CE 15008
Fluid Mechanics

LECTURE NOTES

Module-II

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COURSE CONTENT

CE 15008:
FLUID MECHANICS (3-1-0)
CR-04

Module – II **(8 Hours)**

Fluid kinematics: Steady and unsteady, uniform and non-uniform, laminar and turbulent flows and enclosed flows; Definition of one-, two- and three-dimensional flows, Stream-lines, streak-lines, and path-lines; Stream-tubes; elementary explanation of stream-function and velocity potential; Basic idea of flow nets.

LECTURE NOTES
MODULE 2

FLUID KINEMATICS

Steady flow

A steady flow is one in which all conditions at any point in a stream remain constant with respect to time.

Or

A steady flow is the one in which the quantity of liquid flowing per second through any section, is constant.

This is the definition for the ideal case. True steady flow is present only in Laminar flow. In turbulent flow, there are continual fluctuations in velocity. Pressure also fluctuate at every point. But if this rate of change of pressure and velocity are equal on both sides of a constant average value, the flow is steady flow. The exact term use for this is mean steady flow.
Steady flow may be uniform or non-uniform.

Uniform flow

A truly uniform flow is one in which the velocity is same at a given instant at every point in the fluid.

This definition holds for the ideal case. Whereas in real fluids velocity varies across the section.

But when the size and shape of cross section are constant along the length of channels under consideration, the flow is said to be uniform.

Non-uniform flow

A non-uniform flow is one in which velocity is not constant at a given instant.

Unsteady Flow

A flow in which quantity of liquid flowing per second is not constant, is called unsteady flow.

Unsteady flow is a transient phenomenon. It may be in time become steady or zero flow. For example when a valve is closed at the discharge end of the pipeline. Thus, causing the velocity in the pipeline to decrease to zero. In the meantime, there will be fluctuations in both velocity and pressure within the pipe.

Unsteady flow may also include periodic motion such as that of waves of beaches. The difference between these cases and mean steady flow is that there is so much deviation from the mean. And the time scale is also much longer.

One, Two and Three Dimensional Flows

Term one, two or three dimensional flow refers to the number of space coordinated required to describe a flow. It appears that any physical flow is generally three-dimensional. But these are difficult to calculate and call for as much simplification as possible. This is achieved by ignoring changes to flow in any of the directions, thus reducing the complexity. It may be possible to reduce a three-dimensional problem to a two-dimensional one, even an one-dimensional one at times.

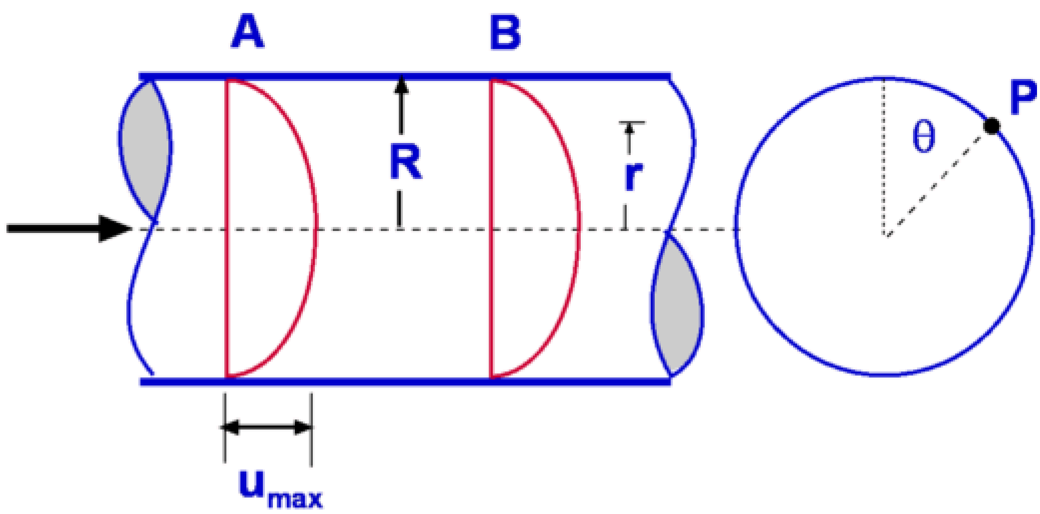


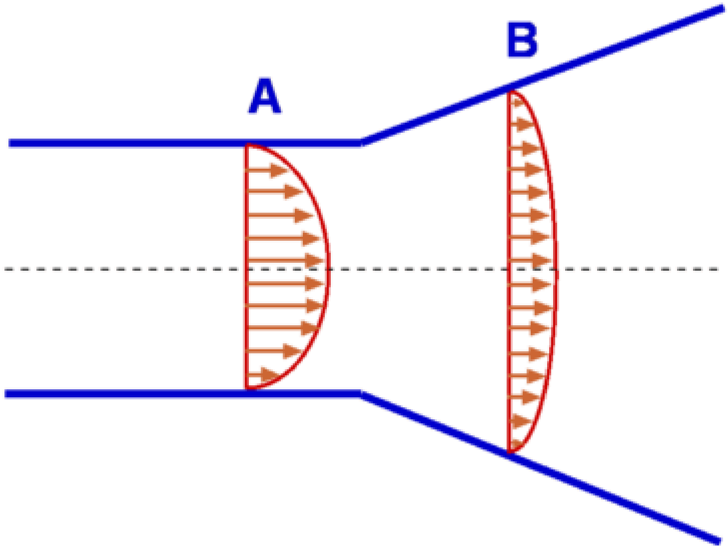
Figure-1: Example of one-dimensional flow

Consider flow through a circular pipe. This flow is complex at the position where the flow enters the pipe. But as we proceed downstream the flow simplifies considerably and attains the state of a fully developed flow. A characteristic of this flow is that the velocity becomes invariant in the flow direction as shown in Fig-1. Velocity for this flow is given by

$$u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right] \quad (3.6)$$

It is readily seen that velocity at any location depends just on the radial distance r from the centreline and is independent of distance, x or of the angular position θ. This represents a typical one-dimensional flow.

Now consider a flow through a diverging duct as shown in Fig. 2. Velocity at any location depends not only upon the radial distance r but also on the x-distance. This is therefore a two-dimensional flow.



Example -2: A two-dimensional flow

Concept of a uniform flow is very handy in analyzing fluid flows. A uniform flow is one where the velocity and other properties are constant independent of directions. We usually assume a uniform flow at the entrance to a pipe, far away from a aerofoil or a motor car as shown in Fig.3.

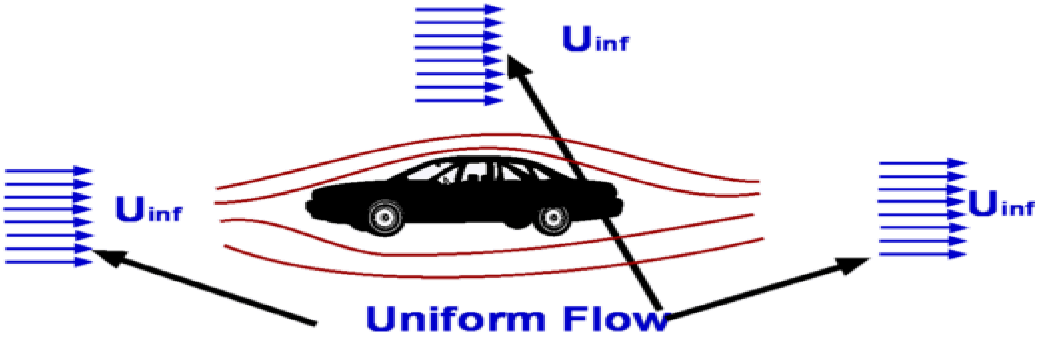


Fig:4 Uniform flow

1. Real fluids

The flow of real fluids exhibits viscous effect that is they tend to "stick" to solid surfaces and have stresses within their body.

You might remember from earlier in the course Newton's law of viscosity:

$$\tau \propto \frac{du}{dy}$$

This tells us that the shear stress, τin a fluid is proportional to the velocity gradient - the rate of change of velocity across the fluid path. For a "Newtonian" fluid we can write:

$$\tau = \mu \frac{du}{dy}$$

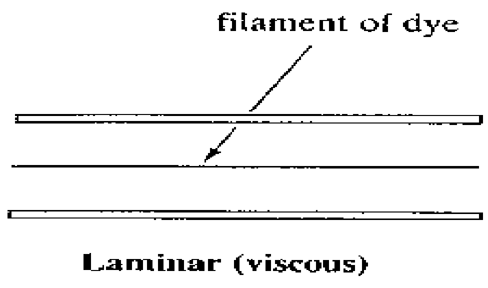
Where the constant of proportionality, μ is known as the coefficient of viscosity (or simply viscosity). We saw that for some fluids - sometimes known as exotic fluids - the value of μ changes with stress or velocity gradient. We shall only deal with Newtonian fluids.

In his lecture we shall look at how the forces due to momentum changes on the fluid and viscous forces compare and what changes take place.

2.Laminar and turbulent flow

If we were to take a pipe of free flowing water and inject a dye into the middle of the stream, what would we expect to happen?

This



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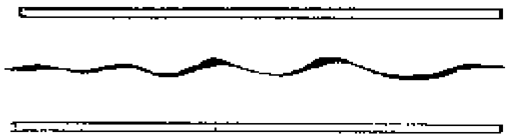
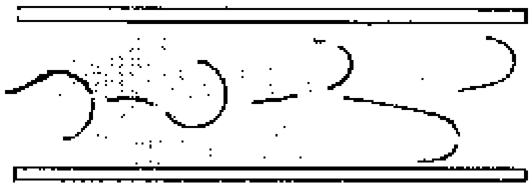


Transitional

Or this



Turbulent

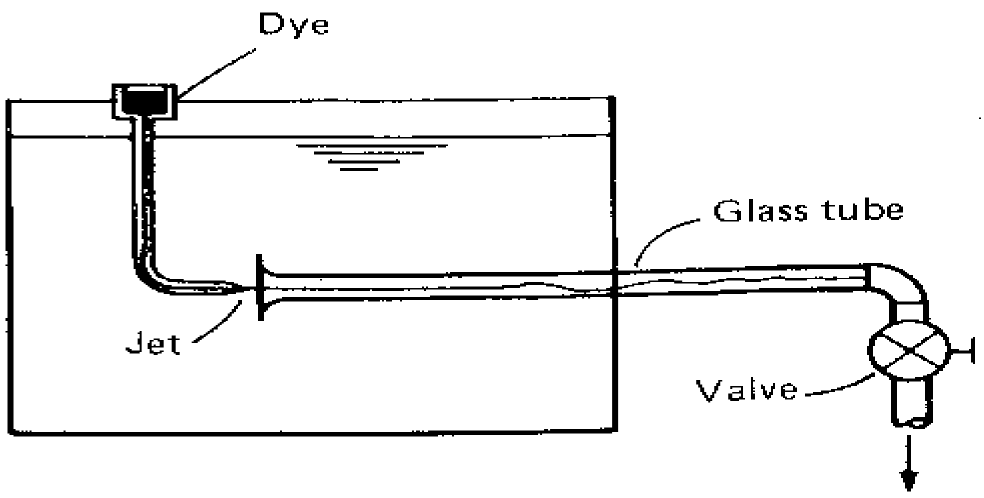
Actually both would happen - but for different flow rates. The top occurs when the fluid is flowing fast and the lower when it is flowing slowly.

The top situation is known as turbulent flow and the lower as laminar flow.

In laminar flow the motion of the particles of fluid is very orderly with all particles moving in straight lines parallel to the pipe walls.

But what is fast or slow? And at what speed does the flow pattern change? And why might we want to know this?

The phenomenon was first investigated in the 1880s by Osbourne Reynolds in an experiment which has become a classic in fluid mechanics.



He used a tank arranged as above with a pipe taking water from the centre into which he injected a dye through a needle. After many experiments he saw that this expression

$$\frac{\rho u d}{\mu}$$

where ρ = density, u = mean velocity, d = diameter and v = viscosity

would help predict the change in flow type. If the value is less than about 2000 then flow is laminar, if greater than 4000 then turbulent and in between these then in the transition zone.

This value is known as the Reynolds number, Re:

$$\mathrm{Re} = \frac{\rho u d}{\mu}$$

Laminar flow: Re < 2000

Transitional flow: 2000 < Re < 4000

Turbulent flow: Re > 4000

What are the units of this Reynolds number? We can fill in the equation with SI units:

$$\rho = kg/m^3, \quad u = m/s, \quad d = m$$
$$\mu = Ns/m^2 = kg/ms$$

$$\mathrm{Re} = \frac{\rho u d}{\mu} = \frac{kg}{m^3}\frac{m}{s}\frac{m}{1}\frac{m}{kg} = 1$$

i.e. it has no units. A quantity that has no units is known as a non-dimensional (or dimensionless) quantity. Thus the Reynolds number, Re, is a non-dimensional number.

We can go through an example to discover at what velocity the flow in a pipe stops being laminar.

If the pipe and the fluid have the following properties:

water density ρ = 1000 kg/m^3

pipe diameter d = 0.5m

(dynamic) viscosity, ν = 0.55x10^{-3} Ns/m^2

We want to know the maximum velocity when the Re is 2000.

$$\mathrm{Re} = \frac{\rho u d}{\mu} = 2000$$

$$u = \frac{2000\mu}{\rho d} = \frac{2000 \times 0.55 \times 10^{-3}}{1000 \times 0.5}$$

$$u = 0.0022\, m/s$$

If this were a pipe in a house central heating system, where the pipe diameter is typically 0.015m, the limiting velocity for laminar flow would be, 0.0733 m/s.

Both of these are very slow. In practice it very rarely occurs in a piped water system - the velocities of flow are much greater. Laminar flow does occur in situations with fluids of greater viscosity - e.g. in bearing with oil as the lubricant.

At small values of Re above 2000 the flow exhibits small instabilities. At values of about 4000 we can say that the flow is truly turbulent. Over the past 100 years since this experiment, numerous more experiments have shown this phenomenon of limits of Re for many different Newtonian fluids - including gasses.

What does this abstract number mean?

We can say that the number has a physical meaning, by doing so it helps to understand some of the reasons for the changes from laminar to turbulent flow.

$$\mathrm{Re} = \frac{\rho u d}{\mu}$$

$$= \frac{\text{inertial f orces}}{\text{viscous fo rces}}$$

It can be interpreted that when the inertial forces dominate over the viscous forces (when the fluid is flowing faster and Re is larger) then the flow is turbulent. When the viscous forces are dominant (slow flow, low Re) they are sufficient enough to keep all the fluid particles in line, then the flow is laminar.

Laminar flow

- Re < 2000
- 'low' velocity
- Dye does not mix with water
- Fluid particles move in straight lines
- Simple mathematical analysis possible
- Rare in practice in water systems.

Transitional flow

- 2000 > Re < 4000
- 'medium' velocity
- Dye stream wavers in water - mixes slightly.

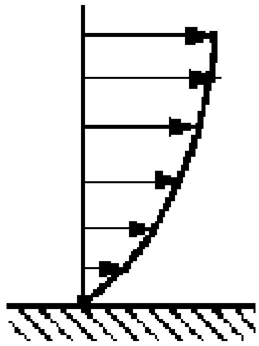
Turbulent flow

- Re > 4000
- 'high' velocity
- Dye mixes rapidly and completely
- Particle paths completely irregular
- Average motion is in the direction of the flow
- Cannot be seen by the naked eye
- Changes/fluctuations are very difficult to detect. Must use laser.
- Mathematical analysis very difficult - so experimental measures are used

3. Pressure loss due to friction in a pipeline

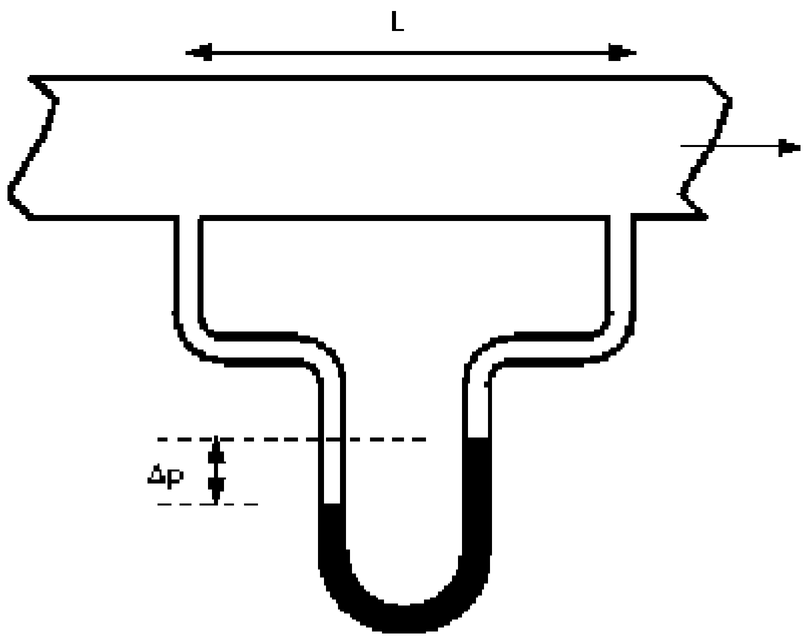
Up to this point on the course we have considered ideal fluids where there have been no losses due to friction or any other factors. In reality, because fluids are viscous, energy is lost by flowing fluids due to friction which must be taken into account. The effect of the friction shows itself as a pressure (or head) loss.

In a pipe with a real fluid flowing, at the wall there is a shearing stress retarding the flow, as shown below.



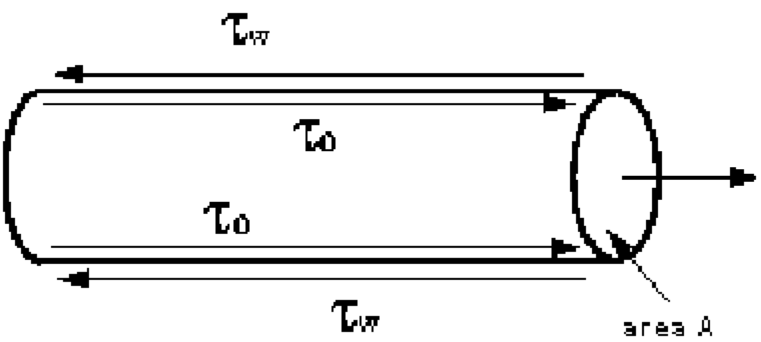
If a manometer is attached as the pressure (head) difference due to the energy lost by the fluid overcoming the shear stress can be easily seen.

The pressure at 1 (upstream) is higher than the pressure at 2.



We can do some analysis to express this loss in pressure in terms of the forces acting on the fluid.

Consider a cylindrical element of incompressible fluid flowing in the pipe, as shown



The pressure at the upstream end is *p*, and at the downstream end the pressure has fallen by Δp to (p-Δp).

The driving force due to pressure (F = Pressure x Area) can then be written

driving force = Pressure force at 1 - pressure force at 2

$$pA - (p - \Delta p)A = \Delta p\, A = \Delta p \frac{\pi d^2}{4}$$

The retarding force is that due to the shear stress by the walls

$$\begin{aligned} &= \text{shear stress} \times \text{area over which it acts} \\ &= \tau_w \times \text{area of pipe wall} \\ &= \tau_w \pi d L \end{aligned}$$

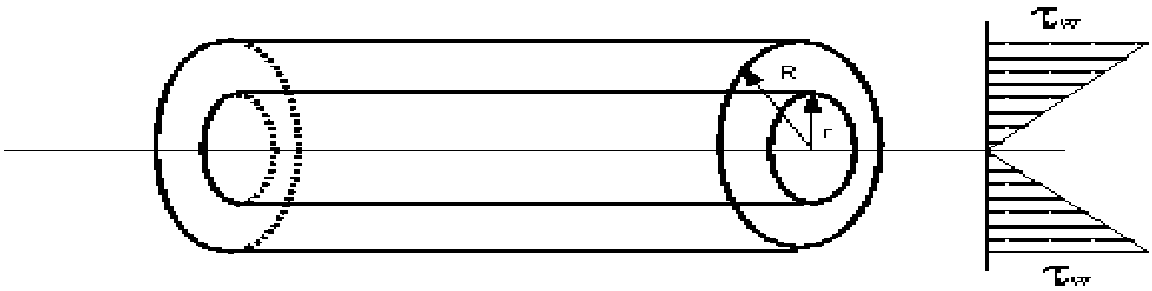
As the flow is in equilibrium,

driving force = retarding force

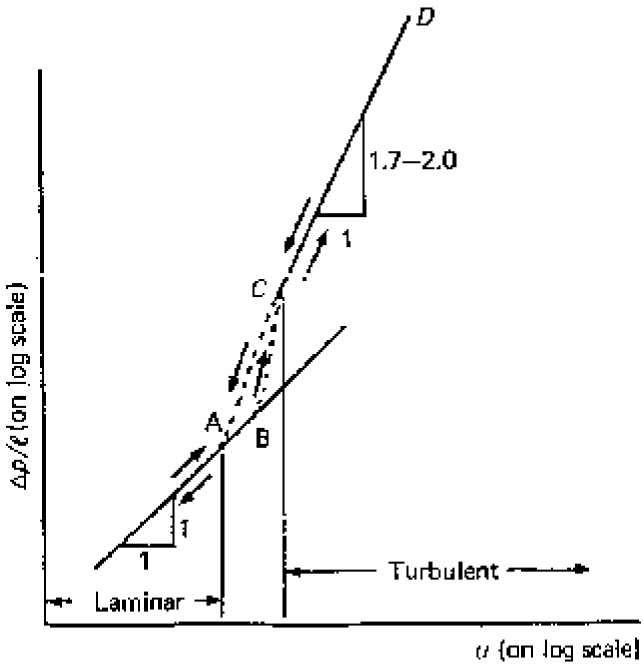
$$\Delta p \frac{\pi d^2}{4} = \tau_w \pi d L$$

$$\Delta p = \frac{\tau_w 4 L}{d}$$

Giving an expression for pressure loss in a pipe in terms of the pipe diameter and the shear stress at the wall on the pipe.



The shear stress will vary with velocity of flow and hence with Re. Many experiments have been done with various fluids measuring the pressure loss at various Reynolds numbers. These results plotted to show a graph of the relationship between pressure loss and Re look similar to the figure below:



This graph shows that the relationship between pressure loss and Re can be expressed as

laminar $\Delta p \propto u$

turbulent $\Delta p \propto u^{1.7 \text{ (or } 2.0)}$

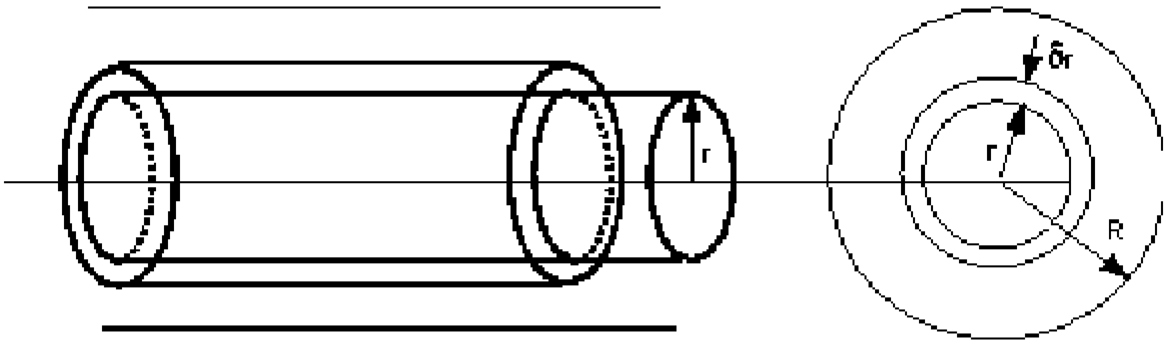
As these are empirical relationships, they help in determining the pressure loss but not in finding the magnitude of the shear stress at the wall on a particular fluid. We could then use it to give a general equation to predict the pressure loss.

4. Pressure loss during laminar flow in a pipe

In general the shear stress τ_w. is almost impossible to measure. But for laminar flow it is possible to calculate a theoretical value for a given velocity, fluid and pipe dimension.

In laminar flow the paths of individual particles of fluid do not cross, so the flow may be considered as a series of concentric cylinders sliding over each other - rather like the cylinders of a collapsible pocket telescope.

As before, consider a cylinder of fluid, length L, radius **r**, flowing steadily in the center of a pipe.



We are in equilibrium, so the shearing forces on the cylinder equal the pressure forces.

$$\tau\ 2\pi r\ L = \Delta p\ A = \Delta p \pi r^2$$
$$\tau = \frac{\Delta p}{L}\frac{r}{2}$$

By Newtons law of viscosity we have $\tau = \mu \frac{du}{dy}$, where y is the distance from the wall. As we are measuring from the pipe centre then we change the sign and replace y with r distance from the centre, giving

$$\tau = -\mu \frac{du}{dr}$$

Which can be combined with the equation above to give

$$\frac{\Delta p}{L}\frac{r}{2} = -\mu \frac{du}{dr}$$
$$\frac{du}{dr} = -\frac{\Delta p}{L}\frac{r}{2\mu}$$

In an integral form this gives an expression for velocity,

$$u = -\frac{\Delta p}{L}\frac{1}{2\mu}\int r\,dr$$

Integrating gives the value of velocity at a point distance r from the centre

$$u_r = -\frac{\Delta p}{L}\frac{r^2}{4\mu} + C$$

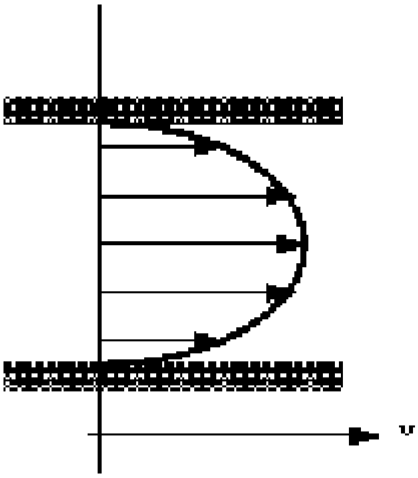
At $r = 0$, (the centre of the pipe), $u = u_{max}$, at $r = R$ (the pipe wall) $u = 0$, giving

$$C = \frac{\Delta p}{L}\frac{R^2}{4\mu}$$

so, an expression for velocity at a point r from the pipe centre when the flow is laminar is

$$u_r = \frac{\Delta p}{L}\frac{1}{4\mu}\left(R^2 - r^2\right)$$

Note how this is a parabolic profile (of the form $y = ax^2 + b$) so the velocity profile in the pipe looks similar to the figure below



What is the discharge in the pipe?

$$Q = u_m A$$
$$u_m = \int_0^R u_r \, dr$$
$$= \frac{\Delta p}{L}\frac{1}{4\mu}\int_0^R \left(R^2 - r^2\right) dr$$
$$= \frac{\Delta p}{L}\frac{R^2}{8\mu} = \frac{\Delta p\, d^2}{32\mu L}$$

So the discharge can be written

$$Q = \frac{\Delta p\, d^2}{32\mu L}\frac{\pi d^2}{4}$$
$$= \frac{\Delta p}{L}\frac{\pi d^2}{128\mu}$$

This is the Hagen-Poiseuille equation for lami $\frac{\partial p}{\partial x} = \frac{\Delta p}{L}$ n a pipe. It expresses the discharge Q in terms of the pressure gradient ( ), diameter of the pipe and the viscosity of the fluid.

We are interested in the pressure loss (head loss) and want to relate this to the velocity of the flow. Writing pressure loss in terms of head loss h_f, i.e. $p = \rho g h_f$

$$u = \frac{\rho g h_f d^2}{32 \mu L}$$

$$h_f = \frac{32 \mu L u}{\rho g d^2}$$

This shows that pressure loss is directly proportional to the velocity when flow is laminar.

It has been validated many time by experiment.

It justifies two assumptions:

1. fluid does not slip past a solid boundary
2. Newton's hypothesis.

Streamline :

This is an imaginary curve in a flow field for a fixed instant of time, tangent to which gives the instantaneous velocity at that point . Two stream lines can never intersect each other, as the instantaneous velocity vector at any given point is unique.

The differential equation of streamline may be written as

$$\frac{du}{u} = \frac{dv}{v} = \frac{dw}{w}$$

where u,v, and w are the velocity components in x, y and z directions respectively as sketched.

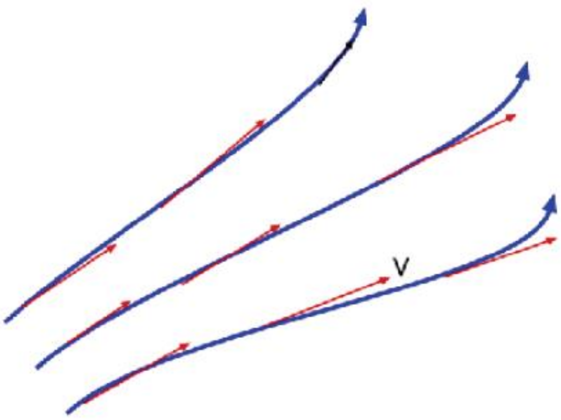


Fig. Streamlines

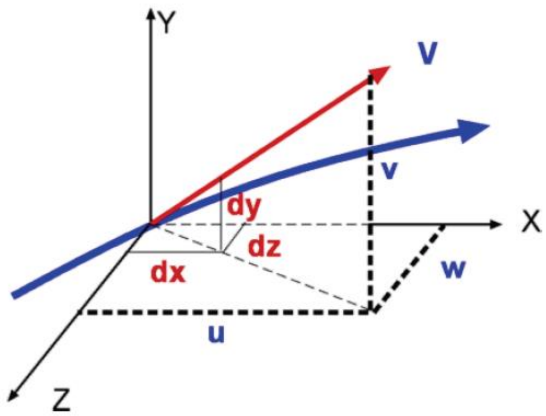


Fig. Streamline function

Stream tube :

If streamlines are drawn through a closed curve, they form a boundary surface across which fluid cannot penetrate. Such a surface bounded by streamlines is a sort of tube, and is known as a stream tube.

From the definition of streamline, it is evident that no fluid can cross the bounding surface of the stream tube. This implies that the quantity(mass) of fluid entering the stream tube at one end must be the same as the quantity leaving it at the other. The stream tube is generally assumed to be a small cross-sectional area so that the velocity over it could be considered uniform.

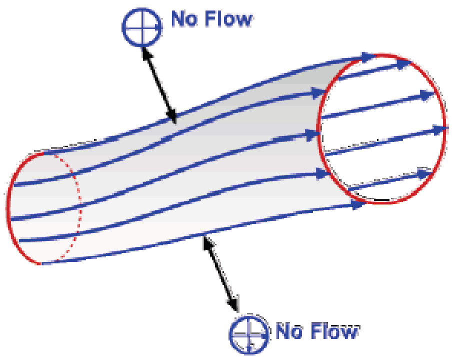


Fig. Streamtube

Pathline :

A pathline is the locus of a fluid particle as it moves along. In others word, a pathline is a curve traced by a single fluid particle during its motion.

Two path lines can intersect each other as or a single path line can form a loop as different particles or even same particle can arrive at the same point at different instants of time.

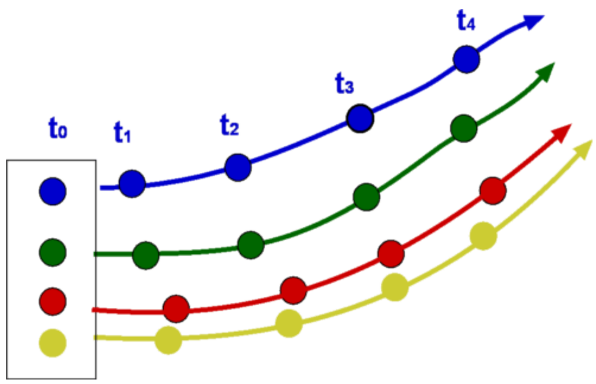


Fig. Pathline

Streak line :

Streakline concentrates on fluid particles that have gone through a fixed station or point. At some instant of time the position of all these particles are marked and a line is drawn through them. Such a line is called a streakline. Thus, a streakline connects all particles passing through a given point.

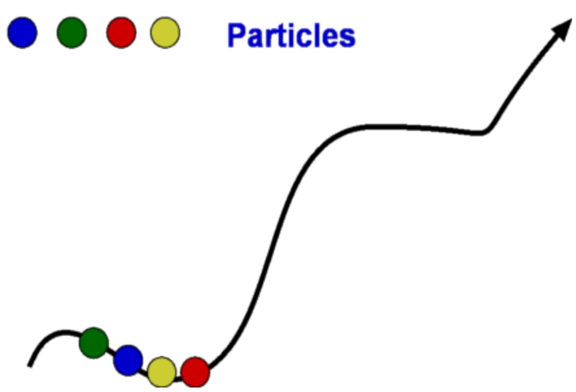


Fig. Streaklines

In a steady flow the streamline, pathline and streakline all coincide. In an unsteady flow they can be different. Streamlines are easily generated mathematically while pathline and streak lines are obtained through experiments.

Stream function :

The idea of introducing stream function works only if the continuity equation is reduced to two terms. There are 4-terms in the continuity equation that one can get by expanding the vector equation i.e.,

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

For a steady, incompressible, plane, two-dimensional flow, this equation reduces to,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Here, the striking idea of stream function works that will eliminate two velocity components u and v into a single variable. So, the *stream function* $\psi(x, y)$ relates to the velocity components in such a way that continuity equation is satisfied.

$$u = \frac{\eth \psi}{\eth y};\ v = -\frac{\eth \psi}{\eth x}$$

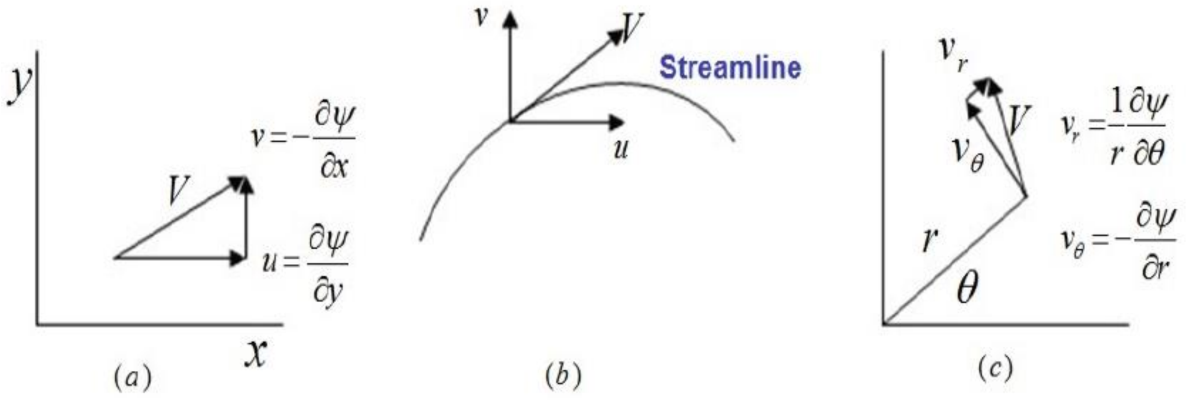


Fig. Velocity components along a streamline

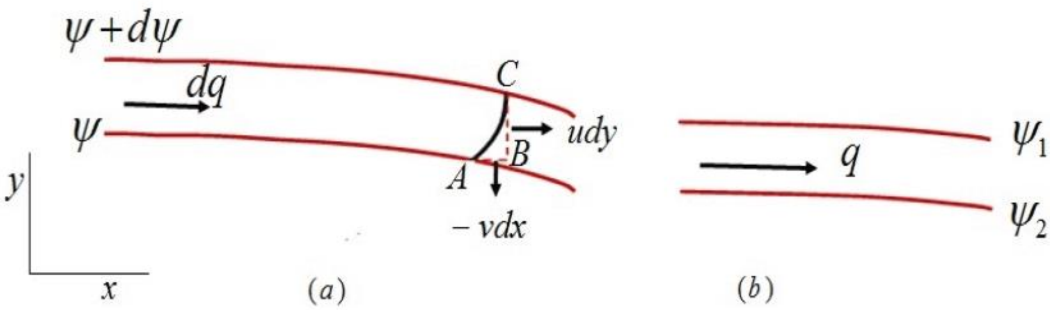


Fig. Flow between two streamlines

Velocity potential :

An irrotational flow is defined as the flow where the vorticity is zero at every point. It gives rise to a scalar function Φ which is similar and complementary to the stream function ψ. Let us consider the equations of irrortional flow and scalar function Φ. In an irrotational flow, there is no vorticity ξ.

The velocity potential is represented by Φ and is defined by the following expression :

$$-\Phi = \int V_s \, ds$$

in which V_s is the velocity along a small length element ds. So we get

$$d\Phi = -V_s \, ds$$

$$\text{or } V_s = -(d\Phi / ds)$$

The velocity potential is a scalar quantity dependent upon space and time. Its negative derivative with respect to any direction gives the velocity in that direction, that is

$$u = -\frac{\partial \Phi}{\partial x}, v = -\frac{\partial \Phi}{\partial y}, w = -\frac{\partial \Phi}{\partial z}$$

In polar co-ordinates (r, θ, z), the velocity components are

$$v_r = -\frac{\partial \Phi}{\partial r}, v_\theta = \frac{\partial \Phi}{r\,\partial r}, v_z = -\frac{\partial \Phi}{\partial z}$$

The velocity potential Φ thus provides an alternative means of expressing velocity components. The minus sign in equation appears because of the convention that the velocity potential decreases in the direction of flow just as the electrical potential decreases in the direction in which the current flows. The velocity potential is not a physical quantity which could be directly measured and, therefore, its zero position may be arbitrarily chosen.

Flownet :

The flownet is a graphical representation of two-dimensional irrotational flow and consists of a family of streamlines intersecting orthogonally a family of equipotential lines (they intersect at right angles) and in the process forming small curvilinear squares.

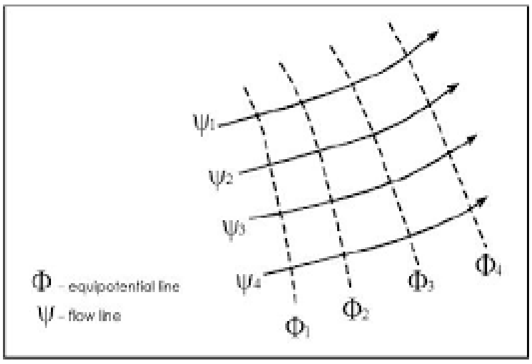


Fig. Flownet

Uses of flownet :

- For given boundaries of flow, the velocity and pressure distribution can be determined, if the velocity distribution and pressure at any reference section are known
- Loss of flow due to seepage in earth dams and unlined canals can be evaluated
- Uplift pressures on the undesirable (bottom) of the dam can be worked out

Relation between stream function & velocity potential

$\emptyset$ exists only in irrotational flow where as φexists in both rotational as well as irrotational flow

$$u= -\frac{\partial \emptyset}{\partial x} = \frac{\partial \varphi}{\partial y} \quad \& \quad v=\frac{\partial \varphi}{\partial x} = -\frac{\partial \emptyset}{\partial y}$$

therefore, $\frac{\partial \varphi}{\partial x} = \frac{\partial \emptyset}{\partial y}$ & $\frac{\partial \emptyset}{\partial x} = -\frac{\partial \varphi}{\partial y}$

References

Text Books:
1. Fluid mechanics by A.K. Jain, Khanna Publishers.

Reference Books:
1. Hydraulics and Fluid Mechanics including Hydraulic Machines by P.N.Modi and S.M. Seth, Standard Book House.
2. Engineering Fluid Mechanics by K.L. Kumar, S. Chand & Co.
3. Fluid Mechanics by V.L. Streeter, MGH