify the symbols, and *(b)* calculate the amount that had to be deposited exactly 1 year ago to earn \$5000 in interest now, if the rate of return is 6% per year.

Solution:

(a) Symbols P (last year is = 1) and F (this year) are needed.
P = ?, i = 6% per year, n = 1 year, $F = P$ + interest = ? + \$5000

(b) Let F = total amount now and P = original amount. It is known that $F - P$ = \$5000 is accrued interest. Now, P can be determined. Refer to Equations [1] through [4].

$F = P + Pi$

The \$5000 interest can be expressed as:

Interest = $F - P = (P + Pi) - P = Pi$
\$5000 = P (0.06)

P = \$5000/0.06 = \$83,333.33

2.7 Cash Flows: Estimation and Diagramming

As mentioned in earlier sections, cash flows are the amounts of money estimated for future projects or observed for project events that have taken place. All cash flows occur during specific time periods, such as 1 month, every 6 months, or 1 year. Annual is the most common time period. For example, a payment of \$10,000 once every year in December for 5 years is a series of 5 outgoing cash flows, and an estimated receipt of \$500 every month for 2 years is a series of 24 incoming cash flows. Engineering economy bases its computations on the timing, size, and direction of cash flows.

Cash inflows are the receipts, revenues, incomes, and savings generated by project and business activity. A **plus sign** indicates a cash inflow.

econos are costs, disbursements, expenses, and taxes caused by projects and business activity. A **negative or minus sign** indicates a cash outflow. When a project involves only costs, the minus sign may be omitted for some techniques, such as benefit/cost analysis.

Once all cash inflows and outflows are estimated (or determined for a completed project), the net cash flow for each time period is calculated as per equations [6] and [7].

Net cash flow = cash inflows - cash outflows [6]

NCF = R - D [7]

where NCF is net cash flow, R is receipts, and D is disbursements.

At the beginning of this section, the timing, size, and direction of cash flows were mentioned as important. Because cash flows may take place at any time during an interest period, as a matter of convention, all cash flows are assumed to occur at the end of an interest period.

The end-of-period convention means that all cash inflows and all cash outflows are assumed to take place at the **end of the interest period** in which they actually occur. When several inflows and outflows occur within the same period, the net cash flow is assumed to occur at the end of the period.

The **cash flow diagram** is a very important tool in an economic analysis, especially when the cash flow series is complex. It is a graphical representation of cash flows drawn on the y axis with a time scale on the x axis. The diagram includes what is known, what is estimated, and what is needed. That is, once the cash flow diagram is complete, another person should be able to work the problem by looking at the diagram.

Cash flow diagram time t = 0 is the present, and t = 1 is the end of time period 1. It is assumed that the periods are in years for now. Since the end-of-year convention places cash flows at the ends of years, the “1” marks the end of year 1.

While it is not necessary to use an exact scale on the cash flow diagram, errors may be avoided if a neat diagram is made to the approximate scale for both time and relative cash flow magnitudes.

The direction of the arrows on the diagram is important to differentiate income from outgo. A vertical arrow pointing up indicates a positive cash flow. Conversely, a down-pointing arrow indicates a negative cash flow.

Example 6:

Each year, a company expends large amounts of funds for mechanical safety features throughout its worldwide operations. A lead engineer for the company plans expenditures of $1 million now and each of the next 4 years just for the improvement of the equipment. Construct the cash flow diagram to find the equivalent value of these expenditures at the end of year 4, using a cost of capital estimate for safety-related funds of 12% per year.

Solution:

Figure 3 indicates the uniform and negative cash flow series (expenditures) for five periods, and the unknown *F* value (positive cash flow equivalent) at exactly the same time as the fifth expenditure. Since the expenditures start immediately, the first $1 million is shown at time 0, not time 1. Therefore, the last negative cash flow occurs at the end of the fourth year, when *F* also occurs. To make this diagram have a full 5 years on the time scale, the addition of the year = -1 completes the diagram. This addition demonstrates that year 0 is the end-of-period point for the year = -1.

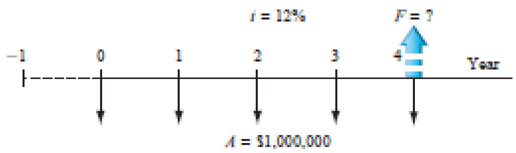

Figure 3 Cash Flow Diagram for Example 6

Example 7:

An electrical engineer wants to deposit an amount *P* now such that she can withdraw an equal annual amount of A_1 = $2000 per year for the first 5 years, starting 1 year after the deposit, and a different annual withdrawal of A_2 = $3000 per year for the following 3 years. How would the cash flow diagram appears if *i* = 8.5% per year?

Solution:

The cash flows are shown in Figure 4. The negative cash outflow *P* occurs now. The withdrawals (positive cash inflow) for the A_1 series occur at the end of years 1 through 5, and A_2 occurs in years 6 through 8.

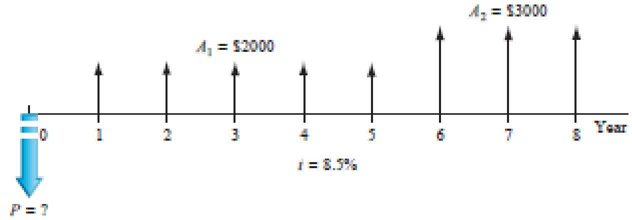

Figure 4 Cash Flow Diagram with two different A Series, Example 7

Example 8:

A rental company spent $2500 on a new air compressor 7 years ago. The annual rental income from the compressor has been $750. The $100 spent on maintenance the first year has increased each year by $25. The company plans to sell the compressor at the end of next year for $150. Construct the cash flow diagram from the company's perspective and indicate where the present worth now is located.

Solution:

Let now be time *t* = 0. The incomes and costs for years -7 through 1 (next year) are tabulated below with net cash flow computed using Equation [6]. The net cash flows (one negative, eight positive) are diagrammed in Figure 5 . Present worth *P* is located at year 0.

End of Year	Income	Cost	Net Cash Flow
−7	$ 0	$2500	$−2500
−6	750	100	650
−5	750	125	625
−4	750	150	600
−3	750	175	575
−2	750	200	550
−1	750	225	525
0	750	250	500
1	750 + 150	275	625

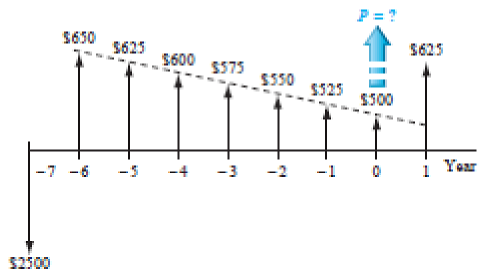

Figure 5 Cash Flow Diagram, Example 8

2.8 Economic Equivalence

Economic equivalence is a fundamental concept upon which engineering economy computations are based.

Economic equivalence is a combination of **interest rate** and **time value of money** to determine the different amounts of money at different points in time that are equal in economic value.
As an illustration, if the interest rate is 6% per year, \$100 today (present time) is equivalent to \$106 one year from today. From equation [4]:

Amount accrued = 100 + 100(0.06) = 100(1 + 0.06) = \$106

If someone receives a gift of \$100 today or \$106 one year from today, it would make no difference which offer is accepted from an economic perspective. In either case \$106 are accumulated one year from today. However, the two sums of money are equivalent to each other only when the interest rate is 6% per year. At a higher or lower interest rate, \$100 today is not equivalent to \$106 one year from today.

In addition to future equivalence, the same logic may be applied to determine equivalence for previous years. A total of \$100 now is equivalent to \$100/1.06 = \$94.34 one year ago at an interest rate of 6% per year. From these illustrations, it can be stated the following: \$94.34 last year, \$100 now, and \$106 one year from now are equivalent at an interest rate of 6% per year.

The cash flow diagram in Figure 6 indicates the amount of interest needed each year to make these three different amounts equivalent at 6% per year.

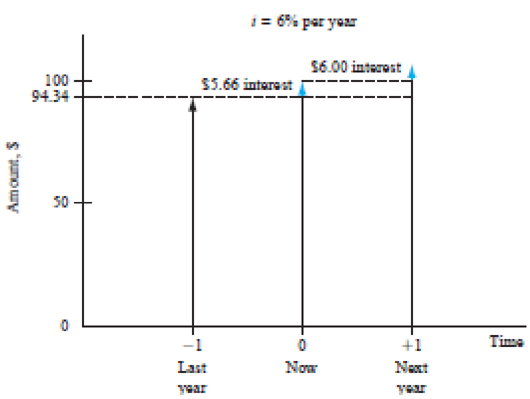

Figure 6 Cash Flow Diagram Showing the Equivalency of Money at 6% per Year

2.9 Simple and Compound Interest

The terms interest, interest period, and interest rate (introduced in previous sections) are useful in calculating equivalent sums of money for one interest period in the past and one period in the future; however, for more than one interest period, the terms simple interest and compound interest become important.

Simple interest is calculated using the principal only, ignoring any interest accrued in preceding interest periods. The total simple interest over several periods is computed as:

Simple Interest = (Principal)(Number of Periods)(Interest Rate) [8]
I = Pni

where *I* is the amount of interest earned or paid and the interest rate *i* is expressed in decimal form.

Example 9:

A Financing company lent an engineering company $100,000 to retrofit an environmentally unfriendly building. The loan is for 3 years at 10% per year simple interest. How much money will the firm repay at the end of 3 years?

Solution:

The interest for each of the 3 years is: Interest per year = $100,000(0.10) = $10,000
Total interest for 3 years from Equation [8] is: Total interest = $100,000(3)(0.10) = $30,000
The amount due after 3 years is: Total due = $100,000 + 30,000 = $130,000
The interest accrued in the first year and in the second year does not earn interest. The interest due each year is $10,000 calculated only on the $100,000 loan principal.

In most financial and economic analyses, **compound interest** calculations are used. For compound interest, the interest accrued for each interest period is calculated on the **principal plus the total amount of interest accumulated in all previous periods**. Thus, compound interest means interest on top of interest.

Compound interest reflects the effect of the time value of money on the interest also. Now the interest for one period is calculated as:

Compound Interest = (Principal + all Accrued Interest)(Interest Rate) [9]

In mathematical terms, the interest I_t for time period t may be calculated using the relation:

$$I_t = \left(P + \sum_{j=1}^{j=t-1} I_j \right)(i) \qquad [10]$$

Total due after n years = (Principal) (1 + Interest Rate)*ⁿ* [11]
= *P* (1 + *i*)*ⁿ*

Example 10:

Assume an engineering company borrows $100,000 at 10% per year compound interest and will pay the principal and all the interest after 3 years. Compute the annual interest and total amount due after 3 years. Graph the interest and total owed for each year and compare with the previous example that involved simple interest.

Solution:
To include compounding of interest, the annual interest and total owed each year are calculated by Equation [9]:

Interest, year 1: 100,000(0.10) = $10,000
Total due, year 1: 100,000 + 10,000 = $110,000
Interest, year 2: 110,000(0.10) = $11,000
Total due, year 2: 110,000 + 11,000 = $121,000
Interest, year 3: 121,000(0.10) = $12,100
Total due, year 3: 121,000 + 12,100 = $133,100

Alternatively, using equation [11]:

The total amount due at the end of each year is:
Year 1: $\$100,000(1.10)^1 = \$110,000$
Year 2: $\$100,000(1.10)^2 = \$121,000$
Year 3: $\$100,000(1.10)^3 = \$133,100$

2.10 Minimum Attractive Rate of Return

For any investment to be profitable, the investor (corporate or individual) expects to receive more money than the amount of capital invested. In other words, a fair rate of return, or return on investment, must be realizable. The definition of ROR in Equation [5] is used in this discussion, that is, amount earned divided by the principal.

The **Minimum Attractive Rate of Return** (MARR) is a reasonable rate of return established for the evaluation and selection of alternatives. A project is not economically viable unless it is expected to return at least the MARR. MARR is also referred to as the hurdle rate, cutoff rate, benchmark rate, and minimum acceptable rate of return.

The MARR is not a rate that is calculated as a ROR. The MARR is established by (financial) managers and is used as a criterion against which an alternative's ROR is measured, when making the accept/reject investment decision.

Although the MARR is used as a criterion to decide on investing in a project, the size of MARR is fundamentally connected to how much it costs to obtain the needed capital funds. It always costs money in the form of interest to raise capital. The interest, expressed as a percentage rate per year, is called the **cost of capital**.

In general, **capital** is developed in two ways: equity financing and debt financing. A combination of these two is very common for most projects and will be discussed in subsequent sections.

Equity Financing: the corporation uses its own funds from cash on hand, stock sales, or retained earnings. Individuals can use their own cash, savings, or investments.

Debt Financing: the corporation borrows from outside sources and repays the principal and interest according to some schedule. Sources of debt capital may be bonds, loans, mortgages, venture capital pools, and many others.

The opportunity cost is the rate of return of a forgone opportunity caused by the inability to pursue a project. Numerically, it is the largest rate of return of all the projects not accepted (forgone) due to the lack of capital funds or other resources. When no specific MARR is established, the de facto MARR is the opportunity cost, i.e., the ROR of the first project not undertaken due to unavailability of capital funds.

2.11 Section Summary

Engineering economy is the application of economic factors and criteria to evaluate alternatives, considering the time value of money. The engineering economy study involves computing a specific economic measure of worth for estimated cash flows over a specific period of time.

The concept of equivalence helps in understanding how different sums of money at different times are equal in economic terms. The differences between simple interest (based on principal only) and compound interest (based on principal and interest upon interest) have been described in formulas, tables, and graphs. This power of compounding is very noticeable, especially over extended periods of time, and for larger sums of money.

The MARR is a reasonable rate of return established as a hurdle rate to determine if an alternative is economically viable. The MARR is always higher than the return from a safe investment and the cost to acquire needed capital.

3 Factors in Engineering Economy

3.1 Single-Amount Factors (*F/P* and *P/F*)

$F = P(1 + i)^n = P(F/P, i, n)$ **[12]**
$P = F(1 + i)^{-n} = F(P/F, i, n)$ **[13]**

The term multiplying P in Equation [12] is called the **Single Payment Compound Amount Factor** (SPCAF). The term multiplying F in Equation [13] is called the **Single Payment Present Worth Factor** (SPPWF).

Example 11:

Mary, a manufacturing engineer, just received a year-end bonus of $10,000 that will be invested immediately. With the expectation of earning at the rate of 8% per year, Mary hopes to take the entire amount out in exactly 20 years to pay for a family vacation when the oldest daughter is due to graduate from college. Find the amount of funds that will be available in 20 years.

Solution:

P = \$10,000 F = ? i = 8% per year n = 20 years
Applying equation [12]: $F = P(1 + i)^n = 10{,}000(1.08)^{20} = 10{,}000(4.6610) =$ \$46,610
Alternatively: $F = P(F/P, i, n)$ = 10,000(4.6610) = \$46,610, where *(F/P,* 8%, 20*)* is obtained from Table 13 at the end of the document.

The equivalency statement is: If Mary invests \$10,000 now and earns 8% per year every year for 20 years, \$46,610 will be available for the family vacation.

Example 12:

A Cement factory required an investment of \$200 million to construct in year 2012. Delays beyond the anticipated implementation year of 2012 will require additional money to construct the factory. Assuming that the cost of money is 10% per year, compound interest, determine the following for the board of directors that plans to develop the plant.

(a) The equivalent investment needed if the plant is built in 2015.
(b) The equivalent investment needed had the plant been constructed in the year 2008.

Solution:

Figure 7 is a cash flow diagram showing the expected investment of \$200 million (\$200 M) in 2012, which we will identify as time $t = 0$. The required investments 3 years in the future and 4 years in the past are indicated by F_3 = ? and P_{-4} = ?, respectively.

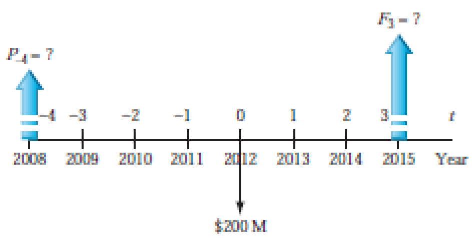

Figure 7 Cash Flow Diagram for Example 12

(a) To find the equivalent investment required in 3 years, apply the *F/P* factor. Use \$1 million units and the tabulated value for 10% interest.

$F_3 = P(F/P, i, n)$ = 200M*(F/P,* 10%, 3*)* = 200M(1.3310) = \$266.2M = \$266,200,000, where *(F/P, 10%, 3)* is obtained from Table 15 at the end of the document.

(b) The year 2008 is 4 years prior to the planned construction date of 2012. To determine the equivalent cost 4 years earlier, consider the \$200 M in 2012 ($t = 0$) as the future value F
and apply the *P*/*F* factor for $n = 4$ to find P_{-4}. (Refer to Figure 7).

$P_{-4} = F(P/F, i, n)$ = 200M(*P*/*F*, 10%, 4) = 200M(0.6830) = \$136.6M = \$136,600,000, where *(F/P,* 10%, 4*)* is obtained from Table 15 at the end of the document.

This equivalence analysis indicates that at $136.6 M in 2008, the plant would have cost about 68% as much as in 2012, and that waiting until 2015, will cause the price tag to increase about 33% to $266 M.

3.2 Uniform Series Present Worth Factor and Capital Recovery Factor (*P/A* and *A/P*)

$$P = A[(1 + i)^n - 1]/i(1 + i)^n = A(P/A, i, n) \quad [14]$$
$$A = P[i(1 + i)^n]/[(1 + i)^n - 1] \quad [15]$$

The term multiplying *A* in Equation [14] is the conversion factor referred to as the **Uniform Series Present Worth Factor** (USPWF). It is the *P/A* factor used to calculate the equivalent *P* value in year 0 for a uniform end-of-period series of *A* values beginning at the end of period 1 and extending for n periods. The cash flow diagram is Figure 8.

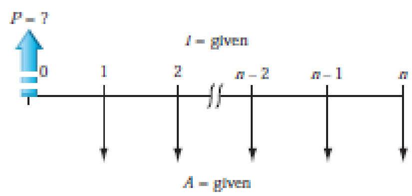

Figure 8 Cash Flow Diagram Used to Determine *P*, Given a Uniform Series *A*

To reverse the situation, the present worth *P* is known and the equivalent uniform series amount *A* is sought (Figure 9). The first *A* value occurs at the end of period 1, that is, one period after *P* occurs. The term multiplying *P* in Equation [15] is called the **Capital Recovery Factor** (CRF).

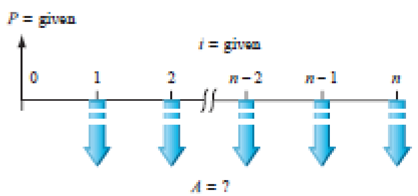

Figure 9 Cash Flow Diagram Used to Determine *A*, Given a Present Worth *P*

Example 13:

How much money should you be willing to pay now for a guaranteed $600 per year for 9 years starting next year, at a rate of return of 16% per year?

Solution:

The cash flows follow the pattern of Figure 8, with A = $600, i = 16%, and n = 9. The present worth is:

P = 600(*P/A*, 16%, 9) = 600(4.6065) = $2763.90, where *(P/A*, 16%, 9*)* is obtained from Table 20 at the end of the document.

3.3 Sinking Fund Factor and Uniform Series Compound Amount Factor (*A/F* and *F/A*)

$A = Fi/[(1 + i)^n - 1] = F(A/F, i, n)$ **[16]**
$F = A[(1 + i)^n - 1]/i = A(F/A, i, n)$ **[17]**

The term multiplying *F* in Equation [16] is called the **Sinking Fund Factor** (SFF). The uniform series *A* begins at the end of year (period) 1 and continues through the year of the given *F*. The last *A* value and *F* occur at the same time as shown in Figure 10.

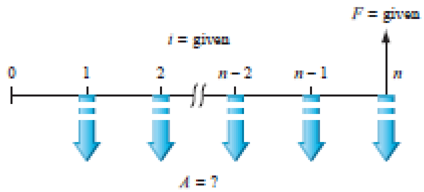

Figure 10 Cash Flow Diagram to Find *A* Given *F*

The term multiplying A in Equation [17] is called the **Uniform Series Compound Amount Factor** (USCAF). It is important to remember that the future amount *F* occurs in the same period as the last *A* as shown in Figure 11.

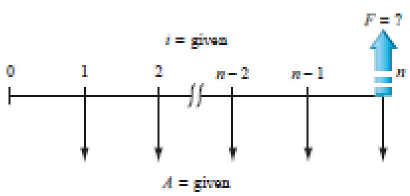

Figure 11 Cash Flow Diagram to Find *F* Given A

Example 14:

The president of a company wants to know the equivalent future worth of a $1 million capital investment each year for 8 years, starting 1 year from now. The company's capital earns at a rate of 14% per year.

Solution:

The cash flow diagram (Figure 12) shows the annual investments starting at the end of year 1 and ending in the year the future worth is desired. In $1000 units, the *F* value in year 8 is found by using the *F/A* factor.

F = 1000(F/*A*, 14%, 8) = 1000(13.2328) = $13,232.80, where *(F/A*, 14%, 8*)* is obtained from Table 18 at the end of the document.

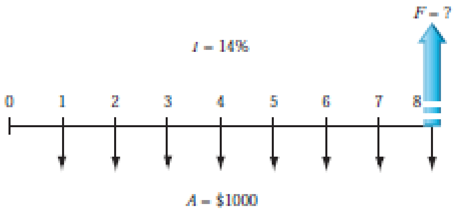

Figure 12 Cash Flow Diagram for Example 14

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3.4 Arithmetic Gradient Factor (P/G and A/G)

An Arithmetic Gradient series is a cash flow series that either increases or decreases by a constant amount each period. The amount of change is called the Gradient.

$P_G = G\{[(1 + i)^n - 1]/i(1 + i)^n - n/(1 + i)^n\} = G(P/G, i, n)$ **[18]**
$A_G = G[1/i - n/[(1 + i)^n - 1] = G(A/G, i, n)$ **[19]**
$F_G = G\{(1/i)[(1 + i)^n - 1]/i - n\}$ **[20]**

Equation [18] converts an **Arithmetic Gradient** *G*, starting with a zero value at year 1, and increasing by an amount *G* every year for n years into a *P* value at year 0 (Figure 13). The factor multiplying *G* in Equation [18] is called the **Arithmetic Gradient Present Worth Factor** (AGPWF).

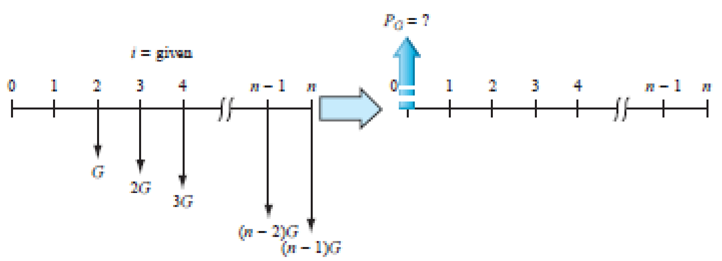

Figure 13 Cash Flow Diagram to Find *P* Given *G*

Equation [19] converts an Arithmetic Gradient *G*, starting with a zero value at year 1, and increasing by an amount G every year for n years into an *A* value, where the first *A* value starts at year 1 (Figure 14). The factor multiplying G in Equation [19] is called the Arithmetic Gradient Uniform Series Factor (AGUSF).

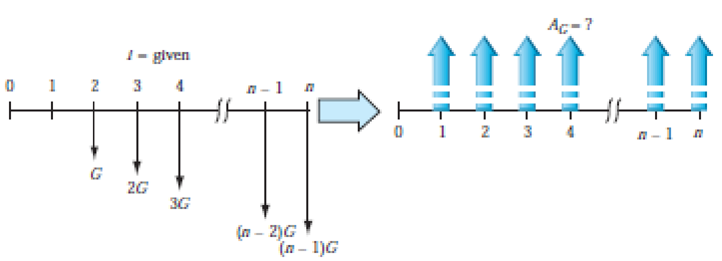

Figure 14 Cash Flow Diagram to Find *A* Given *G*

Equation [20] converts an Arithmetic Gradient G, starting with a zero value at year 1, and increasing by an amount *G* every year for *n* years into an *F* value at year *n*. The factor multiplying *G* in Equation [20] is called the Arithmetic Gradient Future Worth Factor (AGFWF).

Example 15:

The Highway Department expects the cost of maintenance for a piece of heavy construction equipment to be $0 in year 1, to be $500 in year 2, and to increase annually by $500 through year 10. At an interest rate of 10% per year, determine the present worth of 10 years of maintenance costs.

Solution:

P = 500(*P/G, i, n)* = 500(*P/G,* 10%, 10) = 500(22.8913) = $11,445.65, where *(P/G,* 10%, 10*)* is obtained from Table 15 at the end of the document.

Example 16:

The Highway Department expects the cost of maintenance for a piece of heavy construction equipment to be $5000 in year 1, to be $5500 in year 2, and to increase annually by $500 through year 10. At an interest rate of 10% per year, determine the present worth of 10 years of maintenance costs.

Answer:

The cash flow includes a base amount of $5000 starting in year 1 and an increasing gradient with

G = $500, where the first value of G is 0 at year 1.
P = 5000(*P/A*, 10%, 10) + 500(*P/G*, 10%, 10) = 5000(6.1446) + 500(22.8913) = $42,169, where *(P/A*, 10%, 10) and (*P/G*, 10%, 10) are obtained from Table 15 at the end of the document.

Example 17:

Assume the amount planned for investment for 2013 is $100 M with constant decreases of $25M each year thereafter and stops at 2016, and the time value of money for investment capital is 10% per year. Determine *PW* at year 2012.

Solution:

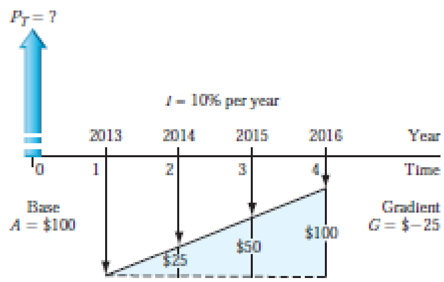

Figure 15 Cash Flow Diagram for Decreasing Gradient for Example 16

This is the case of a **decreasing gradient, which will be solved by deducting an increasing gradient from a uniform series** as shown in Figure 15.

$P_T = P_A - P_G$ = 100(*P/A*, 10%, 4) - 25(*P/G*, 10%, 4) = 100(3.1699) – 25(4.3781) = \$207.537M = \$207,537,000, where (*P/A*, 10%, 4) and (*P/G*, 10%, 4) are obtained from Table 15 at the end of the document.

3.5 Calculations for Cash Flows that are Shifted

When a uniform series begins at a time other than at the end of period 1, it is called a **shifted series**. In this case several methods can be used to find the equivalent present worth *P*. For example, *P* of the uniform series shown in Figure 15 could be determined by any of the following methods:

- Use the *P/F* factor to find the present worth of each disbursement at year 0 and add them.
- Use the *F/P* factor to find the future worth of each disbursement in year 13, add them, and then find the present worth of the total using *P* = *F*(*P/F*, *i*, 13).
- Use the *F/A* factor to find the future amount *F* = *A*(*F/A* ,*i*, 10), and then compute the present worth using *P* = *F*(*P/F*, *i*, 13).
- Use the *P/A* factor to compute the "present worth" (which will be located in year 3 not year 0), and then find the present worth in year 0 by using the (*P/F*, *i* ,3) factor. (Present worth is enclosed in quotation marks here only to represent the present worth as determined by the *P/A* factor in year 3, and to differentiate it from the present worth in year 0.)

Typically, the last method is used. For Figure 16, the "present worth" obtained using the *P/A* factor is located in year 3. This is shown as in Figure 17. **Remember, the present worth is always located one period prior to the first uniform-series amount when using the *P/A* factor.**

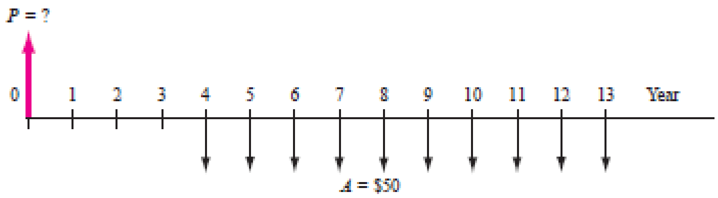

Figure 16 Uniform Series that is Shifted

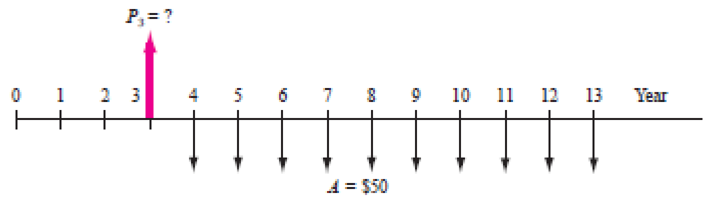

Figure 17 Location of PW for the Shifted Uniform Series in Figure 16

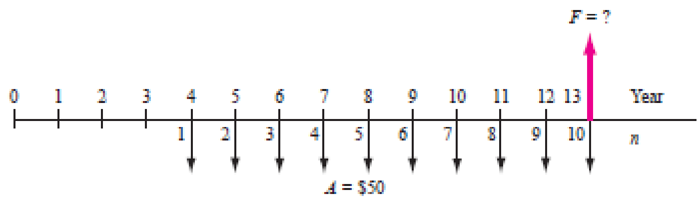

Figure 18 Placement of *F* and Renumbering for *n* for the Shifted Uniform Series

To determine a future worth or *F* value, recall that the *F/A* factor has the *F* located in the *same* period as the last uniform-series amount. Figure 18 shows the location of the future worth when *F/A* is used for Figure 16 cash flows.

Remember, the future worth is always located in the same period as the last uniform-series amount when using the *F/A* factor.

It is also important to remember that the number of periods *n* in the *P/A* or *F/A* factor is equal to the number of uniform-series values. It may be helpful to **renumber** the cash flow diagram to avoid errors in counting. Figure 18 shows Figure 16 renumbered to determine $n = 10$.

As stated above, there are several methods that can be used to solve problems containing a uniform series that is shifted. However, it is generally more convenient to use the uniform-series factors than the single-amount factors. There are specific steps that should be followed in order to avoid errors:

- Draw a diagram of the positive and negative cash flows.
- Locate the present worth or future worth of each series on the cash flow diagram.
- Determine *n* for each series by renumbering the cash flow diagram.
- Set up and solve the equations.

Example 18:

An engineering technology group just purchased new software package for \$5000 now and annual payments of \$500 per year for 6 years starting 3 years from now for annual upgrades. What is the present worth of the payments if the interest rate is 8% per year?

Solution:

The cash flow diagram is shown in Figure 19. The symbol P_A is used throughout this chapter to represent the present worth of a uniform annual series *A*, and P'_A represents the present worth at a time other than period 0. Similarly, P_T represents the total present worth at time 0. The correct placement of P'_A and the diagram renumbering to obtain *n* are also indicated. Note that P'_A is located in actual year 2, not year 3. Also, $n = 6$, not 8, for the *P/A* factor. First find the value of P'_A of the shifted series.

$P'_A = \$500(P/A, 8\%, 6)$

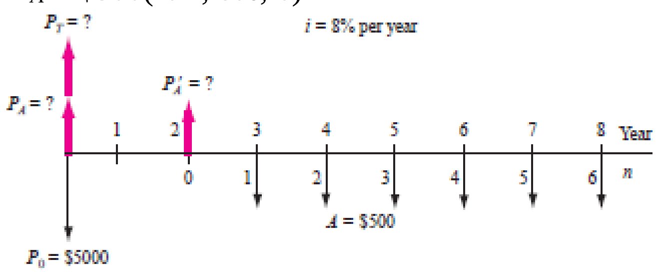

Figure 19 Cash Flow Diagram with Placement of *P* Values, Example 18

Since P'_A is located in year 2, now find P_A in year 0.
$P_A = P'_A(P/F, 8\%, 2)$

The total present worth is determined by adding P_A and the initial payment P_0 in year 0.
$P_T = P_0 + P_A = 5000 + 500(P/A, 8\%, 6)(P/F, 8\%, 2) = 5000 + 500(4.6229)(0.8573) = \$6981.60$, where $(P/A, 8\%, 6)$ and $(P/F, 8\%, 2)$ are obtained from Table 13 at the end of the document.

To determine the present worth for a cash flow that includes both uniform series and single amounts at specific times, use the *P/F* factor for the single amounts and the *P/A* factor for the series. To calculate *A* for the cash flows, first convert everything to a *P* value in year 0, or an *F* value in the last year. Then obtain the *A* value using the *A/P* or *A/F* factor, where *n* is the total number of years over which the *A* is desired.

Many of the considerations that apply to shifted uniform series apply to gradient series as well. Recall that a conventional gradient series starts between periods 1and 2 of the cash flow sequence. A gradient starting at any other time is called a **shifted gradient**. The *n* value in the *P/G* and *A/G* factors for the shifted gradient is determined by renumbering the time scale. The period in which the **gradient first appears is labeled period 2**. The *n* value for the factor is determined by the renumbered period where the last gradient increase occurs. The *P/G* factor values and placement of the gradient series present worth P_G for the shifted arithmetic gradients in Figure 20 are indicated.

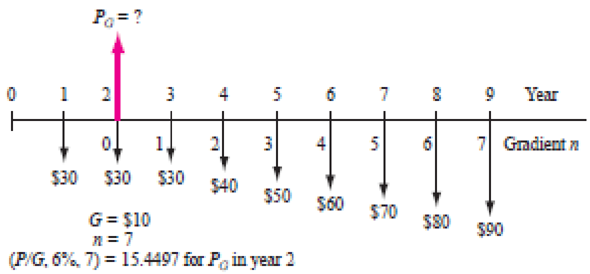

Figure 20 Determination of *G* and *n* Values Used in Factors for Shifted Gradients

It is important to note that the *A/G* factor **cannot** be used to find an equivalent *A* value in periods 1 through *n* for cash flows involving a shifted gradient. Consider the cash flow diagram of Figure 21. To find the equivalent annual series in years 1 through 10 for the gradient series only, first find the present worth of the gradient in year 5, take this present worth back to year 0, and then annualize the present worth for 10 years with the *A/P* factor. If the annual series gradient factor (*A/G*, *i*, 5) is applied directly, the gradient is converted into an equivalent annual series over years 6 through 10 only.

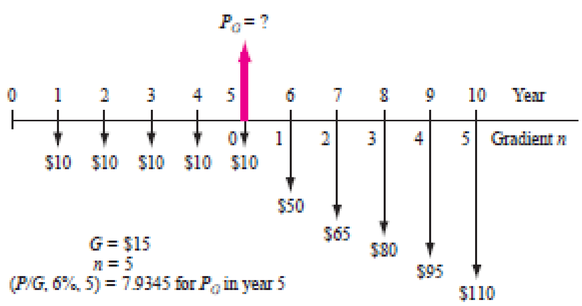

Figure 21 Determination of *G* and *n* Values Used in Factors for Shifted Gradients

Remember, to find the equivalent *A* series of a shifted gradient through all the periods, first find the present worth of the gradient at actual time 0, then apply the (*A/P*, *i*, *n*) factor.

3.6 Section Summary

In this Section, formulas were presented that make it relatively easy to account for the time value of money. In order to use the formulas correctly, certain things must be remembered:

- When using the *P/A* or *A/P* factors, the *P* and the first *A* value are separated by one interest period.
- When using the *F/A* or *AF* factors, the *F* and the last *A* value are in the *same* interest period.
- The *n* in the uniform series formulas is equal to the number of *A* values involved.
- Arithmetic gradients change by a uniform amount from one interest period to the next, and there are two parts to the equation: a uniform series that has an *A* value equal to the

magnitude of the cash flow in period 1 and the gradient that has the same *n* as the uniform series.

- For shifted gradients, the change equal to G occurs between periods 1 and 2. This requires renumbering the cash flows to properly identify which ones are accounted for in the gradient equations.
- For decreasing arithmetic gradients, it is necessary to change the sign in front of the *P/G* or *A/G* factors from plus to minus.

4 Nominal and Effective Interest Rates

4.1 Introduction

In all engineering economy relations developed thus far, the interest rate has been a constant, annual value. For a substantial percentage of the projects evaluated by professional engineers in practice, the interest rate is compounded more frequently than once a year; frequencies such as semiannually, quarterly, and monthly are common. In fact, weekly, daily, and even continuous compounding may be experienced in some project evaluations.

Also, in our own personal lives, many of the financial considerations such as loans of all types (home mortgages, credit cards, automobiles, boats), checking and savings accounts, investments, stock option plans, etc. have interest rates compounded for a time period shorter than 1 year. This requires the introduction of two new terms: **nominal and effective interest rates**.

This section explains how to understand and use nominal and effective interest rates in engineering practice and in daily life situations. Equivalence calculations for any compounding frequency in combination with any cash flow frequency are presented.

4.2 Nominal and Effective Interest Rate Statements

In Section 1, the primary difference between simple interest and compound interest was explained, i.e. compound interest includes interest on the interest earned in the previous period, while simple interest does not. Here, **nominal and effective interest rates**, which have the same basic relationship, will be discussed. The difference here is that the concepts of nominal and effective must be used when interest is compounded more than once each year. For example, if an interest rate is expressed as 1% per month, the terms **nominal and effective** interest rates must be considered.

To understand and correctly handle effective interest rates is important in engineering practice, as well as for individual finances. The interest amounts for loans, mortgages, bonds, and stocks are commonly based upon interest rates compounded more frequently than annually. The engineering economy study must account for these effects. In personal finances, most cash disbursements and receipts are managed on a non-annual time basis. Again, the effect of compounding more frequently than once per year is present.

A **nominal interest rate r** is an interest rate that **does not account** for compounding. By definition:

r **= interest rate per time period x number of periods [21]**

A nominal rate may be calculated for **any time period longer than the time period stated** by using Equation [21]. For example, the interest rate of 1.5% per month is the same as each of the following nominal rates:

	Time Period	Nominal Rate
24 months	1.5 × 24 = 36%	Nominal rate per 2 years
12 months	1.5 × 12 = 18%	Nominal rate per 1 year
6 months	1.5 × 6 = 9%	Nominal rate per 6 months
3 months	1.5 × 3 = 4.5%	Nominal rate per 3 months

Note that none of these rates mention anything about compounding of interest; they are all of the form " **r % per time period**." These nominal rates are calculated in the same way that simple rates are calculated using Equation [8], that is, interest rate *times* number of periods.

After the nominal rate has been calculated, the **compounding period (CP)** must be included in the interest rate statement. As an illustration, again consider the nominal rate of 1.5% per month. If we define the CP as 1 month, the nominal rate statement is 18% per year, **compounded monthly**, or 4.5% per quarter, **compounded monthly**. Now, an **effective interest rate** can be considered.

An effective interest rate *i* is a rate wherein the **compounding of interest is taken into account**. Effective rates are commonly expressed on an annual basis as an effective annual rate; however, any time basis may be used.

The most common form of interest rate statement when compounding occurs over time periods shorter than 1 year is "% per time period, compounded CP-ly," for example, 10% per year, compounded monthly, or 12% per year, compounded weekly. An effective rate may not always include the compounding period in the statement. If the CP is not mentioned, it is understood to be the same as the time period mentioned with the interest rate. For example, an interest rate of "1.5% per month" means that interest is compounded each month; that is, CP is 1 month. An equivalent effective rate statement, therefore, is 1.5% per month, compounded monthly.
All of the following are effective interest rate statements because either **they state they are effective** or the **compounding period is not mentioned**. In the latter case, the CP is the same as the time period of the interest rate.

Statement	CP	What This Is
i = 10% per year	CP not stated; CP = year	Effective rate per year
i = effective 10% per year, compounded monthly	CP stated; CP = month	Effective rate per year
i = $1\frac{1}{2}$% per month	CP not stated; CP = month	Effective rate per month
i = effective $1\frac{1}{2}$% per month, compounded monthly	CP stated; CP = month	Effective rate per month; terms *effective* and *compounded monthly* are redundant
i = effective 3% per quarter, compounded daily	CP stated; CP = day	Effective rate per quarter

All nominal interest rates can be converted to effective rates as will be seen later. All interest formulas, factors, tabulated values, and spreadsheet functions must use an effective interest rate to properly account for the time value of money.

The term **APR (Annual Percentage Rate)** is often stated as the annual interest rate for credit cards, loans, and house mortgages. This is the same as the **nominal rate**. An APR of 15% is the same as a nominal 15% per year or a nominal 1.25% on a monthly basis. Also the term **APY (Annual Percentage Yield)** is a commonly stated annual rate of return for investments, certificates of deposit, and saving accounts. This is the same as an **effective rate**. The names are different, but the interpretations are identical. As will be shown in the following sections, the effective rate is always greater than or equal to the nominal rate, and similarly APR $\leq$ APY.

Based on these descriptions, there are always three time-based units associated with an interest rate statement:

Interest Period (*t*): The period of time over which the interest is expressed. This is the *t* in the statement of *r* % per time period *t* , for example, 1% *per month.* The time unit of 1 year is by far the most common. It is assumed when not stated otherwise.

Compounding Period *(CP)*: The shortest time unit over which interest is charged or earned. This is defined by the compounding term in the interest rate statement, for example, 8% per year, **compounded monthly.** If CP is not stated, it is assumed to be the same as the interest period.

Compounding Frequency *(m)*: The number of times that compounding occurs within the interest period *t.* If the compounding period CP and the time period *t* are the same, the compounding frequency is 1, for example, 1% **per month, compounded monthly.**

Consider the (nominal) rate of 8% per year, compounded monthly. It has an interest period *t* of 1 year, a compounding period CP of 1 month, and a compounding frequency *m* of 12 times per year. A rate of 6% per year, compounded weekly, has $t = 1$ year, CP = 1 week, and $m = 52$, based on the standard of 52 weeks per year.

In previous sections, all interest rates had *t* and CP values of 1 year, so the compounding frequency was always $m = 1$. This made them all effective rates, because the interest period and compounding period were the same. Now, it will be necessary to express a nominal rate as an effective rate on the same time base as the compounding period.

An effective rate can be determined from a nominal rate by using the relation:

Effective rate per CP = *r* % per time period *t/ m* compounding periods per *t* = *r/m* [22]

As an illustration, assume $r = 9\%$ per year, compounded monthly; then $m = 12$. Equation [22] is used to obtain the effective rate of 9%/12 = 0.75% per month, compounded monthly. Note that changing the interest period *t* does not alter the compounding period, which is 1 month in this illustration. Therefore, $r = 9\%$ per year, compounded monthly, and $r = 4.5\%$ per 6 months, compounded monthly, are two expressions of the same interest rate.

Example 19:

Three different bank loan rates for electric generation equipment are listed below. Determine the effective rate on the basis of the compounding period for each rate:

(*a*) 9% per year, compounded quarterly.
(*b*) 9% per year, compounded monthly.
(*c*) 4.5% per 6 months, compounded weekly.

Solution:

Apply Equation [22] to determine the effective rate per CP for different compounding periods. The graphic in Figure 22 indicates the effective rate per CP and how the interest rate is distributed over time.

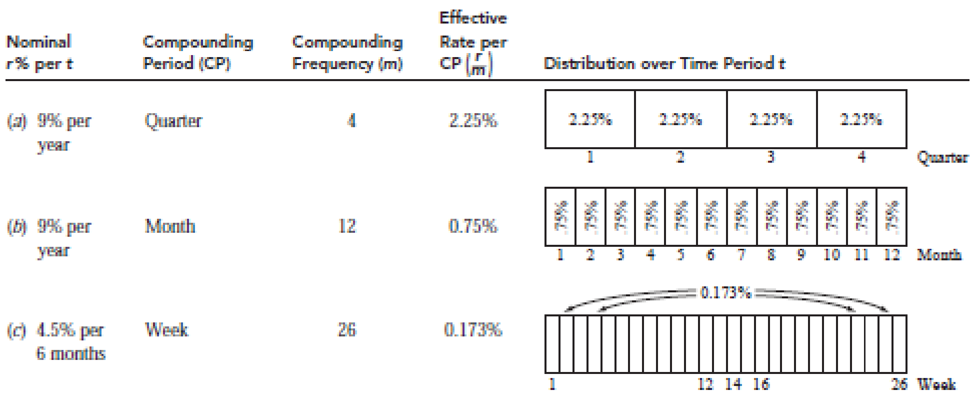

Figure 22 Relations between Interest Period *t*, Compounding Period CP, and Effective Interest Rate per CP

4.3 Effective Interest Rates for any Time Period

Effective i per time period = $(1 + r/m)^m - 1$ [23]

where i = effective rate for specified time period (say, semi-annual)
r = nominal interest rate for same time period (semi-annual)
m = number of times interest is compounded per stated time period (times per 6 months)

The term r/m is always the effective interest rate over a compounding period CP, and m is always the number of times that interest is compounded per the time period on the left of the equals sign in Equation [23].

When the compounding is continuously:

$i = e^r - 1$ [24]

Example 20:

To get a clear understanding of finance costs, a management company asked the engineer to determine the effective semi-annual and annual interest rates for four bids. The bids are as follows:

Bid 1: 9% per year, compounded quarterly
Bid 2: 3% per quarter, compounded quarterly
Bid 3: 8.8% per year, compounded monthly
Bid 4: 18% per year compounded daily
Bid 5: 15% per year compounded continuously

(*a*) Determine the effective annual rate for each bid?
(*b*) What are the effective semi-annual rates for each bid?

Solution:

(a) Bid 1: $i = (1 + 0.09/4)^4 - 1 = 9.31\%$
Bid 2: $i = (1 + 0.03)^4 - 1 = 12.55\%$ (3% per quarter is 12% per year)
Bid 3: $i = (1 + 0.088/12)^{12} - 1 = 9.16\%$
Bid 4: $i = (1 + 0.18/365)^{365} - 1 = 19.716\%$
Bid 5: $i = e^{0.15} - 1 = 16.183\%$

(b) Bid 1: $i = (1 + 0.045/2)^2 - 1 = 4.55\%$ (9% per year is 4.5% per semi-year)
Bid 2: $i = (1 + 0.06/2)^2 - 1 = 6.09\%$ (3% per quarter is 6% per semi-year)
Bid 3: $i = (1 + 0.044/6)^6 - 1 = 4.48\%$
Bid 4: $i = (1 + 0.09/182)^{182} - 1 = 9.415\%$
Bid 5: $i = e^{0.075} - 1 = 7.788\%$

Example 21:

For the past 7 years, a company has paid \$500 every 6 months for a software maintenance contract. What is the equivalent total amount after the last payment, if these funds are taken from a pool that has been returning 8% per year, compounded quarterly?

Solution:
The compounding is quarterly:
F (after 7 years or 14 semi-years) = $500(F/A, i_{\text{eff-semi-year}}, 14)$
$i_{eff\text{-}semi\text{-}year} = (1 + 0.04/2)^2 - 1 = 4.04\%$
$F = \$500(18.3422) = \$9171.09$

Example 22:

Over the past 10 years, a company has placed varying sums of money into a special capital accumulation fund. The company sells compost produced by garbage-to-compost. Figure 23 is the cash flow diagram in \$1000 units. Find the amount in the account after 10 years at an interest rate of 12% per year, compounded semiannually.

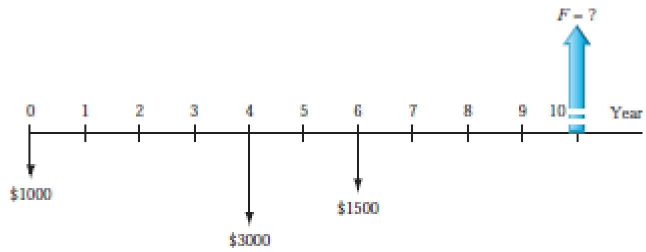

Figure 23 Cash Flow Diagram, Example 22

Solution:

The problem is solved in two ways:

(a) The unit of time is years (All factors are obtained from equation [12]):

$i = (1 + 0.12/2)^2 - 1 = 12.36\%$
$F = 1000(F/P, 12.36\%, 10) + 3000(F/P, 12.36\%, 6) + 1500(F/P, 12.36\%, 4)$
$F = 1000(3.2071) + 3000(2.0122) + 1500(1.5938) = \$11,634$ millions

(b) The unit of time is semi-year:

$i = 12\%/2 = 6\%$ per semi-year (All factors are obtained from Table 11 at the end of the document):

$F = 1000(F/P, 6\%, 20) + 3000(F/P, 6\%, 12) + 1500(F/P, 6\%, 8)$
$F = 1000(3.2071) + 3000(2.0122) + 1500(1.5938) = \$11,634$ millions

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4.4 Summary

Since many real-world situations involve cash flow frequencies and compounding periods other than 1 year, it is necessary to use nominal and effective interest rates.

All engineering economy factors require the use of an effective interest rate. The *i* and *n* values placed in a factor depend upon the type of cash flow series. If only single amounts (*P* and *F*) are present, there are several ways to perform equivalence calculations using the factors. However, when series cash flows (*A*, *G*, and *g*) are present, only one combination of the effective rate *I* and number of periods *n* is correct for the factors. This requires that the relative lengths of PP and CP be considered as *i* and *n* are determined. **The interest rate and payment periods must have the same time unit for the factors to correctly account for the time value of money.** From one year (or interest period) to the next, interest rates will vary. To accurately perform equivalence calculations for *P* and *A* when rates vary significantly, the applicable interest rate should be used, not an average or constant rate.

5 Present Worth Analysis

5.1 Introduction

In this section, techniques for comparing two or more mutually exclusive alternatives by the present worth method are treated.

The nature of the economic proposals is always one of two types:

Mutually exclusive alternatives: Only one of the proposals can be selected. For terminology purposes, each viable proposal is called an *alternative.*

Independent projects: More than one proposal can be selected. Each viable proposal is called a *project.*

The **do-nothing (DN)** proposal is usually understood to be an option when the evaluation is performed. The DN alternative or project means that the current approach is maintained; nothing new is initiated. No new costs, revenues, or savings are generated.

It is important to recognize the nature of the cash flow estimates before starting the computation of a measure of worth that leads to the final selection. Cash flow estimates determine whether the alternatives are revenue or cost-based. All the alternatives or projects must be of the same type when the economic study is performed. Definitions for these types follow:

Revenue: Each alternative generates cost (cash outflow) and revenue (cash inflow) estimates, and possibly savings, also considered cash inflows. Revenues can vary for each alternative.

Cost: Each alternative has only cost cash flow estimates. Revenues or savings are assumed equal for all alternatives; thus they are not dependent upon the alternative selected. These are also referred to as service alternatives.

5.2 Present Worth Analysis of Equal-Life Alternatives

The PW comparison of alternatives with equal lives is straightforward. The present worth *P* is renamed PW of the alternative. The present worth method is quite popular in industry because all future costs and revenues are transformed to **equivalent monetary units NOW;** that is, all future cash flows are converted (discounted) to present amounts (e.g., dollars) at a specific rate of return, which is the MARR. This makes it very simple to determine which alternative has the best economic advantage. The required conditions and evaluation procedure are as follows: If the alternatives have the same capacities for the same time period (life), the **equal-service requirement** is met. Calculate the PW value at the stated MARR for each alternative.

For **mutually exclusive (ME)** alternatives, whether they are revenue or cost alternatives, the following guidelines are applied to justify a single project or to select one from several alternatives.

One alternative: If PW ≥ 0, the requested MARR is met or exceeded and the alternative is economically justified.

Two or more alternatives: Select the alternative with the PW that is **numerically largest,** that is, less negative or more positive. This indicates a lower PW of cost for cost alternatives or a larger PW of net cash flows for revenue alternatives.

For **independent** projects, each PW is considered separately, that is, compared with the DN project, which always has PW = 0. The selection guideline is as follows:

One or more independent projects: Select all projects with PW ≥ 0 at the MARR.

Example 23:

A university lab is a research contractor to a company for in-space fuel cell systems that are hydrogen and methanol-based. During lab research, three equal-service machines need to be evaluated economically. Perform the present worth analysis with the costs shown below. The MARR is 10% per year.

	Electric-Powered	Gas-Powered	Solar-Powered
First cost, $	−4500	3500	−6000
Annual operating cost (AOC), $/year	−900	−700	−50
Salvage value *S*, $	200	350	100
Life, years	8	8	8

Solution:

These are cost alternatives. The salvage values are considered a “negative” cost, so a + sign precedes them. (If it costs money to dispose of an asset, the estimated disposal cost has a - sign.) The PW of each machine is calculated at i = 10% for n = 8 years. Use subscripts E , G , and S.

PW_E = - 4500 - 900(*P/A*, 10%, 8) + 200(*P/F*, 10%, 8) = $ -9208
PW_G = - 3500 - 700(*P/A*, 10%, 8) + 350(*P/F*, 10%, 8) = $ -7071
PW_S = - 6000 - 50(*P/A*, 10%, 8) + 100(*P/F*, 10%, 8) = $ -6220

Where *(P/A*, 10%, 8*)* and (*P/F*, 10%, 8) are obtained from Table 13 at the end of the document. The solar-powered machine is selected since the PW of its costs is the lowest; it has the numerically largest PW value.

5.3 Present Worth Analysis of Different-Life Alternatives

When the present worth method is used to compare mutually exclusive alternatives that have different lives, the equal-service requirement must be met. The procedure of Section 5.1 is followed, with one exception:

The PW of the alternatives must be compared over the same number of years and must end at the same time to satisfy the equal-service requirement.

This is necessary, since the present worth comparison involves calculating the equivalent PW of all future cash flows for each alternative. A fair comparison requires that PW values represent cash flows associated with equal service. For cost alternatives, failure to compare equal service will always favor the shorter-lived mutually exclusive alternative, even if it is not the more economical choice, because fewer periods of costs are involved. The equal-service requirement is satisfied by using either of two approaches:

LCM: Compare the PW of alternatives over a period of time equal to the **least common multiple (LCM)** of their estimated lives.

Study period: Compare the PW of alternatives using a **specified study period of *n* years.** This approach does not necessarily consider the useful life of an alternative. The study period is also called the **planning horizon.**

For either approach, calculate the PW at the MARR and use the same selection guideline as that for equal-life alternatives. The LCM approach makes the cash flow estimates extend to the same period, as required. For example, lives of 3 and 4 years are compared over a 12-year period.

The first cost of an alternative is reinvested at the beginning of each life cycle, and the estimated salvage value is accounted for at the end of each life cycle when calculating the PW values over the LCM period. Additionally, the LCM approach requires that some assumptions be made about subsequent life cycles.

The assumptions when using the LCM approach are that:

(a) The service provided will be needed over the entire LCM years or more.
(b) The selected alternative can be repeated over each life cycle of the LCM in exactly the same manner.
(c) Cash flow estimates are the same for each life cycle.

Example 24:

A construction company plans to purchase new cut-and-finish equipment. Two manufacturers offered the estimates below:

	Vendor A	Vendor B
First cost, $	−15,000	−18,000
Annual M&O cost, $ per year	−3,500	−3,100
Salvage value, $	1,000	2,000
Life, years	6	9

Determine which vendor should be selected on the basis of a present worth comparison, if the MARR is 15% per year.

Solution:

Since the equipment has different lives, compare them over the LCM of 18 years. For life cycles after the first, the first cost is repeated in year 0 of each new cycle, which is the last year of the previous cycle. These are years 6 and 12 for vendor A and year 9 for B. The cash flow diagram is shown in Figure 24. Calculate PW at 15% over 18 years, where *(P/F*, 15%, 6*)* and (*P/F*, 15%, 12), (*P/F*, 15%, 18), and (*P/A*, 15%, 18) are obtained from Table 19 at the end of the document:

PW_A = - 15,000 - 15,000(*P/F*, 15%, 6) + 1000(*P/F*, 15%, 6) - 15,000(*P/F*, 15%, 12) + 1000(*P/F*, 15%, 12) + 1000(*P/F*, 15%, 18) - 3,500(*P/A*, 15%, 18) = $ -45,036

PW_B = - 18,000 - 18,000(*P/F*, 15%, 9) + 2000(*P/F*, 15%, 9) + 2000(*P/F*, 15%, 18) - 3100(*P/A*, 15%, 18) = $ -41,384

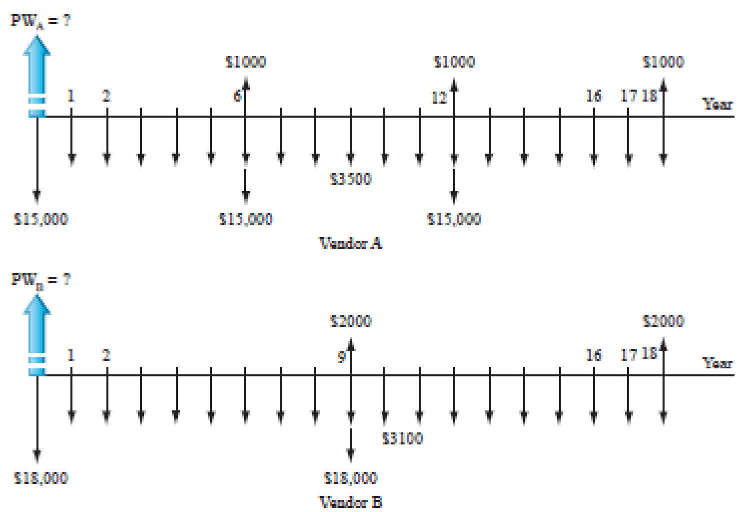

Figure 25 Cash Flow Diagram for Different-Life Alternatives, Example 24

Vendor B is selected, since it costs less in PW terms; that is, the PW_B value is numerically larger than PW_A.

5.4 Future Worth Analysis

The future worth (FW) of an alternative may be determined directly from the cash flows, or by multiplying the PW value by the *F/P* factor, at the established MARR. The *n* value in the *F/P* factor is either the LCM value or a specified study period. Analysis of alternatives using FW values is especially applicable to large capital investment decisions when a prime goal is to maximize **the future wealth** of a corporation's stockholders.

Future worth analysis over a specified study period is often utilized if the asset (equipment, a building, etc.) might be sold or traded at some time before the expected life is reached. Suppose an entrepreneur is planning to buy a company and expects to trade it within 3 years. FW analysis is the best method to help with the decision to sell or keep it 3 years hence.

5.5 Summary

The present worth method of comparing alternatives involves converting all cash flows to present dollars at the MARR. The alternative with the numerically larger (or largest) PW value is selected. When the alternatives have different lives, the comparison must be made for equal-service periods. This is done by performing the comparison over either the LCM of lives or a specific study period. Both approaches compare alternatives in accordance with the equal-service requirement. When a study period is used, any remaining value in an alternative is recognized through the estimated future market value.

6 Annual Worth Analysis

6.1 Introduction

In this section, another alternative comparison tools is added. In section 5, the PW method was explained. In this section, the equivalent annual worth, or AW, method. AW analysis is explained and is commonly considered the more desirable of the two methods because the AW value is easy to calculate; the measure of worth (AW in monetary units per year) is understood by most individuals; and its assumptions are essentially identical to those of the PW method.

Annual worth is also known by other titles. Some are equivalent annual worth (EAW), equivalent annual cost (EAC), annual equivalent (AE), and equivalent uniform annual cost (EUAC). The alternative selected by the AW method will always be the same as that selected by the PW method, and all other alternative evaluation methods, provided they are performed correctly.

An additional application of AW analysis treated here is life-cycle cost (LCC) analysis. This method considers all costs of a product, process, or system from concept to phase-out.

6.2 Annual Worth Analysis

The annual worth method offers a prime computational and interpretation advantage because the AW value needs to be calculated for only one life cycle. The AW value determined over one life cycle is the AW for all future life cycles. Therefore, it is not necessary to use the LCM of lives to satisfy the equal-service requirement.

Example 25:

In Example 24, a company evaluated cut-and-finish equipment from vendor A (6-year life) and vendor B (9-year life). The PW analysis used the LCM of 18 years. Consider only the vendor A option now. The diagram in Figure 26 shows the cash flows for all three life cycles (first cost $ 15,000; annual M&O costs $ -3500; salvage value $1000). Demonstrate the equivalence at i = 15% of PW over three life cycles and AW over one cycle. In Example 24, present worth for vendor A was calculated as PW = $ -45,036.

Solution:
Calculate the equivalent uniform annual worth value for all cash flows in the first life cycle:

AW = -15,000(*A/P*, 15%, 6) + 1000(*A/F*, 15%, 6) - 3500 = $ -7349, where (*A/P*, 15%, 6) and (*A/F*, 15%, 6) are obtained from Table 19 at the end of the document.

When the same computation is performed on each succeeding life cycle, the AW value is $ -7349. Now, the AW Equation is applied to the PW value for 18 years:

AW = -45,036(*A/P*, 15%, 18) = $ -7349, where (*A/P*, 15%, 18) is obtained from Table 19 at the end of the document.

The one-life-cycle AW value and the AW value based on 18 years are equal.

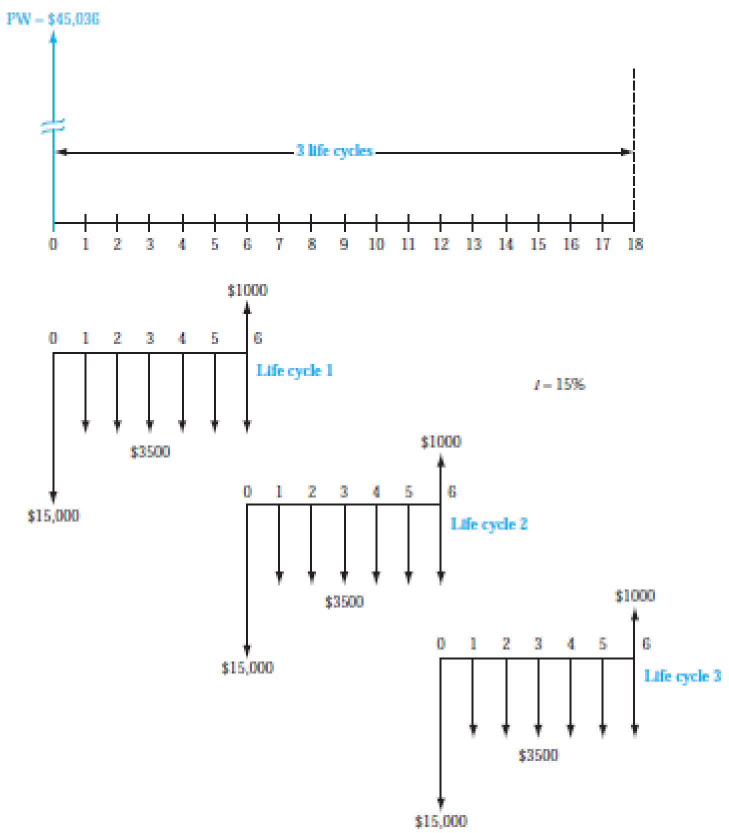

Figure 26 Cash Flow Diagram for Example 25

An alternative should have the following cash flow estimates:

Initial investment: This is the total first cost of all assets and services required to initiate the alternative. When portions of these investments take place over several years, their present worth is an equivalent initial investment. Use this amount as *P*.

Salvage value *S*: This is the terminal estimated value of assets at the end of their useful life.

The *S* is zero if no salvage is anticipated; *S* is negative when it will cost money to dispose of the assets. For study periods shorter than the useful life, *S* is the estimated market value or trade-in value at the end of the study period.

Annual amount *A*: This is the equivalent annual amount (costs only for cost alternatives; costs and receipts for revenue alternatives). Often this is the annual operating cost (AOC) or M&O cost, so the estimate is already an equivalent *A* value.

The annual worth (AW) value for an alternative is comprised of two components: **capital recovery** for the initial investment *P* at a stated interest rate (usually the MARR) and the equivalent annual amount *A*. The symbol CR is used for the capital recovery component. In equation form:

AW = CR + *A*   [25]

Example 26:

Consider below the cash flow diagram of two alternatives and select the better one.

	A	B
Initial cost *P*, $	−15,000	−20,000
Annual M&O, $/year	−6,000	−9,000
Refurbishment cost, $	0	−2,000 every 4 years
Trade-in value *S*, % of *P*	20	40
Life, years	4	12

Solution:

The best evaluation technique for these different-life alternatives is the annual worth method, where AW is taken at 8% per year over the respective lives of 4 and 12 years. (All factors below are obtained from Table 13 at the end of the document).

AW_A = annual equivalent of *P* - annual M&O + annual equivalent of *S*
= -15,000(*A/P*, 8%, 4) - 6000 + 0.2(15,000)(*A/F*, 8%, 4)
= -15,000(0.30192) - 6000 + 3000(0.22192) = $ -9,863

AW = annual equivalent of *P* - annual M&O - annual equivalent of refurbishment + annual equivalent of *S*
= -20,000(*A/P*, 8%, 12) - 9000 - 2000[(*P/F*, 8%, 4) + (*P/F*, 8%, 8)](*A/P*, 8%, 12) + 0.4(20,000)(*A/F*, 8%, 12)
= 20,000(0.13270) – 900 - 2000[0.7350 + 0.5403](0.13270) + 8000(0.05270) = $ -11,571
Alternative A is considerably less costly on an annual equivalent basis, so choose Alternative A.

6.3 Summary

The annual worth method of comparing alternatives is often preferred to the present worth method, because the AW comparison is performed for only one life cycle. This is a distinct advantage when comparing different-life alternatives. The AW for the first life cycle is the AW for the second, third, and all succeeding life cycles, under certain assumptions. When a study period is specified, the AW calculation is determined for that time period, regardless of the lives of the alternatives.

7 Rate of Return Analysis

7.1 Introduction

The most commonly quoted measure of economic worth for a project or alternative is its rate of return (ROR). Whether it is an engineering project with cash flow estimates or an investment in a stock or bond, the rate of return is a well-accepted way of determining if the project or investment is economically acceptable.

The ROR is known by other names such as the internal rate of return (IROR), which is the technically correct term, and return on investment (ROI).

In some cases, more than one ROR value may satisfy the PW or AW equation. This section describes how to recognize this possibility and an approach to find the **multiple values.**

7.2 Rate of Return Analysis

Rate of return (ROR) is the rate paid on the unpaid balance of borrowed money, or the rate earned on the unrecovered balance of an investment, so that the final payment or receipt brings the balance to exactly zero with interest considered.

The **rate of return** is the interest rate that makes **the present worth or annual worth of a cash flow series exactly equal to 0.**

To determine the rate of return, develop the ROR equation using either a PW or AW relation, set it equal to 0 and solve for the interest rate. Alternatively, the present worth of cash outflows (costs and disbursements) PW_O may be equated to the present worth of cash inflows (revenues and savings) PW_I.

The i value that makes these equations numerically correct is called i^*. It is the root of the ROR relation. To determine if the investment project's cash flow series is viable, compare i^* with the established MARR.

The guideline is as follows:

If $i^* \geq$ MARR, accept the project as economically viable.
If $i^* <$ MARR, the project is not economically viable.

Example 27:

Applications of green, lean manufacturing techniques coupled with value stream mapping can make large financial differences over future years while placing greater emphasis on environmental factors. Engineers have recommended to management an investment of $200,000 now in novel methods that will reduce the amount of wastewater, packaging materials, and other solid waste in their consumer paint manufacturing facility. Estimated savings are $15,000 per year for each of the next 10 years and an additional savings of $300,000 at the end of 10 years in facility and equipment upgrade costs. Determine the rate of return.

Solution:
Use the trial-and-error procedure based on a PW equation. Figure 27 shows the cash flow diagram.

Use PW = 0 for the ROR equation:

$0 = -200{,}000 + 15{,}000(P/A, i^*, 10) + 300{,}000(P/F, i^*, 10)$
$200{,}000 = 450{,}000(P/F, i^*, 10)$

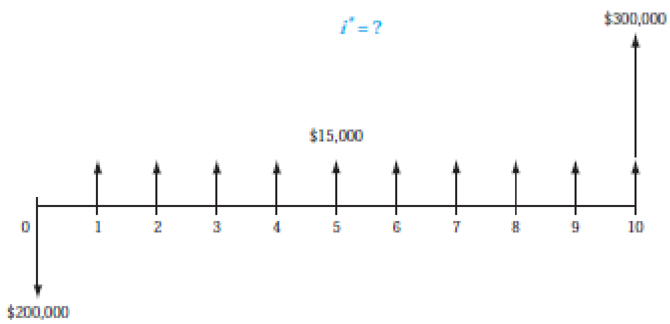

Figure 27 Cash Flow Diagram for Example 27

$(P/F, i^*, 10) = 0.444$
$i^* = 10.58\%$ (by equation [13] or by interpolation between Table 15 and Table 16 at the end of the document).

7.3 Summary

The rate of return of a cash flow series is determined by setting a PW-based or AW-based relation equal to zero and solving for the value of i^*. The ROR is a term used and understood by almost everybody. Most people, however, can have considerable difficulty in calculating a rate of return correctly for anything other than a conventional cash flow series.

8 Benefit/Cost Analysis

8.1 Introduction

The evaluation methods of previous sections are usually applied to alternatives in the private sector, that is, for-profit and not-for-profit corporations and businesses. This section introduces public sector and service sector alternatives and their economic consideration. In the case of public projects, the owners and users (beneficiaries) are the citizens and residents of a government unit (city, county, state, province, or nation). Government units provide the mechanisms to raise capital and operating funds.

Public-private partnerships have become increasingly common, especially for large infrastructure projects such as major highways, power generation plants, water resource developments, and the like.

The benefit/cost (B/C) ratio introduces objectivity into the economic analysis of public sector evaluation, thus reducing the effects of politics and special interests. The different formats of B/C analysis, and associated disbenefits of an alternative, are discussed here. The B/C analysis can use equivalency computations based on PW, AW, or FW values. Performed correctly, the benefit/cost method will always select the same alternative as PW, AW, and ROR analyses.

A **public sector project** is a product, service, or system used, financed, and owned by the citizens of any government level. The primary purpose is to **provide service to the citizenry for the public good at no profit**. Areas such as public health, criminal justice, safety, transportation, welfare, and utilities are publicly owned and require economic evaluation.

To perform a benefit/cost economic analysis of public alternatives, the costs (initial and annual), the benefits, and the disbenefits, if considered, must be estimated as accurately as possible in monetary units.

Costs: estimated expenditures to the government entity for construction, operation, and maintenance of the project, less any expected salvage value.

Benefits: advantages to be experienced by the owners, the public.

Disbenefits: expected undesirable or negative consequences to the owners if the alternative is implemented. Disbenefits may be indirect economic disadvantages of the alternative.

8.2 Benefit/Cost Analysis

The benefit/cost ratio is relied upon as a fundamental analysis method for public sector projects. All cost and benefit estimates must be converted to a common equivalent monetary unit (PW, AW, or FW) at the discount rate (interest rate). The B/C ratio is then calculated using one of these relations:

B/C = PW of benefits/PW of costs = AW of benefits/AW of costs = FW of benefits/FW of costs [26]

The decision guideline is simple:

If B/C ≥ 1.0, accept the project as economically justified for the estimates and discount rate applied.

If B/C < 1.0, the project is not economically acceptable.

The **conventional B/C ratio,** probably the most widely used, is calculated as follows:

B/C = (benefits – disbenefits)/costs = (B – D)/C [27]

In Equation [27], disbenefits are subtracted from benefits, not added to costs.

Example 28:

A company is evaluating a research project where the benefits are $8 million per year, disbenefits are $0.6 million per year, and annual costs are $14.864 million per year. Is this project beneficial?

Answer:

Applying equation [27], B/C = (8M – 0.6M)/14.864M = 0.50 < 1, so project is not beneficial.

8.3 Summary

The benefit/cost method is used primarily to evaluate alternatives in the public sector. All projects with B/C ≥ 1.0 are selected provided there is no budget limitation. It is usually quite **difficult to make accurate estimates of benefits** for public sector projects. The characteristics of public sector projects are substantially different from those of the private sector: initial costs are larger; expected life is longer; additional sources of capital funds include taxation, user fees, and government grants; and interest (discount) rates are lower.

9 Inflation

9.1 Introduction

Inflation is an increase in the amount of money necessary to obtain the same amount of goods or services before the inflated price was present.

Purchasing power, or **buying power**, measures the value of a currency in terms of the quantity and quality of goods or services that one unit of money will purchase. Inflation decreases the purchasing ability of money in that less goods or services can be purchased for the same one unit of money.

9.2 Inflation Analysis

If a cash flow series is expressed in today's (constant-value) dollars, then its PW is the discounted value using the real interest rate *i*.

If the cash flow is expressed in future dollars, the PW value is obtained using i_f as per equation [28], where *f* is the inflation rate:

$$i_f = i + f + if \quad [28]$$

Example 29:

Consider the following cash flow diagram: *n* = 30 years, *F* at 30 years = $ +50,000, *i* = 4% per year compounded annually, *A* = $ +2500 per year, inflation rate *f* = 2.5% per year. Calculate the PW (a) without taking inflation into consideration, and (b) taking inflation into consideration.

Answer:

(a) Without inflation: PW = 2500(*P/A*, 4%, 30) + 50,000(*P/F*, 4%, 30) = $58,645, where all factors are obtained from Table 9 at the end of the document.

(b) With inflation: $i_f = i + f + if$ = 0.04 + 0.025 + (0.04)(0.025) = 0.066 or 6.6% PW = 2500(*P/A*, 6.6%, 30) + 50,000(*P/F*, 6.6%, 30) = $39,660, where all the factors are obtained from equation [13].

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10 Depreciation Methods

10.1 Introduction

Depreciation is a book method (non-cash) to represent the reduction in value of a tangible asset. The method used to depreciate an asset is a way to account for the decreasing value of the asset to the owner and to represent the diminishing value (amount) of the capital funds invested in it. The annual depreciation amount is **not an actual cash flow**, nor does it necessarily reflect the actual usage pattern of the asset during ownership.

Though the term **amortization** is sometimes used interchangeably with the term *depreciation*, they are different. Depreciation is applied to tangible assets, while amortization is used to reflect the decreasing value of intangibles, such as loans, mortgages, patents, trademarks, and goodwill.

10.2 Straight Line (SL) Depreciation

Straight line depreciation derives its name from the fact that the book value decreases **linearly with time.** The depreciation rate (d_t) is the same ($1/n$) each year of the recovery period n. Straight line depreciation is considered the standard against which any depreciation model is compared.

The annual SL depreciation is determined by multiplying the first cost minus the salvage value by d_t. In equation form:

$$D_t = (B - S)\, d_t = (B - S)/n \qquad [29]$$

where t = year ($t = 1, 2, \ldots, n$)
D_t = annual depreciation charge
B = first cost or unadjusted basis
S = estimated salvage value
n = recovery period
d_t = depreciation rate = $1/n$

Since the asset is depreciated by the same amount each year, the book value after t years of service, denoted by BV_t, will be equal to the first cost B minus the annual depreciation times t.

$$BV_t = B - t\, D_t \qquad [30]$$

Example 30:

If an asset has a first cost of $50,000 with a $10,000 estimated salvage value after 5 years, (*a*) calculate the annual depreciation and (*b*) calculate the book value of the asset after each year, using straight line depreciation.

Solution:

(a) The depreciation each year for 5 years can be found by Equation [29]:
$D_t = (B - S)/n = (50{,}000 - 10{,}000)/5 = \$8000$

(b) The book value after each year t is computed using Equation [30]:
$BV_t = B - t\, D_t$
BV_1 (at year 1) = 50,000 – 1(8000) = $42,000
BV_2 (at year 2) = 50,000 – 2(8000) = $34,000
BV_3 (at year 3) = 50,000 – 3(8000) = $26,000
BV_4 (at year 4) = 50,000 – 4(8000) = $18,000
BV_5 (at year 5) = 50,000 – 5(8000) = $10,000

10.3 Declining Balance (DB) and Double Declining Balance (DDB) Depreciation

The **declining balance method** is commonly applied as the book depreciation method. Declining balance is also known as the fixed percentage or uniform percentage method. DB depreciation accelerates the write-off of asset value because the annual depreciation is determined by multiplying the **book value at the beginning of a year** by a fixed (uniform) percentage d, expressed in decimal form.

The maximum annual depreciation rate for the DB method is twice the straight line rate, that is:

$$d_{max} = 2/n \quad [31]$$

In this case the method is called **double declining balance** *(DDB)*.

The depreciation in year t can be calculated using B and d:.

$$D_t = dB(1 - d)^{t-1} \quad [32]$$

The book value in year t is determined in one of two ways: by using the rate d and basis B or by subtracting the current depreciation charge from the previous book value. The equations are:

$$BV_t = B(1 - d)^t \quad [33]$$
$$BV_t = BV_{t-1} - D_t \quad [34]$$

It is important to understand that the book value for the DB method never goes to zero, because the book value is always decreased by a fixed percentage. The implied salvage value after n years is the BV_n amount, that is:

$$\text{Implied } S = BV_n = B(1 - d)^n \quad [35]$$

If a salvage value is estimated for the asset, this **estimated S value is not used in the DB or DDB method** to calculate annual depreciation. However, if the implied $S <$ estimated S, it is necessary to stop charging further depreciation when the book value is at or below the estimated salvage value.

Example 31:

Equipment was purchased for use in specific applications. The equipment will be DDB depreciated over an expected life of 12 years. There is a first cost of \$25,000 and an estimated salvage of \$2500.

(a) Calculate the depreciation and book value for years 1 and 4.

(b) Calculate the implied salvage value after 12 years.

Solution:

(a) The DDB fixed depreciation rate is $d = 2/n = 2/12 = 0.1667$ per year. Use Equations [32] and [33]:
Year 1: $D_1 = (0.1667)(25{,}000)(1 - 0.1667)^{1-1} = \$4167$
$BV_1 = 25{,}000(1 - 0.1667)^1 = \$20{,}833$
Year 4: $D_4 = (0.1667)(25{,}000)(1 - 0.1667)^{4-1} = \$2411$
$BV_4 = 25{,}000(1 - 0.1667)^4 = \$12{,}054$

(b) From Equation [35], the implied salvage value after 12 years is:
Implied $S = 25{,}000(1 - 0.1667)^{12} = \$2803$
Since the estimated $S = \$2500$ is less than \$2803, the asset is not fully depreciated when its 12-year expected life is reached.

10.4 Modified Accelerated Cost Recovery System (MACRS)

In the 1980s, the United States introduced MACRS as the **required tax depreciation method** for all depreciable assets. MACRS determines annual depreciation amounts using the relations:

$$\boldsymbol{D_t = d_t B} \quad \textbf{[36]}$$

$$\mathbf{BV}_t = \mathbf{BV}_{t-1} - \boldsymbol{D_t} \quad \textbf{[37]}$$

$$\mathbf{BV}_t = \textbf{first cost - sum of accumulated depreciation} \quad \textbf{[38]}$$

The basis B (or first cost P) is completely depreciated; salvage is always assumed to be zero, or S = \$0.

Recovery periods are standardized to specific values:
n = 3, 5, 7, 10, 15, or 20 years for personal property (e.g., equipment or vehicles)
n = 27.5 or 39 years for real property (e.g., rental property or structures)

Depreciation rates provide accelerated write-off by incorporating switching between classical methods.
The MACRS personal property depreciation rates (d_t values) for n = 3, 5, 7, 10, 15, and 20 for use in Equations [36], [37], and [38], and are included in Table below:

Year	Depreciation Rate (%) for Each MACRS Recovery Period in Years					
	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 15$	$n = 20$
1	33.33	20.00	14.29	10.00	5.00	3.75
2	44.45	32.00	24.49	18.00	9.50	7.22
3	14.81	19.20	17.49	14.40	8.55	6.68
4	7.41	11.52	12.49	11.52	7.70	6.18
5		11.52	8.93	9.22	6.93	5.71
6		5.76	8.92	7.37	6.23	5.29
7			8.93	6.55	5.90	4.89
8			4.46	6.55	5.90	4.52
9				6.56	5.91	4.46
10				6.55	5.90	4.46
11				3.28	5.91	4.46
12					5.90	4.46
13					5.91	4.46
14					5.90	4.46
15					5.91	4.46
16					2.95	4.46
17–20						4.46
21						2.23

10.5 Summary

Depreciation may be determined for internal company records (book depreciation) or for income tax purposes (tax depreciation). Depreciation does not result in cash flow directly. It is a book method by which the capital investment in tangible property is recovered. The annual depreciation amount is tax deductible, which can result in actual cash flow changes.

Some important points about the straight line and the declining balance methods are presented below:

Straight Line (SL)

• It writes off capital investment linearly over *n* years.

• The estimated salvage value is always considered.

• This is the classical, non-accelerated depreciation model.

Declining Balance (DB)

• The method accelerates depreciation compared to the straight line method.

• The book value is reduced each year by a fixed percentage.

• The most used rate is twice the SL rate, which is called double declining balance (DDB).

• It has an implied salvage that may be lower than the estimated salvage.

- It is not an approved tax depreciation method in the United States. It is frequently used for book depreciation purposes.

Modified Accelerated Cost Recovery System (MACRS)

- It is the only approved tax depreciation system in the United States.
- It automatically switches from DDB or DB to SL depreciation.
- It always depreciates to zero; that is, it assumes S = 0.
- Recovery periods are specified by property classes.
- Depreciation rates are tabulated.
- The actual recovery period is 1 year longer due to the imposed half-year convention.
- MACRS straight line depreciation is an option, but recovery periods are longer than those for regular MACRS.

0.25%		TABLE 1 Discrete Cash Flow: Compound Interest Factors						0.25%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.0025	0.9975	1.00000	1.0000	1.00250	0.9975		
2	1.0050	0.9950	0.49938	2.0025	0.50188	1.9925	0.9950	0.4994
3	1.0075	0.9925	0.33250	3.0075	0.33500	2.9851	2.9801	0.9983
4	1.0100	0.9901	0.24906	4.0150	0.25156	3.9751	5.9503	1.4969
5	1.0126	0.9876	0.19900	5.0251	0.20150	4.9627	9.9007	1.9950
6	1.0151	0.9851	0.16563	6.0376	0.16813	5.9478	14.8263	2.4927
7	1.0176	0.9827	0.14179	7.0527	0.14429	6.9305	20.7223	2.9900
8	1.0202	0.9802	0.12391	8.0704	0.12641	7.9107	27.5839	3.4869
9	1.0227	0.9778	0.11000	9.0905	0.11250	8.8885	35.4061	3.9834
10	1.0253	0.9753	0.09888	10.1133	0.10138	9.8639	44.1842	4.4794
11	1.0278	0.9729	0.08978	11.1385	0.09228	10.8368	53.9133	4.9750
12	1.0304	0.9705	0.08219	12.1664	0.08469	11.8073	64.5886	5.4702
13	1.0330	0.9681	0.07578	13.1968	0.07828	12.7753	76.2053	5.9650
14	1.0356	0.9656	0.07028	14.2298	0.07278	13.7410	88.7587	6.4594
15	1.0382	0.9632	0.06551	15.2654	0.06801	14.7042	102.2441	6.9534
16	1.0408	0.9608	0.06134	16.3035	0.06384	15.6650	116.6567	7.4469
17	1.0434	0.9584	0.05766	17.3443	0.06016	16.6235	131.9917	7.9401
18	1.0460	0.9561	0.05438	18.3876	0.05688	17.5795	148.2446	8.4328
19	1.0486	0.9537	0.05146	19.4336	0.05396	18.5332	165.4106	8.9251
20	1.0512	0.9513	0.04882	20.4822	0.05132	19.4845	183.4851	9.4170
21	1.0538	0.9489	0.04644	21.5334	0.04894	20.4334	202.4634	9.9085
22	1.0565	0.9466	0.04427	22.5872	0.04677	21.3800	222.3410	10.3995
23	1.0591	0.9442	0.04229	23.6437	0.04479	22.3241	243.1131	10.8901
24	1.0618	0.9418	0.04048	24.7028	0.04298	23.2660	264.7753	11.3804
25	1.0644	0.9395	0.03881	25.7646	0.04131	24.2055	287.3230	11.8702
26	1.0671	0.9371	0.03727	26.8290	0.03977	25.1426	310.7516	12.3596
27	1.0697	0.9348	0.03585	27.8961	0.03835	26.0774	335.0566	12.8485
28	1.0724	0.9325	0.03452	28.9658	0.03702	27.0099	360.2334	13.3371
29	1.0751	0.9301	0.03329	30.0382	0.03579	27.9400	386.2776	13.8252
30	1.0778	0.9278	0.03214	31.1133	0.03464	28.8679	413.1847	14.3130
36	1.0941	0.9140	0.02658	37.6206	0.02908	34.3865	592.4988	17.2306
40	1.1050	0.9050	0.02380	42.0132	0.02630	38.0199	728.7399	19.1673
48	1.1273	0.8871	0.01963	50.9312	0.02213	45.1787	1040.06	23.0209
50	1.1330	0.8826	0.01880	53.1887	0.02130	46.9462	1125.78	23.9802
52	1.1386	0.8782	0.01803	55.4575	0.02053	48.7048	1214.59	24.9377
55	1.1472	0.8717	0.01698	58.8819	0.01948	51.3264	1353.53	26.3710
60	1.1616	0.8609	0.01547	64.6467	0.01797	55.6524	1600.08	28.7514
72	1.1969	0.8355	0.01269	78.7794	0.01519	65.8169	2265.56	34.4221
75	1.2059	0.8292	0.01214	82.3792	0.01464	68.3108	2447.61	35.8305
84	1.2334	0.8108	0.01071	93.3419	0.01321	75.6813	3029.76	40.0331
90	1.2520	0.7987	0.00992	100.7885	0.01242	80.5038	3446.87	42.8162
96	1.2709	0.7869	0.00923	108.3474	0.01173	85.2546	3886.28	45.5844
100	1.2836	0.7790	0.00881	113.4500	0.01131	88.3825	4191.24	47.4216
108	1.3095	0.7636	0.00808	123.8093	0.01058	94.5453	4829.01	51.0762
120	1.3494	0.7411	0.00716	139.7414	0.00966	103.5618	5852.11	56.5084
132	1.3904	0.7192	0.00640	156.1582	0.00890	112.3121	6950.01	61.8813
144	1.4327	0.6980	0.00578	173.0743	0.00828	120.8041	8117.41	67.1949
240	1.8208	0.5492	0.00305	328.3020	0.00555	180.3109	19399	107.5863
360	2.4568	0.4070	0.00172	582.7369	0.00422	237.1894	36264	152.8902
480	3.3151	0.3016	0.00108	926.0595	0.00358	279.3418	53821	192.6699

0.5% **TABLE 2 Discrete Cash Flow: Compound Interest Factors** **0.5%**

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.0050	0.9950	1.00000	1.0000	1.00500	0.9950		
2	1.0100	0.9901	0.49875	2.0050	0.50375	1.9851	0.9901	0.4988
3	1.0151	0.9851	0.33167	3.0150	0.33667	2.9702	2.9604	0.9967
4	1.0202	0.9802	0.24813	4.0301	0.25313	3.9505	5.9011	1.4938
5	1.0253	0.9754	0.19801	5.0503	0.20301	4.9259	9.8026	1.9900
6	1.0304	0.9705	0.16460	6.0755	0.16960	5.8964	14.6552	2.4855
7	1.0355	0.9657	0.14073	7.1059	0.14573	6.8621	20.4493	2.9801
8	1.0407	0.9609	0.12283	8.1414	0.12783	7.8230	27.1755	3.4738
9	1.0459	0.9561	0.10891	9.1821	0.11391	8.7791	34.8244	3.9668
10	1.0511	0.9513	0.09777	10.2280	0.10277	9.7304	43.3865	4.4589
11	1.0564	0.9466	0.08866	11.2792	0.09366	10.6770	52.8526	4.9501
12	1.0617	0.9419	0.08107	12.3356	0.08607	11.6189	63.2136	5.4406
13	1.0670	0.9372	0.07464	13.3972	0.07964	12.5562	74.4602	5.9302
14	1.0723	0.9326	0.06914	14.4642	0.07414	13.4887	86.5835	6.4190
15	1.0777	0.9279	0.06436	15.5365	0.06936	14.4166	99.5743	6.9069
16	1.0831	0.9233	0.06019	16.6142	0.06519	15.3399	113.4238	7.3940
17	1.0885	0.9187	0.05651	17.6973	0.06151	16.2586	128.1231	7.8803
18	1.0939	0.9141	0.05323	18.7858	0.05823	17.1728	143.6634	8.3658
19	1.0994	0.9096	0.05030	19.8797	0.05530	18.0824	160.0360	8.8504
20	1.1049	0.9051	0.04767	20.9791	0.05267	18.9874	177.2322	9.3342
21	1.1104	0.9006	0.04528	22.0840	0.05028	19.8880	195.2434	9.8172
22	1.1160	0.8961	0.04311	23.1944	0.04811	20.7841	214.0611	10.2993
23	1.1216	0.8916	0.04113	24.3104	0.04613	21.6757	233.6768	10.7806
24	1.1272	0.8872	0.03932	25.4320	0.04432	22.5629	254.0820	11.2611
25	1.1328	0.8828	0.03765	26.5591	0.04265	23.4456	275.2686	11.7407
26	1.1385	0.8784	0.03611	27.6919	0.04111	24.3240	297.2281	12.2195
27	1.1442	0.8740	0.03469	28.8304	0.03969	25.1980	319.9523	12.6975
28	1.1499	0.8697	0.03336	29.9745	0.03836	26.0677	343.4332	13.1747
29	1.1556	0.8653	0.03213	31.1244	0.03713	26.9330	367.6625	13.6510
30	1.1614	0.8610	0.03098	32.2800	0.03598	27.7941	392.6324	14.1265
36	1.1967	0.8356	0.02542	39.3361	0.03042	32.8710	557.5598	16.9621
40	1.2208	0.8191	0.02265	44.1588	0.02765	36.1722	681.3347	18.8359
48	1.2705	0.7871	0.01849	54.0978	0.02349	42.5803	959.9188	22.5437
50	1.2832	0.7793	0.01765	56.6452	0.02265	44.1428	1035.70	23.4624
52	1.2961	0.7716	0.01689	59.2180	0.02189	45.6897	1113.82	24.3778
55	1.3156	0.7601	0.01584	63.1258	0.02084	47.9814	1235.27	25.7447
60	1.3489	0.7414	0.01433	69.7700	0.01933	51.7256	1448.65	28.0064
72	1.4320	0.6983	0.01157	86.4089	0.01657	60.3395	2012.35	33.3504
75	1.4536	0.6879	0.01102	90.7265	0.01602	62.4136	2163.75	34.6679
84	1.5204	0.6577	0.00961	104.0739	0.01461	68.4530	2640.66	38.5763
90	1.5666	0.6383	0.00883	113.3109	0.01383	72.3313	2976.08	41.1451
96	1.6141	0.6195	0.00814	122.8285	0.01314	76.0952	3324.18	43.6845
100	1.6467	0.6073	0.00773	129.3337	0.01273	78.5426	3562.79	45.3613
108	1.7137	0.5835	0.00701	142.7399	0.01201	83.2934	4054.37	48.6758
120	1.8194	0.5496	0.00610	163.8793	0.01110	90.0735	4823.51	53.5508
132	1.9316	0.5177	0.00537	186.3226	0.01037	96.4596	5624.59	58.3103
144	2.0508	0.4876	0.00476	210.1502	0.00976	102.4747	6451.31	62.9551
240	3.3102	0.3021	0.00216	462.0409	0.00716	139.5808	13416	96.1131
360	6.0226	0.1660	0.00100	1004.52	0.00600	166.7916	21403	128.3236
480	10.9575	0.0913	0.00050	1991.49	0.00550	181.7476	27588	151.7949

0.75% TABLE 3 Discrete Cash Flow: Compound Interest Factors 0.75%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.0075	0.9926	1.00000	1.0000	1.00750	0.9926		
2	1.0151	0.9852	0.49813	2.0075	0.50563	1.9777	0.9852	0.4981
3	1.0227	0.9778	0.33085	3.0226	0.33835	2.9556	2.9408	0.9950
4	1.0303	0.9706	0.24721	4.0452	0.25471	3.9261	5.8525	1.4907
5	1.0381	0.9633	0.19702	5.0756	0.20452	4.8894	9.7058	1.9851
6	1.0459	0.9562	0.16357	6.1136	0.17107	5.8456	14.4866	2.4782
7	1.0537	0.9490	0.13967	7.1595	0.14717	6.7946	20.1808	2.9701
8	1.0616	0.9420	0.12176	8.2132	0.12926	7.7366	26.7747	3.4608
9	1.0696	0.9350	0.10782	9.2748	0.11532	8.6716	34.2544	3.9502
10	1.0776	0.9280	0.09667	10.3443	0.10417	9.5996	42.6064	4.4384
11	1.0857	0.9211	0.08755	11.4219	0.09505	10.5207	51.8174	4.9253
12	1.0938	0.9142	0.07995	12.5076	0.08745	11.4349	61.8740	5.4110
13	1.1020	0.9074	0.07352	13.6014	0.08102	12.3423	72.7632	5.8954
14	1.1103	0.9007	0.06801	14.7034	0.07551	13.2430	84.4720	6.3786
15	1.1186	0.8940	0.06324	15.8137	0.07074	14.1370	96.9876	6.8606
16	1.1270	0.8873	0.05906	16.9323	0.06656	15.0243	110.2973	7.3413
17	1.1354	0.8807	0.05537	18.0593	0.06287	15.9050	124.3887	7.8207
18	1.1440	0.8742	0.05210	19.1947	0.05960	16.7792	139.2494	8.2989
19	1.1525	0.8676	0.04917	20.3387	0.05667	17.6468	154.8671	8.7759
20	1.1612	0.8612	0.04653	21.4912	0.05403	18.5080	171.2297	9.2516
21	1.1699	0.8548	0.04415	22.6524	0.05165	19.3628	188.3253	9.7261
22	1.1787	0.8484	0.04198	23.8223	0.04948	20.2112	206.1420	10.1994
23	1.1875	0.8421	0.04000	25.0010	0.04750	21.0533	224.6682	10.6714
24	1.1964	0.8358	0.03818	26.1885	0.04568	21.8891	243.8923	11.1422
25	1.2054	0.8296	0.03652	27.3849	0.04402	22.7188	263.8029	11.6117
26	1.2144	0.8234	0.03498	28.5903	0.04248	23.5422	284.3888	12.0800
27	1.2235	0.8173	0.03355	29.8047	0.04105	24.3596	305.6387	12.5470
28	1.2327	0.8112	0.03223	31.0282	0.03973	25.1707	327.5416	13.0128
29	1.2420	0.8052	0.03100	32.2609	0.03850	25.9759	350.0867	13.4774
30	1.2513	0.7992	0.02985	33.5029	0.03735	26.7751	373.2631	13.9407
36	1.3086	0.7641	0.02430	41.1527	0.03180	31.4468	524.9924	16.6946
40	1.3483	0.7416	0.02153	46.4465	0.02903	34.4469	637.4693	18.5058
48	1.4314	0.6986	0.01739	57.5207	0.02489	40.1848	886.8404	22.0691
50	1.4530	0.6883	0.01656	60.3943	0.02406	41.5664	953.8486	22.9476
52	1.4748	0.6780	0.01580	63.3111	0.02330	42.9276	1022.59	23.8211
55	1.5083	0.6630	0.01476	67.7688	0.02226	44.9316	1128.79	25.1223
60	1.5657	0.6387	0.01326	75.4241	0.02076	48.1734	1313.52	27.2665
72	1.7126	0.5839	0.01053	95.0070	0.01803	55.4768	1791.25	32.2882
75	1.7514	0.5710	0.00998	100.1833	0.01748	57.2027	1917.22	33.5163
84	1.8732	0.5338	0.00859	116.4269	0.01609	62.1540	2308.13	37.1357
90	1.9591	0.5104	0.00782	127.8790	0.01532	65.2746	2578.00	39.4946
96	2.0489	0.4881	0.00715	139.8562	0.01465	68.2584	2853.94	41.8107
100	2.1111	0.4737	0.00675	148.1445	0.01425	70.1746	3040.75	43.3311
108	2.2411	0.4462	0.00604	165.4832	0.01354	73.8394	3419.90	46.3154
120	2.4514	0.4079	0.00517	193.5143	0.01267	78.9417	3998.56	50.6521
132	2.6813	0.3730	0.00446	224.1748	0.01196	83.6064	4583.57	54.8232
144	2.9328	0.3410	0.00388	257.7116	0.01138	87.8711	5169.58	58.8314
240	6.0092	0.1664	0.00150	667.8869	0.00900	111.1450	9494.12	85.4210
360	14.7306	0.0679	0.00055	1830.74	0.00805	124.2819	13312	107.1145
480	36.1099	0.0277	0.00021	4681.32	0.00771	129.6409	15513	119.6620

1% **TABLE 4 Discrete Cash Flow: Compound Interest Factors** **1%**

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.0100	0.9901	1.00000	1.0000	1.01000	0.9901		
2	1.0201	0.9803	0.49751	2.0100	0.50751	1.9704	0.9803	0.4975
3	1.0303	0.9706	0.33002	3.0301	0.34002	2.9410	2.9215	0.9934
4	1.0406	0.9610	0.24628	4.0604	0.25628	3.9020	5.8044	1.4876
5	1.0510	0.9515	0.19604	5.1010	0.20604	4.8534	9.6103	1.9801
6	1.0615	0.9420	0.16255	6.1520	0.17255	5.7955	14.3205	2.4710
7	1.0721	0.9327	0.13863	7.2135	0.14863	6.7282	19.9168	2.9602
8	1.0829	0.9235	0.12069	8.2857	0.13069	7.6517	26.3812	3.4478
9	1.0937	0.9143	0.10674	9.3685	0.11674	8.5660	33.6959	3.9337
10	1.1046	0.9053	0.09558	10.4622	0.10558	9.4713	41.8435	4.4179
11	1.1157	0.8963	0.08645	11.5668	0.09645	10.3676	50.8067	4.9005
12	1.1268	0.8874	0.07885	12.6825	0.08885	11.2551	60.5687	5.3815
13	1.1381	0.8787	0.07241	13.8093	0.08241	12.1337	71.1126	5.8607
14	1.1495	0.8700	0.06690	14.9474	0.07690	13.0037	82.4221	6.3384
15	1.1610	0.8613	0.06212	16.0969	0.07212	13.8651	94.4810	6.8143
16	1.1726	0.8528	0.05794	17.2579	0.06794	14.7179	107.2734	7.2886
17	1.1843	0.8444	0.05426	18.4304	0.06426	15.5623	120.7834	7.7613
18	1.1961	0.8360	0.05098	19.6147	0.06098	16.3983	134.9957	8.2323
19	1.2081	0.8277	0.04805	20.8109	0.05805	17.2260	149.8950	8.7017
20	1.2202	0.8195	0.04542	22.0190	0.05542	18.0456	165.4664	9.1694
21	1.2324	0.8114	0.04303	23.2392	0.05303	18.8570	181.6950	9.6354
22	1.2447	0.8034	0.04086	24.4716	0.05086	19.6604	198.5663	10.0998
23	1.2572	0.7954	0.03889	25.7163	0.04889	20.4558	216.0660	10.5626
24	1.2697	0.7876	0.03707	26.9735	0.04707	21.2434	234.1800	11.0237
25	1.2824	0.7798	0.03541	28.2432	0.04541	22.0232	252.8945	11.4831
26	1.2953	0.7720	0.03387	29.5256	0.04387	22.7952	272.1957	11.9409
27	1.3082	0.7644	0.03245	30.8209	0.04245	23.5596	292.0702	12.3971
28	1.3213	0.7568	0.03112	32.1291	0.04112	24.3164	312.5047	12.8516
29	1.3345	0.7493	0.02990	33.4504	0.03990	25.0658	333.4863	13.3044
30	1.3478	0.7419	0.02875	34.7849	0.03875	25.8077	355.0021	13.7557
36	1.4308	0.6989	0.02321	43.0769	0.03321	30.1075	494.6207	16.4285
40	1.4889	0.6717	0.02046	48.8864	0.03046	32.8347	596.8561	18.1776
48	1.6122	0.6203	0.01633	61.2226	0.02633	37.9740	820.1460	21.5976
50	1.6446	0.6080	0.01551	64.4632	0.02551	39.1961	879.4176	22.4363
52	1.6777	0.5961	0.01476	67.7689	0.02476	40.3942	939.9175	23.2686
55	1.7285	0.5785	0.01373	72.8525	0.02373	42.1472	1032.81	24.5049
60	1.8167	0.5504	0.01224	81.6697	0.02224	44.9550	1192.81	26.5333
72	2.0471	0.4885	0.00955	104.7099	0.01955	51.1504	1597.87	31.2386
75	2.1091	0.4741	0.00902	110.9128	0.01902	52.5871	1702.73	32.3793
84	2.3067	0.4335	0.00765	130.6723	0.01765	56.6485	2023.32	35.7170
90	2.4486	0.4084	0.00690	144.8633	0.01690	59.1609	2240.57	37.8724
96	2.5993	0.3847	0.00625	159.9273	0.01625	61.5277	2459.43	39.9727
100	2.7048	0.3697	0.00587	170.4814	0.01587	63.0289	2605.78	41.3426
108	2.9289	0.3414	0.00518	192.8926	0.01518	65.8578	2898.42	44.0103
120	3.3004	0.3030	0.00435	230.0387	0.01435	69.7005	3334.11	47.8349
132	3.7190	0.2689	0.00368	271.8959	0.01368	73.1108	3761.69	51.4520
144	4.1906	0.2386	0.00313	319.0616	0.01313	76.1372	4177.47	54.8676
240	10.8926	0.0918	0.00101	989.2554	0.01101	90.8194	6878.60	75.7393
360	35.9496	0.0278	0.00029	3494.96	0.01029	97.2183	8720.43	89.6995
480	118.6477	0.0084	0.00008	11765	0.01008	99.1572	9511.16	95.9200

1.25%	TABLE 5 Discrete Cash Flow: Compound Interest Factors							1.25%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.0125	0.9877	1.00000	1.0000	1.01250	0.9877		
2	1.0252	0.9755	0.49680	2.0125	0.50939	1.9631	0.9755	0.4969
3	1.0380	0.9634	0.32920	3.0377	0.34170	2.9265	2.9023	0.9917
4	1.0509	0.9515	0.24536	4.0756	0.25786	3.8781	5.7569	1.4845
5	1.0641	0.9398	0.19506	5.1266	0.20756	4.8178	9.5160	1.9752
6	1.0774	0.9282	0.16153	6.1907	0.17403	5.7460	14.1569	2.4638
7	1.0909	0.9167	0.13759	7.2680	0.15009	6.6627	19.6571	2.9503
8	1.1045	0.9054	0.11963	8.3589	0.13213	7.5681	25.9949	3.4348
9	1.1183	0.8942	0.10567	9.4634	0.11817	8.4623	33.1487	3.9172
10	1.1323	0.8832	0.09450	10.5817	0.10700	9.3455	41.0973	4.3975
11	1.1464	0.8723	0.08537	11.7139	0.09787	10.2178	49.8201	4.8758
12	1.1608	0.8615	0.07776	12.8604	0.09026	11.0793	59.2967	5.3520
13	1.1753	0.8509	0.07132	14.0211	0.08382	11.9302	69.5072	5.8262
14	1.1900	0.8404	0.06581	15.1964	0.07831	12.7706	80.4320	6.2982
15	1.2048	0.8300	0.06103	16.3863	0.07353	13.6005	92.0519	6.7682
16	1.2199	0.8197	0.05685	17.5912	0.06935	14.4203	104.3481	7.2362
17	1.2351	0.8096	0.05316	18.8111	0.06566	15.2299	117.3021	7.7021
18	1.2506	0.7996	0.04988	20.0462	0.06238	16.0295	130.8958	8.1659
19	1.2662	0.7898	0.04696	21.2968	0.05946	16.8193	145.1115	8.6277
20	1.2820	0.7800	0.04432	22.5630	0.05682	17.5993	159.9316	9.0874
21	1.2981	0.7704	0.04194	23.8450	0.05444	18.3697	175.3392	9.5450
22	1.3143	0.7609	0.03977	25.1431	0.05227	19.1306	191.3174	10.0006
23	1.3307	0.7515	0.03780	26.4574	0.05030	19.8820	207.8499	10.4542
24	1.3474	0.7422	0.03599	27.7881	0.04849	20.6242	224.9204	10.9056
25	1.3642	0.7330	0.03432	29.1354	0.04682	21.3573	242.5132	11.3551
26	1.3812	0.7240	0.03279	30.4996	0.04529	22.0813	260.6128	11.8024
27	1.3985	0.7150	0.03137	31.8809	0.04387	22.7963	279.2040	12.2478
28	1.4160	0.7062	0.03005	33.2794	0.04255	23.5025	298.2719	12.6911
29	1.4337	0.6975	0.02882	34.6954	0.04132	24.2000	317.8019	13.1323
30	1.4516	0.6889	0.02768	36.1291	0.04018	24.8889	337.7797	13.5715
36	1.5639	0.6394	0.02217	45.1155	0.03467	28.8473	466.2830	16.1639
40	1.6436	0.6084	0.01942	51.4896	0.03192	31.3269	559.2320	17.8515
48	1.8154	0.5509	0.01533	65.2284	0.02783	35.9315	759.2296	21.1299
50	1.8610	0.5373	0.01452	68.8818	0.02702	37.0129	811.6738	21.9295
52	1.9078	0.5242	0.01377	72.6271	0.02627	38.0677	864.9409	22.7211
55	1.9803	0.5050	0.01275	78.4225	0.02525	39.6017	946.2277	23.8936
60	2.1072	0.4746	0.01129	88.5745	0.02379	42.0346	1084.84	25.8083
72	2.4459	0.4088	0.00865	115.6736	0.02115	47.2925	1428.46	30.2047
75	2.5388	0.3939	0.00812	123.1035	0.02062	48.4890	1515.79	31.2605
84	2.8391	0.3522	0.00680	147.1290	0.01930	51.8222	1778.84	34.3258
90	3.0588	0.3269	0.00607	164.7050	0.01857	53.8461	1953.83	36.2855
96	3.2955	0.3034	0.00545	183.6411	0.01795	55.7246	2127.52	38.1793
100	3.4634	0.2887	0.00507	197.0723	0.01757	56.9013	2242.24	39.4058
108	3.8253	0.2614	0.00442	226.0226	0.01692	59.0865	2468.26	41.7737
120	4.4402	0.2252	0.00363	275.2171	0.01613	61.9828	2796.57	45.1184
132	5.1540	0.1940	0.00301	332.3198	0.01551	64.4781	3109.35	48.2234
144	5.9825	0.1672	0.00251	398.6021	0.01501	66.6277	3404.61	51.0990
240	19.7155	0.0507	0.00067	1497.24	0.01317	75.9423	5101.53	67.1764
360	87.5410	0.0114	0.00014	6923.28	0.01264	79.0861	5997.90	75.8401
480	388.7007	0.0026	0.00003	31016	0.01253	79.7942	6284.74	78.7619

1.5% **TABLE 6 Discrete Cash Flow: Compound Interest Factors** **1.5%**

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.0150	0.9852	1.00000	1.0000	1.01500	0.9852		
2	1.0302	0.9707	0.49628	2.0150	0.51128	1.9559	0.9707	0.4963
3	1.0457	0.9563	0.32838	3.0452	0.34338	2.9122	2.8833	0.9901
4	1.0614	0.9422	0.24444	4.0909	0.25944	3.8544	5.7098	1.4814
5	1.0773	0.9283	0.19409	5.1523	0.20909	4.7826	9.4229	1.9702
6	1.0934	0.9145	0.16053	6.2296	0.17553	5.6972	13.9956	2.4566
7	1.1098	0.9010	0.13656	7.3230	0.15156	6.5982	19.4018	2.9405
8	1.1265	0.8877	0.11858	8.4328	0.13358	7.4859	25.6157	3.4219
9	1.1434	0.8746	0.10461	9.5593	0.11961	8.3605	32.6125	3.9008
10	1.1605	0.8617	0.09343	10.7027	0.10843	9.2222	40.3675	4.3772
11	1.1779	0.8489	0.08429	11.8633	0.09929	10.0711	48.8568	4.8512
12	1.1956	0.8364	0.07668	13.0412	0.09168	10.9075	58.0571	5.3227
13	1.2136	0.8240	0.07024	14.2368	0.08524	11.7315	67.9454	5.7917
14	1.2318	0.8118	0.06472	15.4504	0.07972	12.5434	78.4994	6.2582
15	1.2502	0.7999	0.05994	16.6821	0.07494	13.3432	89.6974	6.7223
16	1.2690	0.7880	0.05577	17.9324	0.07077	14.1313	101.5178	7.1839
17	1.2880	0.7764	0.05208	19.2014	0.06708	14.9076	113.9400	7.6431
18	1.3073	0.7649	0.04881	20.4894	0.06381	15.6726	126.9435	8.0997
19	1.3270	0.7536	0.04588	21.7967	0.06088	16.4262	140.5084	8.5539
20	1.3469	0.7425	0.04325	23.1237	0.05825	17.1686	154.6154	9.0057
21	1.3671	0.7315	0.04087	24.4705	0.05587	17.9001	169.2453	9.4550
22	1.3876	0.7207	0.03870	25.8376	0.05370	18.6208	184.3798	9.9018
23	1.4084	0.7100	0.03673	27.2251	0.05173	19.3309	200.0006	10.3462
24	1.4295	0.6995	0.03492	28.6335	0.04992	20.0304	216.0901	10.7881
25	1.4509	0.6892	0.03326	30.0630	0.04826	20.7196	232.6310	11.2276
26	1.4727	0.6790	0.03173	31.5140	0.04673	21.3986	249.6065	11.6646
27	1.4948	0.6690	0.03032	32.9867	0.04532	22.0676	267.0002	12.0992
28	1.5172	0.6591	0.02900	34.4815	0.04400	22.7267	284.7958	12.5313
29	1.5400	0.6494	0.02778	35.9987	0.04278	23.3761	302.9779	12.9610
30	1.5631	0.6398	0.02664	37.5387	0.04164	24.0158	321.5310	13.3883
36	1.7091	0.5851	0.02115	47.2760	0.03615	27.6607	439.8303	15.9009
40	1.8140	0.5513	0.01843	54.2679	0.03343	29.9158	524.3568	17.5277
48	2.0435	0.4894	0.01437	69.5652	0.02937	34.0426	703.5462	20.6667
50	2.1052	0.4750	0.01357	73.6828	0.02857	34.9997	749.9636	21.4277
52	2.1689	0.4611	0.01283	77.9249	0.02783	35.9287	796.8774	22.1794
55	2.2679	0.4409	0.01183	84.5296	0.02683	37.2715	868.0285	23.2894
60	2.4432	0.4093	0.01039	96.2147	0.02539	39.3803	988.1674	25.0930
72	2.9212	0.3423	0.00781	128.0772	0.02281	43.8447	1279.79	29.1893
75	3.0546	0.3274	0.00730	136.9728	0.02230	44.8416	1352.56	30.1631
84	3.4926	0.2863	0.00602	166.1726	0.02102	47.5786	1568.51	32.9668
90	3.8189	0.2619	0.00532	187.9299	0.02032	49.2099	1709.54	34.7399
96	4.1758	0.2395	0.00472	211.7202	0.01972	50.7017	1847.47	36.4381
100	4.4320	0.2256	0.00437	228.8030	0.01937	51.6247	1937.45	37.5295
108	4.9927	0.2003	0.00376	266.1778	0.01876	53.3137	2112.13	39.6171
120	5.9693	0.1675	0.00302	331.2882	0.01802	55.4985	2359.71	42.5185
132	7.1370	0.1401	0.00244	409.1354	0.01744	57.3257	2588.71	45.1579
144	8.5332	0.1172	0.00199	502.2109	0.01699	58.8540	2798.58	47.5512
240	35.6328	0.0281	0.00043	2308.85	0.01543	64.7957	3870.69	59.7368
360	212.7038	0.0047	0.00007	14114	0.01507	66.3532	4310.72	64.9662
480	1269.70	0.0008	0.00001	84580	0.01501	66.6142	4415.74	66.2883

2% **TABLE 7 Discrete Cash Flow: Compound Interest Factors** **2%**

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.0200	0.9804	1.00000	1.0000	1.02000	0.9804		
2	1.0404	0.9612	0.49505	2.0200	0.51505	1.9416	0.9612	0.4950
3	1.0612	0.9423	0.32675	3.0604	0.34675	2.8839	2.8458	0.9868
4	1.0824	0.9238	0.24262	4.1216	0.26262	3.8077	5.6173	1.4752
5	1.1041	0.9057	0.19216	5.2040	0.21216	4.7135	9.2403	1.9604
6	1.1262	0.8880	0.15853	6.3081	0.17853	5.6014	13.6801	2.4423
7	1.1487	0.8706	0.13451	7.4343	0.15451	6.4720	18.9035	2.9208
8	1.1717	0.8535	0.11651	8.5830	0.13651	7.3255	24.8779	3.3961
9	1.1951	0.8368	0.10252	9.7546	0.12252	8.1622	31.5720	3.8681
10	1.2190	0.8203	0.09133	10.9497	0.11133	8.9826	38.9551	4.3367
11	1.2434	0.8043	0.08218	12.1687	0.10218	9.7868	46.9977	4.8021
12	1.2682	0.7885	0.07456	13.4121	0.09456	10.5753	55.6712	5.2642
13	1.2936	0.7730	0.06812	14.6803	0.08812	11.3484	64.9475	5.7231
14	1.3195	0.7579	0.06260	15.9739	0.08260	12.1062	74.7999	6.1786
15	1.3459	0.7430	0.05783	17.2934	0.07783	12.8493	85.2021	6.6309
16	1.3728	0.7284	0.05365	18.6393	0.07365	13.5777	96.1288	7.0799
17	1.4002	0.7142	0.04997	20.0121	0.06997	14.2919	107.5554	7.5256
18	1.4282	0.7002	0.04670	21.4123	0.06670	14.9920	119.4581	7.9681
19	1.4568	0.6864	0.04378	22.8406	0.06378	15.6785	131.8139	8.4073
20	1.4859	0.6730	0.04116	24.2974	0.06116	16.3514	144.6003	8.8433
21	1.5157	0.6598	0.03878	25.7833	0.05878	17.0112	157.7959	9.2760
22	1.5460	0.6468	0.03663	27.2990	0.05663	17.6580	171.3795	9.7055
23	1.5769	0.6342	0.03467	28.8450	0.05467	18.2922	185.3309	10.1317
24	1.6084	0.6217	0.03287	30.4219	0.05287	18.9139	199.6305	10.5547
25	1.6406	0.6095	0.03122	32.0303	0.05122	19.5235	214.2592	10.9745
26	1.6734	0.5976	0.02970	33.6709	0.04970	20.1210	229.1987	11.3910
27	1.7069	0.5859	0.02829	35.3443	0.04829	20.7069	244.4311	11.8043
28	1.7410	0.5744	0.02699	37.0512	0.04699	21.2813	259.9392	12.2145
29	1.7758	0.5631	0.02578	38.7922	0.04578	21.8444	275.7064	12.6214
30	1.8114	0.5521	0.02465	40.5681	0.04465	22.3965	291.7164	13.0251
36	2.0399	0.4902	0.01923	51.9944	0.03923	25.4888	392.0405	15.3809
40	2.2080	0.4529	0.01656	60.4020	0.03656	27.3555	461.9931	16.8885
48	2.5871	0.3865	0.01260	79.3535	0.03260	30.6731	605.9657	19.7556
50	2.6916	0.3715	0.01182	84.5794	0.03182	31.4236	642.3606	20.4420
52	2.8003	0.3571	0.01111	90.0164	0.03111	32.1449	678.7849	21.1164
55	2.9717	0.3365	0.01014	98.5865	0.03014	33.1748	733.3527	22.1057
60	3.2810	0.3048	0.00877	114.0515	0.02877	34.7609	823.6975	23.6961
72	4.1611	0.2403	0.00633	158.0570	0.02633	37.9841	1034.06	27.2234
75	4.4158	0.2265	0.00586	170.7918	0.02586	38.6771	1084.64	28.0434
84	5.2773	0.1895	0.00468	213.8666	0.02468	40.5255	1230.42	30.3616
90	5.9431	0.1683	0.00405	247.1567	0.02405	41.5869	1322.17	31.7929
96	6.6929	0.1494	0.00351	284.6467	0.02351	42.5294	1409.30	33.1370
100	7.2446	0.1380	0.00320	312.2323	0.02320	43.0984	1464.75	33.9863
108	8.4883	0.1178	0.00267	374.4129	0.02267	44.1095	1569.30	35.5774
120	10.7652	0.0929	0.00205	488.2582	0.02205	45.3554	1710.42	37.7114
132	13.6528	0.0732	0.00158	632.6415	0.02158	46.3378	1833.47	39.5676
144	17.3151	0.0578	0.00123	815.7545	0.02123	47.1123	1939.79	41.1738
240	115.8887	0.0086	0.00017	5744.44	0.02017	49.5686	2374.88	47.9110
360	1247.56	0.0008	0.00002	62328	0.02002	49.9599	2482.57	49.7112
480	13430	0.0001			0.02000	49.9963	2498.03	49.9643

3% TABLE 8 Discrete Cash Flow: Compound Interest Factors 3%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.0300	0.9709	1.00000	1.0000	1.03000	0.9709		
2	1.0609	0.9426	0.49261	2.0300	0.52261	1.9135	0.9426	0.4926
3	1.0927	0.9151	0.32353	3.0909	0.35353	2.8286	2.7729	0.9803
4	1.1255	0.8885	0.23903	4.1836	0.26903	3.7171	5.4383	1.4631
5	1.1593	0.8626	0.18835	5.3091	0.21835	4.5797	8.8888	1.9409
6	1.1941	0.8375	0.15460	6.4684	0.18460	5.4172	13.0762	2.4138
7	1.2299	0.8131	0.13051	7.6625	0.16051	6.2303	17.9547	2.8819
8	1.2668	0.7894	0.11246	8.8923	0.14246	7.0197	23.4806	3.3450
9	1.3048	0.7664	0.09843	10.1591	0.12843	7.7861	29.6119	3.8032
10	1.3439	0.7441	0.08723	11.4639	0.11723	8.5302	36.3088	4.2565
11	1.3842	0.7224	0.07808	12.8078	0.10808	9.2526	43.5330	4.7049
12	1.4258	0.7014	0.07046	14.1920	0.10046	9.9540	51.2482	5.1485
13	1.4685	0.6810	0.06403	15.6178	0.09403	10.6350	59.4196	5.5872
14	1.5126	0.6611	0.05853	17.0863	0.08853	11.2961	68.0141	6.0210
15	1.5580	0.6419	0.05377	18.5989	0.08377	11.9379	77.0002	6.4500
16	1.6047	0.6232	0.04961	20.1569	0.07961	12.5611	86.3477	6.8742
17	1.6528	0.6050	0.04595	21.7616	0.07595	13.1661	96.0280	7.2936
18	1.7024	0.5874	0.04271	23.4144	0.07271	13.7535	106.0137	7.7081
19	1.7535	0.5703	0.03981	25.1169	0.06981	14.3238	116.2788	8.1179
20	1.8061	0.5537	0.03722	26.8704	0.06722	14.8775	126.7987	8.5229
21	1.8603	0.5375	0.03487	28.6765	0.06487	15.4150	137.5496	8.9231
22	1.9161	0.5219	0.03275	30.5368	0.06275	15.9369	148.5094	9.3186
23	1.9736	0.5067	0.03081	32.4529	0.06081	16.4436	159.6566	9.7093
24	2.0328	0.4919	0.02905	34.4265	0.05905	16.9355	170.9711	10.0954
25	2.0938	0.4776	0.02743	36.4593	0.05743	17.4131	182.4336	10.4768
26	2.1566	0.4637	0.02594	38.5530	0.05594	17.8768	194.0260	10.8535
27	2.2213	0.4502	0.02456	40.7096	0.05456	18.3270	205.7309	11.2255
28	2.2879	0.4371	0.02329	42.9309	0.05329	18.7641	217.5320	11.5930
29	2.3566	0.4243	0.02211	45.2189	0.05211	19.1885	229.4137	11.9558
30	2.4273	0.4120	0.02102	47.5754	0.05102	19.6004	241.3613	12.3141
31	2.5001	0.4000	0.02000	50.0027	0.05000	20.0004	253.3609	12.6678
32	2.5751	0.3883	0.01905	52.5028	0.04905	20.3888	265.3993	13.0169
33	2.6523	0.3770	0.01816	55.0778	0.04816	20.7658	277.4642	13.3616
34	2.7319	0.3660	0.01732	57.7302	0.04732	21.1318	289.5437	13.7018
35	2.8139	0.3554	0.01654	60.4621	0.04654	21.4872	301.6267	14.0375
40	3.2620	0.3066	0.01326	75.4013	0.04326	23.1148	361.7499	15.6502
45	3.7816	0.2644	0.01079	92.7199	0.04079	24.5187	420.6325	17.1556
50	4.3839	0.2281	0.00887	112.7969	0.03887	25.7298	477.4803	18.5575
55	5.0821	0.1968	0.00735	136.0716	0.03735	26.7744	531.7411	19.8600
60	5.8916	0.1697	0.00613	163.0534	0.03613	27.6756	583.0526	21.0674
65	6.8300	0.1464	0.00515	194.3328	0.03515	28.4529	631.2010	22.1841
70	7.9178	0.1263	0.00434	230.5941	0.03434	29.1234	676.0869	23.2145
75	9.1789	0.1089	0.00367	272.6309	0.03367	29.7018	717.6978	24.1634
80	10.6409	0.0940	0.00311	321.3630	0.03311	30.2008	756.0865	25.0353
84	11.9764	0.0835	0.00273	365.8805	0.03273	30.5501	784.5434	25.6806
85	12.3357	0.0811	0.00265	377.8570	0.03265	30.6312	791.3529	25.8349
90	14.3005	0.0699	0.00226	443.3489	0.03226	31.0024	823.6302	26.5667
96	17.0755	0.0586	0.00187	535.8502	0.03187	31.3812	858.6377	27.3615
108	24.3456	0.0411	0.00129	778.1863	0.03129	31.9642	917.6013	28.7072
120	34.7110	0.0288	0.00089	1123.70	0.03089	32.3730	963.8635	29.7737

4%	TABLE 9 Discrete Cash Flow: Compound Interest Factors							4%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.0400	0.9615	1.00000	1.0000	1.04000	0.9615		
2	1.0816	0.9246	0.49020	2.0400	0.53020	1.8861	0.9246	0.4902
3	1.1249	0.8890	0.32035	3.1216	0.36035	2.7751	2.7025	0.9739
4	1.1699	0.8548	0.23549	4.2465	0.27549	3.6299	5.2670	1.4510
5	1.2167	0.8219	0.18463	5.4163	0.22463	4.4518	8.5547	1.9216
6	1.2653	0.7903	0.15076	6.6330	0.19076	5.2421	12.5062	2.3857
7	1.3159	0.7599	0.12661	7.8983	0.16661	6.0021	17.0657	2.8433
8	1.3686	0.7307	0.10853	9.2142	0.14853	6.7327	22.1806	3.2944
9	1.4233	0.7026	0.09449	10.5828	0.13449	7.4353	27.8013	3.7391
10	1.4802	0.6756	0.08329	12.0061	0.12329	8.1109	33.8814	4.1773
11	1.5395	0.6496	0.07415	13.4864	0.11415	8.7605	40.3772	4.6090
12	1.6010	0.6246	0.06655	15.0258	0.10655	9.3851	47.2477	5.0343
13	1.6651	0.6006	0.06014	16.6268	0.10014	9.9856	54.4546	5.4533
14	1.7317	0.5775	0.05467	18.2919	0.09467	10.5631	61.9618	5.8659
15	1.8009	0.5553	0.04994	20.0236	0.08994	11.1184	69.7355	6.2721
16	1.8730	0.5339	0.04582	21.8245	0.08582	11.6523	77.7441	6.6720
17	1.9479	0.5134	0.04220	23.6975	0.08220	12.1657	85.9581	7.0656
18	2.0258	0.4936	0.03899	25.6454	0.07899	12.6593	94.3498	7.4530
19	2.1068	0.4746	0.03614	27.6712	0.07614	13.1339	102.8933	7.8342
20	2.1911	0.4564	0.03358	29.7781	0.07358	13.5903	111.5647	8.2091
21	2.2788	0.4388	0.03128	31.9692	0.07128	14.0292	120.3414	8.5779
22	2.3699	0.4220	0.02920	34.2480	0.06920	14.4511	129.2024	8.9407
23	2.4647	0.4057	0.02731	36.6179	0.06731	14.8568	138.1284	9.2973
24	2.5633	0.3901	0.02559	39.0826	0.06559	15.2470	147.1012	9.6479
25	2.6658	0.3751	0.02401	41.6459	0.06401	15.6221	156.1040	9.9925
26	2.7725	0.3607	0.02257	44.3117	0.06257	15.9828	165.1212	10.3312
27	2.8834	0.3468	0.02124	47.0842	0.06124	16.3296	174.1385	10.6640
28	2.9987	0.3335	0.02001	49.9676	0.06001	16.6631	183.1424	10.9909
29	3.1187	0.3207	0.01888	52.9663	0.05888	16.9837	192.1206	11.3120
30	3.2434	0.3083	0.01783	56.0849	0.05783	17.2920	201.0618	11.6274
31	3.3731	0.2965	0.01686	59.3283	0.05686	17.5885	209.9556	11.9371
32	3.5081	0.2851	0.01596	62.7015	0.05595	17.8736	218.7924	12.2411
33	3.6484	0.2741	0.01510	66.2095	0.05510	18.1476	227.5634	12.5396
34	3.7943	0.2636	0.01431	69.8579	0.05431	18.4112	236.2607	12.8324
35	3.9461	0.2534	0.01358	73.6522	0.05358	18.6646	244.8768	13.1198
40	4.8010	0.2083	0.01052	95.0255	0.05052	19.7928	286.5303	14.4765
45	5.8412	0.1712	0.00826	121.0294	0.04826	20.7200	325.4028	15.7047
50	7.1067	0.1407	0.00655	152.6671	0.04655	21.4822	361.1638	16.8122
55	8.6464	0.1157	0.00523	191.1592	0.04523	22.1086	393.6890	17.8070
60	10.5196	0.0951	0.00420	237.9907	0.04420	22.6235	422.9966	18.6972
65	12.7987	0.0781	0.00339	294.9684	0.04339	23.0467	449.2014	19.4909
70	15.5716	0.0642	0.00275	364.2906	0.04275	23.3945	472.4789	20.1961
75	18.9453	0.0528	0.00223	448.6314	0.04223	23.6804	493.0408	20.8206
80	23.0498	0.0434	0.00181	551.2450	0.04181	23.9154	511.1161	21.3718
85	28.0436	0.0357	0.00148	676.0901	0.04148	24.1085	526.9384	21.8569
90	34.1193	0.0293	0.00121	827.9833	0.04121	24.2673	540.7369	22.2826
96	43.1718	0.0232	0.00095	1054.30	0.04095	24.4209	554.9312	22.7236
108	69.1195	0.0145	0.00059	1702.99	0.04059	24.6383	576.8949	23.4146
120	110.6626	0.0090	0.00036	2741.56	0.04036	24.7741	592.2428	23.9057
144	283.6618	0.0035	0.00014	7066.55	0.04014	24.9119	610.1055	24.4906

5% TABLE 10 Discrete Cash Flow: Compound Interest Factors 5%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.0500	0.9524	1.00000	1.0000	1.05000	0.9524		
2	1.1025	0.9070	0.48780	2.0500	0.53780	1.8594	0.9070	0.4878
3	1.1576	0.8638	0.31721	3.1525	0.36721	2.7232	2.6347	0.9675
4	1.2155	0.8227	0.23201	4.3101	0.28201	3.5460	5.1028	1.4391
5	1.2763	0.7835	0.18097	5.5256	0.23097	4.3295	8.2369	1.9025
6	1.3401	0.7462	0.14702	6.8019	0.19702	5.0757	11.9680	2.3579
7	1.4071	0.7107	0.12282	8.1420	0.17282	5.7864	16.2321	2.8052
8	1.4775	0.6768	0.10472	9.5491	0.15472	6.4632	20.9700	3.2445
9	1.5513	0.6446	0.09069	11.0266	0.14069	7.1078	26.1268	3.6758
10	1.6289	0.6139	0.07950	12.5779	0.12950	7.7217	31.6520	4.0991
11	1.7103	0.5847	0.07039	14.2068	0.12039	8.3064	37.4988	4.5144
12	1.7959	0.5568	0.06283	15.9171	0.11283	8.8633	43.6241	4.9219
13	1.8856	0.5303	0.05646	17.7130	0.10646	9.3936	49.9879	5.3215
14	1.9799	0.5051	0.05102	19.5986	0.10102	9.8986	56.5538	5.7133
15	2.0789	0.4810	0.04634	21.5786	0.09634	10.3797	63.2880	6.0973
16	2.1829	0.4581	0.04227	23.6575	0.09227	10.8378	70.1597	6.4736
17	2.2920	0.4363	0.03870	25.8404	0.08870	11.2741	77.1405	6.8423
18	2.4066	0.4155	0.03555	28.1324	0.08555	11.6896	84.2043	7.2034
19	2.5270	0.3957	0.03275	30.5390	0.08275	12.0853	91.3275	7.5569
20	2.6533	0.3769	0.03024	33.0660	0.08024	12.4622	98.4884	7.9030
21	2.7860	0.3589	0.02800	35.7193	0.07800	12.8212	105.6673	8.2416
22	2.9253	0.3418	0.02597	38.5052	0.07597	13.1630	112.8461	8.5730
23	3.0715	0.3256	0.02414	41.4305	0.07414	13.4886	120.0087	8.8971
24	3.2251	0.3101	0.02247	44.5020	0.07247	13.7986	127.1402	9.2140
25	3.3864	0.2953	0.02095	47.7271	0.07095	14.0939	134.2275	9.5238
26	3.5557	0.2812	0.01956	51.1135	0.06956	14.3752	141.2585	9.8266
27	3.7335	0.2678	0.01829	54.6691	0.06829	14.6430	148.2226	10.1224
28	3.9201	0.2551	0.01712	58.4026	0.06712	14.8981	155.1101	10.4114
29	4.1161	0.2429	0.01605	62.3227	0.06605	15.1411	161.9126	10.6936
30	4.3219	0.2314	0.01505	66.4388	0.06505	15.3725	168.6226	10.9691
31	4.5380	0.2204	0.01413	70.7608	0.06413	15.5928	175.2333	11.2381
32	4.7649	0.2099	0.01328	75.2988	0.06328	15.8027	181.7392	11.5005
33	5.0032	0.1999	0.01249	80.0638	0.06249	16.0025	188.1351	11.7566
34	5.2533	0.1904	0.01176	85.0670	0.06176	16.1929	194.4168	12.0063
35	5.5160	0.1813	0.01107	90.3203	0.06107	16.3742	200.5807	12.2498
40	7.0400	0.1420	0.00828	120.7998	0.05828	17.1591	229.5452	13.3775
45	8.9850	0.1113	0.00626	159.7002	0.05626	17.7741	255.3145	14.3644
50	11.4674	0.0872	0.00478	209.3480	0.05478	18.2559	277.9148	15.2233
55	14.6356	0.0683	0.00367	272.7126	0.05367	18.6335	297.5104	15.9664
60	18.6792	0.0535	0.00283	353.5837	0.05283	18.9293	314.3432	16.6062
65	23.8399	0.0419	0.00219	456.7980	0.05219	19.1611	328.6910	17.1541
70	30.4264	0.0329	0.00170	588.5285	0.05170	19.3427	340.8409	17.6212
75	38.8327	0.0258	0.00132	756.6537	0.05132	19.4850	351.0721	18.0176
80	49.5614	0.0202	0.00103	971.2288	0.05103	19.5965	359.6460	18.3526
85	63.2544	0.0158	0.00080	1245.09	0.05080	19.6838	366.8007	18.6346
90	80.7304	0.0124	0.00063	1594.61	0.05063	19.7523	372.7488	18.8712
95	103.0347	0.0097	0.00049	2040.69	0.05049	19.8059	377.6774	19.0689
96	108.1864	0.0092	0.00047	2143.73	0.05047	19.8151	378.5555	19.1044
98	119.2755	0.0084	0.00042	2365.51	0.05042	19.8323	380.2139	19.1714
100	131.5013	0.0076	0.00038	2610.03	0.05038	19.8479	381.7492	19.2337

6% TABLE 11 Discrete Cash Flow: Compound Interest Factors 6%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.0600	0.9434	1.00000	1.0000	1.06000	0.9434		
2	1.1236	0.8900	0.48544	2.0600	0.54544	1.8334	0.8900	0.4854
3	1.1910	0.8396	0.31411	3.1836	0.37411	2.6730	2.5692	0.9612
4	1.2625	0.7921	0.22859	4.3746	0.28859	3.4651	4.9455	1.4272
5	1.3382	0.7473	0.17740	5.6371	0.23740	4.2124	7.9345	1.8836
6	1.4185	0.7050	0.14336	6.9753	0.20336	4.9173	11.4594	2.3304
7	1.5036	0.6651	0.11914	8.3938	0.17914	5.5824	15.4497	2.7676
8	1.5938	0.6274	0.10104	9.8975	0.16104	6.2098	19.8416	3.1952
9	1.6895	0.5919	0.08702	11.4913	0.14702	6.8017	24.5768	3.6133
10	1.7908	0.5584	0.07587	13.1808	0.13587	7.3601	29.6023	4.0220
11	1.8983	0.5268	0.06679	14.9716	0.12679	7.8869	34.8702	4.4213
12	2.0122	0.4970	0.05928	16.8699	0.11928	8.3838	40.3369	4.8113
13	2.1329	0.4688	0.05296	18.8821	0.11296	8.8527	45.9629	5.1920
14	2.2609	0.4423	0.04758	21.0151	0.10758	9.2950	51.7128	5.5635
15	2.3966	0.4173	0.04296	23.2760	0.10296	9.7122	57.5546	5.9260
16	2.5404	0.3936	0.03895	25.6725	0.09895	10.1059	63.4592	6.2794
17	2.6928	0.3714	0.03544	28.2129	0.09544	10.4773	69.4011	6.6240
18	2.8543	0.3503	0.03236	30.9057	0.09236	10.8276	75.3569	6.9597
19	3.0256	0.3305	0.02962	33.7600	0.08962	11.1581	81.3062	7.2867
20	3.2071	0.3118	0.02718	36.7856	0.08718	11.4699	87.2304	7.6051
21	3.3996	0.2942	0.02500	39.9927	0.08500	11.7641	93.1136	7.9151
22	3.6035	0.2775	0.02305	43.3923	0.08305	12.0416	98.9412	8.2166
23	3.8197	0.2618	0.02128	46.9958	0.08128	12.3034	104.7007	8.5099
24	4.0489	0.2470	0.01968	50.8156	0.07968	12.5504	110.3812	8.7951
25	4.2919	0.2330	0.01823	54.8645	0.07823	12.7834	115.9732	9.0722
26	4.5494	0.2198	0.01690	59.1564	0.07690	13.0032	121.4684	9.3414
27	4.8223	0.2074	0.01570	63.7058	0.07570	13.2105	126.8600	9.6029
28	5.1117	0.1956	0.01459	68.5281	0.07459	13.4062	132.1420	9.8568
29	5.4184	0.1846	0.01358	73.6398	0.07358	13.5907	137.3096	10.1032
30	5.7435	0.1741	0.01265	79.0582	0.07265	13.7648	142.3588	10.3422
31	6.0881	0.1643	0.01179	84.8017	0.07179	13.9291	147.2864	10.5740
32	6.4534	0.1550	0.01100	90.8898	0.07100	14.0840	152.0901	10.7988
33	6.8406	0.1462	0.01027	97.3432	0.07027	14.2302	156.7681	11.0166
34	7.2510	0.1379	0.00960	104.1838	0.06960	14.3681	161.3192	11.2276
35	7.6861	0.1301	0.00897	111.4348	0.06897	14.4982	165.7427	11.4319
40	10.2857	0.0972	0.00646	154.7620	0.06646	15.0463	185.9568	12.3590
45	13.7646	0.0727	0.00470	212.7435	0.06470	15.4558	203.1096	13.1413
50	18.4202	0.0543	0.00344	290.3359	0.06344	15.7619	217.4574	13.7964
55	24.6503	0.0406	0.00254	394.1720	0.06254	15.9905	229.3222	14.3411
60	32.9877	0.0303	0.00188	533.1282	0.06188	16.1614	239.0428	14.7909
65	44.1450	0.0227	0.00139	719.0829	0.06139	16.2891	246.9450	15.1601
70	59.0759	0.0169	0.00103	967.9322	0.06103	16.3845	253.3271	15.4613
75	79.0569	0.0126	0.00077	1300.95	0.06077	16.4558	258.4527	15.7058
80	105.7960	0.0095	0.00057	1746.60	0.06057	16.5091	262.5493	15.9033
85	141.5789	0.0071	0.00043	2342.98	0.06043	16.5489	265.8096	16.0620
90	189.4645	0.0053	0.00032	3141.08	0.06032	16.5787	268.3946	16.1891
95	253.5463	0.0039	0.00024	4209.10	0.06024	16.6009	270.4375	16.2905
96	268.7590	0.0037	0.00022	4462.65	0.06022	16.6047	270.7909	16.3081
98	301.9776	0.0033	0.00020	5016.29	0.06020	16.6115	271.4491	16.3411
100	339.3021	0.0029	0.00018	5638.37	0.06018	16.6175	272.0471	16.3711

7% TABLE 12 Discrete Cash Flow: Compound Interest Factors 7%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.0700	0.9346	1.00000	1.0000	1.07000	0.9346		
2	1.1449	0.8734	0.48309	2.0700	0.55309	1.8080	0.8734	0.4831
3	1.2250	0.8163	0.31105	3.2149	0.38105	2.6243	2.5060	0.9549
4	1.3108	0.7629	0.22523	4.4399	0.29523	3.3872	4.7947	1.4155
5	1.4026	0.7130	0.17389	5.7507	0.24389	4.1002	7.6467	1.8650
6	1.5007	0.6663	0.13980	7.1533	0.20980	4.7665	10.9784	2.3032
7	1.6058	0.6227	0.11555	8.6540	0.18555	5.3893	14.7149	2.7304
8	1.7182	0.5820	0.09747	10.2598	0.16747	5.9713	18.7889	3.1465
9	1.8385	0.5439	0.08349	11.9780	0.15349	6.5152	23.1404	3.5517
10	1.9672	0.5083	0.07238	13.8164	0.14238	7.0236	27.7156	3.9461
11	2.1049	0.4751	0.06336	15.7836	0.13336	7.4987	32.4665	4.3296
12	2.2522	0.4440	0.05590	17.8885	0.12590	7.9427	37.3506	4.7025
13	2.4098	0.4150	0.04965	20.1406	0.11965	8.3577	42.3302	5.0648
14	2.5785	0.3878	0.04434	22.5505	0.11434	8.7455	47.3718	5.4167
15	2.7590	0.3624	0.03979	25.1290	0.10979	9.1079	52.4461	5.7583
16	2.9522	0.3387	0.03586	27.8881	0.10586	9.4466	57.5271	6.0897
17	3.1588	0.3166	0.03243	30.8402	0.10243	9.7632	62.5923	6.4110
18	3.3799	0.2959	0.02941	33.9990	0.09941	10.0591	67.6219	6.7225
19	3.6165	0.2765	0.02675	37.3790	0.09675	10.3356	72.5991	7.0242
20	3.8697	0.2584	0.02439	40.9955	0.09439	10.5940	77.5091	7.3163
21	4.1406	0.2415	0.02229	44.8652	0.09229	10.8355	82.3393	7.5990
22	4.4304	0.2257	0.02041	49.0057	0.09041	11.0612	87.0793	7.8725
23	4.7405	0.2109	0.01871	53.4361	0.08871	11.2722	91.7201	8.1369
24	5.0724	0.1971	0.01719	58.1767	0.08719	11.4693	96.2545	8.3923
25	5.4274	0.1842	0.01581	63.2490	0.08581	11.6536	100.6765	8.6391
26	5.8074	0.1722	0.01456	68.6765	0.08456	11.8258	104.9814	8.8773
27	6.2139	0.1609	0.01343	74.4838	0.08343	11.9867	109.1656	9.1072
28	6.6488	0.1504	0.01239	80.6977	0.08239	12.1371	113.2264	9.3289
29	7.1143	0.1406	0.01145	87.3465	0.08145	12.2777	117.1622	9.5427
30	7.6123	0.1314	0.01059	94.4608	0.08059	12.4090	120.9718	9.7487
31	8.1451	0.1228	0.00980	102.0730	0.07980	12.5318	124.6550	9.9471
32	8.7153	0.1147	0.00907	110.2182	0.07907	12.6466	128.2120	10.1381
33	9.3253	0.1072	0.00841	118.9334	0.07841	12.7538	131.6435	10.3219
34	9.9781	0.1002	0.00780	128.2588	0.07780	12.8540	134.9507	10.4987
35	10.6766	0.0937	0.00723	138.2369	0.07723	12.9477	138.1353	10.6687
40	14.9745	0.0668	0.00501	199.6351	0.07501	13.3317	152.2928	11.4233
45	21.0025	0.0476	0.00350	285.7493	0.07350	13.6055	163.7559	12.0360
50	29.4570	0.0339	0.00246	406.5289	0.07246	13.8007	172.9051	12.5287
55	41.3150	0.0242	0.00174	575.9286	0.07174	13.9399	180.1243	12.9215
60	57.9464	0.0173	0.00123	813.5204	0.07123	14.0392	185.7677	13.2321
65	81.2729	0.0123	0.00087	1146.76	0.07087	14.1099	190.1452	13.4760
70	113.9894	0.0088	0.00062	1614.13	0.07062	14.1604	193.5185	13.6662
75	159.8760	0.0063	0.00044	2269.66	0.07044	14.1964	196.1035	13.8136
80	224.2344	0.0045	0.00031	3189.06	0.07031	14.2220	198.0748	13.9273
85	314.5003	0.0032	0.00022	4478.58	0.07022	14.2403	199.5717	14.0146
90	441.1030	0.0023	0.00016	6287.19	0.07016	14.2533	200.7042	14.0812
95	618.6697	0.0016	0.00011	8823.85	0.07011	14.2626	201.5581	14.1319
96	661.9766	0.0015	0.00011	9442.52	0.07011	14.2641	201.7016	14.1405
98	757.8970	0.0013	0.00009	10813	0.07009	14.2669	201.9651	14.1562
100	867.7163	0.0012	0.00008	12382	0.07008	14.2693	202.2001	14.1703

8% TABLE 13 Discrete Cash Flow: Compound Interest Factors 8%

n	Single Payments		Uniform Series Payments				Arithmetic Gradients	
	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.0800	0.9259	1.00000	1.0000	1.08000	0.9259		
2	1.1664	0.8573	0.48077	2.0800	0.56077	1.7833	0.8573	0.4808
3	1.2597	0.7938	0.30803	3.2464	0.38803	2.5771	2.4450	0.9487
4	1.3605	0.7350	0.22192	4.5061	0.30192	3.3121	4.6501	1.4040
5	1.4693	0.6806	0.17046	5.8666	0.25046	3.9927	7.3724	1.8465
6	1.5869	0.6302	0.13632	7.3359	0.21632	4.6229	10.5233	2.2763
7	1.7138	0.5835	0.11207	8.9228	0.19207	5.2064	14.0242	2.6937
8	1.8509	0.5403	0.09401	10.6366	0.17401	5.7466	17.8061	3.0985
9	1.9990	0.5002	0.08008	12.4876	0.16008	6.2469	21.8081	3.4910
10	2.1589	0.4632	0.06903	14.4866	0.14903	6.7101	25.9768	3.8713
11	2.3316	0.4289	0.06008	16.6455	0.14008	7.1390	30.2657	4.2395
12	2.5182	0.3971	0.05270	18.9771	0.13270	7.5361	34.6339	4.5957
13	2.7196	0.3677	0.04652	21.4953	0.12652	7.9038	39.0463	4.9402
14	2.9372	0.3405	0.04130	24.2149	0.12130	8.2442	43.4723	5.2731
15	3.1722	0.3152	0.03683	27.1521	0.11683	8.5595	47.8857	5.5945
16	3.4259	0.2919	0.03298	30.3243	0.11298	8.8514	52.2640	5.9046
17	3.7000	0.2703	0.02963	33.7502	0.10963	9.1216	56.5883	6.2037
18	3.9960	0.2502	0.02670	37.4502	0.10670	9.3719	60.8426	6.4920
19	4.3157	0.2317	0.02413	41.4463	0.10413	9.6036	65.0134	6.7697
20	4.6610	0.2145	0.02185	45.7620	0.10185	9.8181	69.0898	7.0369
21	5.0338	0.1987	0.01983	50.4229	0.09983	10.0168	73.0629	7.2940
22	5.4365	0.1839	0.01803	55.4568	0.09803	10.2007	76.9257	7.5412
23	5.8715	0.1703	0.01642	60.8933	0.09642	10.3711	80.6726	7.7786
24	6.3412	0.1577	0.01498	66.7648	0.09498	10.5288	84.2997	8.0066
25	6.8485	0.1460	0.01368	73.1059	0.09368	10.6748	87.8041	8.2254
26	7.3964	0.1352	0.01251	79.9544	0.09251	10.8100	91.1842	8.4352
27	7.9881	0.1252	0.01145	87.3508	0.09145	10.9352	94.4390	8.6363
28	8.6271	0.1159	0.01049	95.3388	0.09049	11.0511	97.5687	8.8289
29	9.3173	0.1073	0.00962	103.9659	0.08962	11.1584	100.5738	9.0133
30	10.0627	0.0994	0.00883	113.2832	0.08883	11.2578	103.4558	9.1897
31	10.8677	0.0920	0.00811	123.3459	0.08811	11.3498	106.2163	9.3584
32	11.7371	0.0852	0.00745	134.2135	0.08745	11.4350	108.8575	9.5197
33	12.6760	0.0789	0.00685	145.9506	0.08685	11.5139	111.3819	9.6737
34	13.6901	0.0730	0.00630	158.6267	0.08630	11.5869	113.7924	9.8208
35	14.7853	0.0676	0.00580	172.3168	0.08580	11.6546	116.0920	9.9611
40	21.7245	0.0460	0.00386	259.0565	0.08386	11.9246	126.0422	10.5699
45	31.9204	0.0313	0.00259	386.5056	0.08259	12.1084	133.7331	11.0447
50	46.9016	0.0213	0.00174	573.7702	0.08174	12.2335	139.5928	11.4107
55	68.9139	0.0145	0.00118	848.9232	0.08118	12.3186	144.0065	11.6902
60	101.2571	0.0099	0.00080	1253.21	0.08080	12.3766	147.3000	11.9015
65	148.7798	0.0067	0.00054	1847.25	0.08054	12.4160	149.7387	12.0602
70	218.6064	0.0046	0.00037	2720.08	0.08037	12.4428	151.5326	12.1783
75	321.2045	0.0031	0.00025	4002.56	0.08025	12.4611	152.8448	12.2658
80	471.9548	0.0021	0.00017	5886.94	0.08017	12.4735	153.8001	12.3301
85	693.4565	0.0014	0.00012	8655.71	0.08012	12.4820	154.4925	12.3772
90	1018.92	0.0010	0.00008	12724	0.08008	12.4877	154.9925	12.4116
95	1497.12	0.0007	0.00005	18702	0.08005	12.4917	155.3524	12.4365
96	1616.89	0.0006	0.00005	20199	0.08005	12.4923	155.4112	12.4406
98	1885.94	0.0005	0.00004	23562	0.08004	12.4934	155.5176	12.4480
100	2199.76	0.0005	0.00004	27485	0.08004	12.4943	155.6107	12.4545

9%	TABLE 14 Discrete Cash Flow: Compound Interest Factors							9%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.0900	0.9174	1.00000	1.0000	1.09000	0.9174		
2	1.1881	0.8417	0.47847	2.0900	0.56847	1.7591	0.8417	0.4785
3	1.2950	0.7722	0.30505	3.2781	0.39505	2.5313	2.3860	0.9426
4	1.4116	0.7084	0.21867	4.5731	0.30867	3.2397	4.5113	1.3925
5	1.5386	0.6499	0.16709	5.9847	0.25709	3.8897	7.1110	1.8282
6	1.6771	0.5963	0.13292	7.5233	0.22292	4.4859	10.0924	2.2498
7	1.8280	0.5470	0.10869	9.2004	0.19869	5.0330	13.3746	2.6574
8	1.9926	0.5019	0.09067	11.0285	0.18067	5.5348	16.8877	3.0512
9	2.1719	0.4604	0.07680	13.0210	0.16680	5.9952	20.5711	3.4312
10	2.3674	0.4224	0.06582	15.1929	0.15582	6.4177	24.3728	3.7978
11	2.5804	0.3875	0.05695	17.5603	0.14695	6.8052	28.2481	4.1510
12	2.8127	0.3555	0.04965	20.1407	0.13965	7.1607	32.1590	4.4910
13	3.0658	0.3262	0.04357	22.9534	0.13357	7.4869	36.0731	4.8182
14	3.3417	0.2992	0.03843	26.0192	0.12843	7.7862	39.9633	5.1326
15	3.6425	0.2745	0.03406	29.3609	0.12406	8.0607	43.8069	5.4346
16	3.9703	0.2519	0.03030	33.0034	0.12030	8.3126	47.5849	5.7245
17	4.3276	0.2311	0.02705	36.9737	0.11705	8.5436	51.2821	6.0024
18	4.7171	0.2120	0.02421	41.3013	0.11421	8.7556	54.8860	6.2687
19	5.1417	0.1945	0.02173	46.0185	0.11173	8.9501	58.3868	6.5236
20	5.6044	0.1784	0.01955	51.1601	0.10955	9.1285	61.7770	6.7674
21	6.1088	0.1637	0.01762	56.7645	0.10762	9.2922	65.0509	7.0006
22	6.6586	0.1502	0.01590	62.8733	0.10590	9.4424	68.2048	7.2232
23	7.2579	0.1378	0.01438	69.5319	0.10438	9.5802	71.2359	7.4357
24	7.9111	0.1264	0.01302	76.7898	0.10302	9.7066	74.1433	7.6384
25	8.6231	0.1160	0.01181	84.7009	0.10181	9.8226	76.9265	7.8316
26	9.3992	0.1064	0.01072	93.3240	0.10072	9.9290	79.5863	8.0156
27	10.2451	0.0976	0.00973	102.7231	0.09973	10.0266	82.1241	8.1906
28	11.1671	0.0895	0.00885	112.9682	0.09885	10.1161	84.5419	8.3571
29	12.1722	0.0822	0.00806	124.1354	0.09806	10.1983	86.8422	8.5154
30	13.2677	0.0754	0.00734	136.3075	0.09734	10.2737	89.0280	8.6657
31	14.4618	0.0691	0.00669	149.5752	0.09669	10.3428	91.1024	8.8083
32	15.7633	0.0634	0.00610	164.0370	0.09610	10.4062	93.0690	8.9436
33	17.1820	0.0582	0.00556	179.8003	0.09556	10.4644	94.9314	9.0718
34	18.7284	0.0534	0.00508	196.9823	0.09508	10.5178	96.6935	9.1933
35	20.4140	0.0490	0.00464	215.7108	0.09464	10.5668	98.3590	9.3083
40	31.4094	0.0318	0.00296	337.8824	0.09296	10.7574	105.3762	9.7957
45	48.3273	0.0207	0.00190	525.8587	0.09190	10.8812	110.5561	10.1603
50	74.3575	0.0134	0.00123	815.0836	0.09123	10.9617	114.3251	10.4295
55	114.4083	0.0087	0.00079	1260.09	0.09079	11.0140	117.0362	10.6261
60	176.0313	0.0057	0.00051	1944.79	0.09051	11.0480	118.9683	10.7683
65	270.8460	0.0037	0.00033	2998.29	0.09033	11.0701	120.3344	10.8702
70	416.7301	0.0024	0.00022	4619.22	0.09022	11.0844	121.2942	10.9427
75	641.1909	0.0016	0.00014	7113.23	0.09014	11.0938	121.9646	10.9940
80	986.5517	0.0010	0.00009	10951	0.09009	11.0998	122.4306	11.0299
85	1517.93	0.0007	0.00006	16855	0.09006	11.1038	122.7533	11.0551
90	2335.53	0.0004	0.00004	25939	0.09004	11.1064	122.9758	11.0726
95	3593.50	0.0003	0.00003	39917	0.09003	11.1080	123.1287	11.0847
96	3916.91	0.0003	0.00002	43510	0.09002	11.1083	123.1529	11.0866
98	4653.68	0.0002	0.00002	51696	0.09002	11.1087	123.1963	11.0900
100	5529.04	0.0002	0.00002	61423	0.09002	11.1091	123.2335	11.0930

10%	TABLE 15 Discrete Cash Flow: Compound Interest Factors							10%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.1000	0.9091	1.00000	1.0000	1.10000	0.9091		
2	1.2100	0.8264	0.47619	2.1000	0.57619	1.7355	0.8264	0.4762
3	1.3310	0.7513	0.30211	3.3100	0.40211	2.4869	2.3291	0.9366
4	1.4641	0.6830	0.21547	4.6410	0.31547	3.1699	4.3781	1.3812
5	1.6105	0.6209	0.16380	6.1051	0.26380	3.7908	6.8618	1.8101
6	1.7716	0.5645	0.12961	7.7156	0.22961	4.3553	9.6842	2.2236
7	1.9487	0.5132	0.10541	9.4872	0.20541	4.8684	12.7631	2.6216
8	2.1436	0.4665	0.08744	11.4359	0.18744	5.3349	16.0287	3.0045
9	2.3579	0.4241	0.07364	13.5795	0.17364	5.7590	19.4215	3.3724
10	2.5937	0.3855	0.06275	15.9374	0.16275	6.1446	22.8913	3.7255
11	2.8531	0.3505	0.05396	18.5312	0.15396	6.4951	26.3963	4.0641
12	3.1384	0.3186	0.04676	21.3843	0.14676	6.8137	29.9012	4.3884
13	3.4523	0.2897	0.04078	24.5227	0.14078	7.1034	33.3772	4.6988
14	3.7975	0.2633	0.03575	27.9750	0.13575	7.3667	36.8005	4.9955
15	4.1772	0.2394	0.03147	31.7725	0.13147	7.6061	40.1520	5.2789
16	4.5950	0.2176	0.02782	35.9497	0.12782	7.8237	43.4164	5.5493
17	5.0545	0.1978	0.02466	40.5447	0.12466	8.0216	46.5819	5.8071
18	5.5599	0.1799	0.02193	45.5992	0.12193	8.2014	49.6395	6.0526
19	6.1159	0.1635	0.01955	51.1591	0.11955	8.3649	52.5827	6.2861
20	6.7275	0.1486	0.01746	57.2750	0.11746	8.5136	55.4069	6.5081
21	7.4002	0.1351	0.01562	64.0025	0.11562	8.6487	58.1095	6.7189
22	8.1403	0.1228	0.01401	71.4027	0.11401	8.7715	60.6893	6.9189
23	8.9543	0.1117	0.01257	79.5430	0.11257	8.8832	63.1462	7.1085
24	9.8497	0.1015	0.01130	88.4973	0.11130	8.9847	65.4813	7.2881
25	10.8347	0.0923	0.01017	98.3471	0.11017	9.0770	67.6964	7.4580
26	11.9182	0.0839	0.00916	109.1818	0.10916	9.1609	69.7940	7.6186
27	13.1100	0.0763	0.00826	121.0999	0.10826	9.2372	71.7773	7.7704
28	14.4210	0.0693	0.00745	134.2099	0.10745	9.3066	73.6495	7.9137
29	15.8631	0.0630	0.00673	148.6309	0.10673	9.3696	75.4146	8.0489
30	17.4494	0.0573	0.00608	164.4940	0.10608	9.4269	77.0766	8.1762
31	19.1943	0.0521	0.00550	181.9434	0.10550	9.4790	78.6395	8.2962
32	21.1138	0.0474	0.00497	201.1378	0.10497	9.5264	80.1078	8.4091
33	23.2252	0.0431	0.00450	222.2515	0.10450	9.5694	81.4856	8.5152
34	25.5477	0.0391	0.00407	245.4767	0.10407	9.6086	82.7773	8.6149
35	28.1024	0.0356	0.00369	271.0244	0.10369	9.6442	83.9872	8.7086
40	45.2593	0.0221	0.00226	442.5926	0.10226	9.7791	88.9525	9.0962
45	72.8905	0.0137	0.00139	718.9048	0.10139	9.8628	92.4544	9.3740
50	117.3909	0.0085	0.00086	1163.91	0.10086	9.9148	94.8889	9.5704
55	189.0591	0.0053	0.00053	1880.59	0.10053	9.9471	96.5619	9.7075
60	304.4816	0.0033	0.00033	3034.82	0.10033	9.9672	97.7010	9.8023
65	490.3707	0.0020	0.00020	4893.71	0.10020	9.9796	98.4705	9.8672
70	789.7470	0.0013	0.00013	7887.47	0.10013	9.9873	98.9870	9.9113
75	1271.90	0.0008	0.00008	12709	0.10008	9.9921	99.3317	9.9410
80	2048.40	0.0005	0.00005	20474	0.10005	9.9951	99.5606	9.9609
85	3298.97	0.0003	0.00003	32980	0.10003	9.9970	99.7120	9.9742
90	5313.02	0.0002	0.00002	53120	0.10002	9.9981	99.8118	9.9831
95	8556.68	0.0001	0.00001	85557	0.10001	9.9988	99.8773	9.9889
96	9412.34	0.0001	0.00001	94113	0.10001	9.9989	99.8874	9.9898
98	11389	0.0001	0.00001		0.10001	9.9991	99.9052	9.9914
100	13781	0.0001	0.00001		0.10001	9.9993	99.9202	9.9927

11% TABLE 16 Discrete Cash Flow: Compound Interest Factors 11%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.1100	0.9009	1.00000	1.0000	1.11000	0.9009		
2	1.2321	0.8116	0.47393	2.1100	0.58393	1.7125	0.8116	0.4739
3	1.3676	0.7312	0.29921	3.3421	0.40921	2.4437	2.2740	0.9306
4	1.5181	0.6587	0.21233	4.7097	0.32233	3.1024	4.2502	1.3700
5	1.6851	0.5935	0.16057	6.2278	0.27057	3.6959	6.6240	1.7923
6	1.8704	0.5346	0.12638	7.9129	0.23638	4.2305	9.2972	2.1976
7	2.0762	0.4817	0.10222	9.7833	0.21222	4.7122	12.1872	2.5863
8	2.3045	0.4339	0.08432	11.8594	0.19432	5.1461	15.2246	2.9585
9	2.5580	0.3909	0.07060	14.1640	0.18060	5.5370	18.3520	3.3144
10	2.8394	0.3522	0.05980	16.7220	0.16980	5.8892	21.5217	3.6544
11	3.1518	0.3173	0.05112	19.5614	0.16112	6.2065	24.6945	3.9788
12	3.4985	0.2858	0.04403	22.7132	0.15403	6.4924	27.8388	4.2879
13	3.8833	0.2575	0.03815	26.2116	0.14815	6.7499	30.9290	4.5822
14	4.3104	0.2320	0.03323	30.0949	0.14323	6.9819	33.9449	4.8619
15	4.7846	0.2090	0.02907	34.4054	0.13907	7.1909	36.8709	5.1275
16	5.3109	0.1883	0.02552	39.1899	0.13552	7.3792	39.6953	5.3794
17	5.8951	0.1696	0.02247	44.5008	0.13247	7.5488	42.4095	5.6180
18	6.5436	0.1528	0.01984	50.3959	0.12984	7.7016	45.0074	5.8439
19	7.2633	0.1377	0.01756	56.9395	0.12756	7.8393	47.4856	6.0574
20	8.0623	0.1240	0.01558	64.2028	0.12558	7.9633	49.8423	6.2590
21	8.9492	0.1117	0.01384	72.2651	0.12384	8.0751	52.0771	6.4491
22	9.9336	0.1007	0.01231	81.2143	0.12231	8.1757	54.1912	6.6283
23	11.0263	0.0907	0.01097	91.1479	0.12097	8.2664	56.1864	6.7969
24	12.2392	0.0817	0.00979	102.1742	0.11979	8.3481	58.0656	6.9555
25	13.5855	0.0736	0.00874	114.4133	0.11874	8.4217	59.8322	7.1045
26	15.0799	0.0663	0.00781	127.9988	0.11781	8.4881	61.4900	7.2443
27	16.7386	0.0597	0.00699	143.0786	0.11699	8.5478	63.0433	7.3754
28	18.5799	0.0538	0.00626	159.8173	0.11626	8.6016	64.4965	7.4982
29	20.6237	0.0485	0.00561	178.3972	0.11561	8.6501	65.8542	7.6131
30	22.8923	0.0437	0.00502	199.0209	0.11502	8.6938	67.1210	7.7206
31	25.4104	0.0394	0.00451	221.9132	0.11451	8.7331	68.3016	7.8210
32	28.2056	0.0355	0.00404	247.3236	0.11404	8.7686	69.4007	7.9147
33	31.3082	0.0319	0.00363	275.5292	0.11363	8.8005	70.4228	8.0021
34	34.7521	0.0288	0.00326	306.8374	0.11326	8.8293	71.3724	8.0836
35	38.5749	0.0259	0.00293	341.5896	0.11293	8.8552	72.2538	8.1594
40	65.0009	0.0154	0.00172	581.8261	0.11172	8.9511	75.7789	8.4659
45	109.5302	0.0091	0.00101	986.6386	0.11101	9.0079	78.1551	8.6763
50	184.5648	0.0054	0.00060	1668.77	0.11060	9.0417	79.7341	8.8185
55	311.0025	0.0032	0.00035	2818.20	0.11035	9.0617	80.7712	8.9135
60	524.0572	0.0019	0.00021	4755.07	0.11021	9.0736	81.4461	8.9762
65	883.0669	0.0011	0.00012	8018.79	0.11012	9.0806	81.8819	9.0172
70	1488.02	0.0007	0.00007	13518	0.11007	9.0848	82.1614	9.0438
75	2507.40	0.0004	0.00004	22785	0.11004	9.0873	82.3397	9.0610
80	4225.11	0.0002	0.00003	38401	0.11003	9.0888	82.4529	9.0720
85	7119.56	0.0001	0.00002	64714	0.11002	9.0896	82.5245	9.0790

12% TABLE 17 Discrete Cash Flow: Compound Interest Factors 12%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.1200	0.8929	1.00000	1.0000	1.12000	0.8929		
2	1.2544	0.7972	0.47170	2.1200	0.59170	1.6901	0.7972	0.4717
3	1.4049	0.7118	0.29635	3.3744	0.41635	2.4018	2.2208	0.9246
4	1.5735	0.6355	0.20923	4.7793	0.32923	3.0373	4.1273	1.3589
5	1.7623	0.5674	0.15741	6.3528	0.27741	3.6048	6.3970	1.7746
6	1.9738	0.5066	0.12323	8.1152	0.24323	4.1114	8.9302	2.1720
7	2.2107	0.4523	0.09912	10.0890	0.21912	4.5638	11.6443	2.5512
8	2.4760	0.4039	0.08130	12.2997	0.20130	4.9676	14.4714	2.9131
9	2.7731	0.3606	0.06768	14.7757	0.18768	5.3282	17.3563	3.2574
10	3.1058	0.3220	0.05698	17.5487	0.17698	5.6502	20.2541	3.5847
11	3.4785	0.2875	0.04842	20.6546	0.16842	5.9377	23.1288	3.8953
12	3.8960	0.2567	0.04144	24.1331	0.16144	6.1944	25.9523	4.1897
13	4.3635	0.2292	0.03568	28.0291	0.15568	6.4235	28.7024	4.4683
14	4.8871	0.2046	0.03087	32.3926	0.15087	6.6282	31.3624	4.7317
15	5.4736	0.1827	0.02682	37.2797	0.14682	6.8109	33.9202	4.9803
16	6.1304	0.1631	0.02339	42.7533	0.14339	6.9740	36.3670	5.2147
17	6.8660	0.1456	0.02046	48.8837	0.14046	7.1196	38.6973	5.4353
18	7.6900	0.1300	0.01794	55.7497	0.13794	7.2497	40.9080	5.6427
19	8.6128	0.1161	0.01576	63.4397	0.13576	7.3658	42.9979	5.8375
20	9.6463	0.1037	0.01388	72.0524	0.13388	7.4694	44.9676	6.0202
21	10.8038	0.0926	0.01224	81.6987	0.13224	7.5620	46.8188	6.1913
22	12.1003	0.0826	0.01081	92.5026	0.13081	7.6446	48.5543	6.3514
23	13.5523	0.0738	0.00956	104.6029	0.12956	7.7184	50.1776	6.5010
24	15.1786	0.0659	0.00846	118.1552	0.12846	7.7843	51.6929	6.6406
25	17.0001	0.0588	0.00750	133.3339	0.12750	7.8431	53.1046	6.7708
26	19.0401	0.0525	0.00665	150.3339	0.12665	7.8957	54.4177	6.8921
27	21.3249	0.0469	0.00590	169.3740	0.12590	7.9426	55.6369	7.0049
28	23.8839	0.0419	0.00524	190.6989	0.12524	7.9844	56.7674	7.1098
29	26.7499	0.0374	0.00466	214.5828	0.12466	8.0218	57.8141	7.2071
30	29.9599	0.0334	0.00414	241.3327	0.12414	8.0552	58.7821	7.2974
31	33.5551	0.0298	0.00369	271.2926	0.12369	8.0850	59.6761	7.3811
32	37.5817	0.0266	0.00328	304.8477	0.12328	8.1116	60.5010	7.4586
33	42.0915	0.0238	0.00292	342.4294	0.12292	8.1354	61.2612	7.5302
34	47.1425	0.0212	0.00260	384.5210	0.12260	8.1566	61.9612	7.5965
35	52.7996	0.0189	0.00232	431.6635	0.12232	8.1755	62.6052	7.6577
40	93.0510	0.0107	0.00130	767.0914	0.12130	8.2438	65.1159	7.8988
45	163.9876	0.0061	0.0074	1358.23	0.12074	8.2825	66.7342	8.0572
50	289.0022	0.0035	0.00042	2400.02	0.12042	8.3045	67.7624	8.1597
55	509.3206	0.0020	0.00024	4236.01	0.12024	8.3170	68.4082	8.2251
60	897.5969	0.0011	0.00013	7471.64	0.12013	8.3240	68.8100	8.2664
65	1581.87	0.0006	0.00008	13174	0.12008	8.3281	69.0581	8.2922
70	2787.80	0.0004	0.00004	23223	0.12004	8.3303	69.2103	8.3082
75	4913.06	0.0002	0.00002	40934	0.12002	8.3316	69.3031	8.3181
80	8658.48	0.0001	0.00001	72146	0.12001	8.3324	69.3594	8.3241
85	15259	0.0001	0.00001		0.12001	8.3328	69.3935	8.3278

14%	TABLE 18 Discrete Cash Flow: Compound Interest Factors							14%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.1400	0.8772	1.00000	1.0000	1.14000	0.8772		
2	1.2996	0.7695	0.46729	2.1400	0.60729	1.6467	0.7695	0.4673
3	1.4815	0.6750	0.29073	3.4396	0.43073	2.3216	2.1194	0.9129
4	1.6890	0.5921	0.20320	4.9211	0.34320	2.9137	3.8957	1.3370
5	1.9254	0.5194	0.15128	6.6101	0.29128	3.4331	5.9731	1.7399
6	2.1950	0.4556	0.11716	8.5355	0.25716	3.8887	8.2511	2.1218
7	2.5023	0.3996	0.09319	10.7305	0.23319	4.2883	10.6489	2.4832
8	2.8526	0.3506	0.07557	13.2328	0.21557	4.6389	13.1028	2.8246
9	3.2519	0.3075	0.06217	16.0853	0.20217	4.9464	15.5629	3.1463
10	3.7072	0.2697	0.05171	19.3373	0.19171	5.2161	17.9906	3.4490
11	4.2262	0.2366	0.04339	23.0445	0.18339	5.4527	20.3567	3.7333
12	4.8179	0.2076	0.03667	27.2707	0.17667	5.6603	22.6399	3.9998
13	5.4924	0.1821	0.03116	32.0887	0.17116	5.8424	24.8247	4.2491
14	6.2613	0.1597	0.02661	37.5811	0.16661	6.0021	26.9009	4.4819
15	7.1379	0.1401	0.02281	43.8424	0.16281	6.1422	28.8623	4.6990
16	8.1372	0.1229	0.01962	50.9804	0.15962	6.2651	30.7057	4.9011
17	9.2765	0.1078	0.01692	59.1176	0.15692	6.3729	32.4305	5.0888
18	10.5752	0.0946	0.01462	68.3941	0.15462	6.4674	34.0380	5.2630
19	12.0557	0.0829	0.01266	78.9692	0.15266	6.5504	35.5311	5.4243
20	13.7435	0.0728	0.01099	91.0249	0.15099	6.6231	36.9135	5.5734
21	15.6676	0.0638	0.00954	104.7684	0.14954	6.6870	38.1901	5.7111
22	17.8610	0.0560	0.00830	120.4360	0.14830	6.7429	39.3658	5.8381
23	20.3616	0.0491	0.00723	138.2970	0.14723	6.7921	40.4463	5.9549
24	23.2122	0.0431	0.00630	158.6586	0.14630	6.8351	41.4371	6.0624
25	26.4619	0.0378	0.00550	181.8708	0.14550	6.8729	42.3441	6.1610
26	30.1666	0.0331	0.00480	208.3327	0.14480	6.9061	43.1728	6.2514
27	34.3899	0.0291	0.00419	238.4993	0.14419	6.9352	43.9289	6.3342
28	39.2045	0.0255	0.00366	272.8892	0.14366	6.9607	44.6176	6.4100
29	44.6931	0.0224	0.00320	312.0937	0.14320	6.9830	45.2441	6.4791
30	50.9502	0.0196	0.00280	356.7868	0.14280	7.0027	45.8132	6.5423
31	58.0832	0.0172	0.00245	407.7370	0.14245	7.0199	46.3297	6.5998
32	66.2148	0.0151	0.00215	465.8202	0.14215	7.0350	46.7979	6.6522
33	75.4849	0.0132	0.00188	532.0350	0.14188	7.0482	47.2218	6.6998
34	86.0528	0.0116	0.00165	607.5199	0.14165	7.0599	47.6053	6.7431
35	98.1002	0.0102	0.00144	693.5727	0.14144	7.0700	47.9519	6.7824
40	188.8835	0.0053	0.00075	1342.03	0.14075	7.1050	49.2376	6.9300
45	363.6791	0.0027	0.00039	2590.56	0.14039	7.1232	49.9963	7.0188
50	700.2330	0.0014	0.00020	4994.52	0.14020	7.1327	50.4375	7.0714
55	1348.24	0.0007	0.00010	9623.13	0.14010	7.1376	50.6912	7.1020
60	2595.92	0.0004	0.00005	18535	0.14005	7.1401	50.8357	7.1197
65	4998.22	0.0002	0.00003	35694	0.14003	7.1414	50.9173	7.1298
70	9623.64	0.0001	0.00001	68733	0.14001	7.1421	50.9632	7.1356
75	18530	0.0001	0.00001		0.14001	7.1425	50.9887	7.1388
80	35677				0.14000	7.1427	51.0030	7.1406
85	68693				0.14000	7.1428	51.0108	7.1416

15%		TABLE 19 Discrete Cash Flow: Compound Interest Factors						15%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.1500	0.8696	1.00000	1.0000	1.15000	0.8696		
2	1.3225	0.7561	0.46512	2.1500	0.61512	1.6257	0.7561	0.4651
3	1.5209	0.6575	0.28798	3.4725	0.43798	2.2832	2.0712	0.9071
4	1.7490	0.5718	0.20027	4.9934	0.35027	2.8550	3.7864	1.3263
5	2.0114	0.4972	0.14832	6.7424	0.29832	3.3522	5.7751	1.7228
6	2.3131	0.4323	0.11424	8.7537	0.26424	3.7845	7.9368	2.0972
7	2.6600	0.3759	0.09036	11.0668	0.24036	4.1604	10.1924	2.4498
8	3.0590	0.3269	0.07285	13.7268	0.22285	4.4873	12.4807	2.7813
9	3.5179	0.2843	0.05957	16.7858	0.20957	4.7716	14.7548	3.0922
10	4.0456	0.2472	0.04925	20.3037	0.19925	5.0188	16.9795	3.3832
11	4.6524	0.2149	0.04107	24.3493	0.19107	5.2337	19.1289	3.6549
12	5.3503	0.1869	0.03448	29.0017	0.18448	5.4206	21.1849	3.9082
13	6.1528	0.1625	0.02911	34.3519	0.17911	5.5831	23.1352	4.1438
14	7.0757	0.1413	0.02469	40.5047	0.17469	5.7245	24.9725	4.3624
15	8.1371	0.1229	0.02102	47.5804	0.17102	5.8474	26.6930	4.5650
16	9.3576	0.1069	0.01795	55.7175	0.16795	5.9542	28.2960	4.7522
17	10.7613	0.0929	0.01537	65.0751	0.16537	6.0472	29.7828	4.9251
18	12.3755	0.0808	0.01319	75.8364	0.16319	6.1280	31.1565	5.0843
19	14.2318	0.0703	0.01134	88.2118	0.16134	6.1982	32.4213	5.2307
20	16.3665	0.0611	0.00976	102.4436	0.15976	6.2593	33.5822	5.3651
21	18.8215	0.0531	0.00842	118.8101	0.15842	6.3125	34.6448	5.4883
22	21.6447	0.0462	0.00727	137.6316	0.15727	6.3587	35.6150	5.6010
23	24.8915	0.0402	0.00628	159.2764	0.15628	6.3988	36.4988	5.7040
24	28.6252	0.0349	0.00543	184.1678	0.15543	6.4338	37.3023	5.7979
25	32.9190	0.0304	0.00470	212.7930	0.15470	6.4641	38.0314	5.8834
26	37.8568	0.0264	0.00407	245.7120	0.15407	6.4906	38.6918	5.9612
27	43.5353	0.0230	0.00353	283.5688	0.15353	6.5135	39.2890	6.0319
28	50.0656	0.0200	0.00306	327.1041	0.15306	6.5335	39.8283	6.0960
29	57.5755	0.0174	0.00265	377.1697	0.15265	6.5509	40.3146	6.1541
30	66.2118	0.0151	0.00230	434.7451	0.15230	6.5660	40.7526	6.2066
31	76.1435	0.0131	0.00200	500.9569	0.15200	6.5791	41.1466	6.2541
32	87.5651	0.0114	0.00173	577.1005	0.15173	6.5905	41.5006	6.2970
33	100.6998	0.0099	0.00150	664.6655	0.15150	6.6005	41.8184	6.3357
34	115.8048	0.0086	0.00131	765.3654	0.15131	6.6091	42.1033	6.3705
35	133.1755	0.0075	0.00113	881.1702	0.15113	6.6166	42.3586	6.4019
40	267.8635	0.0037	0.00056	1779.09	0.15056	6.6418	43.2830	6.5168
45	538.7693	0.0019	0.00028	3585.13	0.15028	6.6543	43.8051	6.5830
50	1083.66	0.0009	0.00014	7217.72	0.15014	6.6605	44.0958	6.6205
55	2179.62	0.0005	0.00007	14524	0.15007	6.6636	44.2558	6.6414
60	4384.00	0.0002	0.00003	29220	0.15003	6.6651	44.3431	6.6530
65	8817.79	0.0001	0.00002	58779	0.15002	6.6659	44.3903	6.6593
70	17736	0.0001	0.00001		0.15001	6.6663	44.4156	6.6627
75	35673				0.15000	6.6665	44.4292	6.6646
80	71751				0.15000	6.6666	44.4364	6.6656
85					0.15000	6.6666	44.4402	6.6661

16% TABLE 20 Discrete Cash Flow: Compound Interest Factors 16%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.1600	0.8621	1.00000	1.0000	1.16000	0.8621		
2	1.3456	0.7432	0.46296	2.1600	0.62296	1.6052	0.7432	0.4630
3	1.5609	0.6407	0.28526	3.5056	0.44526	2.2459	2.0245	0.9014
4	1.8106	0.5523	0.19738	5.0665	0.35738	2.7982	3.6814	1.3156
5	2.1003	0.4761	0.14541	6.8771	0.30541	3.2743	5.5858	1.7060
6	2.4364	0.4104	0.11139	8.9775	0.27139	3.6847	7.6380	2.0729
7	2.8262	0.3538	0.08761	11.4139	0.24761	4.0386	9.7610	2.4169
8	3.2784	0.3050	0.07022	14.2401	0.23022	4.3436	11.8962	2.7388
9	3.8030	0.2630	0.05708	17.5185	0.21708	4.6065	13.9998	3.0391
10	4.4114	0.2267	0.04690	21.3215	0.20690	4.8332	16.0399	3.3187
11	5.1173	0.1954	0.03886	25.7329	0.19886	5.0286	17.9941	3.5783
12	5.9360	0.1685	0.03241	30.8502	0.19241	5.1971	19.8472	3.8189
13	6.8858	0.1452	0.02718	36.7862	0.18718	5.3423	21.5899	4.0413
14	7.9875	0.1252	0.02290	43.6720	0.18290	5.4675	23.2175	4.2464
15	9.2655	0.1079	0.01936	51.6595	0.17936	5.5755	24.7284	4.4352
16	10.7480	0.0930	0.01641	60.9250	0.17641	5.6685	26.1241	4.6086
17	12.4677	0.0802	0.01395	71.6730	0.17395	5.7487	27.4074	4.7676
18	14.4625	0.0691	0.01188	84.1407	0.17188	5.8178	28.5828	4.9130
19	16.7765	0.0596	0.01014	98.6032	0.17014	5.8775	29.6557	5.0457
20	19.4608	0.0514	0.00867	115.3797	0.16867	5.9288	30.6321	5.1666
22	26.1864	0.0382	0.00635	157.4150	0.16635	6.0113	32.3200	5.3765
24	35.2364	0.0284	0.00467	213.9776	0.16467	6.0726	33.6970	5.5490
26	47.4141	0.0211	0.00345	290.0883	0.16345	6.1182	34.8114	5.6898
28	63.8004	0.0157	0.00255	392.5028	0.16255	6.1520	35.7073	5.8041
30	85.8499	0.0116	0.00189	530.3117	0.16189	6.1772	36.4234	5.8964
32	115.5196	0.0087	0.00140	715.7475	0.16140	6.1959	36.9930	5.9706
34	155.4432	0.0064	0.00104	965.2698	0.16104	6.2098	37.4441	6.0299
35	180.3141	0.0055	0.00089	1120.71	0.16089	6.2153	37.6327	6.0548
36	209.1643	0.0048	0.00077	1301.03	0.16077	6.2201	37.8000	6.0771
38	281.4515	0.0036	0.00057	1752.82	0.16057	6.2278	38.0799	6.1145
40	378.7212	0.0026	0.00042	2360.76	0.16042	6.2335	38.2992	6.1441
45	795.4438	0.0013	0.00020	4965.27	0.16020	6.2421	38.6598	6.1934
50	1670.70	0.0006	0.00010	10436	0.16010	6.2463	38.8521	6.2201
55	3509.05	0.0003	0.00005	21925	0.16005	6.2482	38.9534	6.2343
60	7370.20	0.0001	0.00002	46058	0.16002	6.2492	39.0063	6.2419

18% **TABLE 21 Discrete Cash Flow: Compound Interest Factors** **18%**

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.1800	0.8475	1.00000	1.0000	1.18000	0.8475		
2	1.3924	0.7182	0.45872	2.1800	0.63872	1.5656	0.7182	0.4587
3	1.6430	0.6086	0.27992	3.5724	0.45992	2.1743	1.9354	0.8902
4	1.9388	0.5158	0.19174	5.2154	0.37174	2.6901	3.4828	1.2947
5	2.2878	0.4371	0.13978	7.1542	0.31978	3.1272	5.2312	1.6728
6	2.6996	0.3704	0.10591	9.4420	0.28591	3.4976	7.0834	2.0252
7	3.1855	0.3139	0.08236	12.1415	0.26236	3.8115	8.9670	2.3526
8	3.7589	0.2660	0.06524	15.3270	0.24524	4.0776	10.8292	2.6558
9	4.4355	0.2255	0.05239	19.0859	0.23239	4.3030	12.6329	2.9358
10	5.2338	0.1911	0.04251	23.5213	0.22251	4.4941	14.3525	3.1936
11	6.1759	0.1619	0.03478	28.7551	0.21478	4.6560	15.9716	3.4303
12	7.2876	0.1372	0.02863	34.9311	0.20863	4.7932	17.4811	3.6470
13	8.5994	0.1163	0.02369	42.2187	0.20369	4.9095	18.8765	3.8449
14	10.1472	0.0985	0.01968	50.8180	0.19968	5.0081	20.1576	4.0250
15	11.9737	0.0835	0.01640	60.9653	0.19640	5.0916	21.3269	4.1887
16	14.1290	0.0708	0.01371	72.9390	0.19371	5.1624	22.3885	4.3369
17	16.6722	0.0600	0.01149	87.0680	0.19149	5.2223	23.3482	4.4708
18	19.6733	0.0508	0.00964	103.7403	0.18964	5.2732	24.2123	4.5916
19	23.2144	0.0431	0.00810	123.4135	0.18810	5.3162	24.9877	4.7003
20	27.3930	0.0365	0.00682	146.6280	0.18682	5.3527	25.6813	4.7978
22	38.1421	0.0262	0.00485	206.3448	0.18485	5.4099	26.8506	4.9632
24	53.1090	0.0188	0.00345	289.4945	0.18345	5.4509	27.7725	5.0950
26	73.9490	0.0135	0.00247	405.2721	0.18247	5.4804	28.4935	5.1991
28	102.9666	0.0097	0.00177	566.4809	0.18177	5.5016	29.0537	5.2810
30	143.3706	0.0070	0.00126	790.9480	0.18126	5.5168	29.4864	5.3448
32	199.6293	0.0050	0.00091	1103.50	0.18091	5.5277	29.8191	5.3945
34	277.9638	0.0036	0.00065	1538.69	0.18065	5.5356	30.0736	5.4328
35	327.9973	0.0030	0.00055	1816.65	0.18055	5.5386	30.1773	5.4485
36	387.0368	0.0026	0.00047	2144.65	0.18047	5.5412	30.2677	5.4623
38	538.9100	0.0019	0.00033	2988.39	0.18033	5.5452	30.4152	5.4849
40	750.3783	0.0013	0.00024	4163.21	0.18024	5.5482	30.5269	5.5022
45	1716.68	0.0006	0.00010	9531.58	0.18010	5.5523	30.7006	5.5293
50	3927.36	0.0003	0.00005	21813	0.18005	5.5541	30.7856	5.5428
55	8984.84	0.0001	0.00002	49910	0.18002	5.5549	30.8268	5.5494
60	20555			114190	0.18001	5.5553	30.8465	5.5526

20% TABLE 22 Discrete Cash Flow: Compound Interest Factors 20%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.2000	0.8333	1.00000	1.0000	1.20000	0.8333		
2	1.4400	0.6944	0.45455	2.2000	0.65455	1.5278	0.6944	0.4545
3	1.7280	0.5787	0.27473	3.6400	0.47473	2.1065	1.8519	0.8791
4	2.0736	0.4823	0.18629	5.3680	0.38629	2.5887	3.2986	1.2742
5	2.4883	0.4019	0.13438	7.4416	0.33438	2.9906	4.9061	1.6405
6	2.9860	0.3349	0.10071	9.9299	0.30071	3.3255	6.5806	1.9788
7	3.5832	0.2791	0.07742	12.9159	0.27742	3.6046	8.2551	2.2902
8	4.2998	0.2326	0.06061	16.4991	0.26061	3.8372	9.8831	2.5756
9	5.1598	0.1938	0.04808	20.7989	0.24808	4.0310	11.4335	2.8364
10	6.1917	0.1615	0.03852	25.9587	0.23852	4.1925	12.8871	3.0739
11	7.4301	0.1346	0.03110	32.1504	0.23110	4.3271	14.2330	3.2893
12	8.9161	0.1122	0.02526	39.5805	0.22526	4.4392	15.4667	3.4841
13	10.6993	0.0935	0.02062	48.4966	0.22062	4.5327	16.5883	3.6597
14	12.8392	0.0779	0.01689	59.1959	0.21689	4.6106	17.6008	3.8175
15	15.4070	0.0649	0.01388	72.0351	0.21388	4.6755	18.5095	3.9588
16	18.4884	0.0541	0.01144	87.4421	0.21144	4.7296	19.3208	4.0851
17	22.1861	0.0451	0.00944	105.9306	0.20944	4.7746	20.0419	4.1976
18	26.6233	0.0376	0.00781	128.1167	0.20781	4.8122	20.6805	4.2975
19	31.9480	0.0313	0.00646	154.7400	0.20646	4.8435	21.2439	4.3861
20	38.3376	0.0261	0.00536	186.6880	0.20536	4.8696	21.7395	4.4643
22	55.2061	0.0181	0.00369	271.0307	0.20369	4.9094	22.5546	4.5941
24	79.4968	0.0126	0.00255	392.4842	0.20255	4.9371	23.1760	4.6943
26	114.4755	0.0087	0.00176	567.3773	0.20176	4.9563	23.6460	4.7709
28	164.8447	0.0061	0.00122	819.2233	0.20122	4.9697	23.9991	4.8291
30	237.3763	0.0042	0.00085	1181.88	0.20085	4.9789	24.2628	4.8731
32	341.8219	0.0029	0.00059	1704.11	0.20059	4.9854	24.4588	4.9061
34	492.2235	0.0020	0.00041	2456.12	0.20041	4.9898	24.6038	4.9308
35	590.6682	0.0017	0.00034	2948.34	0.20034	4.9915	24.6614	4.9406
36	708.8019	0.0014	0.00028	3539.01	0.20028	4.9929	24.7108	4.9491
38	1020.67	0.0010	0.00020	5098.37	0.20020	4.9951	24.7894	4.9627
40	1469.77	0.0007	0.00014	7343.86	0.20014	4.9966	24.8469	4.9728
45	3657.26	0.0003	0.00005	18281	0.20005	4.9986	24.9316	4.9877
50	9100.44	0.0001	0.00002	45497	0.20002	4.9995	24.9698	4.9945
55	22645		0.00001		0.20001	4.9998	24.9868	4.9976

22% **TABLE 23 Discrete Cash Flow: Compound Interest Factors** **22%**

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.2200	0.8197	1.00000	1.0000	1.22000	0.8197		
2	1.4884	0.6719	0.45045	2.2200	0.67045	1.4915	0.6719	0.4505
3	1.8158	0.5507	0.26966	3.7084	0.48966	2.0422	1.7733	0.8683
4	2.2153	0.4514	0.18102	5.5242	0.40102	2.4936	3.1275	1.2542
5	2.7027	0.3700	0.12921	7.7396	0.34921	2.8636	4.6075	1.6090
6	3.2973	0.3033	0.09576	10.4423	0.31576	3.1669	6.1239	1.9337
7	4.0227	0.2486	0.07278	13.7396	0.29278	3.4155	7.6154	2.2297
8	4.9077	0.2038	0.05630	17.7623	0.27630	3.6193	9.0417	2.4982
9	5.9874	0.1670	0.04411	22.6700	0.26411	3.7863	10.3779	2.7409
10	7.3046	0.1369	0.03489	28.6574	0.25489	3.9232	11.6100	2.9593
11	8.9117	0.1122	0.02781	35.9620	0.24781	4.0354	12.7321	3.1551
12	10.8722	0.0920	0.02228	44.8737	0.24228	4.1274	13.7438	3.3299
13	13.2641	0.0754	0.01794	55.7459	0.23794	4.2028	14.6485	3.4855
14	16.1822	0.0618	0.01449	69.0100	0.23449	4.2646	15.4519	3.6233
15	19.7423	0.0507	0.01174	85.1922	0.23174	4.3152	16.1610	3.7451
16	24.0856	0.0415	0.00953	104.9345	0.22953	4.3567	16.7838	3.8524
17	29.3844	0.0340	0.00775	129.0201	0.22775	4.3908	17.3283	3.9465
18	35.8490	0.0279	0.00631	158.4045	0.22631	4.4187	17.8025	4.0289
19	43.7358	0.0229	0.00515	194.2535	0.22515	4.4415	18.2141	4.1009
20	53.3576	0.0187	0.00420	237.9893	0.22420	4.4603	18.5702	4.1635
22	79.4175	0.0126	0.00281	356.4432	0.22281	4.4882	19.1418	4.2649
24	118.2050	0.0085	0.00188	532.7501	0.22188	4.5070	19.5635	4.3407
26	175.9364	0.0057	0.00126	795.1653	0.22126	4.5196	19.8720	4.3968
28	261.8637	0.0038	0.00084	1185.74	0.22084	4.5281	20.0962	4.4381
30	389.7579	0.0026	0.00057	1767.08	0.22057	4.5338	20.2583	4.4683
32	580.1156	0.0017	0.00038	2632.34	0.22038	4.5376	20.3748	4.4902
34	863.4441	0.0012	0.00026	3920.20	0.22026	4.5402	20.4582	4.5060
35	1053.40	0.0009	0.00021	4783.64	0.22021	4.5411	20.4905	4.5122
36	1285.15	0.0008	0.00017	5837.05	0.22017	4.5419	20.5178	4.5174
38	1912.82	0.0005	0.00012	8690.08	0.22012	4.5431	20.5601	4.5256
40	2847.04	0.0004	0.00008	12937	0.22008	4.5439	20.5900	4.5314
45	7694.71	0.0001	0.00003	34971	0.22003	4.5449	20.6319	4.5396
50	20797		0.00001	94525	0.22001	4.5452	20.6492	4.5431
55	56207				0.22000	4.5454	20.6563	4.5445

24% TABLE 24 Discrete Cash Flow: Compound Interest Factors 24%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.2400	0.8065	1.00000	1.0000	1.24000	0.8065		
2	1.5376	0.6504	0.44643	2.2400	0.68643	1.4568	0.6504	0.4464
3	1.9066	0.5245	0.26472	3.7776	0.50472	1.9813	1.6993	0.8577
4	2.3642	0.4230	0.17593	5.6842	0.41593	2.4043	2.9683	1.2346
5	2.9316	0.3411	0.12425	8.0484	0.36425	2.7454	4.3327	1.5782
6	3.6352	0.2751	0.09107	10.9801	0.33107	3.0205	5.7081	1.8898
7	4.5077	0.2218	0.06842	14.6153	0.30842	3.2423	7.0392	2.1710
8	5.5895	0.1789	0.05229	19.1229	0.29229	3.4212	8.2915	2.4236
9	6.9310	0.1443	0.04047	24.7125	0.28047	3.5655	9.4458	2.6492
10	8.5944	0.1164	0.03160	31.6434	0.27160	3.6819	10.4930	2.8499
11	10.6571	0.0938	0.02485	40.2379	0.26485	3.7757	11.4313	3.0276
12	13.2148	0.0757	0.01965	50.8950	0.25965	3.8514	12.2637	3.1843
13	16.3863	0.0610	0.01560	64.1097	0.25560	3.9124	12.9960	3.3218
14	20.3191	0.0492	0.01242	80.4961	0.25242	3.9616	13.6358	3.4420
15	25.1956	0.0397	0.00992	100.8151	0.24992	4.0013	14.1915	3.5467
16	31.2426	0.0320	0.00794	126.0108	0.24794	4.0333	14.6716	3.6376
17	38.7408	0.0258	0.00636	157.2534	0.24636	4.0591	15.0846	3.7162
18	48.0386	0.0208	0.00510	195.9942	0.24510	4.0799	15.4385	3.7840
19	59.5679	0.0168	0.00410	244.0328	0.24410	4.0967	15.7406	3.8423
20	73.8641	0.0135	0.00329	303.6006	0.24329	4.1103	15.9979	3.8922
22	113.5735	0.0088	0.00213	469.0563	0.24213	4.1300	16.4011	3.9712
24	174.6306	0.0057	0.00138	723.4610	0.24138	4.1428	16.6891	4.0284
26	268.5121	0.0037	0.00090	1114.63	0.24090	4.1511	16.8930	4.0696
28	412.8642	0.0024	0.00058	1716.10	0.24058	4.1566	17.0365	4.0987
30	634.8199	0.0016	0.00038	2640.92	0.24038	4.1601	17.1369	4.1193
32	976.0991	0.0010	0.00025	4062.91	0.24025	4.1624	17.2067	4.1338
34	1500.85	0.0007	0.00016	6249.38	0.24016	4.1639	17.2552	4.1440
35	1861.05	0.0005	0.00013	7750.23	0.24013	4.1664	17.2734	4.1479
36	2307.71	0.0004	0.00010	9611.28	0.24010	4.1649	17.2886	4.1511
38	3548.33	0.0003	0.00007	14781	0.24007	4.1655	17.3116	4.1560
40	5455.91	0.0002	0.00004	22729	0.24004	4.1659	17.3274	4.1593
45	15995	0.0001	0.00002	66640	0.24002	4.1664	17.3483	4.1639
50	46890		0.00001		0.24001	4.1666	17.3563	4.1653
55					0.24000	4.1666	17.3593	4.1663

25% TABLE 25 Discrete Cash Flow: Compound Interest Factors 25%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.2500	0.8000	1.00000	1.0000	1.25000	0.8000		
2	1.5625	0.6400	0.44444	2.2500	0.69444	1.4400	0.6400	0.4444
3	1.9531	0.5120	0.26230	3.8125	0.51230	1.9520	1.6640	0.8525
4	2.4414	0.4096	0.17344	5.7656	0.42344	2.3616	2.8928	1.2249
5	3.0518	0.3277	0.12185	8.2070	0.37185	2.6893	4.2035	1.5631
6	3.8147	0.2621	0.08882	11.2588	0.33882	2.9514	5.5142	1.8683
7	4.7684	0.2097	0.06634	15.0735	0.31634	3.1611	6.7725	2.1424
8	5.9605	0.1678	0.05040	19.8419	0.30040	3.3289	7.9469	2.3872
9	7.4506	0.1342	0.03876	25.8023	0.28876	3.4631	9.0207	2.6048
10	9.3132	0.1074	0.03007	33.2529	0.28007	3.5705	9.9870	2.7971
11	11.6415	0.0859	0.02349	42.5661	0.27349	3.6564	10.8460	2.9663
12	14.5519	0.0687	0.01845	54.2077	0.26845	3.7251	11.6020	3.1145
13	18.1899	0.0550	0.01454	68.7596	0.26454	3.7801	12.2617	3.2437
14	22.7374	0.0440	0.01150	86.9495	0.26150	3.8241	12.8334	3.3559
15	28.4217	0.0352	0.00912	109.6868	0.25912	3.8593	13.3260	3.4530
16	35.5271	0.0281	0.00724	138.1085	0.25724	3.8874	13.7482	3.5366
17	44.4089	0.0225	0.00576	173.6357	0.25576	3.9099	14.1085	3.6084
18	55.5112	0.0180	0.00459	218.0446	0.25459	3.9279	14.4147	3.6698
19	69.3889	0.0144	0.00366	273.5558	0.25366	3.9424	14.6741	3.7222
20	86.7362	0.0115	0.00292	342.9447	0.25292	3.9539	14.8932	3.7667
22	135.5253	0.0074	0.00186	538.1011	0.25186	3.9705	15.2326	3.8365
24	211.7582	0.0047	0.00119	843.0329	0.25119	3.9811	15.4711	3.8861
26	330.8722	0.0030	0.00076	1319.49	0.25076	3.9879	15.6373	3.9212
28	516.9879	0.0019	0.00048	2063.95	0.25048	3.9923	15.7524	3.9457
30	807.7936	0.0012	0.00031	3227.17	0.25031	3.9950	15.8316	3.9628
32	1262.18	0.0008	0.00020	5044.71	0.25020	3.9968	15.8859	3.9746
34	1972.15	0.0005	0.00013	7884.61	0.25013	3.9980	15.9229	3.9828
35	2465.19	0.0004	0.00010	9856.76	.025010	3.9984	15.9367	3.9858
36	3081.49	0.0003	0.00008	12322	0.25008	3.9987	15.9481	3.9883
38	4814.82	0.0002	0.00005	19255	0.25005	3.9992	15.9651	3.9921
40	7523.16	0.0001	0.00003	30089	0.25003	3.9995	15.9766	3.9947
45	22959		0.00001	91831	0.25001	3.9998	15.9915	3.9980
50	70065				0.25000	3.9999	15.9969	3.9993
55					0.25000	4.0000	15.9989	3.9997

30% TABLE 26 Discrete Cash Flow: Compound Interest Factors 30%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.3000	0.7692	1.00000	1.0000	1.30000	0.7692		
2	1.6900	0.5917	0.43478	2.3000	0.73478	1.3609	0.5917	0.4348
3	2.1970	0.4552	0.25063	3.9900	0.55063	1.8161	1.5020	0.8271
4	2.8561	0.3501	0.16163	6.1870	0.46163	2.1662	2.5524	1.1783
5	3.7129	0.2693	0.11058	9.0431	0.41058	2.4356	3.6297	1.4903
6	4.8268	0.2072	0.07839	12.7560	0.37839	2.6427	4.6656	1.7654
7	6.2749	0.1594	0.05687	17.5828	0.35687	2.8021	5.6218	2.0063
8	8.1573	0.1226	0.04192	23.8577	0.34192	2.9247	6.4800	2.2156
9	10.6045	0.0943	0.03124	32.0150	0.33124	3.0190	7.2343	2.3963
10	13.7858	0.0725	0.02346	42.6195	0.32346	3.0915	7.8872	2.5512
11	17.9216	0.0558	0.01773	56.4053	0.31773	3.1473	8.4452	2.6833
12	23.2981	0.0429	0.01345	74.3270	0.31345	3.1903	8.9173	2.7952
13	30.2875	0.0330	0.01024	97.6250	0.31024	3.2233	9.3135	2.8895
14	39.3738	0.0254	0.00782	127.9125	0.30782	3.2487	9.6437	2.9685
15	51.1859	0.0195	0.00598	167.2863	0.30598	3.2682	9.9172	3.0344
16	66.5417	0.0150	0.00458	218.4722	0.30458	3.2832	10.1426	3.0892
17	86.5042	0.0116	0.00351	285.0139	0.30351	3.2948	10.3276	3.1345
18	112.4554	0.0089	0.00269	371.5180	0.30269	3.3037	10.4788	3.1718
19	146.1920	0.0068	0.00207	483.9734	0.30207	3.3105	10.6019	3.2025
20	190.0496	0.0053	0.00159	630.1655	0.30159	3.3158	10.7019	3.2275
22	321.1839	0.0031	0.00094	1067.28	0.30094	3.3230	10.8482	3.2646
24	542.8008	0.0018	0.00055	1806.00	0.30055	3.3272	10.9433	3.2890
25	705.6410	0.0014	0.00043	2348.80	0.30043	3.3286	10.9773	3.2979
26	917.3333	0.0011	0.00033	3054.44	0.30033	3.3297	11.0045	3.3050
28	1550.29	0.0006	0.00019	5164.31	0.30019	3.3312	11.0437	3.3153
30	2620.00	0.0004	0.00011	8729.99	0.30011	3.3321	11.0687	3.3219
32	4427.79	0.0002	0.00007	14756	0.30007	3.3326	11.0845	3.3261
34	7482.97	0.0001	0.00004	24940	0.30004	3.3329	11.0945	3.3288
35	9727.86	0.0001	0.00003	32423	0.30003	3.3330	11.0980	3.3297

35% **TABLE 27 Discrete Cash Flow: Compound Interest Factors** **35%**

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount *F/P*	Present Worth *P/F*	Sinking Fund *A/F*	Compound Amount *F/A*	Capital Recovery *A/P*	Present Worth *P/A*	Gradient Present Worth *P/G*	Gradient Uniform Series *A/G*
1	1.3500	0.7407	1.00000	1.0000	1.35000	0.7407		
2	1.8225	0.5487	0.42553	2.3500	0.77553	1.2894	0.5487	0.4255
3	2.4604	0.4064	0.23966	4.1725	0.58966	1.6959	1.3616	0.8029
4	3.3215	0.3011	0.15076	6.6329	0.50076	1.9969	2.2648	1.1341
5	4.4840	0.2230	0.10046	9.9544	0.45046	2.2200	3.1568	1.4220
6	6.0534	0.1652	0.06926	14.4384	0.41926	2.3852	3.9828	1.6698
7	8.1722	0.1224	0.04880	20.4919	0.39880	2.5075	4.7170	1.8811
8	11.0324	0.0906	0.03489	28.6640	0.38489	2.5982	5.3515	2.0597
9	14.8937	0.0671	0.02519	39.6964	0.37519	2.6653	5.8886	2.2094
10	20.1066	0.0497	0.01832	54.5902	0.36832	2.7150	6.3363	2.3338
11	27.1439	0.0368	0.01339	74.6967	0.36339	2.7519	6.7047	2.4364
12	36.6442	0.0273	0.00982	101.8406	0.35982	2.7792	7.0049	2.5205
13	49.4697	0.0202	0.00722	138.4848	0.35722	2.7994	7.2474	2.5889
14	66.7841	0.0150	0.00532	187.9544	0.35532	2.8144	7.4421	2.6443
15	90.1585	0.0111	0.00393	254.7385	0.35393	2.8255	7.5974	2.6889
16	121.7139	0.0082	0.00290	344.8970	0.35290	2.8337	7.7206	2.7246
17	164.3138	0.0061	0.00214	466.6109	0.35214	2.8398	7.8180	2.7530
18	221.8236	0.0045	0.00158	630.9247	0.35158	2.8443	7.8946	2.7756
19	299.4619	0.0033	0.00117	852.7483	0.35117	2.8476	7.9547	2.7935
20	404.2736	0.0025	0.00087	1152.21	0.35087	2.8501	8.0017	2.8075
22	736.7886	0.0014	0.00048	2102.25	0.35048	2.8533	8.0669	2.8272
24	1342.80	0.0007	0.00026	3833.71	0.35026	2.8550	8.1061	2.8393
25	1812.78	0.0006	0.00019	5176.50	0.35019	2.8556	8.1194	2.8433
26	2447.25	0.0004	0.00014	6989.28	0.35014	2.8560	8.1296	2.8465
28	4460.11	0.0002	0.00008	12740	0.35008	2.8565	8.1435	2.8509
30	8128.55	0.0001	0.00004	23222	0.35004	2.8568	8.1517	2.8535
32	14814	0.0001	0.00002	42324	0.35002	2.8569	8.1565	2.8550
34	26999		0.00001	77137	0.35001	2.8570	8.1594	2.8559
35	36449		0.00001		0.35001	2.8571	8.1603	2.8562

40% TABLE 28 Discrete Cash Flow: Compound Interest Factors 40%

	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.4000	0.7143	1.00000	1.0000	1.40000	0.7143		
2	1.9600	0.5102	0.41667	2.4000	0.81667	1.2245	0.5102	0.4167
3	2.7440	0.3644	0.22936	4.3600	0.62936	1.5889	1.2391	0.7798
4	3.8416	0.2603	0.14077	7.1040	0.54077	1.8492	2.0200	1.0923
5	5.3782	0.1859	0.09136	10.9456	0.49136	2.0352	2.7637	1.3580
6	7.5295	0.1328	0.06126	16.3238	0.46126	2.1680	3.4278	1.5811
7	10.5414	0.0949	0.04192	23.8534	0.44192	2.2628	3.9970	1.7664
8	14.7579	0.0678	0.02907	34.3947	0.42907	2.3306	4.4713	1.9185
9	20.6610	0.0484	0.02034	49.1526	0.42034	2.3790	4.8585	2.0422
10	28.9255	0.0346	0.01432	69.8137	0.41432	2.4136	5.1696	2.1419
11	40.4957	0.0247	0.01013	98.7391	0.41013	2.4383	5.4166	2.2215
12	56.6939	0.0176	0.00718	139.2348	0.40718	2.4559	5.6106	2.2845
13	79.3715	0.0126	0.00510	195.9287	0.40510	2.4685	5.7618	2.3341
14	111.1201	0.0090	0.00363	275.3002	0.40363	2.4775	5.8788	2.3729
15	155.5681	0.0064	0.00259	386.4202	0.40259	2.4839	5.9688	2.4030
16	217.7953	0.0046	0.00185	541.9883	0.40185	2.4885	6.0376	2.4262
17	304.9135	0.0033	0.00132	759.7837	0.40132	2.4918	6.0901	2.4441
18	426.8789	0.0023	0.00094	1064.70	0.40094	2.4941	6.1299	2.4577
19	597.6304	0.0017	0.00067	1491.58	0.40067	2.4958	6.1601	2.4682
20	836.6826	0.0012	0.00048	2089.21	0.40048	2.4970	6.1828	2.4761
22	1639.90	0.0006	0.00024	4097.24	0.40024	2.4985	6.2127	2.4866
24	3214.20	0.0003	0.00012	8033.00	0.40012	2.4992	6.2294	2.4925
25	4499.88	0.0002	0.00009	11247	0.40009	2.4994	6.2347	2.4944
26	6299.83	0.0002	0.00006	15747	0.40006	2.4996	6.2387	2.4959
28	12348	0.0001	0.00003	30867	0.40003	2.4998	6.2438	2.4977
30	24201		0.00002	60501	0.40002	2.4999	6.2466	2.4988
32	47435		0.00001		0.40001	2.4999	6.2482	2.4993
34	92972				0.40000	2.5000	6.2490	2.4996
35					0.40000	2.5000	6.2493	2.4997

50%			TABLE 29 Discrete Cash Flow: Compound Interest Factors					50%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.5000	0.6667	1.00000	1.0000	1.50000	0.6667		
2	2.2500	0.4444	0.40000	2.5000	0.90000	1.1111	0.4444	0.4000
3	3.3750	0.2963	0.21053	4.7500	0.71053	1.4074	1.0370	0.7368
4	5.0625	0.1975	0.12308	8.1250	0.62308	1.6049	1.6296	1.0154
5	7.5938	0.1317	0.07583	13.1875	0.57583	1.7366	2.1564	1.2417
6	11.3906	0.0878	0.04812	20.7813	0.54812	1.8244	2.5953	1.4226
7	17.0859	0.0585	0.03108	32.1719	0.53108	1.8829	2.9465	1.5648
8	25.6289	0.0390	0.02030	49.2578	0.52030	1.9220	3.2196	1.6752
9	38.4434	0.0260	0.01335	74.8867	0.51335	1.9480	3.4277	1.7596
10	57.6650	0.0173	0.00882	113.3301	0.50882	1.9653	3.5838	1.8235
11	86.4976	0.0116	0.00585	170.9951	0.50585	1.9769	3.6994	1.8713
12	129.7463	0.0077	0.00388	257.4927	0.50388	1.9846	3.7842	1.9068
13	194.6195	0.0051	0.00258	387.2390	0.50258	1.9897	3.8459	1.9329
14	291.9293	0.0034	0.00172	581.8585	0.50172	1.9931	3.8904	1.9519
15	437.8939	0.0023	0.00114	873.7878	0.50114	1.9954	3.9224	1.9657
16	656.8408	0.0015	0.00076	1311.68	0.50076	1.9970	3.9452	1.9756
17	985.2613	0.0010	0.00051	1968.52	0.50051	1.9980	3.9614	1.9827
18	1477.89	0.0007	0.00034	2953.78	0.50034	1.9986	3.9729	1.9878
19	2216.84	0.0005	0.00023	4431.68	0.50023	1.9991	3.9811	1.9914
20	3325.26	0.0003	0.00015	6648.51	0.50015	1.9994	3.9868	1.9940
22	7481.83	0.0001	0.00007	14962	0.50007	1.9997	3.9936	1.9971
24	16834	0.0001	0.00003	33666	0.50003	1.9999	3.9969	1.9986
25	25251		0.00002	50500	0.50002	1.9999	3.9979	1.9990
26	37877		0.00001	75752	0.50001	1.9999	3.9985	1.9993
28	85223		0.00001		0.50001	2.0000	3.9993	1.9997
30					0.50000	2.0000	3.9997	1.9998
32					0.50000	2.0000	3.9998	1.9999
34					0.50000	2.0000	3.9999	2.0000
35					0.50000	2.0000	3.9999	2.0000

11 References

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