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VECTORS IN PHYSICS

Chapter 3

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Units of Chapter 3

- Scalars Versus Vectors
- The components of a vector
- Adding and subtracting vectors
- Unit vectors
- Position, Displacement, Velocity, and Acceleration Vectors
- Relative Motion

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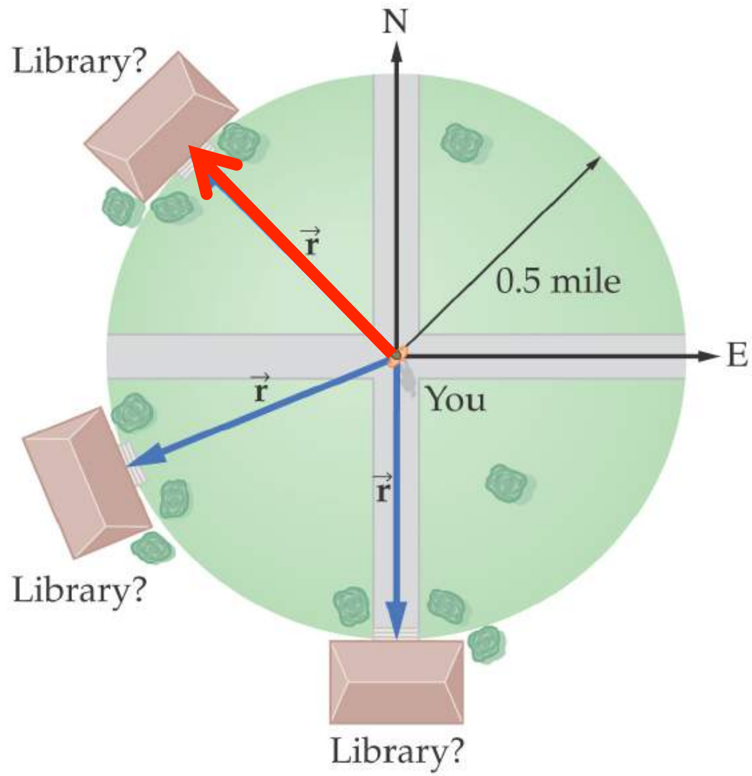
3-1 Scalars versus vectors

- **Scalar**: number with units.
- **Vector**: quantity with magnitude and direction.
- How to get to the library: need to know how far and which way



0.5 mi northwest

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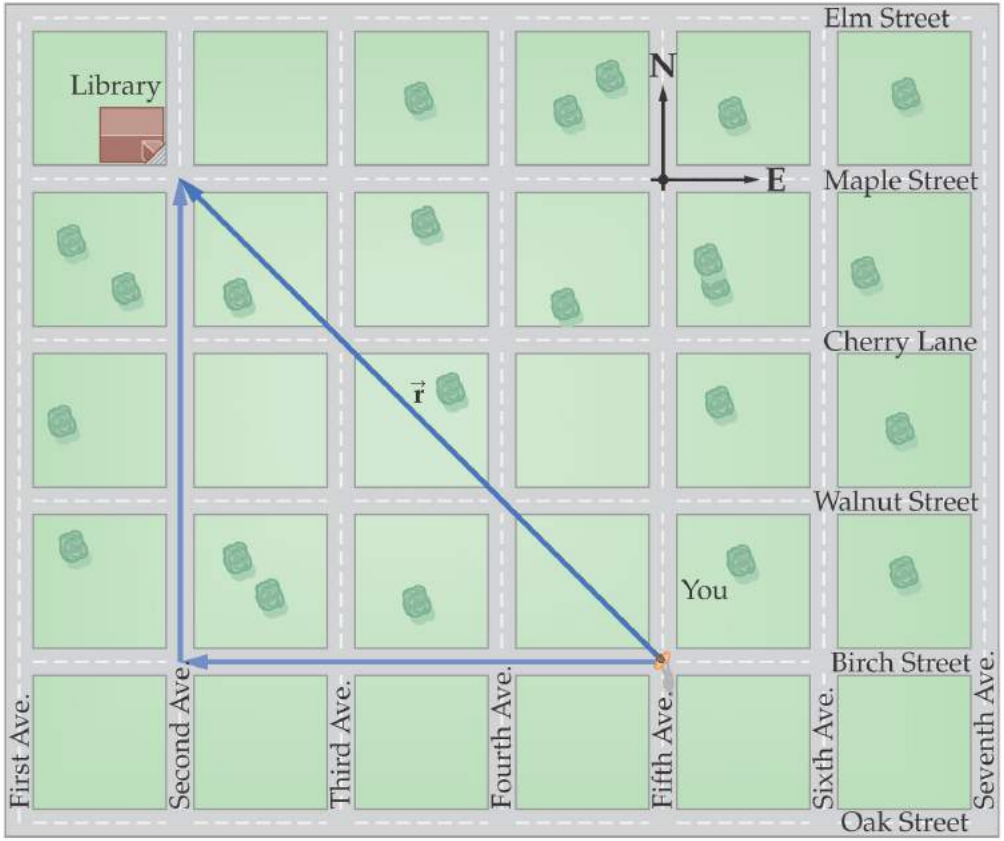
3-2 The Components of a Vector

- Even though you know how far and in which direction the library is, you may not be able to walk there in a straight line:



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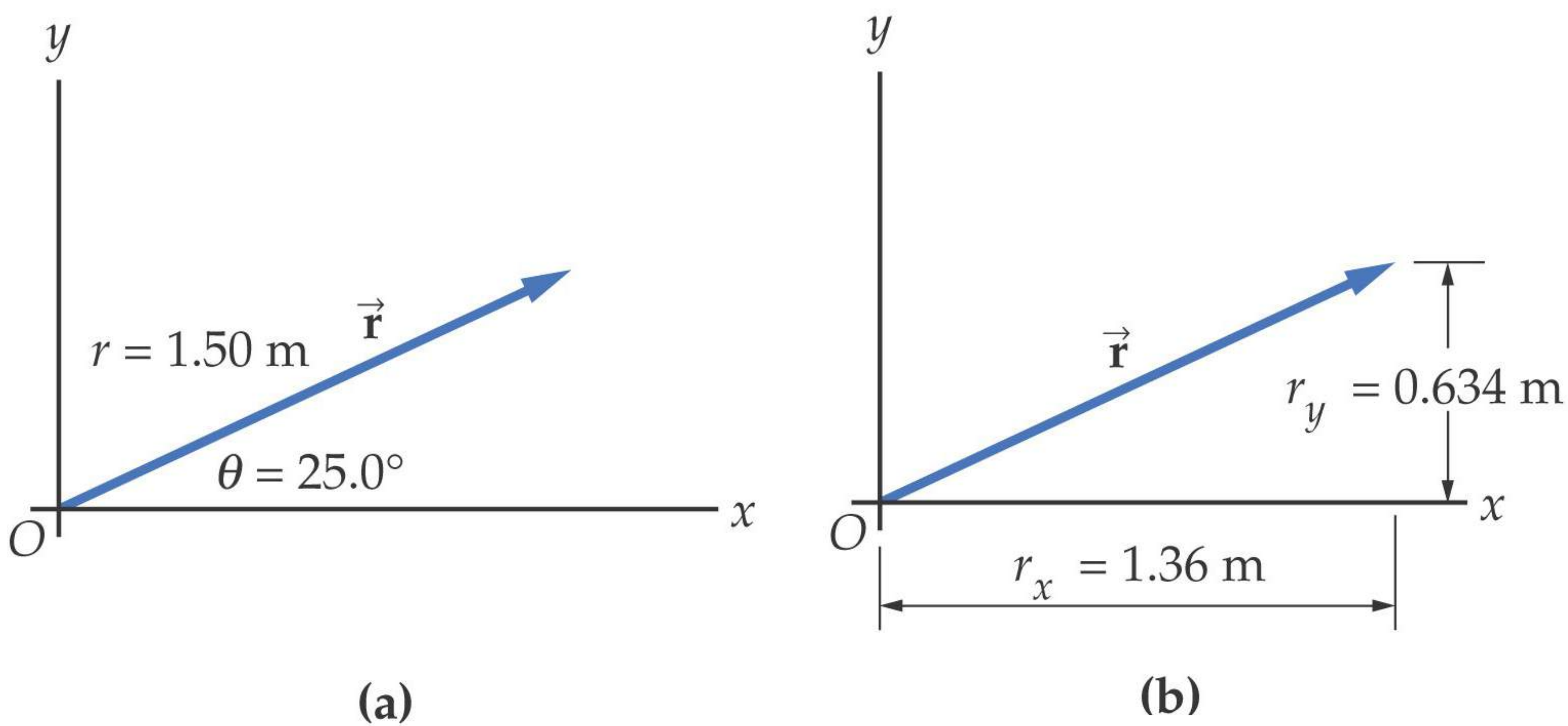
3-2 The Components of a Vector

- Can resolve vector into **perpendicular components** using a two-dimensional coordinate system:



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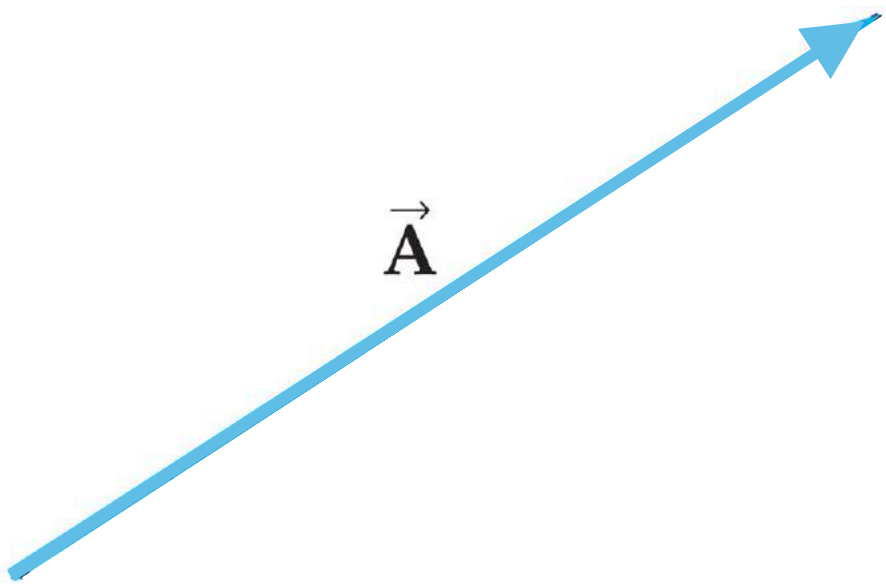
3-2 The Components of a Vector

- Length, angle, and components can be calculated from each other using trigonometry:



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Given the magnitude and direction of a vector, find its components:

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

Given the components of a vector, find its magnitude and direction:

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1}(A_y / A_x)$$

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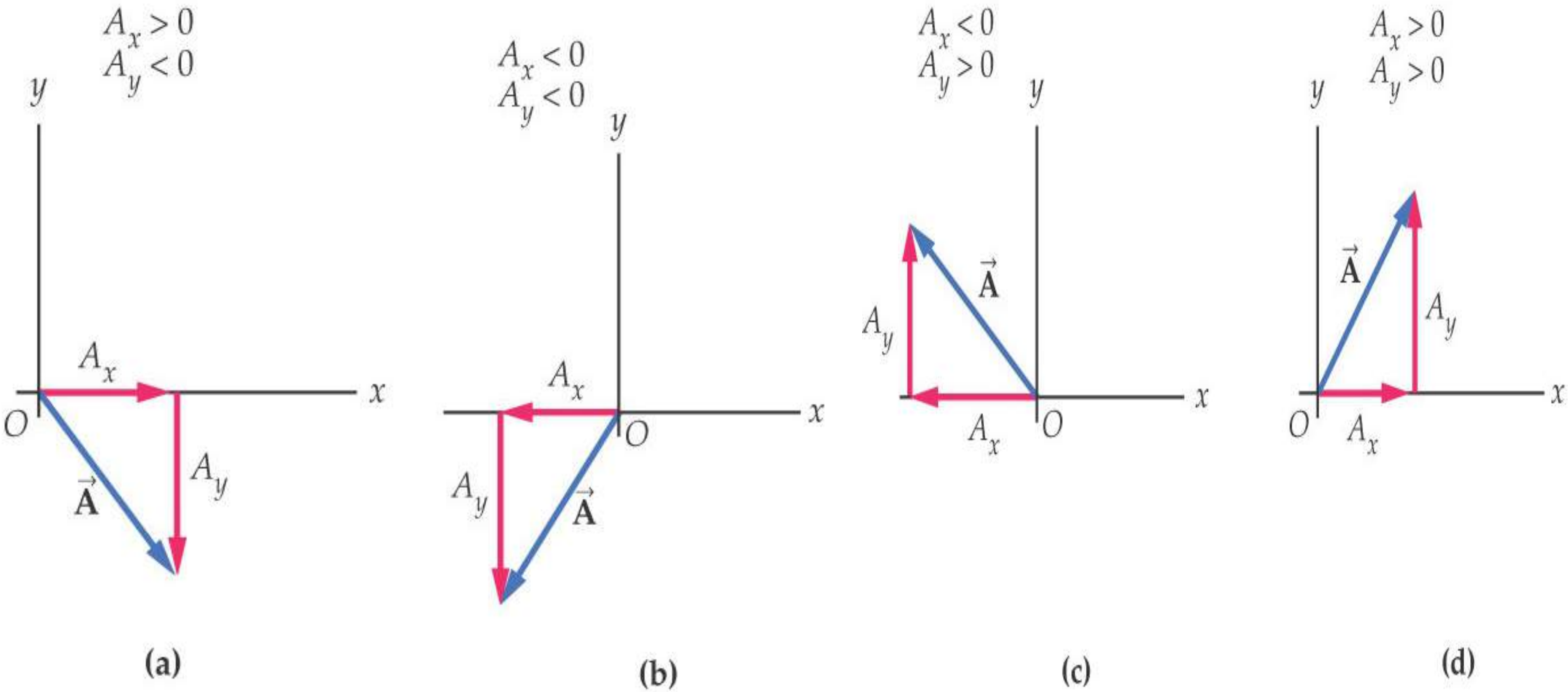
3-2 The Components of a Vector

- Signs of vector components:



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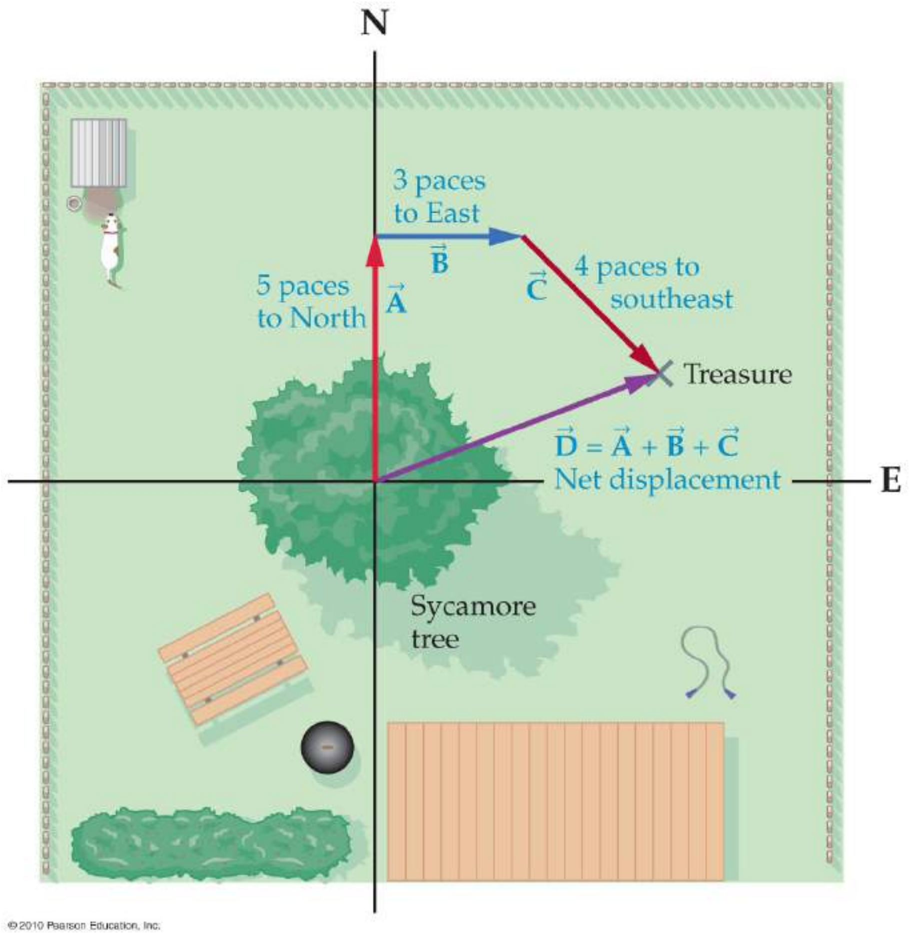
3-3 Adding and Subtracting Vectors

- Adding vectors graphically: Place the tail of the second at the head of the first. The sum points from the tail of the first to the head of the last.



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3-3 Adding and Subtracting Vectors

- Adding vectors using components:

1. Find the components of each vector to be added.

2. Add the *x*- and *y*-components separately.

3. Find the resultant vector.

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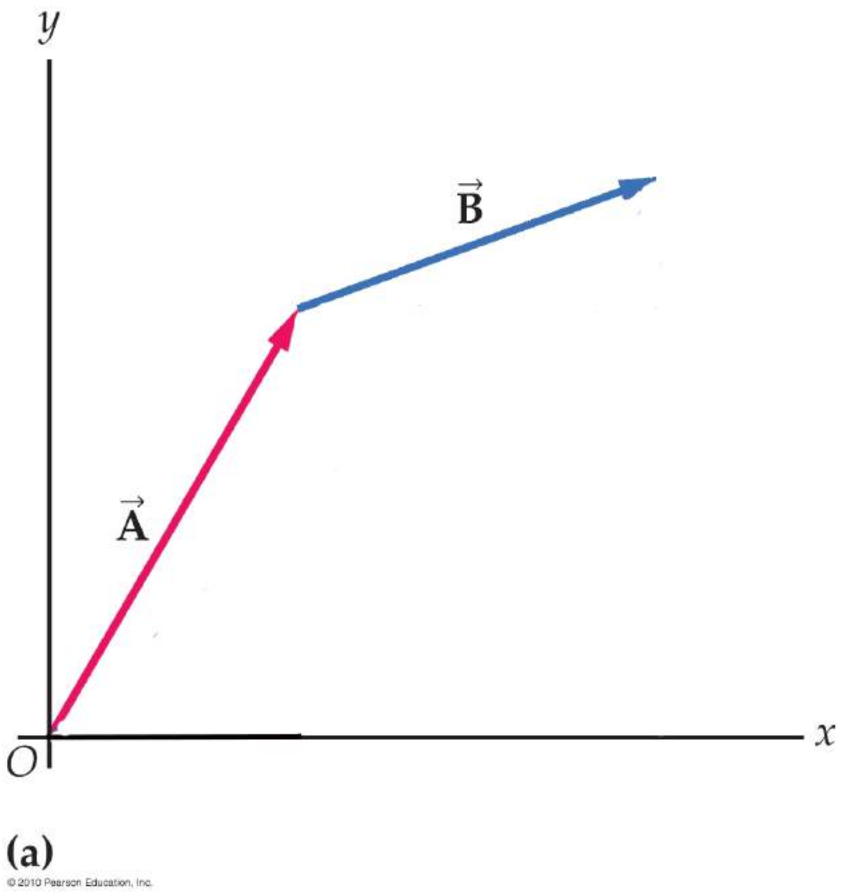
3-3 Adding and Subtracting Vectors

- Adding vectors using components:



(a)

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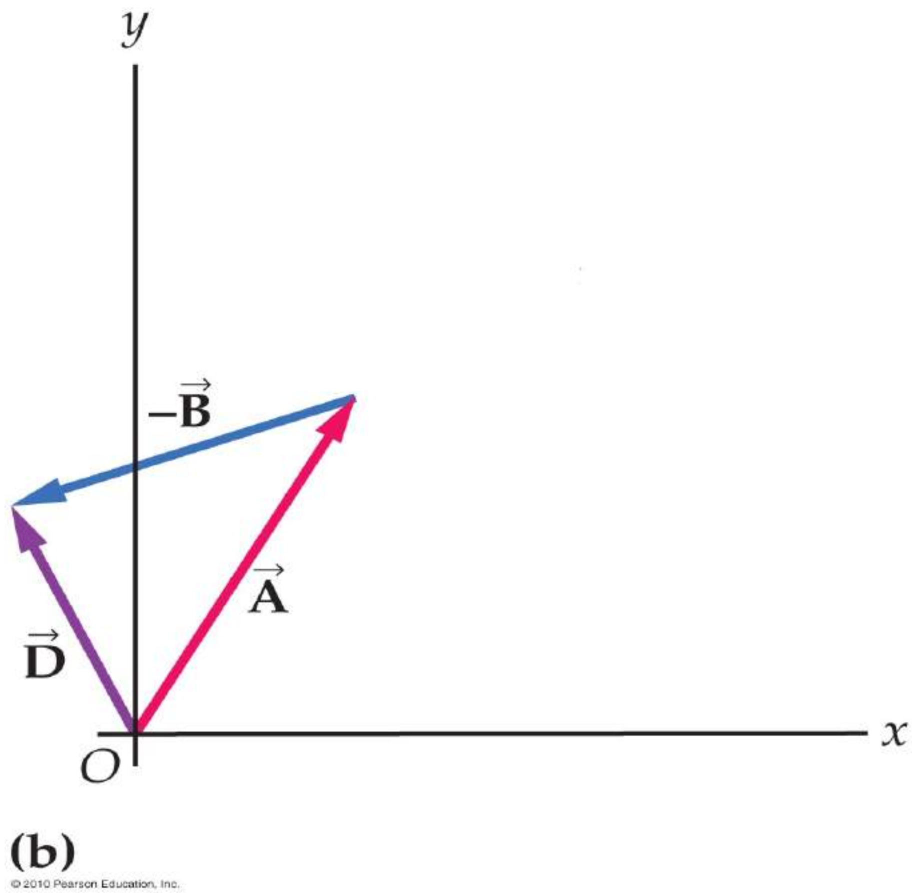
3-3 Adding and Subtracting Vectors

- Subtracting Vectors: The negative of a vector is a vector of the same magnitude pointing in the opposite direction.

Here,

$$\vec{D} = \vec{A} - \vec{B}$$



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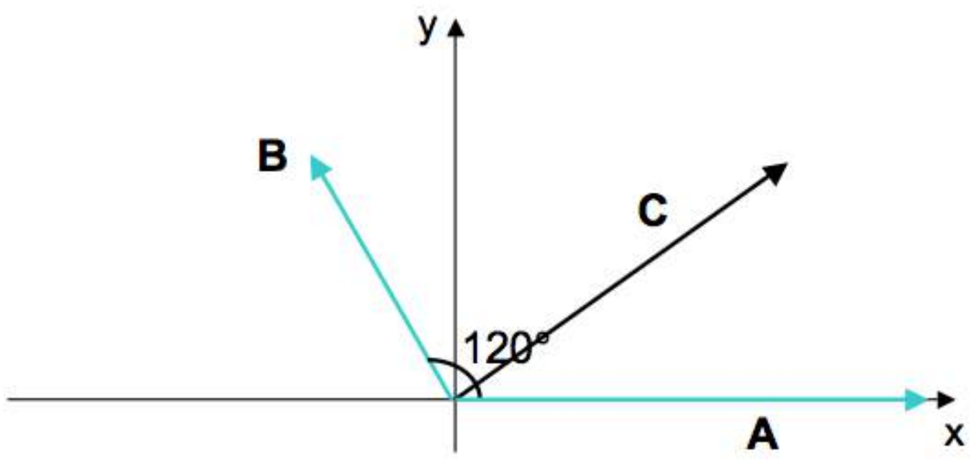
Example

Vector **A** has a length of 5.00 meters and points along the x-axis. Vector **B** has a length of 3.00 meters and points 120° from the +x-axis. Compute **A**+**B** (=**C**)



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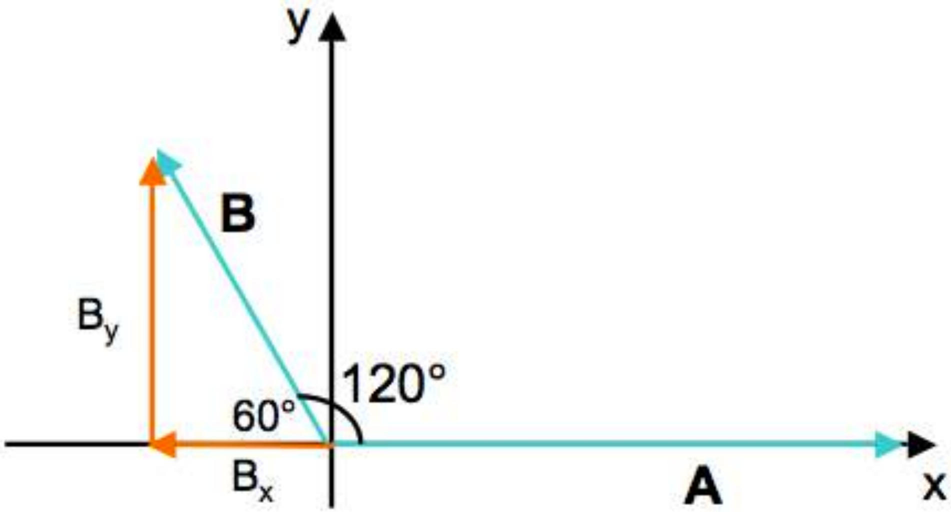
Example continued



$$\sin 60° = \frac{B_y}{B} \Rightarrow B_y = B\sin 60° = (3.00\text{m})\sin 60° = 2.60\text{ m}$$

$$\cos 60° = \frac{-B_x}{B} \Rightarrow B_x = -B\cos 60° = -(3.00\text{m})\cos 60° = -1.50\text{ m}$$

and A_x = 5.00 m and A_y = 0.00 m

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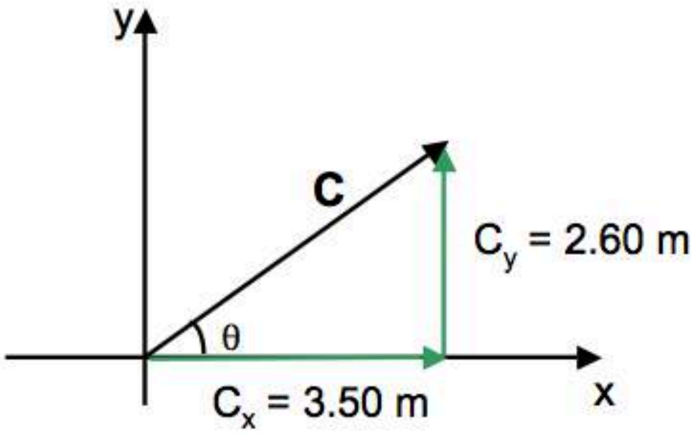
Example continued

The components of **C**:

$$C_x = A_x + B_x = 5.00\,\text{m} + (-1.50\,\text{m}) = 3.50\,\text{m}$$

$$C_y = A_y + B_y = 0.00\,\text{m} + 2.60\,\text{m} = 2.60\,\text{m}$$



The length of C is:

$$C = |\mathbf{C}| = \sqrt{C_x^2 + C_y^2}$$

$$= \sqrt{(3.50\,\text{m})^2 + (2.60\,\text{m})^2}$$

$$= 4.36\,\text{m}$$

The direction of C is: $\tan\theta = \dfrac{C_y}{C_x} = \dfrac{2.60\,\text{m}}{3.50\,\text{m}} = 0.7429$

$\theta = \tan^{-1}(0.7429) = 36.6°$ From the +x-axis

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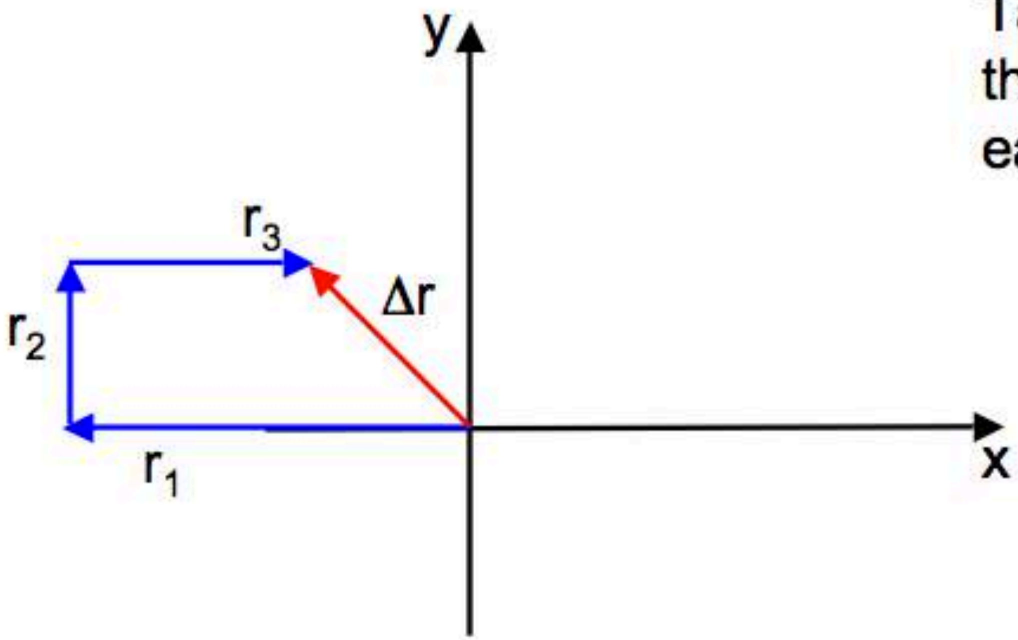
Example

- Margaret walks to the store using the following path: 0.500 miles west, 0.200 miles north, 0.300 miles east. What is her total displacement? Give the magnitude and direction.



Take north to be in the +y direction and east to be along +x.

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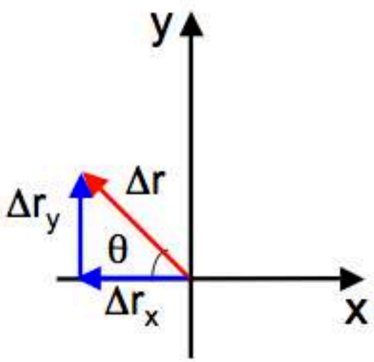
Example continued

The displacement is $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$. The initial position is the origin; what is $\mathbf{r}_f$?

The final position will be $\mathbf{r}_f = \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3$. The components are $r_{fx} = -r_1 + r_3 = -0.2$ miles and $r_{fy} = +r_2 = +0.2$ miles.



Using the figure, the magnitude and direction of the displacement are

$$|\Delta\mathbf{r}| = \sqrt{\Delta r_x^2 + \Delta r_y^2} = 0.283 \text{ miles}$$

$$\tan\theta = \frac{|\Delta r_y|}{|\Delta r_x|} = 1 \text{ and } \theta = 45^\circ \qquad \text{N of W.}$$

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Example

During Sunday's superbowl, the middle linebacker for the Seattle Sea Hawks made the following movements after the ball was snapped on third down. First, he back-pedaled in the southern direction for 2.6 meters. He then shuffled to his left (west) for a distance of 2.2 meters. Finally, he made a half-turn and ran downfield a distance of 4.8 meters in a direction of 240° counter-clockwise from east (30° W of S) before finally knocking the wind out of New England Patriots' wide receiver. Determine the magnitude and direction of his overall displacement.

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Example

- Vector **A** is 5.5 cm long and points along the East. Vector **B** is 7.5 cm long and points at +30° North of West. Determine the sum of these two vectors in terms of magnitude and direction.

- A) 2.0 cm at 15° above the x-axis
- B) 3.9 cm at 75° above the x-axis
- C) 7.8 cm at 33° above the x-axis
- D) 13 cm at 17° above the x-axis
- E) 7.5 cm at 30°above the x-axis

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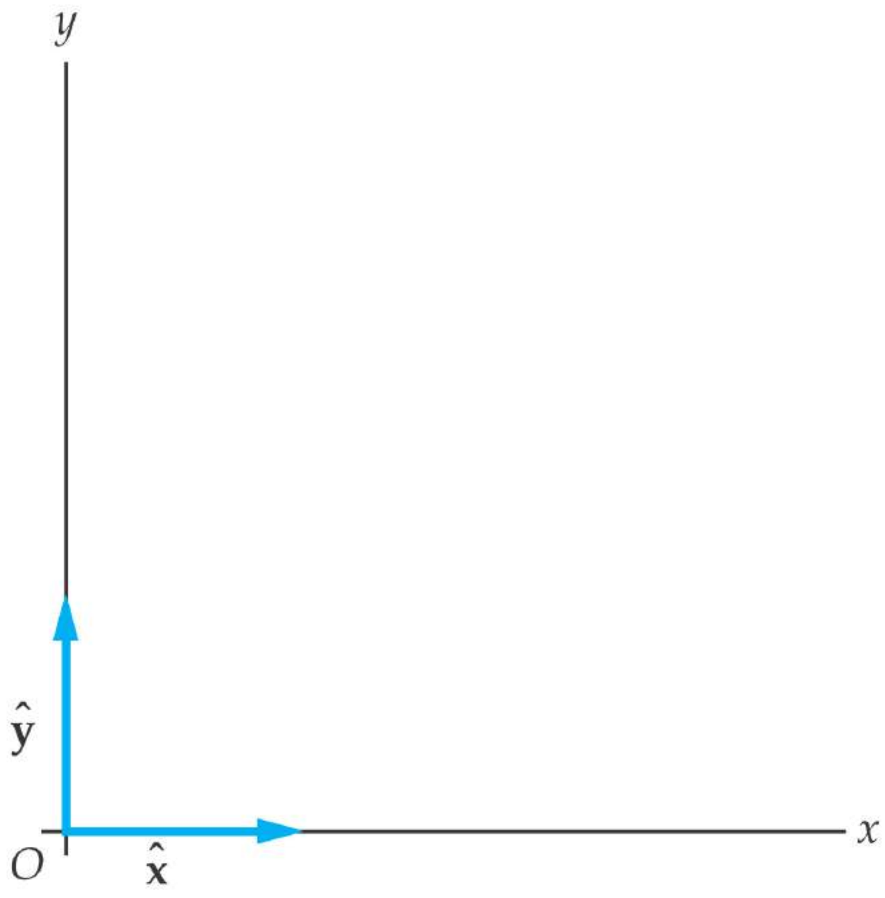
3-4 Unit Vectors

- Unit vectors are dimensionless vectors of unit length.



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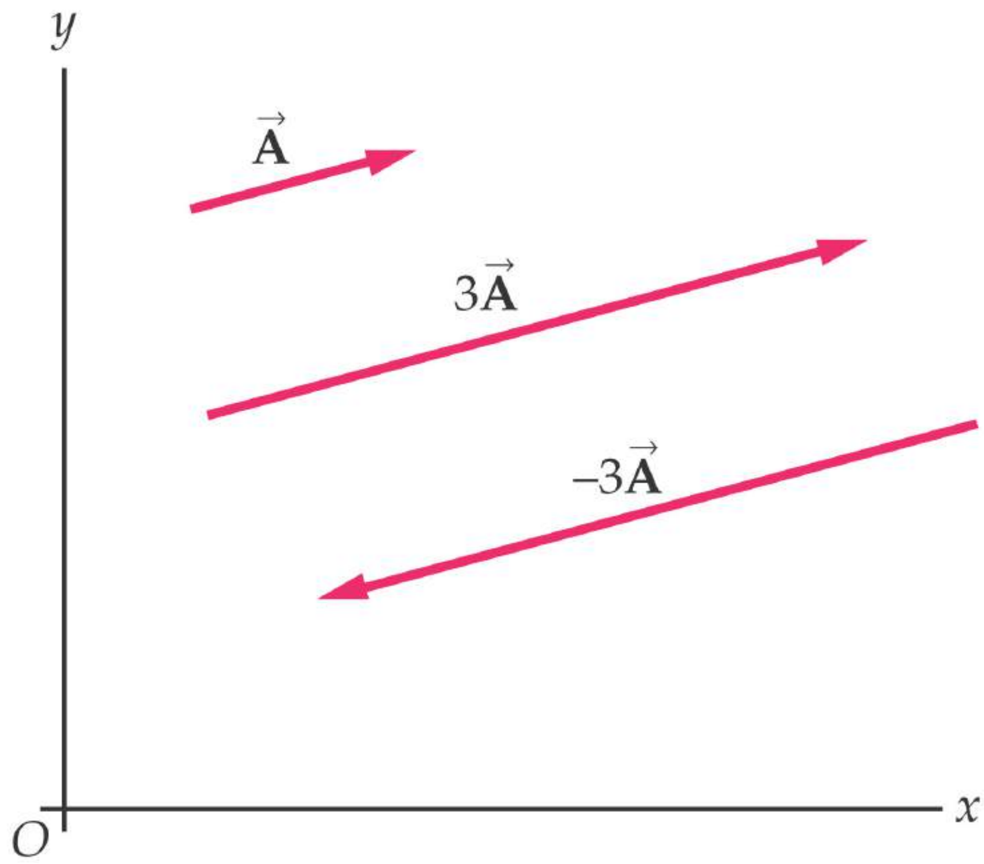
3-4 Unit Vectors

- Multiplying unit vectors by scalars: the multiplier changes the length, and the sign indicates the direction.



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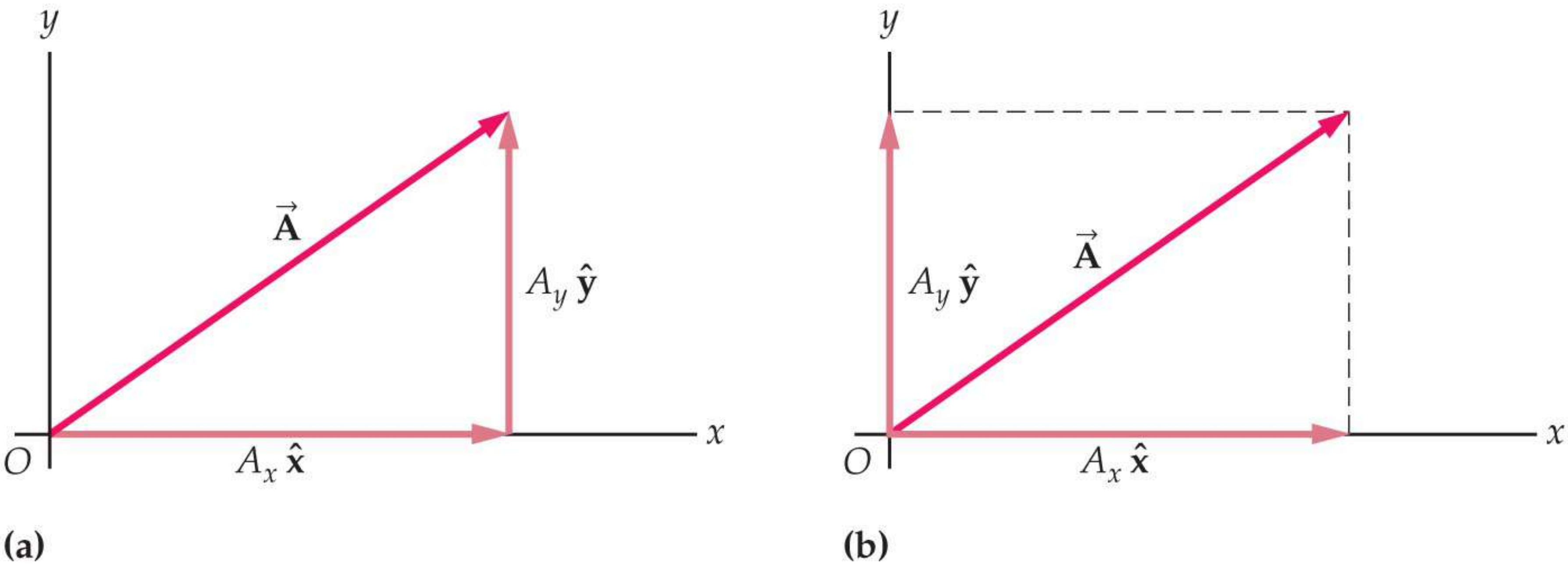
3-4 Unit Vectors

- Unit vectors provide a useful way to keep track of the x and y components of a vector:



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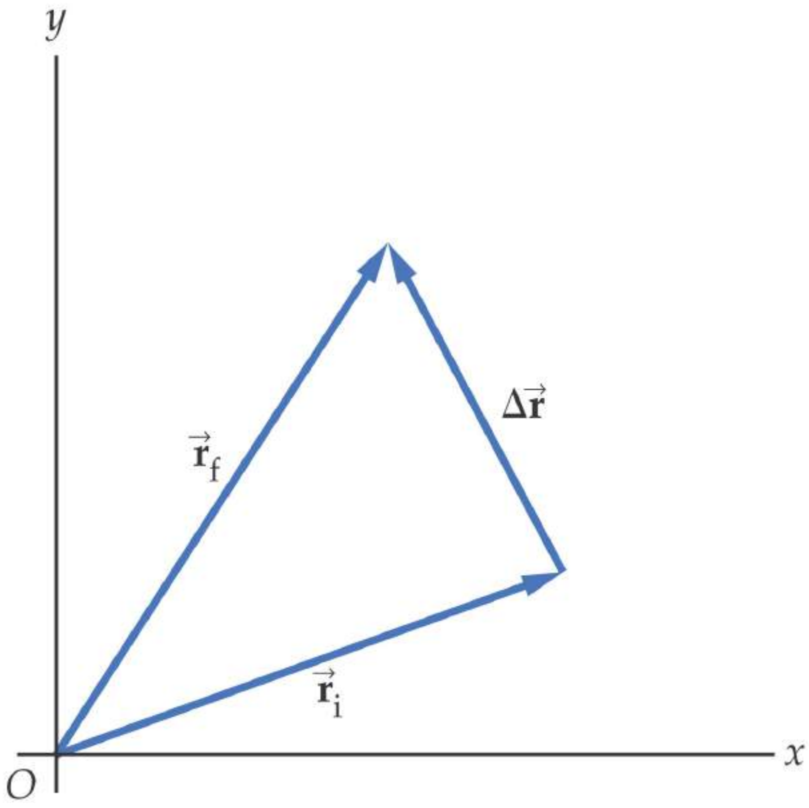
3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Position vector $\vec{r}_f$ points from the origin to the location in question.



The displacement vector $\Delta\vec{r}$ points from the original position to the final position

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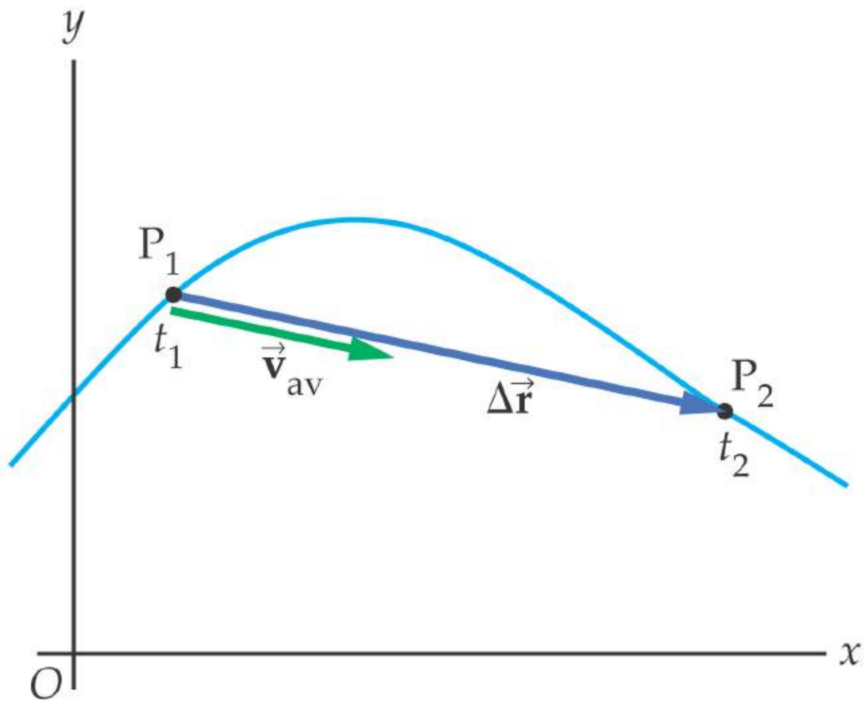
3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Average velocity vector:

$$\vec{\mathbf{v}}_{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

So $\vec{v}_{av}$ is in the same direction as $\Delta\vec{r}$.



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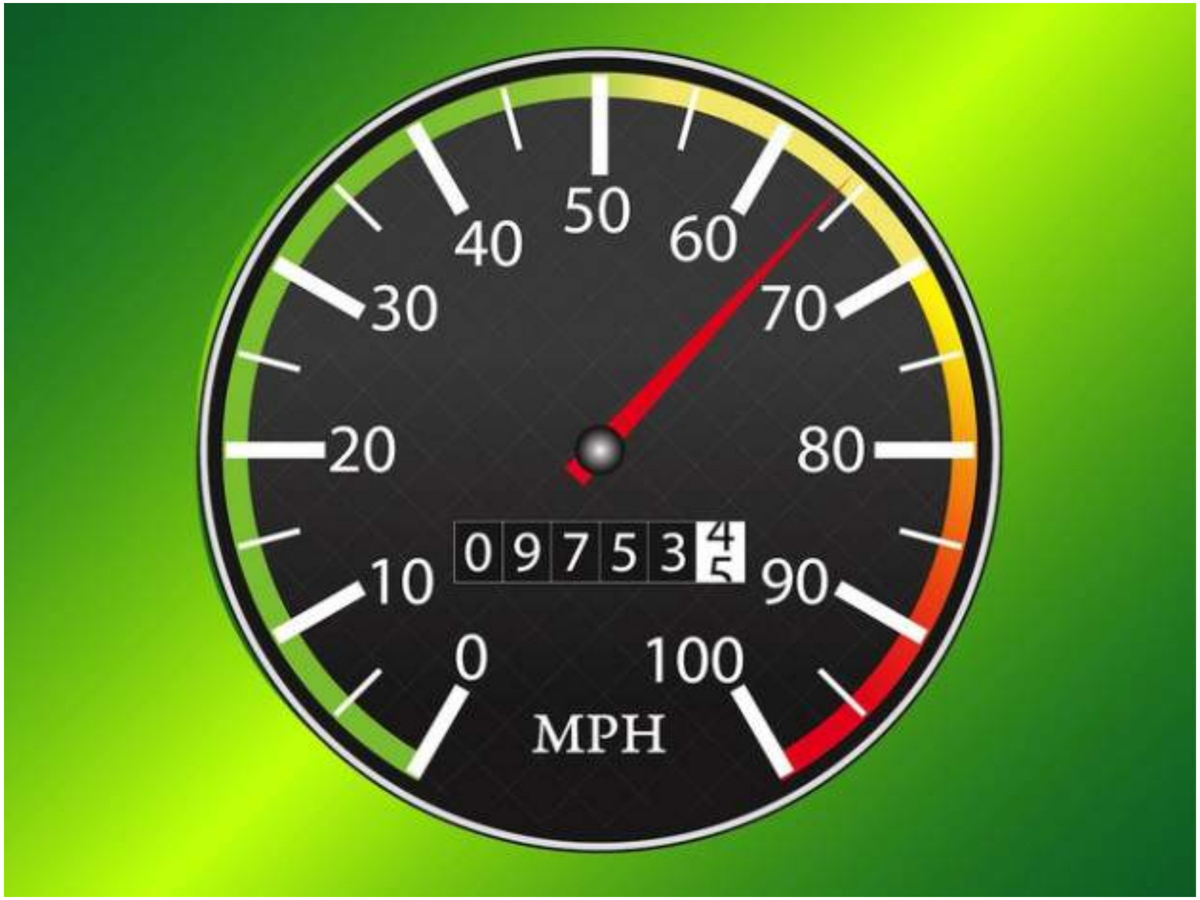
3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Instantaneous velocity vector is tangent to the path:



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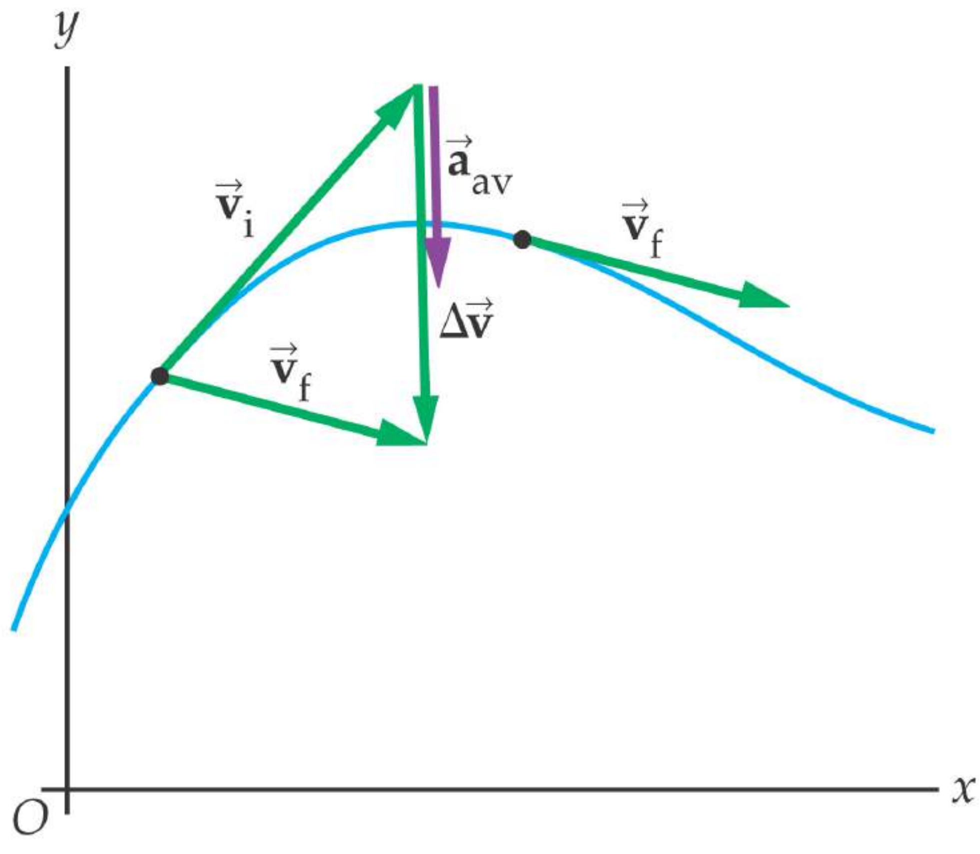
3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Average acceleration vector is in the direction of the change in velocity:

$$\vec{\mathbf{a}}_{av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$



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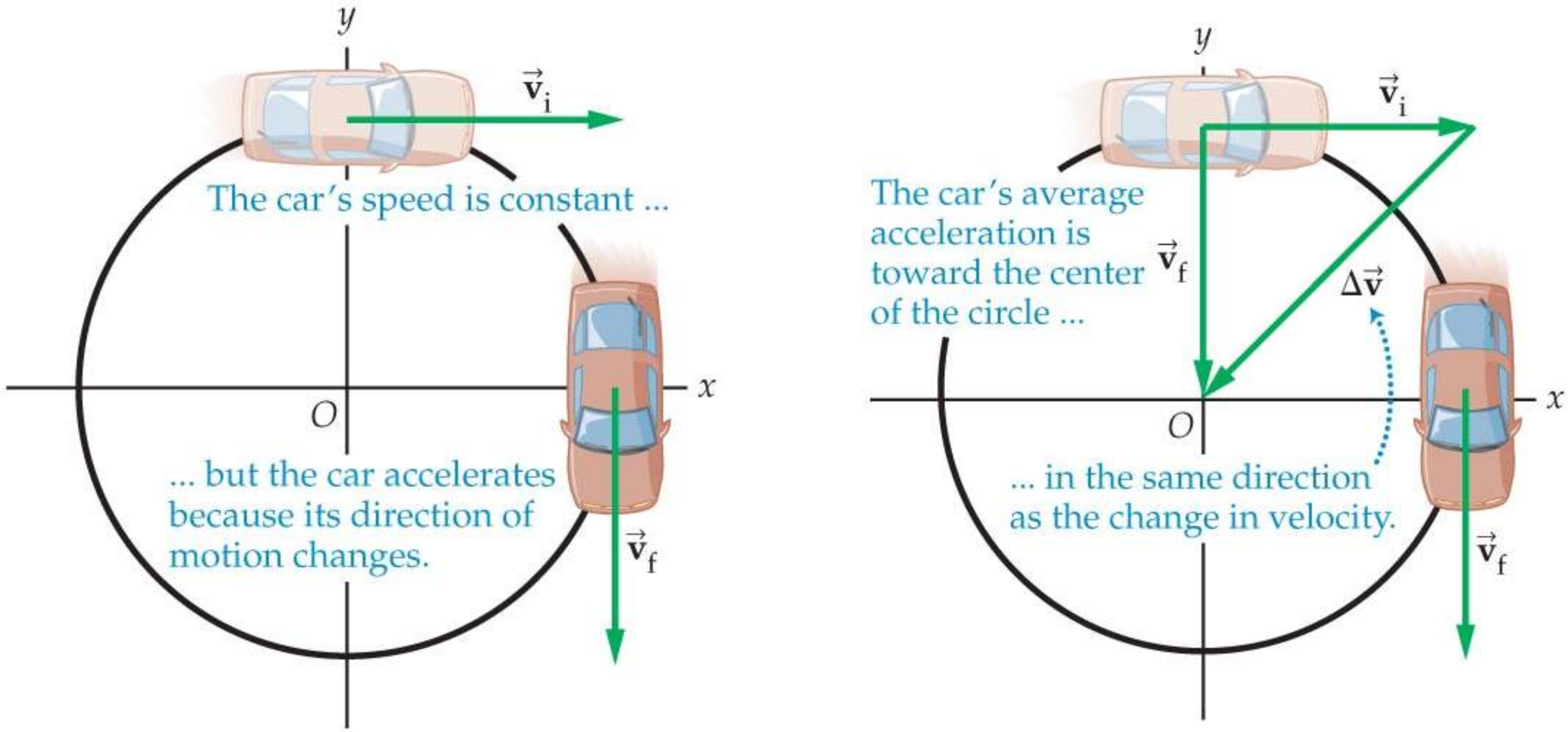
3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Average acceleration is non-zero for constant speed but change of direction:



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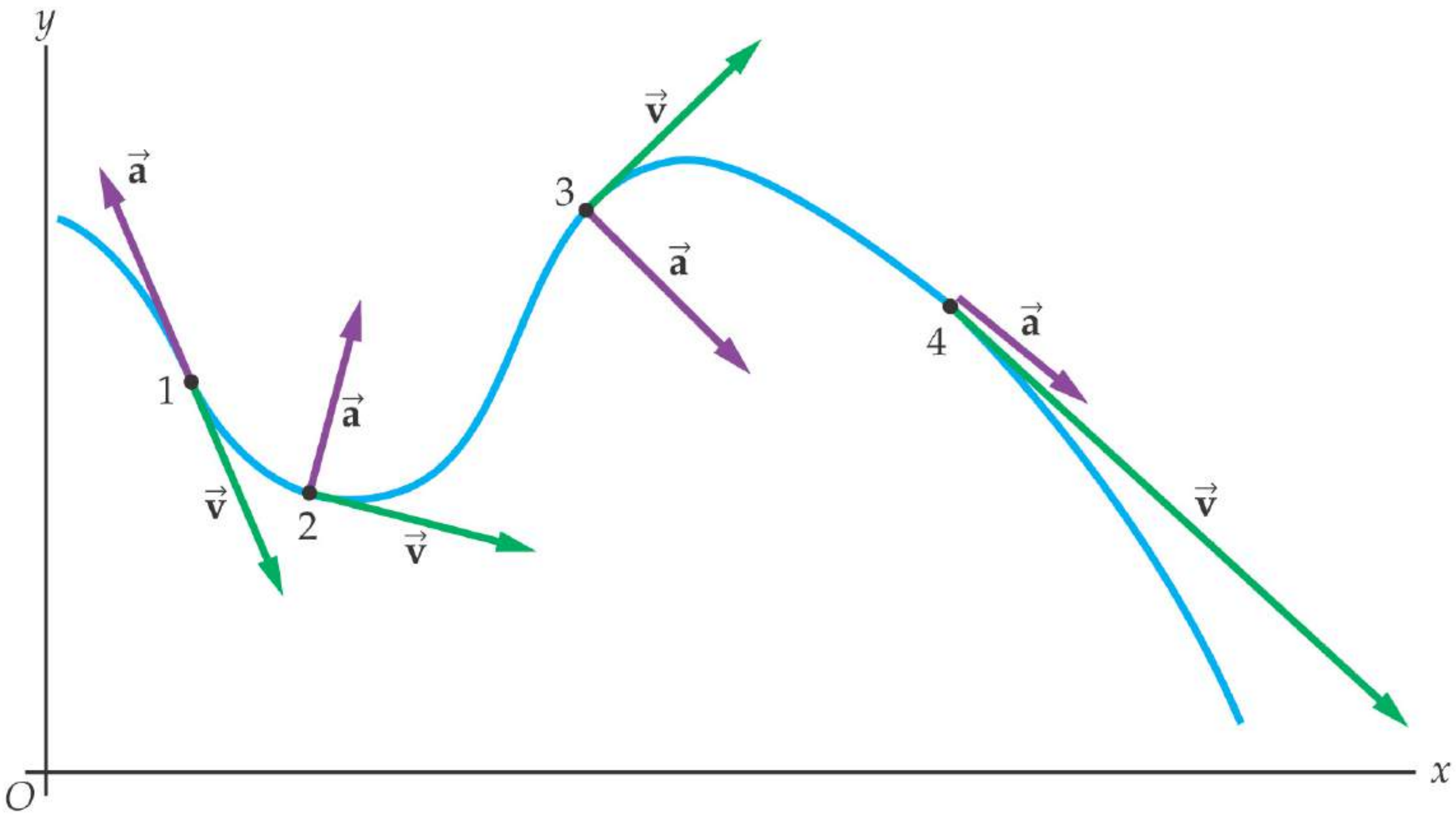
3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Velocity vector is always in the direction of motion; acceleration vector can point anywhere:



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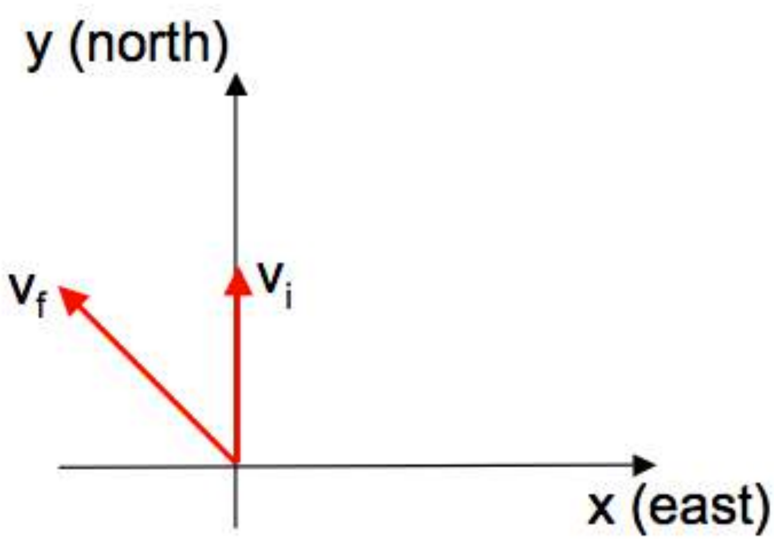
Example

- At the beginning of a 3 hour plane trip you are traveling due north at 192 km/hour. At the end, you are traveling 240 km/hour at 45° west of north.

(a) Draw the initial and final velocity vectors.



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Example continued

(b) Find $\Delta \mathbf{v}$

The components are

$$\Delta v_x = v_{fx} - v_{ix} = -v_f \sin 45° - 0 = -170\ \text{km/hr}$$

$$\Delta v_y = v_{fy} - v_{iy} = +v_f \cos 45° - v_i = -22.3\ \text{km/hr}$$

The magnitude and direction are:

$$|\Delta \mathbf{v}| = \sqrt{\Delta v_x{}^2 + \Delta v_y{}^2} = 171\ \text{km/hr}$$

$$\tan\varphi = \frac{|\Delta v_y|}{|\Delta v_x|} = 0.1312 \Rightarrow \phi = \tan^{-1}(0.1312) = 7.5°$$

South of west

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Example continued

(c) What is $\mathbf{a}_{av}$ during the trip?

$$\mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t} \qquad a_{x,av} = \frac{\Delta v_x}{\Delta t} = \frac{-170\ \text{km/hr}}{3\ \text{hr}} = -56.7\ \text{km/hr}^2$$

$$a_{y,av} = \frac{\Delta v_y}{\Delta t} = \frac{-22.3\ \text{km/hr}}{3\ \text{hr}} = -7.43\ \text{km/hr}^2$$

The magnitude and direction are:

$$\left|\mathbf{a}_{av}\right| = \sqrt{a_{x,av}^2 + a_{y,av}^2} = 57.2\ \text{km/hr}^2$$

$$\tan\phi = \frac{\left|a_{y,av}\right|}{\left|a_{x,av}\right|} = 0.1310 \Rightarrow \phi = \tan^{-1}(0.1310) = 7.5°$$

South of west

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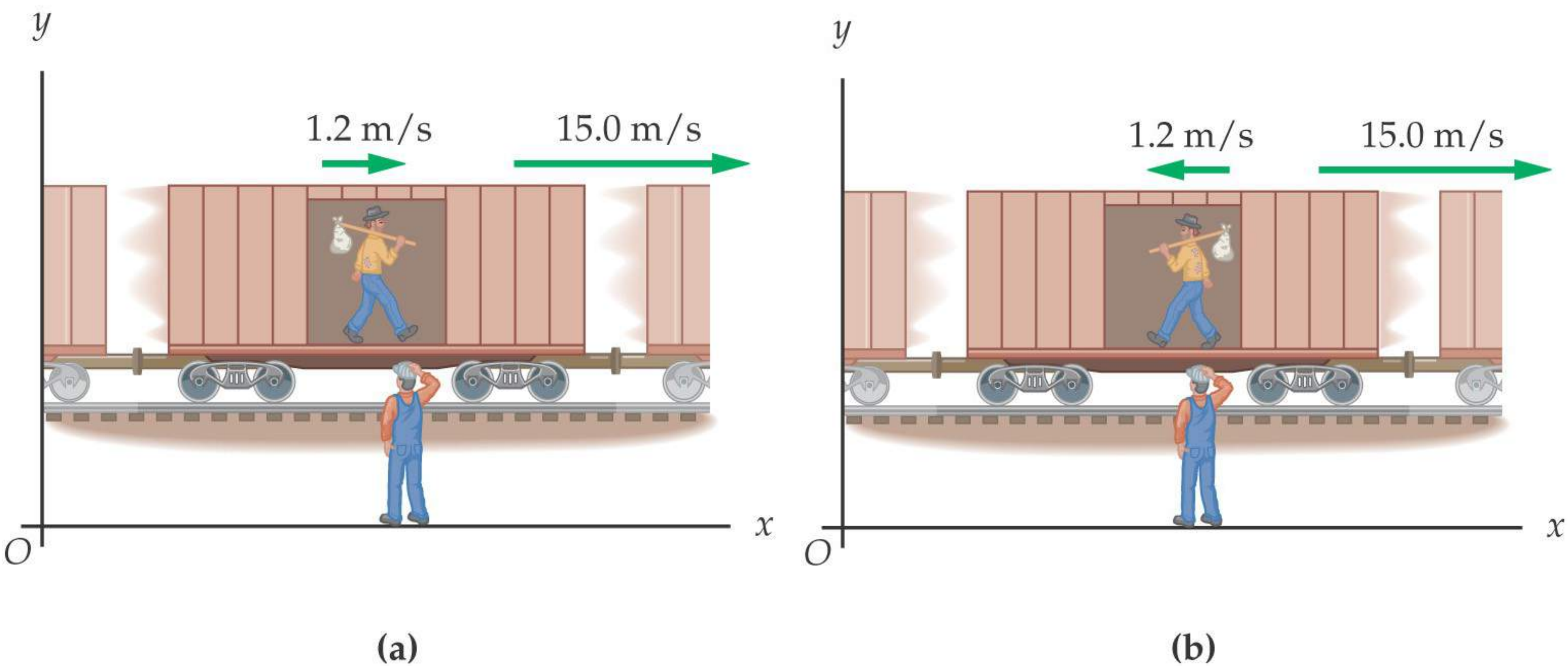
3-6 Relative motion

- The speed of the passenger with respect to the ground depends on the relative directions of the passenger's and train's speeds:



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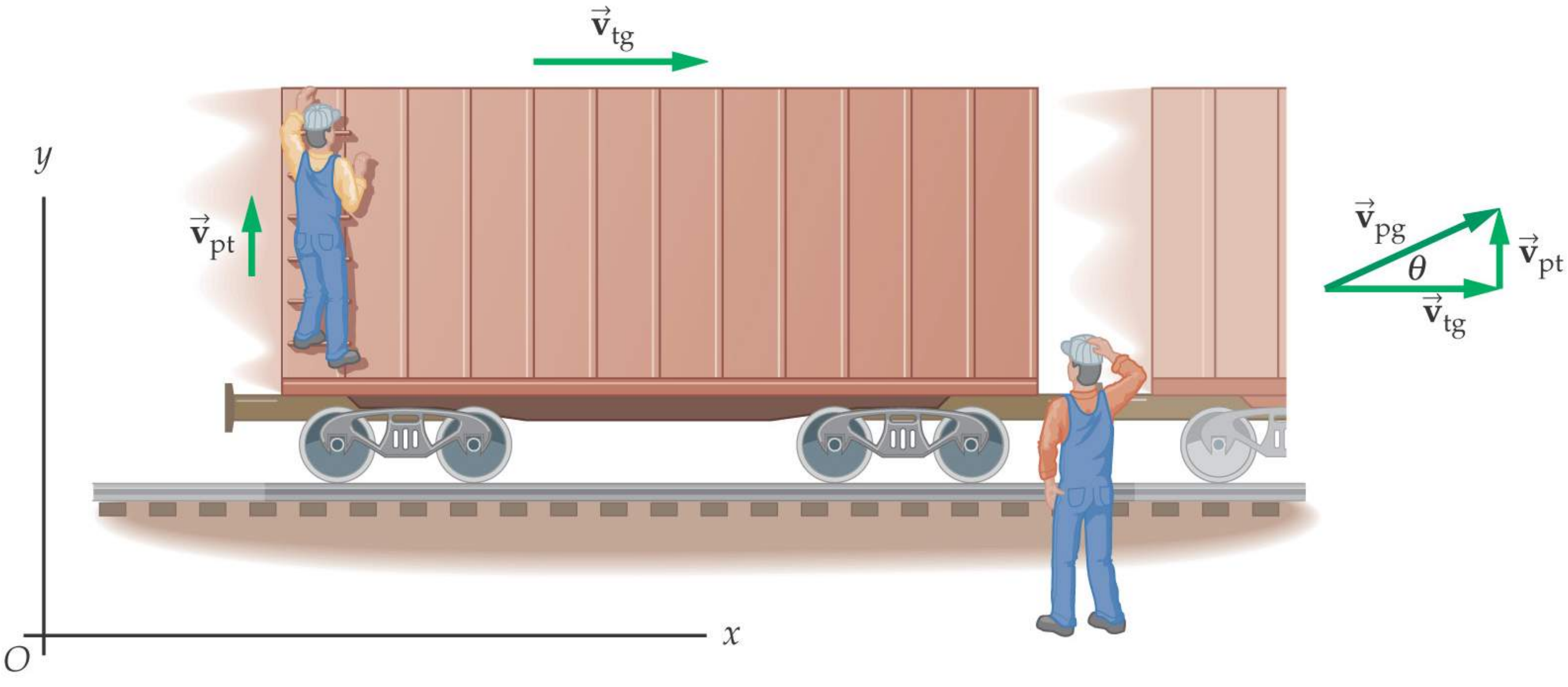
3-6 Relative motion

$$\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$$

- This also works in two dimensions:



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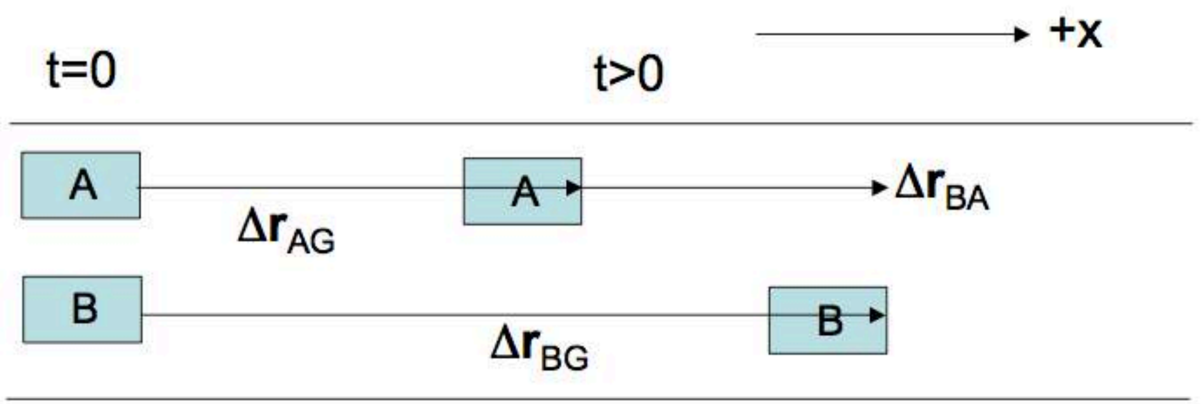
Example

- You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour east. What is the speed of car B relative to car A?



From the picture: $\Delta\mathbf{r}_{BG} = \Delta\mathbf{r}_{AG} + \Delta\mathbf{r}_{BA}$

$$\Delta\mathbf{r}_{BA} = \Delta\mathbf{r}_{BG} - \Delta\mathbf{r}_{AG}$$

Divide by Δt: $\mathbf{v}_{BA} = \mathbf{v}_{BG} - \mathbf{v}_{AG}$

$$\mathbf{v}_{BA} = 65 \text{ miles/hr east} - 60 \text{ miles/hr east}$$

$$= 5 \text{ miles/hour east}$$

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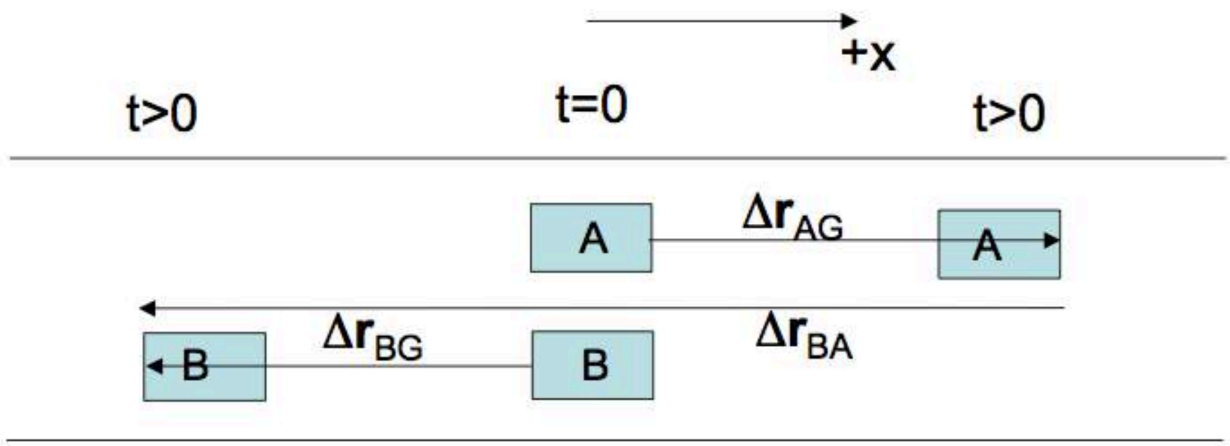
Example

- You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour west. What is the speed of car B relative to car A?



From the picture: $\Delta \mathbf{r}_{BA} = \Delta \mathbf{r}_{BG} - \Delta \mathbf{r}_{AG}$

Divide by Δt: $\mathbf{v}_{BA} = \mathbf{v}_{BG} - \mathbf{v}_{AG}$

$= 65 \text{ miles/hr west} - 60 \text{ miles/hr east}$

$= 125 \text{ miles/hr west}$

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Example

- A plane is headed eastward at a speed of 156 m/s relative to the wind. A 20.0 m/s wind is blowing southward at the same time as the plane is flying. The velocity of the plane relative to the ground is:

- A) 155 m/s at an angle 7.36° south of east
- B) 155 m/s at an angle 7.36° east of south
- C) 157 m/s at an angle 7.36° south of east
- D) 157 m/s at an angle 7.31° south of east
- E) 157 m/s at an angle 7.31° east of south

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Summary of Chapter 3

- Scalar: number, with appropriate units
- Vector: quantity with magnitude and direction
- Vector components: $A_x = A \cos \theta$, $B_y = B \sin \theta$
- Magnitude: $A = (A_x^2 + A_y^2)^{1/2}$
- Direction: $\theta = \tan^{-1} (A_y / A_x)$
- Graphical vector addition: Place tail of second at head of first; the sum points from tail of first to head of last

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Summary of Chapter 3

- Component method: add components of individual vectors, then find magnitude and direction
- Unit vectors are dimensionless and of unit length
- Position vector points from origin to location
- Displacement vector points from original position to final position
- Velocity vector points in direction of motion
- Acceleration vector points in direction of change of motion
- Relative motion: $\mathbf{v}_{13} = \mathbf{v}_{12} + \mathbf{v}_{23}$