LECTURE NOTES

ON

STRUCTURAL ANALYSIS (ACE008)

III B. Tech I Semester (IARE-R16)

By

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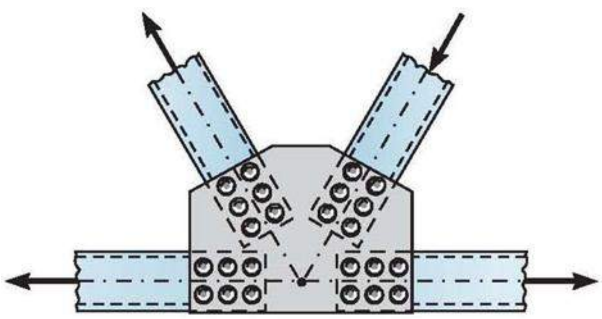
UNIT- I
ANALYSIS OF PLANE FRAMES

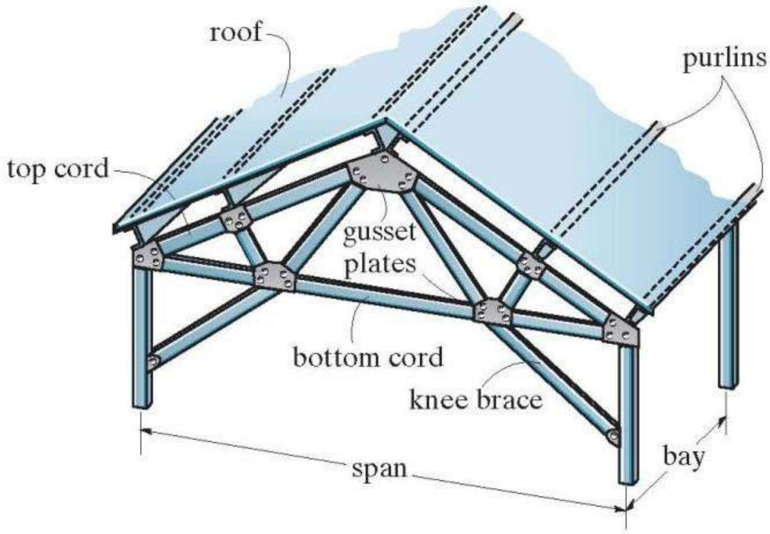
INTRODUCTION TO TRUSSES

- A truss is one of the major types of engineering structures which provides a practical and economical solution for many engineering constructions, especially in the design of bridges and buildings that demand large spans.
- A truss is a structure composed of slender members joined together at their end points
- The joint connections are usually formed by bolting or welding the ends of the members to a common plate called gusset
- Planar trusses lie in a single plane & is often used to support roof or bridges

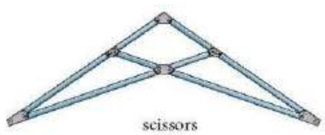


Common Types of Trusses

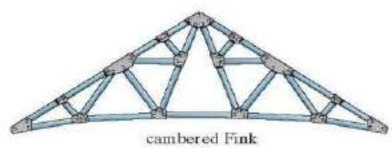
- Roof Trusses
  - They are often used as part of an industrial building frame
  - Roof load is transmitted to the truss at the joints by means of a series of purlins
  - To keep the frame rigid & thereby capable of resisting horizontal wind forces, knee braces are sometimes used at the supporting column



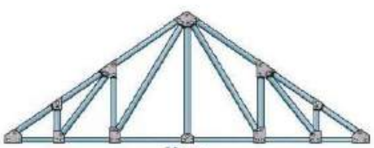
- Roof Trusses



scissors
(a)

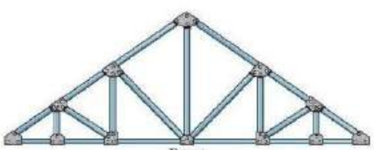


cambered Fink
(f)

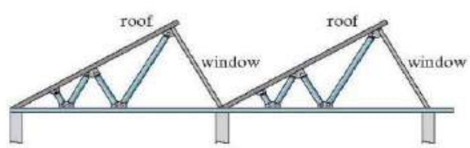


Howe
(b)

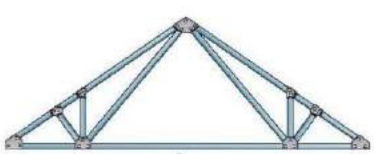
Warren
(g)



Pratt
(c)

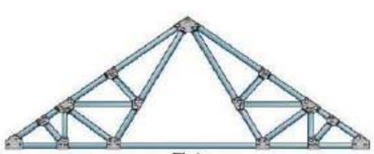


sawtooth
(h)

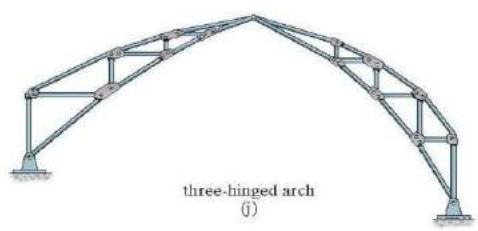


fan
(d)

bowstring
(i)



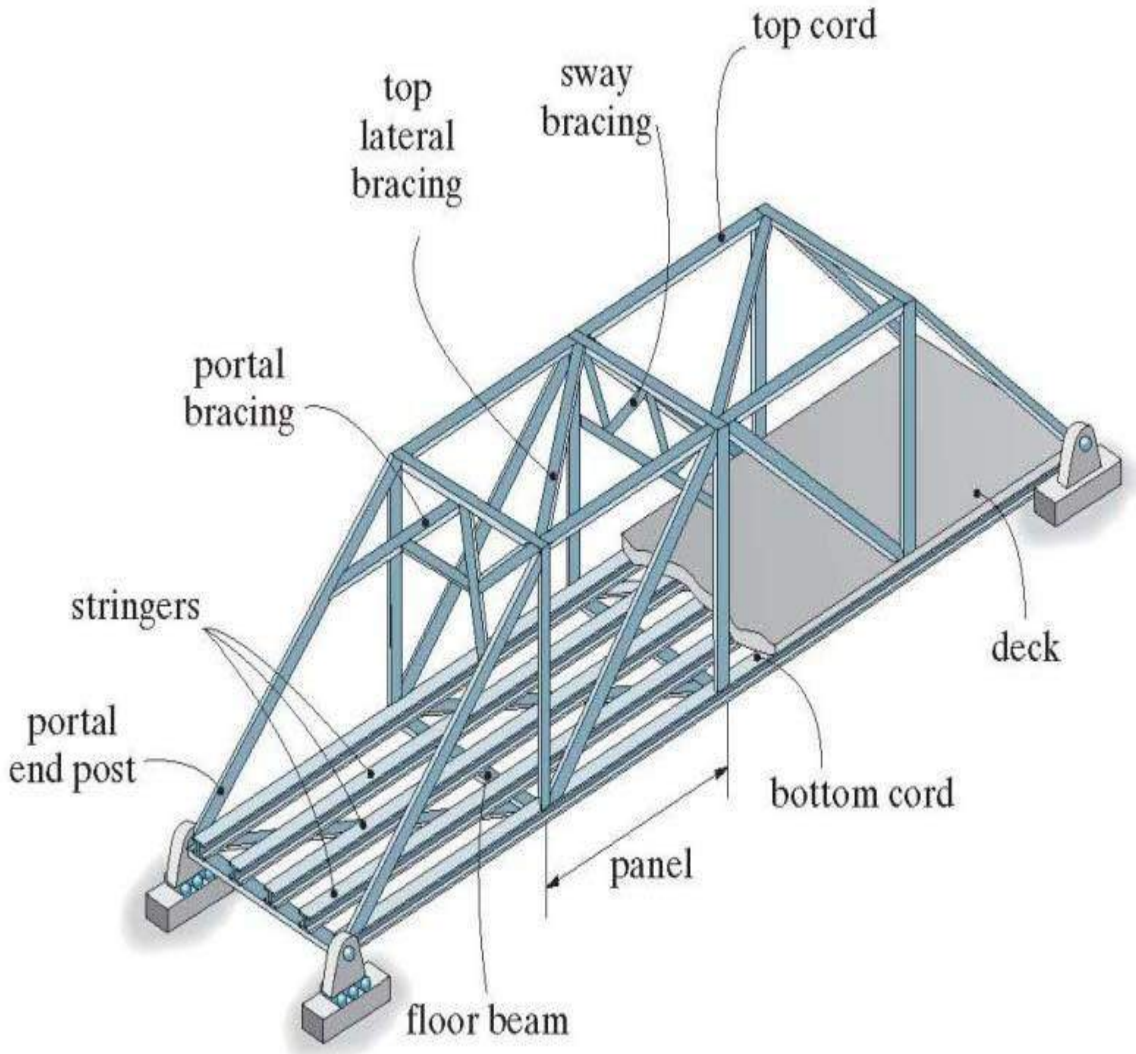
Fink
(e)



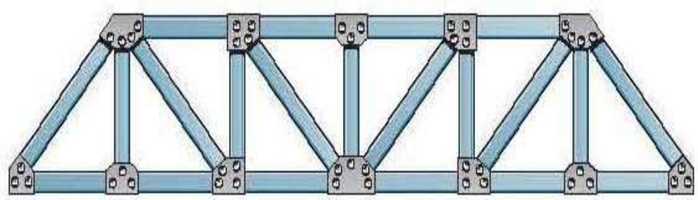
three-hinged arch
(j)

➢ **Bridge Trusses**

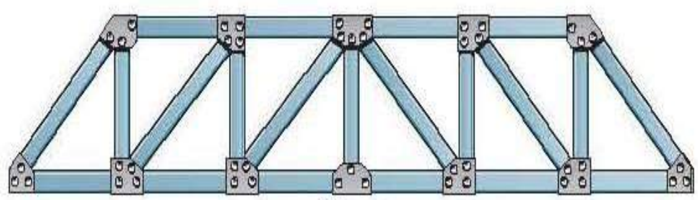
- The main structural elements of a typical bridge truss are shown in figure. Here it is seen that a load on the deck is first transmitted to stringers, then to floor beams, and finally to the joints of the two supporting side trusses.
- The top and bottom cords of these side trusses are connected by top and bottom lateral bracing, which serves to resist the lateral forces caused by wind and the sidesway caused by moving vehicles on the bridge.
- Additional stability is provided by the portal and sway bracing. As in the case of many long-span trusses, a roller is provided at one end of a bridge truss to allow for thermal expansion.



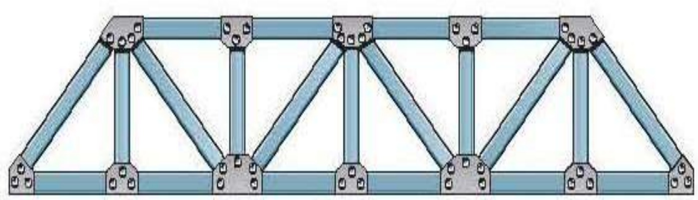
- **Bridge Trusses**
  - In particular, the Pratt, Howe, and Warren trusses are normally used for spans up to 61 m in length. The most common form is the Warren truss with verticals.
  - For larger spans, a truss with a polygonal upper cord, such as the Parker truss, is used for some savings in material.
  - The Warren truss with verticals can also be fabricated in this manner for spans up to 91 m.



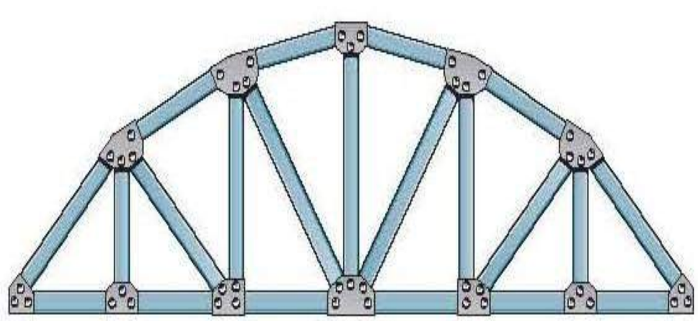
Pratt
(a)



Howe
(b)

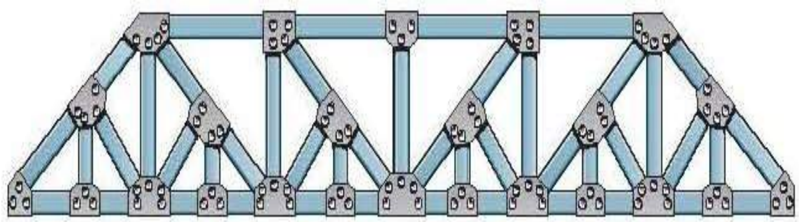


Warren (with verticals)
(c)

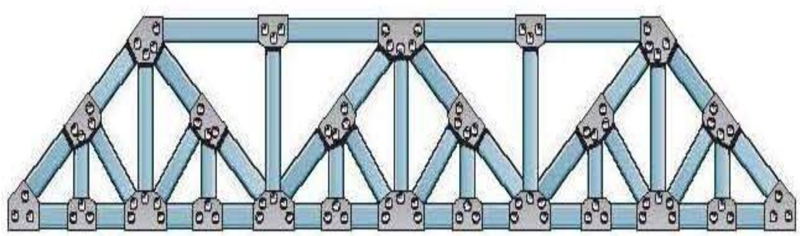


Parker
(d)

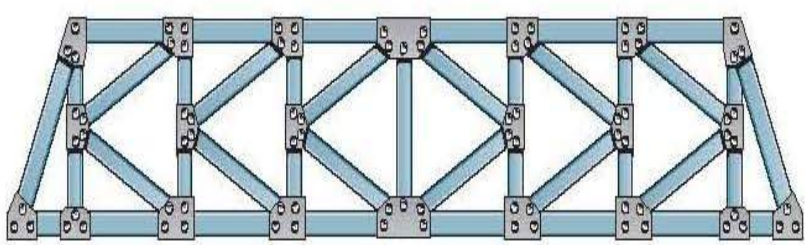
- **Bridge Trusses**
  - The greatest economy of material is obtained if the diagonals have a slope between 45° and 60° with the horizontal. If this rule is maintained, then for spans greater than 91 m, the depth of the truss must increase and consequently the panels will get longer.
  - This results in a heavy deck system and, to keep the weight of the deck within tolerable limits, subdivided trusses have been developed. Typical examples include the Baltimore and subdivided Warren trusses.
  - The K-truss shown can also be used in place of a subdivided truss, since it accomplishes the same purpose.



Baltimore
(e)



subdivided Warren
(f)



K-truss
(g)

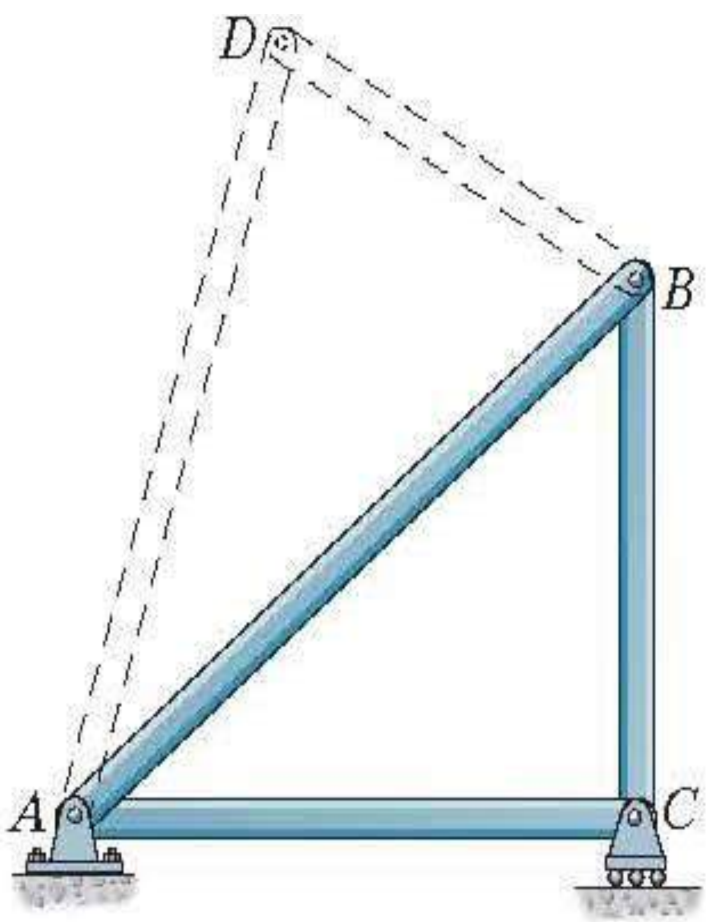
Assumptions for Design

- The members are joined together by smooth pins
- All loadings are applied at the joints

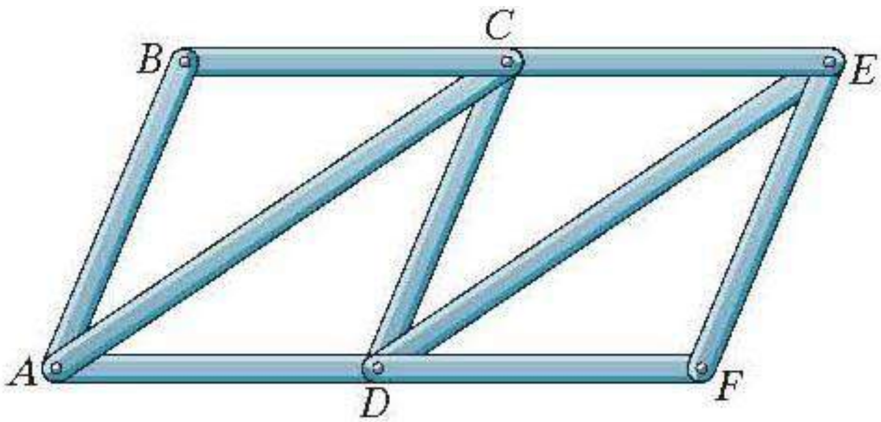
Due to the 2 assumptions, each truss member acts as an axial force member

Classification of Coplanar Trusses

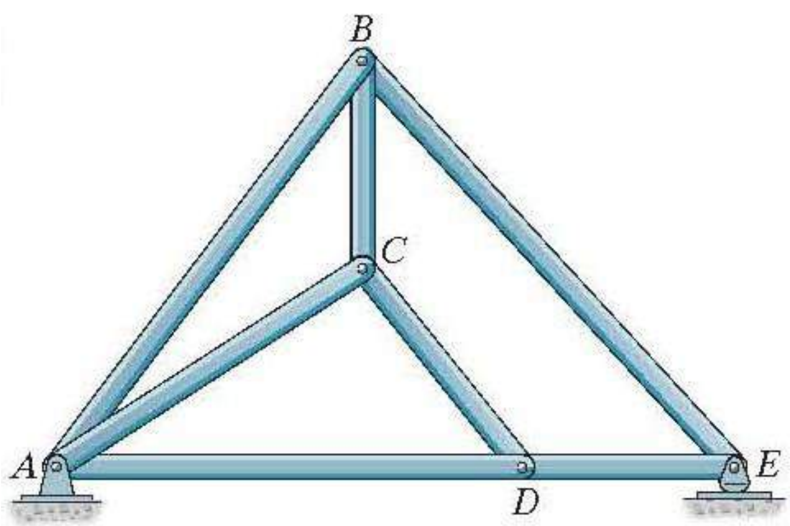
- **Simple , Compound or Complex Truss**
- **Simple Truss**
- **To prevent collapse, the framework of a truss must be rigid**
- **The simplest framework that is rigid or stable is a triangle**
- **The members are joined together by smooth pins**
- **All loadings are applied at the joints**



- **Simple Truss**
  - The basic "stable" triangle element is ABC
  - The remainder of the joints D, E & F are established in alphabetical sequence
  - Simple trusses do not have to consist entirely of triangles



simple truss



simple truss

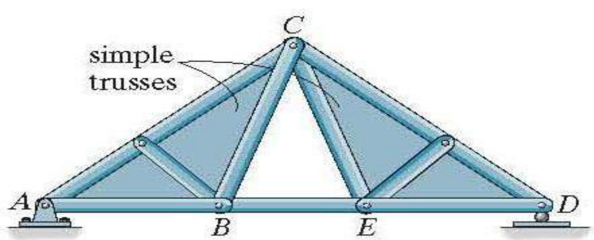
- **Compound Truss**
    - It is formed by connecting 2 or more simple truss together
    - Often, this type of truss is used to support loads acting over a larger span as it is cheaper to construct a lighter compound truss than a heavier simple truss

      Types of compound truss:
  - Type 1

    The trusses may be connected by a common joint & bar
  - Type 2

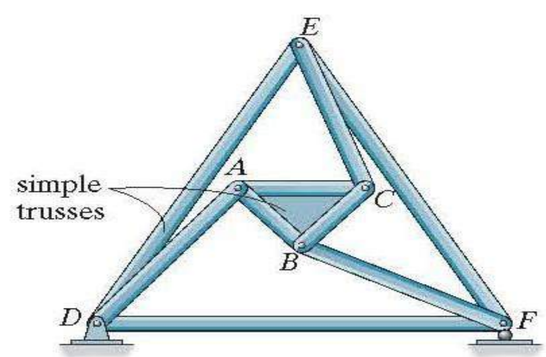
    The trusses may be joined by 3 bars
  - Type 3

    The trusses may be joined where bars of a large simple truss, called the main truss, have been substituted by simple truss, called secondary trusses



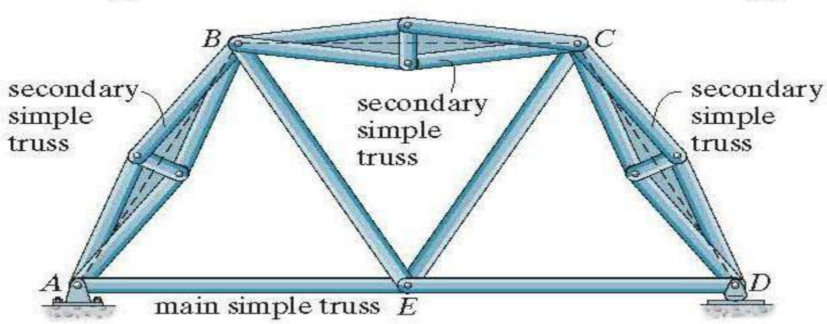
compound truss

(a)



compound truss

(b)

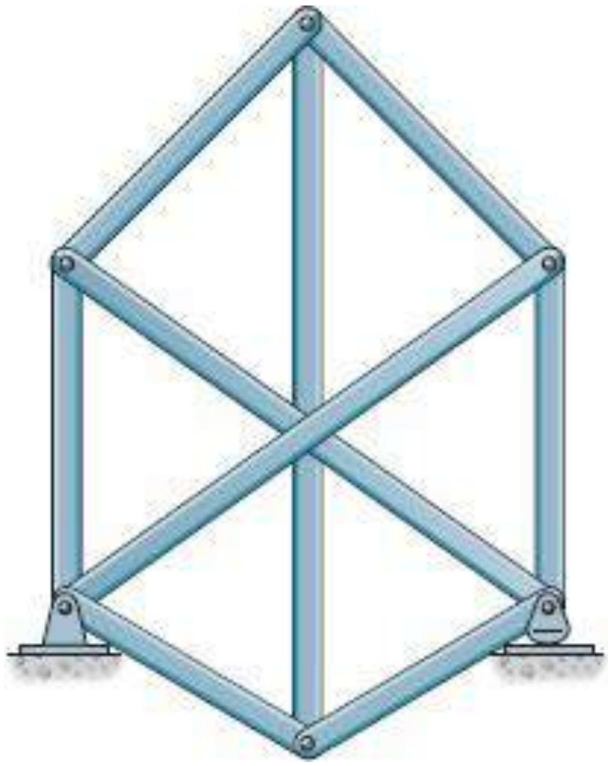


compound truss

(c)

- Complex Truss

- A complex truss is one that cannot be classified as being either simple or compound



- **Determinacy**
  - The total number of unknowns includes the forces in b number of bars of the truss and the total number of external support reactions r.
  - Since the truss members are all straight axial force members lying in the same plane, the force system acting at each joint is coplanar and concurrent.
  - Consequently, rotational or moment equilibrium is automatically satisfied at the joint (or pin).

Therefore only

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

By comparing the total unknowns with the total number of available equilibrium equations, we have:

$$b + r = 2j \quad \text{statically determinate}$$
$$b + r > 2j \quad \text{statically indeterminate}$$

- **Stability**

  If b + r < 2j => collapse

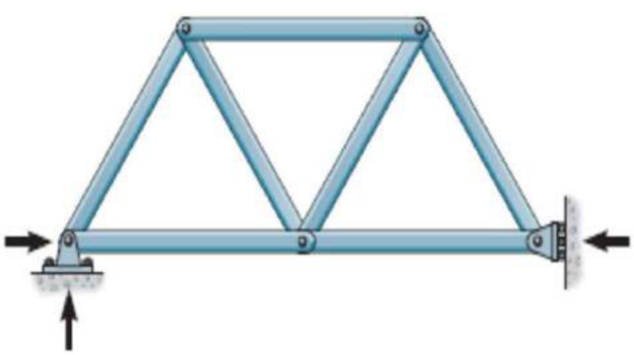
  A truss can be unstable if it is statically determinate or statically indeterminate

  Stability will have to be determined either through inspection or by force analysis

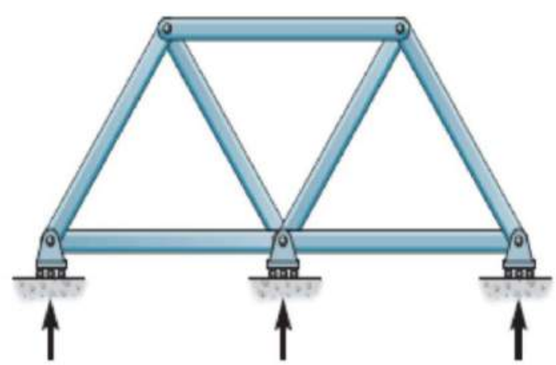
- **External Stability**

  A structure is externally unstable if all of its reactions are concurrent or parallel

  The trusses are externally unstable since the support reactions have lines of action that are either concurrent or parallel



unstable concurrent reactions



unstable parallel reactions

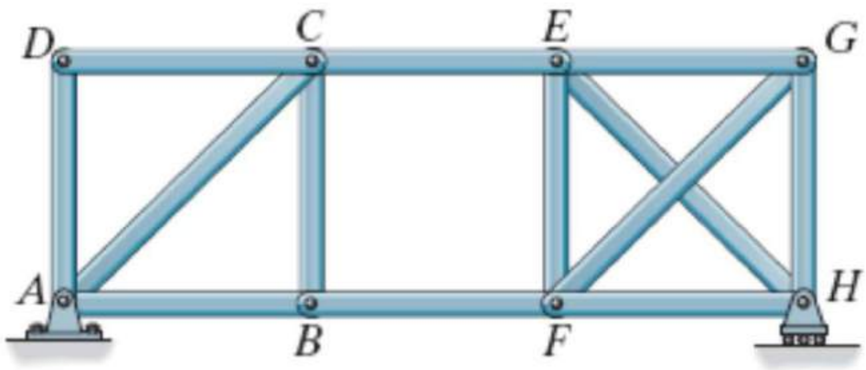
- Internal Stability

  The internal stability can be checked by careful inspection of the arrangement of its members

  If it can be determined that each joint is held fixed so that it cannot move in a "rigid body" sense with respect to the other joints, then the truss will be stable

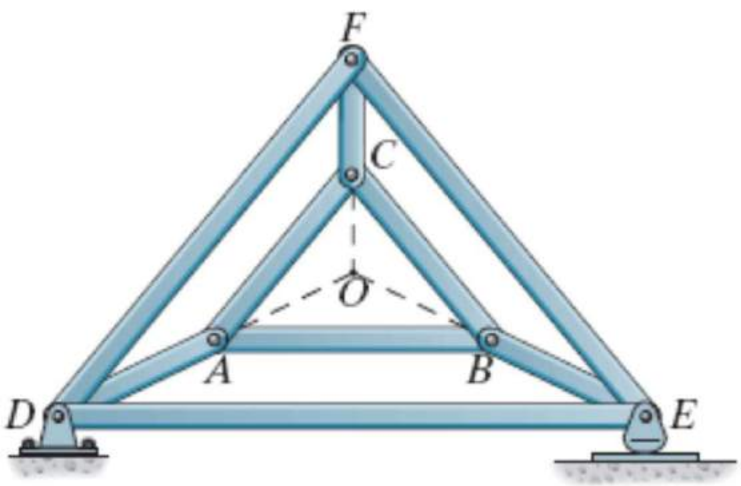
A simple truss will always be internally stable

If a truss is constructed so that it does not hold its joints in a fixed position, it will be unstable or have a "critical form"



To determine the internal stability of a compound truss, it is necessary to identify the way in which the simple truss are connected together

The truss shown is unstable since the inner simple truss ABC is connected to DEF using 3 bars which are concurrent at point O



Thus an external load can be applied at A, B or C & cause the truss to rotate slightly

For complex truss, it may not be possible to tell by inspection if it is stable

The instability of any form of truss may also be noticed by using a computer to solve the 2j simultaneous equations for the joints of the truss

If inconsistent results are obtained, the truss is unstable or have a critical form

DIFFERENCE BETWEEN FORCE & DISPLACEMENT METHODS

FORCE METHODS	DISPLACEMENT METHODS
1. Method of consistent deformation 2. Theorem of least work 3.Column analogy method 4. Flexibility matrix method	1. Slope deflection method 2.Moment distribution method 3.Kani's method 4. Stiffness matrix method
Types of indeterminacy- static indeterminacy	Types of indeterminacy-indeterminacy kinematic
Governing equations-compatibility equations	Governing equations-equilibriumequations
Force displacement relations-flexibility Matrix	Force displacement relations- stiffness matrix

DETERMINATION OF THE MEMBER FORCES

- The Method of Joints
- The Method of Sections (Ritter Method)
- The Graphical Method (Cremona Method)

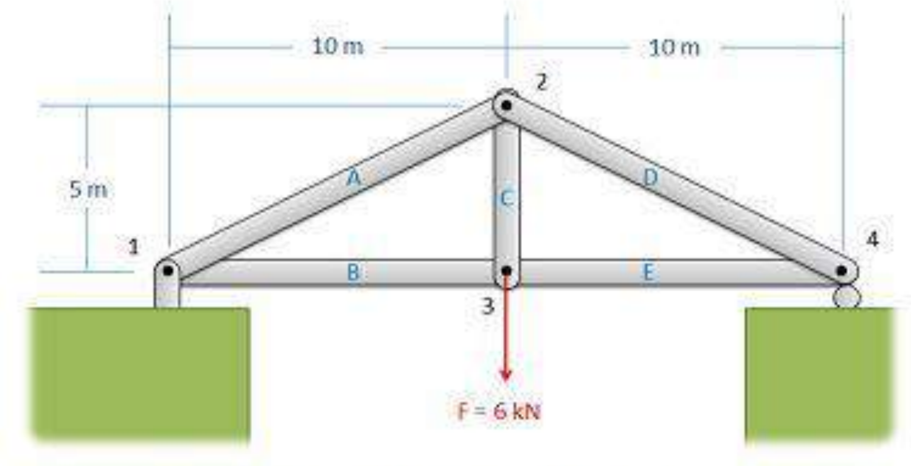
The Method of Joints

The method of joints is a process used to solve for the unknown forces acting on members of a truss. The method centers on the joints or connection points between the members, and it is usually the fastest and easiest way to solve for all the unknown forces in a truss structure

Using the Method of Joints:

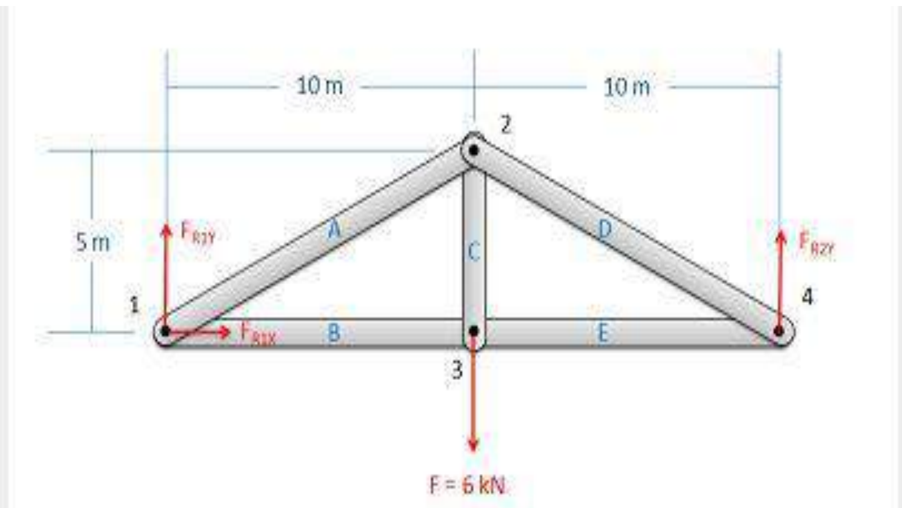
The process used in the method of joints is outlined below:

1. In the beginning it is usually useful to label the members and the joints in your truss. This will help you keep everything organized and consistent in later analysis. In this book, the members will be labeled with letters and the joints will be labeled with numbers.



The first step in the method of joints is to label each joint and each member.

2. Treating the entire truss structure as a rigid body, draw a free body diagram, write out the equilibrium equations, and solve for the external reacting forces acting on the truss structure. This analysis should not differ from the analysis of a single rigid body.

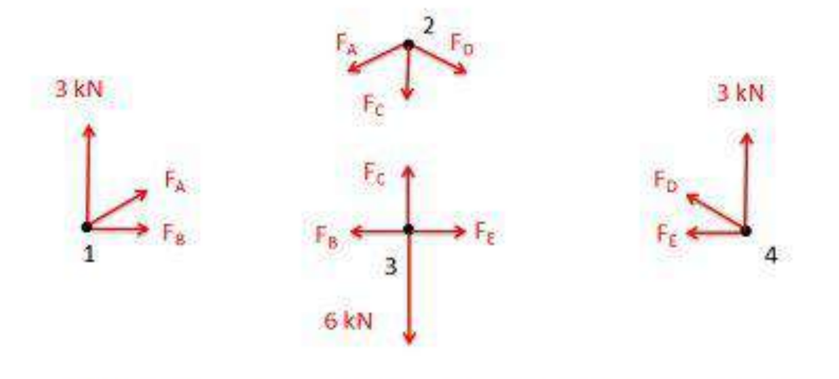


Treat the entire truss as a rigid body and solve for the reaction forces supporting the truss structure.

3. Assume there is a pin or some other small amount of material at each of the connection points between the members. Next you will draw a free body diagram for each connection point.

Remember to include:

- Any external reaction or load forces that may be acting at that joint.
- A normal force for each two force member connected to that joint. Remember that for a two force member, the force will be acting along the line between the two connection points on the member. We will also need to guess if it will be a tensile or a compressive force. An incorrect guess now though will simply lead to a negative solution later on. A common strategy then is to assume all forces are tensile, then later in the solution any positive forces will be tensile forces and any negative forces will be compressive forces.
- Label each force in the diagram. Include any known magnitudes and directions and provide variable names for each unknown.



Drawing a free body diagram of each joint, we draw in the known forces as well as tensile forces from each two force member.

4. Write out the equilibrium equations for each of the joints. You should treat the joints as particles, so there will be force equations but no moment equations. With either two (for 2D problems) or three (for 3D problems) equations for each joint, this should give you a large number of equations.

$$\sum \mathbf{F} = 0$$

$$\sum F_x = 0 \quad \sum F_y = 0$$

In planar trusses, the sum of the forces in the x direction will be zero and the sum of the forces in the y direction will be zero for each of the joints.

$$\sum \mathbf{F} = 0$$

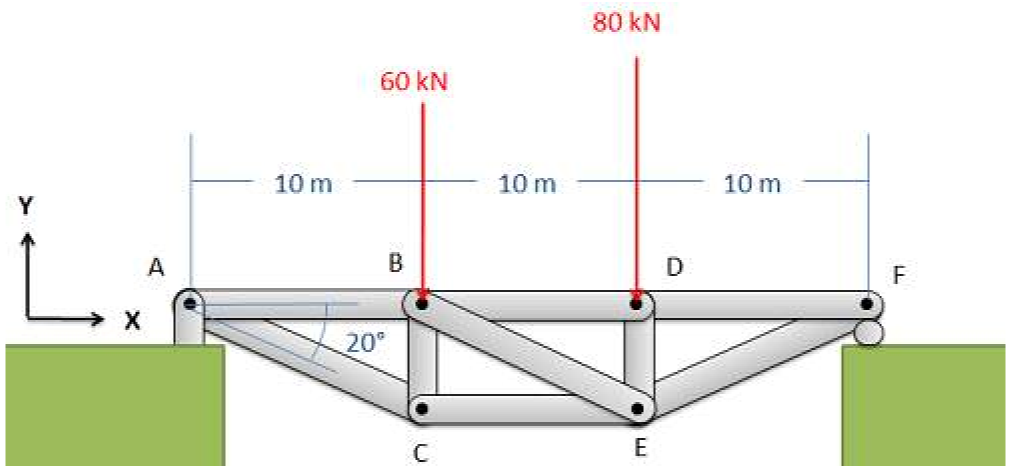
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

In space trusses, the sum of the forces in the x direction will be zero, the sum of the forces in the y direction will be zero, and the sum of forces in the z direction will be zero for each of the joints.

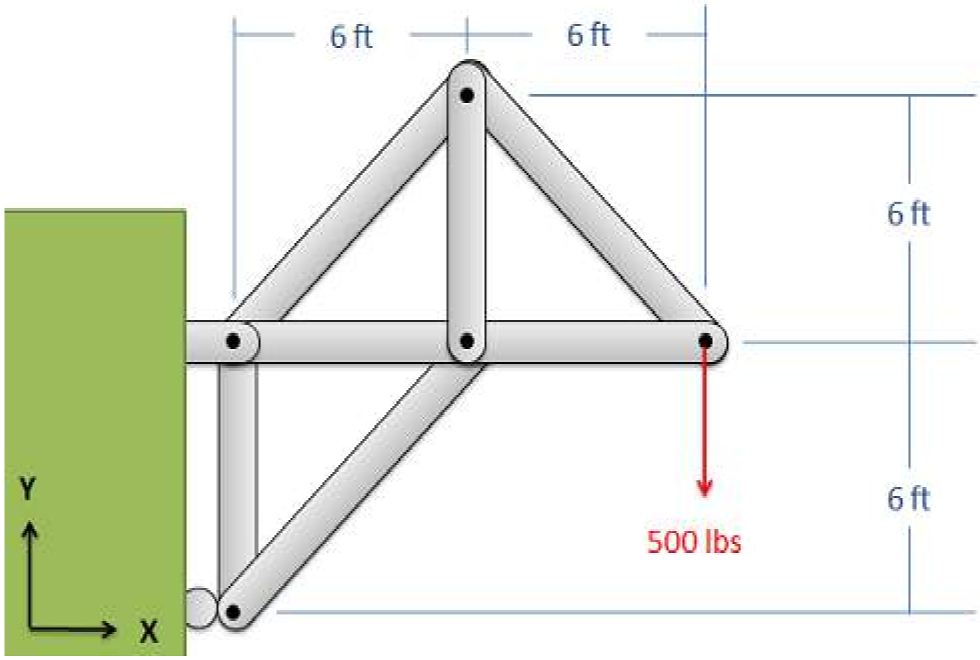
5.Finally, solve the equilibrium equations for the unknowns. You can do this algebraically, solving for one variable at a time, or you can use matrix equations to solve for everything at once. If you assumed that all forces were tensile earlier, remember that negative answers indicate compressive forces in the members.

PROBLEMS

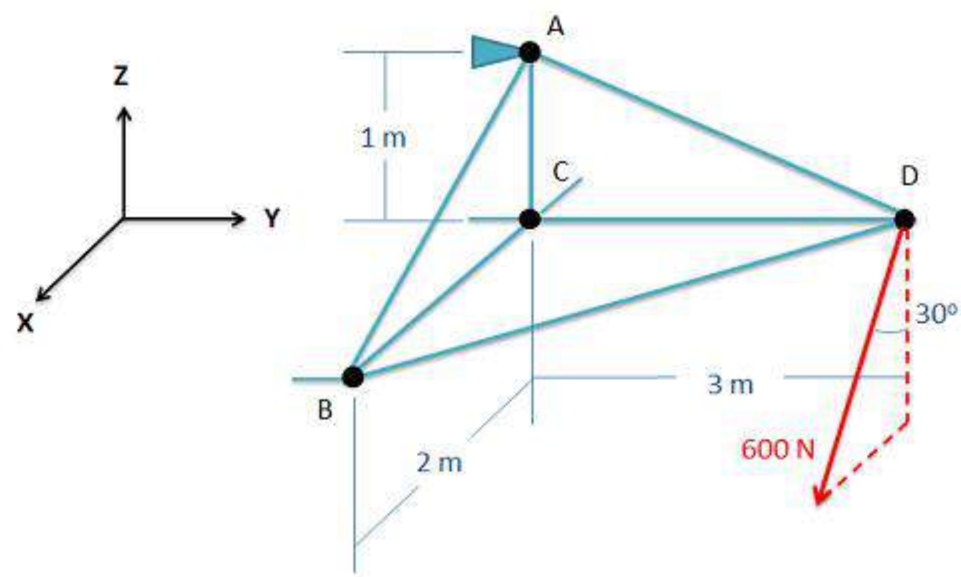
1. Find the force acting in each of the members in the truss bridge shown below. Remember to specify if each member is in tension or compression



2. Find the force acting in each of the members of the truss shown below. Remember to specify if each member is in tension or compression.



3. Find the force acting in each of the members of the truss shown below. Remember to specify if each member is in tension or compression.



THE METHOD OF SECTIONS

APPLICATIONS

Long trusses are often used to construct bridges.
The method of joints requires that many joints be analyzed before we can determine the forces in the middle part of the truss.

In the method of sections, a truss is divided into two parts by taking an imaginary "cut" (shown here as a-a) through the truss.
Since truss members are subjected to only tensile or compressive forces along their length, the internal forces at the cut member will also be either tensile or compressive with the same magnitude.

This result is based on the equilibrium principle and Newton's third law.

STEPS FOR ANALYSIS

- Decide how you need to "cut" the truss. This is based on where you need to determine forces, and, b) where the total number of unknowns does not exceed three (in general).
- Decide which side of the cut truss will be easier to work with (minimize the number of reactions you have to find).
- If required, determine the necessary support reactions by drawing the FBD of the entire truss and applying the E-of-E.

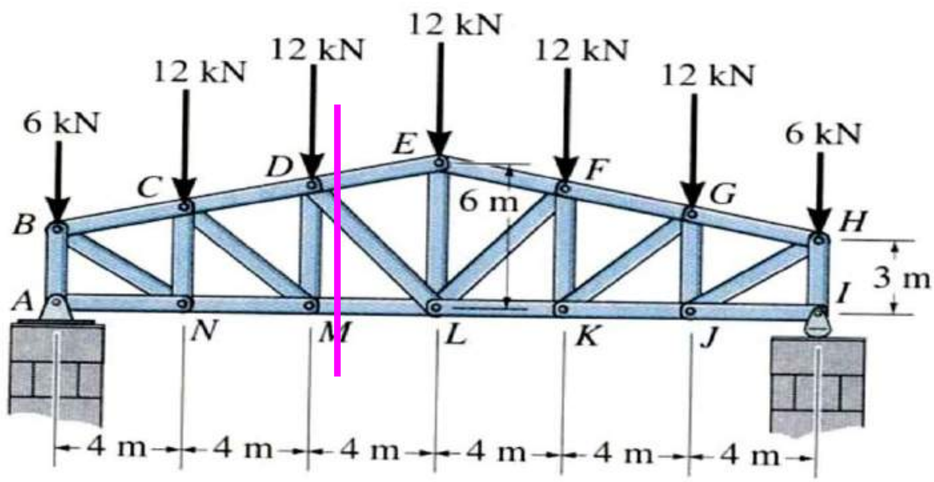
- Draw the FBD of the selected part of the cut truss. We need to indicate the unknown forces at the cut members. Initially we may assume all the members are in tension, as we did when using the method of joints. Upon solving, if the answer is positive, the member is in tension as per our assumption. If the answer is negative, the member must be in compression.

    - Apply the equations of equilibrium (E-of-E) to the selected cut section of the truss to solve for the unknown member forces. Please note that in most cases it is possible to write one equation to solve for one unknown directly.

PROBLEMS

Given: Loads as shown on the roof truss.

Find:The force in members DE, DL, and ML.



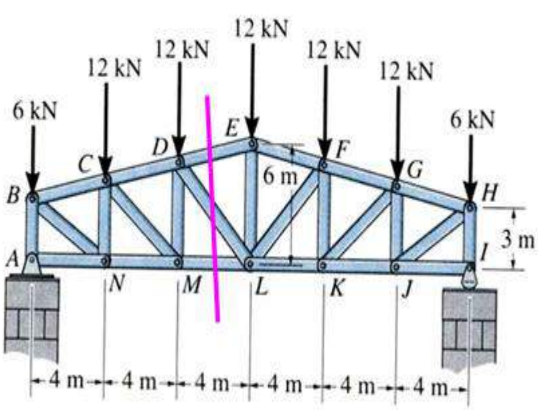
Plan:

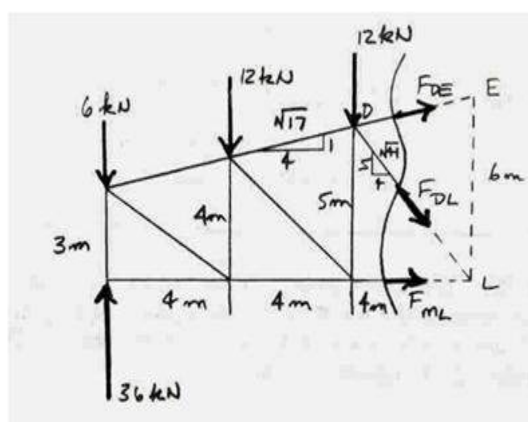
a) Take a cut through the members DE, DL, and ML.

b) Work with the left part of the cut section. Why?

c) Determine the support reaction at A.

d) Apply the EofE to find the forces in DE, DL, and ML.





Analyzing the entire truss, we get FX = AX = 0.

By symmetry, the vertical support reactions are

AY = IY = 36 kN

+ MD = – 36 (8) + 6 (8) + 12 (4) + FML (5) = 0

FML = 38.4 kN(T)

+ML = –36 (12) + 6 (12) + 12 (8) + 12 (4) – FDE (4/17)(6) = 0

FDE = –37.11 kN or 37.1 kN (C)

+ FX = 38.4 + (4/17) (–37.11) + (4/41) FDL = 0

FDL = –3.84 kN or 3.84 kN (C)

INTERNAL FORCES

In order to obtain the internal forces at a specified point, we should make section cut perpendicular to the axis of the member at this point. This section cut divides the structure in two parts. The portion of the structure removed from the part in to consideration should be replaced by the internal forces. The internal forces ensure the equilibrium of the isolated part subjected to the action of external loads and support reactions. A free body diagram of either segment of the cut member is isolated and the internal loads could be derived by the six equations of equilibrium applied to the segment in to consideration.

ANALYSIS OF SPACE TRUSSES USING METHOD OF TENSION COEFFICIENTS

1. Tension Co-efficient Method

The tension co efficient for a member of a frame is defined as the pull or tension in that member is divided by its length.

t = T/l Where t = tension co efficient for the member

T= Pull in the member

l = Length

2. Analysis Procedure Using Tension Co-efficient - 2D Frames

1. List the coordinates of each joint (node)of the truss.
2. Determine the projected lengths Xij and Yij of each member of the truss. Determine the support lengths lij of each member using the equation lij =?Xij2+Yij2

3. Resolve the the applied the forces at the joint in the X and Y directions. Determine the support reactions and their X and Y components.
4. Identify a node with only two unknown member forces and apply the equations of equilibrium. The solution yields the tension co efficient for the members at the node.
5. Select the next joint with only two unknown member forces and apply the equations of equilibrium and apply the tension co efficient.
6. Repeat step 5 till the tension co efficient of all the members are obtained. 7. Compute the member forces from the tension co efficient obtained as above using

Tij= tijxlij

3. Analysis Procedure Using Tension Co-efficient - Space Frames

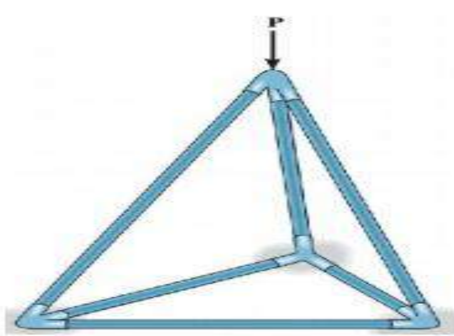
1. In step 2 above the projected lengths Zij in the directions are also computed. Determine the support lengths lij of each member using the equation lij =?Xij2+Yij2 +Zij2
2. In step 3 above the components of forces and reactions in the Z directions are also to be determined.
3. In step 4 and 5, each time, nodes with not more than three unknown member forces are to be considered.

Tetrahedron: simplest element of stable space truss (six members, four joints) expand by adding 3members and 1 joint each time

Determinacy and Stability b+r <3j unstable

b +r=3j statically determinate (check stability)

b+r>3j statically indeterminate (check stability)



<u>INTERNAL FORCES</u>

In order to obtain the internal forces at a specified point, we should make section cut perpendicular to the axis of the member at this point. This section cut divides the structure in two parts. The portion of the structure removed from the part in to consideration should be replaced by the internal forces. The internal forces ensure the equilibrium of the isolated part subjected to the action of external loads and support reactions. A free body diagram of either segment of the cut member is isolated and the internal loads could be derived by the six equations of equilibrium applied to the segment in to consideration.

UNIT- II

ARCHES

Till now, we had been studying two-dimensional (plane) structures like beams, frames and trusses which were mostly linear in their geometry or comprised of elements which were formed out of straight lines. Now in this unit, we are being introduced to a class of structures which will be composed of curved-members instead of straight ones. The simplest member of this class is the arch. Arches as such are not a new mode of construction and have been in use as a load bearing structure since ancient times. Although it is more difficult to construct a curved structure like an arch, there are certain advantages, apart from their aesthetic look, which have made them popular among civil engineers and architects. It will be shown here in this unit that the bending moment in an arch section is generally less than that in a corresponding beam section, of similar span and loading.

Hence, the all-important bending stresses are less in arches. However, in an arch section, there is in additional normal thrust which is not present in beam sections (with transverse loading). But normally the net effect is not critical as concrete and masonry are usually stronger in compression and the total stresses are generally well within limits. So overall speaking, an arch is lighter and stronger than a similar or similarly-loaded beam.

Some kinds of arch used in civil engineering.

(a) Fixed Arch

(b) Linear Arch

(c) Trussed Arch.

An arch could be defined in simple terms as a two-dimensional structure element which is curved in elevation and is supported at ends by rigid or hinged supports which are capable of developing the desired thrust to resist the loads.

It could also be defined as a two-dimensional element which resists external loads through its profile. This is achieved by its characteristic horizontal reaction developed at the supports. The horizontal thrust causes hogging bending moment which tend to reduce the sagging moment due to loading and thus, the net bending moment is much smaller.

TWO HINGED ARCHES

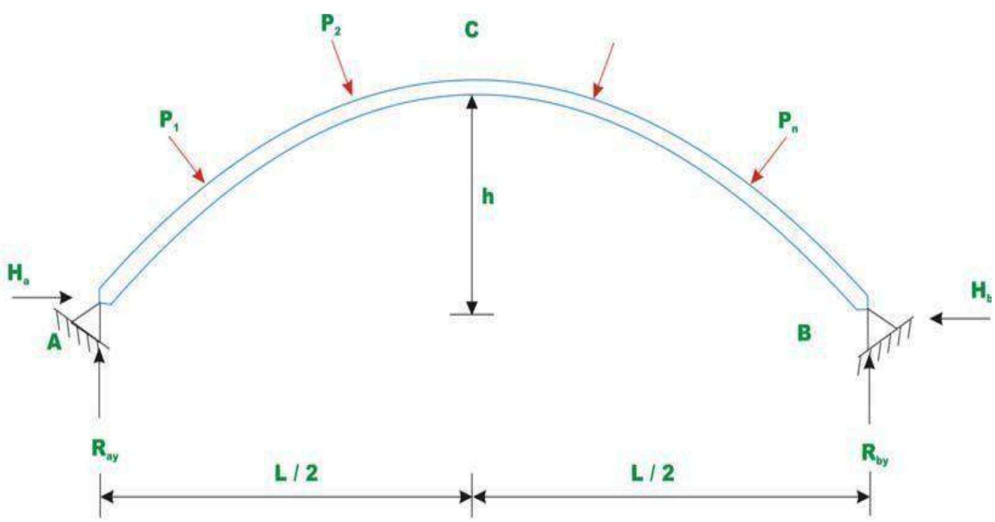
INTRODUCTION

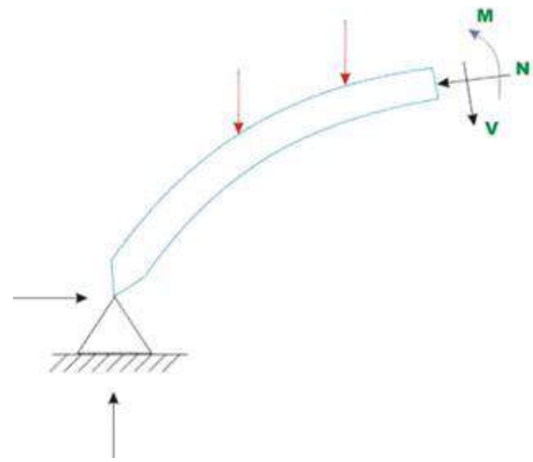
Mainly three types of arches are used in practice: three-hinged, two-hinged and hinge less arches. In the early part of the nineteenth century, three-hinged arches were commonly used for the long span structures as the analysis of such arches could be done with confidence. However, with the development in structural analysis, for long span structures starting from late nineteenth century engineers adopted two-hinged and hinge less arches.

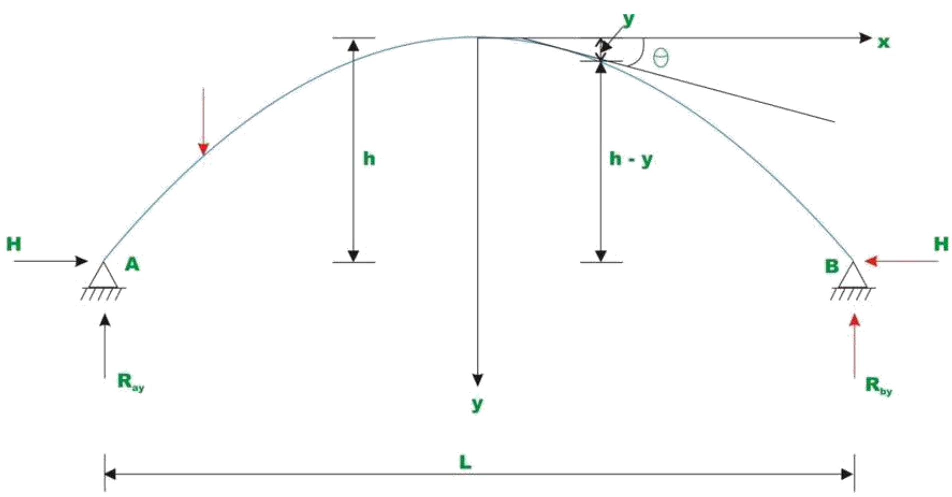
Two-hinged arch is the statically indeterminate structure to degree one. Usually, the horizontal reaction is treated as the redundant and is evaluated by the method of least work. In this lesson, the analysis of two-hinged arches is discussed and few problems are solved to illustrate the procedure for calculating the internal forces.

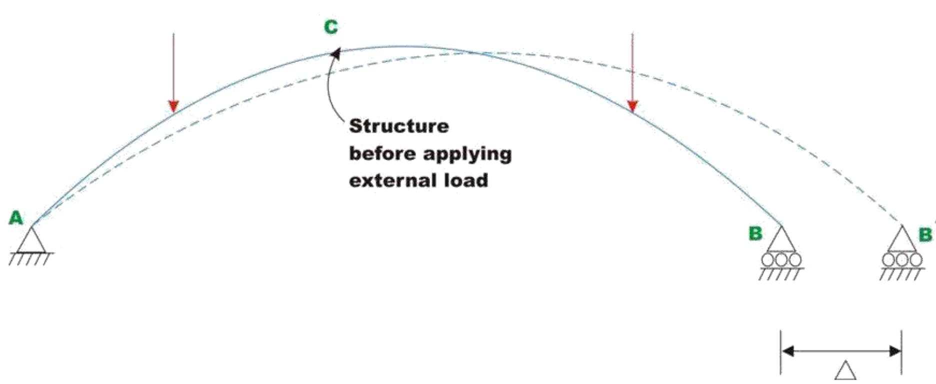
ANALYSIS OF TWO-HINGED ARCH

A typical two-hinged arch is shown in Figure below. In the case of two-hinged arch, we have four unknown reactions, but there are only three equations of equilibrium available. Hence, the degree of statically indeterminacy is one for two- hingedarch.




M
N
V


y
x
Θ
h
h - y
H
A
B
H
R_{ay}
y
R_{by}
L


C
Structure
before applying
external load
A
B
B
△

The fourth equation is written considering deformation of the arch. The unknown redundant reaction $H_b$is calculated by noting that the horizontal displacement of hinge *B* is zero.

In general the horizontal reaction in the two hinged arch is evaluated by straightforward application of the theorem of least work, which states that the partial derivative of the strain energy of a statically indeterminate structure with respect to statically indeterminate action should vanish.

Hence to obtain, horizontal reaction, one must develop an expression for strain energy. Typically, any section of the arch (vide Fig 33.1b) is subjected to shear force*V*, bending moment *M* and the axial compression *N*. The strain energy due to bending $U_b$is calculated from the followingexpression.

$$U_b = \int_0^s \frac{M^2}{2EI} ds$$

The above expression is similar to the one used in the case of straight beams. However, in this case, the integration needs to be evaluated along the curved arch length. In the above equation, *s* is the length of the centerline of the arch, *I is* the moment of inertia of the arch cross section, *E*is the Young's modulus of the arch material.

The strain energy due to shear is small as compared to the strain energy due to bending and is usually neglected in the analysis. In the case of flat arches, the strain energy due to axial compression can be appreciable and is givenby,

$$U_a = \int_0^s \frac{N^2}{2AE} ds$$

The total strain energy of the arch is given by

$$U = \int_0^s \frac{M^2}{2EI} ds + \int_0^s \frac{N^2}{2AE} ds$$

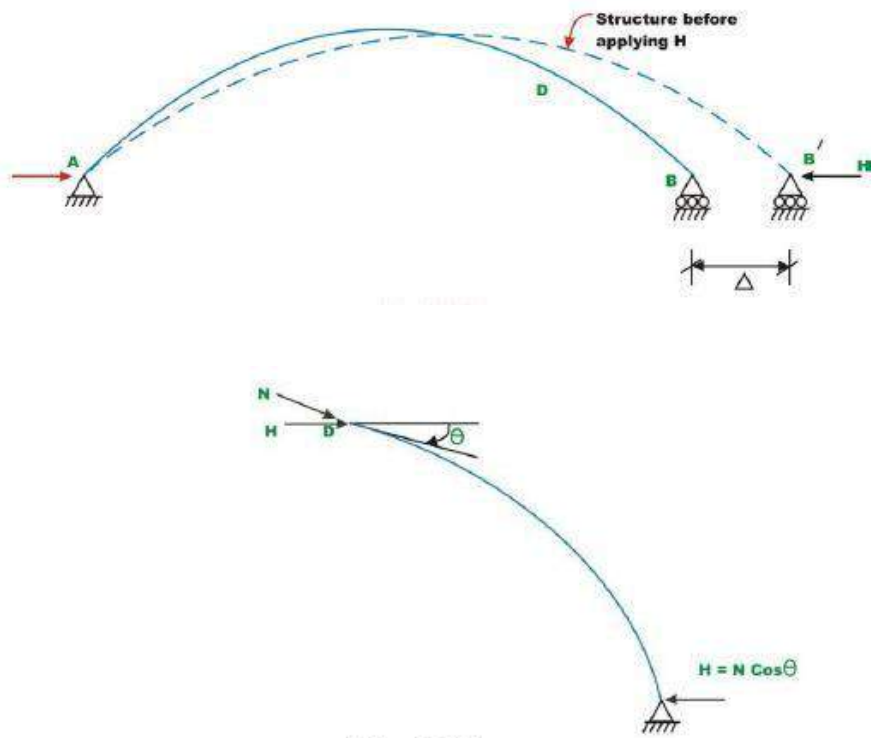
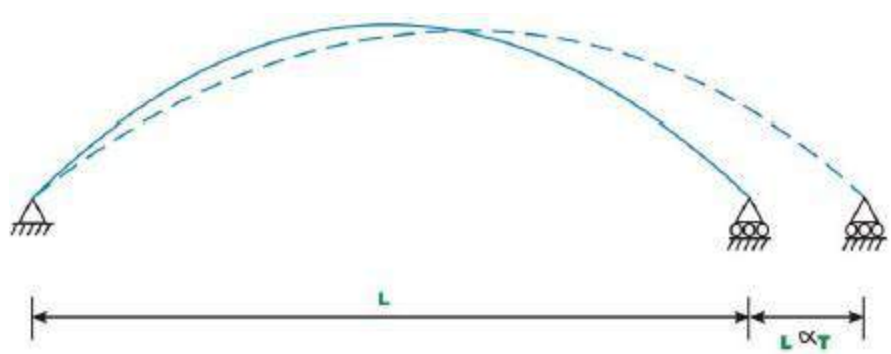
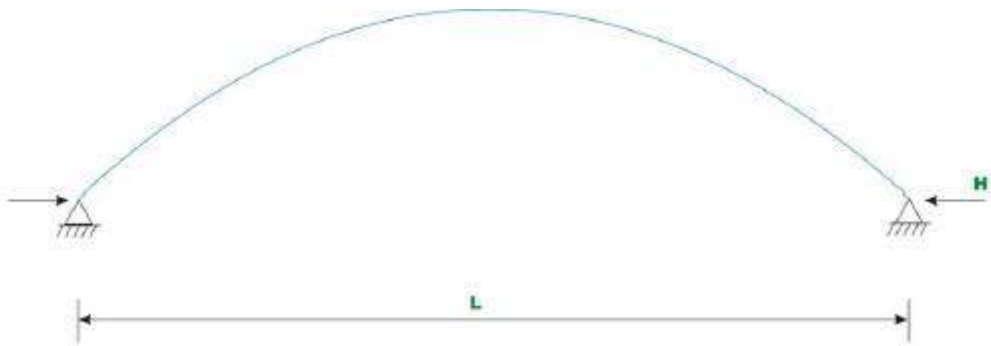


Fig. 33.2d.

TEMPERATURE EFFECT

Consider an unloaded two-hinged arch of span *L*. When the arch undergoes a uniform temperature change of *T*, then its span would increase by $C^{\circ}TL\alpha$ if it were allowed to expand freely (vide Fig 33.3a). α is the co-efficient of thermal expansion of the arch material. Since the arch is restrained from the horizontal movement, a horizontal force is induced at the support as the temperature is increase





Now applying the Castigliano's first theorem,

Solving for H,

$$H = \frac{\alpha LT}{\int_0^s \frac{\tilde{y}^2}{EI} ds + \int_0^s \frac{\cos^2\theta}{EA} ds}$$

The second term in the denominator may be neglected, as the axial rigidity is quite high.

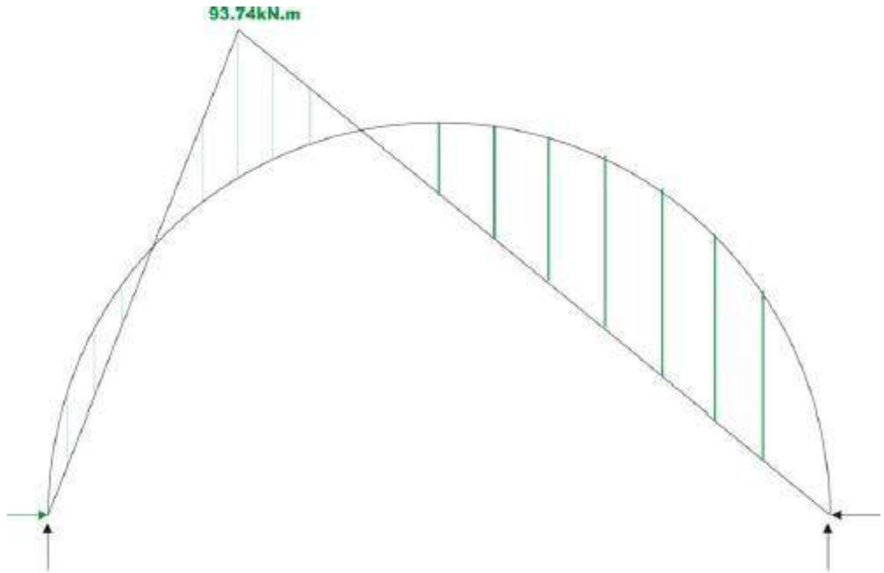
Neglecting the axial rigidity, the above equation can be written as

$$\frac{\partial U}{\partial H} = \alpha LT = \int_0^s \frac{H\tilde{y}^2}{EI} ds + \int_0^s \frac{H\cos^2\theta}{EA} ds$$

is given by,

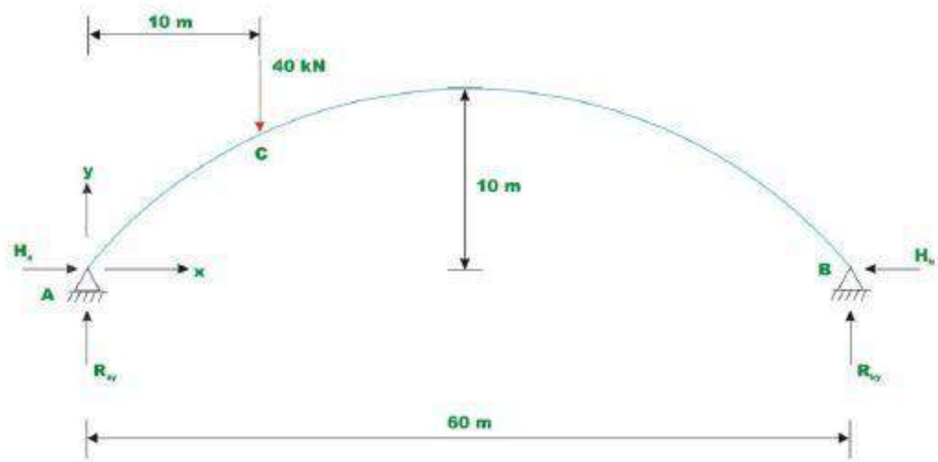
$$H = \frac{\alpha LT}{\int_0^s \frac{\tilde{y}^2}{EI} ds}$$

Using equations bending moment at any angle θ can be computed. The bending moment diagram is shown



A two hinged parabolic arch of constant cross section has a span of 60m and a rise of 10m. It is subjected to loading as shown . Calculate reactions of the arch if the temperature of the arch is raised by. Assume co-efficient of thermal expansion as

$$\alpha = 12 \times 10^{-6} / {}^{\circ}C.$$



Taking *A*as the origin, the equation of two hinged parabolic arch may be written as,

The given problem is solved in two steps.

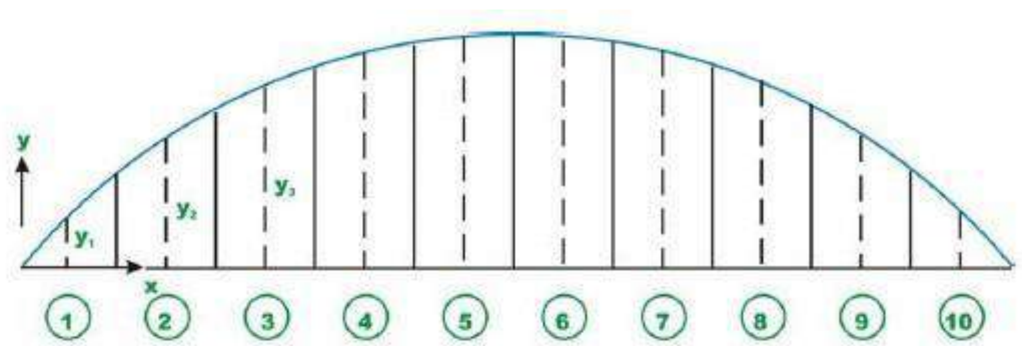
In the first step calculate the horizontal reaction due to 40kN load applied at C.

In the next step calculate the horizontal reaction due to rise in temperature.

Adding both, one gets the horizontal reaction at the hinges due to 40kN combined external loading and temperature change. The horizontal reaction due to load may be calculated by the following equation,

Please note that in the above equation, the integrations are carried out along the *x*- axis instead of the curved arch axis. The error introduced by this change in the variables in the case of flat arches is negligible. Using equation (1), the above equation (3) can be easily evaluated.

The vertical reaction *A* is calculated by taking moment of all forces about *B*. Hence,



	from A (m)		(kNm)		
1	3	1.9	99.99	1139.886	21.66
2	9	5.1	299.97	9179.082	156.06
3	15	7.5	299.95	13497.75	337.5
4	21	9.1	259.93	14192.18	496.86
5	27	9.9	219.91	13062.65	588.06
6	33	9.9	179.89	10685.47	588.06
7	39	9.1	139.87	7636.902	496.86
8	45	7.5	99.85	4493.25	337.5
9	51	5.1	59.83	1830.798	156.06
10	57	1.9	19.81	225.834	21.66
			$\sum$	75943.8	3300.3

$$H_1 = \frac{\sum (M_0)_i y_i (\Delta s)}{\sum (y)_i^2 (\Delta s)_i} = \frac{75943.8}{3200.3} = 23.73 \quad \text{kN} \qquad (10)$$

This compares well with the horizontal reaction computed from the exact integration.

Two-hinged arch is the statically indeterminate structure to degree one. Usually, the horizontal reaction is treated as the redundant and is evaluated by the method of least work. Towards this end, the strain energy stored in the two-hinged arch during deformation is given. The reactions developed due to thermal loadings are discussed. Finally, a few numerical examples are solved to illustrate theprocedure.

UNIT- III

FORCE METHOD OF ANALYSIS OF INDETERMINATE BEAMS

INTRODUCTION TO FORCE AND DISPLACEMENT METHODS OF STRUCTURAL ANALYSIS

Since twentieth century, indeterminate structures are being widely used for its obvious merits. It may be recalled that, in the case of indeterminate structures either the reactions or the internal forces cannot be determined from equations of statics alone. In such structures, the number of reactions or the number of internal forces exceeds the number of static equilibrium equations. In addition to equilibrium equations, compatibility equations are used to evaluate the unknown reactions and internal forces in statically indeterminate structure.

In the analysis of indeterminate structure it is necessary to satisfy theequilibrium equations (implying that the structure is in equilibrium) compatibility equations(requirement if for assuring the continuity of the structure without any breaks) and force displacement equations (the way in which displacement are related to forces). We have two distinct method of analysis for statically indeterminate structure depending upon how the above equations are satisfied:

1. Force method of analysis

2. Displacement method of analysis

In the force method of analysis,primary unknown are forces.In this method compatibility equations are written for displacement and rotations (which are calculated by force displacement equations). Solving these equations, redundant forces are calculated.

Once the redundant forces are calculated, the remaining reactions are evaluated by equations of equilibrium. In the displacement method of analysis,the primary unknowns are the isplacements. In this method, first force -displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure.

After determining the unknown displacements, the other forces are calculated satisfying the compatibility conditions and force displacement relations The displacement-based method is amenable to computer programming and hence the method is being widely used in the modern day structural analysis.

DIFFERENCE BETWEEN FORCE & DISPLACEMENT METHODS

FORCE METHODS	DISPLACEMENT METHODS
1. Method of consistent deformation 2. Theorem of least work 3. Column analogy method 4. Flexibility matrix method	1. Slope deflection method 2. Moment distribution method 3. Kani’s method 4. Stiffness matrix method
Types of indeterminacy- static indeterminacy	Types of indeterminacy- kinematic indeterminacy
Governing equations-compatibility equations	Governing equations-equilibrium equations
Force displacement relations-flexibilitymatrix	Force displacement relations- stiffness matrix

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure.

It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

SLOPE DEFLECTION METHOD

In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements. The slope-deflection method can be used to analyze statically determinate and indeterminate beams and frames.

In this method it is assumed that all deformations are due to bending only. In other words deformations due to axial forces are neglected. In the force method of analysis compatibility equations are written in terms of unknown reactions. It must be noted that all the unknown reactions appear in each of the compatibility equations making it difficult to solve resulting equations.

The slope-deflection equations are not that lengthy in comparison. The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment-displacement relations, moments are then known. The structure is thus reduced to a determinate structure. The slope-deflection method was originally developed by Heinrich Manderla and Otto Mohr for computing secondary stresses in trusses. The method as used today was presented by G.A.Maney in 1915 for analyzing rigid jointed structures.

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

FUNDAMENTAL SLOPE-DEFLECTION EQUATIONS:

The slope deflection method is so named as it relates the unknown slopes and deflections to the applied load on a structure. In order to develop general form of slope deflection equations, we will consider the typical span AB of a continuous beam which is subjected to arbitrary loading and has a constant EI. We wish to relate the beams internal end moments in terms of its three degrees of freedom, namely its angular displacements and linear displacement which could be caused by relative settlements between the supports.

Since we will be developing a formula, moments and angular displacements will be considered positive, when they act clockwise on the span. The linear displacement will be considered positive since this displacement causes the chord of the span and the span's chord angle to rotate clockwise.

The slope deflection equations can be obtained by using principle of superposition by considering separately the moments developed at each supports due to each of the displacements & then the loads.

GENERAL PROCEDURE OF SLOPE-DEFLECTION METHOD

- Find the fixed end moments of each span (both ends left & right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
  - Determine the end moments and then treat each span as simply supported beam subjected to given load & end moments so we can work out the reactions & draw the bending moment & shear force diagram.

CONTINUOUS BEAMS

Introduction

Beams are made continuous over the supports to increase structural integrity. Acontinuous beam provides an alternate load path in the case of failure at a section. Inregions with high seismic risk, continuous beams and frames are preferred in buildingsand bridges. A continuous beam is a statically indeterminate structure.The advantages of a continuous beam as compared to a simply supported beam are asfollows.

1) For the same span and section, vertical load capacity is more.

2) Mid span deflection is less.

3) The depth at a section can be less than a simply supported beam for the samespan.

Else, for the same depth the span can be more than a simply supportedbeam.

⇒The continuous beam is economical in material.

4) There is redundancy in load path.

⇒ Possibility of formation of hinges in case of an extreme event.

5) Requires less number of anchorages of tendons.

6) For bridges, the number of deck joints and bearings are reduced.

⇒ Reduced maintenance

There are of course several disadvantages of a continuous beam as compared to asimply supported beam.

1) Difficult analysis and design procedures.

2) Difficulties in construction, especially for precast members.

3) Increased frictional loss due to changes of curvature in the tendon profile.

4) Increased shortening of beam, leading to lateral force on the supporting columns.

5) Secondary stresses develop due to time dependent effects like creep andshrinkage, settlement of support and variation of temperature.

6) The concurrence of maximum moment and shear near the supports needsproper detailing of reinforcement.

7) Reversal of moments due to seismic force requires proper analysis and design.

THE ANALYSIS OF CONTINUOUS BEAMS IS BASED ON ELASTIC THEORY.

For pre-stressed beams the following aspects are important.

1) Certain portions of a span are subjected to both positive and negative moments.

These moments are obtained from the envelop moment diagram.

2) The beam may be subjected to partial loading and point loading. The envelop moment diagrams are developed from "pattern loading". The pattern loading refers to the placement of live loads in patches only at the locations with positive or negative values of the influence line diagram for a moment at a particular location.

3) For continuous beams, pre-stressing generates reactions at the supports. These reactions cause additional moments along the length of a beam.

The analysis of a continuous beam is illustrated to highlight the aspects stated earlier. The bending moment diagrams for the following load cases are shown schematically in the following figures.

1) Dead load (DL)

2) Live load (LL) on every span

3) Live load on a single span.

For moving point loads as in bridges, first the influence line diagram is drawn. The influence line diagram shows the variation of the moment or shear for a particular location in the girder, due to the variation of the position of a unit point load. The vehicle load is placed based on the influence line diagram to get the worst effect. An influence line diagram is obtained by the Müller-Breslau Principle.

IS:456 - 2000, Clause 22.4.1, recommends the placement of live load as follows.

1) LL in all the spans.

2) LL in adjacent spans of a support for the support moment. The effect of LL in the alternate spans beyond is neglected.

3) LL in a span and in the alternate spans for the span moment.

The envelop moment diagrams are calculated from the analysis of each load case and their combinations. The analysis can be done by moment distribution method or by computer analysis.

In lieu of the analyses, the moment coefficients in Table 12 of IS: 456 - 2000 can be used under conditions of uniform cross-section of the beams in the several spans, uniform loads and similar lengths of span.

The envelop moment diagrams provide the value of a moment due to the external loads. It is to be noted that the effect of pre-stressing force is not included in the envelop moment diagrams.

Consider a continuous beam over several supports carrying arbitrary loads, w(x).Using the Moment-Area Theorem, we will analyze two adjoining spans of this beam to find the relationship between the internal moments at each of the supports and the loads applied to the beam. We will label the left, center ,and right supports of this two-span segment L, C, and R. The left span has length LL and flexural rigidity EIL; the right span has length LR and flexural rigidity EIR.

Applying the principle of superposition to this two-span segment, we can separate the moments caused by the applied loads from the internal moments at the supports. The two-span segment can be represented by two simply supported spans (with zero moment at L, C, and R) carrying the external loads plus two simply-supported spans carrying the internal moments ML,MC, and MR . The applied loads are illustrated below the beam, so as not to confuse the loads with the moment diagram. Note that we are being consistent with our sign convention: positive moments create positive curvature in the beam. The internal moments ML, MC, and MR are drawn in the positive directions. The areas under the moment diagrams due to the applied loads on the simply supported spans are AL and AR; ¯xL represents the distance from the left support to the centroid of AL, and ¯xR represents the distance from the right support to the centroid of AR, as shown. The moment diagrams due to the unknown moments, ML, MC, and MR are triangular, All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations. These degrees of freedom are specified at supports, joints and at the free ends.

Examining the elastic curve of the continuous beam, we recognize that the rotation of the beam at the center support, θC, is continuous across support C. In other words, θC just to the left of point C is the same as θC

Just to the right of point C. This continuity condition may be expressed

ΔLtan C LL = − ΔR tan C LR , (1)

Where Δ Ltan C is the distance from the tangent at C to point L, and ΔR tan C is the distance from the tangent at C to point R.

Using the second Moment-Area Theorem, and assuming that the flexural rigidity (EI) is constant within each span, we can find the terms ΔLtanC, and ΔR tan C in terms of the unknown moments, ML, MC, and MR and the known applied loads.

To apply the three-moment equation numerically, the lengths, moments of inertia, and applied loads must be specified for each span. Two commonly applied loads are point loads and uniformly distributed loads.

For point loads PL and PR acting a distance xL and xR from the left and right supports respectively, the right hand side of the three-moment equation becomes the equation of three moments is set up for each pair of adjacent spans with all pairs considered in succession. Consequently the number of equations for a multi-span beam is equal to the degree of static indeterminacy.

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are

commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations. These degrees of freedom are specified at supports, joints and at the free ends.

Application of the equation of three moments.

(1) First of all we determine the degree of static indeterminacy according to the formula

$$K = - m n ,$$

where K is the degree of static indeterminacy, m is the number of unknown reactions, n is the number of equations of static equilibrium. So, m n = = 4; 3 and

K =1. The fact of the beam being singly statically indeterminate gets obvious.

Bearing of Lintel

The bearing provided should be the minimum of following 3 cases.

i. 10 cm
ii. Height of beam
iii. 1/10th to 1/12th of span of the lintel.

Types of Lintel used in Building Construction

Lintels are classified based on the material of construction as:

1. Timber Lintel

In olden days of construction, Timber lintels were mostly used. But now a days they are replaced by several modern techniques, however in hilly areas these are using. The main disadvantages with timber are more cost and less durable and vulnerable to fire.

If the length of opening is more, then it is provided by joining multiple number of wooden pieces with the help of steel bolts In case of wider walls, it is composed of two wooden pieces kept at a distance with the help of packing pieces made of wood. Sometimes, these are strengthened by the provision of mild steel plates at their top and bottom, called as flitched lintels.

Stone Lintel

These are the most common type, especially where stone is abundantly available. The thickness of these are most important factor of its design. These are also provided over the openings in brick walls. Stone lintel is provided in the form of either one single piece or more than one piece.

The depth of this type is kept equal to 10 cm / meter of span, with a minimum value of 15 cm. They are used up to spans of 2 meters. In the structure is subjected to vibratory loads, cracks are formed in the stone lintel because of its weak tensile nature. Hence caution is needed.

Brick Lintel

These are used when the opening is less than 1m and lesser loads are acting. Its depth varies from 10 cm to 20 cm, depending up on the span. Bricks with frogs are more suitable than normal bricks because frogs when filled with mortar gives more shear resistance of end joints which is known as joggled brick lintel.

Reinforced Brick Lintel

These are used when loads are heavy and span is greater than 1m. The depth of reinforced brick lintel should be equal to 10 cm or 15 cm or multiple of 10 cm. the bricks are so arranged that 2 to 3 cm wide space is left length wise between adjacent bricks for the insertion of mild steel bars as reinforcement. 1:3 cement mortar is used to fill up the gaps.

Vertical stirrups of 6 mm diameter are provided in every 3rd vertical joint. Main reinforcement is provided at the bottom consists 8 to 10 mm diameter bars, which are cranked up at the ends.

Steel Lintel

These are used when the superimposed loads are heavy and openings are large. These consist of channel sections or rolled steel joists. We can use one single section or in combinations depending up on the requirement.

When used singly, the steel joist is either embedded in concrete or cladded with stone facing to keep the width same as width of wall. When more than one units are placed side by side, they are kept in position by tube separators.

Reinforced Cement Concrete Lintel

At present, the lintel made of reinforced concrete are widely used to span the openings for doors, windows, etc. in a structure because of their strength, rigidity, fire resistance, economy and ease in construction. These are suitable for all the loads and for any span. The width is equal to width of wall and depth depends on length of span and magnitude of loading.

Main reinforcement is provided at the bottom and half of these bars are cranked at the ends. Shear stirrups are provided to resist transverse shear

ARCH

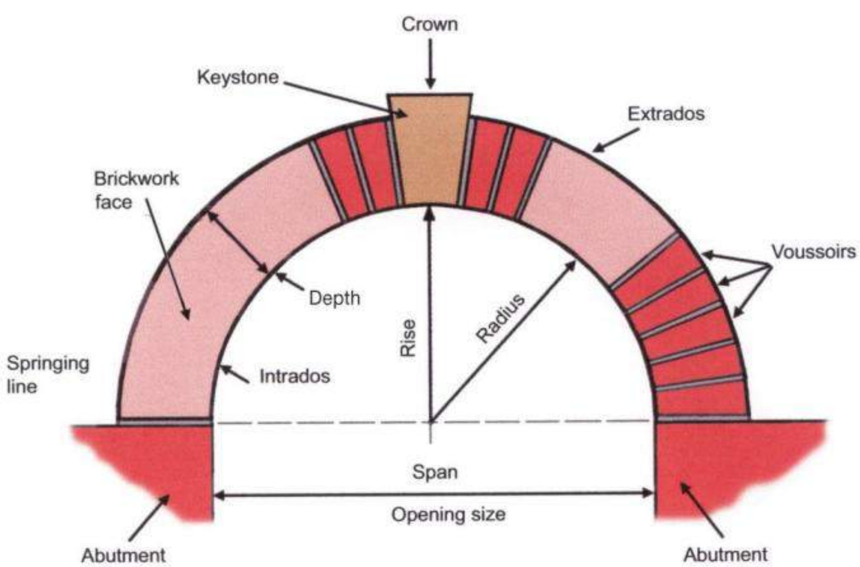
An arch is a structure constructed in curved shape with wedge shaped units (either bricks or stones), which are jointed together with mortar, and provided at openings to support the weight of the wall above it along with other superimposed loads.

Because of its shape the loads from above gets distributed to supports (pier or abutment).

Different Components of an Arch

The following are the different components of arches and terms used in arch construction:

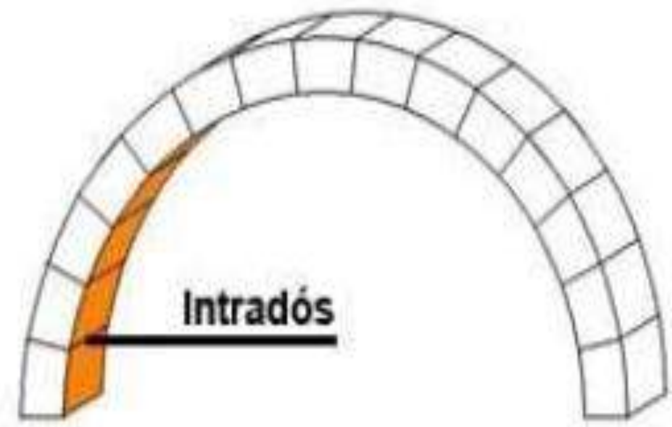
:



Intrados

The curve which bounds the lower edge of the arch OR The inner curve of an arch is called as intrados.

The distinction between soffit and intrados is that the intrados is a line, while the soffit is a surface.



Extrados

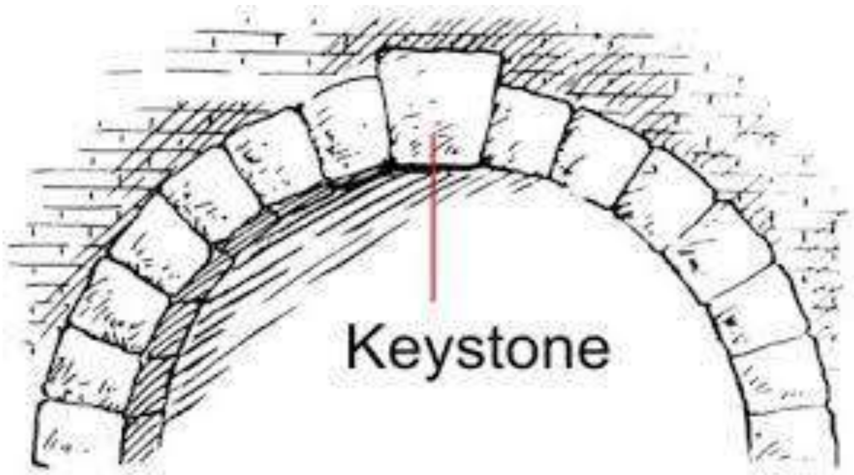
The outer curve of an arch is termed as extrados.

Crown

The apex of the arch's extrados. In symmetrical arches, the crown is at the mid span.

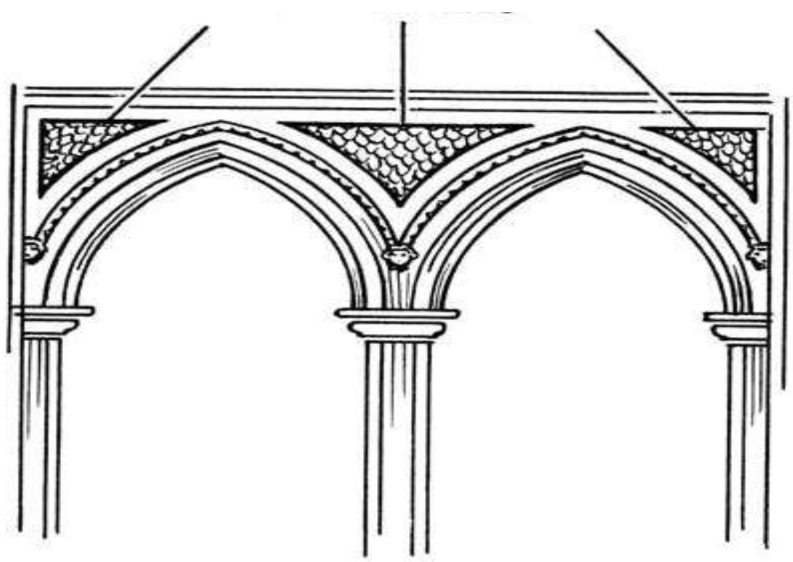
Keystone

The wedge shaped unit which is fixed at the crown of the arch is called keystone.



Spandrel in an Arch

If two arches are constructed side by side, then a curved triangular space is formed between the extrados with the base as horizontal line through the crown. This space is called as spandrel.



Skew Back

The surface on which the arch joins the supporting abutment.

The upper surface of an abutment or pier from which an arch springs; its face is on a line radiating from the center of the arch.

Springing Points

The imaginary points which are responsible for the springing of curve of an arch are called as springing points.

Springing Line

The imaginary line joining the springing points of either ends is called as springing line.

Springer in Arches

The first voussoir at springing level which is immediately adjacent to the skewback is called as springer.

Haunch

The lower half of the arch between the crown and skewback is called haunch. Highlighted area in the below fig is haunch.Span of an Arch

The clear horizontal distance between the supports or abutments or piers is termed as span of an arch.

Rise of an Arch

The clear vertical distance between the highest point on the intrados and the springing line is called as rise.

Pier and Abutment of an Arch

The intermediate support of an arch is called as pier. The end support of an arch is called as abutment.

Muram or Mud Floors:

The ground floor having its topping consisting of muram or mud is called Muram or Mud Floors

These floors are easily and cheaply repairable Method of Construction:

- The surface of earth filling is properly consolidated
- 20cm thick layer of rubble or broken bats is laid, hand packed, wet and rammed
- 15cm thick layer of muram or good earth is laid
- 2.5cm thick layer of powdery variety of muram earth is uniformly spread
- The whole surface is well watered and rammed until the cream of muram earth rises to the earth surface
- After 12 hours the surface is again rammed for three days.
- The surface is smeared with a thick paste of cow-dung and rammed for two days
- Thin coat of mixture of 4 parts of cow-dung and 1 part of Portland cement is evenly applied the surface is wiped clean by hand.
- For maintaining this type of floor properly, leaping is done once a week

Suitability: These floors are generally used for unimportant building in rural areas

Cement Concrete Floor:

The floor having its topping consisting of cement concrete is called Cement Concrete Floor or Conglomerate Floor Types of Cement Concrete Floor:

According to the method of finishing the topping, Cement Concrete Floor can be classified into the following two types

1. Non-monolithic or bonded floor finish concrete floor
2. Monolithic floor finish concrete floor

Non-monolithic or bonded floor finish concrete floor:

The type of Cement Concrete Floor in which the topping is not laid monolithically with the base concrete is known as Non-monolithic or bonded floor finish concrete floor.

Method of Construction:

1. The earth is consolidated.
2. 10cm thick layer of clean sand is spread.
3. 10cm thick Lime Concrete (1:4:8) or Lean Cement Concrete (1:8:16) is laid thus forming base concrete
4. The topping {4cm thick Cement Concrete (1:2:4)} is laid on the third day of laying base cement concrete, thus forming Non-monolithic construction.

This type of construction is mostly adopted in the field

The topping is laid by two methods:

I- Topping laid in single layer:

The topping consists of single layer of Cement Concrete (1:2:4), having its thickness 4cm

II- Topping laid two layers:

The topping consists of 1.5cm thick Cement Concrete (1:2:3), which is laid monolithically over 2.5cm thick Cement Concrete (1:3:6)

Monolithic Floor Finish Concrete Floor:

The Cement Concrete Floor in which the topping consisting of 2cm thick Cement Concrete (1:2:4) is laid monolithically with the Base Concrete is known as Monolithic Floor Finish Concrete Floor.

Method of Construction:

1. The surface of muram or earth filling is leveled, well watered and rammed
2. 10cm layer of clean and dry sand is spread over
3. When the sub soil conditions are not favorable and monolithic construction is desired, then, 5cm to 10cm thick hard core of dry brick or rubble filling is laid.
4. 10cm thick layer of Base Concrete consisting of Cement Concrete (1:4:8) or Lean Cement Concrete (1:8:16) is laid.
5. The topping {2cm thick layer of Cement Concrete(1:2:4)} is laid after 45 minutes to 4 hours of laying Base Concrete.

Tile Floor: The floor having its topping consisting of tiles is called tile floor. Method of Construction:

1. The muram or earth filling is properly consolidated.
2. 10cm thick layer of dry clean sand is evenly laid
3. 10cm thick layer of Lime Concrete (1:4:8) or Lean Cement Concrete (1:8:16) is laid, compacted and cured to form a base concrete.
4. A thin layer of lime or cement mortar is spread with the help of screed battens.
5. Then the screed battens are properly leveled and fixed at the correct height.
6. When the surface mortar is harden sufficiently, 6mm thick bed of wet cement (1:5) is laid and then over this the specified tiles are laid.
7. The surplus mortar which comes out of the joints is cleaned off.
8. After 3 days, the joints are well rubbed with a corborundum stone to chip off all the projecting edges.
9. Rubbing should not be done in case of glazed tiles.
10. The surface is polished by rubbing with a softer variety of a corborundum or a pumice stone.
11. The surface is finally washed with soap.

Suitability: This type of floor is suitable for courtyard of buildings. Glazed tiles are used in modern buildings where a high class finish is desired.

Mosaic Floors:

The floors having its topping consisting of mosaic tiles or small regular cubes, square or hexagons, embedded into a cementing mixture is known as Mosaic Floors

Method of Construction:

1. The earth is consolidated.
2. 10cm thick layer of clean sand is spread.
3. 10cm thick Lime Concrete (1:4:8) or Lean Cement Concrete (1:8:16) is laid thus forming base concrete.
4. Over this base course 5cm thick Lime Mortar or Cement Mortar or Lime and Surkhi mortar (1:2) is laid.
5. The mortar is laid in small area so that the mortar may not get dried before finishing the wearing course.
6. 3mm thick cementing mixture is spread.
7. The cementing mixture consists of one part of pozzolana, one part of marble chips and two parts of slacked lime.
8. After nearing 4 hours, patterns are formed on the top of the cementing material.
9. Now the tiles of regular shaped marble cubes are hammered in the mortar along the outline of the pattern.
10. The inner spaces are then filled with colored pieces of marble.
11. A roller 30cm in diameter and 50cm in length is passed gently over the surface.
12. Water is sprinkled to work up the mortar between the marble pieces.
13. The surface is then rubbed with pumice stone fixed to a wooden handle about 1.5m long.
14. The surface is then allowed to dry up for 2 weeks.

Double Flag Stone floor

Two layer of flagstones are used to build this type of floor, this is why it is called double flag stone floor.

Materials used to build this type of floor are –

- Flagstone (about 40 mm thickness)
- Rolled steel joist
- Rolled Steel beam (for span above 4 meter)

Procedure:

For span above 4 meter, a framework is built consist of rolled steel beam and rolled steel joist. To make formwork, beam are place at 10 feet centre to centre distance then joists are placed at right angle to the beam. And then two layers of flagstone are fixed with the joist. One layer is at top flanged of joist and another layer is at bottom flanged of joist. The gap between the two layers of flagstone is filled with earth or concrete before fixing the top layer of flagstone.

Jack arch floor

You'll find the following components/materials in this type of floor –

- Arch (brick arch or concrete arch)
- Rolled steel joist
- Rolled steel beam
- Wall
- Tie rod

Mechanism

Joists are placed on wall or beam and tied together with the tie rod. And then concrete arches or brick arches are constructed and rest on lower flanged of Joists.

Non-Composite Floor

Non composite type of floors are those which are built using one material only. Mostly used material for non-composite floor is timber.

Timber floor can further be divided into 3 types

- Single joist floor
- Double joist floor

- Tripple joist floor,

Floor board: Floor board are fixed at the top of bridging joist. It acts as the wearing of the top surface of the floor.

Floor ceiling: To make the bottom of the floor flat and increasing the aesthetic look floor ceiling is provided. For this purpose plaster board or sheet of asbestors or some other suitable materials are used. Floor ceiling rests on bridging joist. To make the ceiling more durable and strong ceiling joist may be provided at the right angle to the bridging joist.

Single joist floor

In this type of floor single joist is placed below floor board. This joist is supported by wall-plate at both end.

Double joist floor

In this type of floor binders are provided to support the bridging joist. Binders are then rest on the walls at both end.

Triple joist floor

Triple joist floor is also called framed timber floor. In this type of floor another member is added that is girder, which we didn’t use in double joist floor.

These girders are placed on the wall to support the binders. And then joist are placed on the binders.

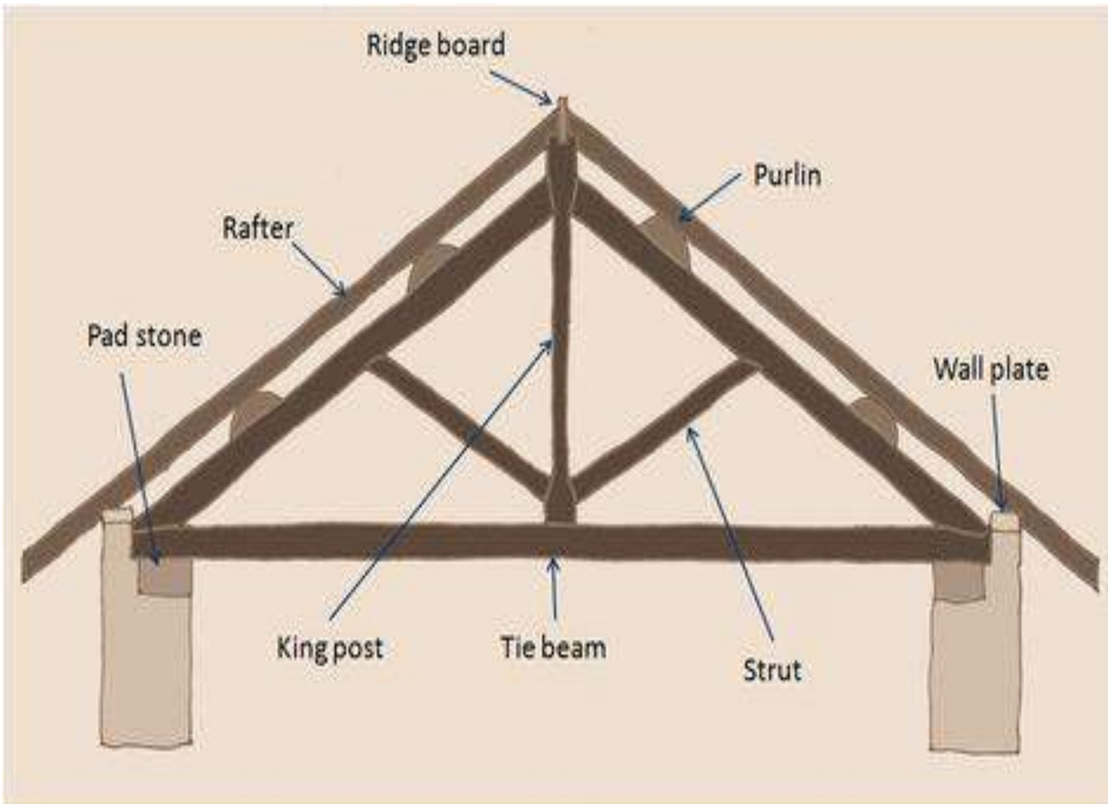
In this post, I did not include floors which are built on ground as a type of floor. Because, I think, it does not need serious mechanism to build a floor on ground. Because there is a ground itself to support the floor.

Roof truss

A roof truss is basically a structure that includes one or multiple triangular units that include straight slender members with their ends connected via nodes.

King Post Truss

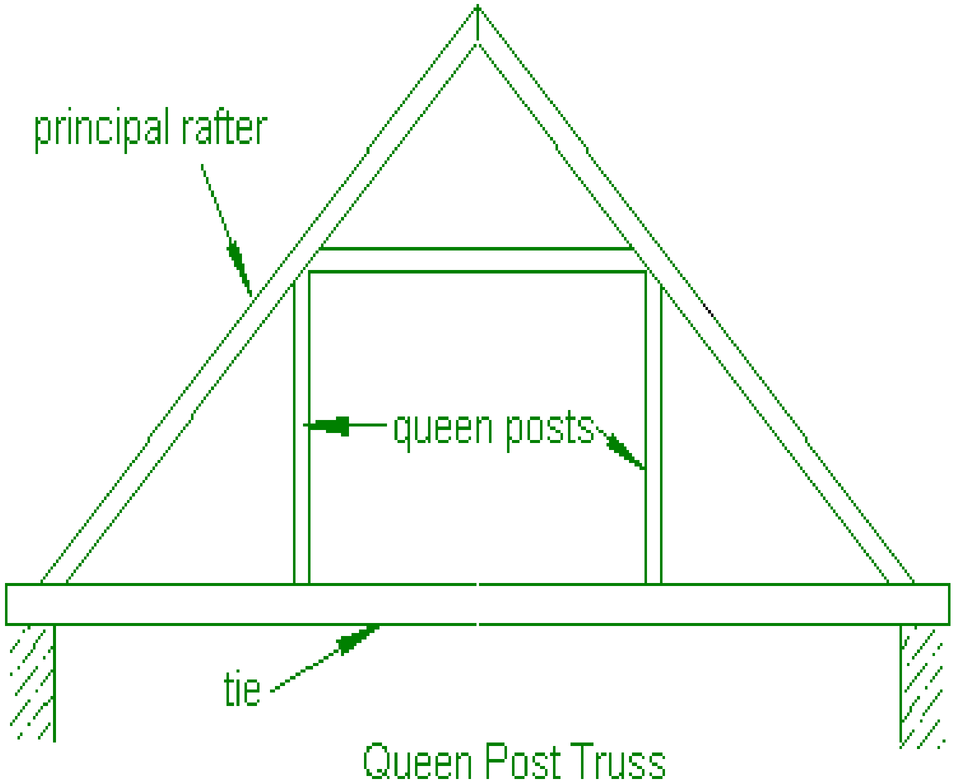
This particular truss is made out of wood most of the time, but it can also be built out of a combination of steel and wood. It all comes down to the architect and the building structure. The King Post Truss spans up to 8m, which makes it perfect for multiple types of houses, especially the smaller ones.



Queen Post Truss

The Queen Post Truss is designed to be a very reliable, simple and versatile type of roof truss that you can use at any given time.

It offers a good span, around 10m, and it has a simple design which makes it perfect for a wide range of establishments.



North Light Roof Truss

The North Light Roof Truss is suitable for the larger spans that go over 20m and get up to 30m. This happens because it's cheaper to add a truss that has a wide, larger set of lattice girders that include support trusses.

Mechanism

Joists are placed on wall or beam and tied together with the tie rod. And then concrete arches or brick arches are constructed and rest on lower flanged of Joists.

Non-Composite Floor

Non composite type of floors are those which are built using one material only. Mostly used material for non-composite floor is timber.

Timber floor can further be divided into 3 types

- Single joist floor

- Double joist floor
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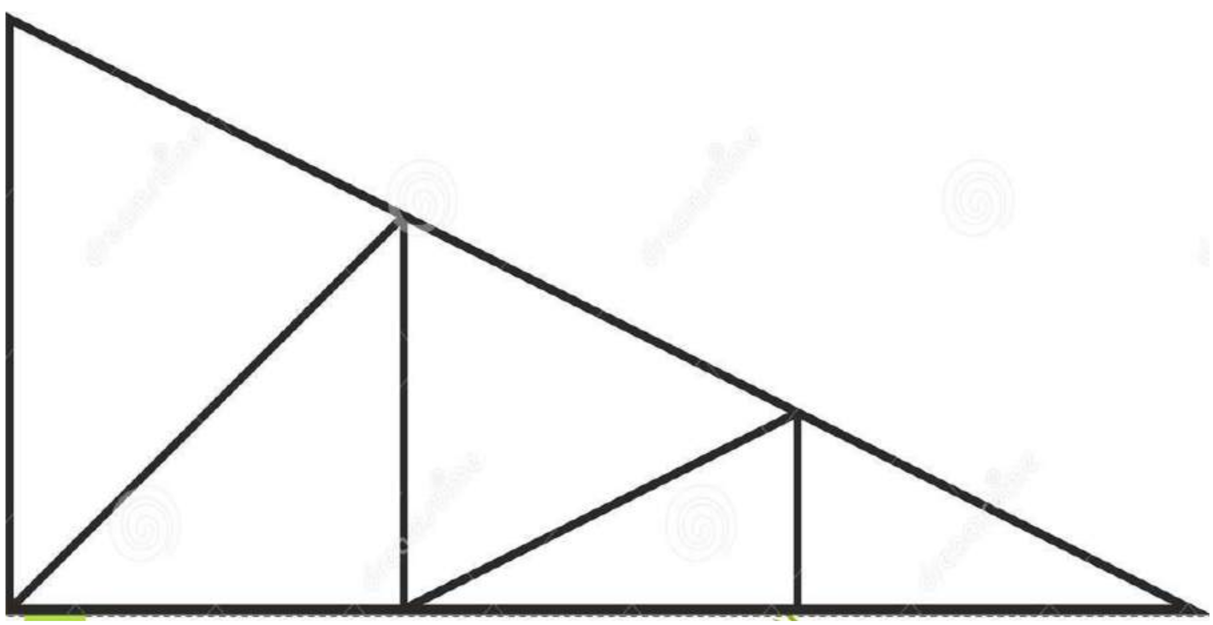
Double joist floor

In this type of floor binders are provided to support the bridging joist. Binders are then rest on the walls at both end.



This method is one of the oldest, as well as most economical ones that you can find on the market, as it allows you to bring in proper ventilation. Plus, the roof has more resistance too because of that.

If you are looking for types of roof trusses design that bring in durability and versatility, this is a very good one to check out. You can use it for industrial buildings, but this truss also works for drawing rooms and in general those spaces that are very large.

UNIT- IV

DISPLACEMENT METHOD OF ANALYSIS: SLOPE DEFLECTION ANDMOMENT DISTRIBUTION

Since twentieth century, indeterminate structures are being widely used for its obvious merits. It may be recalled that, in the case of indeterminate structures either the reactions or the internal forces cannot be determined from equations of statics alone. In such structures, the number of reactions or the number of internal forces exceeds the number of static equilibrium equations. In addition to equilibrium equations, compatibility equations are used to evaluate the unknown reactions and internal forces in statically indeterminate structure. In the analysis of indeterminate structure it is necessary to satisfy theequilibrium equations (implying that the structure is in equilibrium) compatibility equations(requirement if for assuring the continuity of the structure without any breaks) and force displacement equations (the way in which displacement are related to forces). We have two distinct method of analysis for statically indeterminate structure depending upon how the above equations are satisfied:

1. Force method of analysis

2. Displacement method of analysis

In the force method of analysis, primary unknown are forces. In this method compatibility equations are written for displacement and rotations (which are calculated by force displacement equations). Solving these equations, redundant forces are calculated. Once the redundant forces are calculated, the remaining reactions are evaluated by equations of equilibrium. In the displacement method of analysis, the primary unknowns are the displacements. In this method, first force -displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure. After determining the unknown displacements, the other forces are calculated satisfying the compatibility conditions and force displacement relations.

The displacement-based method is amenable to computer programming and hence the method is being widely used in the modern day structural analysis.

DIFFERENCE BETWEEN FORCE & DISPLACEMENT METHODS

FORCE METHODS	DISPLACEMENT METHODS
1. Method of consistent deformation 2. Theorem of least work 3. Column analogy method 4. Flexibility matrix method	1. Slope deflection method 2. Moment distribution method 3. Kani’s method 4. Stiffness matrix method
Types of indeterminacy- static indeterminacy	Types of indeterminacy- kinematic indeterminacy
Governing equations-compatibility equations	Governing equations-equilibrium equations
Force displacement relations- flexibility matrix	Force displacement relations- stiffness matrix

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

SLOPE DEFLECTION METHOD

In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements. The slope-deflection method can be used to analyze statically determinate and indeterminate beams and frames.

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In this method it is assumed that all deformations are due to bending only. In other words deformations due to axial forces are neglected. In the force method of analysis compatibility equations are written in terms of unknown reactions. It must be noted that all the unknown reactions appear in each of the compatibility equations making it difficult to solve resulting equations. The slope-deflection equations are not that lengthy in comparison. The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment-displacement relations, moments are then known.

The structure is thus reduced to a determinate structure. The slope-deflection method was originally developed by Heinrich Manderla and Otto Mohr for computing secondary stresses in trusses. The method as used today was presented by G.A.Maney in 1915 for analyzing rigid jointed structures.

FUNDAMENTAL SLOPE-DEFLECTION EQUATIONS:

The slope deflection method is so named as it relates the unknown slopes and deflections to the applied load on a structure. In order to develop general form of slope deflection equations, we will consider the typical span AB of a continuous beam which is subjected to arbitrary loading and has a constant EI. We wish to relate the beams internal end moments in terms of its three degrees of freedom, namely its angular displacements and linear displacement which could be caused by relative settlements between the supports. Since we will be developing a formula, moments and angular displacements will be considered positive, when they act clockwise on the span. The linear displacement will be considered positive since this displacement causes the chord of the span and the span's chord angle to rotate clockwise. The slope deflection equations can be obtained by using principle of superposition by considering separately the moments developed at each supports due to each of the displacements & then the loads.

GENERAL PROCEDURE OF SLOPE-DEFLECTION METHOD

- Find the fixed end moments of each span (both ends left & right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
- Determine the end moments and then treat each span as simply supported beamsubjected to given load & end moments so we can work out the reactions & drawthe bending moment & shear force diagram.

Loads and displacements are vector quantities and hence a proper coordinate system is required to specify their correct sense of direction. Consider a planar truss, In this truss each node is identified by a number and each member is identified by a number enclosed in a circle. The displacements and loads acting on the truss are defined with respect to global co-ordinate system *xyz.*

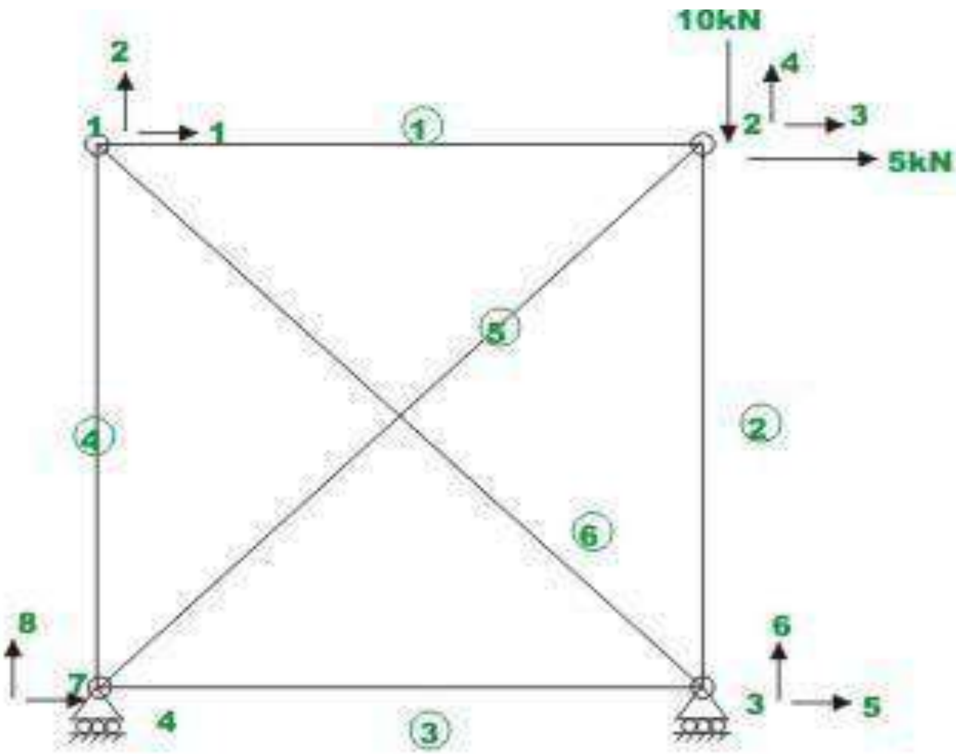
The same co ordinate system is used to define each of the loads and displacements of all loads. In a global co-ordinate system, each node of a planer truss can have only two displacements: one along *x* -axis and another along *y*-axis. The truss shown in figure has eight displacements. Each displacement (degree of freedom) in a truss is shown by a number in the figure at the joint.

The direction of the displacements is shown by an arrow at the node. However out of eight displacements, five are unknown. The displacements indicated by numbers 6, 7 and 8 are zero due to support conditions.

The displacements denoted by numbers 1-5 are known as unconstrained degrees of freedom of the truss and displacements denoted by 6-8 represent constrained degrees of freedom. In this course, unknown displacements are denoted by lower numbers and the known displacements are denoted by higher code numbers.

Consider a planar truss, In this truss each node is identified by a number and each member is identified by a number enclosed in a circle.

The displacements and loads acting on the truss are defined with respect to global co-ordinate system *xyz.*



MEMBER STIFFNESS MATRIX INTRODUCTION

To analyse the truss shown in, the structural stiffness matrix *K* need to be evaluated for the given truss. This may be achieved by suitably adding all the member stiffness matrices *k*', which is used to express the force-displacement relation of the member in local co-ordinate system. Since all members are oriented at different directions, it is required to transform member displacements and forces from the local co-ordinate system to global co-ordinate system so that a global load-displacement relation may be written for the complete truss.

MEMBER STIFFNESS MATRIX ANALYSIS

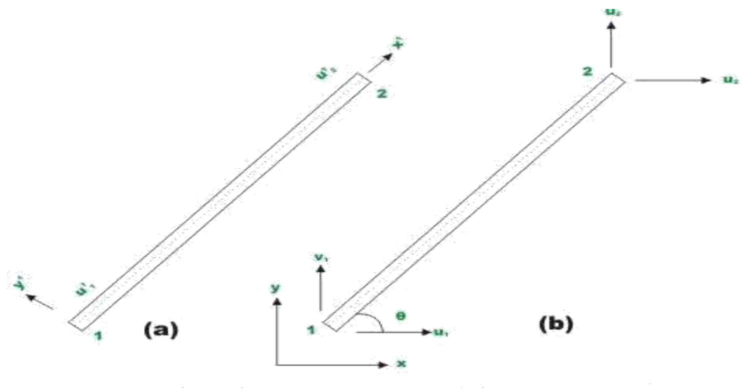
Consider a member of the truss in local co-ordinate system *x*'*y*'. As the loads are applied along the centroidal axis, only possible

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

refers to node 1 of the truss member and subscript 2 refers to node 2 of the truss member. Give displacement u'_1 at node 1 of the member in the positive *x*' direction, keeping all other displacements to zero. This displacement in turn **TRANSFORMATION FROM LOCAL TO GLOBAL CO-ORDINATE SYSTEM.**

Displacement Transformation Matrix

A truss member is shown in local and global co ordinate system in Figure. Let *x*' *y*' *z*'be in local co ordinate system and*xyz* be the global co ordinate system.



The nodes of the truss member be identified by 1 and 2. Let u'_1 and u'_2 be the displacement of nodes 1 and 2 in local co ordinate system. In global co ordinate system, each node has two degrees of freedom. Thus, u_1,v_1 and u_2,v_2 are the nodal displacements at nodes 1 and 2 respectively along x - and y - directions.

Let the truss member be inclined to x axis by θ as shown in figure. It is observed from the figure that u'_1 is equal to the projection of u_1 on x' axis plus projection of $v_1$on x'-axis. Thus, (vide Fig. 24.7)

u'_1□ $u_1\cos\theta$□ $v_1\sin\theta$

u'_2□ $u_2\cos\theta$□ $v_2\sin\theta$This may be written as

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

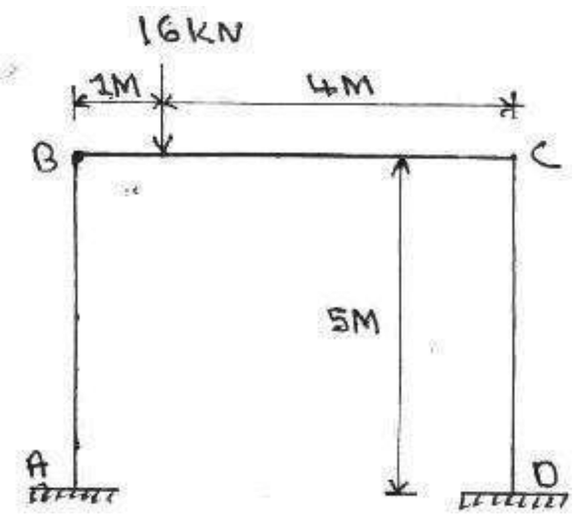
All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

MOMENT DISTRIBUTION METHOD FOR FRAMES WITH SIDE SWAY

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Analyze the frame shown in figure by moment distribution method. Assume EI is constant.

Non SwayAnalysis:



First consider the frame without side sway

$$M_{FAB} = M_{FBA} = M_{FCD} = 0$$

$$M_{FBC} = -\frac{16\times1\times4^2}{5^2} = -10.24KNm$$

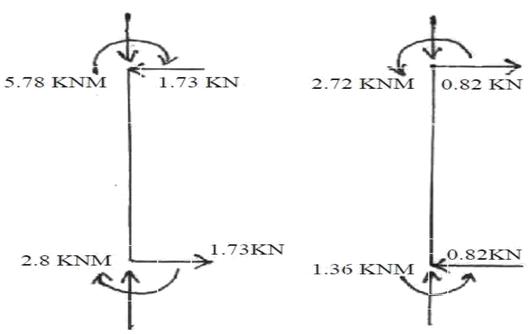
$$M_{FCB} = \frac{16\times4\times1^2}{5^2} = 2.56KNm$$

DISTRIBUTION FACTOR

Jt.	Member	Relative stiffness K	ΣK	DF = $\frac{K}{\sum K}$
B	BA BC	I/5 = 0.2 I I/5 = 0.2 I	0.4 I	0.5 0.5
C	CB CD	I/5 = 0.2I I/5 = 0.2I	0.4I	0.5 0.5

DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

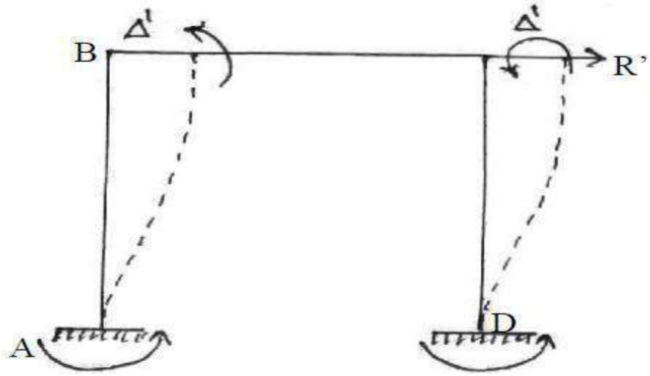
Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	0	0.5	0.5	0.5	0.5	0
FEM	0		-10.24	2.56	0	0
Balance		5.12	5.12	-1.28	-1.28	
CO	2.56		-0.64	2.56		-0.64
Balance		0.32	0.32	-0.08	-0.08	
CO	0.16		-0.64	0.16		-0.64
Balance		0.32	0.32	-0.08	-0.08	
C.O	0.16		-0.04	0.16		-0.04
Balance		0.02	0.02	-0.08	-0.08	
C.O	0.01					-0.04
Final moments	**2.89**	**5.78**	**-5.78**	**2.72**	**-2.72**	**-1.36**



By seeing of the FBD of columns R = 1.73 – 0.82

(Using F_x =0 for entire frame) = 0.91 KN ⟵

Now apply R = 0.91 KN acting opposite as shown in the above figure for the sway analysis. Sway analysis: For this we will assume a force R' is applied at C causing the frame to deflect as shown in the following figure.



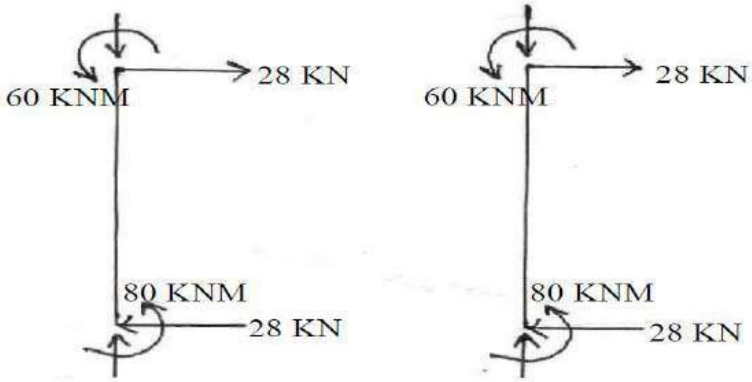
Since both ends are fixed, columns are of same length & I and assuming joints B & C are temporarily restrained from rotating and resulting fixed end moment are assumed.

$$M'_{AB} = M'_{BA} = M'_{CD} = M'_{DC} = \frac{6EI}{L^2}\Delta$$
$$M'_{BA} = -100KNm$$
$$M'_{AB} = M'_{CD} = M'_{DC} = -100KNm$$

Moment distribution table for sway analysis:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	01	0.5	0.5	0.5	0.5	0
FEM	–100	–100	0	0	–100	–100
Balance		50	50	50	50	
CO	25		25	25		25
Balance		–12.5	–12.5	–12.5	–12.5	
CO	-6.25		-6.25	-6.25		–6.25
Balance		3.125	3.125	3.125	3.125	
C.O	1.56		1.56	1.56		1.56
Balance		–0.78	–0.78	–0.78	–0.78	
C.O	-0.39		–0.39	–0.39		0.39
Balance		0.195	0.195	0.195	0.195	
C.O	0.1					0.1
Final moments	**- 80**	**- 60**	**60**	**60**	**– 60**	**– 80**

Free body diagram of columns



Using F x = 0 for the entire frame R = 28 + 28 = 56 KN

Hence R' = 56KN creates the sway moments shown in above moment distribution table. Corresponding moments caused by R = 0.91KN can be determined by proportion. Thus final moments are calculated by adding non sway moments and sway.

Moments calculated for R = 0.91KN, as shown below.

$$M_{AB} = 2.89 + \frac{0.91}{56}(-80) = 1.59KNm$$

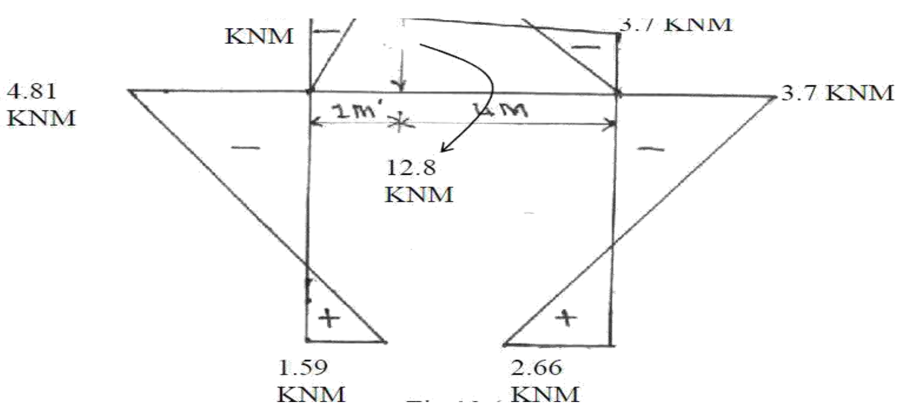
$$M_{BA} = 5.78 + \frac{0.91}{56}(-60) = 4.81KNm$$

$$M_{BC} = -5.78 + \frac{0.91}{56}(60) = -4.81KNm$$

$$M_{CB} = 2.72 + \frac{0.91}{56}(60) = 3.7KNm$$

$$M_{CD} = -2.72 + \frac{0.91}{56}(-60) = -3.7KNm$$

$$M_{DC} = -1.36 + \frac{0.91}{56}(-80) = -2.66KNm$$



BMD

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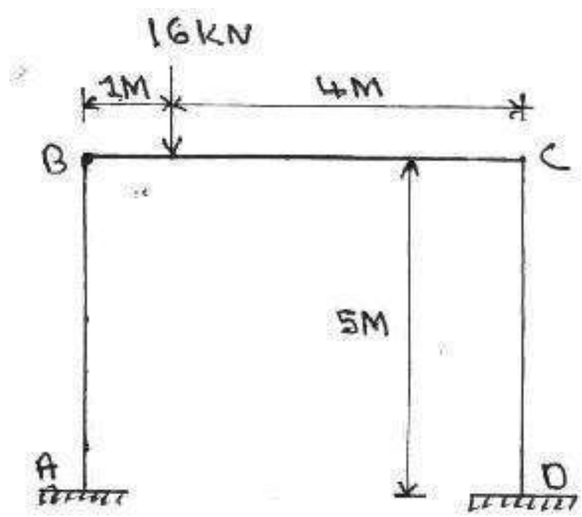

Non SwayAnalysis:

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$$M_{FAB=}\ M_{FBA=}\ M_{FCD=}\ 0$$

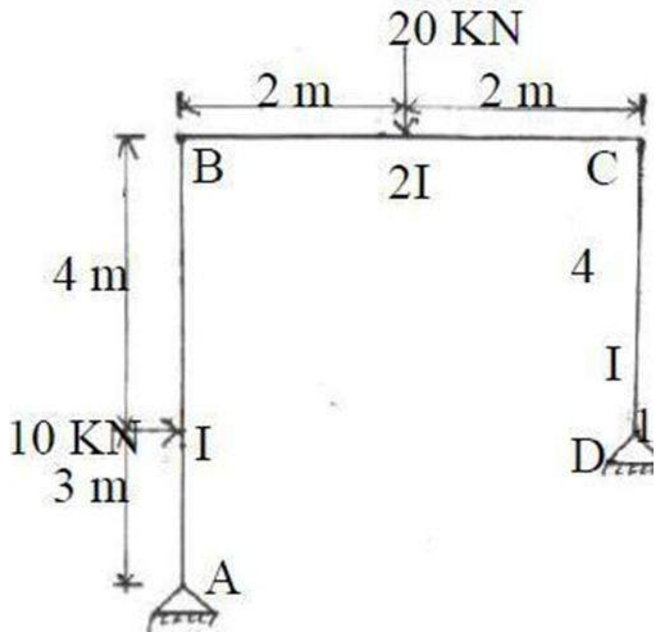
$$M_{FBC=} -\frac{16\times1\times4^2}{5^2} = -10.24KNm$$

$$M_{FCB=} \frac{16\times4\times1^2}{5^2} = 2.56KNm$$

Hence $R^{'}$ = 56KN creates the sway moments shown in above moment distribution table. Corresponding moments caused by R = 0.91KN can be determined by proportion. Thus final moments are calculated by adding non sway moments and sway.

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Q) Analysis the rigid frame shown in figure by moment distribution method and draw BMD



A. Non Sway Analysis:

First consider the frame held from side sway

FEMS

$$M_{FAB=} - \frac{10 \times 3 \times 4^2}{7^2} = -9.8KNm$$

$$M_{FBA=} \frac{10 \times 4 \times 3^2}{7^2} = 7.3KNm$$

$$M_{FBC=} -\frac{20\times4}{8} = -10KNm$$

$$M_{FCB=} \frac{20 \times 4}{8} = 10KNm$$

$$M_{FCD=}\ M_{FDC=}\ 0$$

Calculation of distribution factors:

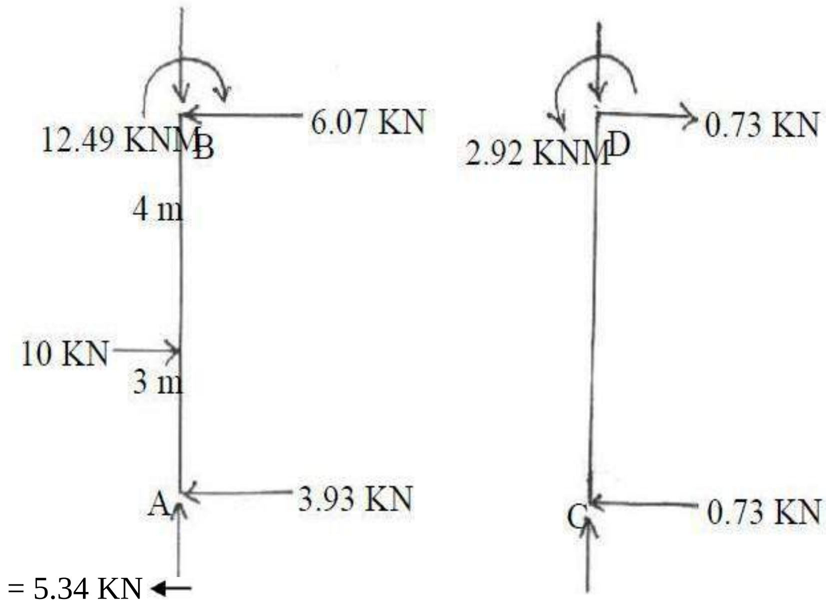
Joint	Member	Relative stiffness k	Σk	$DF = \frac{K}{\sum K}$
B	BA	$\frac{3}{4}\times\frac{I}{7} = 0.11I$	0.61 I	0.18
	BC	2I/4 = 0.5I		0.82
C	CB	2I/4 = 0.5I		0.72
	CD	$\frac{3}{4}\times\frac{I}{4} = 0.19\ I$	0.69 I	0.28

DISTRIBUTION FACTOR

DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	-9.8	7.3	-10	10	0	0
Release jt. 'D'	+9.8					
CO		4.9				
Initial moments	0	12.2	-10	10	0	0
Balance		-0.4	-1.8	-7.2	-2.8	
CO			-3.6	-0.9		
Balance		0.65	2.95	0.65	0.25	
C.O			0.33	1.48		
Balance		-0.06	-0.27	-1.07	-0.41	
C.O			-0.54	-0.14		
Balance		0.1	0.44	0.1	0.04	
Final moments	**0**	**12.49**	**-12.49**	**2.92**	**-2.92**	**0**

FREE BODY DIAGRAM OF COLUMNS



Applying Fx = 0 for frame as a Whole, R = 10 – 3.93 – 0.73

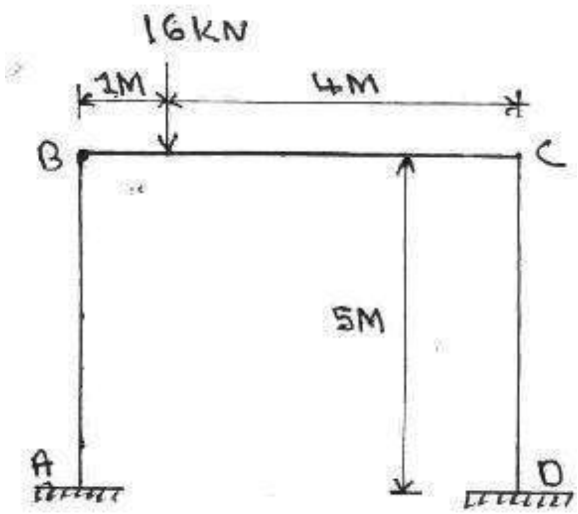
Now apply R = 5.34KN acting opposite

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Analyze the frame shown in figure by moment distribution method. Assume EI is constant.



Non SwayAnalysis:

First consider the frame without side sway

$$M_{FAB} = M_{FBA} = M_{FCD} = 0$$

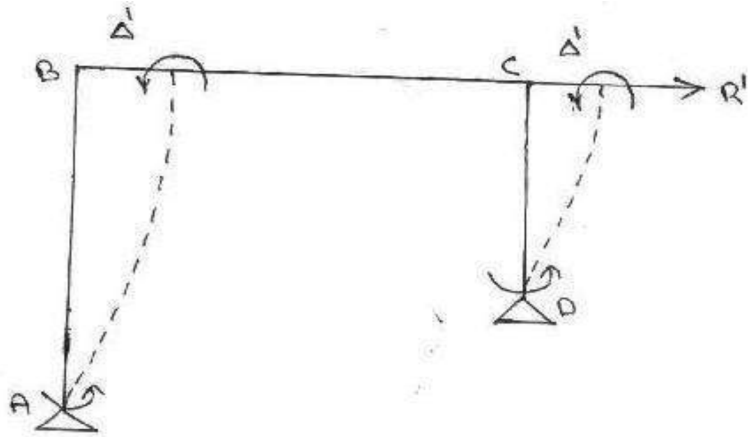
$$M_{FBC} = -\frac{16\times1\times4^2}{5^2} = -10.24KNm$$

$$M_{FCB} = \frac{16\times4\times1^2}{5^2} = 2.56KNm$$

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

Sway analysis: For this we will assume a force R' is applied at C causing the frame todeflect as shown in figure



Since ends A & D are hinged and columns AB & CD are of different lengths

$M'_{BA} = -\frac{3EI}{L_1^2}\Delta', \quad M'_{CD} = -\frac{3EI}{L_2^2}\Delta',$

$$\frac{M'_{BA}}{M'_{CD}} = \frac{\frac{3EI}{L_1^2}\Delta'}{\frac{3EI}{L_2^2}\Delta'} = \frac{L_2^2}{L_1^2} = \frac{4^2}{7^2} = \frac{16}{49}$$

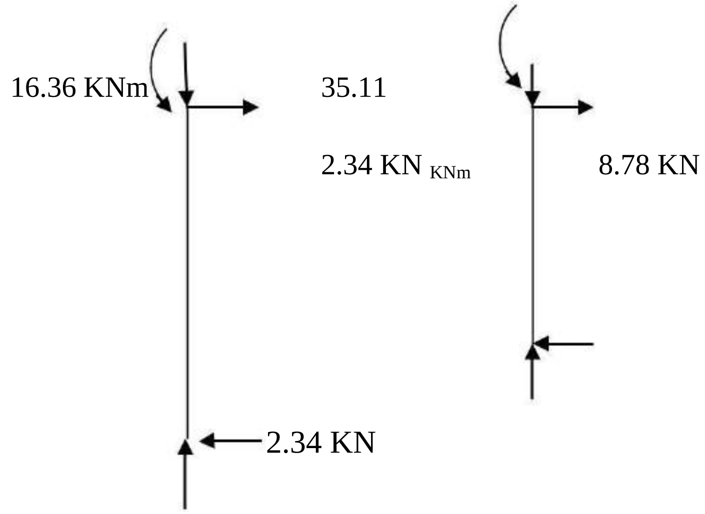
Assume

$$M'_{BA} = -16KNm, M'_{AB} = 0$$
$$M'_{CD} = -49KNm, \ M'_{DC} = 0$$

MOMENT DISTRIBUTION FOR SWAY ANALYSIS

Joint	A		B		C	D
Member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	0	-16	0	0	-49	0
Balance		2.88	13.12	35.28	13.72	
CO			17.64	6.56		
Balance		-3.18	-14.46	-4.72	-1.84	
CO			-2.36	-7.23		
Balance		0.42	1.94	5.21	2.02	
C.O			2.61	0.97		
Balance		-0.47	-2.14	-0.7	-0.27	
C.O			0.35	-1.07		
Balance		0.06	0.29	0.77	0.3	
C.O			0.39	0.15		
Balance		-0.07	-0.32	-0.11	-0.04	
Final moments	**0**	**-16.36**	**16.36**	**35.11**	**-35.11**	**0**

FREE BODY DIAGRAMS OF COLUMNS AB &CD



Using Fx = 0 for the entire frame

$R^{'}$ = 11.12 k →

Hence $R^{'}$ = 11.12 KN creates the sway moments shown in the above moment distribution table. Corresponding moments caused by R = 5.34 kN can be determined by proportion. Thus final moments are calculated by adding non-sway moments and sway moments determined for R = 5.34 KN as shown below.

$$M_{AB} = 0$$

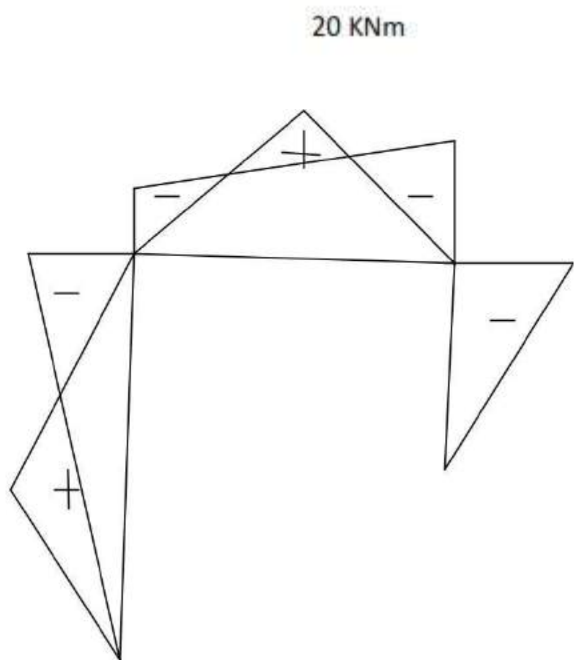
$$M_{BA} = 12.49 + \frac{5.34}{11.12}(-16.36) = 4.63\text{KNm}$$

$$M_{BC} = -12.49 + \frac{5.34}{11.12}(16.36) = -4.63\text{KNm}$$

$$M_{CB} = 2.92 + \frac{5.34}{11.12}(35.11) = 19.78\text{KNm}$$

$$M_{CD} = -2.92 + \frac{5.34}{11.12}(-35.11) = -19.78\text{KNm}$$

$$M_{DC} = 0$$



B.M.D

APPROXIMATE LATERAL LOAD ANALYSIS BY PORTAL METHOD

Portal Frame

Portal frames, used in several Civil Engineering structures like buildings, factories, bridges have the primary purpose of transferring horizontal loads applied at their tops to their foundations. Structural requirements usually necessitate the use of statically indeterminate layout for portal frames, and approximate solutions are often used in their analyses.

Assumptions for the Approximate Solution

In order to analyze a structure using the equations of statics only, the number of independent force components must be equal to the number of independent equations of statics.

If there are *n* more independent force components in the structure than there are independent equations of statics, the structure is statically indeterminate to the *nth* degree. Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make *n* additional independent assumptions. A solution based on statics will not be possible by making fewer than *n* assumptions, while more than *n* assumptions will not in general be consistent.

Thus, the first step in the approximate analysis of structures is to find its degree of statically indeterminacy (dosi) and then to make appropriate number of assumptions.

For example, the dosi of portal frames shown in (i), (ii), (iii) and (iv) are 1, 3, 2 and 1 respectively. Based on the type of frame, the following assumptions can be made for portal structures with a vertical axis of symmetry that are loaded horizontally at thetop

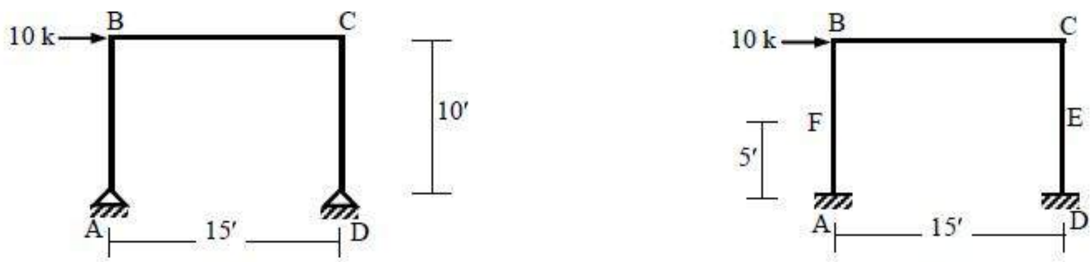
1. The horizontal support reactions areequal
2. There is a point of inflection at the center of the unsupported height of each fixed basedcolumn

Assumption 1 is used if dosi is an odd number (i.e., = 1 or 3) and Assumption 2 is used if dosi 1.

Some additional assumptions can be made in order to solve the structure approximately for different loading and support conditions.

3. Horizontal body forces not applied at the top of a column can be divided into two forces (i.e., applied at the top and bottom of the column) based on simple supports
4. For hinged and fixed supports, the horizontal reactions for fixed supports can be assumed to be four times the horizontal reactions for hinged supports Example

Draw the axial force, shear force and bending moment diagrams of the frames loaded as shown below.



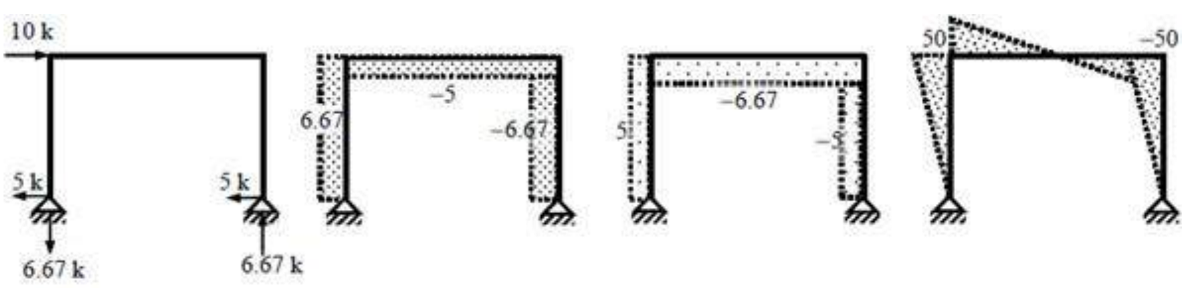
(i) (ii)

Solution

(i) For this frame, dosi = 3 × 3 + 4 – 3 × 4 = 1; i.e., Assumption 1 ⇒ $H_A = H_D = 10/2 = 5$ k

$\therefore \sum M_A = 0 \Rightarrow 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 6.67$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -6.67$ k



Reactions AFD (k) SFD (k) BMD

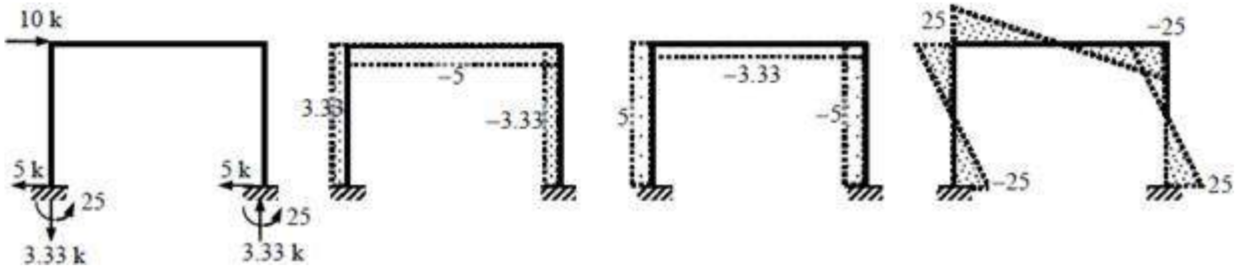
(ii) dosi = 3 × 3 + 6 – 3 × 4 = 3

Assumption 1 ⇒ $H_A = H_D = 10/2 = 5$ k, Assumption 2 ⇒ $BM_E = BM_F = 0$

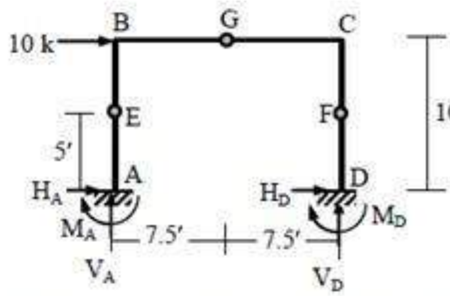
$\therefore BM_F = 0 \Rightarrow H_A \times 5 + M_A = 0 \Rightarrow M_A = -25$ k-ft; Similarly $BM_E = 0 \Rightarrow M_D = -25$

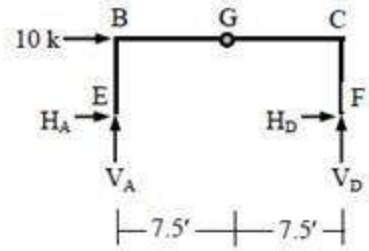
$\therefore \sum M_A = 0 \Rightarrow -25 - 25 + 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -3.33$ k



(iii)





dosi = 3 × 4 + 6 − 3 × 5 − 1 = 2; ∴ Assumption 1 and 2 ⇒ $BM_E = BM_F = 0$

∴ $BM_E = 0$ (bottom) ⇒ $-H_A \times 5 + M_A = 0 \Rightarrow M_A = 5H_A$; Similarly $BM_F = 0 \Rightarrow M_D = 5H_D$

Also $BM_E = 0$ (free body of EBCF) ⇒ $10 \times 5 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

∴ $\Sigma F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -V_D = -3.33$ k

$BM_G = 0$ (between E and G) ⇒ $V_A \times 7.5 - H_A \times 5 = 0 \Rightarrow H_A = -5$ k ⇒ $M_A = 5H_A = -25$

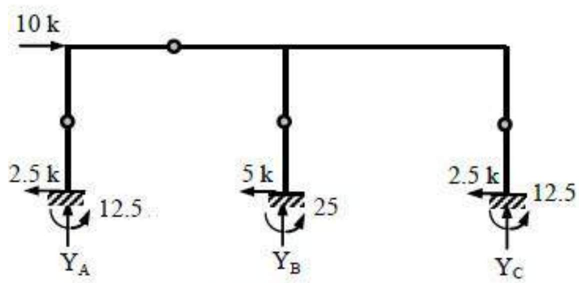
$\Sigma F_x = 0$ (entire structure) ⇒ $H_A + H_D + 10 = 0 \Rightarrow -5 + H_D + 10 = 0 \Rightarrow H_D = -5$ k ⇒ $M_D = 5H_D = -25$

(iv) dosi = 3 × 5 + 9 − 3 × 6 = 6 ⇒ 6 Assumptions needed to solve the structure

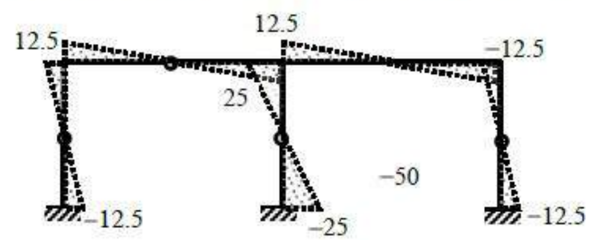
Assumption 1 and 2 ⇒ $H_A: H_B: H_C = 1: 2: 1 \Rightarrow H_A = 10/4 = 2.5$ k, $H_B = 5$ k, $H_C = 2.5$ k

∴ $M_A = M_C = 2.5 \times 5 = 12.5$ k-ft, $M_B = 5 \times 5 = 25$

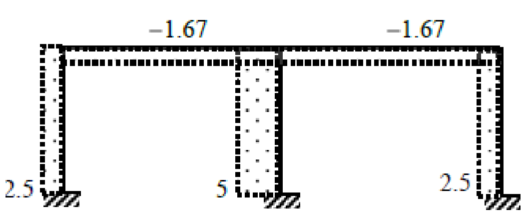
The other 4 assumptions are the assumed internal hinge locations at midpoints of columns and one beam



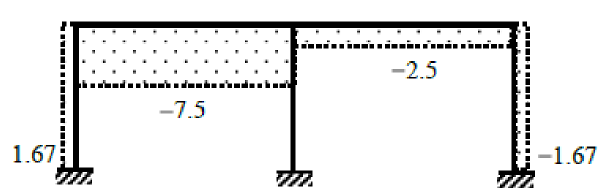
Reactions



BMD



SFD (k)



AFD (k)

Analysis of Multi-storied Structures by Portal Method

Approximate methods of analyzing multi-storied structures are important because such structures are statically highly indeterminate. The number of assumptions that must be made to permit an analysis by statics alone is equal to the degree of statically indeterminacy of the structure.

Assumptions

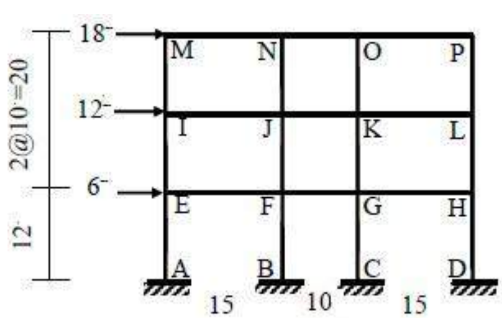
The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures. The *Portal Method* thus formulated is based on three assumptions

1. The shear force in an interior column is twice the shear force in an exterior column.
2. There is a point of inflection at the center of eachcolumn.
3. There is a point of inflection at the center of eachbeam.

Assumption 1 is based on assuming the interior columns to be formed by columns of two adjacent bays or portals. Assumption 2 and 3 are based on observing the deflected shape of thestructure.

Example

Use the Portal Method to draw the axial force, shear force and bending moment diagrams of the three-storied frame structure loaded as shown below.



Column shear forces are at the ratio of 1:2:2:1.

∴ Shear force in (V) columns IM, JN, KO, LP are [18 × 1/(1 + 2 + 2 + 1) =] 3, [18 × 2/(1 + 2 + 2 + 1) =] 6, 6, 3 respectively. Similarly,

$V_{EI} = 30 \times 1/(6) = 5$, $V_{FJ} = 10$, $V_{GK} = 10$, $V_{HL} = 5$; and

$V_{AE} = 36 \times 1/(6) = 6$, $V_{BF} = 12$, $V_{CG} = 12$, $V_{DH} = 6$

Bending moments are

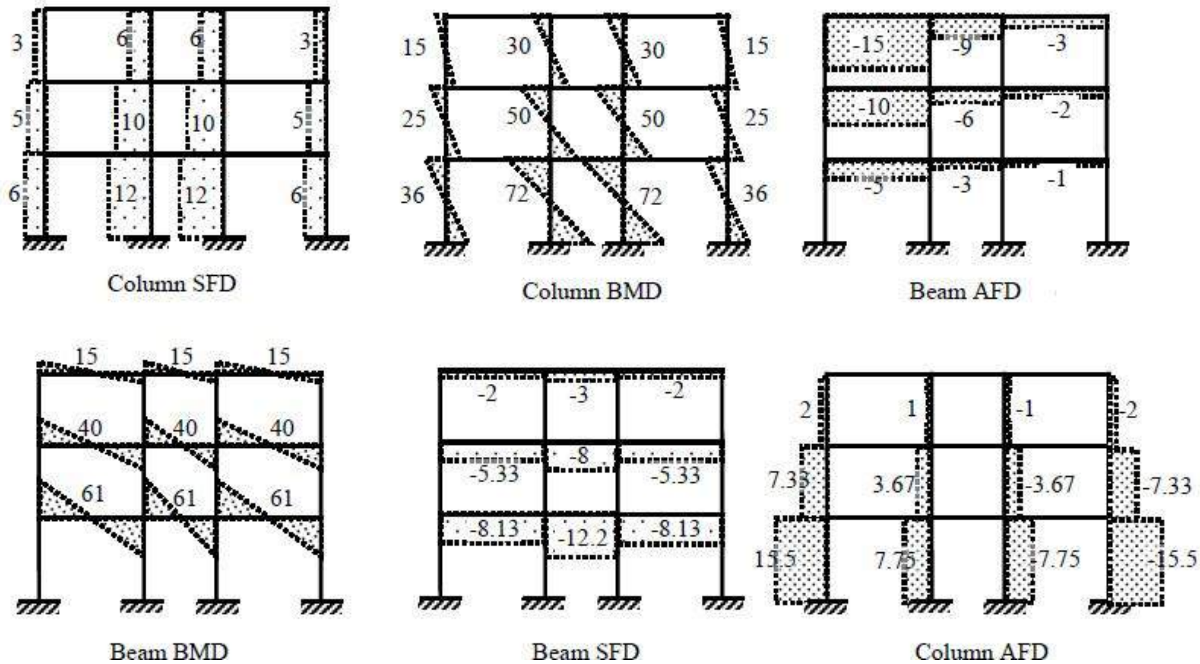
$M_{IM} = 3 \times 10/2 = 15$, $M_{JN} = 30$, $M_{KO} = 30$, $M_{LP} = 15$

$M_{EI} = 5 \times 10/2 = 25$, $M_{FJ} = 50$, $M_{GK} = 50$, $M_{HL} = 25$

$M_{AE} = 6 \times 10/2 = 30$, $M_{FJ} = 60$, $M_{GK} = 60$, $M_{HL} = 30$

The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures.

The rest of the calculations follow from the free-body diagrams



Analysis of Multi-storied Structures by Cantilever Method

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members.

The *Cantilever Method* attempts to rectify this limitation by considering the cross-sectional areas of columns in distributing the axial forces in various columns of a story.

Assumptions

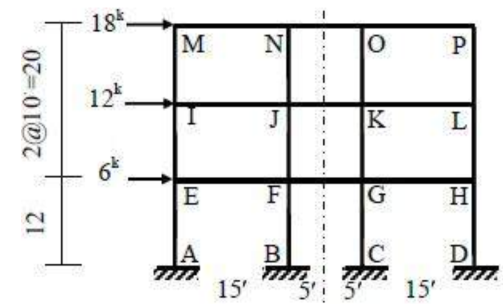
The Cantilever Method is based on three assumptions

1. The axial force in each column of a storey is proportional to its horizontal distance from the centroidal axis of all the columns of thestorey.
2. There is a point of inflection at the center of eachcolumn.
3. There is a point of inflection at the center of eachbeam.

Assumption 1 is based on assuming that the axial stresses can be obtained by a method analogous to that used for determining the distribution of normal stresses on a transverse section of a cantilever beam. Assumption 2 and 3 are based on observing the deflected shape of the structure.

Example

Use the Cantilever Method to draw the axial force, shear force and bending moment diagrams of the three -storied frame structure loaded as shown below.



The dotted line is the column centerline (at all floors)

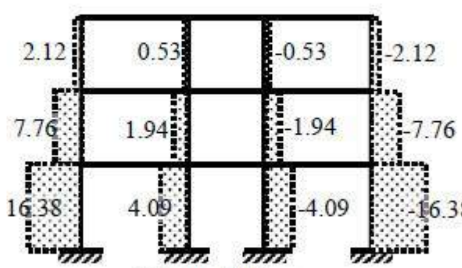
∴Column axial forces are at the ratio of 20: 5: −5: −20.

∴Axial force in (P) columns IM, JN, KO, LP are $[18 \times 5 \times 20/\{20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 2.12 , $[18 \times 5 \times 5/(20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 0.53, −0.53, −2.12 respectively.

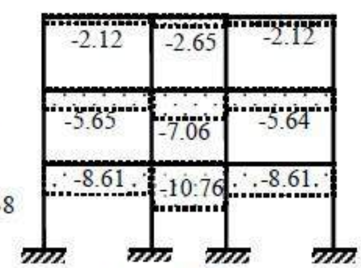
Similarly, $P_{EI} = 330\times20/(850) = 7.76$, $P_{FJ} = 1.94$, $P_{GK} = -1.94$, $P_{HL} = -7.76$; and

$P_{AE} = 696\times20/(850) = 16.38$, $P_{BF} = 4.09$, $P_{CG} = -4.09$, $P_{DH} = 16.38$

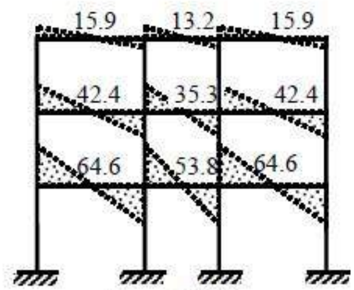
The rest of the calculations follow from the free-body diagrams



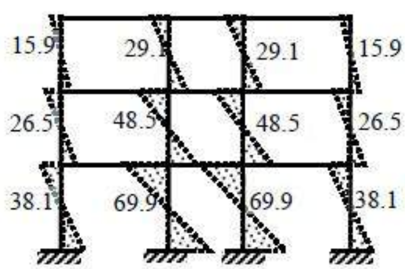
Column AFD



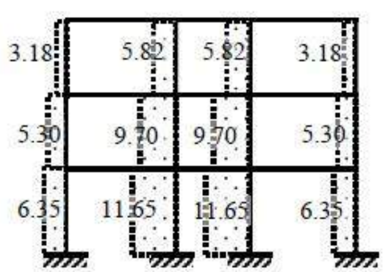
Beam SFD



Beam BMD



Column BMD



Column SFD

Beam AFD

Introducing direction cosines $l\cos\theta$; $m\sin\theta$; the above equation is written as

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} \quad (24.10a)$$

Or, $$\{u'\} = [T]\{u\} \quad (24.10b)$$

In the above equation $T$is the displacement transformation matrix whichtransforms the four global displacement components to two displacement component in local coordinate system.

UNIT -V

INFLUENCE LINES AND MOVING LOADS

Influence Lines

An influence line represents the variation of either the reaction, shear, moment or deflection at a specific point in a member as a concentrated force moves over the member. Example bridges, industrial crane rails, conveyors, etc

Influence lines are important in the design of structures that resist large live loads. ¬In our work up to this point, we have discussed analysis techniques for structures subjected to dead or fixed loads.

We learned that shear and moment diagrams are important in determining the maximum internal force in a structure. ¬If a structure is subjected to a live or moving load, the variation in shear and moment is best described using influence lines.

Since beams or girders are usually major load– carrying members in large structures, it is important to draw influence lines for reaction, shear, and moment at specified points. ¬ Once an influence line has been drawn, it is possible to locate the live loads on the beam so that the maximum value of the reaction, shear, or moment is produced. ¬ This is very important in the design procedure.

Concentrated Force - Since we use a unit force (a dimensionless load), the value of the function (reaction, shear, or moment) can be found by multiplying the ordinate of the influence line at the position x by the magnitude of the actual force P.

One can tell at a glance where the moving load should be placed on the structure so that it creates the greatest influence at the specified point.

Influence lines for statically determinate structures are piecewise linear.

statically indeterminate example

shear& moment diagrams:

effect of fixed loads at all points along the axis of the member , influence lines:

effect of a moving load only at a specified point on the member

Rounded Aggregate

The rounded aggregates are completely shaped by attrition and available in the form of seashore gravel. Rounded aggregates result the minimum percentage of voids (32 – 33%) hence gives more workability. They require lesser amount of water-cement ratio. They are not considered for high strength concrete because of poor interlocking behavior and weak bond strength.

Irregular Aggregates

The irregular or partly rounded aggregates are partly shaped by attrition and these are available in the form of pit sands and gravel. Irregular aggregates may result 35- 37% of voids. These will give lesser workability when compared to rounded aggregates. The bond strength is slightly higher than rounded aggregates but not as required for high strength concrete.

Angular Aggregates

The angular aggregates consist well defined edges formed at the intersection of roughly planar surfaces and these are obtained by crushing the rocks. Angular aggregates result maximum percentage of voids (38-45%) hence gives less workability. They give 10-20% more compressive strength due to development of stronger aggregate-mortar bond. So, these are useful in high strength concrete manufacturing.

Flaky Aggregates

When the aggregate thickness is small when compared with width and length of that aggregate it is said to be flaky aggregate. Or in the other, when the least dimension of aggregate is less than the 60% of its mean dimension then it is said to be flaky aggregate.

Elongated Aggregates

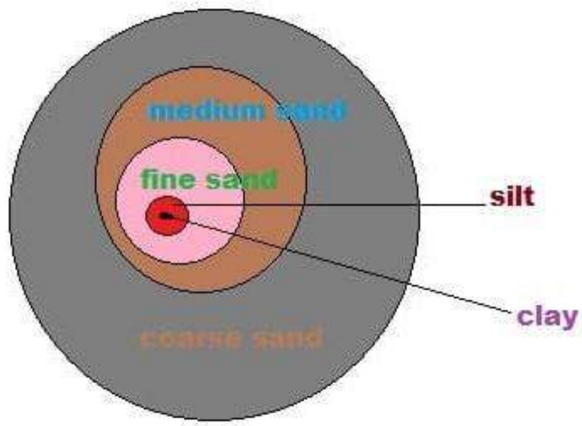
When the length of aggregate is larger than the other two dimensions then it is called elongated aggregate or the length of aggregate is greater than 180% of its mean dimension.

Flaky and Elongated Aggregates

When the aggregate length is larger than its width and width is larger than its thickness then it is said to be flaky and elongated aggregates. The above 3 types of aggregates are not suitable for concrete mixing. These are generally obtained from the poorly crushed rocks.

Classification of Aggregates Based on Size

Aggregates are available in nature in different sizes. The size of aggregate used may be related to the mix proportions, type of work etc. the size distribution of aggregates is called grading of aggregates.



Following are the classification of aggregates based on size:

Aggregates are classified into 2 types according to size

1. Fine aggregate
2. Coarse aggregate

Fine Aggregate

When the aggregate is sieved through 4.75mm sieve, the aggregate passed through it called as fine aggregate. Natural sand is generally used as fine aggregate, silt and clay are also come under this category. The soft deposit consisting of sand, silt and clay is termed as loam. The purpose of the fine aggregate is to fill the voids in the coarse aggregate and to act as a workability agent.

Coarse Aggregate

When the aggregate is sieved through 4.75mm sieve, the aggregate retained is called coarse aggregate. Gravel, cobble and boulders come under this category. The maximum size aggregate used may be dependent upon some conditions. In general, 40mm size aggregate used for normal strengths and 20mm size is used for high strength concrete. the size range of various coarse aggregates given below.

Grading of Aggregates

Grading is the particle-size distribution of an aggregate as determined by a sieve analysis using wire mesh sieves with square openings. As per IS:2386(Part-1)

Fine aggregate—6 standard sieves with openings from 150 μm to 4.75 mm.

Coarse aggregate—5 sieves with openings from 4.75mm to 80 mm.

Gradation (grain size analysis)

Grain size distribution for concrete mixes that will provide a dense strong mixture. Ensure that the voids between the larger particles are filled with medium particles. The remaining voids are filled with still smaller particles until the smallest voids are filled with a small amount of fines. Ensure maximum density and strength using a maximum density curve

Good Gradation:

Concrete with good gradation will have fewer voids to be filled with cement paste (economical mix) Concrete with good gradation will have fewer voids for water to permeate (durability)

Particle size distribution affects:

1. Workability
2. Mixproportioning

Fine Aggregate effect on concrete:

2. Over sanded (More than requiredsand)

- Over cohesivemix.
- Water reducers may be lesseffective.
- Air entrainment may be moreeffective.

3. Under sanded (deficit ofsand)

- Prone to bleed andsegregation.
- May get high levels of waterreduction.
- Air entrainers may be lesseffective.

Shape and surface texture of aggregates:

The shape of aggregate is an important characteristic since it affects the workability of concrete.

It is difficult to measure the shape of irregular shaped aggregates. Not only the type of parent rock but also the type of crusher used also affects the shape of the aggregate produced.

Good Granite rocks found near Bangalore will yield cuboidal aggregates. Many rocks contain planes of jointing which is characteristics of its formation and hence tend to yield more flaky aggregates.

The shape of the aggregates produced is also dependent on type of crusher and the reduction ratio of the crusher.

Quartzite which does not possess cleavage planes tend to produce cubical shape aggregates.

From the standpoint of economy in cement requirement for a given water cement ratio rounded aggregates are preferable to angular aggregates.

On the other hand, the additional cement required for angular aggregates is offset to some extent by the higher strengths and some times greater durability as a result of greater Interlocking texture of the hardened concrete.

Flat particles in concrete will have objectionable influence on the workability of concrete, cement requirement, strength and durability.

In general excessively flaky aggregates make poor concrete.

While discussing the shape of the aggregates, the texture of the aggregate also enters the discussion because of its close association with the shape.

Generally round aggregates are smooth textured and angular aggregates are rough textured. Therefore some engineers argue against round aggregates from the point of bond strength between aggregates and cement.

But the angular aggregates are superior to rounded aggregates from the following two points:

Angular aggregates exhibit a better interlocking effect in concrete, which property makes it superior in concrete used for road and pavements.

The total surface area of rough textured angular aggregate is more than smooth rounded aggregates for the given volume.

By having greater surface area, the angular aggregates may show higher bond strength than rounded aggregates.

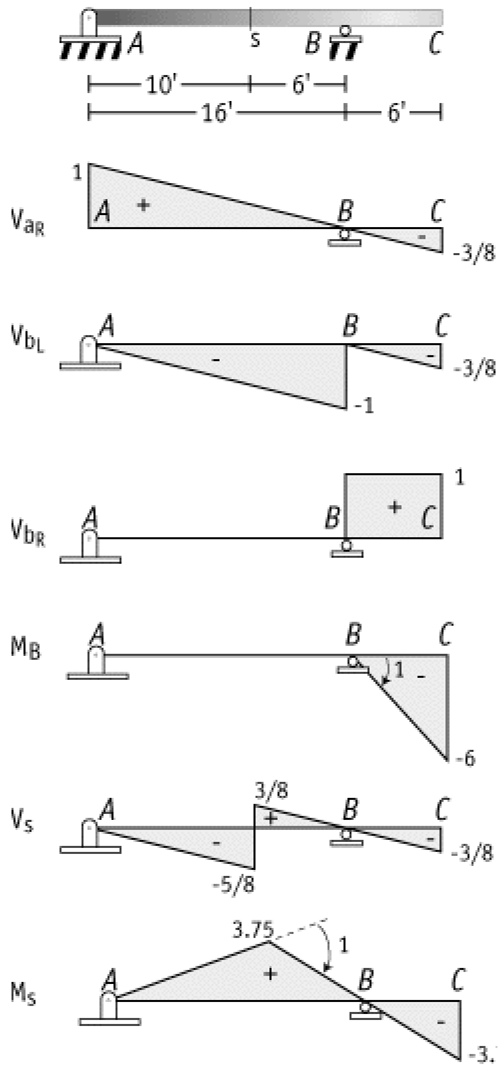
The shape of the aggregates becomes all the more important in case of high strength and high performance concrete where very low water/cement ratio is required to be used . In such cases cubical aggregates are required for better workability.

Surface texture is the property, the measure of which depends upon the relative degree to which particle surface are polished or dull, smooth or rough.

Surface texture depends upon hardness, grain size, pore structure, structure of the rock and the degree to which the forces acting on it have smoothened the surface or roughened. Experience and laboratory experiments have shown that the adhesion between cement paste and the aggregate is influenced by several complex factors in

Procedure of Analysis

1. tabulate values
2. influence-line equations



Assumptions for the Approximate Solution

In order to analyze a structure using the equations of statics only, the number of independent force components must be equal to the number of independent equations of statics.

If there are *n* more independent force components in the structure than there are independent equations of statics, the structure is statically indeterminate to the *nth* degree. Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make *n* additional independent assumptions. A solution based on statics will not be possible by making fewer than *n*

assumptions, while more than *n* assumptions will not in general be consistent.
Thus, the first step in the approximate analysis of structures is to find its degree of statically indeterminacy (dosi) and then to make appropriate number of assumptions.
For example, the dosi of portal frames shown in (i), (ii), (iii) and (iv) are 1, 3, 2 and 1 respectively. Based on the type of frame, the following assumptions can be made for portal structures with a vertical axis of symmetry that are loaded horizontally at thetop

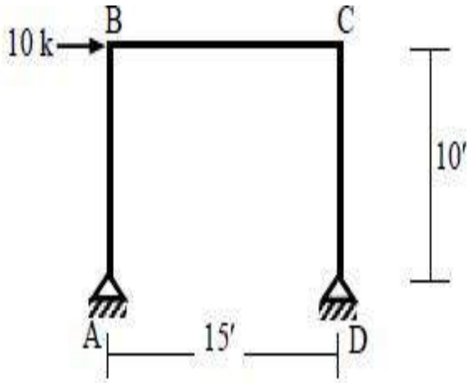
The horizontal support reactions areequal

There is a point of inflection at the center of the unsupported height of each fixed basedcolumn

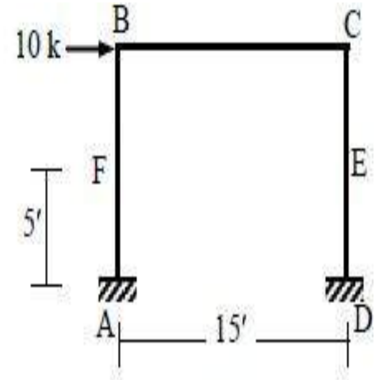
Assumption 1 is used if dosi is an odd number (i.e., = 1 or 3) and Assumption 2 is used if dosi 1.

Some additional assumptions can be made in order to solve the structure approximately for different loading and support conditions.
Horizontal body forces not applied at the top of a column can be divided into two forces (i.e., applied at the top and bottom of the column) based on simple supports
For hinged and fixed supports, the horizontal reactions for fixed supports can be assumed to be four times the horizontal reactions for hinged supports Example
Draw the axial force, shear force and bending moment diagrams of the frames loaded as shown below.

Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make *n* additional independent assumptions. A solution based on statics will not be possible by making fewer than *n* assumptions, while more than *n* assumptions will not in general be consistent.



(i)



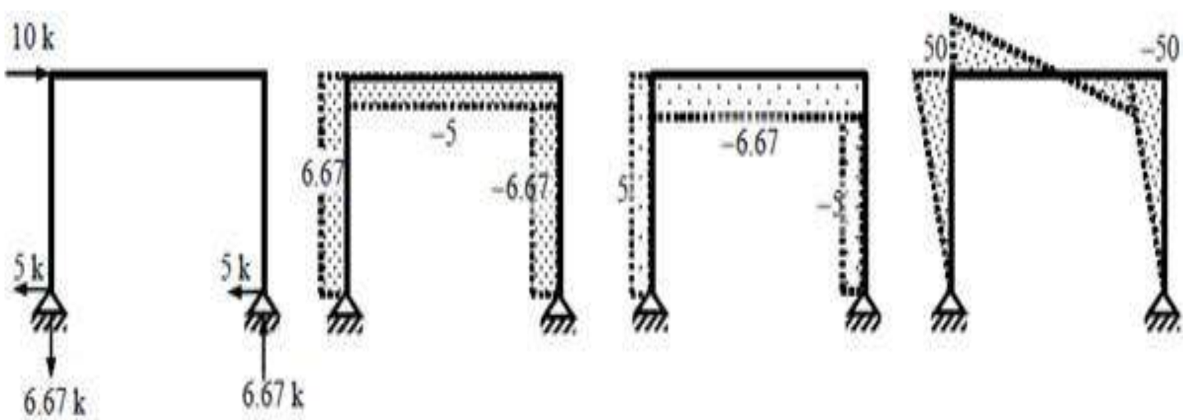
(ii)

<u>Solution</u>

(i) For this frame, dosi = $3 \times 3 + 4 - 3 \times 4 = 1$; i.e., Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5$ k

$\therefore \sum M_A = 0 \Rightarrow 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 6.67$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -6.67$ k



Reactions AFD (k) SFD (k) BMD

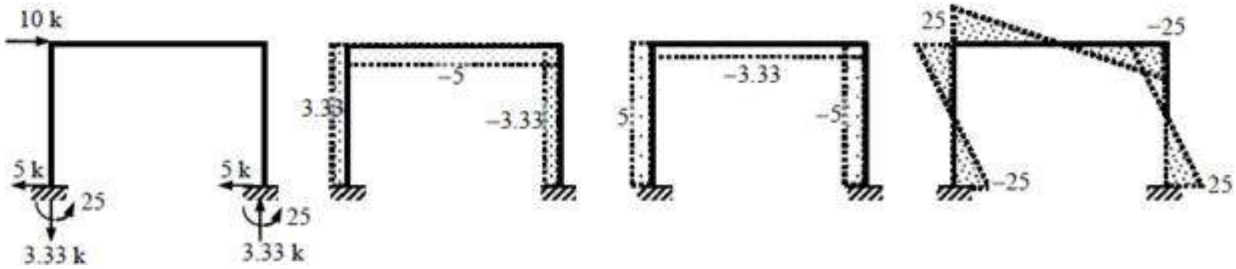
(ii) dosi = $3 \times 3 + 6 - 3 \times 4 = 3$

Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5$ k, Assumption 2 $\Rightarrow BM_E = BM_F = 0$

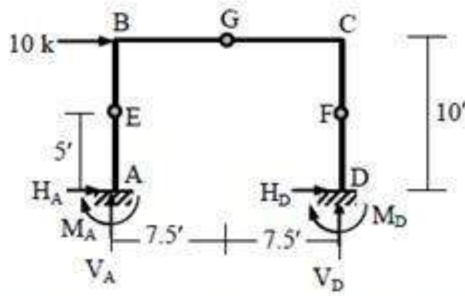
$\therefore BM_F = 0 \Rightarrow H_A \times 5 + M_A = 0 \Rightarrow M_A = -25$ k-ft; Similarly $BM_E = 0 \Rightarrow M_D = -25$

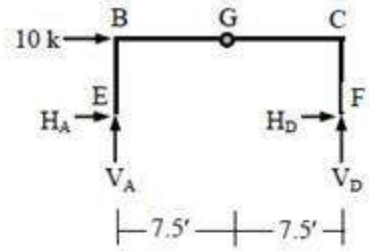
$\therefore \sum M_A = 0 \Rightarrow -25 - 25 + 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -3.33$ k



(iii)





dosi = 3 × 4 + 6 − 3 × 5 − 1 = 2; ∴ Assumption 1 and 2 ⇒ $BM_E = BM_F = 0$

∴ $BM_E = 0$ (bottom) ⇒ $-H_A \times 5 + M_A = 0 \Rightarrow M_A = 5H_A$; Similarly $BM_F = 0 \Rightarrow M_D = 5H_D$

Also $BM_E = 0$ (free body of EBCF) ⇒ $10 \times 5 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

∴ $\Sigma F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -V_D = -3.33$ k

$BM_G = 0$ (between E and G) ⇒ $V_A \times 7.5 - H_A \times 5 = 0 \Rightarrow H_A = -5$ k ⇒ $M_A = 5H_A = -25$

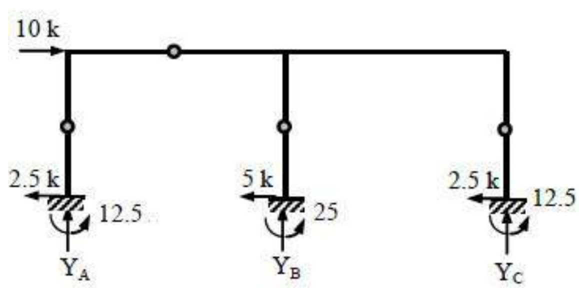
$\Sigma F_x = 0$ (entire structure) ⇒ $H_A + H_D + 10 = 0 \Rightarrow -5 + H_D + 10 = 0 \Rightarrow H_D = -5$ k ⇒ $M_D = 5H_D = -25$

(iv) dosi = 3 × 5 + 9 − 3 × 6 = 6 ⇒ 6 Assumptions needed to solve the structure

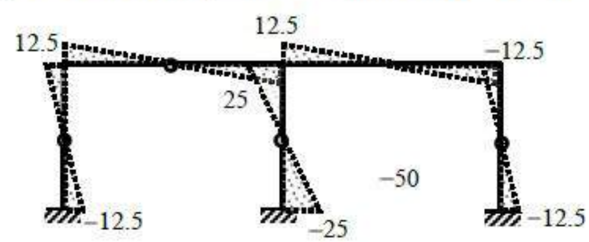
Assumption 1 and 2 ⇒ H_A: H_B: H_C = 1: 2: 1 ⇒ H_A = 10/4 = 2.5 k, H_B = 5 k, H_C = 2.5 k

∴ $M_A = M_C = 2.5 \times 5 = 12.5$ k-ft, $M_B = 5 \times 5 = 25$

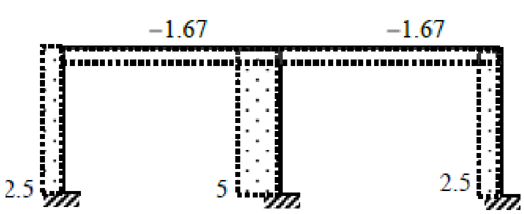
The other 4 assumptions are the assumed internal hinge locations at midpoints of columns and one beam



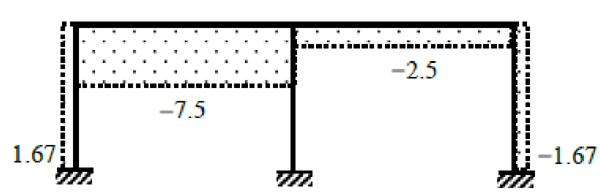
Reactions



BMD



SFD (k)



AFD (k)

Analysis of Multi-storied Structures by Portal Method

Approximate methods of analyzing multi-storied structures are important because such structures are statically highly indeterminate. The number of assumptions that must be made to permit an analysis by statics alone is equal to the degree of statically indeterminacy of the structure.

Assumptions

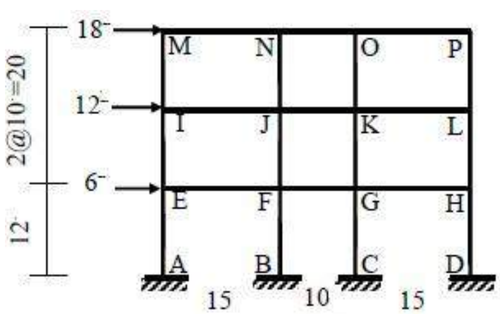
The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures. The *Portal Method* thus formulated is based on three assumptions

4. The shear force in an interior column is twice the shear force in an exterior column.
5. There is a point of inflection at the center of eachcolumn.
6. There is a point of inflection at the center

Assumption 1 is based on assuming the interior columns to be formed by columns of two adjacent bays or portals. Assumption 2 and 3 are based on observing the deflected shape of thestructure.

Example

Use the Portal Method to draw the axial force, shear force and bending moment diagrams of the three-storied frame structure loaded as shown below.



Column shear forces are at the ratio of 1:2:2:1.

∴ Shear force in (V) columns IM, JN, KO, LP are $[18 \times 1/(1+2+2+1) =]\ 3$, $[18 \times 2/(1+2+2+1) =]\ 6$, 6, 3 respectively. Similarly,

$V_{EI} = 30 \times 1/(6) = 5$, $V_{FJ} = 10$, $V_{GK} = 10$, $V_{HL} = 5$; and

$V_{AE} = 36 \times 1/(6) = 6$, $V_{BF} = 12$, $V_{CG} = 12$, $V_{DH} = 6$

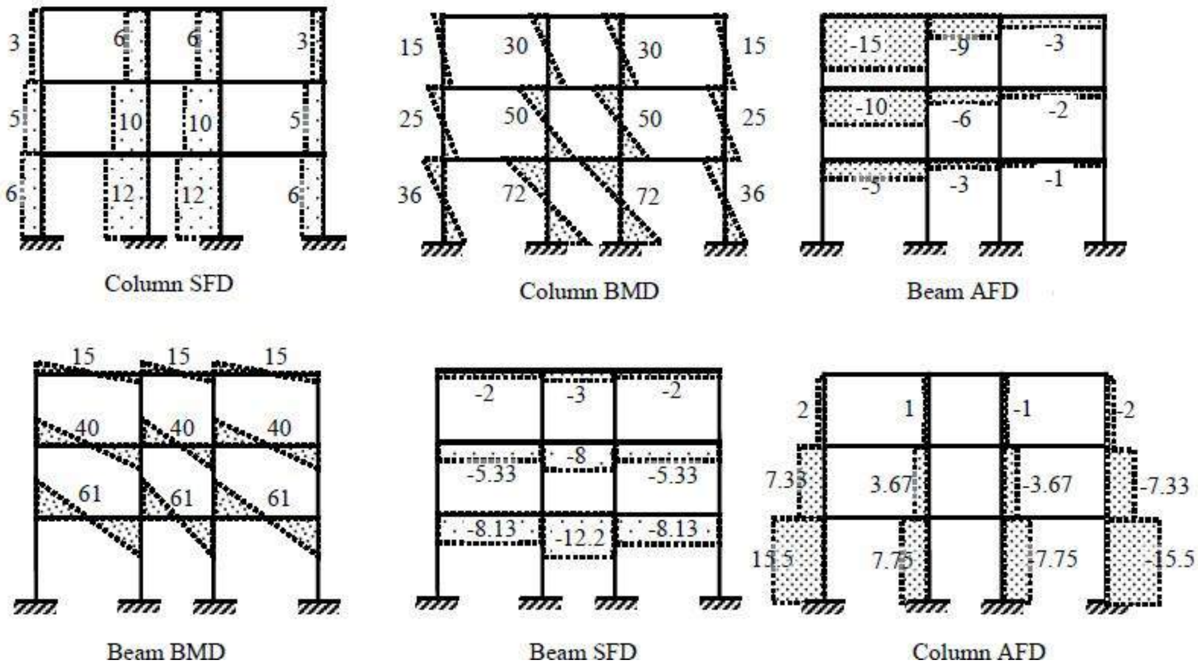
Bending moments are

$M_{IM} = 3 \times 10/2 = 15$, $M_{JN} = 30$, $M_{KO} = 30$, $M_{LP} = 15$

$M_{EI} = 5 \times 10/2 = 25$, $M_{FJ} = 50$, $M_{GK} = 50$, $M_{HL} = 25$

$M_{AE} = 6 \times 10/2 = 30$, $M_{FJ} = 60$, $M_{GK} = 60$, $M_{HL} = 30$

The rest of the calculations follow from the free-body diagrams



Analysis of Multi-storied Structures by Cantilever Method

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members. The *Cantilever Method* attempts to rectify this limitation by considering the cross- sectional areas of columns in distributing the axial forces in various columns of a story.

Assumptions

The Cantilever Method is based on three assumptions

The axial force in each column of a storey is proportional to its horizontal distance from the centroidal axis of all the columns of thestorey.

There is a point of inflection at the center of eachcolumn.

There is a point of inflection at the center of eachbeam. Assumption 1 is based on assuming that the axial stresses can be obtained by a method analogous to that used for determining the distribution of normal stresseson a transverse section of a cantilever beam. Assumption

2 and 3 are based on observing the deflected shape of the structure.

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members. The *Cantilever Method* attempts to rectify this limitation by considering the cross- sectional areas of columns in distributing the axial forces in various columns of a story.

Approximate methods of analyzing multi-storied structures are important because such structures are statically highly indeterminate. The number of assumptions that must be made to permit an analysis by statics alone is equal to the degree of statically indeterminacy of the structure.

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members

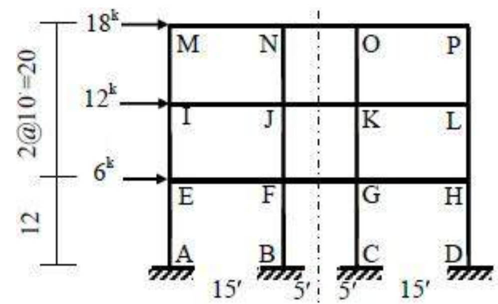
If there are *n* more independent force components in the structure than there are independent equations of statics, the structure is statically indeterminate to the *nth* degree. Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make *n* additional independent assumptions. A solution based on statics will not be possible by making fewer than *n*

Since beams or girders are usually major load– carrying members in large structures, it is important to draw influence lines for reaction, shear, and moment at specified points. ¬ Once an influence line has been drawn, it is possible to locate the live loads on the beam so that the maximum value of the reaction, shear, or moment is produced. ¬ This is very important in the design procedure.

Concentrated Force - Since we use a unit force (a dimensionless load), the value of the function (reaction, shear, or moment) can be found by multiplying the ordinate of the influence line at the position x by the magnitude of the actual force P.

Example

Use the Cantilever Method to draw the axial force, shear force and bending moment diagrams of the three -storied frame structure loaded as shown below.



The dotted line is the column centerline (at all floors)

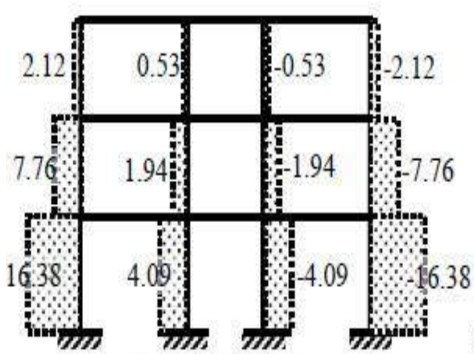
∴ Column axial forces are at the ratio of 20: 5: −5: −20.

∴ Axial force in (P) columns IM, JN, KO, LP are $[18 \times 5 \times 20/\{20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 2.12 , $[18 \times 5 \times 5/(20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 0.53, −0.53, −2.12 respectively.

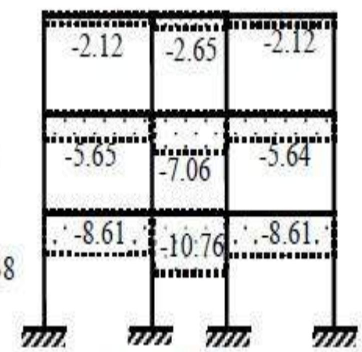
Similarly, $P_{EI} = 330 \times 20/(850) = 7.76$, $P_{FJ} = 1.94$, $P_{GK} = -1.94$, $P_{HL} = -7.76$; and

$P_{AE} = 696 \times 20/(850) = 16.38$, $P_{BF} = 4.09$, $P_{CG} = -4.09$, $P_{DH} = 16.38$

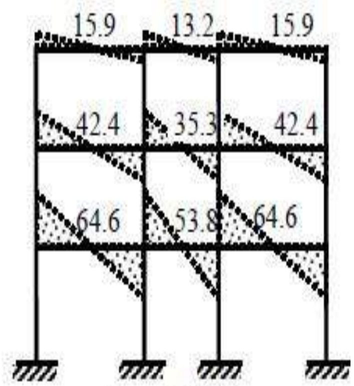
The rest of the calculations follow from the free-body diagrams



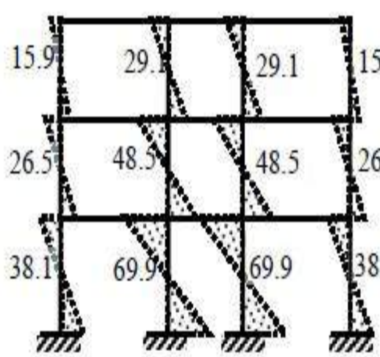
Column AFD



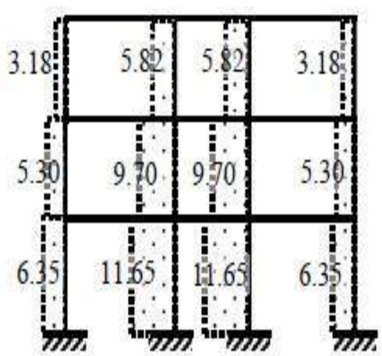
Beam SFD



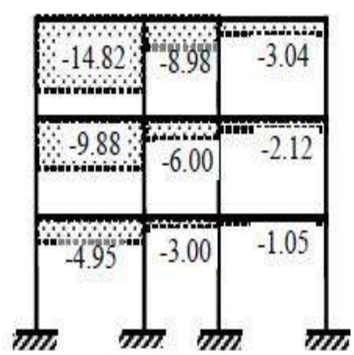
Beam BMD



Column BMD



Column SFD



Beam AFD

Introducing direction cosines $l = \cos\theta; m = \sin\theta$; the above equation is written as

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

Or, $$\{u'\} = [T]\{u\}$$

In the above equation *T*is the displacement transformation matrix which transforms the four global displacement components to two displacement component in local coordinate system

GENERAL PROCEDURE OF SLOPE-DEFLECTION METHOD

- Find the fixed end moments of each span (both ends left & right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
- Determine the end moments and then treat each span as simply supported beamsubjected to given load & end moments so we can work out the reactions & drawthe bending moment & shear force diagram.

Loads and displacements are vector quantities and hence a proper coordinate system is required to specify their correct sense of direction. Consider a planar truss, In this truss each node is identified by a number and each member is identified by a number enclosed in a circle. The displacements and loads acting on the truss are defined with respect to global co-ordinate system *xyz.*

ASSUMPTIONS FOR THE APPROXIMATE SOLUTION

In order to analyze a structure using the equations of statics only, the number of independent force components must be equal to the number of independent equations of statics.

If there are *n* more independent force components in the structure than there are independent equations of statics, the structure is statically indeterminate to the *nth* degree. Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make *n* additional independent assumptions. A solution based on statics will not be possible by making fewer than *n* assumptions, while more than *n* assumptions will not in general be consistent.

Thus, the first step in the approximate analysis of structures is to find its degree of statically indeterminacy (dosi) and then to make appropriate number of assumptions.

For example, the dosi of portal frames shown in (i), (ii), (iii) and (iv) are 1, 3, 2 and 1 respectively. Based on the type of frame, the following assumptions can be made for portal structures with a vertical axis of symmetry that are loaded horizontally at thetop

The horizontal support reactions areequal

There is a point of inflection at the center of the unsupported height of each fixed basedcolumn

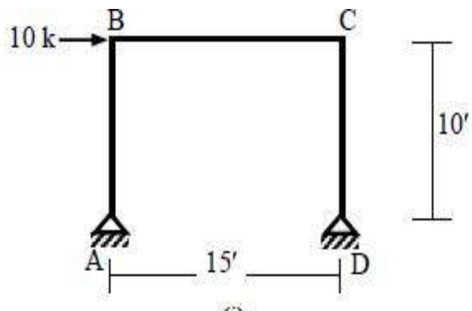
Assumption 1 is used if dosi is an odd number (i.e., = 1 or 3) and Assumption 2 is used if dosi 1.

Some additional assumptions can be made in order to solve the structure approximately for different loading and support conditions.

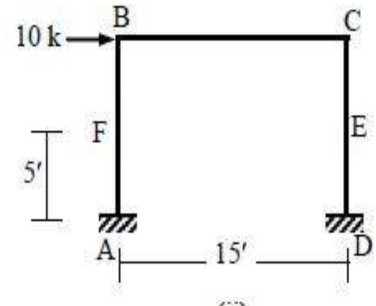
Horizontal body forces not applied at the top of a column can be divided into two forces (i.e., applied at the top and bottom of the column) based on simple supports

For hinged and fixed supports, the horizontal reactions for fixed supports can be assumed to be four times the horizontal reactions for hinged supports Example

Draw the axial force, shear force and bending moment diagrams of the frames loaded as shown below.



(i)



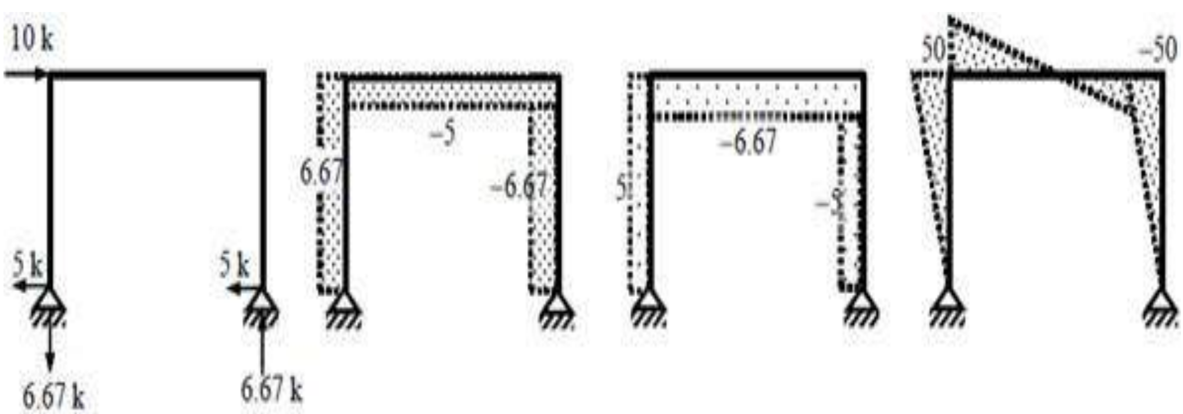
(ii)

Solution

(i) For this frame, dosi = $3 \times 3 + 4 - 3 \times 4 = 1$; i.e., Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5$ k

$\therefore \sum M_A = 0 \Rightarrow 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 6.67$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -6.67$ k



Reactions AFD (k) SFD (k) BMD

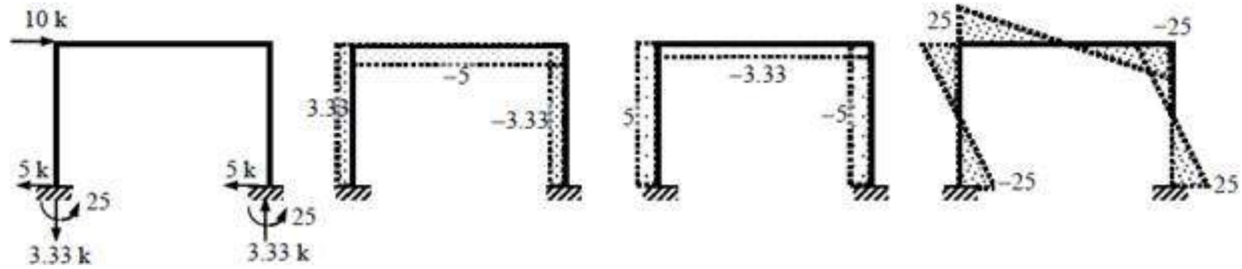
(ii) dosi = $3 \times 3 + 6 - 3 \times 4 = 3$

Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5$ k, Assumption 2 $\Rightarrow BM_E = BM_F = 0$

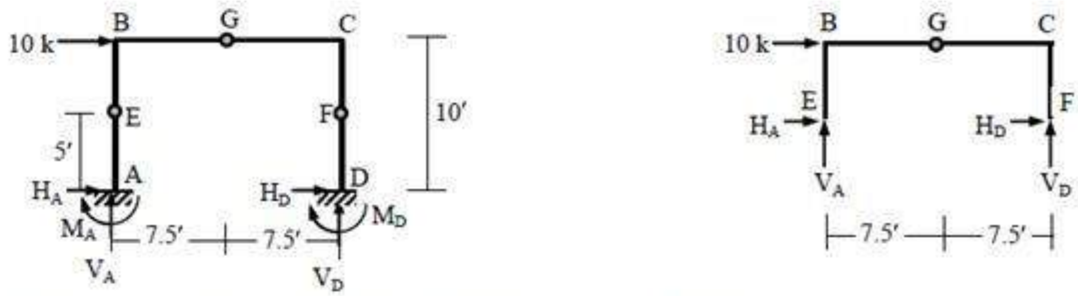
$\therefore BM_F = 0 \Rightarrow H_A \times 5 + M_A = 0 \Rightarrow M_A = -25$ k-ft; Similarly $BM_E = 0 \Rightarrow M_D = -25$

$\therefore \sum M_A = 0 \Rightarrow -25 - 25 + 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -3.33$ k



(iii)



dosi = $3 \times 4 + 6 - 3 \times 5 - 1 = 2$; $\therefore$ Assumption 1 and 2 $\Rightarrow BM_E = BM_F = 0$

$\therefore BM_E = 0$ (bottom) $\Rightarrow -H_A \times 5 + M_A = 0 \Rightarrow M_A = 5H_A$; Similarly $BM_F = 0 \Rightarrow M_D = 5H_D$

Also $BM_E = 0$ (free body of EBCF) $\Rightarrow 10 \times 5 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

$\therefore \Sigma F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -V_D = -3.33$ k

$BM_G = 0$ (between E and G) $\Rightarrow V_A \times 7.5 - H_A \times 5 = 0 \Rightarrow H_A = -5$ k $\Rightarrow M_A = 5H_A = -25$

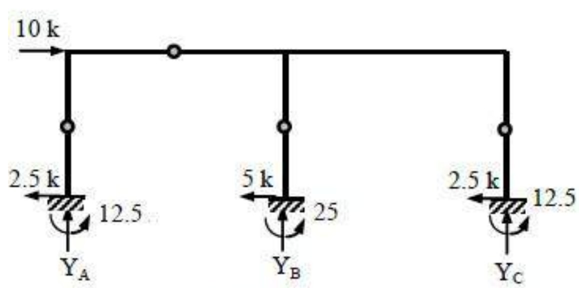
$\Sigma F_x = 0$ (entire structure) $\Rightarrow H_A + H_D + 10 = 0 \Rightarrow -5 + H_D + 10 = 0 \Rightarrow H_D = -5$ k $\Rightarrow M_D = 5H_D = -25$

(iv) dosi = $3 \times 5 + 9 - 3 \times 6 = 6 \Rightarrow$ 6 Assumptions needed to solve the structure

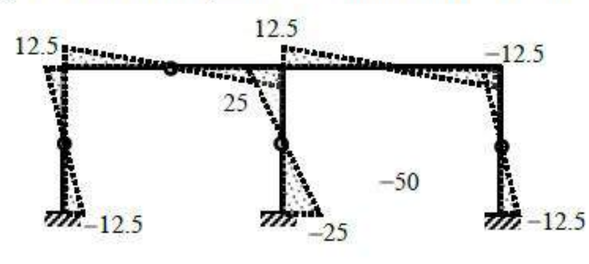
Assumption 1 and 2 $\Rightarrow H_A: H_B: H_C = 1: 2: 1 \Rightarrow H_A = 10/4 = 2.5$ k, $H_B = 5$ k, $H_C = 2.5$ k

$\therefore M_A = M_C = 2.5 \times 5 = 12.5$ k-ft, $M_B = 5 \times 5 = 25$

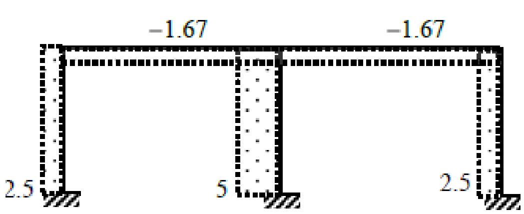
The other 4 assumptions are the assumed internal hinge locations at midpoints of columns and one beam



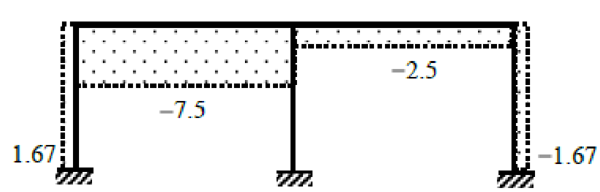
Reactions



BMD



SFD (k)



AFD (k)

Analysis of Multi-storied Structures by Portal Method

Approximate methods of analyzing multi-storied structures are important because such structures are statically highly indeterminate. The number of assumptions that must be made to permit an analysis by statics alone is equal to the degree of statical indeterminacy of the structure.

Assumptions

The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures. The *Portal Method* thus formulated is based on three assumptions

The shear force in an interior column is twice the shear force in an exterior column.

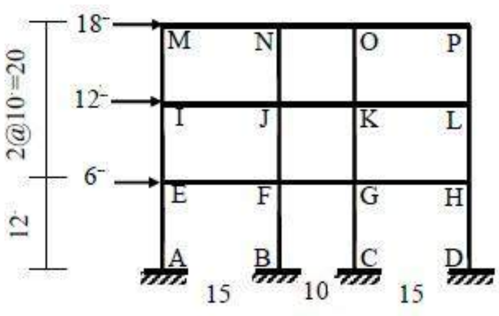
There is a point of inflection at the center of eachcolumn.

There is a point of inflection at the center of eachbeam.

Assumption 1 is based on assuming the interior columns to be formed by columns of two adjacent bays or portals. Assumption 2 and 3 are based on observing the deflected shape of thestructure.

Example

Use the Portal Method to draw the axial force, shear force and bending moment diagrams of the three-storied frame structure loaded as shown below.



Column shear forces are at the ratio of 1:2:2:1.

∴ Shear force in (V) columns IM, JN, KO, LP are

[18 × 1/(1 + 2 + 2 + 1) =] 3 , [18 × 2/(1 + 2 + 2 + 1) =] 6, 6, 3 respectively. Similarly,

$V_{EI} = 30\times1/(6) = 5$, $V_{FJ} = 10$, $V_{GK} = 10$, $V_{HL} = 5$; and

$V_{AE} = 36\times1/(6) = 6$, $V_{BF} = 12$, $V_{CG} = 12$, $V_{DH} = 6$

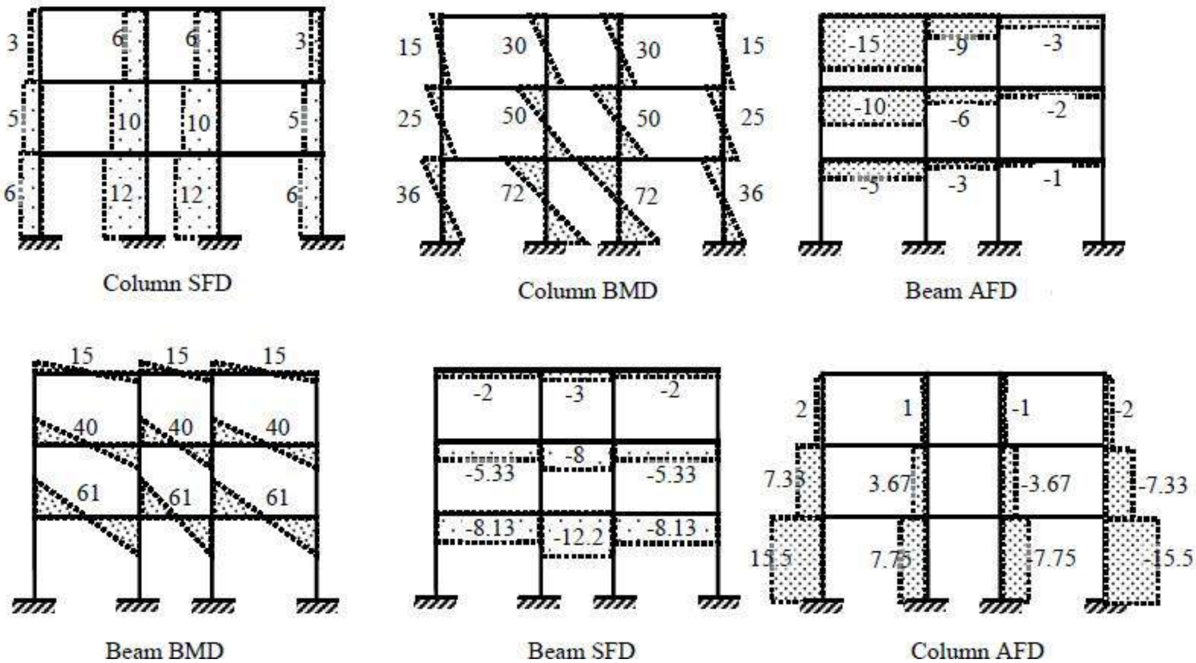
Bending moments are

$M_{IM} = 3\times10/2 = 15$, $M_{JN} = 30$, $M_{KO} = 30$, $M_{LP} = 15$

$M_{EI} = 5\times10/2 = 25$, $M_{FJ} = 50$, $M_{GK} = 50$, $M_{HL} = 25$

$M_{AE} = 6\times10/2 = 30$, $M_{FJ} = 60$, $M_{GK} = 60$, $M_{HL} = 30$

The rest of the calculations follow from the free-body diagrams



Analysis of Multi-storied Structures by Cantilever Method

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members. The *Cantilever Method* attempts to rectify this limitation by considering the cross- sectional areas of columns in distributing the axial forces in various columns of a story.

Assumptions

The Cantilever Method is based on three assumptions

The axial force in each column of a storey is proportional to its horizontal distance from the centroidal axis of all the columns of thestorey.

There is a point of inflection at the center of eachcolumn.

There is a point of inflection at the center of eachbeam.