

# Mathematics of Finance



With the help of the topics covered in this unit, students will be familiar with basic knowledge and importance of the mathematics of finance in the time value of money. The unit surveys interest, depreciation, present value and future value of money and different types of annuities followed by examples.

**Blank Page**

## Lesson-1: Interest

After studying this lesson, you should be able to

- State the nature of interest;
- Calculate the simple interest;
- Calculate the compound interest in various situations.

### Nature of Interest

When  $x$  borrows money from  $y$ , then  $x$  has to pay certain amount to  $y$  for the use of the money. The amount paid by  $x$  is called interest. The amount borrowed by  $x$  from  $y$  is called principal. The sum of the interest and principal is usually called the total amount. When interest is payable on the principal only, it is termed as simple interest. On the other hand, when interest is calculated on the amount of the previous year or period, then it is called compound interest.

*When interest is payable on the principal only, it is termed as simple interest.*

### Calculation of Simple Interest

Let  $P$  = Principal i.e., the initial sum of money invested.

$I$  = Interest per unit money/ per unit time.

$X$  = Period i.e., unit of time for which the interest is calculated.

$A$  = Amount i.e., principal plus interest accrued.

The interest on 1 unit of money for 1 unit of time =  $i$

The interest on 1 unit of money for 'n' unit of time =  $ni$

The interest on  $P$  unit of money for 'n' unit of time =  $Pni$

Hence  $A = P + Pni = P(1+ni)$

The simple interest obtained on principal ( $P$ ) after  $n$  years will be

$$= A - P$$

$$= P(1+ni) - P = (P + Pni - P) = Pni$$

For example, the rate of simple interest is 10% per annum means that the interest payable on Tk.100 for one year is Tk.10, i.e., at the end of one year, total amount will be Tk.110, at the end of second year, it will be Tk.120 and so on.

### Example-1:

Mr. Rahim has invested Tk.30,000 for 5 years at 10% rate of interest. What will be the simple interest and amount after 5 years?

### Solution:

We know that the simple interest on principal ( $P$ ) for 'n' year at a rate 'i'  
=  $Pni$

Here  $P = 30,000$ ,  $N = 5$ ,  $i = 10\% = 0.10$

Substituting the given values we have,

$$\text{Simple Interest} = 30,000 \times 5 \times 0.10$$

$$= \text{Tk.15,000}$$

Hence the required simple interest of 5 years is Tk.15,000

$$\begin{aligned} \text{Amount after 5 years at simple interest, } A &= P (1+ni) \\ &= 30,000 (1 + 5 \times 0.10) \\ &= 30,000 (1.50) \\ &= \text{Tk.45,000} \end{aligned}$$

### Calculation of Compound Interest

*At the end of every period, the interest earned is added to the principal to become the principal earning interest for the next period.*

If  $i$  be the rate of interest per unit per period, a principal 1 accumulates at compound interest in the following manner. At the end of every period, the interest earned is added to the principal to become the principal earning interest for the next period, For example

		<u>Amount</u>
Principal (P)	$1$	$1$
Interest for the first period	$i$	
Principal for the 2 <sup>nd</sup> period	$(1+i)$	$(1+i)$
Interest for the 2 <sup>nd</sup> period	$i (1+i)$	
Principal for the 3 <sup>rd</sup> period	$(1+i) (1+i)$	$= (1+i)^2$
Interest for the 3 <sup>rd</sup> period	$i (1+i)^2$	
Principal for the 4 <sup>th</sup> period	$(1+i)^2 (1+i)$	$= (1+i)^3$

And so on.

Hence the amount at the end of  $n$  period  $= (1+i)^n$

Thus the amount (A) of Principal (P) at the end of  $n$  periods is,

$$A = P (1+i)^n$$

The fundamental formula of compound interest, namely  $A = P (1+i)^n$  is easily adopted to logarithmic calculation, where

$$\log A = \log P + n \log (1+i)$$

$$\begin{aligned} \text{Now, the compound interest} &= A - P \\ &= P (1+i)^n - P \\ &= P [(1+i)^n - 1] \end{aligned}$$

Let  $P$  = Principal,  $A$  = the total amount,  $t$  = total interest,  $i$  = annual rate of interest,  $n$  = number of period; then the compound interest can be computed by using the following formula, which may be changed on the basis of the number of compounding time.

Compounding Time	Total amount	I = Total amount (A) - Principal amount (P)
Weekly	$A = P (1 + \frac{i}{52})^{52n}$	$I = P [(1 + \frac{i}{52})^{52n} - 1]$
Monthly	$A = P (1 + \frac{i}{12})^{12n}$	$I = P [(1 + \frac{i}{12})^{12n} - 1]$

Compounding Time	Total amount	I = Total amount (A) – Principal amount (P)
Quarterly	$A = P \left(1 + \frac{i}{4}\right)^{4n}$	$I = P \left[\left(1 + \frac{i}{4}\right)^{4n} - 1\right]$
Half yearly	$A = P \left(1 + \frac{i}{2}\right)^{2n}$	$I = P \left[\left(1 + \frac{i}{2}\right)^{2n} - 1\right]$
Annually	$A = P (1 + i)^n$	$I = P[(1 + i)^n - 1]$

If the interest is  $i$  per unit per annum, nominal convertible ' $m$ ' times a year;  $i/m$  is converted into the principal at the end of every such compounding time; and 1 will accumulate to  $(1+i/m)^m$  in a year. The difference  $[(1+i/m)^m - 1]$  a year on a principal 1 is known as the effective rate of interest per annum.

The difference  $[(1+i/m)^m - 1]$  a year on a principal 1 is known as the effective rate of interest per annum.

A Principal  $m$  accumulates to  $A = (1+i/m)^{nm}$  in  $n$  year at the above rate.

**Example-2:**

Mr. Rahim has invested Tk.30,000 for 4 years at 12% rate of interest.

1. What will be the compound interest and amount after 4 years if it is compounding (a) Yearly; or (b) Monthly?
2. Find the number of years in which the sum will double itself at annual compound interest.
3. What should be the annual compound interest rate to make the amount Tk.60,000 after 4 years?

**Solution:**

We are given,  $P = 30,000$ ,  $n = 4$  and  $i = 0.12$

(1)

(a) In the case of Yearly Compounding:

$$\begin{aligned} \text{Compound interest after 4 years} &= P [(1+i)^n - 1] \\ &= 30,000 [(1 + 0.12)^4 - 1] \\ &= 30,000 [(1.12)^4 - 1] \\ &= 30,000 \times 0.5735 = \text{Tk.}17,205 \end{aligned}$$

$$\begin{aligned} \text{Amount after 4 years, } A &= P (1+i)^4 \\ &= 30,000 (1 + 0.12)^4 \\ &= 30,000 \times 1.5735 \\ &= \text{Tk.}47, 205 \end{aligned}$$

(b) In the case of Monthly Compounding: [then  $m = 12$ ]

Compound interest after 4 years =  $P [(1 + \frac{i}{m})^{mn} - 1]$

$$= 30,000 \left[ \left( 1 + \frac{0.12}{12} \right)^{12 \times 4} - 1 \right]$$

$$= 30,000 [1.6122 - 1]$$

$$= 30,000 \times 0.6122 = \text{Tk.}18, 366$$

Amount after 4 years,  $A = P (1 + \frac{i}{m})^{mn}$

$$= 30,000 \left( 1 + \frac{0.12}{12} \right)^{12 \times 4}$$

$$= 30,000 (1.01)^{48}$$

$$= 30,000 \times 1.6122$$

$$= \text{Tk.}48, 366$$

2. Let the sum will be  $\text{Tk.}(30,000 \times 2) = \text{Tk.}60, 000$  is  $n$  years.

So,  $P = 30,000$ ,  $A = 60,000$ ,  $i = .12$  and  $n = ?$

Now,  $A = P (1+i)^n$

$$\text{Or, } 60,000 = 30,000 (1 + 0.12)^n$$

$$\text{Or, } (1.12)^n = 60,000/30,000$$

$$\text{Or, } (1.12)^n = 2$$

Taking logarithm both sides, we have

$$\text{or, } n \log 1.12 = \log 2$$

$$n = \frac{\log 2}{\log 1.12} = \frac{0.3010}{0.0492} = 6.12 \text{ years.}$$

Hence it will take 6.12 years for  $\text{Tk.}30, 000$  to be doubled to  $\text{Tk.}60,000$ .

3. We have,  $P = 30,000$ ;  $A = 60,000$ ,  $n = 4$  and  $i = ?$

Now  $A = P (1+i)^n$

$$\text{or, } 60,000 = 30,000(1+i)^4$$

$$\text{or, } (1+i)^4 = 60,000/30,000$$

$$\text{or, } (1+i)^4 = 2$$

Taking logarithm both sides, we have

$$\text{or, } 4\log(1+i) = \log 2$$

$$\text{or, } \log(1+i) = \frac{\log 2}{4} = \frac{0.3010}{4}$$

$$\text{or, } \log(1+i) = 0.0753$$

$$\text{or, } (1+i) = \text{antilog } 0.0753$$

$$\text{or, } (1+i) = 1.1893$$

$$\text{or, } i = (1.1893 - 1) = 0.1893 \text{ or } 18.93\%$$

Hence the rate of interest should be 18.93% to make the amount Tk.60,000 after 4 years.

### Calculation of compound interest with growing investment (withdrawals)

Let  $A$  invested at the beginning of the first year and an additional sum  $B$  be added to the investment in each subsequent year. No withdrawals are to be made and whose sum invested is to be allowed to accumulate at a compound rate.

$$\text{Hence } A = (A_0 + B/i) (1+i)^n - B/i$$

Here,  $A$  = the sum of amount

$i$  = the rate of interest

$A_0$  = invested at the beginning of the year.

$B$  = additional sum to be added / (withdrawn) in investment.

$n$  = number of periods.

Therefore compound interest = Total Amount ( $A$ ) – Principal Amount.

Let us illustrate it by the following examples.

$$\text{Compound interest} = \text{Total Amount (A)} - \text{Principal Amount.}$$

#### Example – 3:

Tk.10,000 is invested at the beginning of 1999. It remains invested and, on 1<sup>st</sup> January in each subsequent year, another Tk.500 is added to it. What sum will be available on 1<sup>st</sup> January 2005 if interest is compounded each year at the rate of 5% per annum?

#### Solution:

$$\text{We know that, } A = (A_0 + B/i) (1+i)^n - B/i$$

$$\text{Here, } A_0 = 10,000, B = 500, i = .05, n = 6$$

Substituting the values we have,

$$\begin{aligned} A &= (10,000 + 500/0.05) (1 + .05)^6 - 500/0.05 \\ &= (10,000 + 10,000) (1.05)^6 - 10,000 \\ &= 20,000 \times 1.3401 - 10,000 \end{aligned}$$

$$= 26,802 - 10,000 = \text{Tk.}16, 802$$

Therefore, the sum of amount on 1<sup>st</sup> January 2005 is Tk.16, 802.

**Example-4:**

A man invests Tk.10,000 once at how and withdraws Tk.1500 at the end of each year starting at the end of the first year. How much will have left after seven years if the money is invested at 4% per annum?

**Solution:**

We know that,  $A = (A_0 + B/i) (1+i)^n - B/i$

Here,  $A_0 = 10,000, n = 7, i = 0.04, B = - 1,500$

Substituting the values we have

$$\begin{aligned} A &= \{10,000 + (-1500)/0.04\} (1 + 0.04)^7 - (-1500/0.04) \\ &= (10,000 - 37,500) (1.04)^7 + 37,500 \\ &= (-27,500) (1.3159) + 37,500 \\ &= - 36187.25 + 37,500 \\ &= \text{Tk.}1, 312.75 \end{aligned}$$

Drawing at this rate he will only have Tk.1, 312.75 at the end of seven years.



### Questions For Review:

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. Define the following: Simple interest, Compound interest, Effective rate of interest.
2. Compare between simple interest and compound interest.
3. Mr. Asif has invested Tk.1,00,000 for 5 years at 10% rate of interest.
  - a. What will be the simple interest and amount after 5 years?
  - b. What will be the compound interest and amount after 5 years if interest is paid
    - (i) Monthly, or           (ii) Quarterly?
  - c. What should be annual compound interest rate to make the amount Tk.2,00,000 after 5 years?
4. At what rate of interest an amount of investment will be thrice as much as at the end of 6 years?
5. How many years will it take at 12% interest compounding annually for Tk.6000 to grow to Tk.11, 000?
6. Tk.50,000 invested at the beginning of 1997. It remains invested and, on 1<sup>st</sup> January of each subsequent year, another Tk.5000 is added to it. What sum will be available on 1<sup>st</sup> January 2005 if interest is compounded yearly @ 10% per annum?

### Multiple choice questions (✓ the appropriate answers)

1. A man borrows Tk.2000 and pays back after 3 years at 10% simple interest. The amount paid by the man is:
  - a) 2400                   b) 2600                   c) 2750
2. The difference between the simple interest and compound interest for 2years at 4% p.a. is Tk.20. The principle amount will be
  - a) 12500                   b) 12000                   c) 13000
3. A person takes a loan of Tk.200 at 5% simple interest. He returns Tk.100 at the end of 1 year. In order to clear his dues at the end of 2 years, he will pay
  - a) 110                   b) 115                   c) 115.50
4. Two equal amount of money are deposited in two banks, each at 15% simple interest p.a. for 3.5 years and 5years. If the difference between their interest is Tk.144, each sum is
  - a) 640                   b) 500                   c) 720

5. In what time will a sum of money double itself at  $6\frac{1}{4}$  p.a. simple interest  
a) 16years                      b) 12 years                      c) 8 years
6. A sum of money will triple itself in 15 years at simple interest with yearly rate of  
a)  $12\frac{2}{3}\%$                       b)  $13\frac{1}{3}\%$                       c)  $16\frac{2}{3}\%$
7. A sum was put at simple interest at a certain rate for 2 years. Had it been put at 3% higher rate, it would have fetched Tk.72 more. The sum is  
a) Tk.1600                      b) Tk.1800                      c) Tk.1200
8. What annual payment will discharge a debt of Tk.580 due in 5 years, the rate being 8% p.a.?  
a) 100                      b) 120                      c) 166.40
9. A sum of Tk.400 would become Tk.441 after 2 years if the rate of compound interest were:  
a) 5%                      b) 7.5%                      c) 2.5%
10. A sum of Tk.12,000 deposited at compound interest becomes double after 5 years. After 20years it will become  
a) Tk.1, 92,00                      b) Tk.1, 24,00                      c) Tk.1,20,00

## Lesson-2: Depreciation

After studying this lesson, you should be able to:

- Explain the nature of depreciation and depreciated value;
- Calculate the amount of depreciation under the different methods of depreciation.

### Nature of Depreciation

In case of depreciation, the principal value is diminished every year by some amount, and in the subsequent period the diminished value becomes the principal value. In case of uniform decrease or depreciation, 'i' is to be substituted by '-i' in the formula of future value. In that case depreciated value and accumulated depreciation is calculated by using the following formula:

*In case of uniform decrease or depreciation, 'i' is to be substituted by '-i' in the formula of future value.*

Depreciated value =  $P (1-i)^n$  [Under reducing balance method]

Accumulated depreciation =  $P [1 - (1+i)^n]$

Where,  $P$  = Cost price of the asset

$i$  = Rate of depreciation

$n$  = Number of periods the asset has been depreciated.

For calculation of depreciated value and accumulated depreciation the following examples are highlighted here.

### Example-1:

A machine has been purchased in 1999 at a cost of Tk.3,00,000. The machine is depreciated @8% per annum on reducing balance method. Compute-

- i. What would be the depreciated value of the machine at the end of 2005?
- ii. What amount should be charged as depreciation of the machine for 2006?
- iii. Would it be profitable to sale the machine for Tk.1,20,000 at the end of 2007?
- iv. When the depreciated value of the machine will be Tk.1,02,550?

### Solution:

We are given,  $P = 3,00,000$ ,  $i = 0.08$

- i. The machine has been purchased in 1999. At the end of 2005, it will be 7 years' old. Hence, the depreciated value of the machine at the end of 2005 would be,

$$= P (1-i)^n$$

$$= 3,00,000 (1 - 0.08)^7 = (3,00,000 \times 0.5578) = \text{Tk.}1,67,340.$$

- ii. The depreciated value of the machine at the end of 2006, would be,

$$\begin{aligned} &= P (1-i)^n \\ &= 3,00,000 (1 - 0.08)^8 \\ &= (3,00,000 \times 0.5132) = \text{Tk.}1,53,960 \end{aligned}$$

Therefore depreciation for 2006 would be:

$$(1,67,340 - 1,53,960) = \text{Tk.}1,41,630$$

- iii. The depreciated value of the machine at the end of 2007 would be,

$$\begin{aligned} &= P (1-i)^n \\ &= 3,00,000 (1 - 0.08)^9 \\ &= 3,00,000 \times 0.4721 = \text{Tk.}1,41,630 \end{aligned}$$

Hence, it would not be profitable to sale the machine for Tk.1,20,000 at the end of 2007.

- iv. Let after  $n$  years the depreciated value of the machine would be Tk.1,02,550.

$$\text{Now } 3,00,000 (1 - 0.08)^n = 1,02,550$$

$$\text{or, } (0.92)^n = 1,02,000/3,00,000 = 0.3418$$

Taking logarithm both sides we have

$$\text{or, } n \log 0.92 = \log 0.3418$$

$$\text{So, } n = \frac{\log 0.3418}{\log 0.92} = \frac{-0.4662}{-0.0362} = 12.88 \text{ years.}$$

Therefore, after 12.88 years the depreciated value of the machine would be Tk.1,02,550.

### Different Methods for Calculation of Depreciation

*The depreciation calculations have a significant impact on cash flows after taxes (CFAT).*

The depreciation calculations have a significant impact on cash flows after taxes (CFAT). It is because a firm can legitimately deduct depreciation from its gross income to arrive at its before tax income. Different methods of depreciation affect tax liability, and hence the cash flows differently. Generally, there are three methods of depreciation calculations which are discussed as under.

- (1) **Straight Line Method:** Under this method, depreciation charges are allocated equally over the asset's economic life. The amount of annual depreciation charge is given by the formula:

$$\text{Amount of Annual Depreciation} = (\text{Original cost} - \text{Salvage Value}) \div \text{Economic life of an asset.}$$

- (2) **Sum-of-the-year's-Digits Method:** In this method, the depreciation base in each year is the same as in the straight-line method - *original cost less salvage value*. However, depreciation factor changes in each year. The depreciation factor for any year is the number of useful years remaining in the life of the project taken from the beginning of the year divided by the sum of a

series of numbers representing the years of service life. The depreciation factor of multiplier is calculated by the following formula:

$$\text{Multiplier} = \frac{n(n+1)}{2} \text{ where } n = \text{Economic life of asset.}$$

- (3) **Double Declining Balance Method:** It is more popularly known as the twice straight line depreciation method. Under this method, the amount of depreciation to be charged is twice the straight line rate. For example, a machine which has been purchased for Tk.2,20,000 has a salvage value of Tk.20,000 and economic life of 5 years. The straight line depreciation would be Tk.40,000 per year and, therefore, annual depreciation will be  $\left(\frac{40,000}{220000 - 20000}\right) = 20\%$  of the depreciable value. Then the rate would be 40% under the double declining balance method. This 40% depreciation rate would be applied to the book value of the asset each year until book value equals salvage value (Tk.20,000).

$$\text{Here, annual depreciation} = \left[ \frac{\text{Book value}}{\text{Economic life}} \times 2 \right].$$

Let us take an example to illustrate the depreciation calculations under different methods of depreciation.

**Example-2:**

A firm purchased a machine for Tk.4,00,000. Its useful life was 5 years and salvage value of Tk.10,000. Calculate the amount of depreciation under different methods of depreciation.

**(1) Straight Line Depreciation Method**

Depreciation Annual =  $(4,00,000 - 10,000) / 5 = 3,90,000 / 5 = 78,000$  (From 1<sup>st</sup> year to 5<sup>th</sup> Year)

**(2) Sum-of-the-Years Digit Depreciation Method.**

$$\text{Depreciation factor/Multiplier (S)} = \frac{n(n+1)}{2} = \frac{5(5+1)}{2} = 5 \times 3 = 15$$

<u>Year</u>	<u>Multiplier</u>	<u>Depreciation (in Tk.)</u>
1	5/15	$(\frac{5}{15} \times 3,90,000) = 1,30,000$
2	4/15	$(\frac{4}{15} \times 3,90,000) = 1,04,000$
3	3/15	$(\frac{3}{15} \times 3,90,000) = 78,000$
4	2/15	$(\frac{2}{15} \times 3,90,000) = 52,000$
5	1/15	$(\frac{1}{15} \times 3,90,000) = 26,000$

**(3) Double Declining (reducing) Balance Depreciation Method.**

<u>Year</u>	<u>Total Investment</u>	<u>Depreciation (in Tk.)</u>	<u>Year-end Book value</u>
1	3,90,000	$(3,90,000 / 5) \times 2 = 1,56,000$	2,34,000
2	2,34,000	$(2,34,000 / 5) \times 2 = 93,600$	1,40,400
3	1,40,400	$(1,40,400 / 5) \times 2 = 56,160$	84,240

<u>Year</u>	<u>Total Investment</u>	<u>Depreciation (in Tk.)</u>	<u>Year-end Book value</u>
4	84,240	$(84,240/5) \times 2 = 33,696$	50,544
5	50,544	$= 50544$	00

**Example-3:**

A machine depreciates @ 10% of its value at the beginning of the year. The machine was purchased for Tk.5,810 and the scrap value realized when sold was Tk.2,250. Find out the number of years during which the machine was in use.

**Solution:**

We know,  $A = P (1-i)^n$

Here  $A = 2,250$ ,  $P = 5,810$ ,  $i = 0.10$ ,  $n = ?$

Substituting the given value we have

$$2,250 = 5,810 (1-i)^n$$

$$\text{or, } (1 - 0.10)^n = \frac{2250}{5810}$$

$$\text{or, } (0.90)^n = 0.38726$$

Taking logarithm both sides we get

$$n \log 0.90 = \log 0.38726$$

$$\text{or, } n(-0.04576) = -0.412$$

$$\text{or } n = \frac{-0.412}{-0.04576} = 9$$

**Example-4:**

The life of A machine is estimated to be 10 years and the machine costs Tk.10,000. Calculate the scarp value at the end of its life; depreciation on the reducing balance method being charged @ 10% per annum.

**Solution:**

We know that,  $A = P (1-i)^n$

Here  $P = 10,000$ ,  $n = 10$ ,  $i = 0.1$

Putting the values we have,

$$\text{So, } A = 10,000 (1 - i)^{10}$$

$$\text{or, } A = 10,000 (0.90)^{10} \text{ [By using calculator, here, } A = 3486.78]$$

$$\text{or, } \log A = \log 10,000 + 10 \log 0.90$$

$$\text{or, } \log A = 4 + 10(-0.0458)$$

$$\text{or, } \log A = 4 - 0.458$$

$$\text{or, } \log A = 3.542$$

$$\text{or, } A = \text{antilog} 3.542 = 3483.37$$

Hence the scarp value is Tk.3,483.37

**Questions for review**

1. A machine has been purchased in 1995 at a cost of Tk.1,00,000. The machine is depreciated @12% p.a. on reducing balance method.
  - A. What would be depreciated value of the machine at the end of 2005?
  - B. What amount should be charged as depreciation of the machine for 2001?
  - C. Would it be profitable to sale the machine for Tk.5,00,000 at the end of 2002?
  - D. When the depreciated value of the machine will be Tk.4,20,550?
2. The value of a machine depreciates @ 10% p.a. If its present value is Tk.81000, what will be its worth after 2 years? What was the value of the machine 2 years ago?
3. A machine depreciates @ 12% p.a of its value at the beginning of a year. The machine was purchased for Tk.58,100 and the scrape value released when sold was Tk.10,000 Find out the number of years during which the machine was in use?

**Multiple choice questions (✓ the appropriate answers)**

1. The value of a machine depreciates @ 16% p.a. If the price of the new machine is Tk.62,000, its value after 5 years will be:
  - a) Tk.12400
  - b) Tk.25929
  - c) Tk.30868
2. The value of a machine depreciates @ 10% p.a. It was purchased 3years ago. If the present value is Tk.26,389.80, the purchase price of the machine was:
  - a) Tk.37500
  - b) Tk.35600
  - c) Tk.36200
- 3) A firm purchased a machine for Tk.3,00,000. Its useful life was 6 years with salvage value of Tk.30,000. The yearly amount of depreciation using straight line method was:
  - a) Tk.45, 000
  - b) Tk.44, 000
  - c) Tk.47, 000

### Lesson-3: Present Value and Future Value of Money

After studying this lesson, you should be able to:

- Explain the importance of present value;
- Future value as a tool of mathematics of finance;
- Apply formula for calculating the present value;
- Future value of the money.

#### Introduction

*The present value of a taka received after sometime will be less than a taka received today.*

Generally time value of money means that the value of a sum of money received today is more than its value received after some time. Conversely, the sum of money received in future is less valuable than it is today. In other words, the present value of a taka received after sometime will be less than a taka received today.

The time value of money can also be referred to as time preference for money. The main reasons for time preference for money are to be found in the re-investment opportunities for funds which are received early. The funds so invested will earn a rate of return which will not be possible in case they are received later. The time preference for money is, therefore, expressed generally in terms of a rate of return or more popularly as a discount rate.

#### Nature of Present Value and Future Value

*The procedure finding present values is commonly called discounting and the future values is commonly called compounding.*

The concept of present value is the exact opposite of that of compound or future value. While in the later approach money invested now appreciates in value because compound interest is added, in the former approach (Present Value Approach) money is received at same future date from now and will be worth less because we have lost the corresponding interest during the period. In other words, the present value of money that will be received in the future will be less than the value of money in hand today. Thus, in contrast to the corresponding approach where we convert present value into future value, in present value approach future values are converted into present value. Given a positive rate of interest the present value of future money will always be lower. It is for this reason, therefore, that the procedure finding present values is commonly called discounting and the future values is commonly called compounding.

Let a certain amount will be received after  $n$  years; now, if the discounting rate is  $i$ , then the present value of money will be calculated by using the following formula:

$$\begin{aligned} \text{Present value (PV)} &= A / (1+i)^n \\ &= A (1+i)^{-n} \end{aligned}$$

Again if the different amounts of a series are received (paid) at the end of each year, then present value of money will be calculated by using the following formula:

$$PV = A_1 / (1+i)^1 + A_2 / (1+i)^2 + A_3 / (1+i)^3 + \dots + A_n / (1+i)^n$$



Where, PV = the sum of the individual present values of a separate cash flows and  $A_1, A_2, A_3, \dots, A_n$  refer to cash flows at the end of time periods 1, 2, 3, ...,  $n$  respectively.

But if the different amount of a series are received / (paid) at the beginning of each year, then the present value will be computed by using the following formula:

$$PV = A_1 + A_2/(1+i)^1 + A_3/(1+i)^2 + \dots + A_n/(1+i)^{n-1}$$

On the other hand, let  $A$  is a present amount in corresponding to compounding rate  $i$ , then the future value after  $n$ th years will be calculated by using the following formula:

$$\text{Future Value (FV)} = A (1+i)^n$$

If the different amount of a series are received / (paid) at the beginning of each year, the future value after  $n$ th year will be computed by using following formula:

$$FV = A_1 (1+i)^n + A_2 (1+i)^{n-1} + A_3 (1+i)^{n-2} + \dots + A_n (1+i)$$

Where  $A_1, A_2, A_3, \dots, A_n$  refer to cash inflows/(outflows) at the beginning of different periods 1, 2, 3, ...,  $n$  respectively.

But if the different amount of a series are received/(paid) at the end of each year, then future value after  $n$ th year will be calculated by using the following formula:

$$FV = A_1 (1+i)^{n-1} + A_2 (1+i)^{n-2} + A_3 (1+i)^{n-3} + \dots + A_n$$

The following section of this lesson contains some model application of concepts relating to the present value and future value of money.

### Example-1:

Mr. Khan has following three alternatives after investing Tk.2,00,000 at now:

- (a) Collecting Tk.3,00,000 after 3 years;
- (b) Collecting Tk.1,40,000, Tk.1,20,000, and Tk.80,000 at the end of each year next.
- (c) Collecting Tk.1,20,000, Tk.1,10,000 and Tk.80,000 at the beginning of each year next.

What alternative would be profitable for Mr. Khan if his expected rate of return is 15% p.a.?

### Solution:

In case of alternative (a)

$$A = 3,00,000; n = 3; i = 0.15$$

$$\begin{aligned} \text{So, Present Value (PV)} &= A/(1+i)^n \\ &= 3,00,000/(1+0.15)^3 \\ &= 3,00,000/(1.15)^3 \\ &= 3,00,000/1.5209 \\ &= \text{Tk.1,97,251.63.} \end{aligned}$$

In case of alternative (b)

$$A_1 = 1,40,000; A_2 = 1,20,000; A_3 = 80,000; i = 0.15$$

$$\begin{aligned}\text{So, PV} &= A_1/(1+i)^1 + A_2/(1+i)^2 + A_3/(1+i)^3 \\ &= 1,40,000/(1+0.15)^1 + 1,20,000/(1+0.15)^2 + 80,000/(1+0.15)^3 \\ &= 1,40,000/1.15 + 1,20,000/1.3225 + 80,000/1.5209 \\ &= 1,21,739.13 + 90,737.24 + 52600.43 \\ &= \text{Tk.}2,65,076.80.\end{aligned}$$

In case of alternative (c)

$$A_1 = 1,20,000; A_2 = 1,10,000; A_3 = 80,000$$

$$\begin{aligned}\text{So, PV} &= A_1 + A_2/(1+i)^1 + A_3/(1+i)^2 \\ &= 1,20,000 + 1,10,000/(1+0.15)^1 + 80,000/(1+0.15)^2 \\ &= 1,20,000 + 1,10,000/1.15 + 80,000/1.3225 \\ &= 1,20,000 + 95,652.17 + 60,491.49 \\ &= \text{Tk.}2,76,143.66.\end{aligned}$$

Hence, Mr. Khan should select the alternative (c) due to having higher present value of expected future cash inflows.

### Example-2:

Mr. Rafiq wants to invest Tk.1,00,000, Tk.1,60,000, Tk.2,00,000 and Tk.2,40,000 at the beginning of the following 4 years respectively. If the expected rate of return is 12%, would be it possible to get Tk.10,00,000 after 4 years?

### Solution:

We know that,

$$\text{Future Value} = A_1 (1+i)^n + A_2 (1+i)^{n-1} + A_3 (1+i)^{n-2} + A_4 (1+i)^{n-3}$$

$$\text{Here } A_1 = 1,00,000; A_2 = 1,60,000; A_3 = 2,00,000; A_4 = 2,40,000; n = 4; i = 0.12$$

Substituting the given values we have

$$\begin{aligned}\text{FV} &= 1,00,000 (1 + 0.12)^4 + 1,60,000 (1 + 0.12)^{4-1} + 2,00,000 (1 + 0.12)^{4-2} \\ &\quad + 2,40,000 (1+0.12)^{4-3} \\ &= 1,00,000(1.12)^4 + 1,60,000(1.12)^3 + 2,00,000(1.12)^2 + 2,40,000(1.12)^1 \\ &= 1,00,000(1.5735) + 1,60,000(1.4049) + 2,00,000(1.25444) + 2,40,000(1.12) \\ &= 1,57,350 + 2,24,784 + 2,50,880 + 2,68,800 \\ &= \text{Tk.}9,01,814.\end{aligned}$$

Hence it will not be possible for Mr. Rafiq to get Tk.10,00,000 after 4 years from his planned investment.

**Questions for Review:**

These questions are designed to help you assess how far you have understood and apply the learning you have accomplished by answering (in written form) the following questions:

1. What do you mean by present value and future value of the money? Determine the present value of Tk.10,000 to be received 5 years from now, assuming 10% annual interest.
2. Find the present value of Tk.2,500 payable 4 years from now at 8% discounting (i) Quarterly (ii) Monthly.
3. Calculate the present values of the following alternatives:

Alternative	Rate	Annual flow starting after one year	Number of Years
(a)	10%	Tk.10,000	8
(b)	8%	Tk.8,000	10
(c)	14%	Tk.15,000	6

4. Mr. Nuruddin wants to invest Tk.20,000, Tk.30,000, Tk.40,000, Tk.50,000 and Tk.60,000 at the end of the following 5 years respectively. If the expected rate of return is 10%, would it be possible to get Tk.4,00,000 after 5 years?
5. Mr. Jewel has following two alternatives after investing Tk.1,50,000 at now:
  - (a) Collecting Tk.3,20,000 after 5 years.
  - (b) Collecting Tk.50,000, Tk.60,000, Tk.70,000, Tk.80,000 and Tk.85,000 at the end of each year next.

What alternative would be profitable for Mr. Nuruddin if his expected rate of return is 12% p.a.?

6. Mr. Karim owes to Mr. Rahim Tk10, 000 due in 4 years and Tk.6,000 due in seven years. Mr. Rahim agrees to pay Tk.3,000 at now, how much should have to be paid 5 years from now to settle his entire debt, assuring that money is worth 13% compounded semi-annually.

**Multiple choice questions (✓ the appropriate answers)**

1. Tk.5000 due 5 years from now. If interest is at 4% compounding semi-annually, the amount of present value is:
  - a) Tk.4202
  - b) Tk.4102
  - c) Tk.4505
2. An individual expects to receive Tk.700 after 8 years. What is the present value of the expected receipt assuming an annual compound interest rate of 8%?
  - a) Tk.378
  - b) Tk.380
  - c) Tk.490
3. How much amount should be deposited now at 5% annual compounding, if the account grows Tk.500 in 10 years?
  - a) Tk.301
  - b) Tk.307
  - c) Tk.320

## Lesson-4: Annuity

After studying this lesson, you should be able to:

- Explain the different types of annuities,
- Calculate annuity by yourself.

### Nature of Annuity

*A series of uniform payments is called an annuity.*

A series of uniform payments is called an annuity. In other words, an annuity is a series of payments of a fixed amount at regular intervals generally. The interval is a year, but it may be six months, or a quarter or a month.

Annuities can be divided into two classes – (1) Annuity certain and (2) Annuity contingent. In annuity certain, the payments are to be made unconditionally, for a certain or fixed number of years. In annuity contingent, the payments are to be made till the happening of some contingent event such as the death of a person, the marriage of a girl, the education of a child reaching a specified age. Life annuity is an example of annuity contingent.

Annuity certain can be divided into (i) annuity due; and (ii) immediate annuity. When the payment of an annuity is at the beginning of each period, it is said to be an annuity due. When the payment is at the end of each period, the annuity is termed as immediate annuity.

### The Present Value of an Annuity

*The present value of an annuity is the sum of the present values of its installments.*

The present value of an annuity is the sum of the present values of its installments. In calculating the present value of an annuity it is always customary to reckon compound interest.

Let  $A$  be the annuity,  $V$  is the present value,  $i$  is the rate of interest per year and  $n$  the number of years to continue, and then the present value of an immediate annuity is calculated by the following formula:

$$V = \frac{A}{i} \left[ 1 - \frac{1}{(1+i)^n} \right]$$

On the other hand the present value of an annuity due is calculated by the following formula:

$$V = \frac{A}{i} (1+i) \left[ 1 - \frac{1}{(1+i)^n} \right]$$

$$\text{or, } V = A/i (1+i) [1 - 1/(1+i)^{n-1}]$$

Let us illustrate it by two examples.

**Example-1:**

An investment will yield Tk.10,000 per annum for 8 years. If finance can be obtained at 7% per annum and the investment costs Tk.50,000, is it worth undertaking?

**Solution:**

We know that the present value of the immediate annuity would be

$$V = A/i [1-1/(1+i)^n ]$$

Here  $A = \text{Tk.}10,000$

$$i = 0.07$$

$$n = 8$$

Substituting the given values we have

$$\begin{aligned} V &= 10,000/0.07 [1-1/(1+0.07)^8] \\ &= 10,000/0.07 [1-1(1.07)^8] \\ &= 1,42,857 [1-1/1.7182] \\ &= 1,42,857 [1- 0.5820] \\ &= 1,42,857 \times 0.4180 \\ &= \text{Tk.}59,714.23 \text{ (App)} \end{aligned}$$

Since the investment's actual cost is Tk.50,000 and the present value of the annuity is Tk.59,714.23; the investment should be made.

**Example-2:**

Mr. Karim can purchase a machine by paying Tk.40,000 in cash at now. He can also purchase the machine by 8 equals' yearly installments to be paid at the beginning of each year. If the interest rate is 12%, what should be amount of each installment?

**Solution**

Let  $A$  be the annual installment. Then Tk.40,000 is the present value of this annuity due. We are given,

$$V = 40,000, n = 8, i = 0.12 \text{ and } A = ?$$

Using the formula

$$V = \frac{A(1+i)}{i} \left[ 1 - \frac{1}{(1+i)^n} \right]$$

$$\text{Or, } 40,000 = \frac{A(1+0.12)}{0.12} \left[ 1 - \frac{1}{(1+0.12)^8} \right]$$

$$\text{Or, } 40,000 = A \left[ \frac{1.12}{0.12} \right] \left[ 1 - \frac{1}{2.4760} \right]$$

$$\text{Or, } 40,000 = A (9.3333) (1-0.4039)$$

$$\text{Or, } 40,000 = A (9.3333) (0.5961)$$

$$\text{Or, } 40,000 = A (5.5636)$$

$$\text{Or, } A = (40000/5.5636) = 7189.59$$

Hence the amount of each investment should be Tk.7189.59

### Amount of an Annuity

Let  $A$  be the annuity,  $i$  the rate of interest per year,  $n$  the total time period of an annuity and  $M$  the future amount of annuity after  $n$  years, then the total amount of an immediate annuity is calculated by the following formula:

$$M = \frac{A}{i} [(1+i)^n - 1]$$

On the other hand, the total amount of an annuity due is calculated by the following formula:

$$M = \frac{A(1+i)}{i} [(1+i)^n - 1]$$

*The difference between amount of installment and interest is termed as amortization.*

In the repayment of a loan, it is sometimes arranged that the repayment is to be made in equal periodical installments, including repayment of principal and interest. The difference between amount of installment and interest is termed as amortization.

The following section of this lesson contains some model applications of amount of an annuity.

#### Example-3:

A machine costs the company Tk.98,000 and its effective life is estimated to be 12 years. If the scrap realizes Tk.3,000 only, what amount should be retained out of profits at the end of each year to accumulate at compound interest at 5% per annum?

#### Solution:

Let  $A$  be the annual installment. Evidently the amount of the annuity  $A$  to continue for 12 years, i.e. the balance amount to be retained =  $(98,000 - 3,000) = 95,000$

We know that  $M = A/i [(1+i)^n - 1]$

Here,  $M = 95,000$ ;  $i = 0.05$ ;  $n = 12$  and  $A = ?$

Now putting the values we get,

$$95,000 = A/0.05 [(1+0.05)^{12} - 1]$$

$$\text{Or, } 95,000 = A/0.05 [(1.05)^{12} - 1]$$

$$\text{Or, } 95,000 = A/0.05 [1.7959 - 1]$$

$$\text{Or, } 95000 \cdot 0.05 = A (0.7959)$$

$$\text{Or, } A = \frac{4750}{0.7959} = 5968.09$$

So, Tk.5968.83 should be retained out of profits at the end of each year.

**Example-4:**

Mr. Zahad wants to purchase a machine after 10 years when it will cost Tk.6,00,000. From now, he wants to save money for the machine and plans to deposit money into bank in 10 equal installments, the first deposit is to be made immediately. Calculate the amount of each installment reckoning compound interest at 10% p.a.

**Solution:**

Here the deposit pattern is an annuity due. We are given,

$$M = 6,00,000; i = 0.10; n = 10 \text{ and } A = ?$$

Now using the formula

$$M = A/i (1+i) [(1+i)^n - 1]$$

$$\text{or, } 6,00,000 = A/0.10(1+0.10) [(1+0.10)^{10} - 1]$$

$$\text{or, } 6,00,000 = A/0.10(1.10) [(1.10)^{10} - 1]$$

$$\text{or, } 6,00,000 = A(11) (2.5937 - 1)$$

$$\text{or, } 6,00,000 = A(11) (1.5937)$$

$$\text{or, } 6,00,000 = 17.5307 A$$

$$\text{or, } A = \frac{600000}{17.5307} = 34225.67$$

Hence Mr. Zahad has to deposit Tk.34,225.67 in each installment at the beginning of each year.

### **Questions for Review**

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. Define the following with example:  
Annuity, Annuity certain, Annuity due, Contingent annuity, Immediate annuity and Amortization.
2. Mr. Karim buys a house worth Tk.3,50,000. The contract is that Karim will pay Tk.1,00,000 immediately and the balance in 15 annual equal installments with 10% per annum compound interest. How much he has to pay annually?
3. A man wishes to have Tk.1,50,000 available in a bank account when his daughter's first year college expenses begin. How much must he deposit now at 12% compounded annually if the girl is to start in college five years from now?
4. A man retires at the age of 55 years from active service and his employer gives him pension of Tk.1,500 a year paid in half yearly installments for the rest of his life. Assuming his expectation of remaining life to be 15 years and that interest is 12% p.a. payable half yearly? What single sum is equivalent to his pension?
5. Hena borrowed Tk.10,000 to buy a refrigerator. She will amortize the loan by monthly payment of Tk.R each over a period of 3 years. Find the monthly payment if interest is 12% compounding monthly. Also find the total amount Hena will pay.
6. A firm intends to invest Tk.2,50,000 at the end of each year and to receive interest on the amounts at 10% per annum. What sum of money will be available at the end of fifth year?
7. You borrowed Tk.22,000 at 12% to be repaid over the next 6 years. Equal installment payments are required at the end of each year and these payments must be significant in amount to repay the Tk.22,000 together with providing the lender at 12% return.

Based on these data prepare an amortization schedule.



**Multiple choice questions (✓ the appropriate answers)**

1. A house is offered for sale for Tk.25,000. The seller agrees to accept Tk.200 at the end of each month for 8 years provided the proper down payment is made. Assuming annual interest is @ 6% compounding monthly, what should be the down payment?  
a) Tk.9781                      b) Tk.9652                      c) Tk.10,000
2. How much should be the deposit each year @ 5% compounding annually to accumulate Tk.1000 in 10 years from now?  
a) Tk.79.50                      b) Tk.75                      c) Tk.85
3. A of Tk.1000 is to be paid in 5 equal annual installment loan interest at 6% p. a. compounding annually and the 1<sup>st</sup> payment is to be made after one years. What amount is to be paid in each installment?  
a) Tk.237                      b) Tk.245                      c) Tk.250
4. Monir buys a house worth Tk.2,50,000. The contract is that Monir will pay Tk.1,00,000 immediately and the balance in 15 annual equal installments with 10% p.a. compound interest. How much he has to pay annually?  
a) Tk.19710                      b) Tk.18000                      c) Tk.19550